Tilted universes and the deceleration parameter

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Motivation

Observational considerations

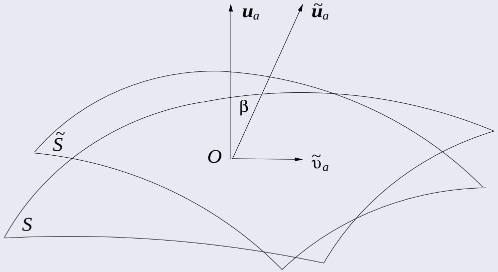
- Bulk peculiar motions appear to be the norm rather than the exception.
- Typical bulk-flow sizes and speeds are $\sim$100 Mpc and $\sim$100 km/sec.
- No “real” observer in the universe seems to follow the CMB frame.
- Relative motions can “contaminate” the observations.

Theoretical considerations

- Most theoretical cosmological studies bypass peculiar motions.
- The few and sparse studies of peculiar flows are typically Newtonian.

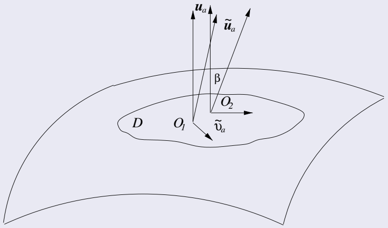
Method & aims

- Employ a “tilted”, almost-FRW universe.
- Use relativistic linear cosmological perturbation theory.
- Introduce CMB and bulk-flow observers, with u_a and $\tilde{u}_a$.



- Compare the mean kinematics of the two frames.
- Focus on the deceleration parameters.

Spacetime splitting



Bulk flows (D) in tilted universes

4-velocity boost

$$\tilde{u}_a = \gamma(u_a + \tilde{v}_a)\,,$$

where $u_a u^a = -1 = \tilde{u}_a \tilde{u}^a, \quad u_a \tilde{v}^a = 0$

and $\cosh\beta = \gamma$

When $\tilde{v}^2 = \tilde{v}_a \tilde{v}^a \ll 1$, $\cosh\beta = \gamma \simeq 1$.

Temporal and spatial derivative operators

Time derivatives:

$$^{\cdot} = u^a \nabla_a \qquad \textit{and} \qquad ' = \tilde{u}^a \nabla_a\,.$$

Spatial derivatives:

$$\mathrm{D}_a = h_a{}^b \nabla_b \qquad \textit{and} \qquad \tilde{\mathrm{D}}_a = \tilde{h}_a{}^b \nabla_b\,,$$

with $h_{ab} = g_{ab} + u_a u_b$ and $\tilde{h}_{ab} = g_{ab} + \tilde{u}_a \tilde{u}_b$.

Dynamic variables

CMB frame

$$T_{ab} = \rho u_a u_b + p h_{ab} + 2q_{(a}u_{b)} + \pi_{ab}\,,$$

where ρ is the density, p is the pressure, q_a is the flux, π_{ab} is the viscosity.

Tilted frame

$$T_{ab} = \tilde{\rho}\tilde{u}_a\tilde{u}_b + \tilde{p}\tilde{h}_{ab} + 2\tilde{q}_{(a}\tilde{u}_{b)} + \tilde{\pi}_{ab}\,.$$

Linear relations

On Friedmann backgrounds, to first approximation

$$\tilde{\rho} = \rho\,, \qquad \tilde{p} = p\,, \qquad \tilde{q}_a = q_a - (\rho + p)\tilde{v}_a \qquad \textit{and} \qquad \tilde{\pi}_{ab} = \pi_{ab}\,.$$

Only one observer "sees" the cosmic fluid as perfect (unless $p = -\rho$).

Assuming that $q_a = 0 = \pi_{ab}$ in the CMB frame and also setting $p = 0$, gives

$$\tilde{\rho} = \rho\,, \qquad \tilde{p} = 0\,, \qquad \tilde{q}_a = -\rho\tilde{v}_a \qquad \textit{and} \qquad \tilde{\pi}_{ab} = 0\,,$$

in the tilted coordinate system.

Conservation laws

Energy & momentum conservation

On an FRW background in the absence of pressure but in the presence of peculiar flows,

$$\tilde{\rho}' = -3H\tilde{\rho} - \tilde{\mathrm{D}}^a\tilde{q}_a \qquad \textit{and} \qquad \rho\tilde{A}_a = -\tilde{q}'_a - 4H\tilde{q}_a\,,$$

relative to the tilted frame.

The terms in red reflect the peculiar-flux contribution to the gravitational field.

(*i.e. fluxes gravitate*)

The 4-acceleration

The 3-gradient of the energy conservation combines with the momentum conservation to give

$$\tilde{A}_a = \frac{1}{3H}\,\tilde{\mathrm{D}}_a\tilde{\vartheta} - \frac{1}{3aH}\left(\tilde{\Delta}'_a + \tilde{\mathcal{Z}}_a\right) \neq 0\,, \qquad (1)$$

to linear order and in the coordinate system of the bulk-flow observers.

Here, $\tilde{\Delta}_a$ and $\tilde{\mathcal{Z}}_a$ monitor spatial inhomogeneities in the density of the matter and in the volume expansion of the universe respectively. Also, $\tilde{\vartheta} = \tilde{\mathrm{D}}^a\tilde{v}_a$ with $\tilde{\vartheta} \gtrless 0$.

The Raychaudhuri equation(s)

CMB frame

In the absence of pressure

$$\dot{\Theta} = -\frac{1}{3}\,\Theta^2 - \frac{1}{2}\,\rho \;\Rightarrow\; \frac{1}{3}\Theta^2 q = \frac{1}{2}\,\rho\,,$$

where $\Theta = \mathrm{D}^a u_a > 0$ and $q = -[1 + (3\dot{\Theta}/\Theta^2)]$ is the deceleration parameter.

Tilted frame

In the absence of pressure, but in the presence of peculiar motions

$$\tilde{\Theta}' = -\frac{1}{3}\,\tilde{\Theta}^2 - \frac{1}{2}\,\tilde{\rho} - \tilde{\mathrm{D}}^a\tilde{A}_a \;\Rightarrow\; \frac{1}{3}\tilde{\Theta}^2\tilde{q} = \frac{1}{2}\,\tilde{\rho} - \tilde{\mathrm{D}}^a\tilde{A}_a\,,$$

with $\tilde{\Theta} = \tilde{\mathrm{D}}^a\tilde{u}_a > 0$ and $\tilde{q} = -[1 + (3\tilde{\Theta}'/\tilde{\Theta}^2)]$.

Relating the deceleration parameters

Given that $\tilde{\Theta} = \Theta + \tilde{\vartheta}$ to linear order,

$$\tilde{q} = q - \frac{1}{3H^2}\,\tilde{\mathrm{D}}^a\tilde{A}_a\,.$$

Therefore, $\tilde{q} \neq q$ due to relative-motion effects.

Relative-motion corrections to $\tilde{q}$

Comparing the deceleration parameters

Employing the linear expression (1) of $\tilde{A}_a$, leads to

$$\tilde{q} = q - \frac{1}{9H^3}\,\tilde{\mathrm{D}}^2\tilde{\vartheta} + \frac{1}{9}\left(\frac{\lambda_H}{\lambda_K}\right)^2\left(\frac{\tilde{\Delta}'}{H} + \frac{\tilde{\mathcal{Z}}}{H}\right),$$

where $\tilde{\mathrm{D}}^2 = \tilde{\mathrm{D}}^a\tilde{\mathrm{D}}_a$, $\lambda_H = 1/H$ and $\lambda_K = a/|K|$ (with $K = \pm 1$ and $\lambda_H/\lambda_K \ll 1$).

The scale-dependence of $\tilde{q}$

The 3-D Laplacian ensures a scale-dependent correction term, so that

$$\tilde{q} = q + \frac{1}{9}\left(\frac{\lambda_H}{\lambda}\right)^2\frac{\tilde{\vartheta}}{H}, \qquad \textit{where} \qquad \tilde{\vartheta}/H \ll 1\,.$$

Qualitative results

- On large enough scales (with $\lambda \geq \lambda_H$), we find $\tilde{q} \to q$ (as expected).
- On sub-Hubble scales (with $\lambda \ll \lambda_H$) the correction term dominates at the critical length

$$\lambda_T = \sqrt{\frac{1}{9q}\frac{|\tilde{\vartheta}|}{H}}\,\lambda_H\,.$$

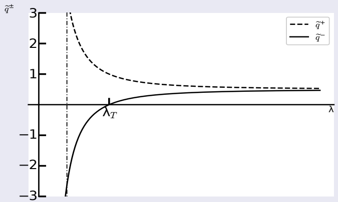
The "Transition Scale"

$\tilde{q} = \tilde{q}(\lambda_T)$

Employing the critical length λ_T, gives

$$\tilde{q} = q\left[1 \pm \left(\frac{\lambda_T}{\lambda}\right)^2\right],$$

where $\pm$ denotes locally expanding/contracting (i.e. with $\tilde{\vartheta} \gtrless 0$ respectively) bulk flows.



Having set $q = 1/2$

Local over-deceleration vs local acceleration

- When $\lambda < \lambda_T$ and $\tilde{\vartheta} > 0$, we have

$$\tilde{q} > 2q \qquad \rightarrow \qquad over-decelerated\ expansion\,.$$

- When $\lambda < \lambda_T$ and $\tilde{\vartheta} < 0$, we have

$$\tilde{q} < 0 \qquad \rightarrow \qquad accelerated\ expansion\,.$$

In the latter case λ_T marks the "Transition Scale", where $\tilde{q}$ turns negative.

Generalising the FRW background

Tilted almost-FRW universes with $p \neq 0$ and $\Omega \neq 1$

Assuming that $\Lambda = 0$, to linear order,

$$\tilde{q} = q + \frac{2}{3}\left[1 - \frac{3}{2}c_s^2 + \frac{1}{6}\left(\frac{\lambda_H}{\lambda}\right)^2\right]\frac{\tilde{\vartheta}}{H} + \frac{|1-\Omega|}{9(1+w)}\left[\frac{\tilde{\Delta}'}{H} - 3w\tilde{\Delta} + (1+w)\frac{\tilde{\mathcal{Z}}}{H}\right],$$

where $c_s^2 = \mathrm{d}p/\mathrm{d}\rho < 1$ and $w = p/\rho$ (with $-1 < w < 1$). Also, $0 < \Omega < 1$ in open FRW models and $\Omega > 1$ in those with closed spatial sections.

On sub-Hubble scales with $\lambda \ll \lambda_H$

To leading order,

$$\tilde{q} = q + \frac{1}{9}\left(\frac{\lambda_H}{\lambda}\right)^2\frac{\tilde{\vartheta}}{H},$$

unless $\Omega \gg 1$ (i.e. for unrealistically high positive curvature).

The above reproduces the tilted EdS result.

Generalising to anisotropic tilted universes

Tilted almost-Bianchi universes with $\sigma \neq 0$

Assuming zero pressure and setting $p = 0$ and $\Omega_K \ll 1$,

$$\tilde{q} = q + \frac{2}{3}\left[1 + \frac{1}{6}\left(\frac{\lambda_H}{\lambda}\right)^2 + \frac{1}{2}\,\frac{\zeta}{H}\right]\frac{\tilde{\vartheta}}{H}\,,$$

where ζ is the shear eigenvalue along $\tilde{v}_a$ (i.e. $\sigma_{ab}\tilde{v}^b = \zeta\tilde{v}_a$).

The ratio ζ/H measures the anisotropy of the Bianchi universe.

On sub-Hubble scales with $\lambda \ll \lambda_H$

To leading order,

$$\tilde{q} = q + \frac{1}{9}\left(\frac{\lambda_H}{\lambda}\right)^2\frac{\tilde{\vartheta}}{H}\,,$$

unless $\zeta/H \gg 1$ (i.e. for unrealistically high anisotropy).

The above also reproduces the tilted EdS result.

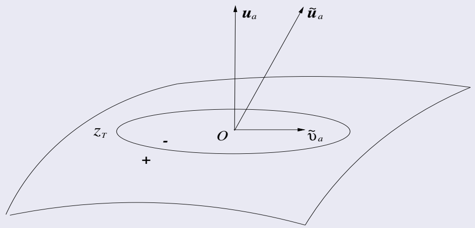
Estimating $\tilde{q}$ and λ_T

Estimating $\tilde{\vartheta}$

On average, within the bulk flow,

$$\tilde{\vartheta} \simeq \pm \frac{\langle \tilde{v} \rangle}{\lambda}\,,$$

where $\langle \tilde{v} \rangle$ is the mean bulk velocity and λ is the size of the bulk flow.



$\tilde{q} = 0$ at $z = z_T$ and $\tilde{q} \to q$ when $z \gg z_T$

Using the bulk-flow surveys

for $q = 1/2$ and $H_0 \simeq 70\ \mathrm{km/secMpc}$

Survey	λ (Mpc)	$\langle v \rangle$ (km/s)	$\tilde{q}^{(+)}$	$\tilde{q}^{(-)}$	λ_T (Mpc)
Nusser & Davis (2011)	280	260	$+1.01$	-0.01	282
Colin, et al (2011)	250	260	$+1.24$	-0.24	304
Scrimgeour, et al (2016)	200	240	$+1.81$	-0.81	323
Watkins, et al (2023)	250	400	$+1.48$	-0.48	350

Summary

Qualitative results

- Relative-motion effects can increase/decrease the local value of q.
- $\tilde{q} > q$ when $\tilde{\vartheta} > 0$.
- $\tilde{q} < q$ when $\tilde{\vartheta} < 0$.

Quantitative results

- Relative-motion effects introduce a characteristic length scale λ_T.
- For $\lambda < \lambda_T$ and $\tilde{\vartheta} > 0$, we have "local" over-deceleration ($\tilde{q} > 2q$).
- For $\lambda < \lambda_T$ and $\tilde{\vartheta} < 0$, we have "local" acceleration ($\tilde{q} < 0$).

The effect is "local", but the affected scales are large enough ($\sim$ few hundred Mpc) to make it look as a recent global event.

The bulk-flow contraction appears as acceleration of the surrounding universe.

Outlook

A possible scenario

- If there is no natural bias for expanding, or contracting, bulk flows on cosmological scales, there is 50% chance of living in one of them.
- Nearly half the observers in the universe will believe that the cosmos is over-decelerated.
- The other half will think that their universe is under-decelerated, or even accelerated in some cases.

Main predictions

- The $\tilde{q}$-profile: $\tilde{q} > 0$ when $z > z_T$ and $\tilde{q} < 0$ when $z < z_T$.
- The sky-distribution of $\tilde{q}$ should contain a dipolar anisotropy.
- The $\tilde{q}$-dipole axis should lie fairly close to that of the CMB dipole.
- The magnitude of the dipole should drop with increasing scale/redshift.

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Anisotropies in the $\tilde{q}$-distribution

The deceleration tensor

Anisotropies in the universal deceleration/acceleration are monitored by the 3-D tensor

$$Q_{ab} = -\left(h_{ab} + \frac{9}{\Theta^2}\, h_a{}^c h_b{}^d \dot{\Theta}_{cd}\right), \qquad \text{with} \qquad Q_a{}^a = 3q\,,$$

where q is the (scalar) deceleration parameter and

$$\Theta_{ab} = \mathrm{D}_b u_a = \frac{1}{3}\,\Theta h_{ab} + \sigma_{ab} + \omega_{ab}\,, \qquad \text{with} \qquad \Theta_a{}^a = \Theta = 3H\,,$$

is the expansion tensor.

Selecting a spatial direction

The deceleration/acceleration rate along any given spatial direction (n_a) is given by

$$Q_{ab}n^a n^b = q + \frac{9}{\Theta^2}\,\dot{\sigma}_{ab}n^a n^b\,.$$

In Friedmann models

$$Q_{ab} = qh_{ab} \qquad \text{and} \qquad Q_{ab}n^a n^b = q\,,$$

in all directions (as expected).

Apparent dipole in the q-distribution

The deceleration tensor(s) in tilted universes

To linear order, comparing between the tilted and the CMB frames,

$$\begin{aligned} \tilde{Q}_{ab}n^a n^b &= Q_{ab}n^a n^b + \frac{1}{H}n^a\tilde{\mathrm{D}}_a\left(\tilde{v}_b n^b\right) - \frac{1}{H^2}n^a\tilde{\mathrm{D}}_a\left(\tilde{v}'_b n^b\right) \\ &= q + \frac{1}{H}n^a\tilde{\mathrm{D}}_a\left(\tilde{v}\cos\phi\right) - \frac{1}{H^2}n^a\tilde{\mathrm{D}}_a\left(\tilde{v}'\cos\psi\right)\,, \end{aligned}$$

where ϕ is the angle between $\tilde{v}_a$ and n_a and ψ between $\tilde{v}'_a$ and n_a.

Doppler-like dipole due to relative motion.

In the simplest case,

$$\tilde{Q}_{ab}n^a n^b = q + \frac{1}{H}n^a\tilde{\mathrm{D}}_a\left(\tilde{v}\cos\phi\right)\,,$$

Therefore,

$$\tilde{Q}_{ab}n^a n^b = q \pm \frac{1}{H}n^a\tilde{\mathrm{D}}_a\tilde{v}\,,$$

when $\tilde{v}_a \uparrow\uparrow n_a$ (+) and $\tilde{v}_a \uparrow\downarrow n_a$ (-).

Dipolar anisotropy along n_a.