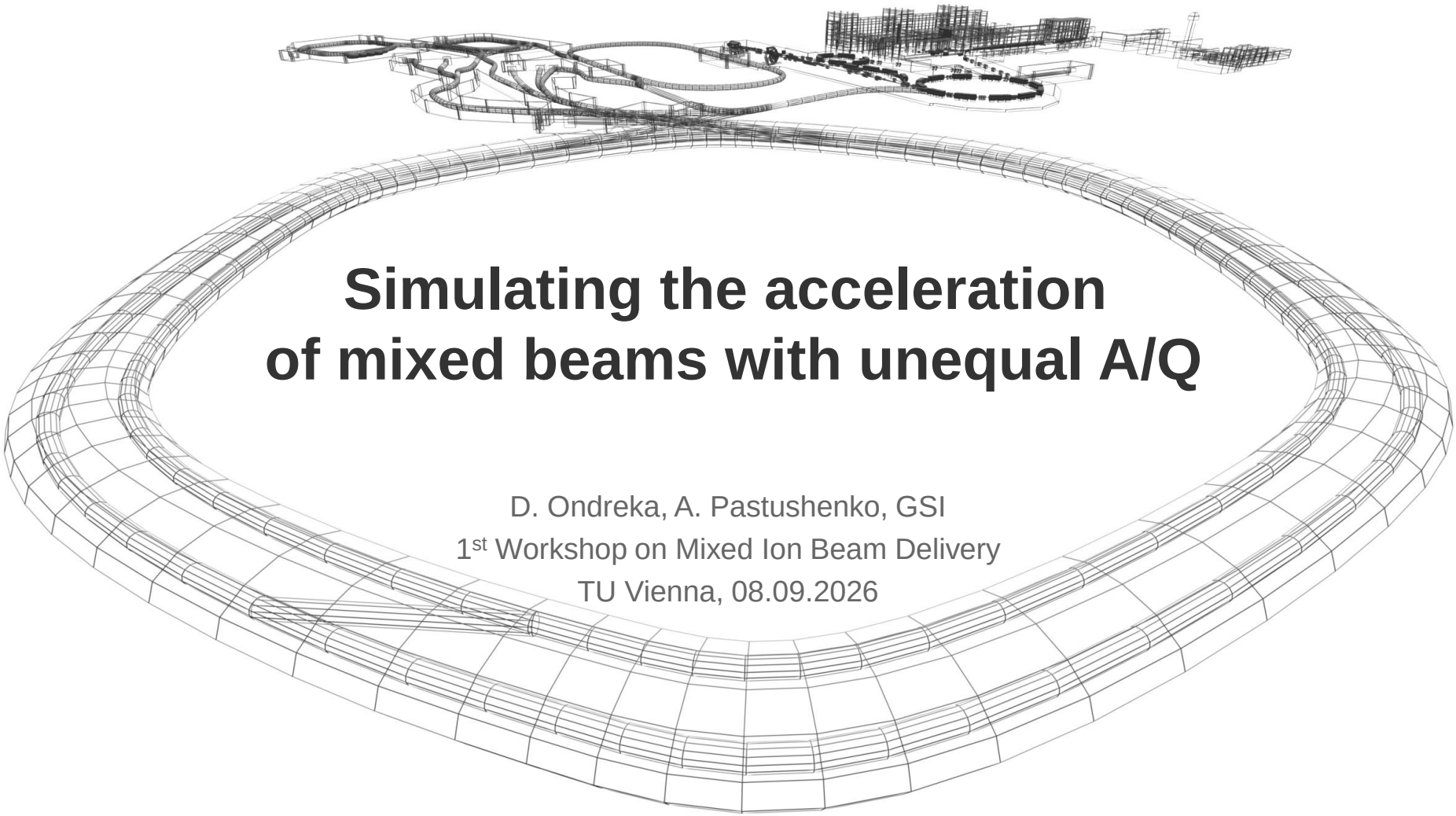

Simulating the acceleration of mixed beams with unequal A/Q

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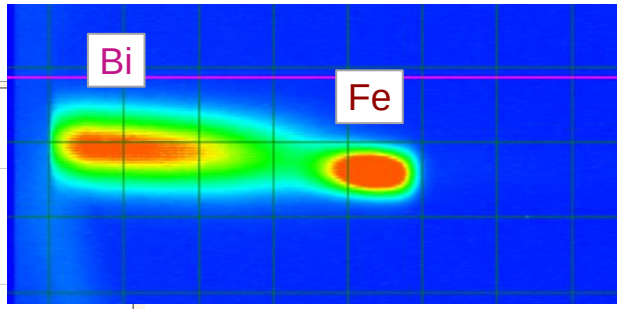
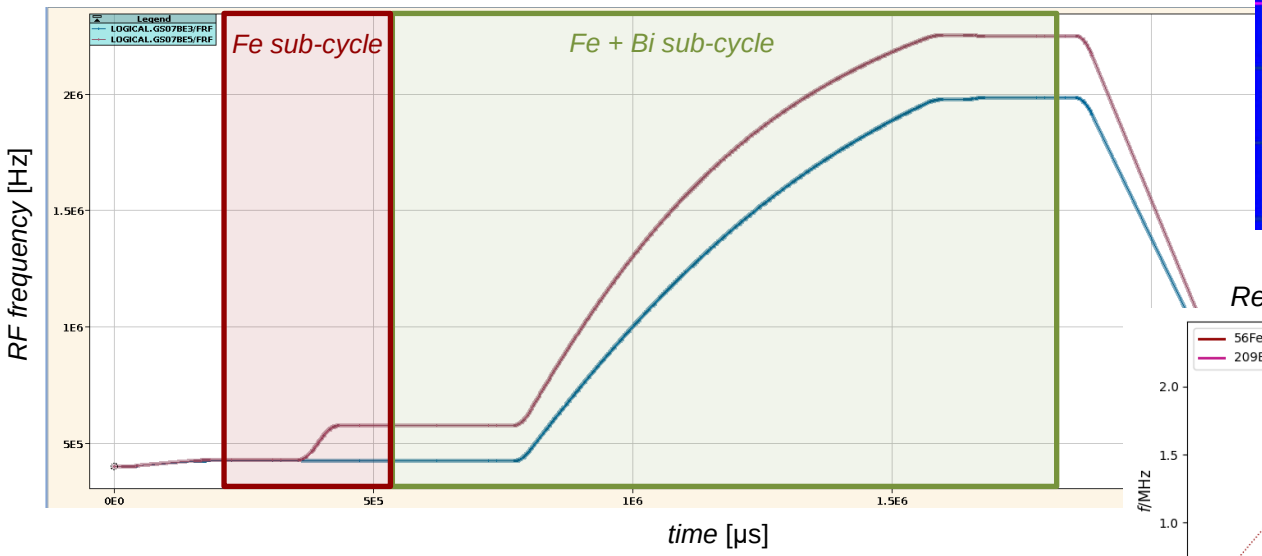
1st Workshop on Mixed Ion Beam Delivery
TU Vienna, 08.09.2026

Acceleration with unequal A/Q

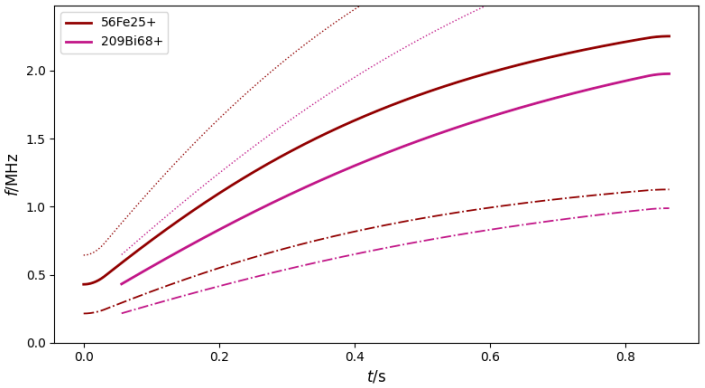
- Ions with unequal A/Q can in principle be accelerated in same cycle with two independent RF programs
  - Influence of second RF expected to average out as long as it is not a harmonic of first beam
  - Influence of second beam expected to be negligible for low intensities
- Successful demonstration at SIS18 with $^{56}Fe^{25+}$ and $^{209}Bi^{68+}$ in May 2025
  - Fast extraction of both ions verified, but…

Beam spots on screen behind magnetic septum





Revolution frequencies and RF frequencies



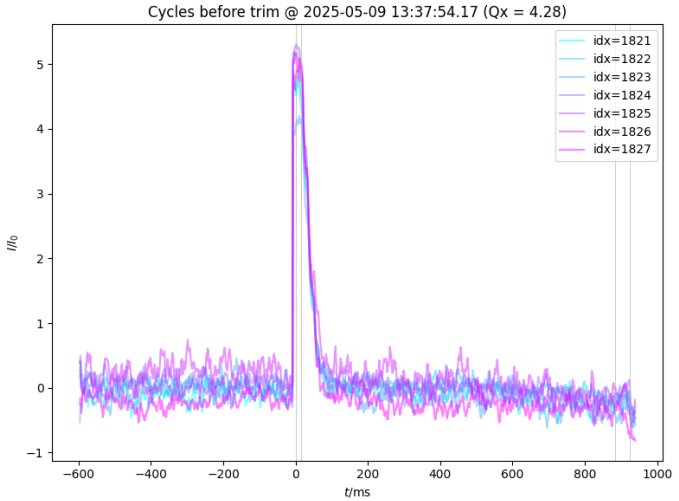
More details: 'Acceleration of mixed beams with unequal A/Q' (tomorrow)

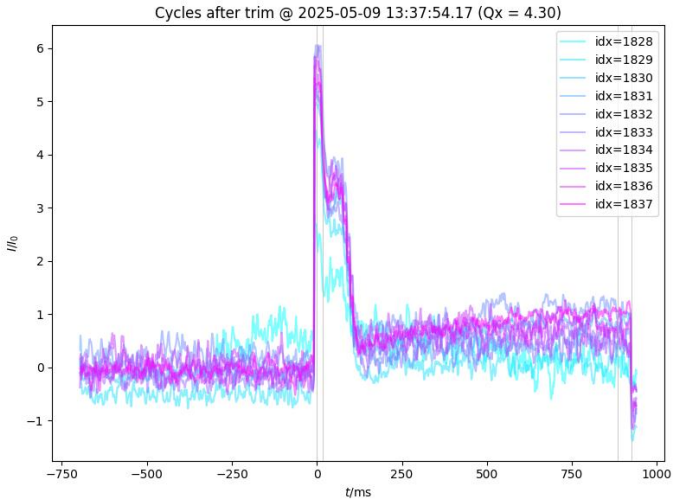
Fe/Bi Experiment: Expectations and reality

- Expectations
  - No influence of ‘alien’ RF since harmonics okay
  - Interactions negligible for low intensities

- Reality: large losses for Bi, but not for Fe
  - Bi lost when Fe-RF on, even without Fe injected
  - Bi losses observed to depend on
    - RF voltage amplitude
    - Horizontal phase space filling
    - Horizontal tune

- *(How) can we simulate what happens?*
  - Are losses longitudinal or transverse?
  - What is the effect of the ‘alien’ RF?
  - Why are there high losses for Bi, but not for Fe?





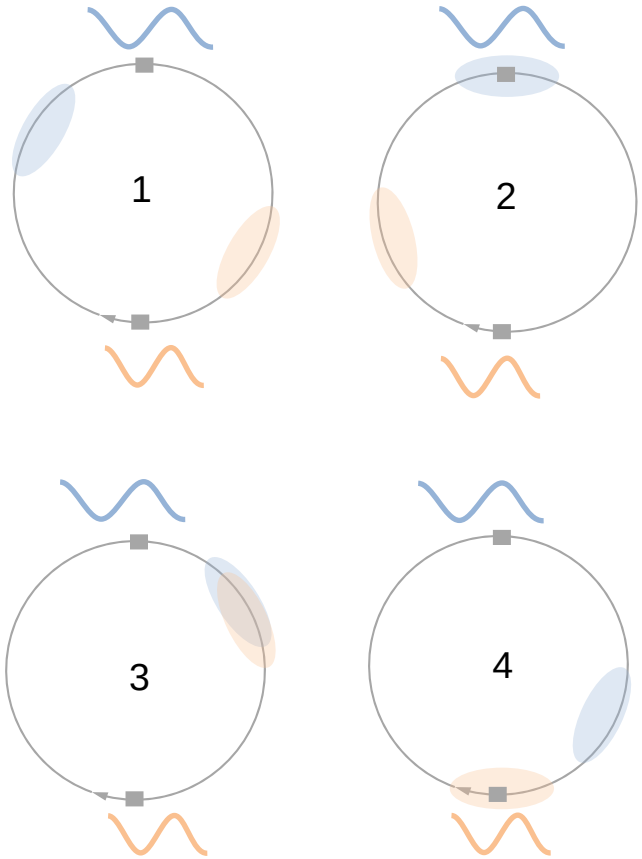
Simulation strategy

- First step (this talk)
  - Ignore presence of second beam
  - Simulate influence of ‘alien’ RF
  - Goals
    - Understand different losses of Bi (large) and Fe (negligible)
    - Be able to predict behavior quantitatively
  - Separation of effects of ‘alien’ RF on phase space directions
    - Longitudinal phase space
    - Combined longitudinal and horizontal phase space

- Second step (outlook)
  - Study influence of second beam (beam-beam effects)

The ion race

- Faster ion continuously overtakes the slower
  - Bunch crossings at difference frequency $|f_2 - f_1|$
- RF programs
  - Determined by phase functions $\varphi_1(t)$ and $\varphi_2(t)$
  - Voltage given by $V_i = V_{i,0} \cdot \sin(\varphi_i(t))$
  - Frequency is derivative of phase: $2\pi \cdot f_i(t) = d/dt\ \varphi_i(t)$
- Cavities
  - Single cavity suffices if bandwidth large enough
  - Otherwise two cavities required
- Each ion will see both gap voltages



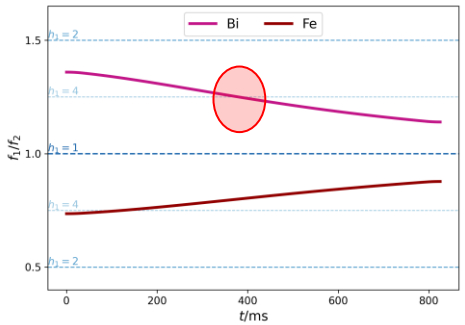
Longitudinal phase space (1/2)

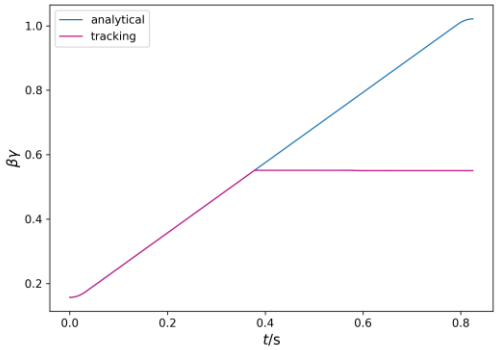
- Simple ***time-based*** *longitudinal tracking*

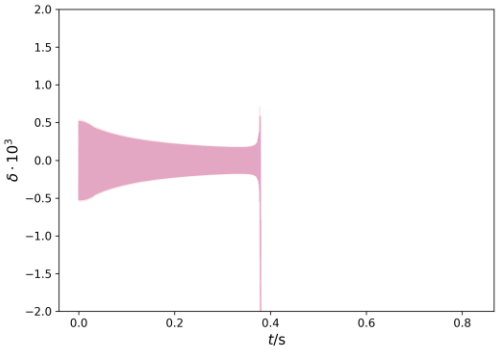
- Reference ramps defined analytically
  - Magnetic rigidity, momenta, frequencies
  - RF cavity phases $\varphi_1(t)$ and $\varphi_2(t)$
- Time tracked for each particle
  - Velocity and path length with mom. compaction
- Energy kick from phases and arrival time
  - Mom. deviation by subtracting ref. momentum

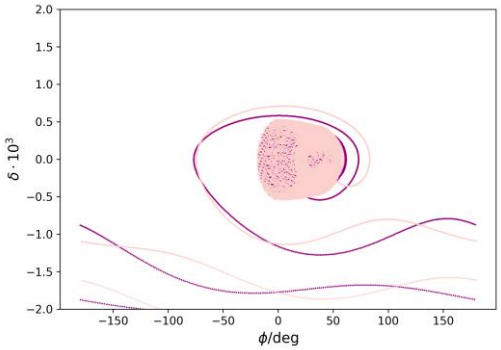
- Expected strong impact when ‘alien’ RF is harmonic of rev. frequency (h=4)

```python
def _track_turn(self, time0: float, energy0: float) -> tuple[float, float, float]:
    voltage_output = self.func_voltage(time0)
    voltage = voltage_output.voltage
    phase = voltage_output.phase
    if self.alien:
        volt_alien_output = self.func_volt_alien(time0)
        voltage += volt_alien_output.voltage
    delta_energy = voltage * self.charge_over_mass
    energy = energy0 + delta_energy
    bg = _r.calc_betagamma_from_energy(energy)
    bg_ref = self.func_bg(time0)
    delta = bg / bg_ref - 1
    path_length = (1 + self.alphac * delta) * self.circumference
    velocity = _r.speed_of_light * _r.calc_beta_from_betagamma(bg)
    delta_time = path_length / velocity
    time = time0 + delta_time
    return time, energy, phase
```





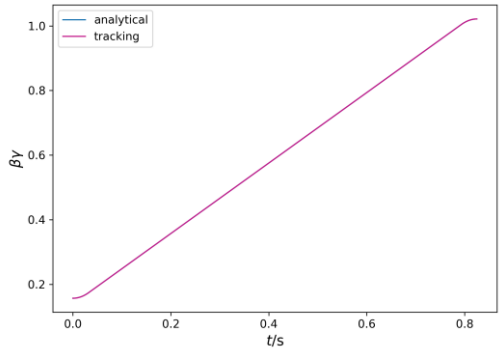


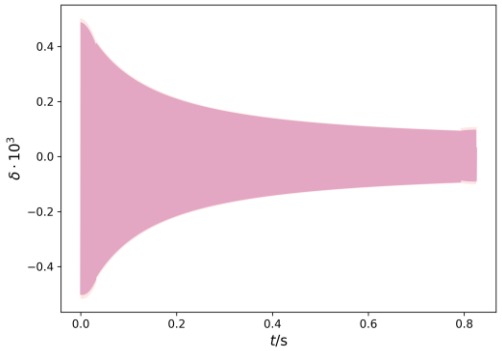


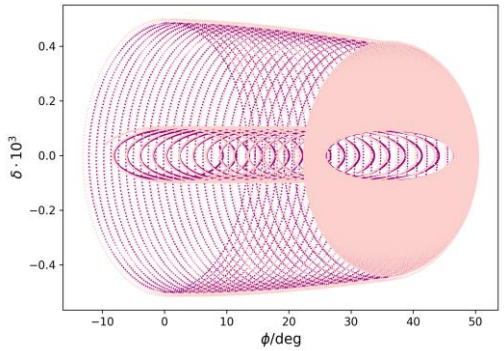
Longitudinal phase space (2/2)

- No sign of emittance blow-up or loss under experiment conditions (h=2)

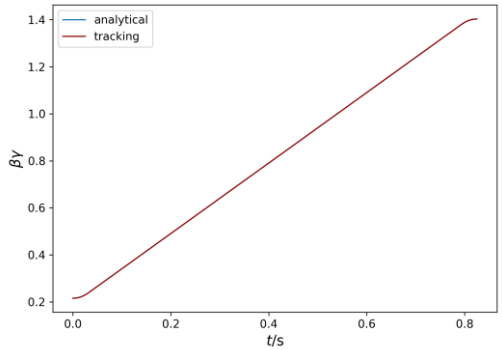
'Alien' RF off

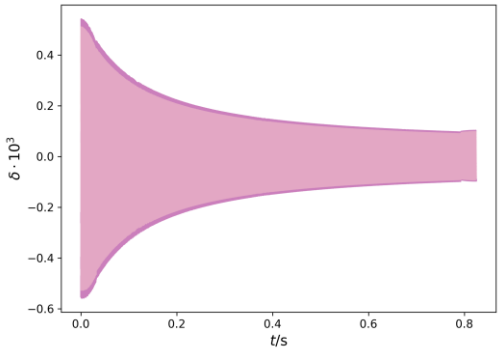


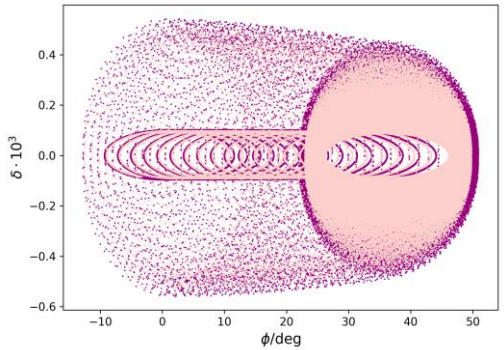




'Alien' RF on







Losses not caused by purely longitudinal effects → must simulate 6D

6D phase space: Xsuite (1/2)

- ***Turn-based tracking*** *using Xsuite*
  - Need to translate from time-based definition

- Standard set-up for simulation of acceleration
  - Set reference particle to desired ion species
  - Define energy program (e.g. via momentum) with small enough time steps
  - Configure acc. cavity (e.g. via f(t) and V(t))

- Configuration of cavity for 'alien' RF
  - Phase offset **relative** to acceleration cavity
    - 'Seems to imply setting phase to $\varphi_2(t) - \varphi_1(t)$
    - But $\varphi_1(t_r) = n \cdot 2\pi$ for reference particle!
    - **Set phase to $\varphi_2(t)$**
  - Frequency and voltage like acceleration cavity
    - Set frequency to $f_2(t)$
    - Set voltage to $V_2(t)$

- Phase obtained by integrating frequency
  - Simple numerical integration sufficient
  - Toy examples with analytical integrals exist

Time-dependent RF voltages

$$V_k(t) = \hat{V}_k\ (t)\sin\left(\varphi_k(t)\right)$$

Parameterization of cavity voltage in Xsuite

$$V(t_r,\zeta) = \hat{V}(t_r)\sin\left(\varphi_0(t_r) - 2\pi f_0(t_r)\frac{\zeta}{c\beta_0(t_r)}\right)$$

Voltage of acceleration cavity in Xsuite

$$V_1(\zeta) = \hat{\boldsymbol{V}}_{\boldsymbol{1}}\ \sin\left(-2\pi \boldsymbol{f}_1\frac{\zeta}{c\beta_0}\right)$$

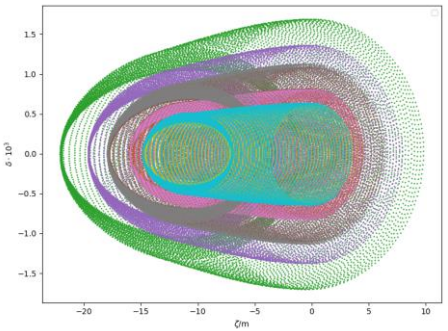
Voltage of 'alien' cavity in Xsuite

$$V_2(\zeta) = \hat{\boldsymbol{V}}_{\boldsymbol{2}}\ \sin\left(\boldsymbol{\varphi_2} - 2\pi \boldsymbol{f}_{\boldsymbol{2}}\frac{\zeta}{c\beta_0}\right)$$

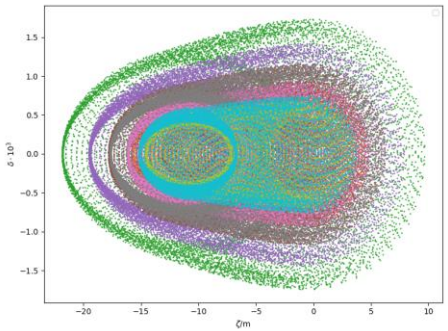
6D phase space: Xsuite (2/2)

- Benchmarking with test cycles
  - Simplified ramps for SIS18
  - Longitudinal results compared to simple tracking
  - Acceleration without ‘alien’ RF looks reasonable
  - Effect of ‘alien’ RF small as expected, but…

‘Alien’ RF off

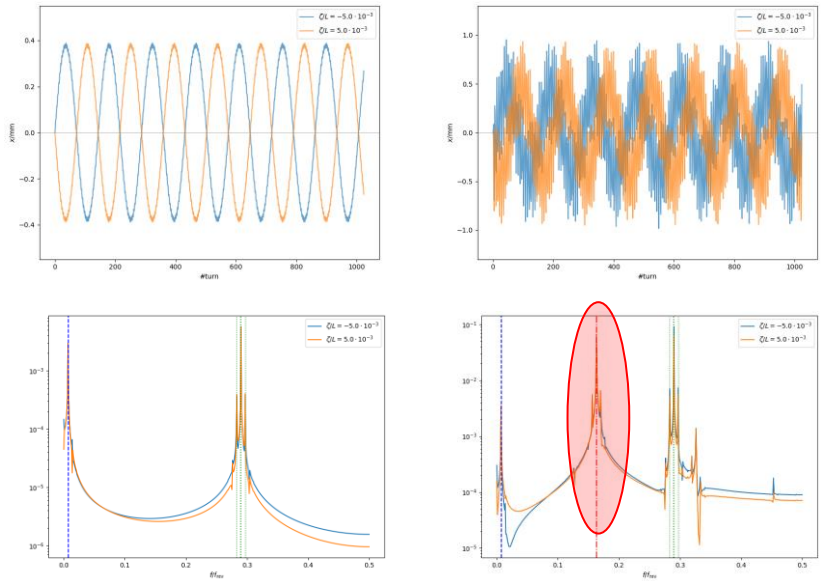


‘Alien’ RF on



- Horizontal oscillations caused by ‘alien’ RF
  - Coupling from longitudinal to horizontal motion
  - Mediated by **non-zero dispersion at cavity**
  - Visible in horizontal spectrum as separate peak

Horizontal oscillations without and with ‘alien’ RF



- Python project for simulation of unequal A/Q
  - https://github.com/dondreka/dualbeam-project
  - Presently private, access upon request

More details on results of experiment and simulations in talk ‘Acceleration of mixed beams with unequal A/Q’ (tomorrow)

Beam-beam effects

- Beam-beam effects expected to become relevant for large enough intensities
  - Space charge
  - Intra-beam scattering

- Xsuite not suitable for unequal A/Q mixes
  - No useful definition of turn for both ions
  - ‘Alien’ particles get lost if one insists on trying

- Challenges for simulation tool
  - Time-based tracking for rings
  - Inclusion of particle interactions
  - Computation speed allowing O(10^5) turns

If you know such a tool, please let us know!

Summary

- Simulation of mixed beam with unequal A/Q started
  - Motivated by observations in demonstration experiment at SIS18

- Presently, only influence of ‘alien’ RF can be simulated
  - Simple purely longitudinal time-based code implemented
    - Based on phase functions $\varphi_1(t)$ and $\varphi_2(t)$ for acceleration and ‘alien’ RF
  - Xsuite for 6D tracking by relatively simple configuration of ‘alien’ cavity
    - Only non-trivial ingredient is phase $\varphi_2(t)$ of ‘alien’ RF
  - Codes reproduce observed behavior qualitatively
    - Insufficient data quality for quantitative comparison

- Simulation of beam-beam effects is an open question
  - No data for comparison so far due to low intensity in demonstration experiment

Thanks for your attention!

Number of turns from energy program

- Xsuite won't track beyond end of energy program
  - Must specify number of turns <= max number
  - Otherwise Xsuite fails with an error
  - How to determine max. number of turns?

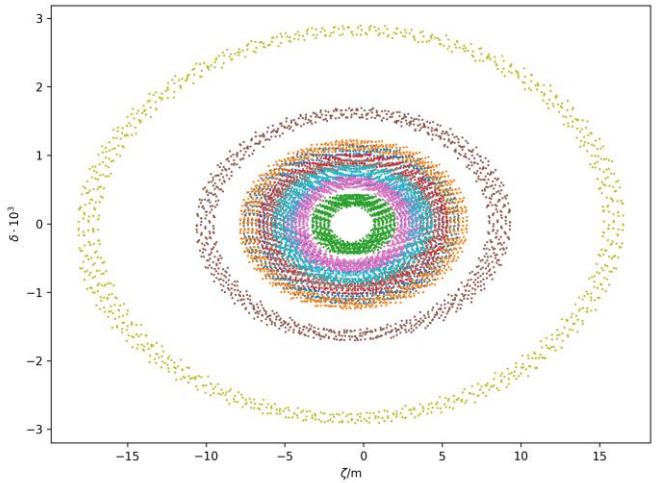
- Option 1: ask Xsuite
  - Xsuite's energy program provides 't_at_turn_interpolator'
  - Xsuite stops at t > t_at_turn_interpolator.y[-1]
  - Use t_at_turn_interpolator.x[-1] as max number of turns

- Option 2: use your phase function φ(t)
  - Number of turns given by $N \cdot h \cdot 2\pi = \varphi(t_{max})$
  - Drop fractional part of N, and Xsuite will agree,
    if your energy program has small enough time steps

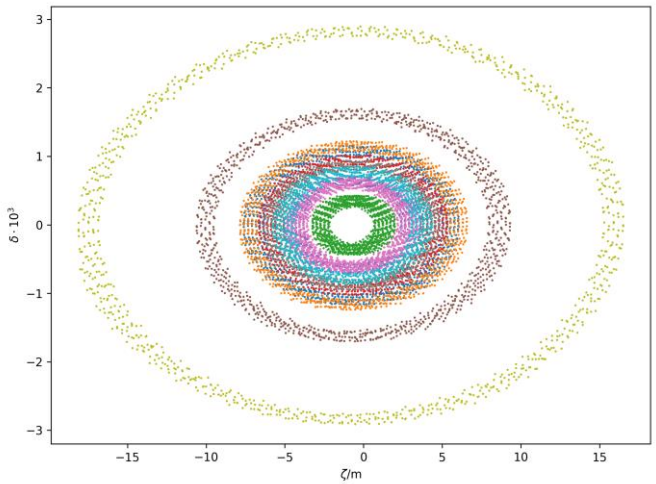
‘Alien’ phase vs. phase difference

- Time-based tracking
  - Cavity phase defined by phase function φ(t)
  - Obtained by integrating f(t) ~ β(t)
- Xsuite reference particle
  - Has sin(φ) = 0 at each cavity by construction
  - Equivalent to zero-crossings of φ(t) / h
- ‘Alien’ RF cavity relative to first cavity
  - Generally defined via phase difference $\varphi_2(t) - \varphi_1(t)$
  - But for reference particle $\varphi_1(t_r) = n \cdot h \cdot 2\pi$
  - **Phase can be given as $\varphi_2(t) - \varphi_1(t)$ or just $\varphi_2(t)$**
  - Verified in in Xsuite tracking

Tracking with ‘alien’ phase set to $\varphi_2 - \varphi_1$



Tracking with ‘alien’ phase set to φ_2



Calculation of the phase function

- Simulation of ‘alien’ RF requires phase function φ(t)
  - Requires integrating f(t) ~ β(t)
  - No analytical solution except for special cases
- Practical approach
  - Can numerically integrate frequency function f(t)
  - Discretize f(t) with time step smaller than revolution time
  - Use simple sum formula for approximating integral
- Analytical toy model
  - Define β as function of field or βγ
  - Linear change in βγ leads to analytical integral
  - For roundings, use cosh (start) and cos (end), which also both lead to analytical integrals
  - Phase then becomes piecewise analytical function

Analytical integrals for roundings in toy model

$$\phi = \int \frac{2\pi c}{L}\beta dt = \int \frac{2\pi c}{L}\frac{\beta\gamma}{\sqrt{1+(\beta\gamma)^2}}dt$$

$$\int \frac{\cosh(a\,x)\,dx}{\sqrt{b+\cosh^2(a\,x)}} = \frac{1}{a}\sinh^{-1}\left(\frac{\sinh(a\,x)}{\sqrt{b+1}}\right)+C$$

$$\int \frac{\cos(a\,x)\,dx}{\sqrt{b+\cos^2(a\,x)}} = \frac{1}{a}\sin^{-1}\left(\frac{\sin(a\,x)}{\sqrt{b+1}}\right)+C$$

Time derivative of βγ in toy model for SIS18

