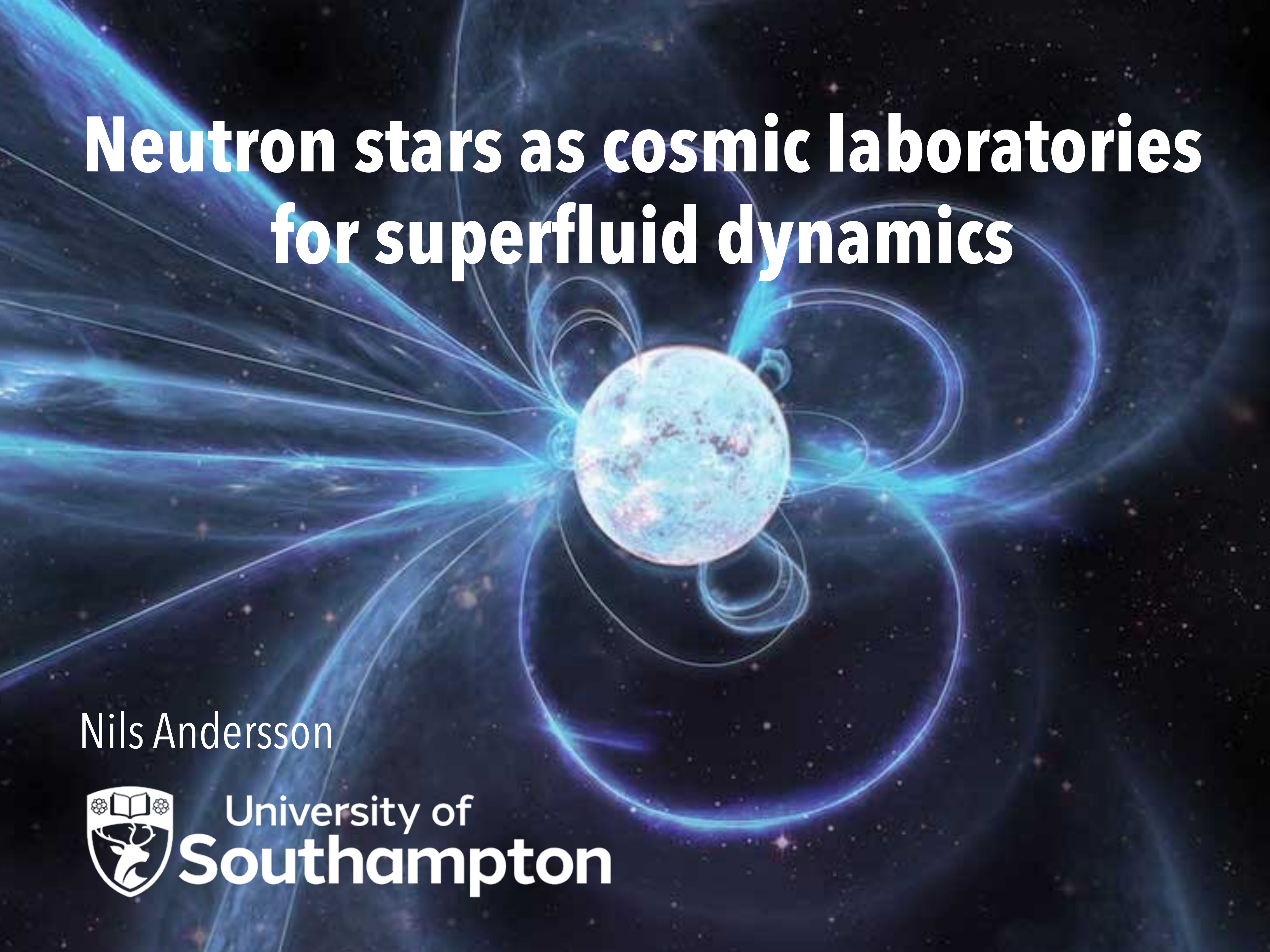

Neutron stars as cosmic laboratories for superfluid dynamics

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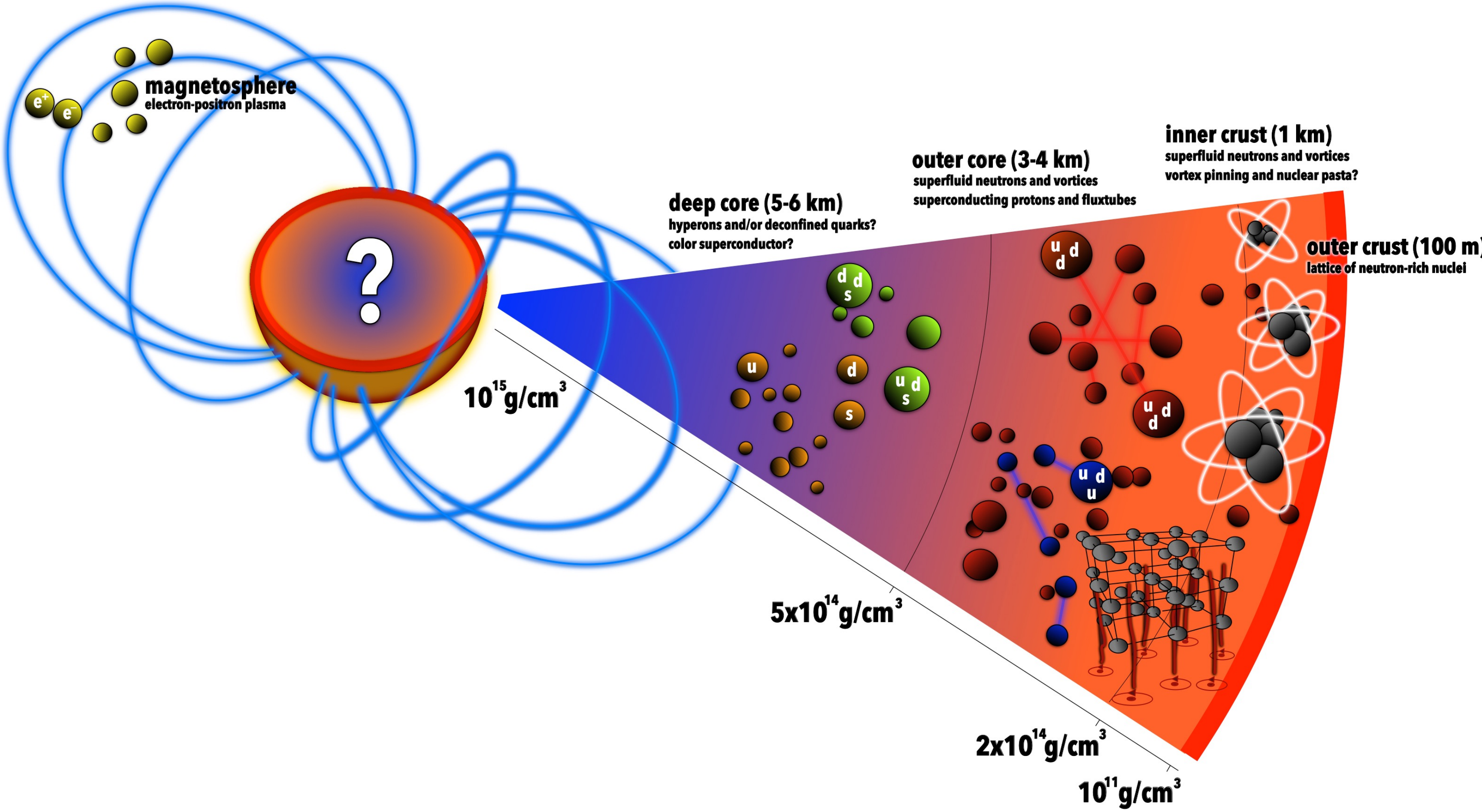


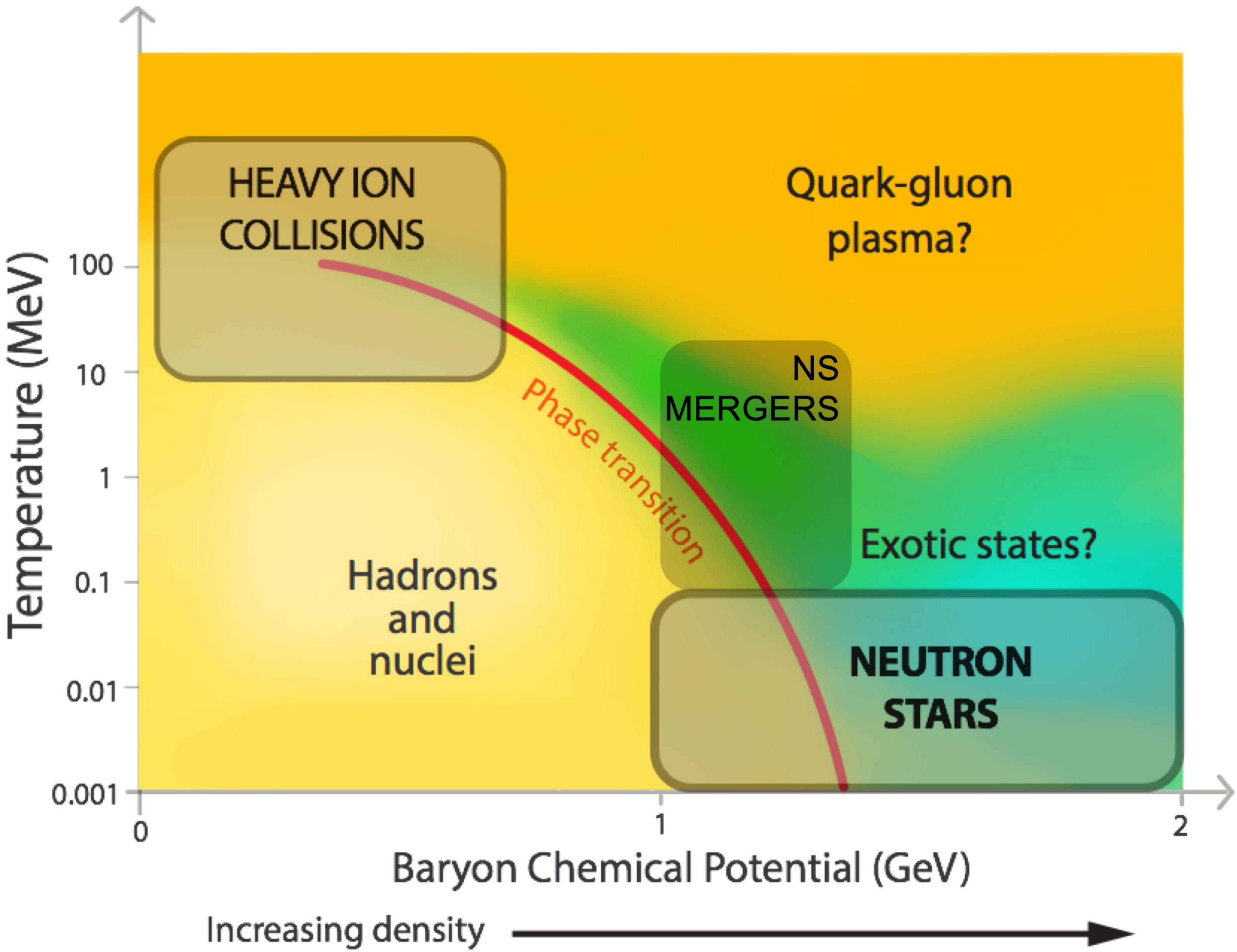
Four fundamental forces:

Gravity, holds the star together (local inertial frames…)

Electromagnetism, makes pulsars pulse (entrainment + vorticity)

Strong interaction, determines internal composition

Weak interaction, affects reaction rates (cooling and internal viscosity)


HEAVY ION COLLISIONS
Quark-gluon plasma?
NS MERGERS
Phase transition
Exotic states?
Hadrons and nuclei
NEUTRON STARS
Temperature (MeV)
100
10
1
0.1
0.01
0.001
0
1
2
Baryon Chemical Potential (GeV)
Increasing density

Seven states of matter:

Solid The outer kilometer of the star freezes to form an elastic crust.

Liquid The star's core remains fluid & accreted matter forms an ocean.

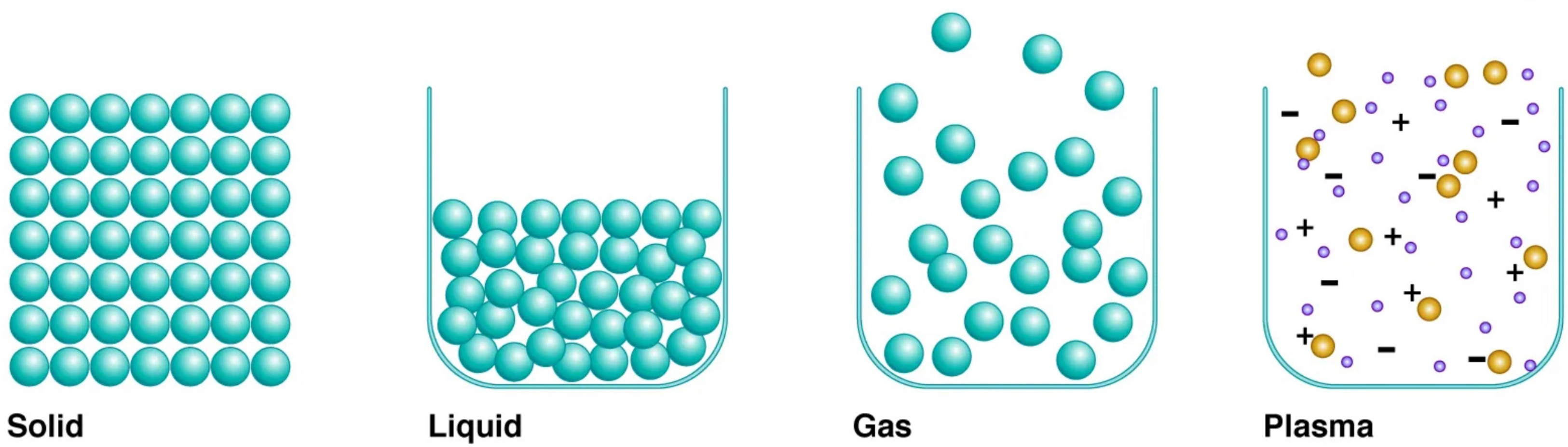
Gas There is a dilute atmosphere.

Plasma The star's exterior is dominated by an electron-positron plasma.

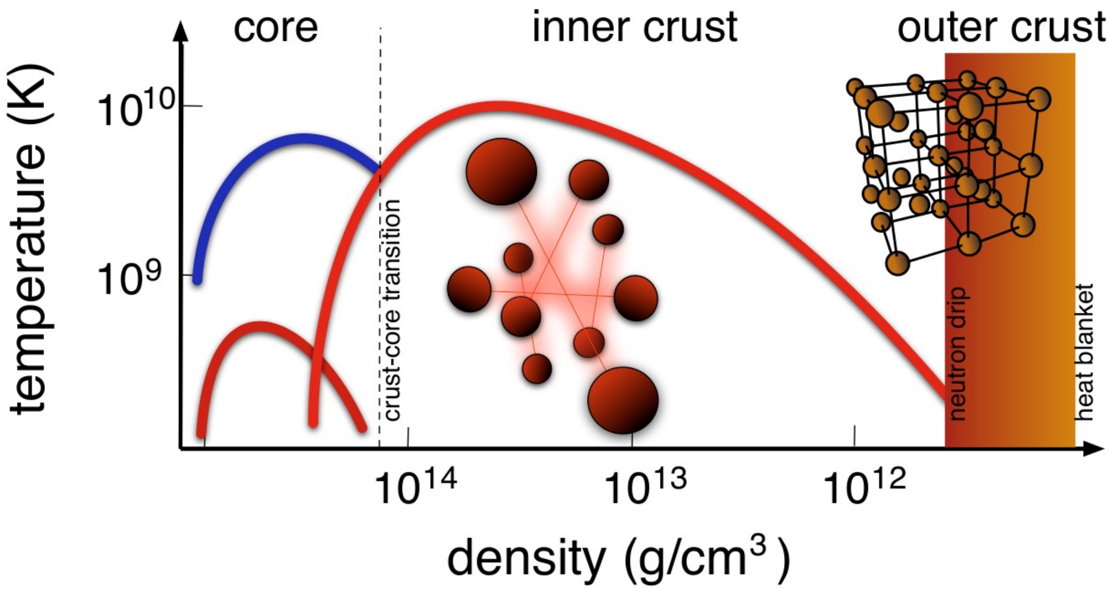
Quark-gluon plasma Neutrons and protons disintegrate.

Superfluid The star's core is cold enough for neutrons to be superfluid.

Superconductor At high densities, protons form a superconductor.



Mature neutron stars are “cold” (10^8K<< $T_{Fermi}=10^{12}$K) so they **should** be either solid or superfluid.



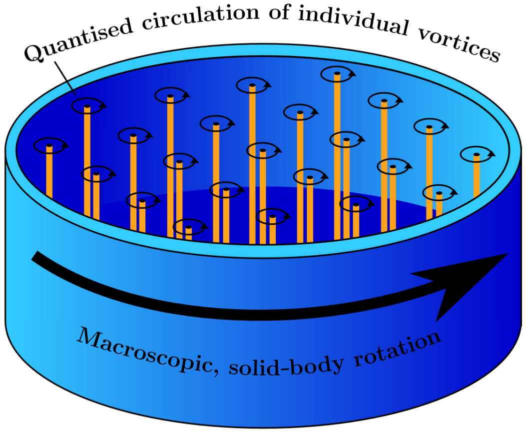
Since late 1950s, nuclear physics calculations indicate “BCS-like” pairing gaps for neutrons and protons.

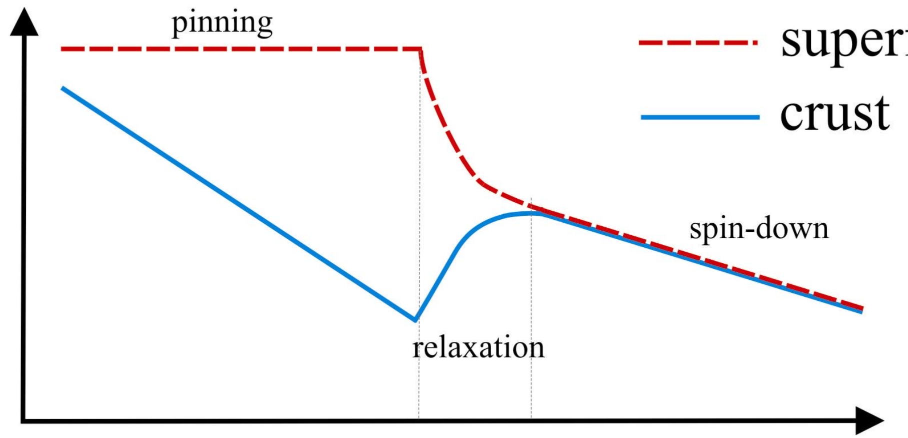
Observational evidence mainly from radio pulsar. Many pulsars are perfect ”clocks”, but in some cases the spin is not so regular. “Glitches” (=sudden spin-up events, observed in more than 100 systems) provide evidence for superfluidity.

Simple “story” but rich phenomenology - mechanism not understood.

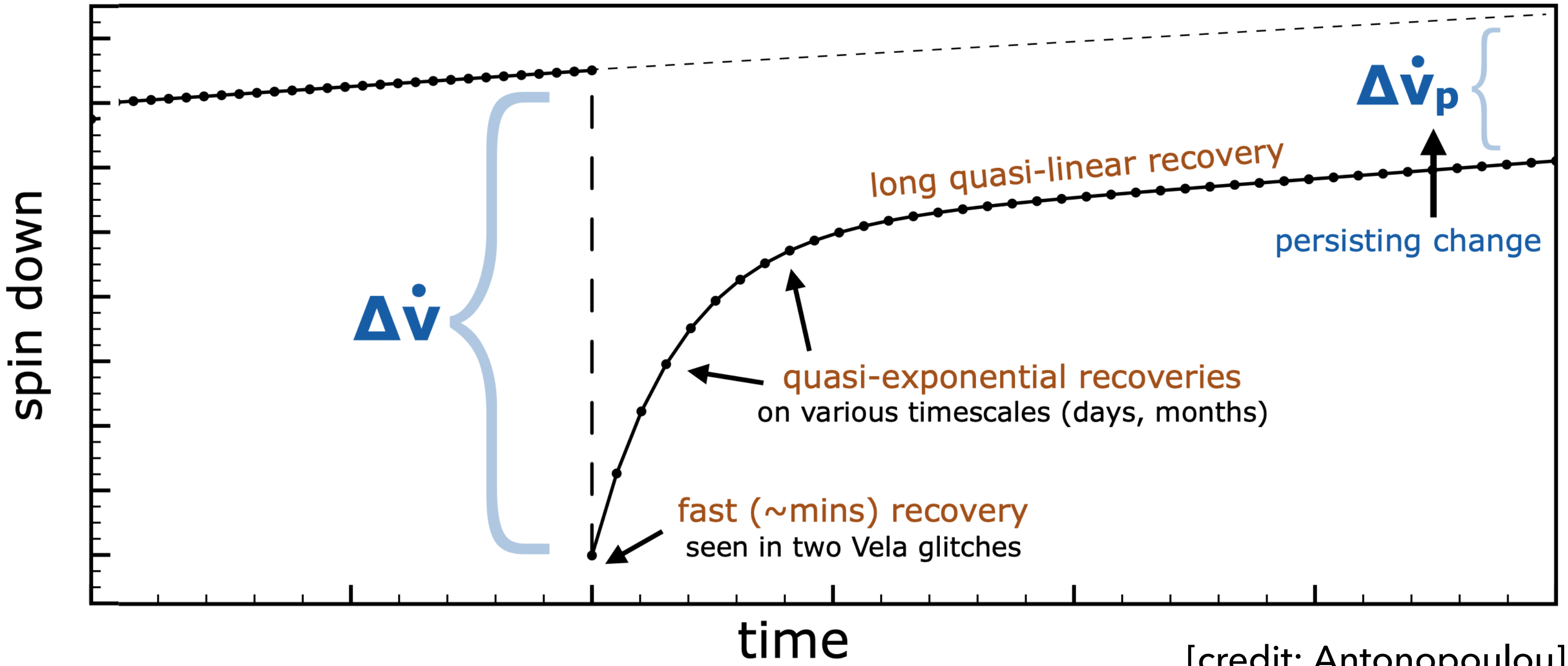
The common view is that glitches are a manifestation of the (singlet) superfluid that permeates the star's crust. The interaction with the crust nuclei is supposed to provide the required vortex pinning.

At the "cartoon level" a glitch can be explained by transfer of angular momentum from an internal superfluid component to the crust.

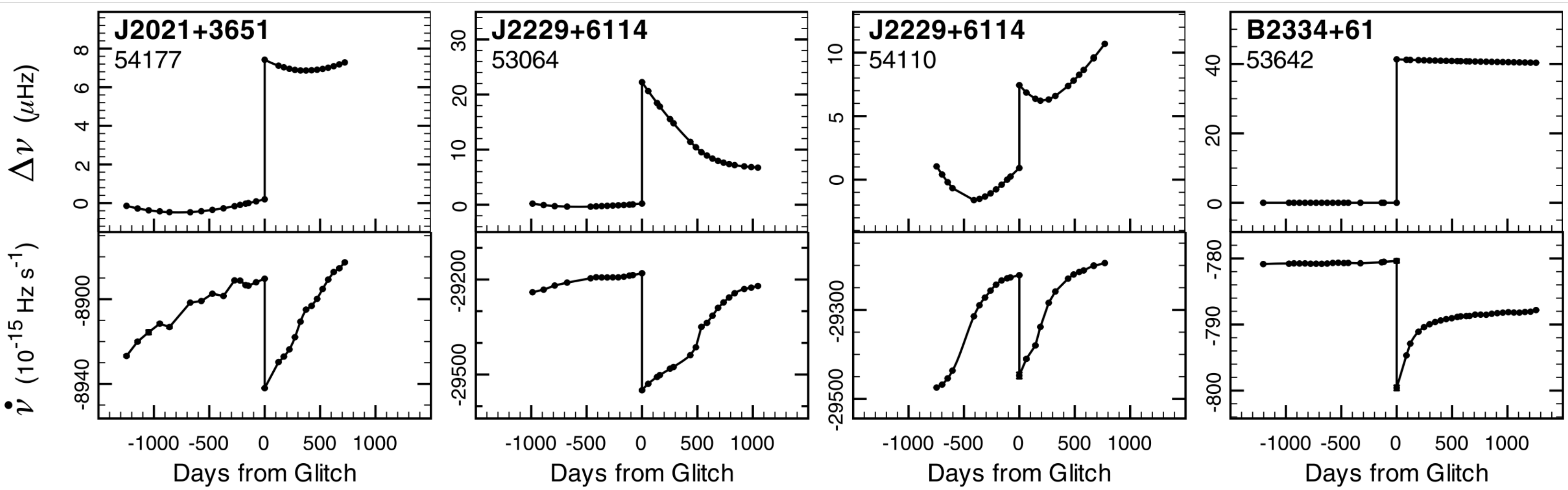




- the crust slows down due to magnetic braking
- the superfluid can only spin down if vortices move outwards
- if the vortices are pinned (to the crust), the superfluid lags behind
- at some critical level, a large number of vortices are released. They couple to the crust due to mutual friction and, as a result, the star is seen to spin up.



[credit: Antonopoulou]



[credit: Espinoza et al]

“Explanations” inspired by two-fluid model for superfluid Helium, but the motivation is a little bit different.

For Helium, the “orthodox” model separates the normal fluid, with density $\rho_{\rm N}$, from the superfluid, with density $\rho_{\rm S}$. However, these quantities only have **statistical meaning**. There are no specific atoms that can be identified with either density.

An alternative is to separate all atoms - after all, they are all paired! - with density ρ and velocity $v_{\rm n}^i$ from the excitations, which are then represented by the (massless) entropy (taken to be the second “fluid”).

Identifying the normal fluid velocity with the entropy flow, we then have the total mass current (=measurable)

$$\rho v_{\rm n}^i = \frac{\rho}{1-\varepsilon} v_{\rm S}^i - \frac{\varepsilon\rho}{1-\varepsilon} v_{\rm N}^i$$

where ε represents the **entropy entrainment** - a measure of how easy it is for the entropy to flow relative to the matter. In effect,

$$\rho_{\rm S} = \frac{\rho}{1-\varepsilon} \qquad \rho_{\rm N} = -\frac{\varepsilon\rho}{1-\varepsilon} \Longrightarrow \rho v_{\rm n}^i = \rho_{\rm S} v_{\rm S}^i + \rho_{\rm N} v_{\rm N}^i$$

which is the usual result.

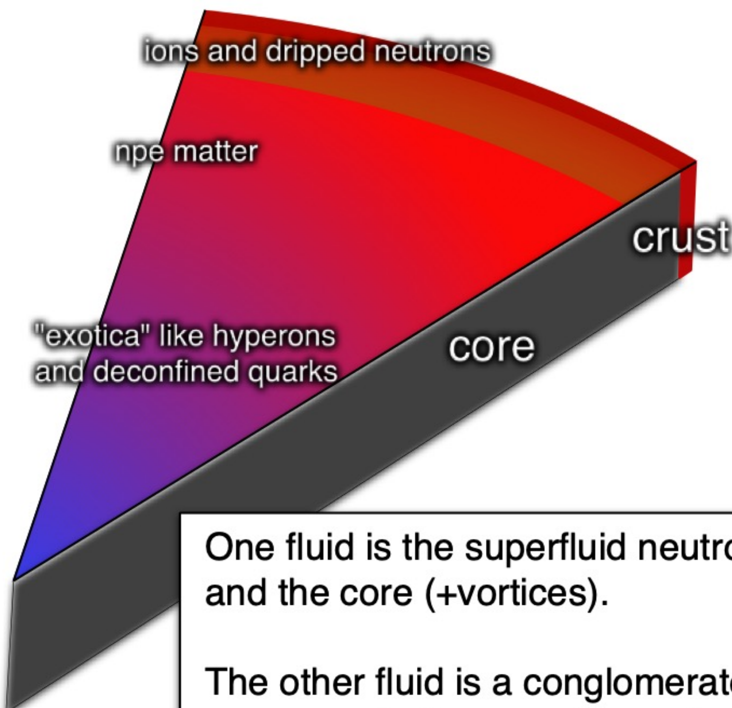
The “simplest” model for a superfluid neutron star assumes that:

– electrons/muons in the core are coupled (electromagnetically) to the protons on very short timescales

– vortices (and magnetic fluxtubes) are sufficiently dense that **smooth averaging** can be performed (”fat” fluid elements)

– entrainment, due to the strong interaction, plays key role

– at high densities, we may need to consider additional condensates; hyperon and/or quark superfluidity/superconductivity



One fluid is the superfluid neutrons in the crust and the core (+vortices).

The other fluid is a conglomerate of all charged components (ions, protons, electrons...)

A more realistic description involves (at least) four fluids…

Need to model the dynamics of two entrained/interacting fluids. Labelling the fluids by $\mathrm{x} = \mathrm{n}, \mathrm{p}$, we have two continuity equations

$$\partial_t n_\mathrm{x} + \nabla_i \left(n_\mathrm{x} v_\mathrm{x}^i \right) = 0$$

and two coupled Euler equations

$$\left(\partial_t + v_\mathrm{x}^j \nabla_j \right) p_i^\mathrm{x} + \nabla_i \left(\Phi + \mu_\mathrm{x} \right) + \varepsilon_\mathrm{x} w_j^\mathrm{yx} \nabla_i v_\mathrm{x}^j = f_i^\mathrm{x}$$

Here the relative velocity is $w_\mathrm{yx}^i = v_\mathrm{y}^i - v_\mathrm{x}^i$ and the momenta are given by

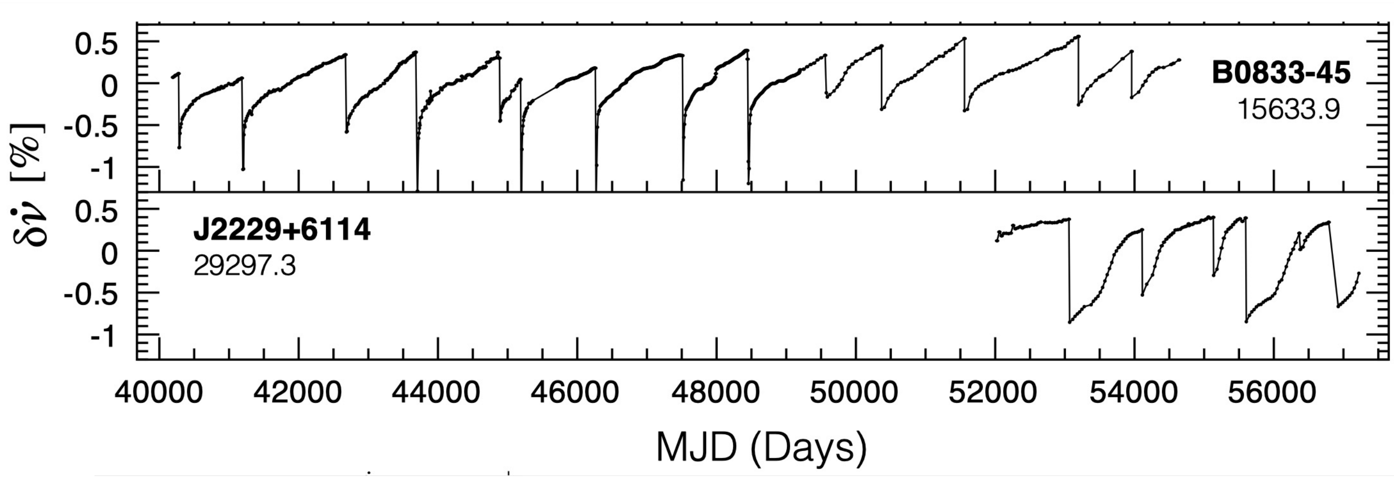
$$p_i^\mathrm{x} = v_i^\mathrm{x} + \varepsilon_\mathrm{x} w_i^\mathrm{yx}$$

This encodes the entrainment, due to which the velocity of each fluid does not have to be parallel to its momentum. Finally, the force f_i^x can be used to add in, for example, vortex mutual friction.

It is often useful to think of the entrainment as an “effective mass”;

$$\rho_\mathrm{p} \varepsilon_\mathrm{p} = n_\mathrm{p} \left(m_\mathrm{p} - m_\mathrm{p}^* \right)$$

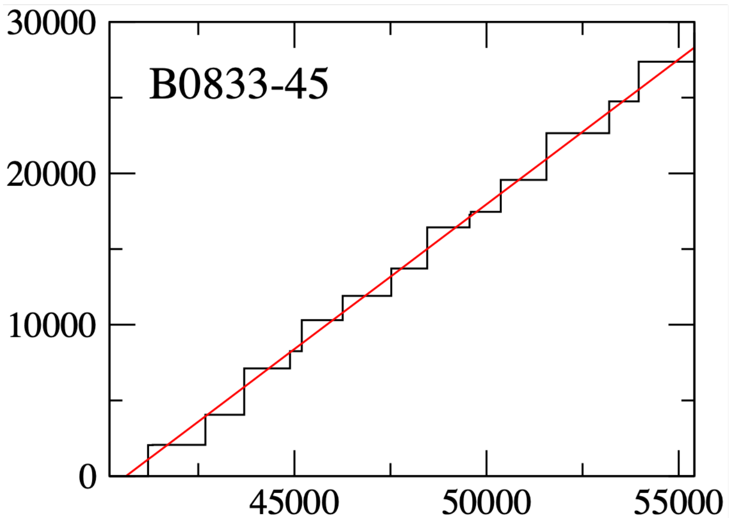
Note: The effective neutron mass may be very large in the inner crust of the star (due to Bragg scattering with the lattice of nuclei).



For systems that glitch regularly, one can **estimate the inertia** of the superfluid component.

Need to involve up to 2% of the total moment of inertia.

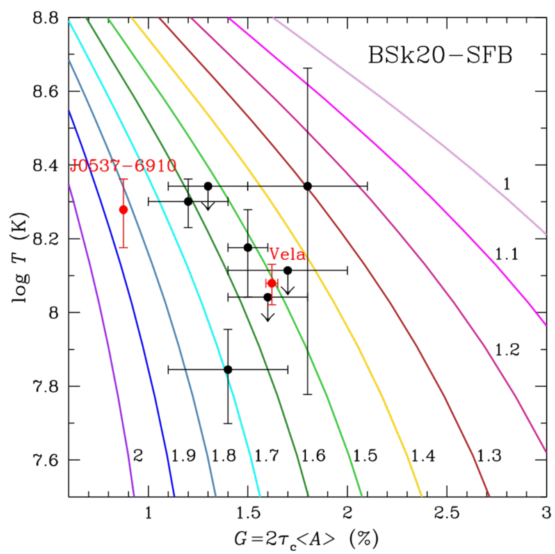
The crust superfluid model accords with observations as long as we do not worry about the entrainment.

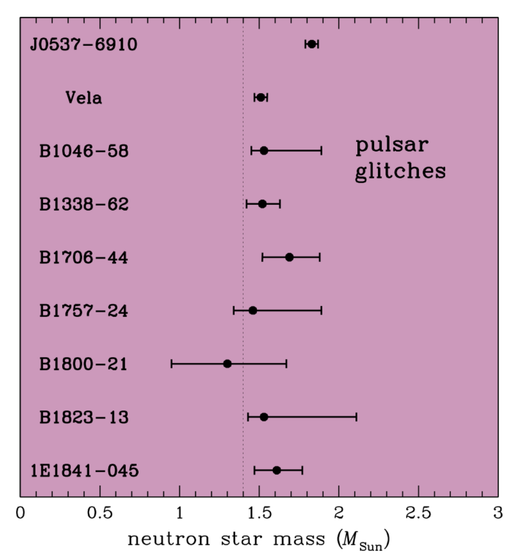


However… the **large effective neutron mass in the crust** lowers the effective superfluid moment of inertia by a factor of 5 or so.

This is "problematic":

1. A fraction of the core superfluid could be involved, but why would the glitches then be "the same size"?
2. The (singlet) pairing gap could lead to a smaller superfluid region, just large enough to explain the observations.
3. Lack of "precision": Need more accurate superfluid parameters.





Interestingly… may provide an opportunity to "weigh" isolated pulsars.

Need to consider dissipation.

In a superfluid, the presence of vortices leads to "mutual friction" (Hall+Vinen 1950s).

Standard form (for a straight vortex array);

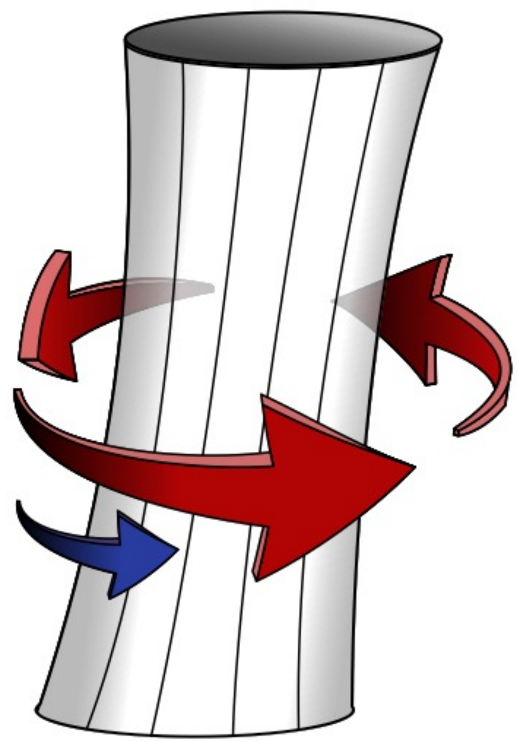
$$f_i^{\mathrm{mf}} = \frac{R}{1+R^2}\epsilon_{ijk}\hat{\omega}_{\mathrm{n}}^j \epsilon^{klm}\omega_l^{\mathrm{n}} w_m^{\mathrm{np}} + \frac{R^2}{1+R^2}\epsilon_{ijk}\omega_{\mathrm{n}}^j w_{\mathrm{np}}^k$$

where

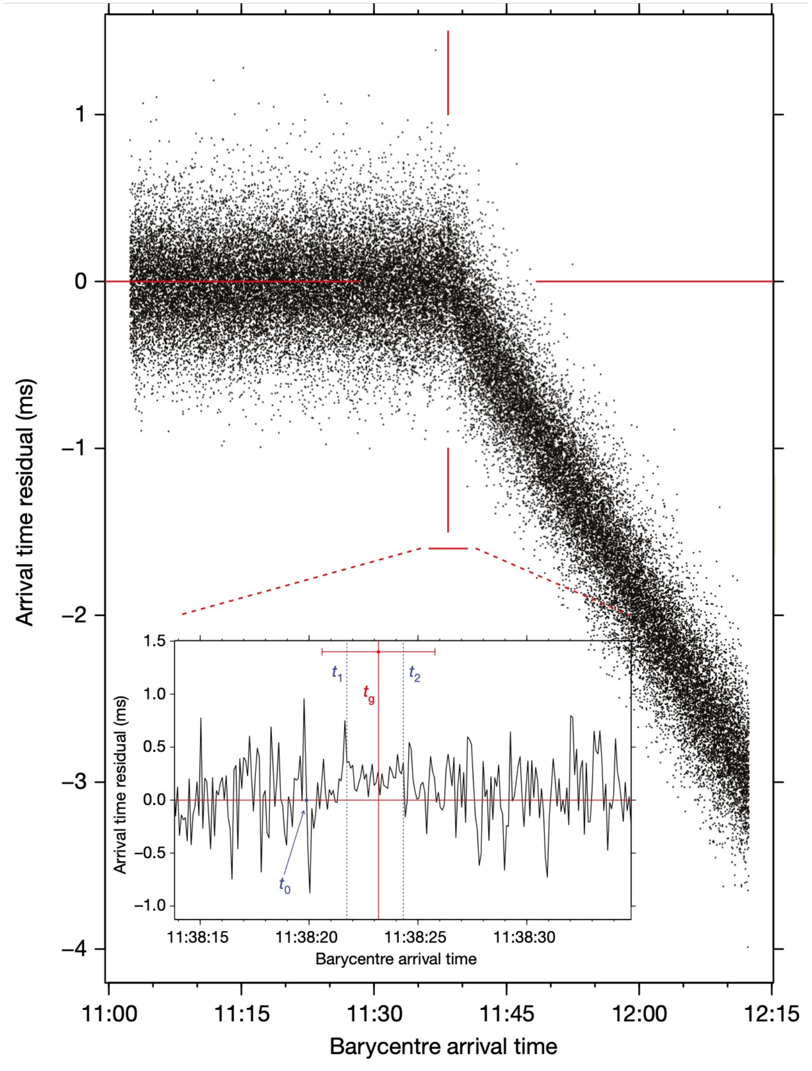
– electron scattering off (magnetised) vortices leads to

$$R \ll 1$$

– vortex/fluxtube interactions may lead to a stronger – velocity dependent – effect



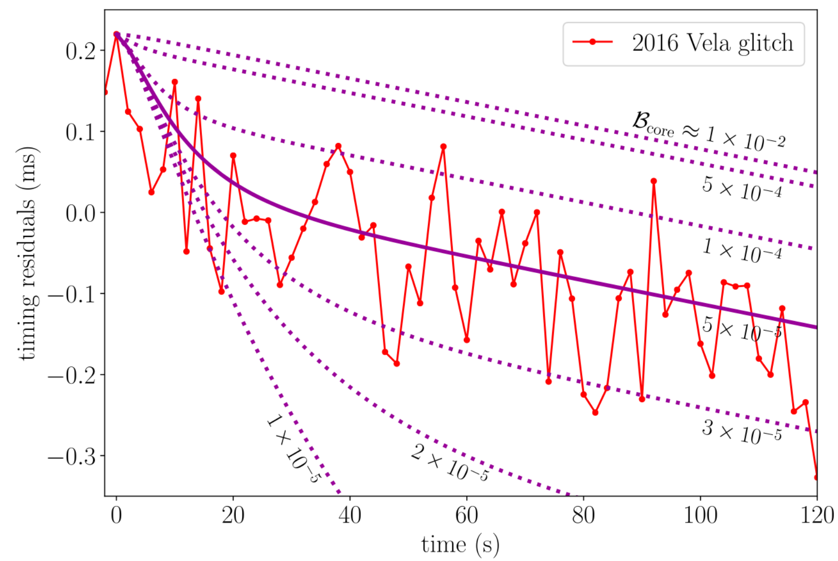
Mutual friction is key to modelling glitch dynamics as it dictates the timescales involved.



[credit: Palfreyman et al]

Example: The "resolved" Vela glitch from 2016.

The fast glitch rise (< 40s) and subsequent relaxation, provide an opportunity to contrast different models for the mutual friction.



[credit: Graber et al]

Rapidly driven systems only glitch once they exceed maximum pinning force.

- ”snow plow” model (Pizzochero+)
- instabilities (Glampedakis+)
- vortex “avalanches” may lead to scale independent behaviour (Melatos+)

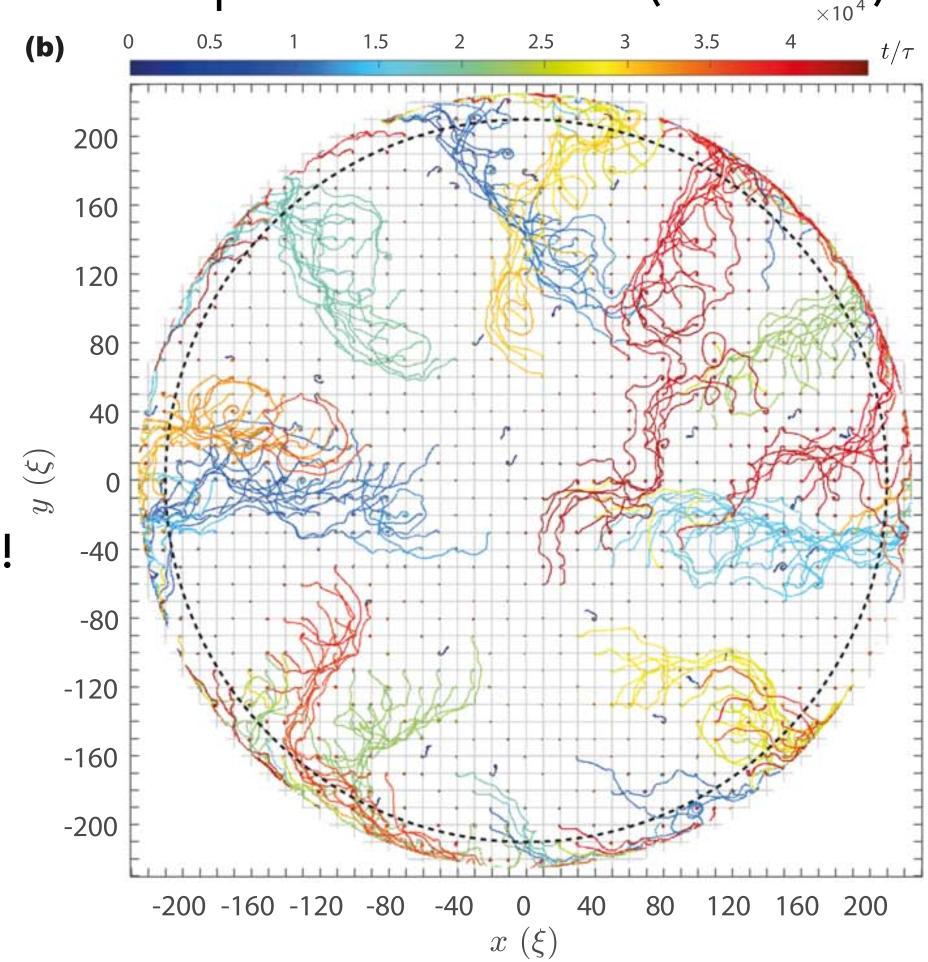
Simulations (mostly 2D) suggest glitch-like behaviour.

But… need to extrapolate by many, many, many… orders of magnitude (100 fm to 10 km!)

From the fluid dynamics perspective, we are often only simulating individual fluid elements!

Raises the issue of averaging/filtering of small-scale features in the fluid model.

Leads to issues similar to classic turbulence (and obviously, its quantum cousin).



[credit: Liu et al]

final remarks

After more than 50 years of observing pulsar glitches we have not made much progress on explaining the phenomenon.

We (think we) know that the explanation involves superfluidity-superconductivity and large-scale vortex dynamics.

Superfluidity is also relevant for neutron star cooling, free precession, seismology (gravitational waves/magnetar flares), magnetic field evolution…

We do not have clear answers to the what/why/how of the glitch problem, but there have been "useful" results;

— relevance of entrainment and the mutual friction damping;

— two-stream instability (onset of turbulence);

— pinning and impact of of core superconductivity

The main challenge relates to the range of scales - from the 100 fm coherence length of a vortex to the 10-100 m which can be resolved in numerical (relativity) simulations…

Laboratory systems provide insight, but… need "translation" to the neutron star context.

additional thoughts

Most current glitch “models” are essentially phenomenological and involve a number of simplifying assumptions.

For example; “axisymmetry” with a straight vortex array, averaging over the “height” and vortex pinning only in the inner crust. Temperature effects are ignored…

Each of these assumptions is likely to be wrong.

- Dynamics is likely to trigger vortex turbulence leading to tangles, impacting on the mutual friction (polarisation).
- Vortices will interact with fluxtubes in the superconducting region, which could influence both pinning and friction.
- Due the temperature dependence of the pairing gaps, there will **always** be a region near the transition density in which thermal effects must be accounted for. This ought to impact on vortex creation/destruction and angular momentum transfer.

additional thoughts

- Should perhaps worry about the nature of vortices in the singlet/triplet pairing regions and how they “move” across the boundary.
- Experiments suggest anti-vortices; what role could they play in the neutron star context?
- ...

At the end of the day, we have more questions than answers.

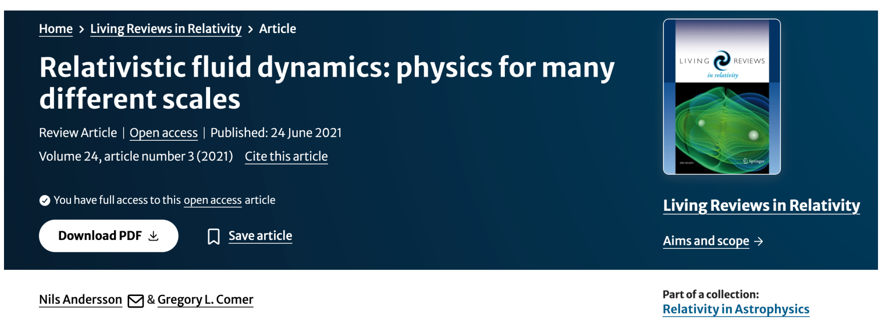
My own, not at all biased (!), view is that we need to;

- develop more detailed “multi-fluid” models based on large-scale averaging of the expect small-scale behaviour.
- keep asking if and how phenomenology observed in laboratory systems should be expected to be present also in neutron stars and how this might affect observations.

in memoriam

Gregory Lee Comer

11/4 1962 – 19/5 2026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