Quantum Turbulence in a Bose gas close to an Anomalous Non-thermal Fixed Point

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Synthetic Quantum Systems

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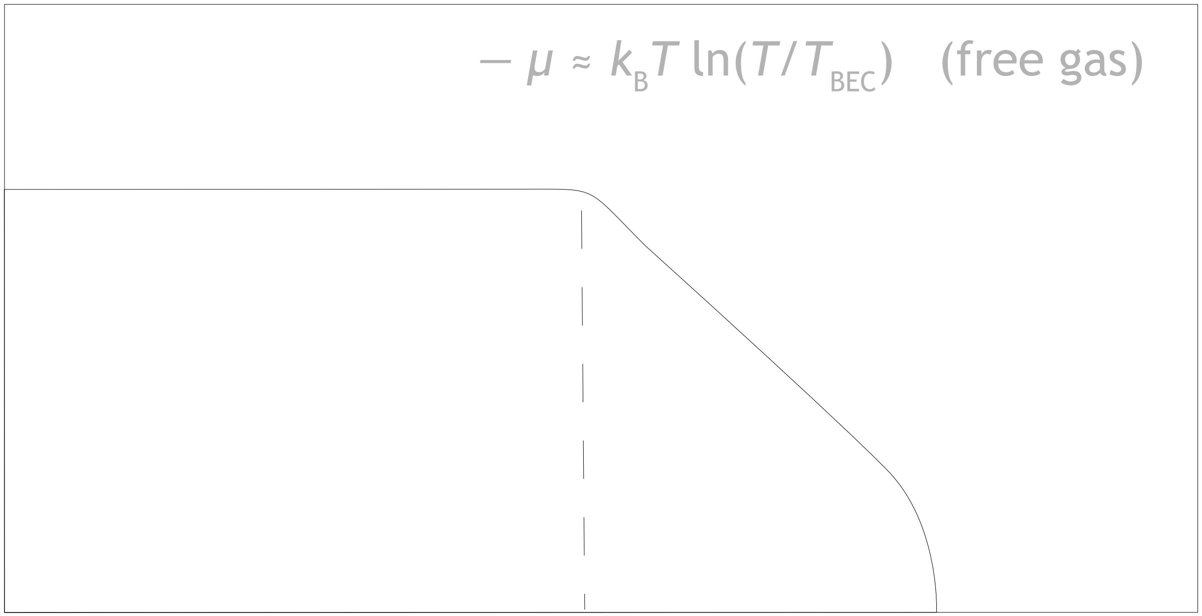
Universal dynamics

Let us consider an example in equilibrium:

Bose-Einstein distribution... — **it's universal !** —

log $n(\omega)$



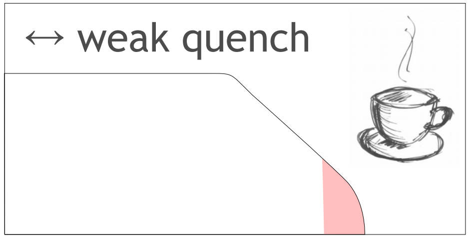
$-\mu$ T

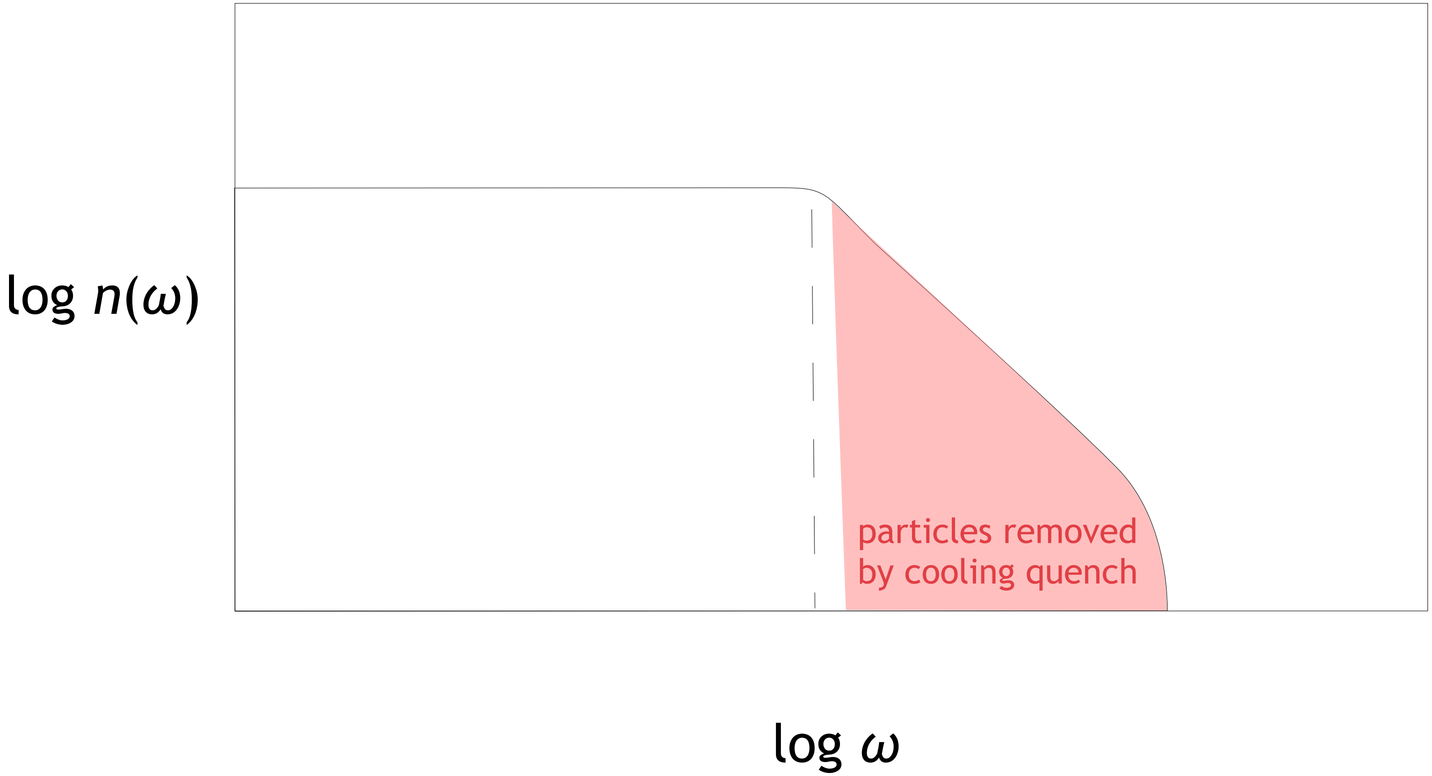
log ω

$$n(\omega) = \frac{1}{e^{(\omega-\mu)/T} - 1}$$

(**double-log!** — here: $T_{BEC} < T \ll 2T_{BEC}$)

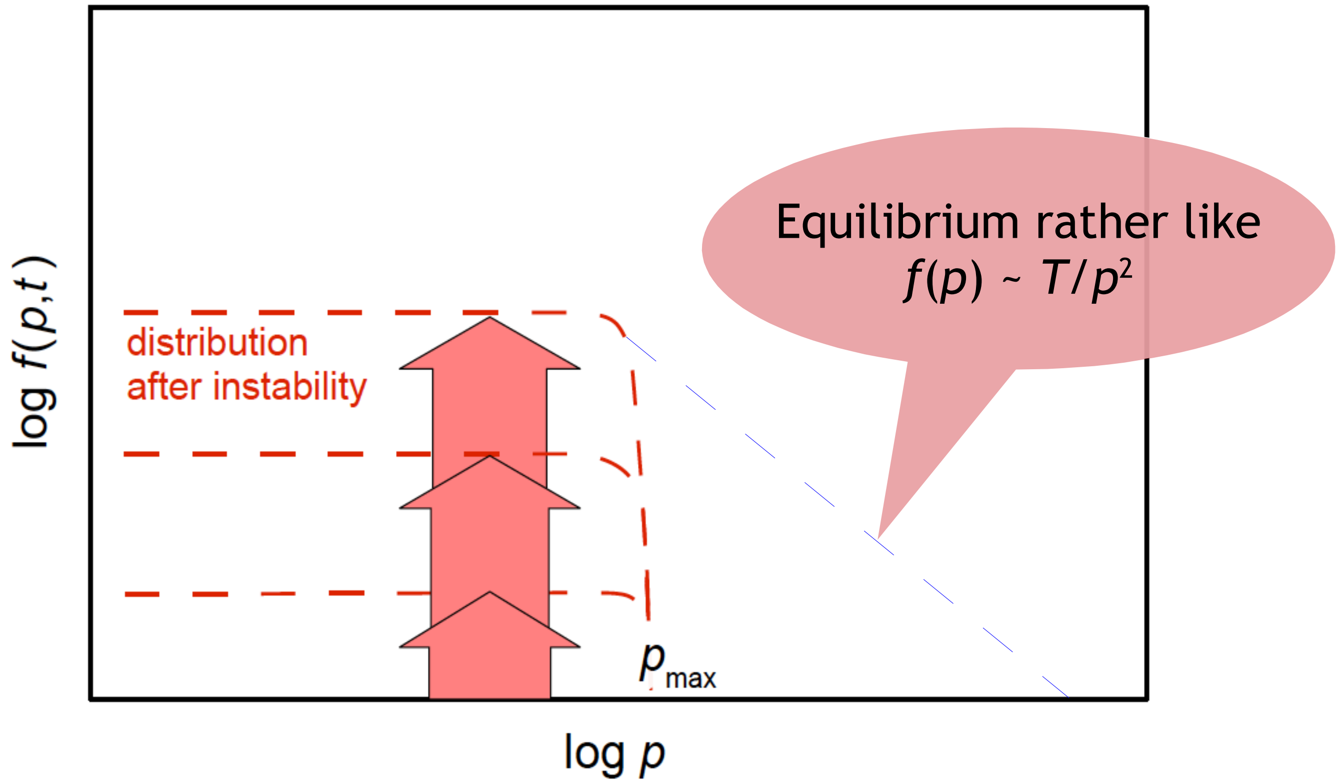
Apply **strong** cooling quench...





Alternatively: fast growth …

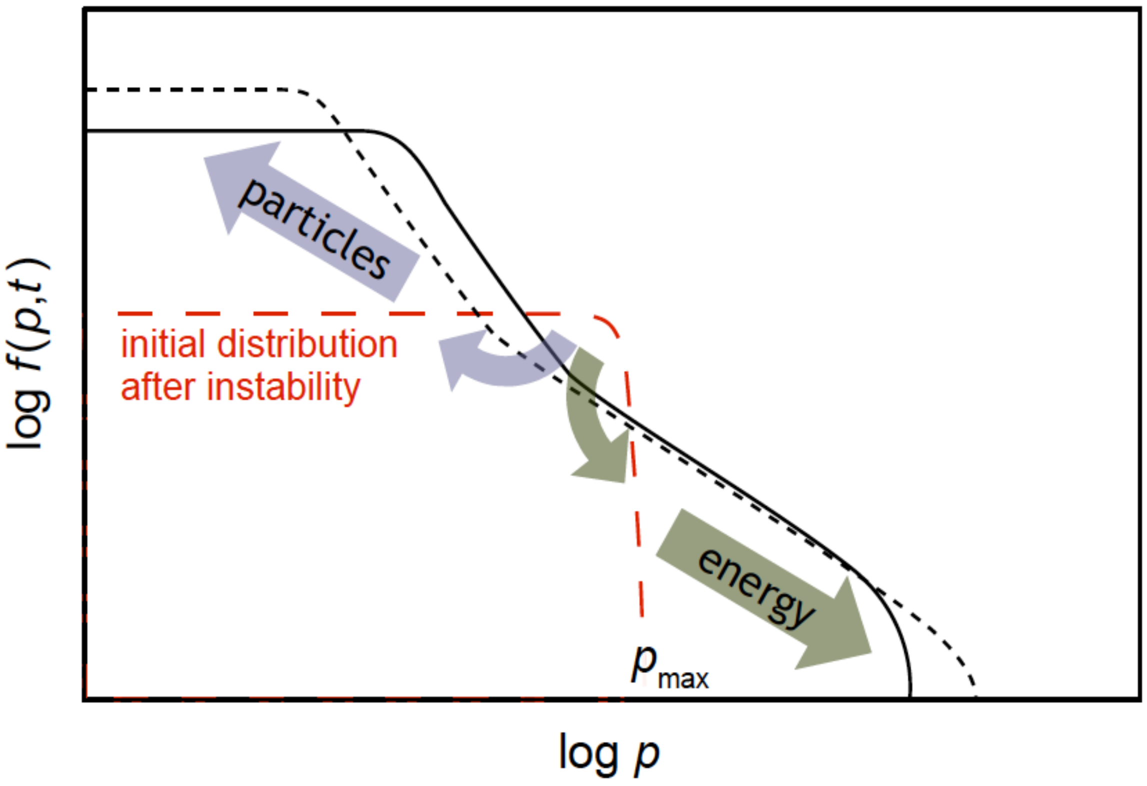
...to create a far-from-equilibrium state:



I. Chantesana, A. Piñeiro Orioli, TG, PRA **99**, 043620 (2019)

An instability/strong cooling quench…

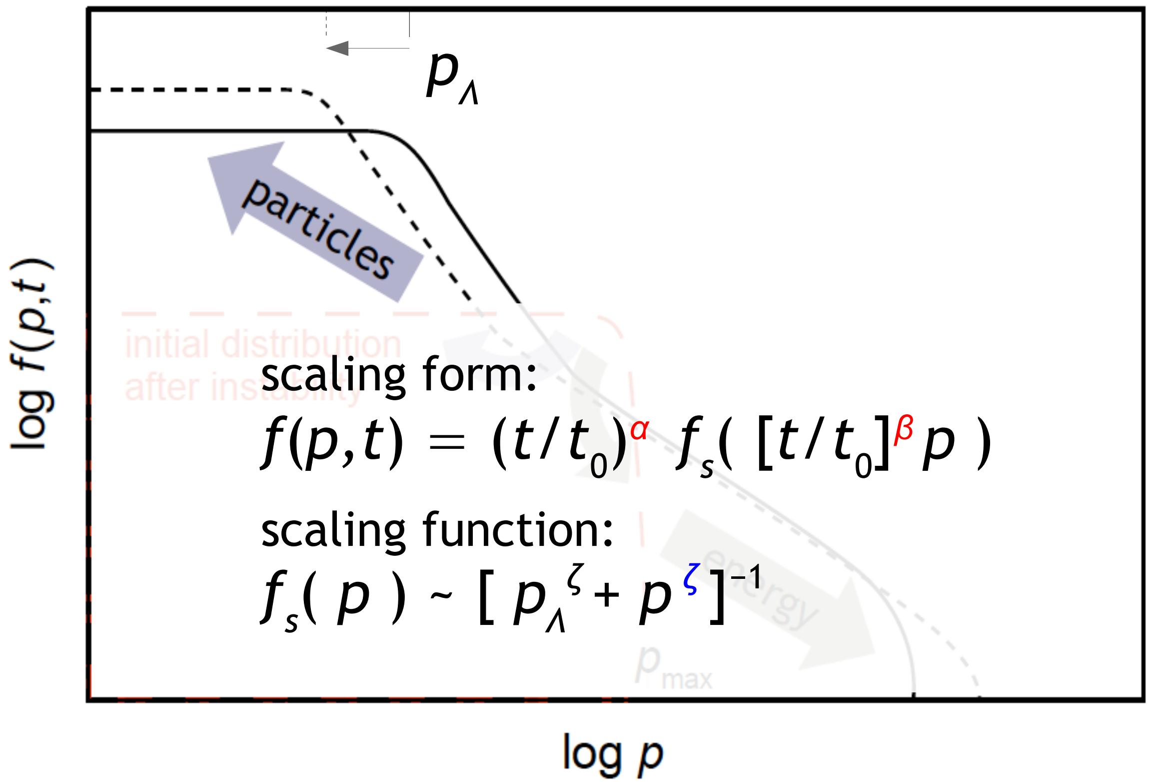
...induces **far-from-equilibrium** bidirectional transport, which...



I. Chantesana, A. Piñeiro Orioli, TG, PRA **99**, 043620 (2019)

An instability/strong cooling quench…

...can lead to self-similar **scaling dynamics**, in the IR...



scaling form:

$$f(p,t) = (t/t_0)^{\alpha}\ f_s(\,[t/t_0]^{\beta} p\,)$$

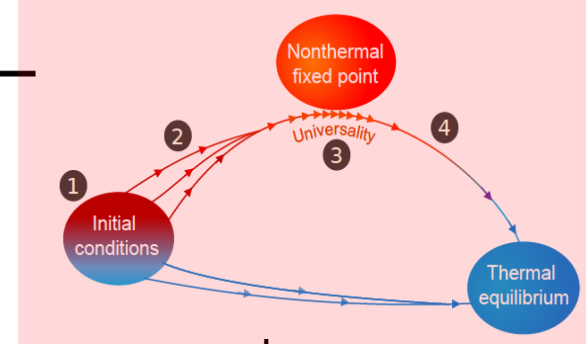
scaling function:

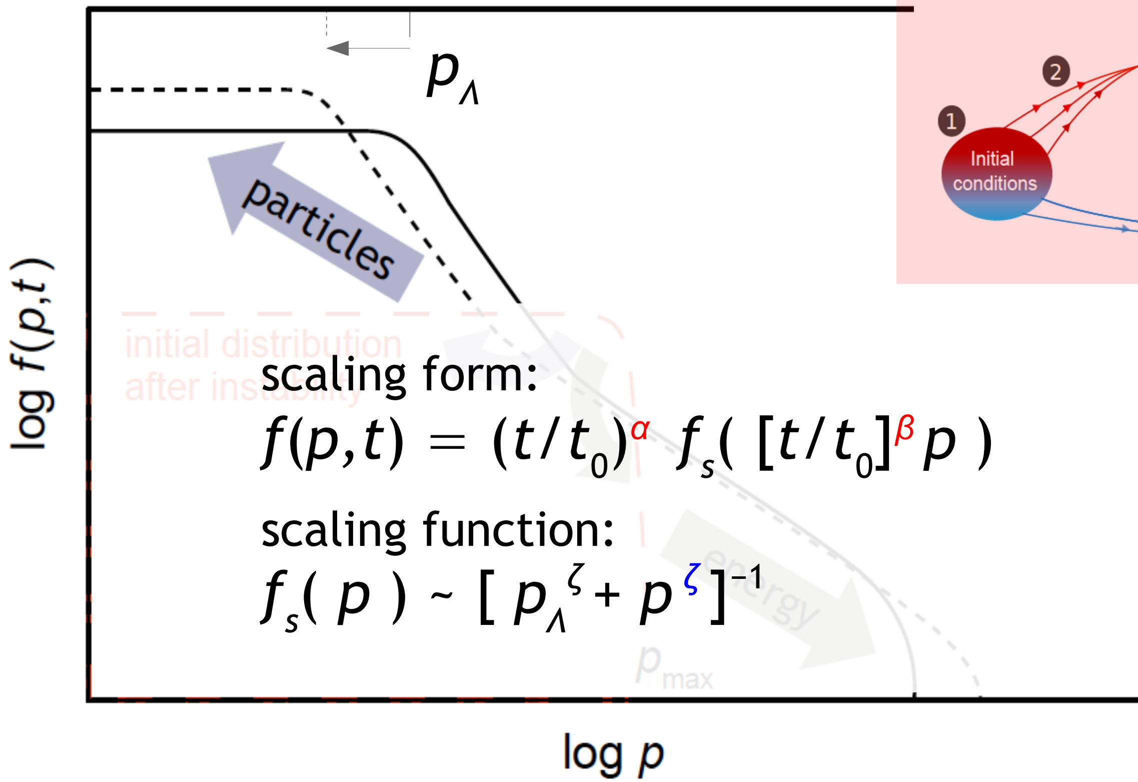
$$f_s(\,p\,) \sim [\,p_\Lambda^{\,\zeta} + p^{\,\zeta}\,]^{-1}$$

I. Chantesana, A. Piñeiro Orioli, TG, PRA **99**, 043620 (2019)

An instability/strong cooling quench…

…can lead to self-similar **scaling dynamics**, near a **Non-thermal Fixed Point.**





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Non-thermal Fixed Points: Berges, Rothkopf, Schmidt: PRL **101**, 041603 (08), Hoffmeister, Sexty, Schlichting, Piñeiro Orioli, Boguslavski, Noel, … Berges (09-) Scheppach, Berges, TG: PRA **81**, 033611 (10), Nowak, Sexty, Schole, Schmidt, Erne, Karl, Schmied, Mikheev, ... TG (11-)

Kinetic theory summary: Chantesana, Piñeiro Orioli, TG: PRA **99**, 043620 (19);
Low-energy effective theory: Mikheev, Schmied, TG: PRA **99**, 063622 (19)
Review article: Mikheev, Siovitz, TG, Eur. Phys. J. ST **232**: 3393 (23)

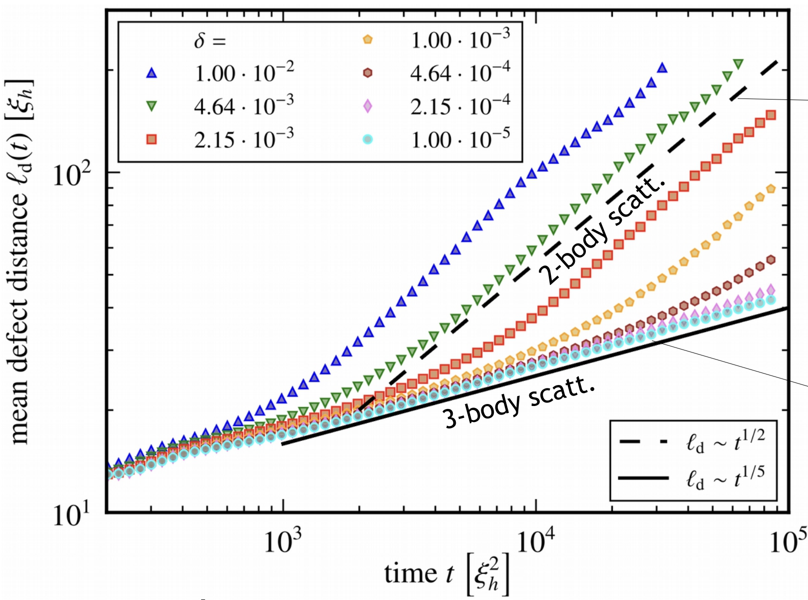
Non-thermal fixed points in other systems

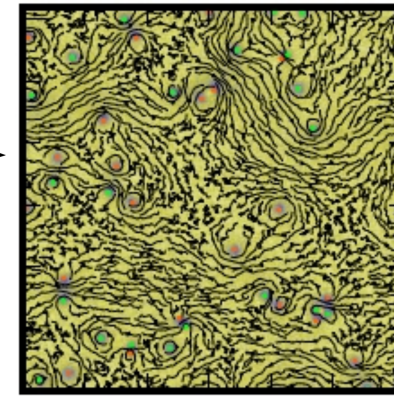
Video: Approach of a non-thermal fixed point in a 1-component 2D gas

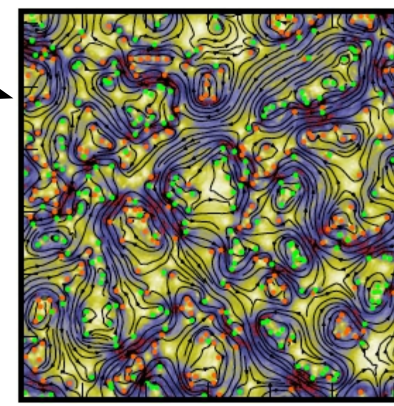
https://www.kip.uni-heidelberg.de/gasenzer/projects/anomalousntfp

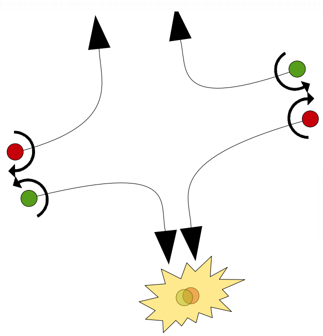
Anomalous NTFP

Vortex coarsening in a 2D Bose gas: mutual annihilation









$$\partial_t N_\mathrm{d} \sim -\Gamma_3 N_\mathrm{d} \sim -\ \mathrm{const.} \times N_\mathrm{d}^{7/2}$$

$$\Rightarrow \quad N_\mathrm{d}(t) \sim t^{-2/5}$$

M. Karl, TG, NJP 19, 093014 (2017) — also: J. Deng, S. Schlichting, R. Venugopalan, Q. Wang, PRA 97, 053606 (18); D. Spitz, J. Berges, M. Oberthaler, A. Wienhard, SciPost Phys. 11, 060 (21); V. Noel, TG, K. Boguslavski, PRR 7, 033220 (25); N. Rasch, L. Chaumaz, TG, 2506.01653

Anomalous NTFP

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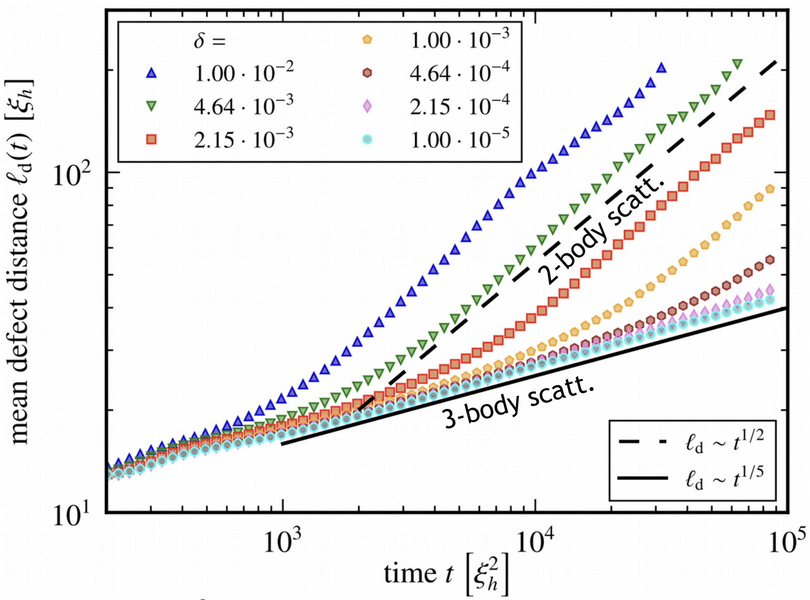
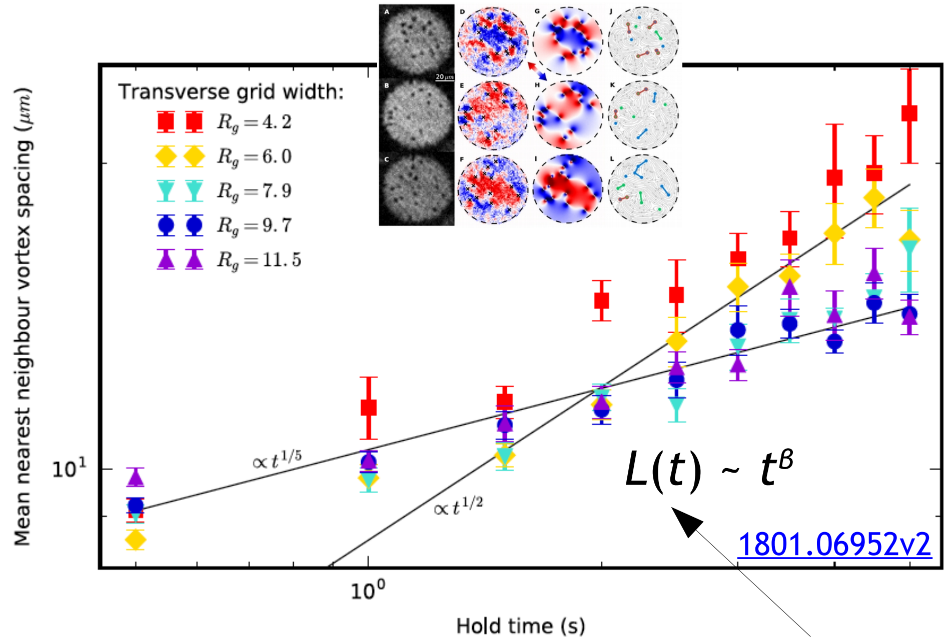
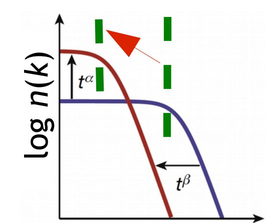
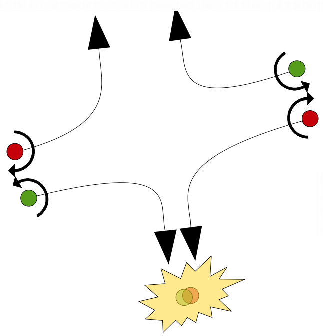




$k_L(t) \sim L(t)^{-1}$





$$\partial_t N_d \sim -\Gamma_3 N_d \sim -\text{ const.} \times N_d^{7/2}$$

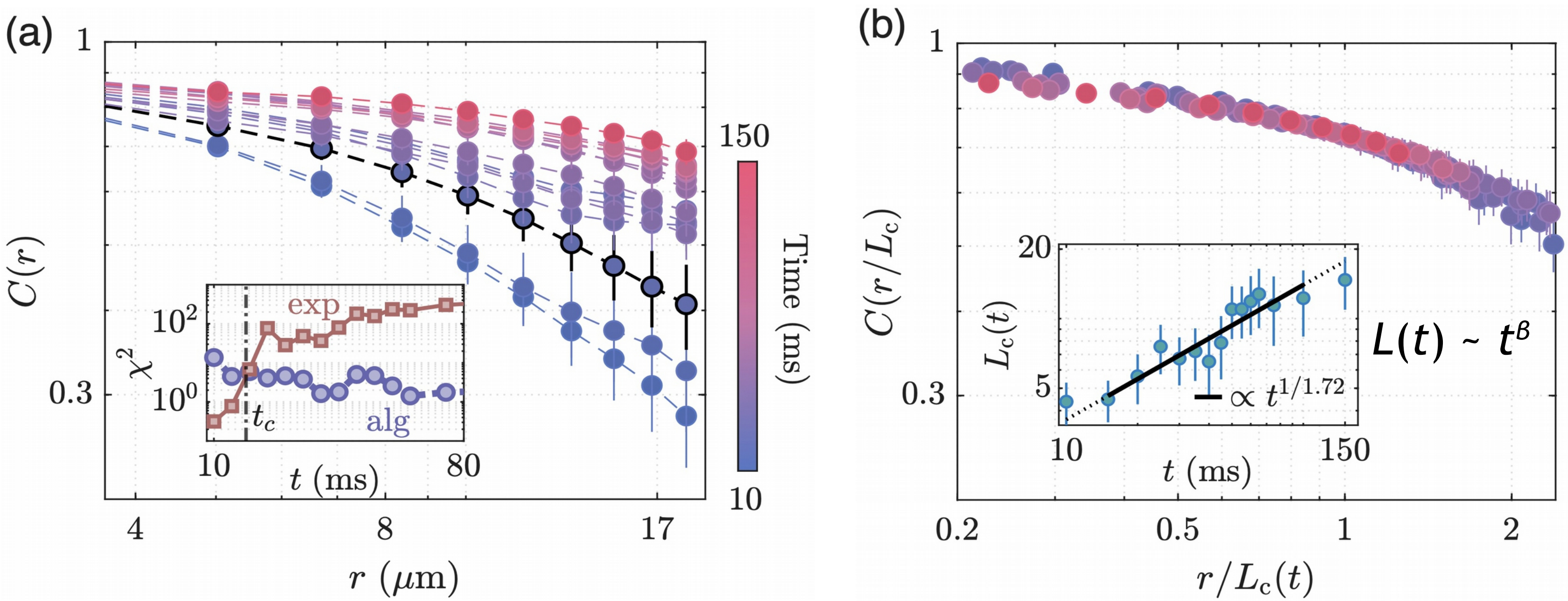
$$\Rightarrow \quad N_d(t) \sim t^{-2/5}$$

M. Karl, TG, NJP 19, 093014 (2017)

S.P. Johnstone, A.J. Groszek, P.T. Starkey, C.J. Billington, T.P. Simula, K. Helmerson, Science **364**, 1267 (2019); 1801.06952v2

Experiment (E. Chang et al., Oxford)

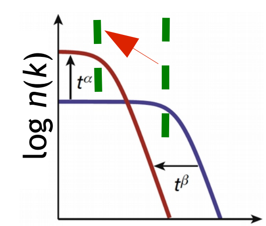
Experimental observation in a 2D two-layer Bose condensate: Vortex decay



$$C(r) = \langle \varphi^{+}(\mathbf{x})\varphi(\mathbf{x}-\mathbf{r})\rangle_{\mathbf{x},\Omega} \sim \mathcal{FT}[n(\mathbf{k})](r)$$

$$\beta = 1/z$$

$$z = 1.73 \pm 0.09$$

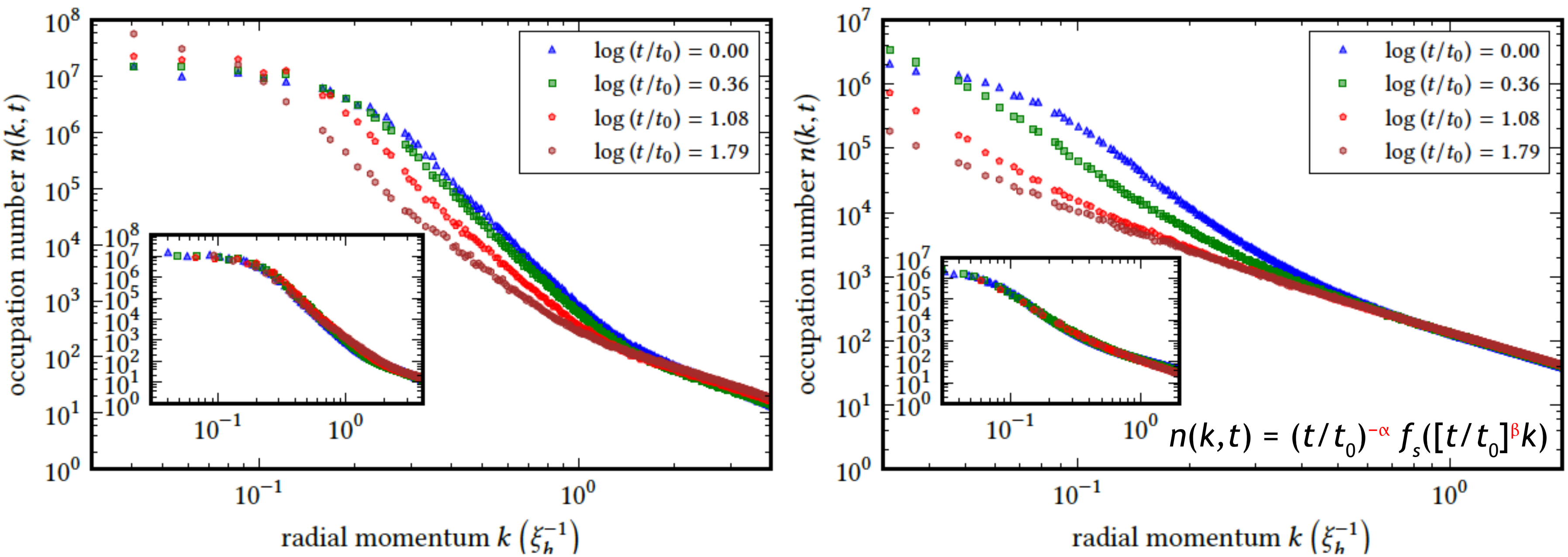


En Chang, Vijay Pal Singh, Abel Beregi, Erik Rydow, Ludwig Mathey, Christopher J. Foot, Shinichi Sunami; arXiv: 2510.23600

Anomalous vs near-equilibrium NTFP

Coarsening evolution of single-particle spectrum

(TW-simulation from two different initial conditions)



$\alpha_a = 0.402 \pm 0.05$

$\beta_a = 0.193 \pm 0.05$

$z \approx 5$

Analytic prediction:
(loop resummed self-energy)

$\alpha = 2 \cdot \beta$

$\beta = 1/z$

[Chantesana, Piñeiro-Orioli, TG, PRA **99** (19) 043620]

$\alpha_g = 1.10 \pm 0.08$

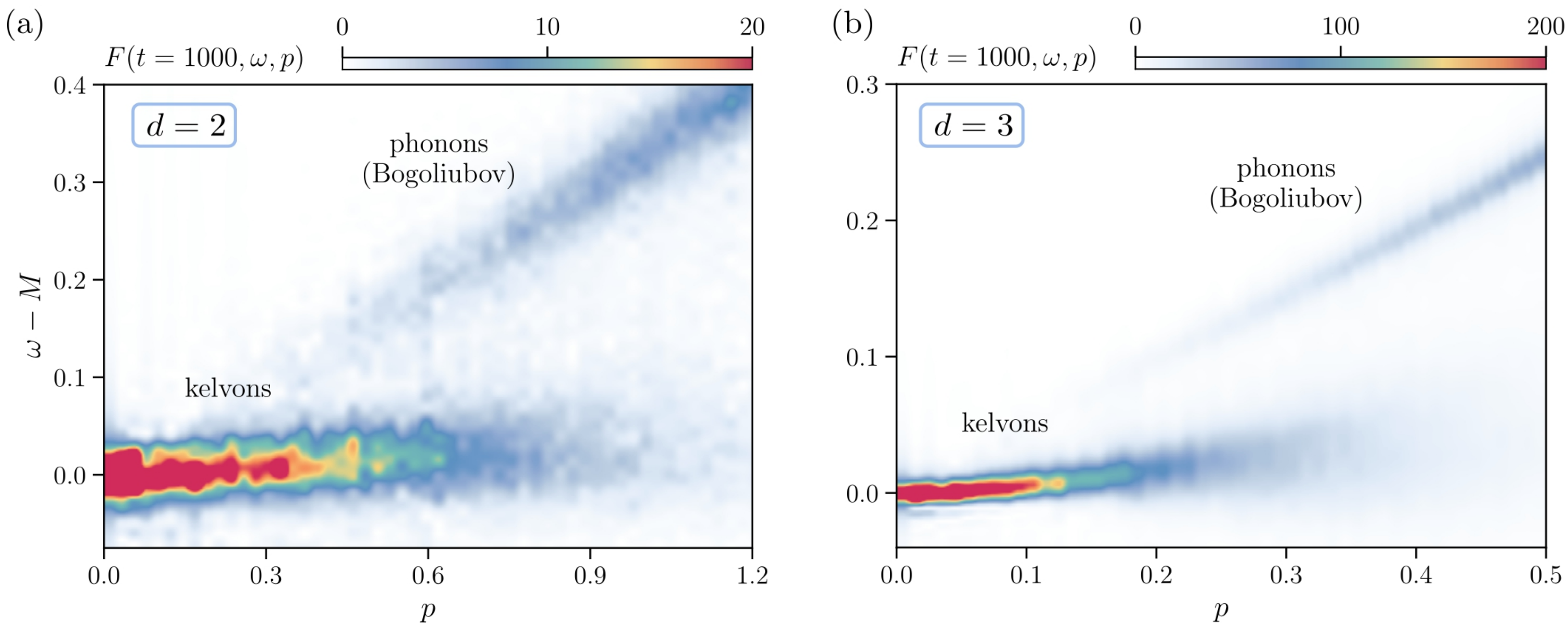
$\beta_g = 0.56 \pm 0.08$

$z = 1.79 \pm 0.3$

M. Karl, TG, NJP **19,** 093014 (2017), 1611.01163 [cond-mat.quant-gas]

Role of Kelvons

Vortex coarsening in a 2+3D Bose gases: mutual annihilation driven by Kelvin waves



V. Noel, TG, K. Boguslavski, PRR 7, 033220 (2025)

Quantum turbulence

Inverse Cascade

of energy in 2D classical turbulence

— together with direct k^{-3} *enstrophy* (~ *vorticity density*) cascade —

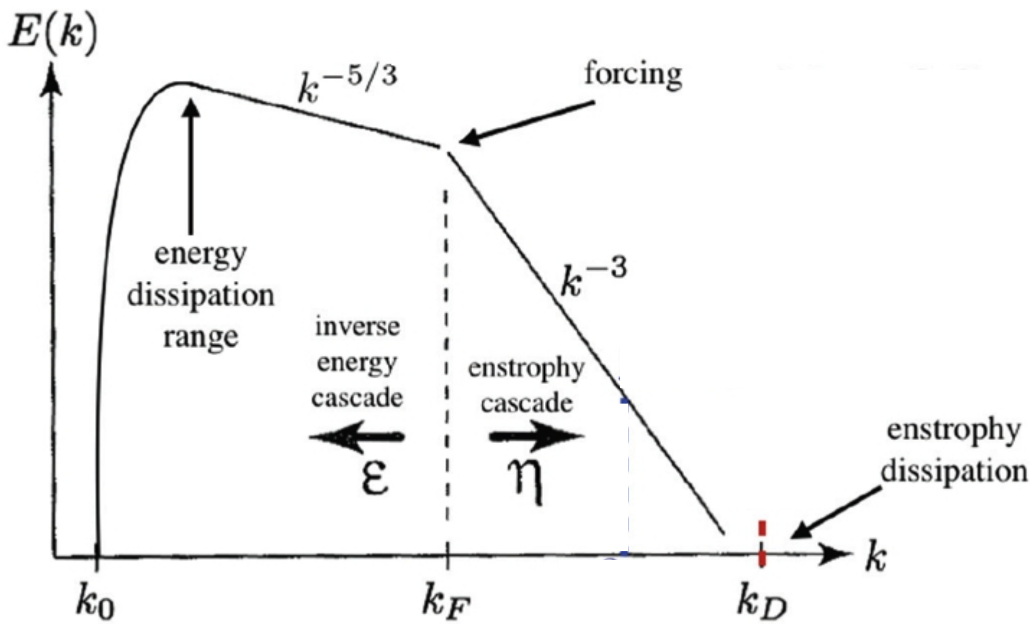


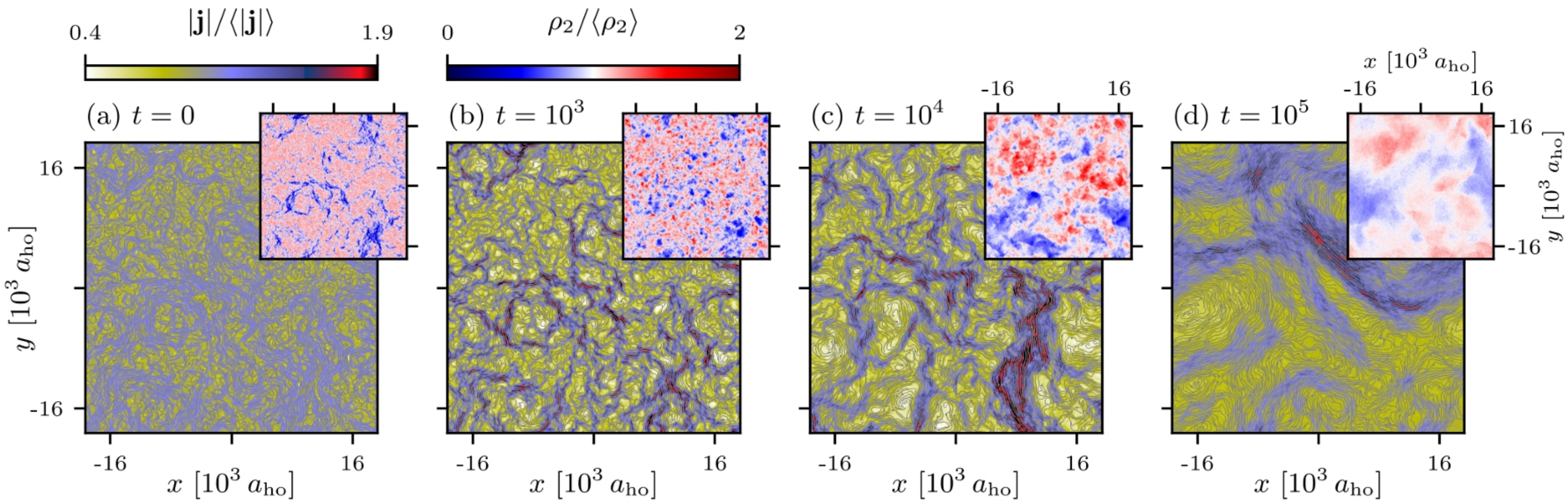
Fig. adapted from Fox-Kemper et al., CLIVAR Exchanges 65 (14) 42.

Robert H. Kraichnan, *Inertial Ranges in Two-Dimensional Turbulence*. Phys. Fl. **10**, 1417 (1967)

Equivalent in nearly incompressible flow in 2D quantum turbulence (GPE):

N. P. Müller, G. Krstulovic, PRL **132**, 094002 (2024)
M. Reeves et al., PRL **110**, 104501 (2013)

Pattern in position space



Decreased UV resolution

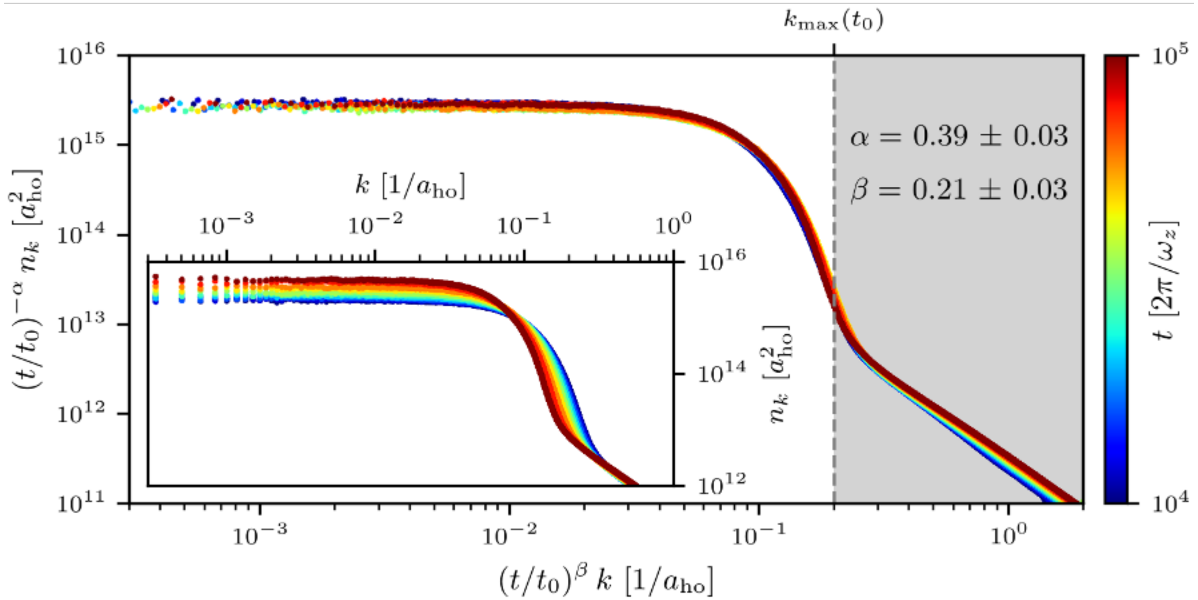
$\xi \approx 2\,dx$

Increased IR resolution

- Larger grid 16384^2
- Higher initial vortex density → increase k_v ($1.4 \cdot 10^6$ vortices & antivortices)

Niklas Rasch, TG, Phys. Rev. A 113: L051302, 2026.

Momentum spectra



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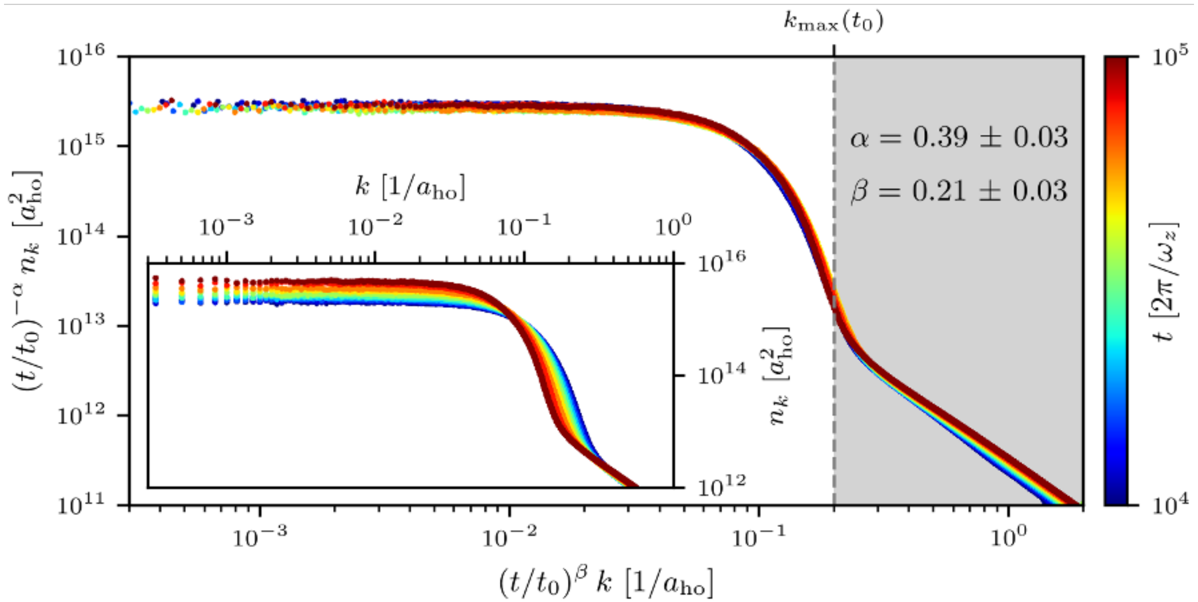
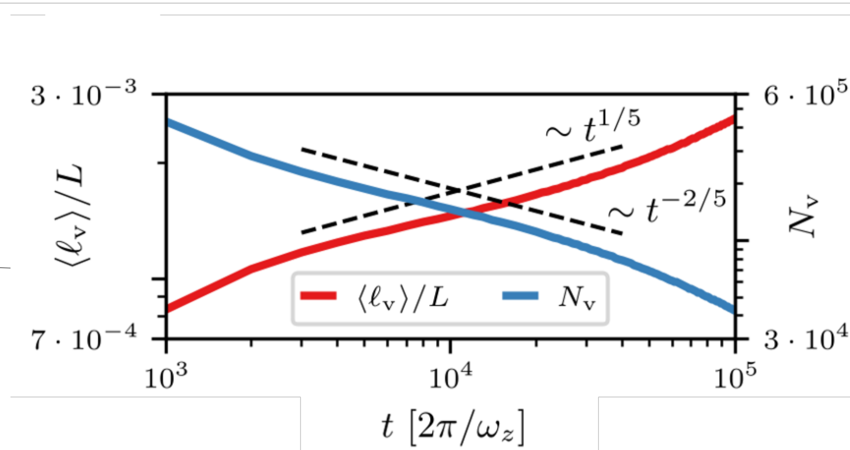
Niklas Rasch, TG, Phys. Rev. A 113: L051302, 2026.

Momentum spectra & Vortex number/distance



inter-vortex distance



total vortex number

Niklas Rasch, TG, Phys. Rev. A 113: L051302, 2026.

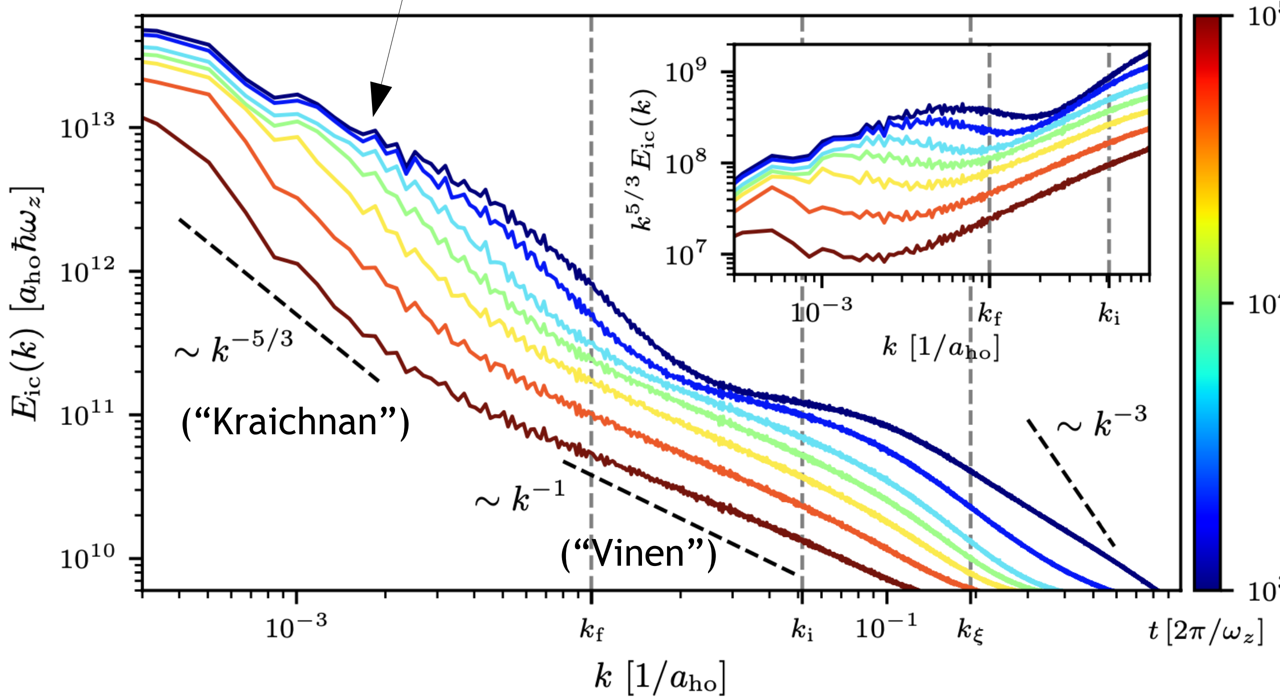
Spectrum of incompressible kinetic energy

Split kinetic energy:

$$E_{\mathrm{kin}} = \frac{1}{2}\int \mathrm{d}^2\boldsymbol{x}\,\Big(\underbrace{|\nabla\sqrt{n}|^2}_{E_{\mathrm{q}}} + \underbrace{|\sqrt{n}\nabla\varphi|^2}_{E_{\mathrm{cl}}}\Big)$$

Incompressible energy in point-vortex model

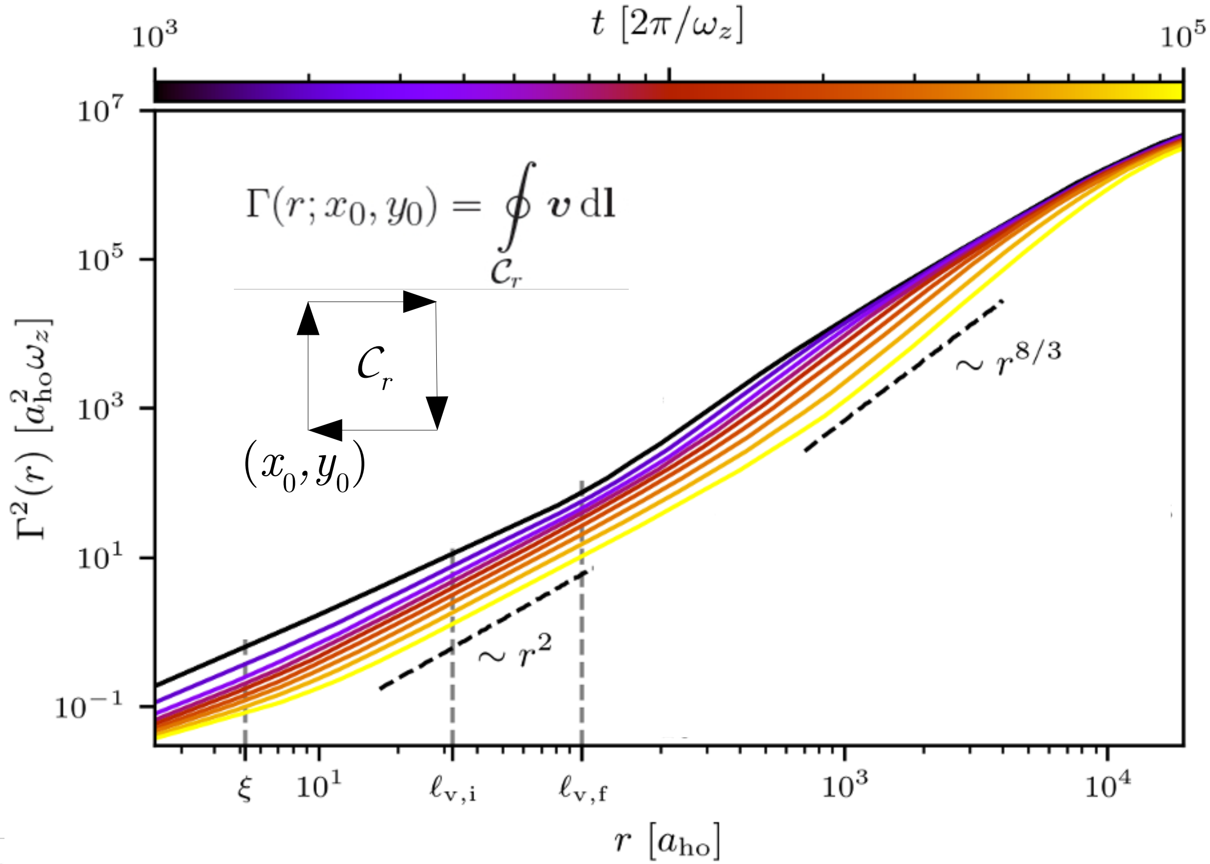
$$E_{\mathrm{cl}}^{\mathrm{incomp}}(k) = \frac{(2\pi)^2 n}{2k}\sum_{ij} q_i q_j J_0(k l_{ij})$$



Niklas Rasch, TG, Phys. Rev. A 113: L051302, 2026.

Analysis in position space

Velocity circulation:

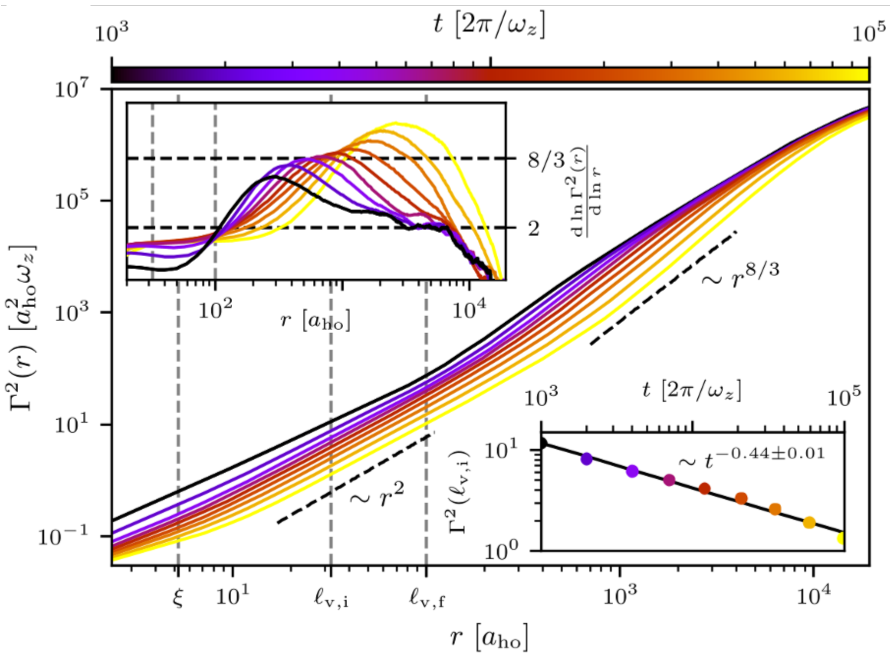


$$\langle|\Gamma(r)|^p\rangle = \frac{1}{V}\int |\Gamma(r; x_0, y_0)|^p\, \mathrm{d}x_0\, \mathrm{d}y_0$$

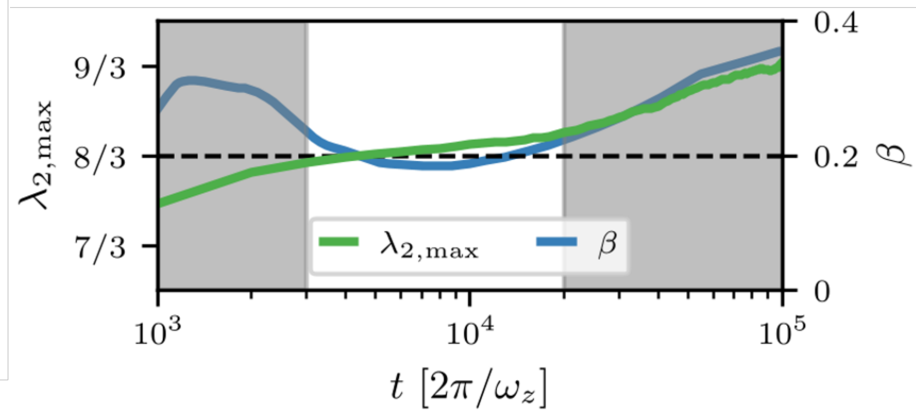
Niklas Rasch, TG, Phys. Rev. A 113: L051302, 2026.

See also: forced/quenched 2D superfluid turbulence (IEC):
Experiment: H.-Y. Zhu, J.-H. Xie, and K.-Q. Xia, PRL **130**, 214001 (2023)
Simulation: N. P. Müller, G. Krstulovic, PRL **132**, 094002 (2024)

Analysis in position space



Compare maximal local slope $\lambda_{(2,\max)}(t)$ with time-local coarsening $\beta(t)$:



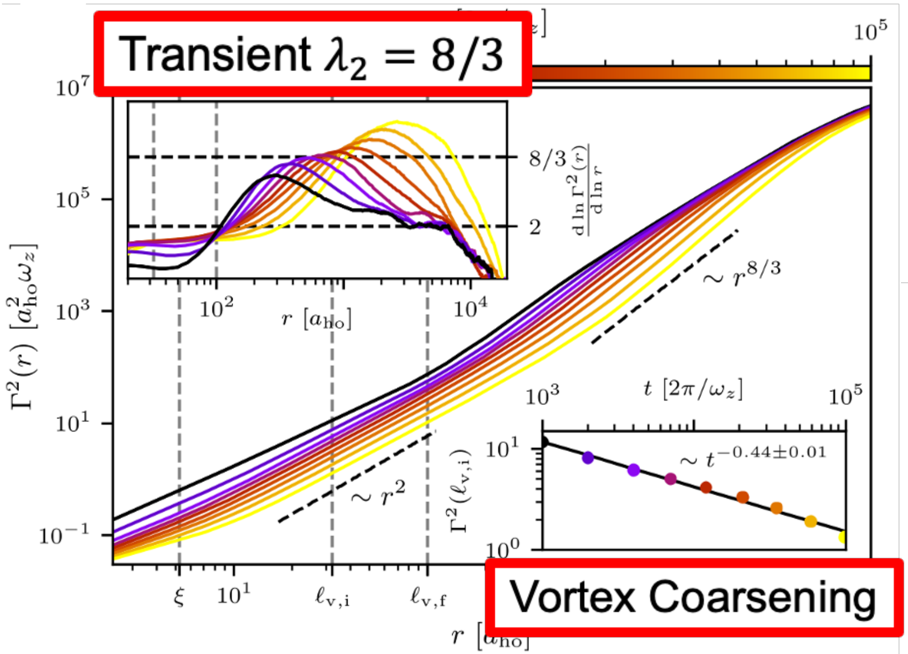
Single vortex scaling: $\langle|\Gamma(r)|^p\rangle \sim r^2$ for $r < \ell_v$

Inverse energy cascade: $\langle|\Gamma(r)|^p\rangle \sim r^{\lambda_p}$ for $r > \ell_v$

Kraichnan Batchelor: $\lambda_p = 4p/3$ (non-intermittent)

Niklas Rasch, TG, Phys. Rev. A 113: L051302, 2026.

Analysis in position space



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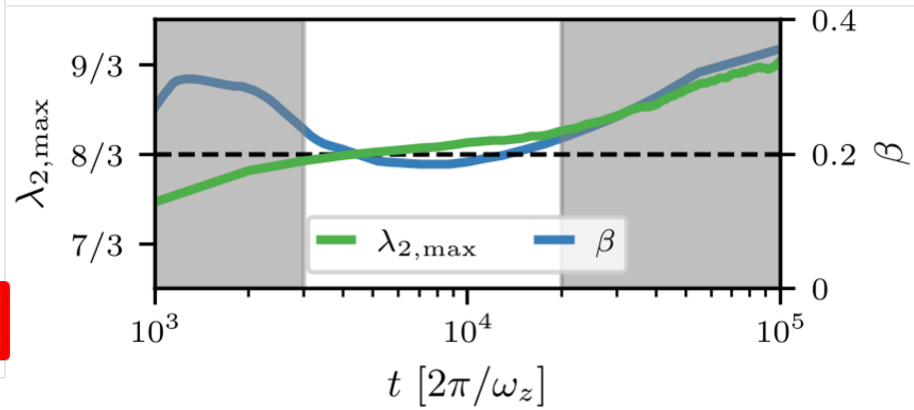


Indications of simultaneity of transient inverse energy cascade and anomalous NTFP in universal interval

Niklas Rasch, TG, Phys. Rev. A 113: L051302, 2026.

Intermittency in higher-order moments of circulation

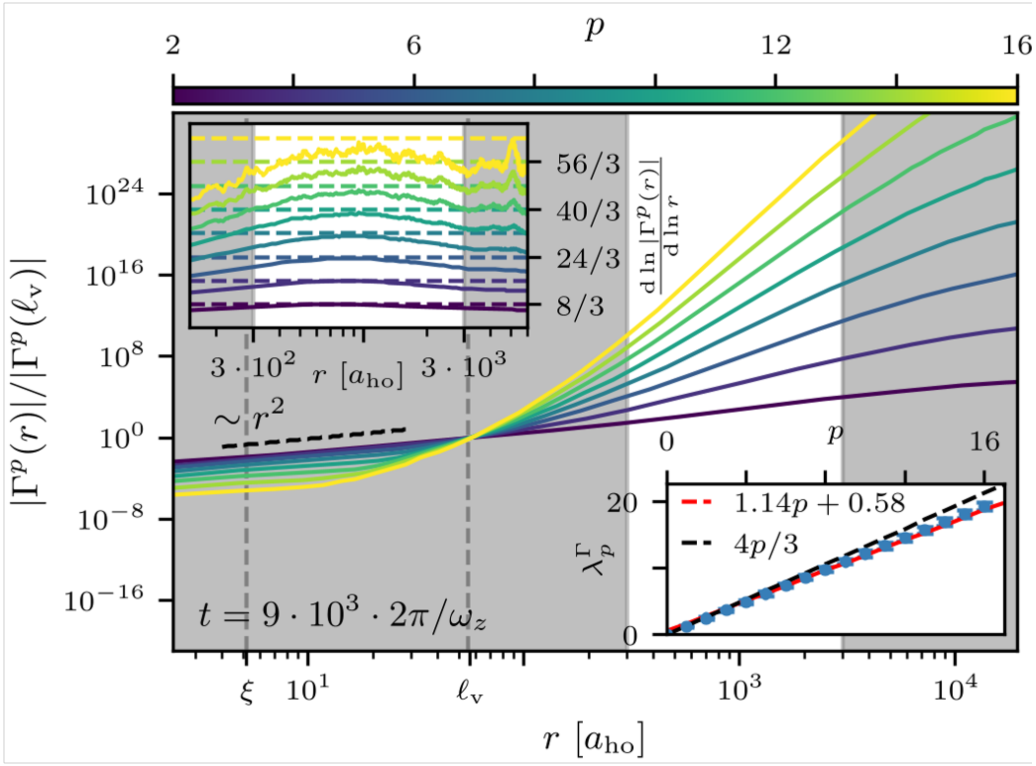
- Intermittency reflected in deviations from

$$\lambda_p = 4p/3$$

- Logarithmic slopes flatten in inertial range

Extracted scaling exponents can be fit with
bifractal intermittency model

$$\lambda_p \approx 1.14\,p + 0.58$$



Measured in thin fluid layers:
H.-Y. Zhu et al., PRL **130**, 214001 (2023)
Simulated in nearly incompressible QT:
N. P. Müller and G. Krstulovic, PRL **132**, 094002 (2024)

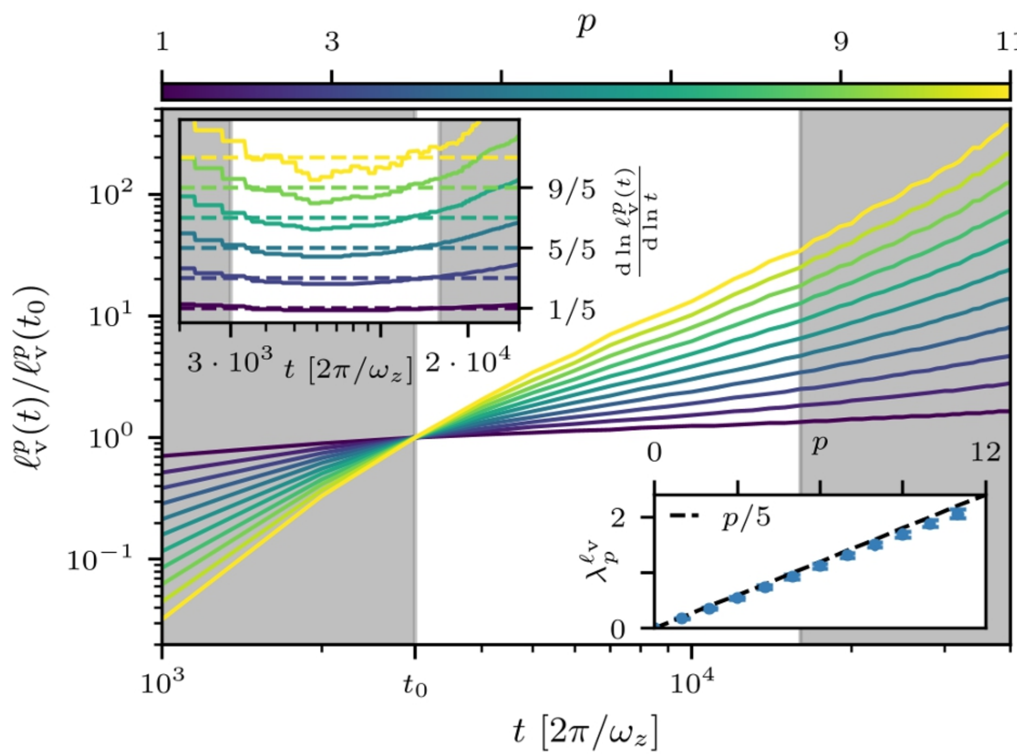
Niklas Rasch, TG, Phys. Rev. A 113: L051302, 2026.

Higher-order moments of intervortex distance distr.

- Logarithmic slopes flatten in universal interval
- Extracted scaling exponents follow

$$\lambda_p \approx p/5$$

- Inter-defect distribution does not show deviations from Gaussianity



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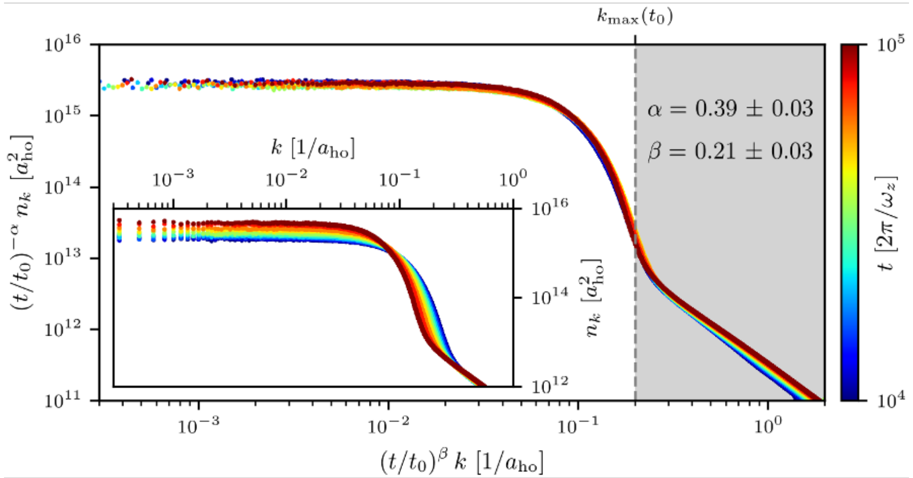
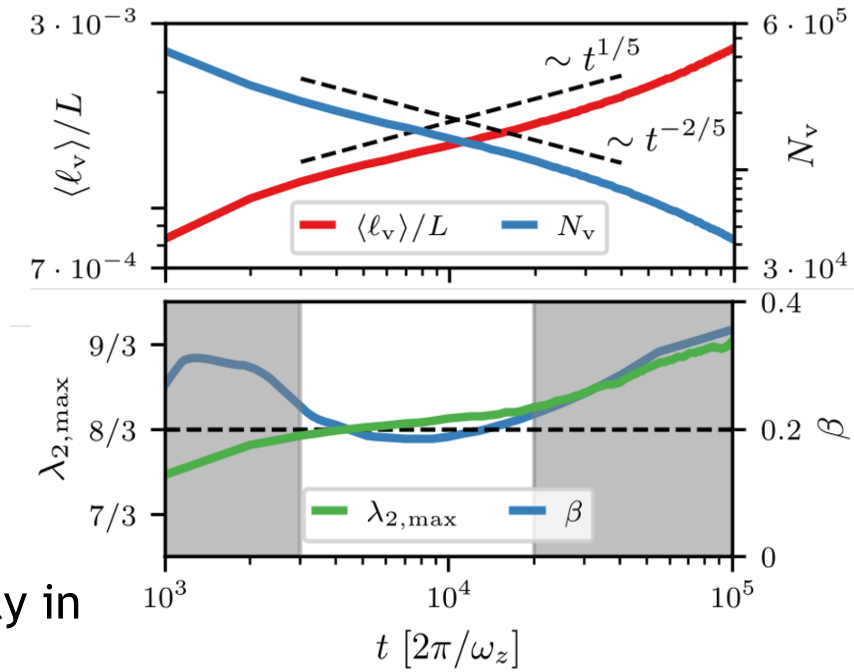
Simulated in nearly incompressible QT:
N. P. Müller and G. Krstulovic, PRL **132**, 094002 (2024)

Niklas Rasch, TG, Phys. Rev. A 113: L051302, 2026.

Decaying 2D turbulence: quantum vs. classical





Power law exponent of vortex density decay in classical decaying 2D Turbulence:

Simulations:

Reference	$\alpha\ = 2\beta$
Carnevale *et al.*[7]	0.75
Weiss and McWilliams[2]	0.72±0.03
Dritschel[5]	0.29±0.03
Clercx and Nielsen[21]	1.03±0.10 [c]
Bracco *et al.*[8]	0.76±0.03

Experiments:

Reference	$\alpha\ = 2\beta$
Carnevale *et al.*[7]	0.75
Tabeling *et al.*[9]	0.70±0.1
Cardoso *et al.*[28]	0.44±0.1
Hansen *et al.*[10]	0.70±0.1
Clercx *et al.*[29]	0.70±0.1

For these tables & Refs., see van Bokhoven et al., Phys. Fl. **19**, 046601 (07)

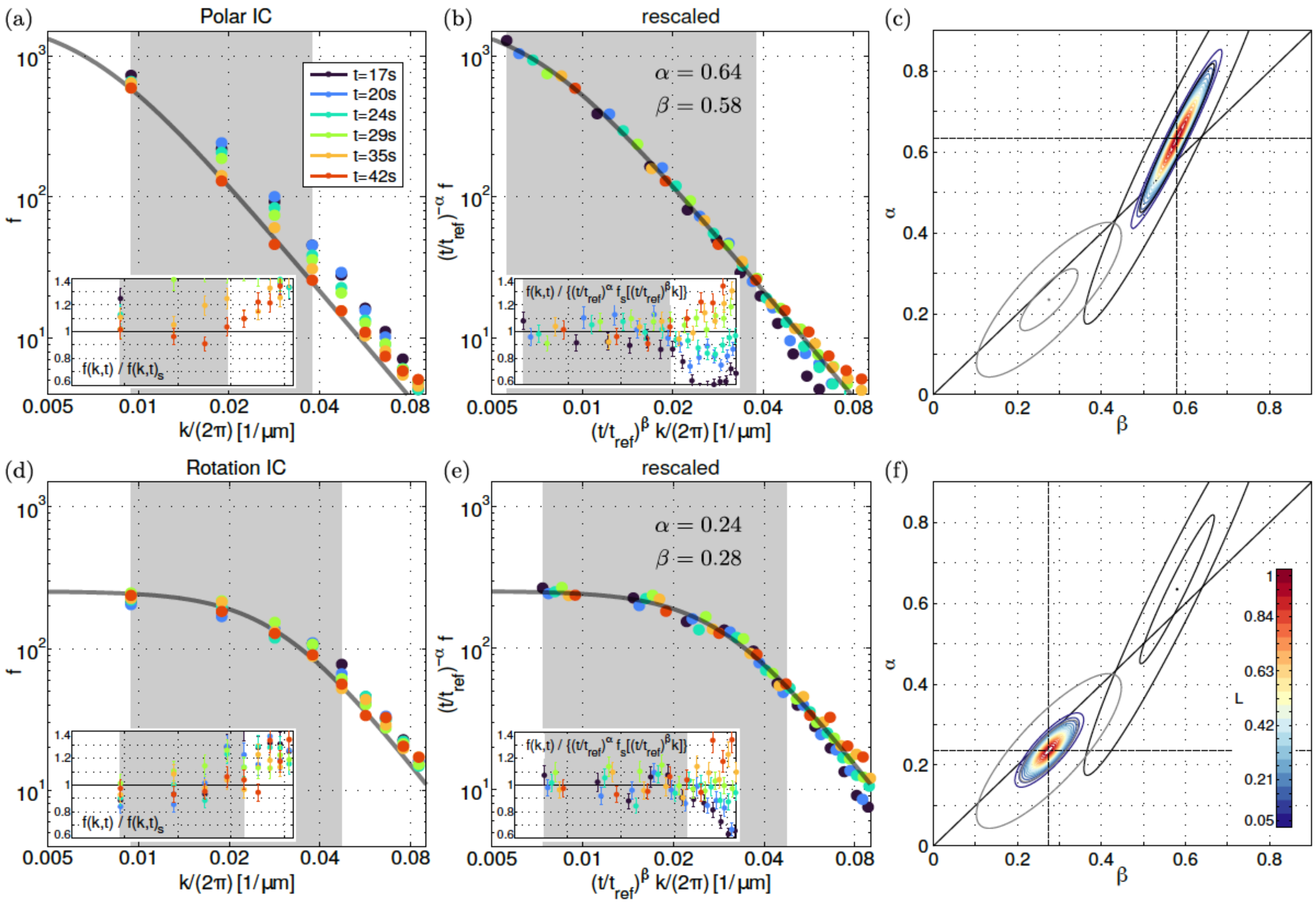

AND NOW FOR
SOMETHING
COMPLETELY
DIFFERENT

Universal dynamics in **experiment**

SynQS

Oberthaler labs

Experimental identification of two non-thermal fixed points in a Spin-1 Bose gas



$\beta \approx 1/2$

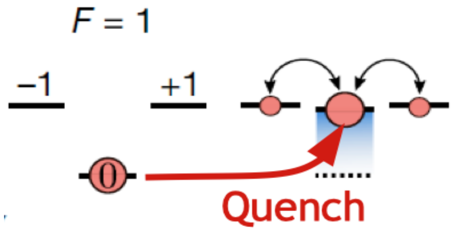
$\alpha = d\,\beta$

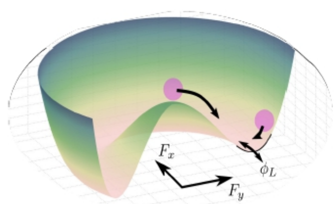
$\beta \approx 1/4$

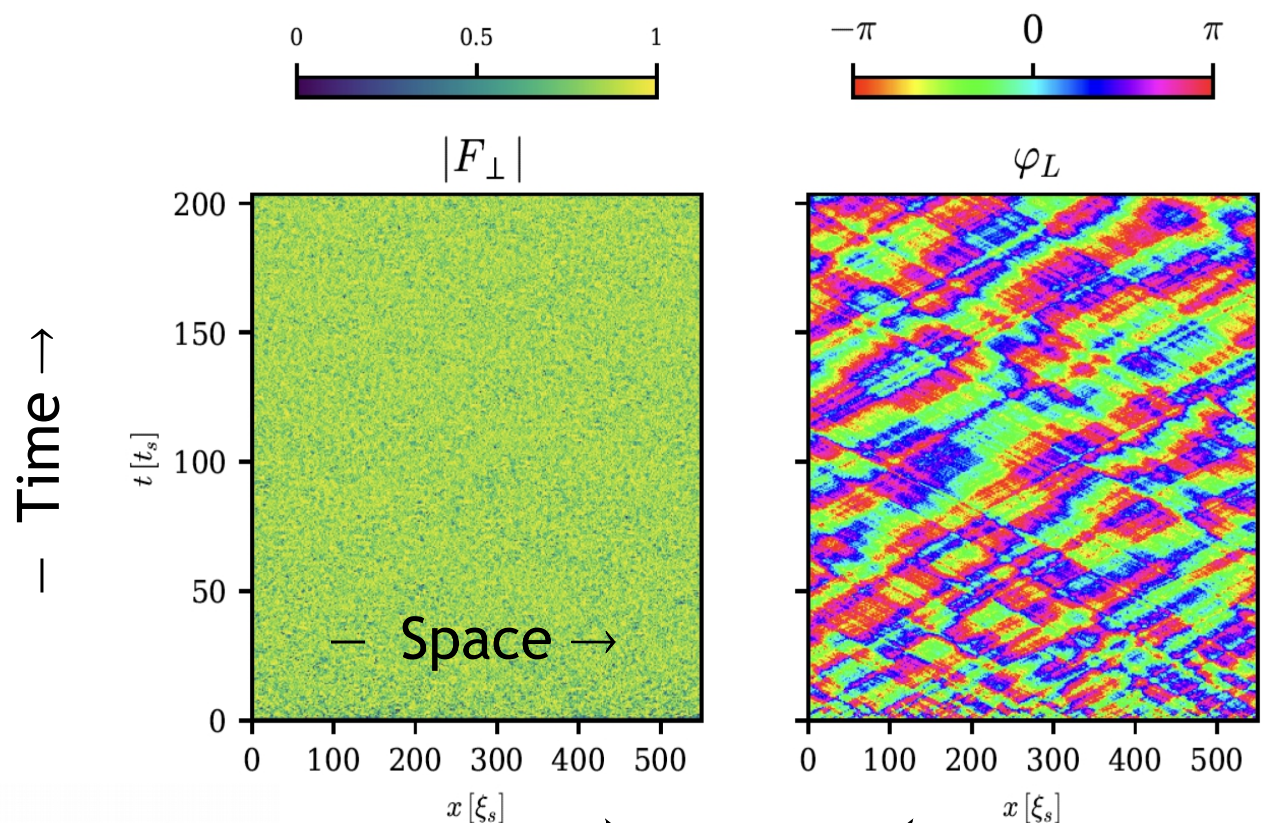
$\alpha = d\,\beta$

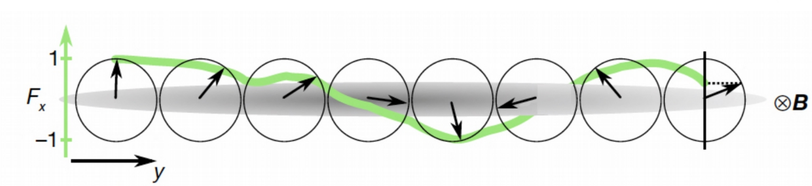
S. Lannig et al., arXiv:2306.16497 [cond-mat.quant-gas]; (also: M. Prüfer, et al., Nature 563, 217 (18))

1D Spin-1 gas: Pattern formation in F_x-F_y–plane:









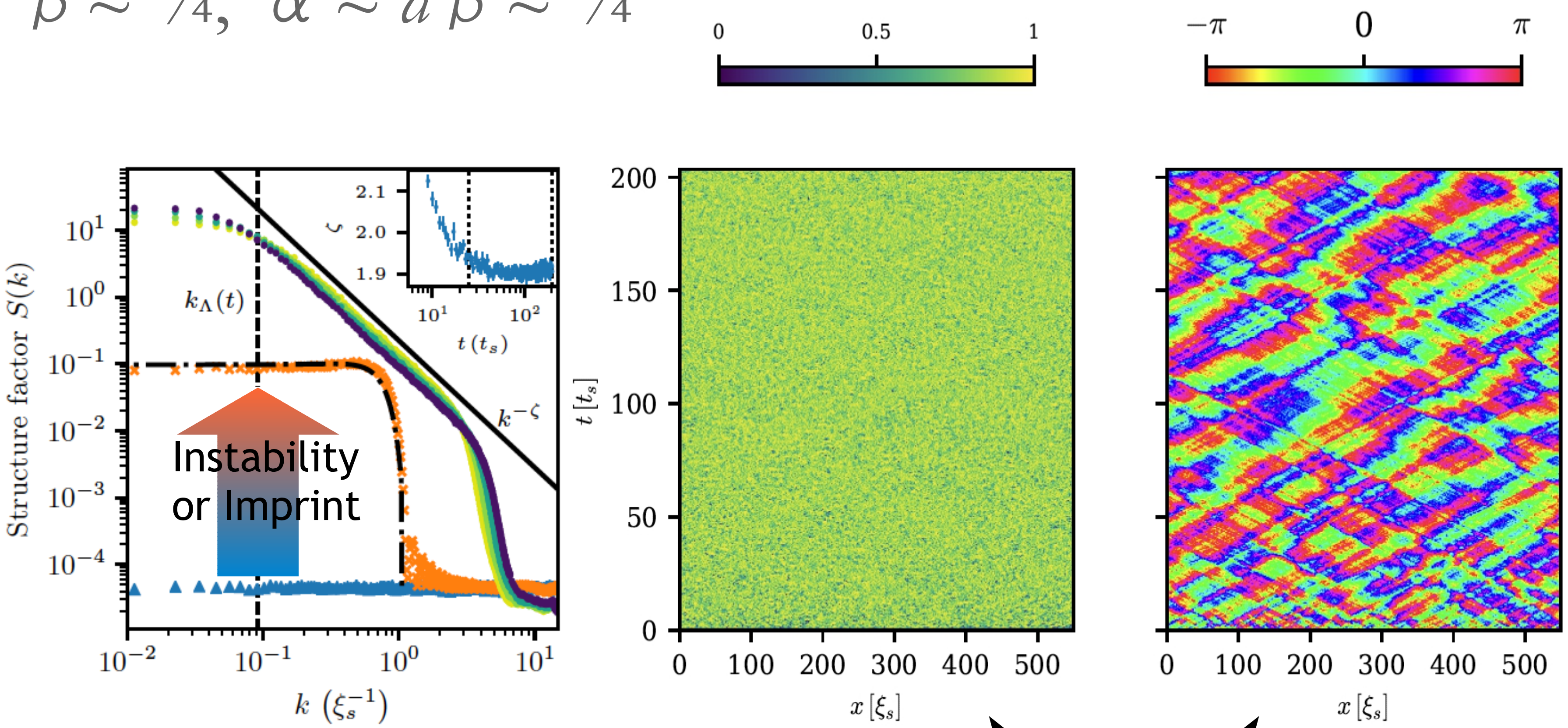
Amplitude & Larmor phase

$$F_\perp = F_x + iF_y = |F_\perp| e^{i\varphi_L}$$

C.M. Schmied, M. Prüfer, M.K. Oberthaler, TG, PRA **99**, 033611 (2019); arXiv:1812.08571 [cond-mat.quant-gas]
I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]

Coarsening: Universal dynamics!

$\beta \approx 1/4, \;\; \alpha \approx d\,\beta \approx 1/4$

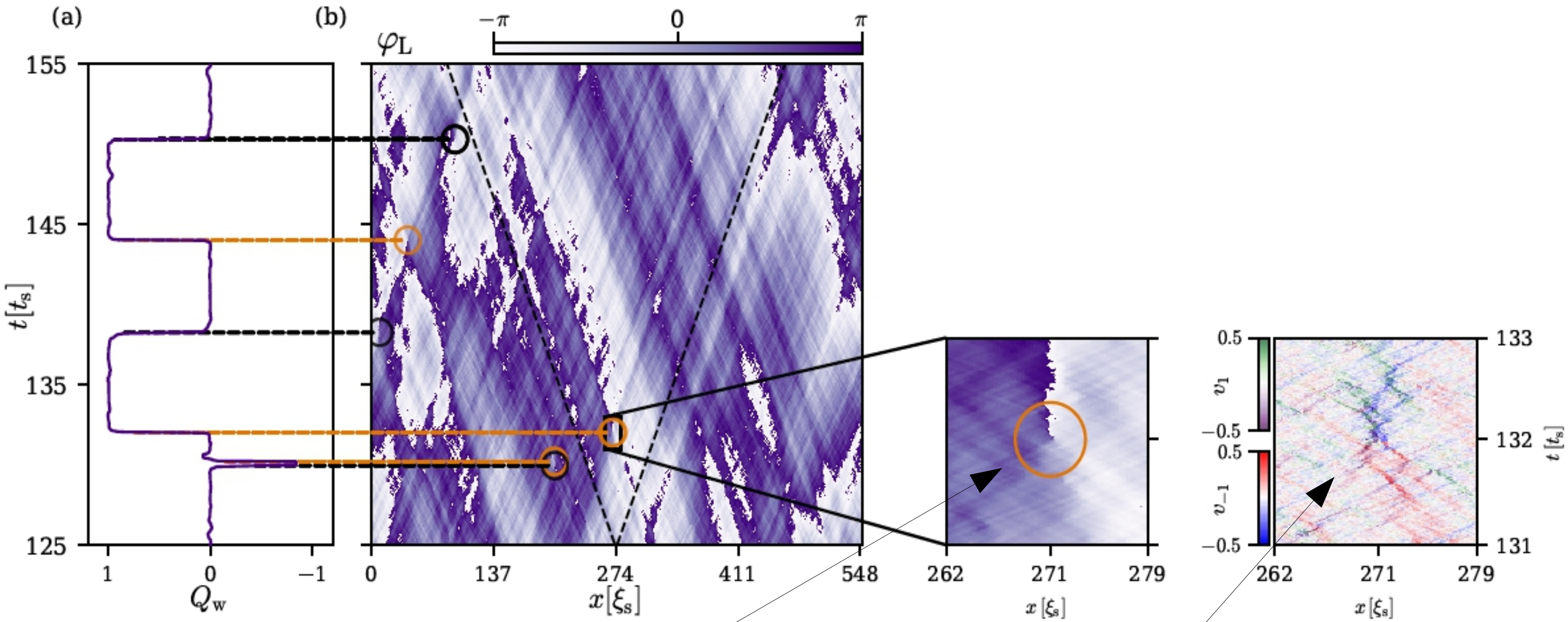


Structure factor … of … Amplitude & Larmor phase

$$S(k,t) = \langle |F_\perp(k,t)|^2 \rangle \qquad F_\perp = F_x + iF_y = |F_\perp| e^{i\varphi_L}$$

C.M. Schmied, M. Prüfer, M.K. Oberthaler, TG, PRA **99**, 033611 (2019); arXiv:1812.08571 [cond-mat.quant-gas]
I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:2304.09293 [cond-mat.quant-gas]

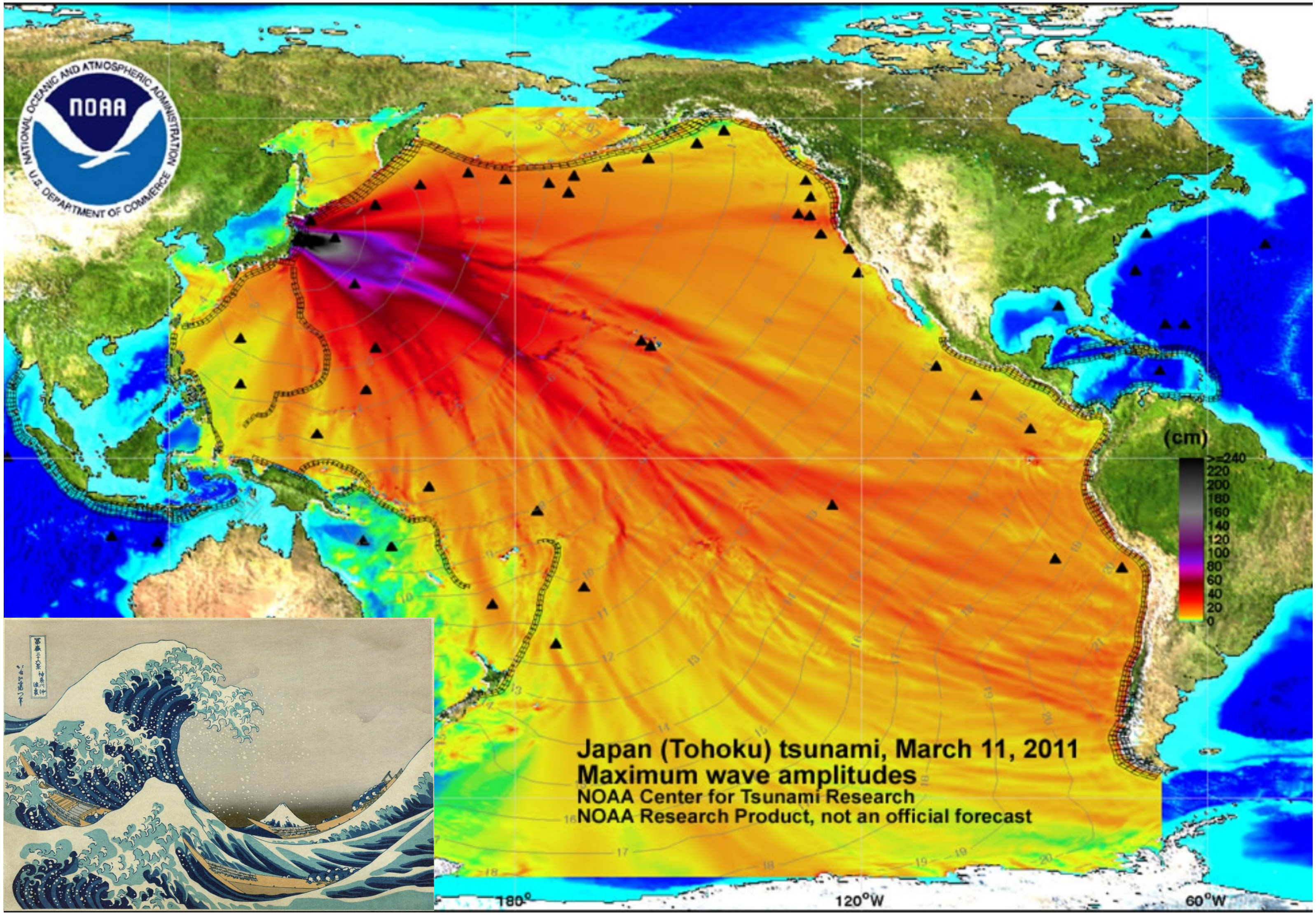
Instantons induced by rogue waves



Instanton = phase slip, induced by **rogue wave** in spinor phase

I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M.K. Oberthaler, TG, PRL **131**, 183402 (23); arXiv:[2304.09293 [cond-mat.quant-gas]]

Caustics lead to Rogue Waves

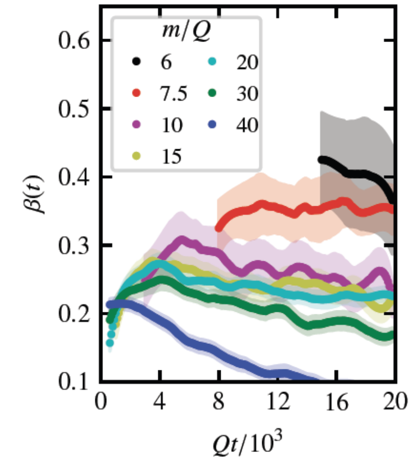


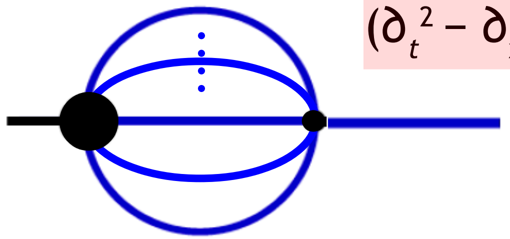
Source: NOAA https://nctr.pmel.noaa.gov/honshu20110311/

Analytic results from non-equilibr. QFT

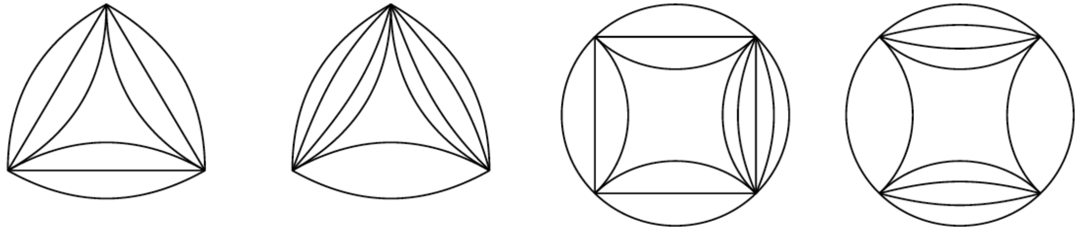
Kadanoff-Baym equation for the **sine-Gordon** equation

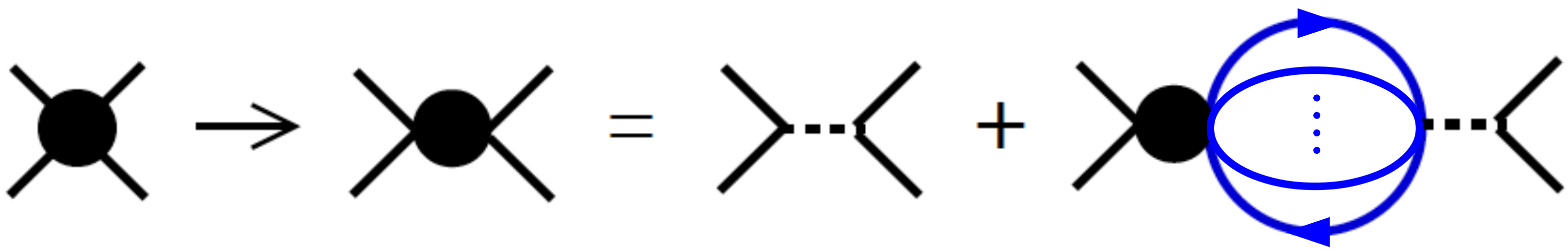
$(\partial_t^2 - \partial_x^2)\varphi + \lambda \sin\varphi = 0$



$\partial_t n(p) =$ 

from **2PI (two-particle-irreducible) effective action**, resummed as

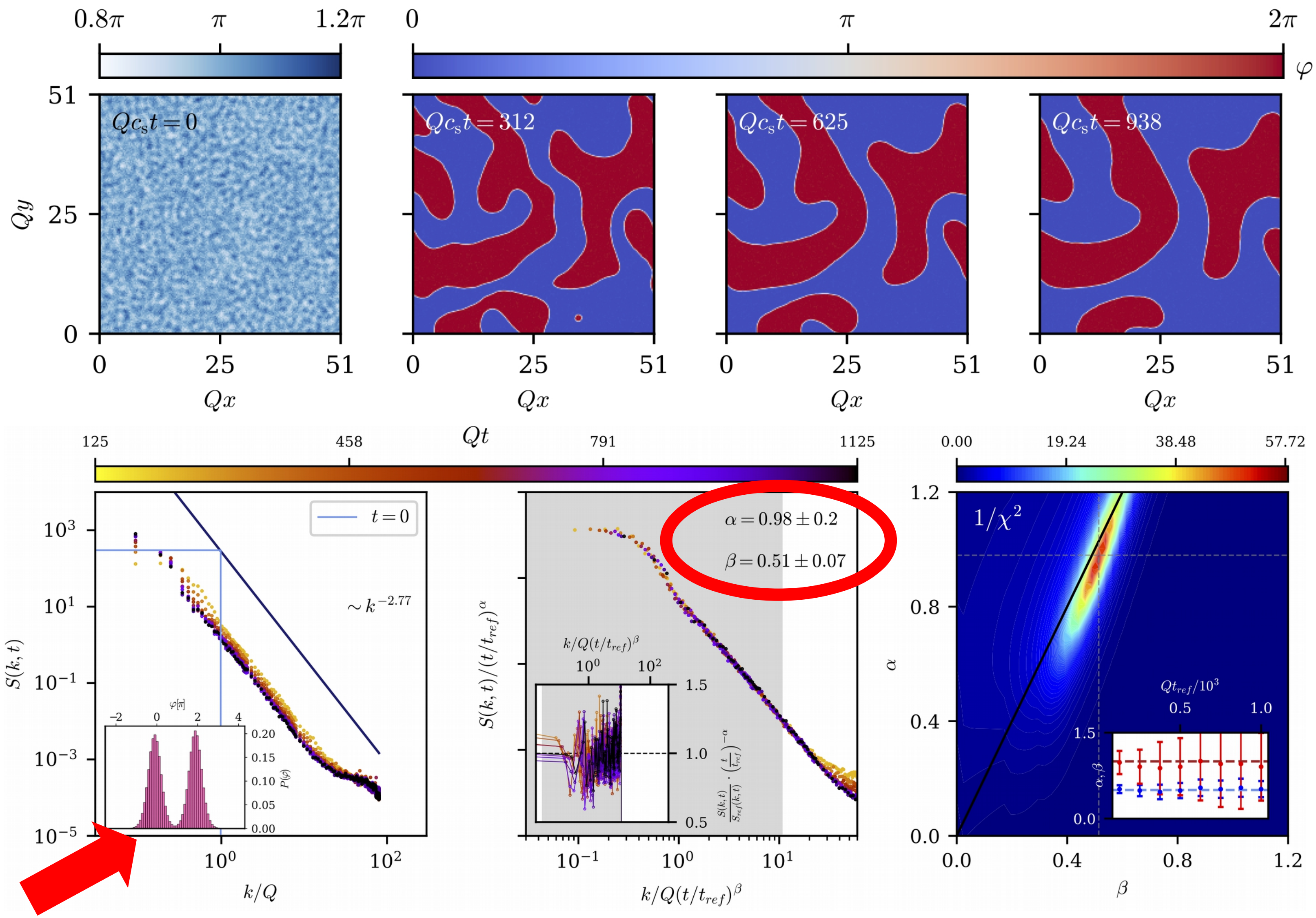




Scaling analysis: $\beta = 1/2$ or $\beta = 1/(2+d)$

[P. Heinen, A. Mikheev, TG, Phys. Rev. A **107**, 043303 (23) & arXiv:2212.01162; perturbative calc.: Tavora & A. Mitra (2013)]

Universal scaling from Sine^2-Gordon: 2D



I. Siovitz, A.-M. E. Glück, Y. Deller, A. Schmutz, F. Klein, H. Strobel, M.K. Oberthaler, TG, PRA **112,** 023304 (25); 2412.13986 [cond-mat.quant-gas]

Alexander Baum Martin Zboron Nils Becker Julian Mayr Florian Schmitt Niklas Rasch Wyatt Kirkby Hannes Köper TG

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Universal dynamics and non-thermal fixed points in quantum fluids far from equilibrium
A. N. Mikheev, I. Siovitz, TG,
EPJST **232**, 3393 (2023), arXiv:2304.12464 [cond-mat.quant-gas]

Universal dynamics of rogue waves in a quenched spinor Bose condensate
I. Siovitz, S. Lannig, Y. Deller, H. Strobel, M. K. Oberthaler, TG,
Phys. Rev. Lett. **131**, 183402; arXiv:2304.09293 [cond-mat.quant-gas]

Observation of two non-thermal fixed points for the same microscopic symmetry
S. Lannig, M. Prüfer, Y. Deller, I. Siovitz, J. Dreher, TG, H. Strobel, M. K. Oberthaler,
arXiv:2306.16497 [cond-mat.quant-gas]

Anomalous scaling at non-thermal fixed points of the sine-Gordon model
P. Heinen, A.N. Mikheev, TG
Phys. Rev. A **107**, 043303 (2019); arXiv:2212.01163 [cond-mat.quant-gas], 2212.01162 [cond-mat.quant-gas]

Low-energy effective theory of non-thermal fixed points in a multicomponent Bose gas
A. N. Mikheev, C.-M. Schmied, TG,
Phys. Rev. A **99**, 063622 (2019); arXiv:1807.10228 [cond-mat.quant-gas]

Prescaling in a far-from-equilibrium Bose gas
C.-M. Schmied, A. N. Mikheev, TG,
Phys. Rev. Lett. **122**: 170404 (2019); arXiv:1807.07514 [cond-mat.quant-gas]

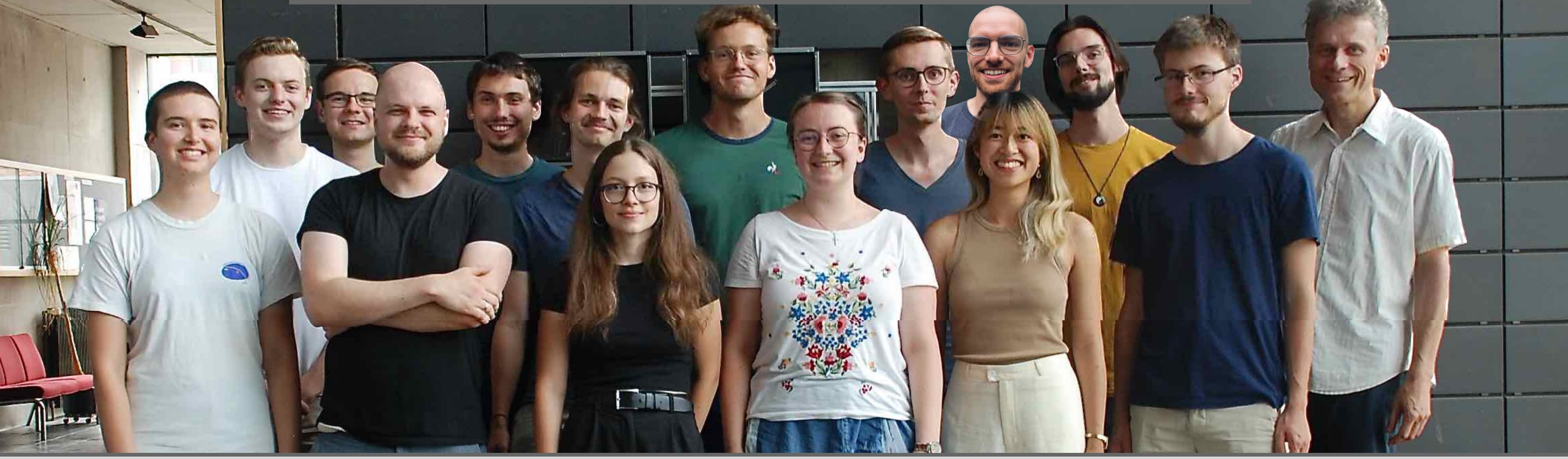
Kinetic theory of non-thermal fixed points in a Bose gas
I. Chantesana, A. Piñeiro Orioli, TG,
Phys. Rev. A **99**, 043620 (2019); arXiv:1801.09490 [cond-mat.quant-gas]

Prethermalization and universal dynamics in near-integrable quantum systems
T. Langen, TG, J. Schmiedmayer,
JSTAT **064009**, 2016; arXiv:1603.09385 [cond-mat.quant-gas]

The End

[& Alexander Flamm]

Alex Schmutz | Raphael Schäfer | Luise Heiland | Yannik Deller | Helmut Strobel | Markus Oberthaler



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Coulomb gas (of vortices) vs. Sine-Gordon (BKT)

2+0D euclidean Sine-Gordon partition sum:

$$\left\langle \mathrm{e}^{2\lambda\int_{\mathbf{x}}\cos(\theta_{\mathbf{x}})}\right\rangle_{\mathcal{L}_0} \sim \left\langle 1+\sum_{k=1}^{\infty}\frac{\lambda^k}{k!}\int_{\mathbf{x}_1,\dots,\mathbf{x}_k}\prod_{j=1}^{k}\left(\mathrm{e}^{\mathrm{i}\theta_{\mathbf{x}_j}}+\mathrm{e}^{-\mathrm{i}\theta_{\mathbf{x}_j}}\right)\right\rangle_{\mathcal{L}_0}$$

$$\mathcal{L}_0 \sim (\partial_{\mathbf{x}}\theta)^2$$

2+0D euclidean 2-point correlations:

$$\left\langle \mathrm{e}^{\mathrm{i}q_i\theta_{\mathbf{x}_i}}\mathrm{e}^{\mathrm{i}q_j\theta_{\mathbf{x}_j}}\right\rangle_{\mathcal{L}_0} \sim \left|\mathbf{x}_i-\mathbf{x}_j\right|^{-\eta q_i q_j} = \mathrm{e}^{-\eta q_i q_j \ln|\mathbf{x}_i-\mathbf{x}_j|}$$

↑ Coulomb potential

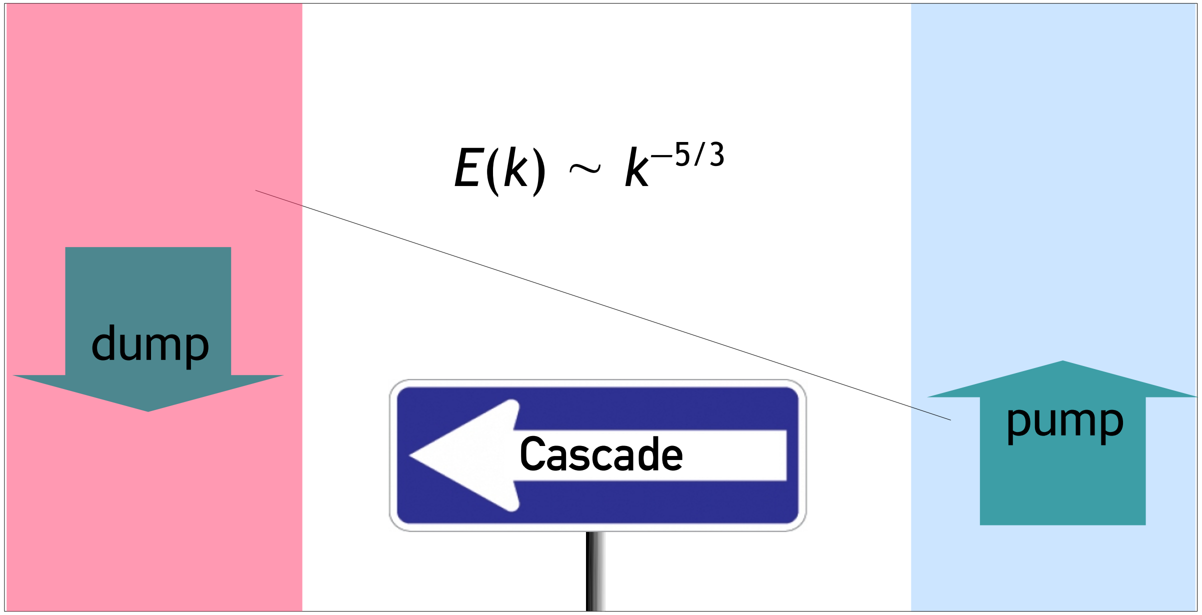
Berezinskii, Sov. Phys. JETP 32, 493 (71); 34, 610 (72); Kosterlitz & Thouless, J. Phys. C 6, 1181 (73)

Inverse Cascade

e.g. energy in 2D classical turbulence

— together with direct k^{-3} *enstrophy* ($\sim$ *vorticity density*) cascade —

log $E(k)$



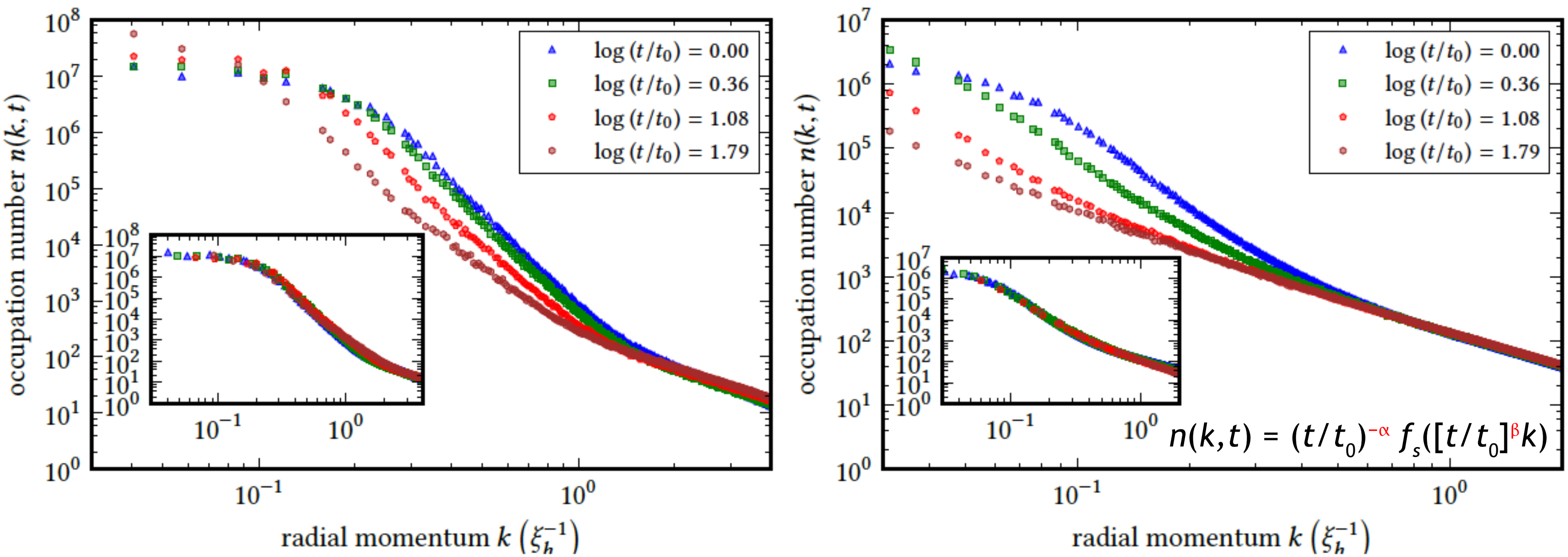
log k

Robert H. Kraichnan, *Inertial Ranges in Two-Dimensional Turbulence*. Phys. Fl. **10**, 1417 (1967)

Anomalous vs near-equilibrium NTFP

Coarsening evolution of single-particle spectrum

(TW-simulation from two different initial conditions)



$\alpha = 0.4_{\pm 0.05}$
$\beta = 0.2_{\pm 0.05}$
?

Analytic prediction:
(loop resummed self-energy)

$\alpha = 2 \cdot \beta$
$\beta = 1/z$

[Chantesana, Piñeiro-Orioli, TG, PRA 99 (19) 043620]

$\alpha = 1.0_{\pm 0.05}$
$\beta = 0.5_{\pm 0.05}$
$z = 2$

M. Karl, TG, NJP **19,** 093014 (2017), 1611.01163 [cond-mat.quant-gas]