Pondering Cosmic Rays (among other things) with Subir

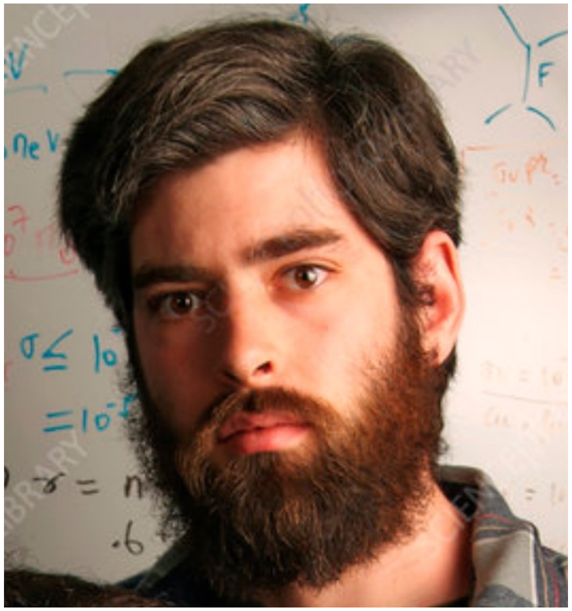
Dan Hooper – Fermilab and the University of Chicago

SubirFest

Oxford University

September 11, 2023

Astro-Particle Physics in 2003



Me (Circa 2003)

- In 2003, Astro-particle physics was still something of a fledgling field
- HESS, VERITAS, MAGIC, IceCube, Fermi, PAMELA, etc. were all still years away
- These days, most particle theorists have written at least a few papers on dark matter or other topics in particle-cosmology; in 2003, working at the interface of particle-astrophysics was comparatively rare
- Around this time, Subir was writing papers with titles like:

No cosmological domain wall problem for weakly coupled fields

POSSIBLE ASTROPHYSICAL PROBES OF QUANTUM GRAVITY

Low scale inflation

The high energy cosmic ray spectrum from relic particle decay

- This was a big part of why I decided to go to Oxford for my first postdoc

Subir the Contrarian

The conventional wisdom regarding ultra-high energy cosmic rays (2003):

1) UHECRs were protons

2) UHECRs exceeding the GZK cutoff had been observed

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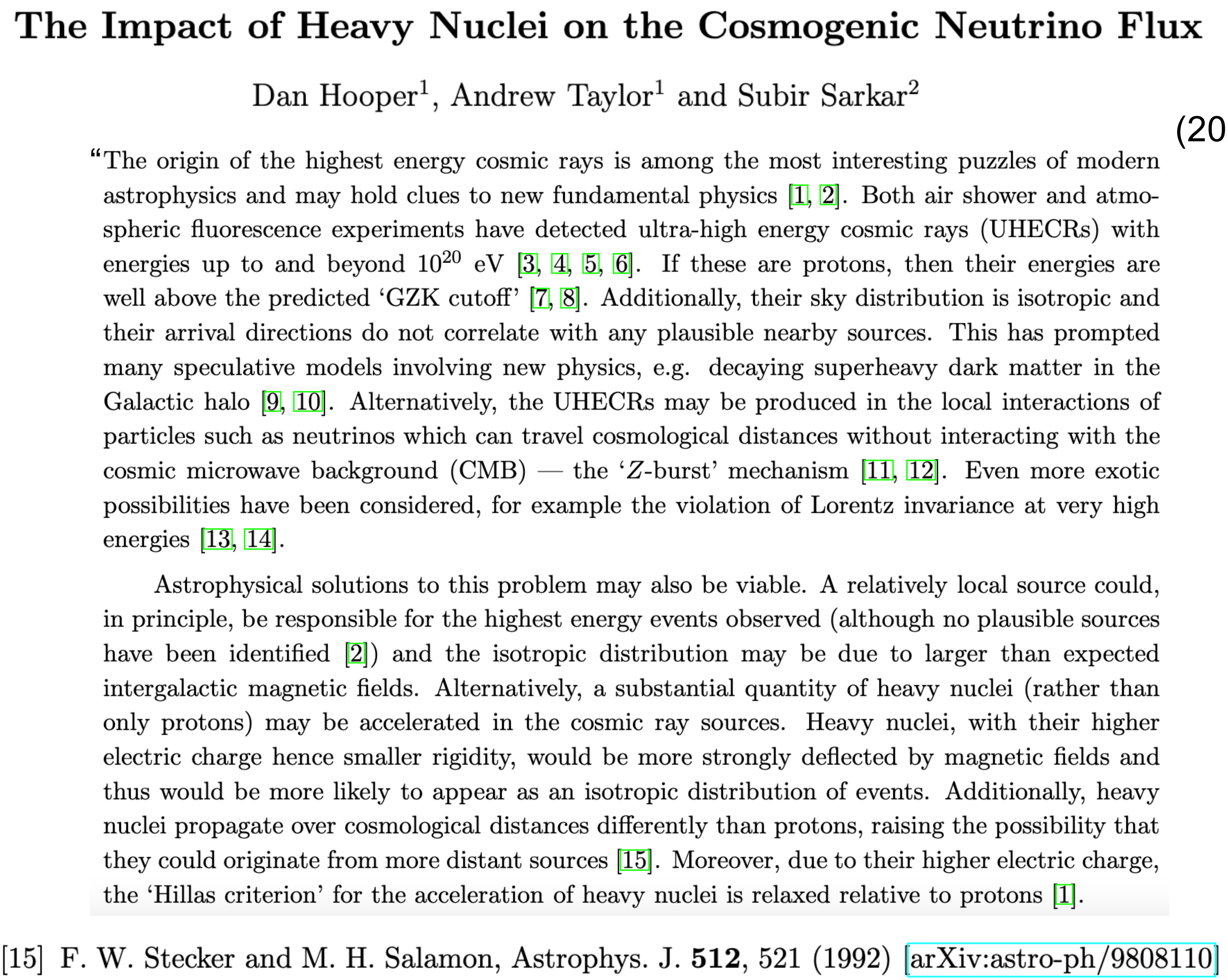
Why was this conventional wisdom so blindly accepted at the time?



The Impact of Heavy Nuclei on the Cosmogenic Neutrino Flux

Dan Hooper[1], Andrew Taylor[1] and Subir Sarkar[2]

(2004)

"The origin of the highest energy cosmic rays is among the most interesting puzzles of modern astrophysics and may hold clues to new fundamental physics [1, 2]. Both air shower and atmospheric fluorescence experiments have detected ultra-high energy cosmic rays (UHECRs) with energies up to and beyond 10^{20} eV [3, 4, 5, 6]. If these are protons, then their energies are well above the predicted 'GZK cutoff' [7, 8]. Additionally, their sky distribution is isotropic and their arrival directions do not correlate with any plausible nearby sources. This has prompted many speculative models involving new physics, e.g. decaying superheavy dark matter in the Galactic halo [9, 10]. Alternatively, the UHECRs may be produced in the local interactions of particles such as neutrinos which can travel cosmological distances without interacting with the cosmic microwave background (CMB) — the 'Z-burst' mechanism [11, 12]. Even more exotic possibilities have been considered, for example the violation of Lorentz invariance at very high energies [13, 14].

Astrophysical solutions to this problem may also be viable. A relatively local source could, in principle, be responsible for the highest energy events observed (although no plausible sources have been identified [2]) and the isotropic distribution may be due to larger than expected intergalactic magnetic fields. Alternatively, a substantial quantity of heavy nuclei (rather than only protons) may be accelerated in the cosmic ray sources. Heavy nuclei, with their higher electric charge hence smaller rigidity, would be more strongly deflected by magnetic fields and thus would be more likely to appear as an isotropic distribution of events. Additionally, heavy nuclei propagate over cosmological distances differently than protons, raising the possibility that they could originate from more distant sources [15]. Moreover, due to their higher electric charge, the 'Hillas criterion' for the acceleration of heavy nuclei is relaxed relative to protons [1].

[15] F. W. Stecker and M. H. Salamon, Astrophys. J. **512**, 521 (1992) [arXiv:astro-ph/9808110]

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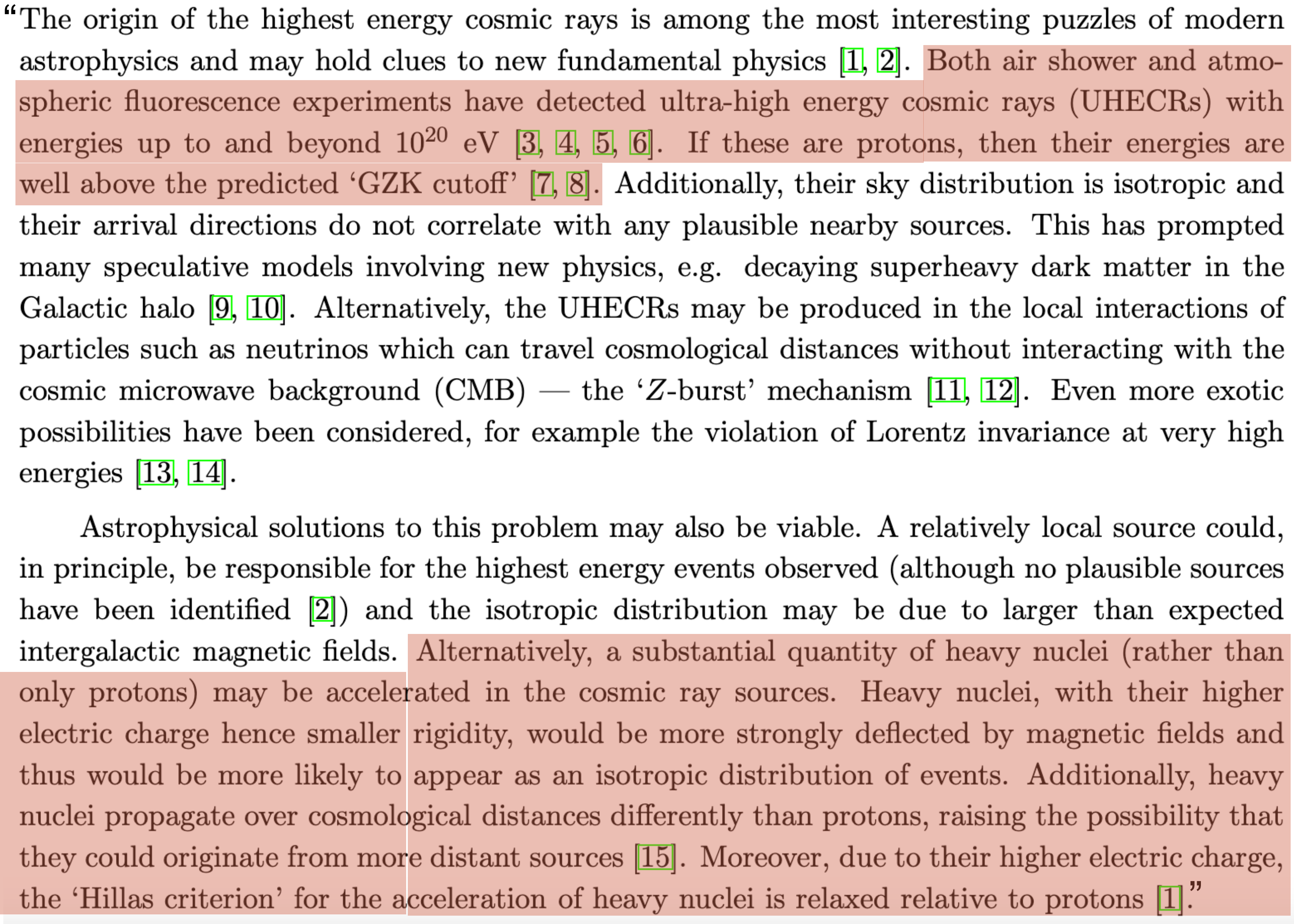

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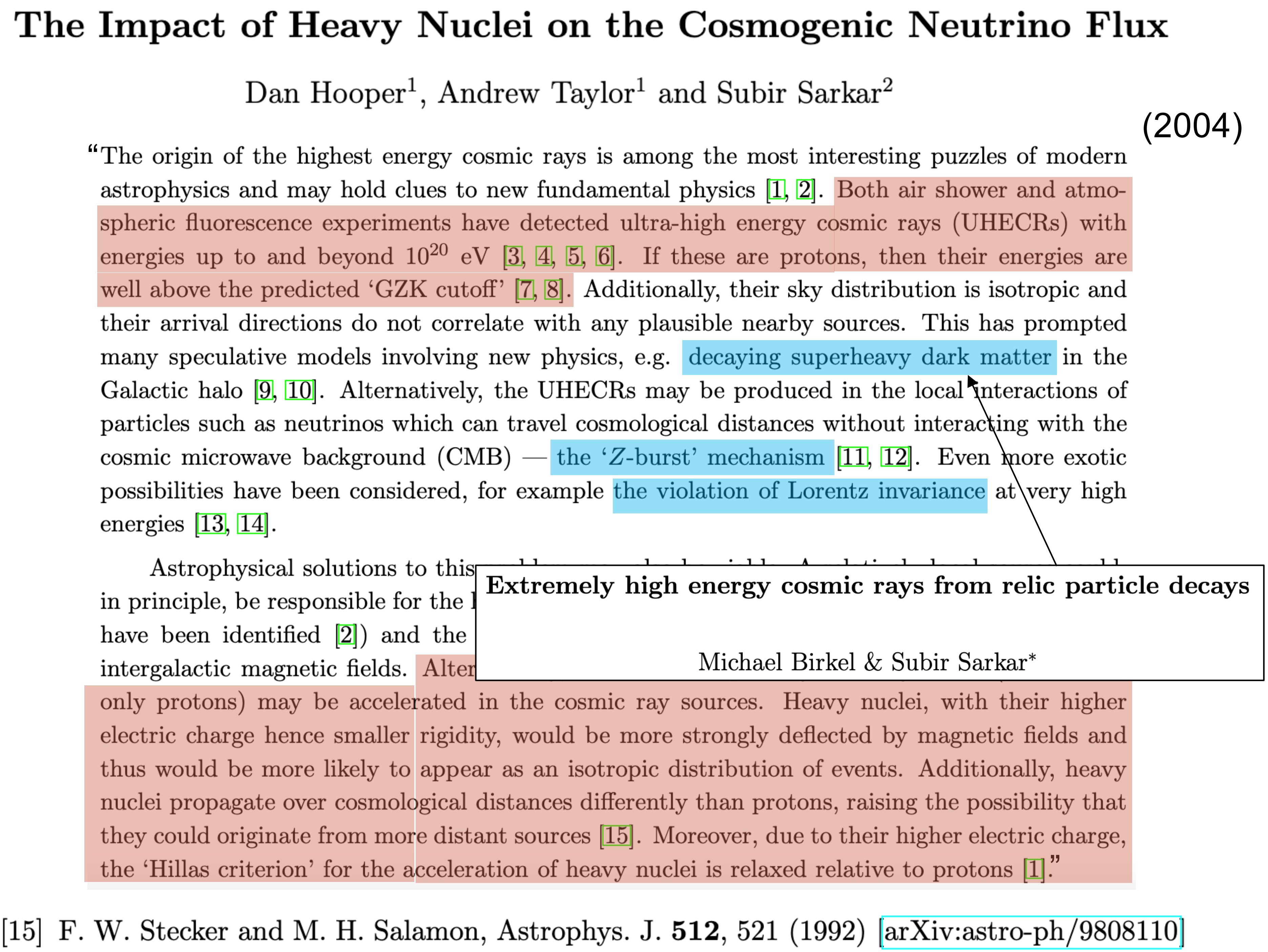

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Extremely high energy cosmic rays from relic particle decays

Michael Birkel & Subir Sarkar*

[15] F. W. Stecker and M. H. Salamon, Astrophys. J. **512**, 521 (1992) [arXiv:astro-ph/9808110]

Cosmogenic Neutrinos

- Because of scattering with the CMB, our universe is opaque to protons with more energy than:

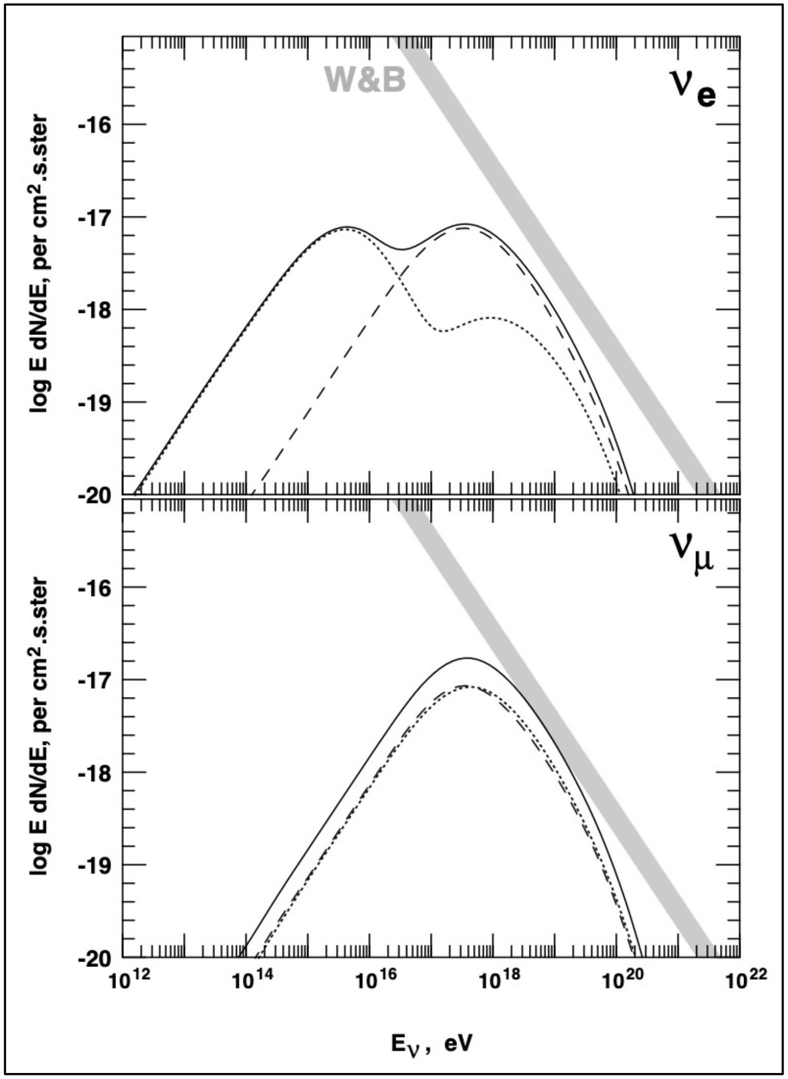
$$E_p > \frac{(m_\Delta^2 - m_p^2)}{4\epsilon_{\rm CMB}}$$

$$\sim 2.5 \times 10^{11}\,{\rm GeV} \times \left(\frac{6.3 \times 10^{-4}\,{\rm eV}}{\epsilon_{\rm CMB}}\right)$$

$$p + \gamma \rightarrow \Delta^+ \rightarrow \pi^+ + n$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ \bar{\nu}_\mu \nu_e \nu_\mu$$

- If the ultra-high energy cosmic rays were mostly protons, this would lead to a large and “guaranteed” flux of $\sim 10^8 - 10^9$ GeV neutrinos
- This “guarantee” was often cited in the context of IceCube, which was projected to be sensitive to this predicted flux



Engle, Seckel, Stanev (2001)

The Impact of Heavy Nuclei on the Cosmogenic Neutrino Flux

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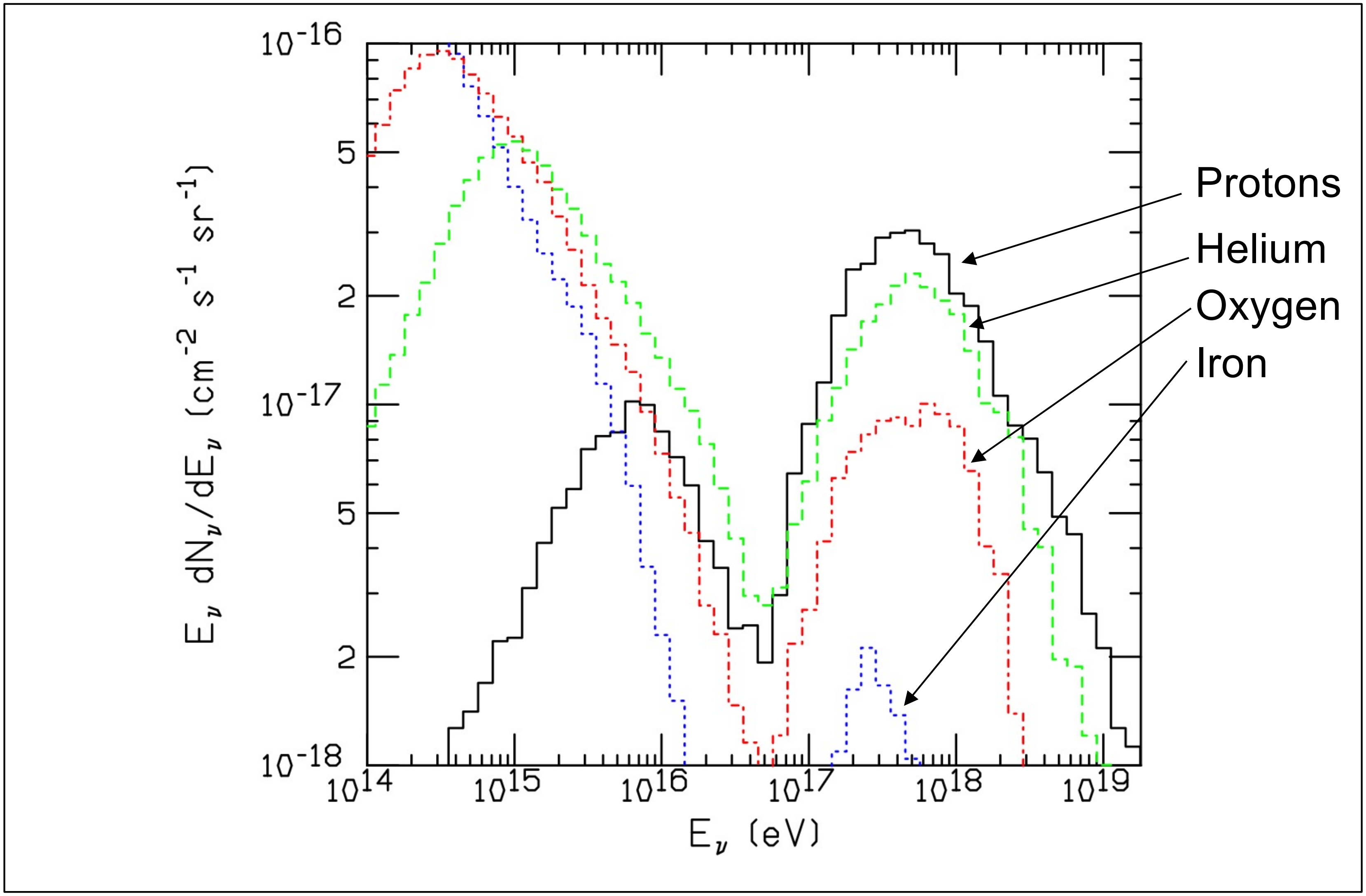
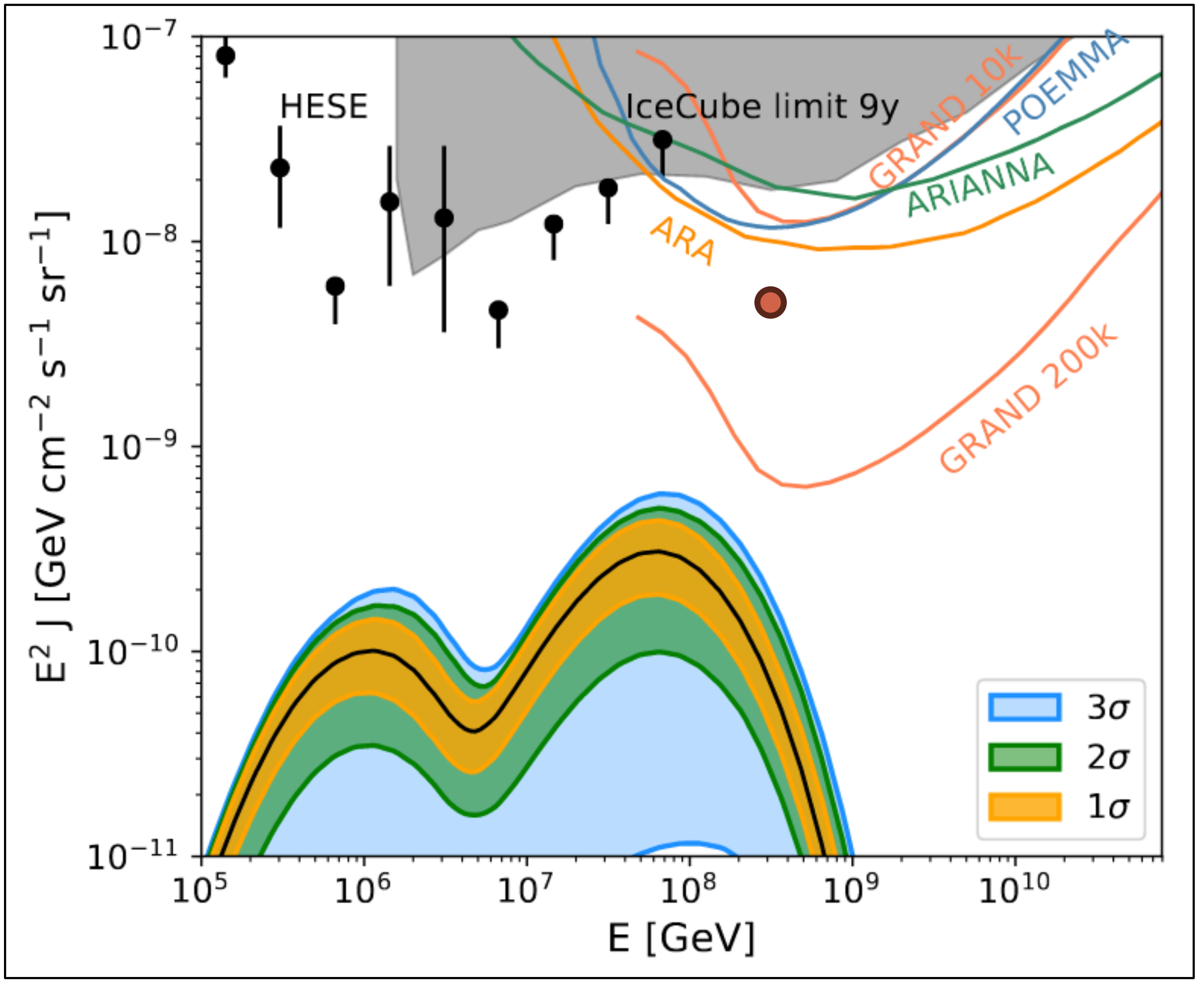


Protons
Helium
Oxygen
Iron

Looking Back, We Were Bearers of Bad News!



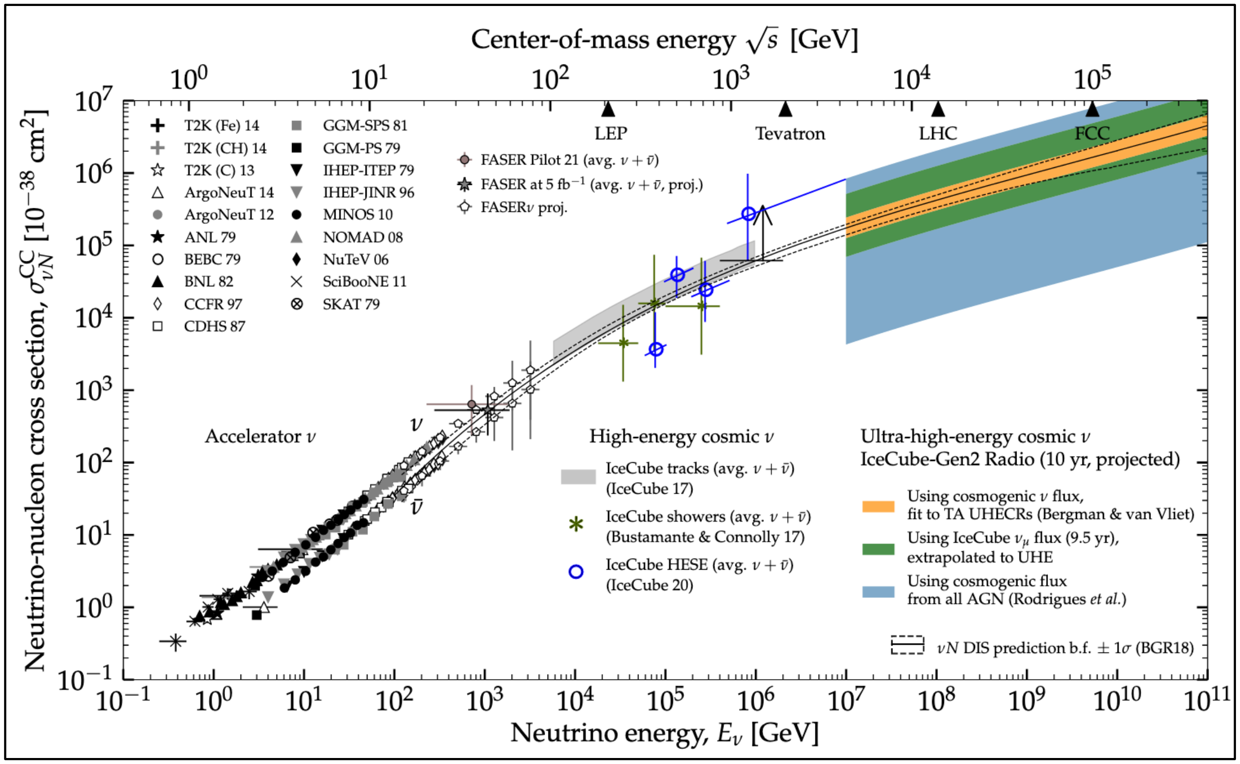
Heinze, et al, arXiv:1901.03338

High-Energy Neutrinos as Probes of New Physics

- Deep down, I am not particularly excited by most “astrophysical questions” (and I suspect that Subir is not either)
- High-energy astrophysics, however, is not only about astrophysics – it is also a powerful vehicle for testing the fundamental laws of physics
- In 2003, this was still a rather fringe view

High-Energy Neutrinos as Probes of New Physics

- Neutrino telescopes allow us to measure the interactions of neutrinos at ***higher energies*** and over ***longer baselines*** than is possible in any existing laboratory experiment



Ackermann et al (Snowmass White Paper), arXiv:2203.08096

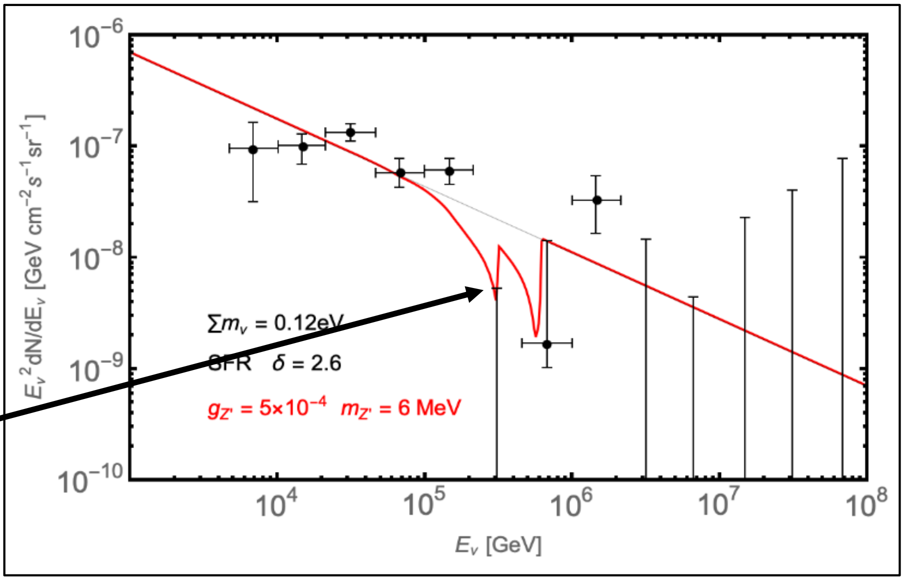
Probes of New Interactions?

- As an example, consider a light Z’ that couples to muons (or muons and taus), with a gauge coupling selected to explain the FNAL/BNL measurements of g_μ-2
- Over cosmological distances, such a Z’ would cause high-energy neutrinos to scatter with the cosmic neutrino background, leading to resonant absorption features at

$$E_\nu \approx \frac{m_{Z'}^2}{2m_{\nu,i}\,(1+z_{\rm abs})}$$

$$\approx 1\,{\rm PeV} \times \left(\frac{m_{Z'}}{10\,{\rm MeV}}\right)^2 \left(\frac{0.05\,{\rm eV}}{m_{\nu,i}}\right) \left(\frac{1}{1+z_{\rm abs}}\right)$$

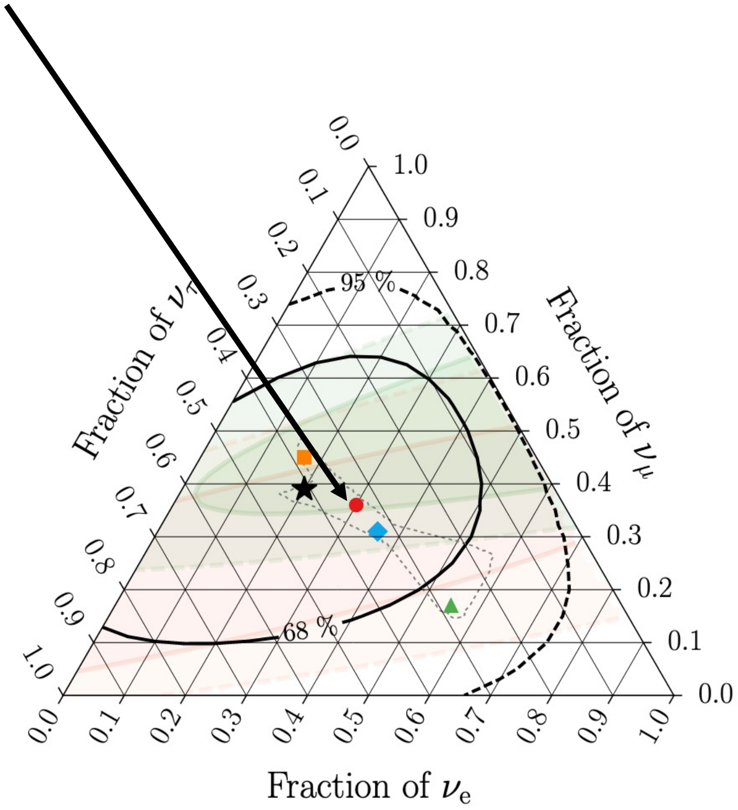
- This could even provide an explanation for the observed dip at ~0.2-1 PeV



DH, Iguaz, Serpico, arXiv:2302.03571
DiFranzo, DH, arXiv:1507.03015
DH, arXiv:0701194

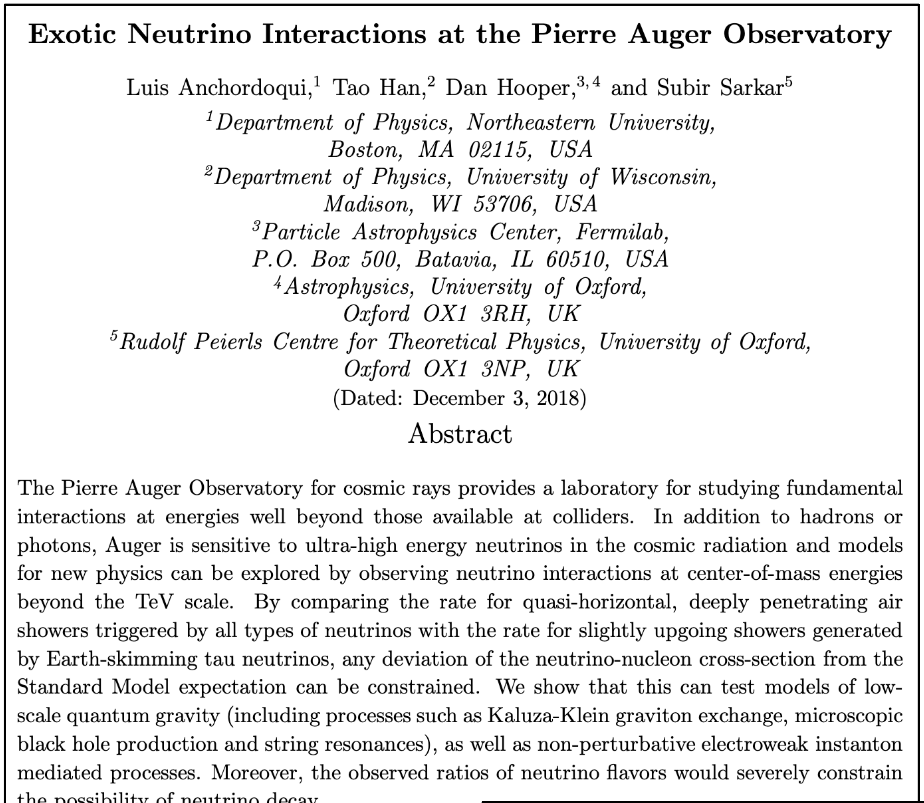
Neutrino Decay?

- Perhaps one or more neutrino species are unstable?
- If one or two neutrino species decay into lighter neutrinos, this would impact the ratio of neutrino flavors that reach Earth
- Neutrinos from pion decay are produced with $\nu_e:\nu_\mu:\nu_\tau = 1:2:0$, which after oscillations becomes $\nu_e:\nu_\mu:\nu_\tau = 0.90:1.18:1.02$
- Current measurements are consistent with these predictions, allowing us to constrain $\tau/m > 10$ s/eV



Beacom, Bell, DH, Pakvasa, Weiler, arXiv:0211305
IceCube, arXiv:2011.03561

High-Energy Neutrinos as Probes of New Physics



Exotic Neutrino Interactions at the Pierre Auger Observatory

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[5] *Rudolf Peierls Centre for Theoretical Physics, University of Oxford,*
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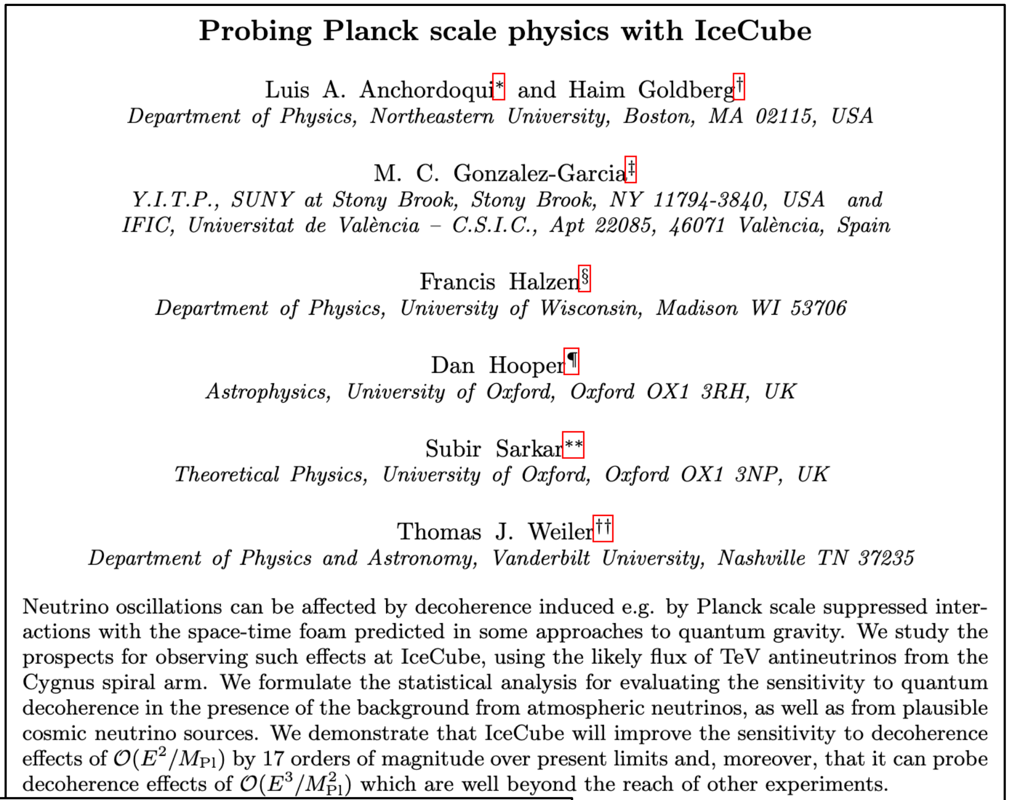
(Dated: December 3, 2018)

Abstract

The Pierre Auger Observatory for cosmic rays provides a laboratory for studying fundamental interactions at energies well beyond those available at colliders. In addition to hadrons or photons, Auger is sensitive to ultra-high energy neutrinos in the cosmic radiation and models for new physics can be explored by observing neutrino interactions at center-of-mass energies beyond the TeV scale. By comparing the rate for quasi-horizontal, deeply penetrating air showers triggered by all types of neutrinos with the rate for slightly upgoing showers generated by Earth-skimming tau neutrinos, any deviation of the neutrino-nucleon cross-section from the Standard Model expectation can be constrained. We show that this can test models of low-scale quantum gravity (including processes such as Kaluza-Klein graviton exchange, microscopic black hole production and string resonances), as well as non-perturbative electroweak instanton mediated processes. Moreover, the observed ratios of neutrino flavors would severely constrain the possibility of neutrino decay.



Probing Planck scale physics with IceCube

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Subir Sarkar**
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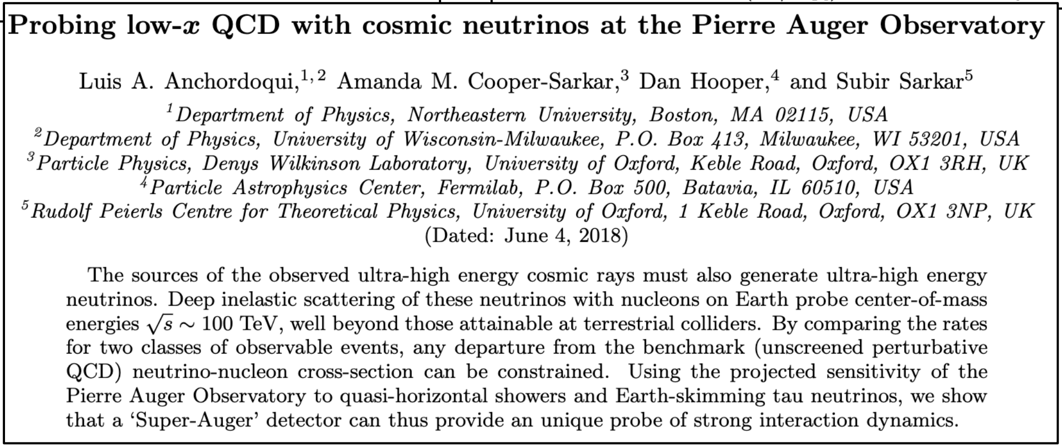
Thomas J. Weiler††
Department of Physics and Astronomy, Vanderbilt University, Nashville TN 37235

Neutrino oscillations can be affected by decoherence induced e.g. by Planck scale suppressed interactions with the space-time foam predicted in some approaches to quantum gravity. We study the prospects for observing such effects at IceCube, using the likely flux of TeV antineutrinos from the Cygnus spiral arm. We formulate the statistical analysis for evaluating the sensitivity to quantum decoherence in the presence of the background from atmospheric neutrinos, as well as from plausible cosmic neutrino sources. We demonstrate that IceCube will improve the sensitivity to decoherence effects of $\mathcal{O}(E^2/M_{\rm Pl})$ by 17 orders of magnitude over present limits and, moreover, that it can probe decoherence effects of $\mathcal{O}(E^3/M_{\rm Pl}^2)$ which are well beyond the reach of other experiments.



Probing low-x QCD with cosmic neutrinos at the Pierre Auger Observatory

Luis A. Anchordoqui,[1,2] Amanda M. Cooper-Sarkar,[3] Dan Hooper,[4] and Subir Sarkar[5]
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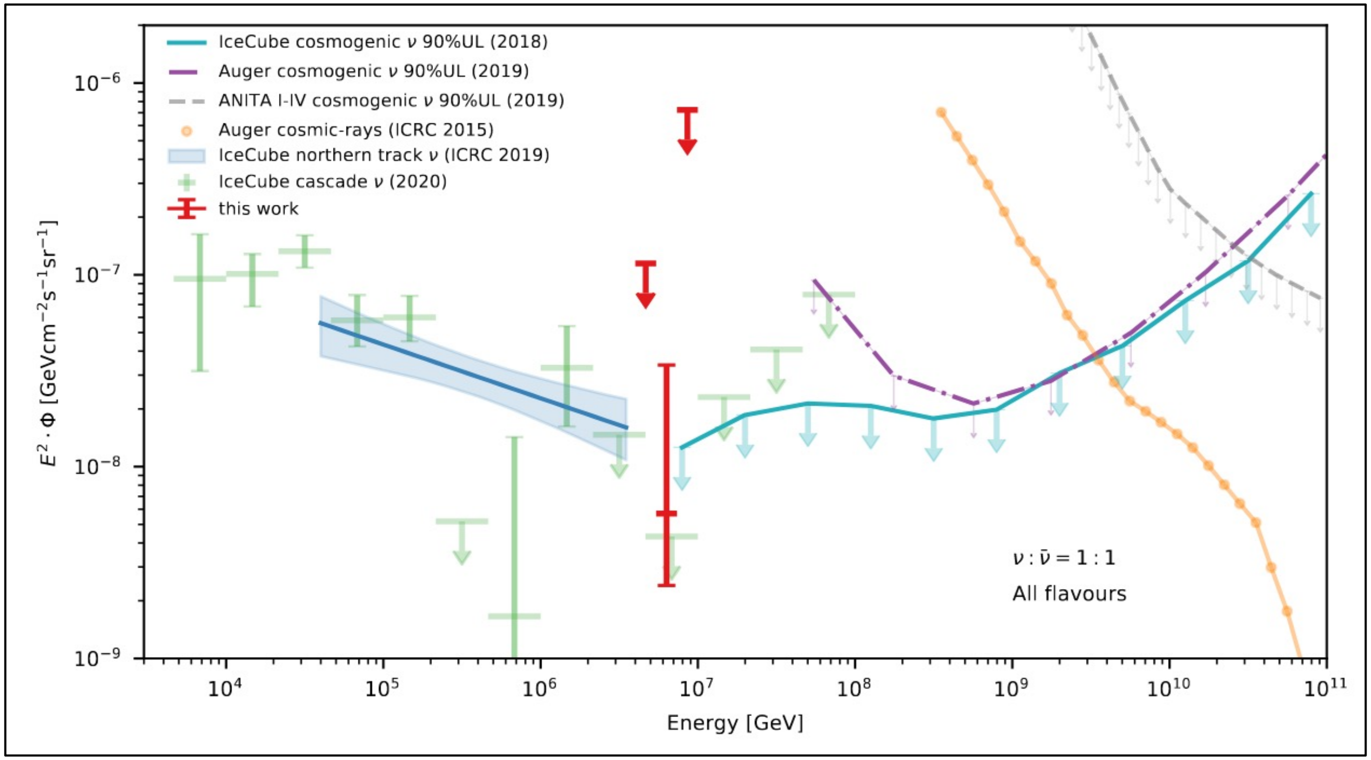
[2] *Department of Physics, University of Wisconsin-Milwaukee, P.O. Box 413, Milwaukee, WI 53201, USA*
[3] *Particle Physics, Denys Wilkinson Laboratory, University of Oxford, Keble Road, Oxford, OX1 3RH, UK*
[4] *Particle Astrophysics Center, Fermilab, P.O. Box 500, Batavia, IL 60510, USA*
[5] *Rudolf Peierls Centre for Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP, UK*
(Dated: June 4, 2018)

The sources of the observed ultra-high energy cosmic rays must also generate ultra-high energy neutrinos. Deep inelastic scattering of these neutrinos with nucleons on Earth probe center-of-mass energies $\sqrt{s} \sim 100$ TeV, well beyond those attainable at terrestrial colliders. By comparing the rates for two classes of observable events, any departure from the benchmark (unscreened perturbative QCD) neutrino-nucleon cross-section can be constrained. Using the projected sensitivity of the Pierre Auger Observatory to quasi-horizontal showers and Earth-skimming tau neutrinos, we show that a 'Super-Auger' detector can thus provide an unique probe of strong interaction dynamics.

Living in the Age of High-Energy Neutrino Astrophysics

- IceCube has measured a diffuse spectrum of astrophysical neutrinos, ranging in energy from tens of TeV to several PeV (at least)
- Approximately isotropic, with a roughly power-law spectrum dN/dE ~ $E^{-2.3}$
- The origins of these particles remains unknown, but they are almost certainly connected to the origin of the (high-energy) cosmic ray spectrum



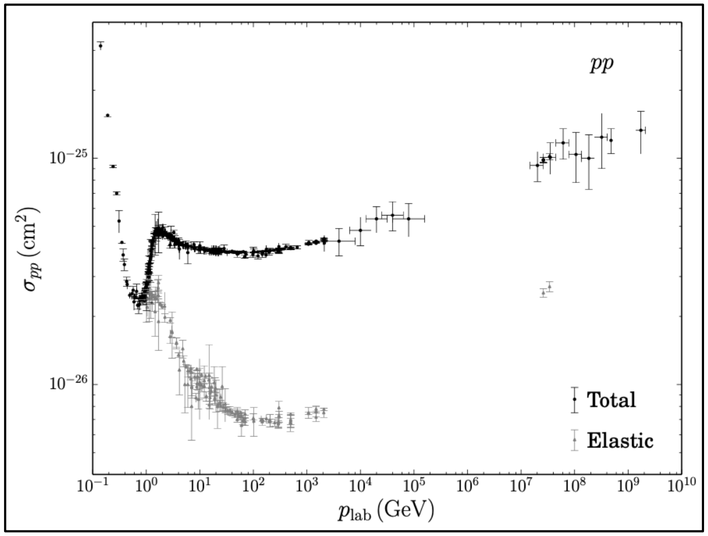
IceCube, arXiv:2110.15051

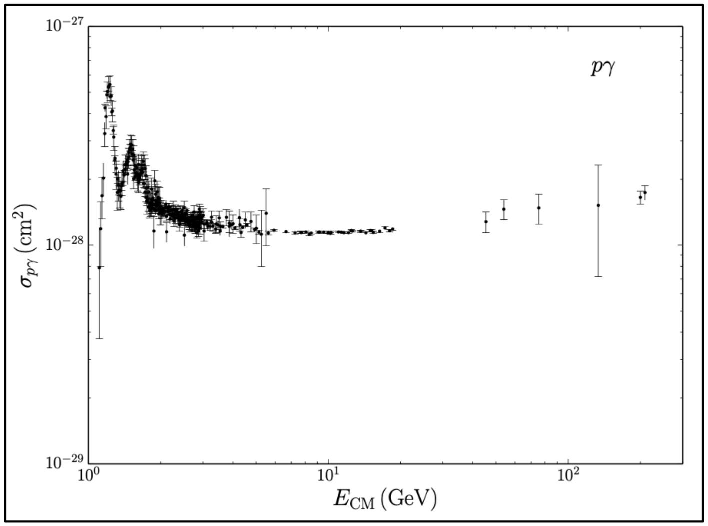
The Neutrino/Gamma Ray/Cosmic Ray Connection

- High-energy protons produce pions through their inelastic scattering with gas and radiation, and those pions decay to produce photons and neutrinos

$$\pi^0 \to \gamma\gamma$$
$$\pi^+ \to \mu^+ + \nu_\mu \to e^+\bar{\nu}_\mu\nu_e\nu_\mu$$
$$\pi^- \to \mu^- + \bar{\nu}_\mu \to e^-\nu_\mu\bar{\nu}_e\bar{\nu}_\mu.$$

- Sources of cosmic rays are inevitably sources of gamma rays and high-energy neutrinos





From the Cosmic Ray Spectrum to the High-Energy Neutrino Spectrum

- There is an argument that goes back to the late 1990s which relates the expected neutrino flux to the observed cosmic-ray spectrum (Waxman-Bahcall)
- For sources that are optically-thin (with little absorption), these are related as follows:

$$E_\nu^2 \frac{dN_\nu}{dE_\nu} = \int E_{\rm CR}^2 \frac{d\dot{N}_{\rm CR}}{dE_{\rm CR}}(z)\, \epsilon\, f_{\pi^\pm} f_\nu \frac{c}{4\pi} \frac{dt}{dz} \frac{dz}{(1+z)}$$

$$E_{\rm CR}^2 \frac{d\dot{N}_{\rm CR}}{dE_{\rm CR}}(z) = E_{\rm CR}^2 \frac{d\dot{N}_{\rm CR}}{dE_{\rm CR}}\bigg|_{z=0} \times f(z)$$

The fraction of energy in CRs that goes into π's

$f_\pi \sim \frac{1}{2}, \frac{2}{3}$

$f_\nu \sim \frac{3}{4}$

$\xi \sim 1-6$ (for realistic z dist.)

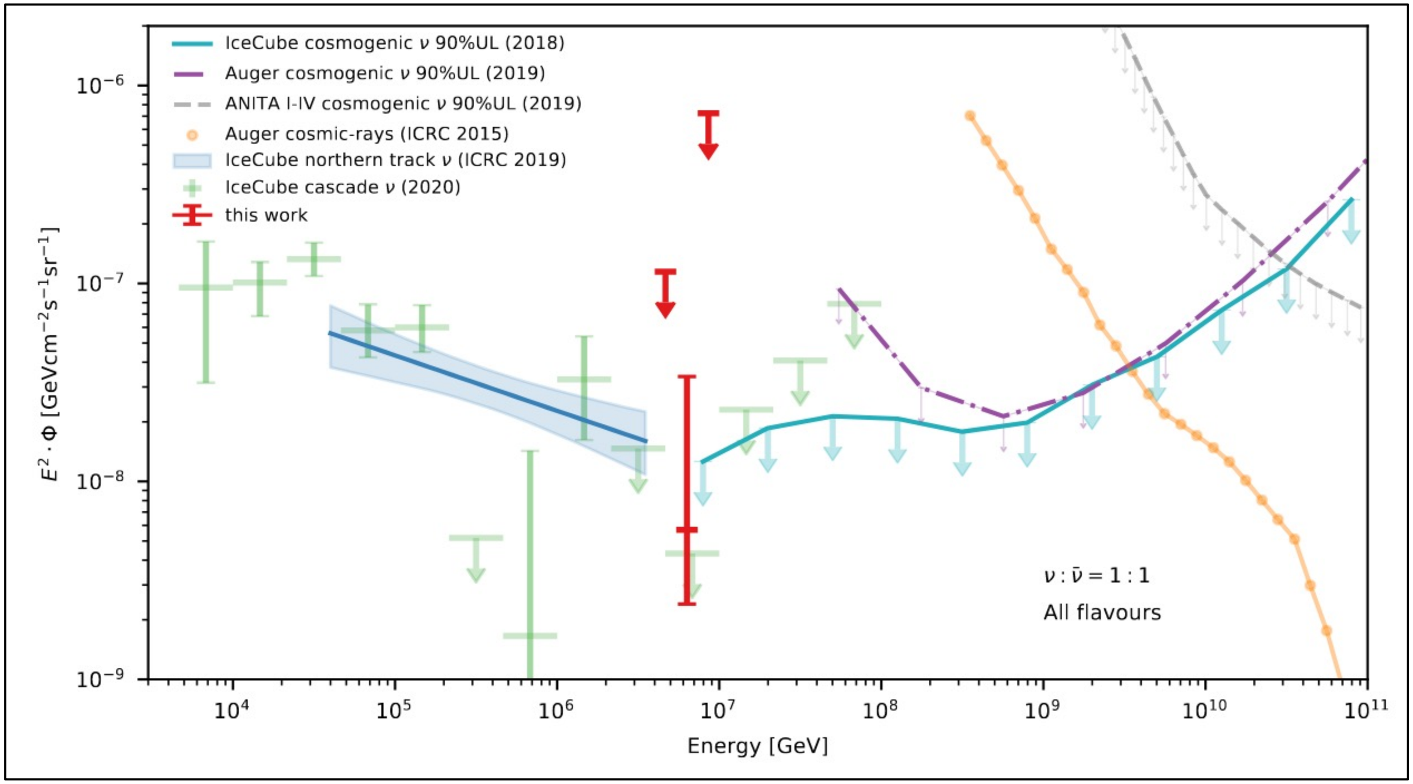
$$E_\nu^2 \frac{dN_\nu}{dE_\nu} = E_{\rm CR}^2 \frac{d\dot{N}_{\rm CR}}{dE_{\rm CR}}\bigg|_{z=0} \frac{\epsilon\, f_{\pi^\pm} f_\nu c}{4\pi}\, \xi \int_0^\infty \frac{dz}{H(z)(1+z)}$$

$$\approx 2.5 \times 10^{-8}\,{\rm GeV/cm^2/s/sr} \times \left(\frac{\epsilon}{1}\right)\left(\frac{f_{\pi^\pm}}{0.5}\right)\left(\frac{f_\nu}{0.75}\right)\left(\frac{\xi}{1}\right)$$

"Waxman-Bahcall Bound" (for optically-thin sources)

IceCube's High-Energy Neutrinos

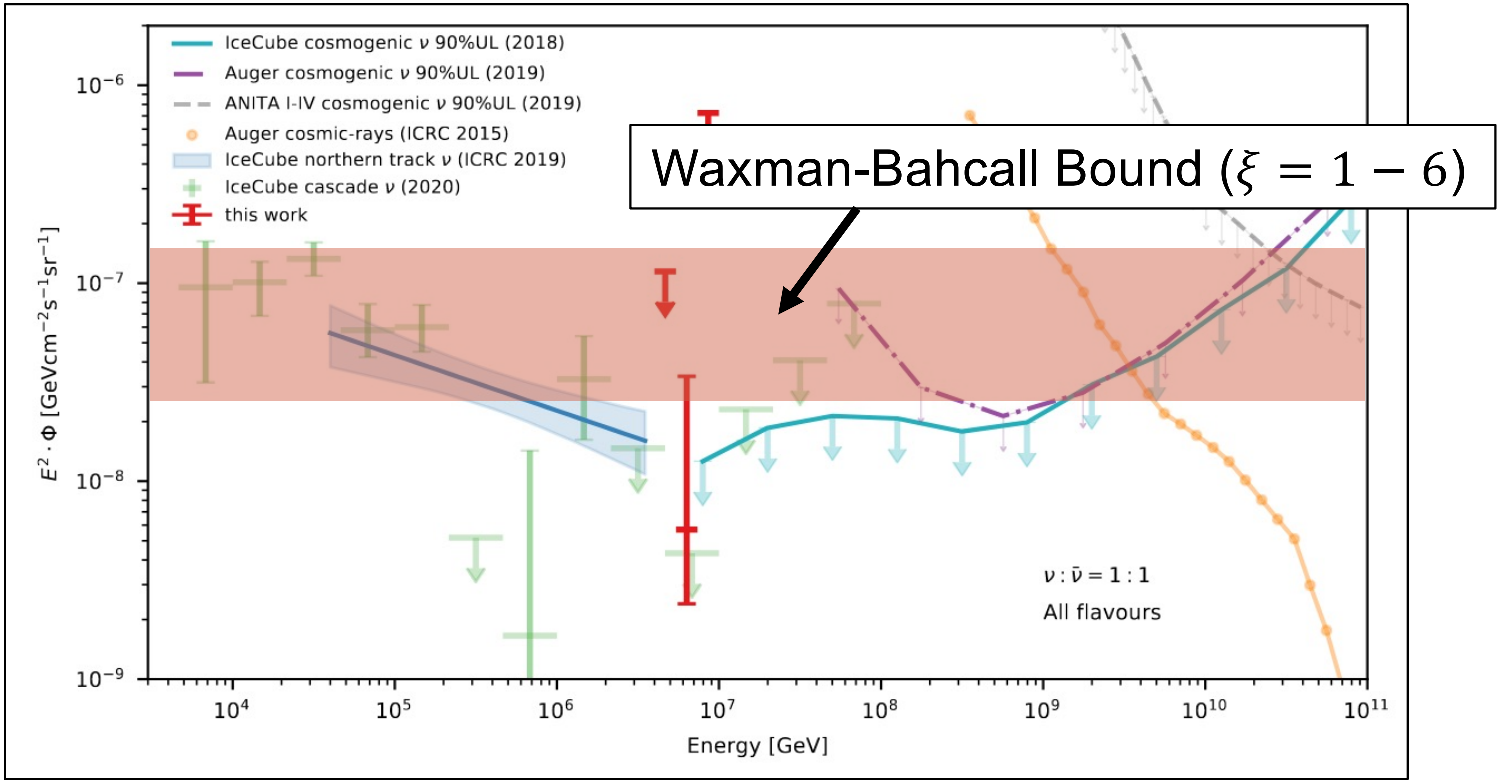
- In this context, let's consider the diffuse neutrino spectrum measured by IceCube



IceCube, arXiv:2110.15051

IceCube's High-Energy Neutrinos

- In this context, let's consider the diffuse neutrino spectrum measured by IceCube
- This suggests that IceCube's neutrinos originate from the sources of the high and ultra-high energy cosmic rays



Waxman-Bahcall Bound ($\xi = 1 - 6$)

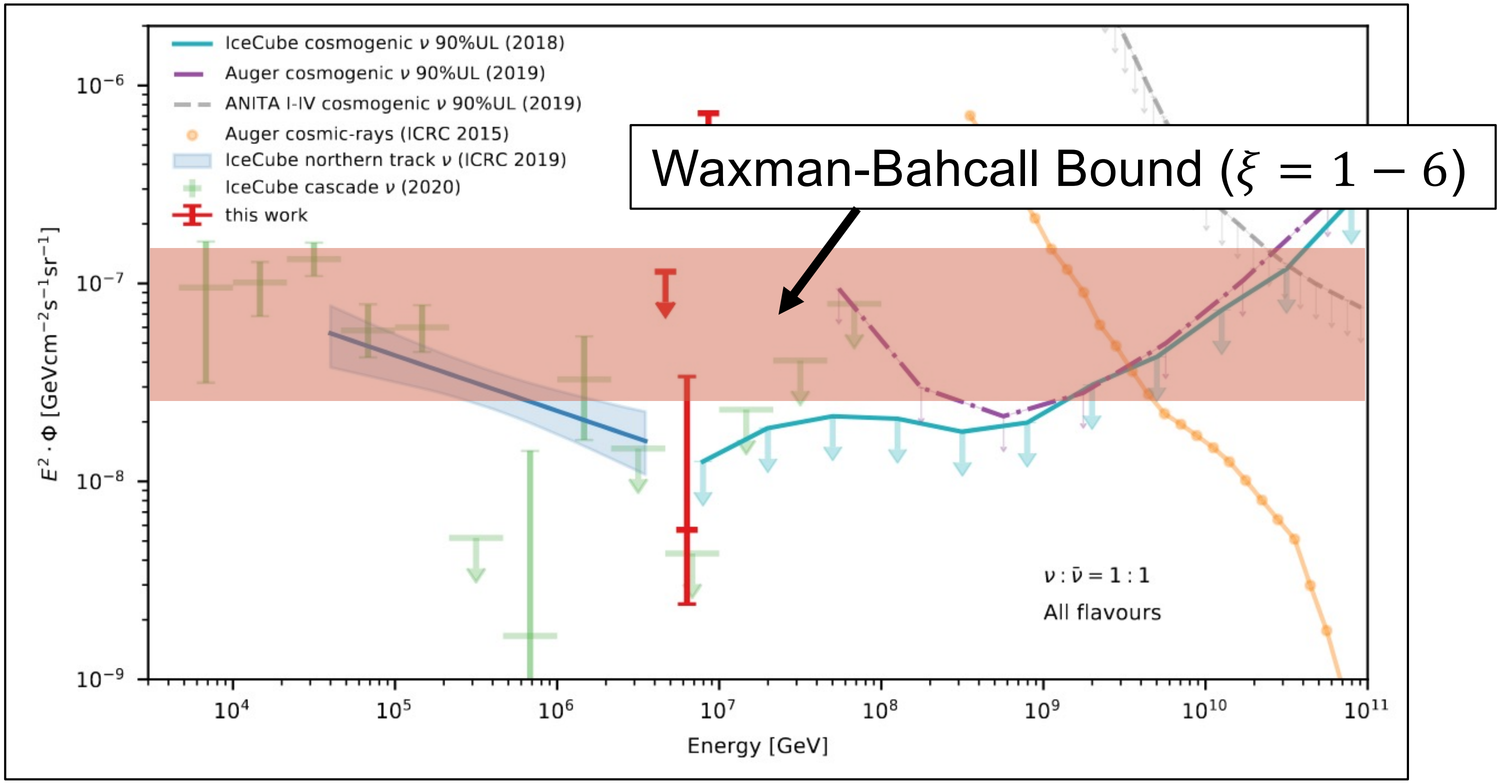
IceCube, arXiv:2110.15051

IceCube’s High-Energy Neutrinos

Two Possibilities:

1) IceCube’s neutrinos come from the main sources of the cosmic rays, which feature $\epsilon \sim 0.1 - 1$

2) IceCube’s neutrinos come from optically thick sources (which absorb most of the particles they accelerate before they escape)



Waxman-Bahcall Bound ($\xi = 1 - 6$)

IceCube, arXiv:2110.15051

IceCube's High-Energy Neutrinos

Where do these neutrinos come from? Some long-standing hypotheses include:

-Gamma-Ray Bursts (GRB)

-Blazars

-Other Active Galactic Nuclei (AGN)

-Star-Forming/Starburst Galaxies

IceCube's High-Energy Neutrinos

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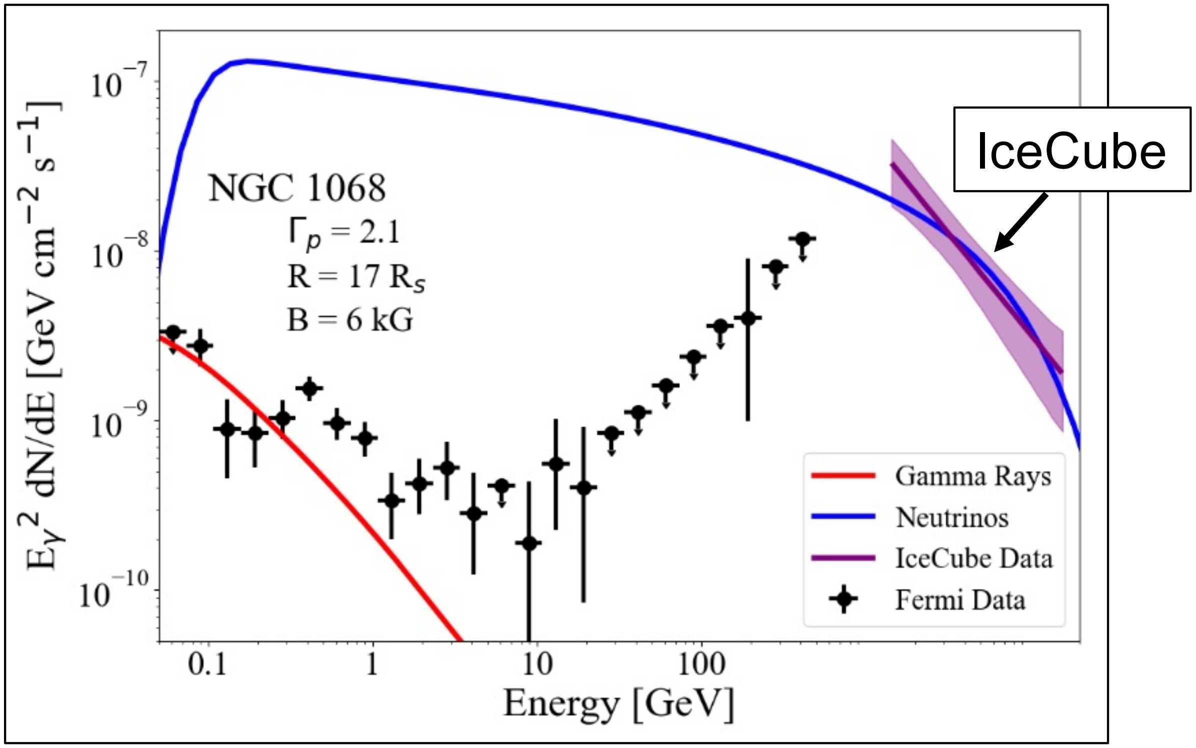
Neutrinos from NGC 1068

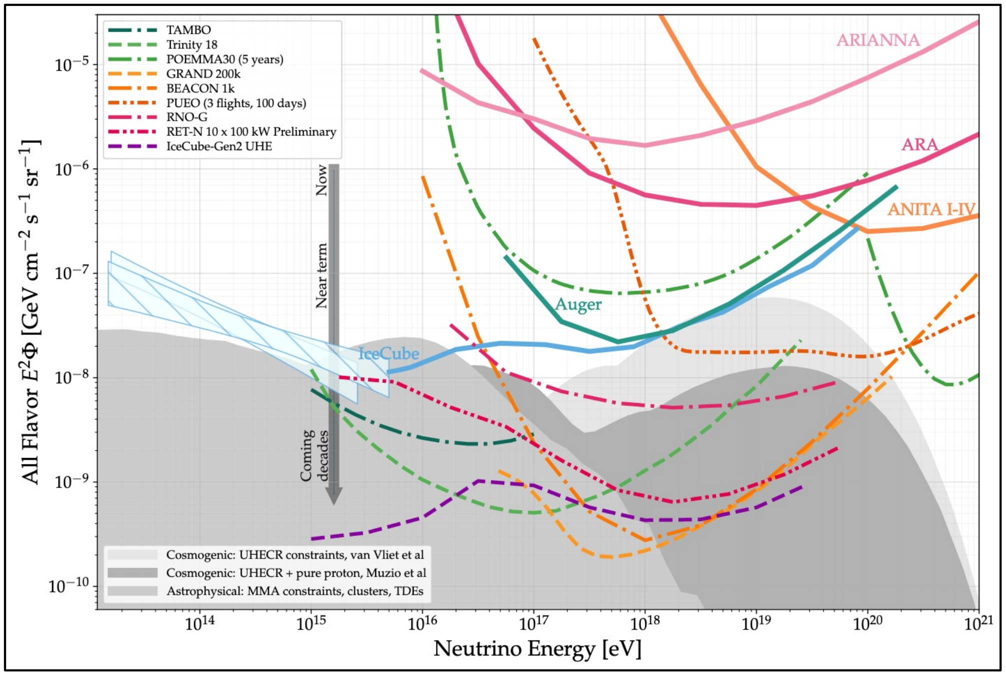
- Last year, IceCube reported an excess of 79 ± 22 TeV-scale neutrino events from the direction of the active galaxy NGC 1068 (post-trials significance of 4.2σ)
- Upper limits on the very high-energy gamma-ray emission from NGC 1068 indicate that the ~TeV photons from this source must be very efficiently absorbed and reemitted at energies below ~10-100 MeV, making this an example of an optically thick "Hidden Source"

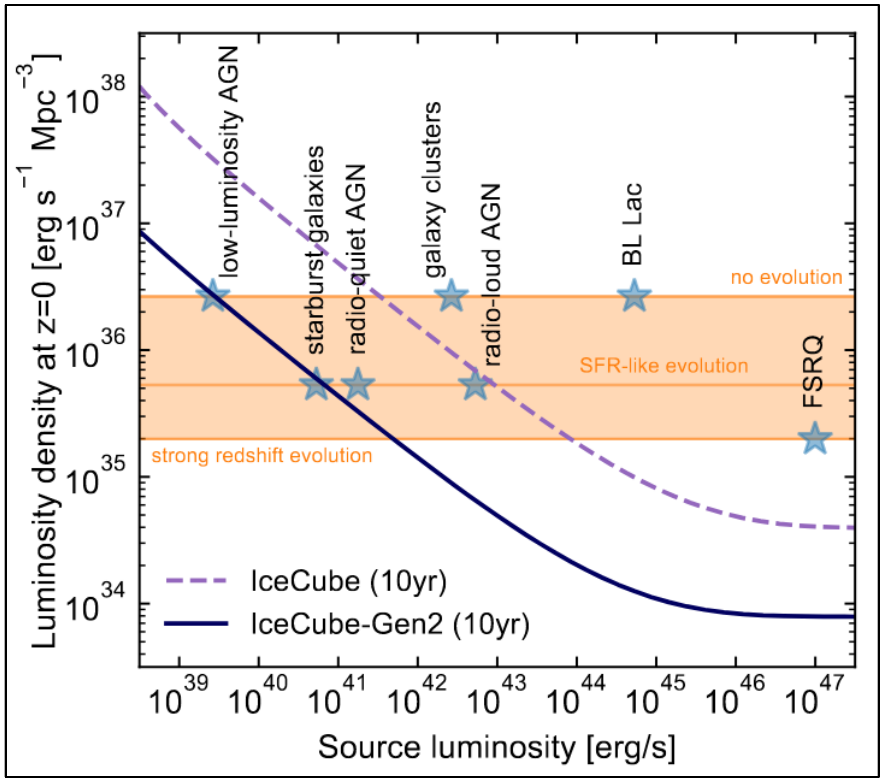


IceCube, Science, arXiv:2211.09972; Blanco, DH, Linden, Pinetti, arXiv:2307.03259

The Future

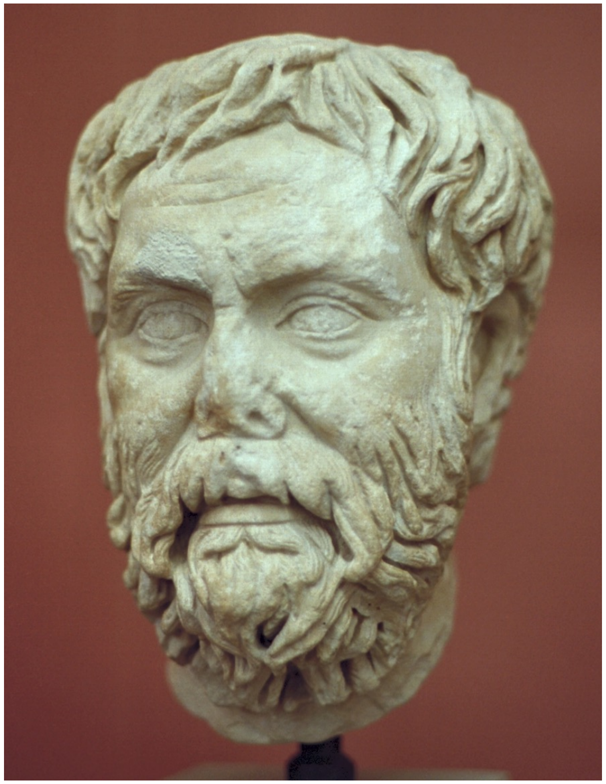
- Existing data indicates that IceCube's neutrinos come from many relatively faint sources (ruling out blazars and GRB, for example)
- Testing our best remaining hypotheses (non-blazar AGN, starburst galaxies) is beyond the current reach of IceCube, but within the reach of proposed next-generation efforts



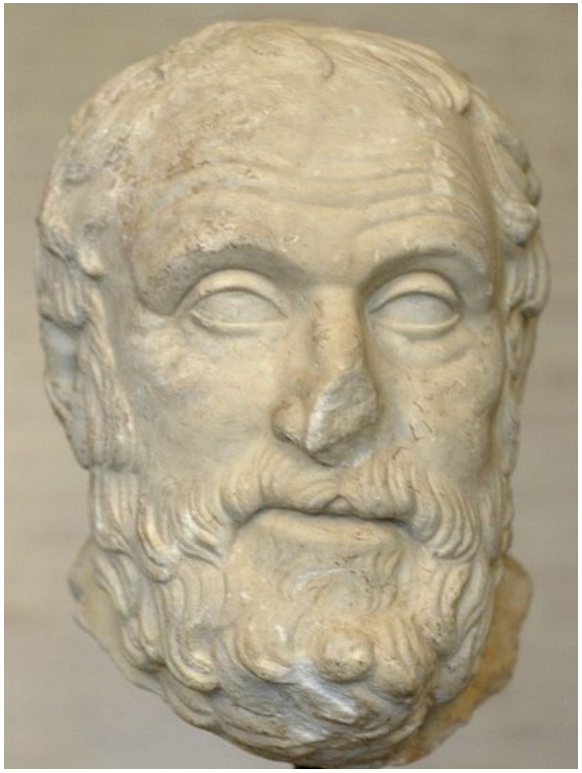


Ackermann et al., Snowmass White Paper, 2203.08096

Subir the Skeptic



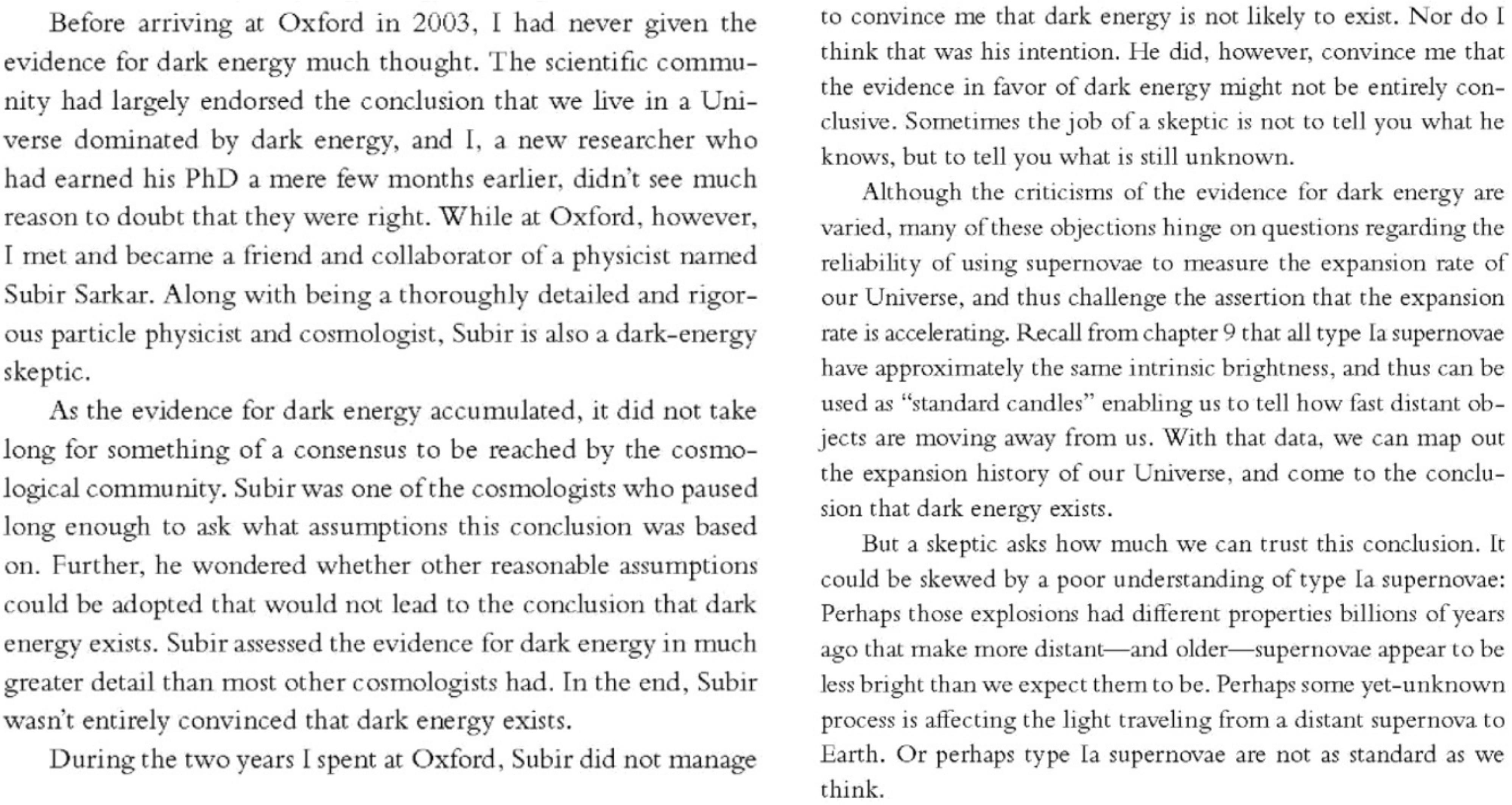
Pyrrho of Elis (360-270 BCE)



Carneades (213-128 BCE)

Subir of Oxford

Subir the Skeptic



Before arriving at Oxford in 2003, I had never given the evidence for dark energy much thought. The scientific community had largely endorsed the conclusion that we live in a Universe dominated by dark energy, and I, a new researcher who had earned his PhD a mere few months earlier, didn't see much reason to doubt that they were right. While at Oxford, however, I met and became a friend and collaborator of a physicist named Subir Sarkar. Along with being a thoroughly detailed and rigorous particle physicist and cosmologist, Subir is also a dark-energy skeptic.

As the evidence for dark energy accumulated, it did not take long for something of a consensus to be reached by the cosmological community. Subir was one of the cosmologists who paused long enough to ask what assumptions this conclusion was based on. Further, he wondered whether other reasonable assumptions could be adopted that would not lead to the conclusion that dark energy exists. Subir assessed the evidence for dark energy in much greater detail than most other cosmologists had. In the end, Subir wasn't entirely convinced that dark energy exists.

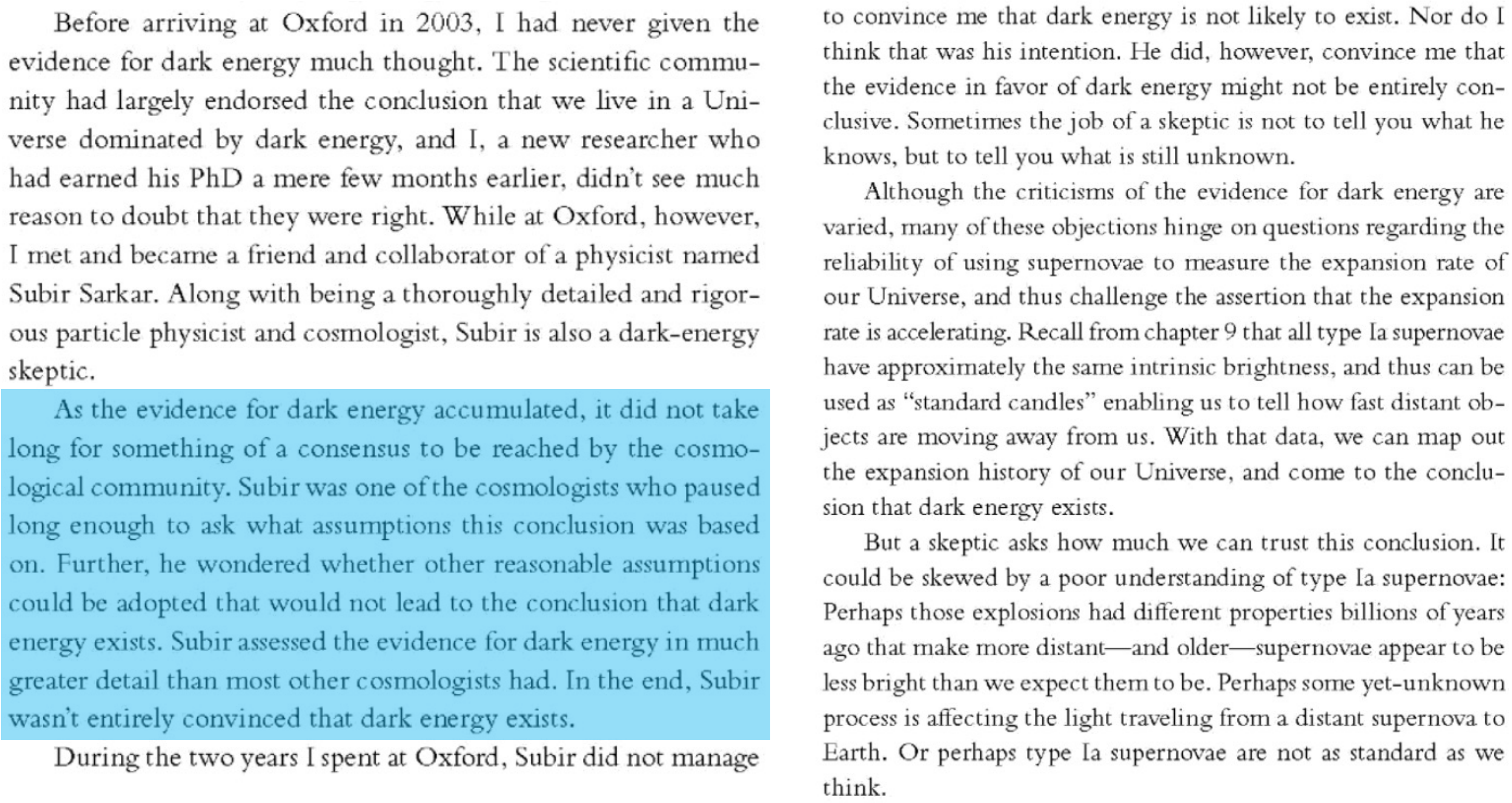
During the two years I spent at Oxford, Subir did not manage to convince me that dark energy is not likely to exist. Nor do I think that was his intention. He did, however, convince me that the evidence in favor of dark energy might not be entirely conclusive. Sometimes the job of a skeptic is not to tell you what he knows, but to tell you what is still unknown.

Although the criticisms of the evidence for dark energy are varied, many of these objections hinge on questions regarding the reliability of using supernovae to measure the expansion rate of our Universe, and thus challenge the assertion that the expansion rate is accelerating. Recall from chapter 9 that all type Ia supernovae have approximately the same intrinsic brightness, and thus can be used as "standard candles" enabling us to tell how fast distant objects are moving away from us. With that data, we can map out the expansion history of our Universe, and come to the conclusion that dark energy exists.

But a skeptic asks how much we can trust this conclusion. It could be skewed by a poor understanding of type Ia supernovae: Perhaps those explosions had different properties billions of years ago that make more distant—and older—supernovae appear to be less bright than we expect them to be. Perhaps some yet-unknown process is affecting the light traveling from a distant supernova to Earth. Or perhaps type Ia supernovae are not as standard as we think.

From *Dark Cosmos*, Pg. 213-14 (2006)

Subir the Skeptic



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Subir the Friend