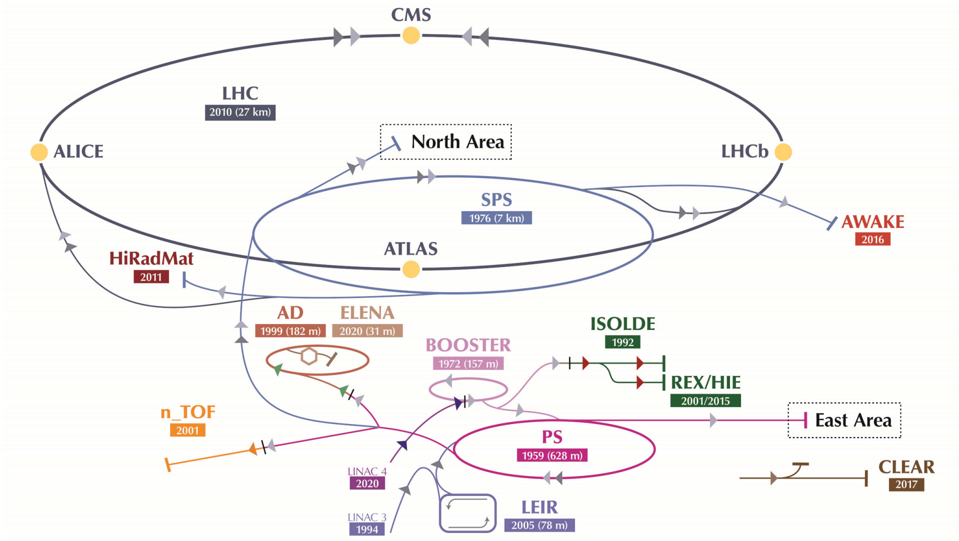
Brightness optimization in strong space charge machines

Martín Atria Paublo

Motivation & Fundamentals

Motivation



- **High-Luminosity LHC:** Luminosity depends on beam quality delivered by the injector chain.
- **The PSB Role:** Accelerates protons from 160 MeV to 2 GeV, feeding the Proton Synchrotron (PS).

The goal: Brightness

Beam brightness (B) is a metric for collider luminosity:

$$B = \frac{N}{0.5(\varepsilon_x + \varepsilon_y)}$$

- N: Beam intensity (number of protons per bunch)
- $\varepsilon_x, \varepsilon_y$: Transverse emittances

Challenge: At low energy, space charge forces induce emittance blow-up ($\varepsilon_\perp$ up) and intensity losses (N down), restricting overall brightness.

The Space Charge Effect

It is a collective effect, so it only applies when considering a bunch of particles, not particles alone by themselves.

It is only present at low energies, caused by the repulsion of protons from Coulomb's force, but it vanishes in the light-speed limit $v = c$.

$$F_{\perp} \propto \frac{q^2}{2\pi\varepsilon_0}(1 - \beta^2)\nabla\rho$$

At injection energy in the PSB (160 MeV, $\beta^2 \approx 0.2756$), this cancellation is small, making space charge a significant force.

How does it affect the beam

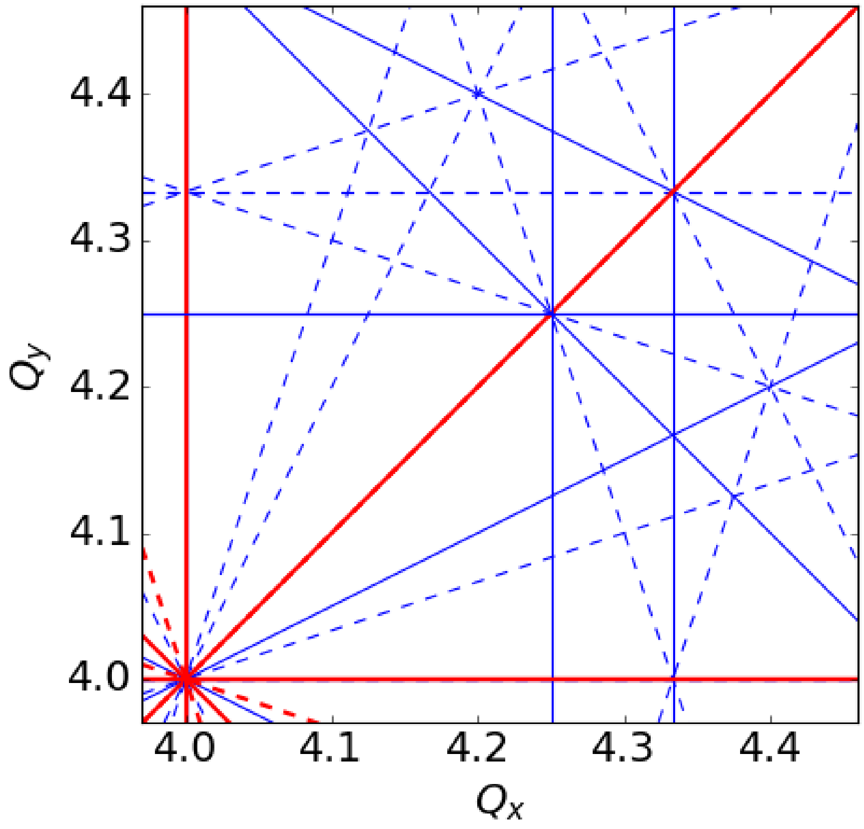
Space Charge defocuses particles non-uniformly across the beam, producing different *tune shifts*.

The maximum tune shift occurs at the beam core, and is given by:

$$\delta Q_{x,y} = -\frac{r_0 \lambda}{2\pi e \beta_r^2 \gamma_r^3} \oint \frac{\beta_{x,y}}{\sigma_{x,y}(s)\left[\sigma_x(s) + \sigma_y(s)\right]}$$

All the different tune shifts together produce something called a **tune spread** across the tune space.

Resonances: What are they

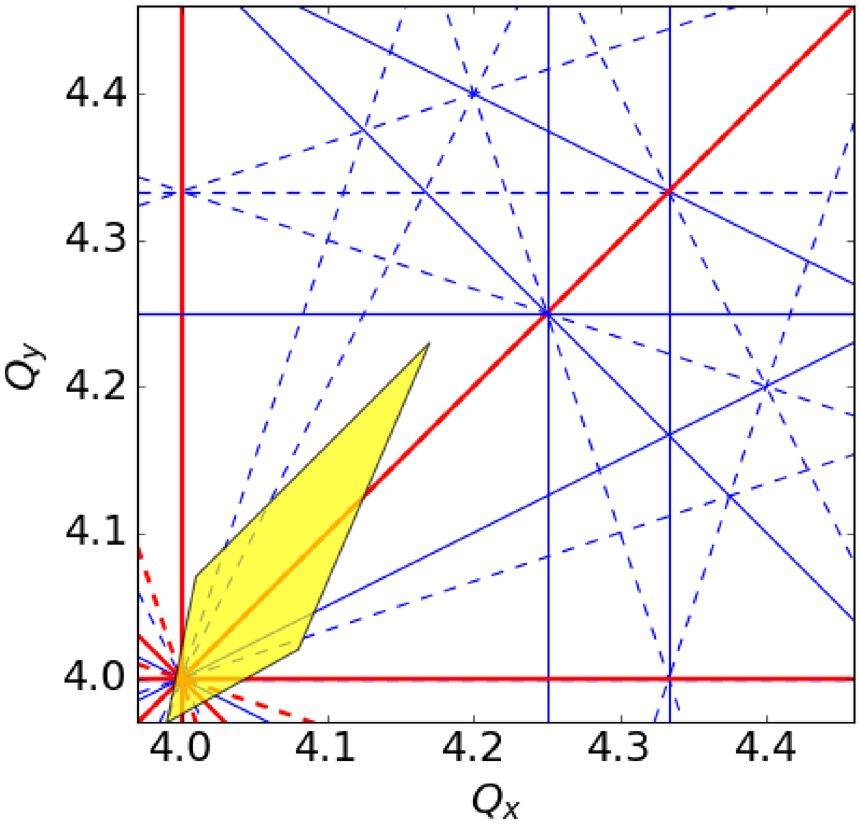


- They are the lines produced when the transverse tunes satisfy:
  $nQ_x + mQ_y = p, \;\; p \in \mathbb{Z}.$
- Two types of resonances:
  - **Structural resonances**

    $$p = 16k, \;\; k \in \mathbb{Z}$$

  - **Random resonances**

Resonances: What are they



The yellow necktie is an example of how a tune spread could look like (not an actual result).

We can see that some of the particles cross the resonance lines
$4Q_x = 16$,
$4Q_y = 16$,
$2kQ_x - 2kQ_y = 0$,
among others.

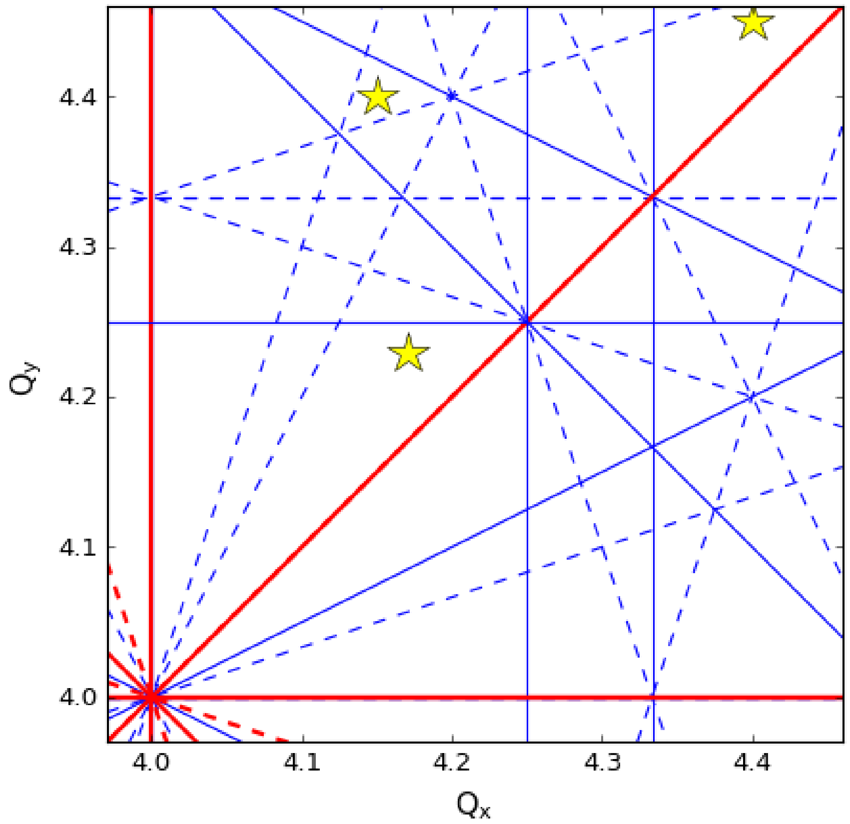
Resonances: Why are they bad?

Particles that periodically cross the resonances are driven towards larger transverse amplitudes. This increase of the amplitude leads to two unwanted phenomenons:

- **Emittance blow-up**
- **Intensity losses**

Both of these contribute to lower the beam brightness, so one way to optimize the brightness is to avoid periodic resonance crossings.

Resonances: How to avoid them



Intensity and energy are constrained, so our main control knob is selecting the *target* transverse tune - the **Working Point (WP)**

This allows us to move the working point away from the resonances. It doesn't have to be static, it can be changed mid run.

Simulation Setup & Results

Simulation Modeling Framework

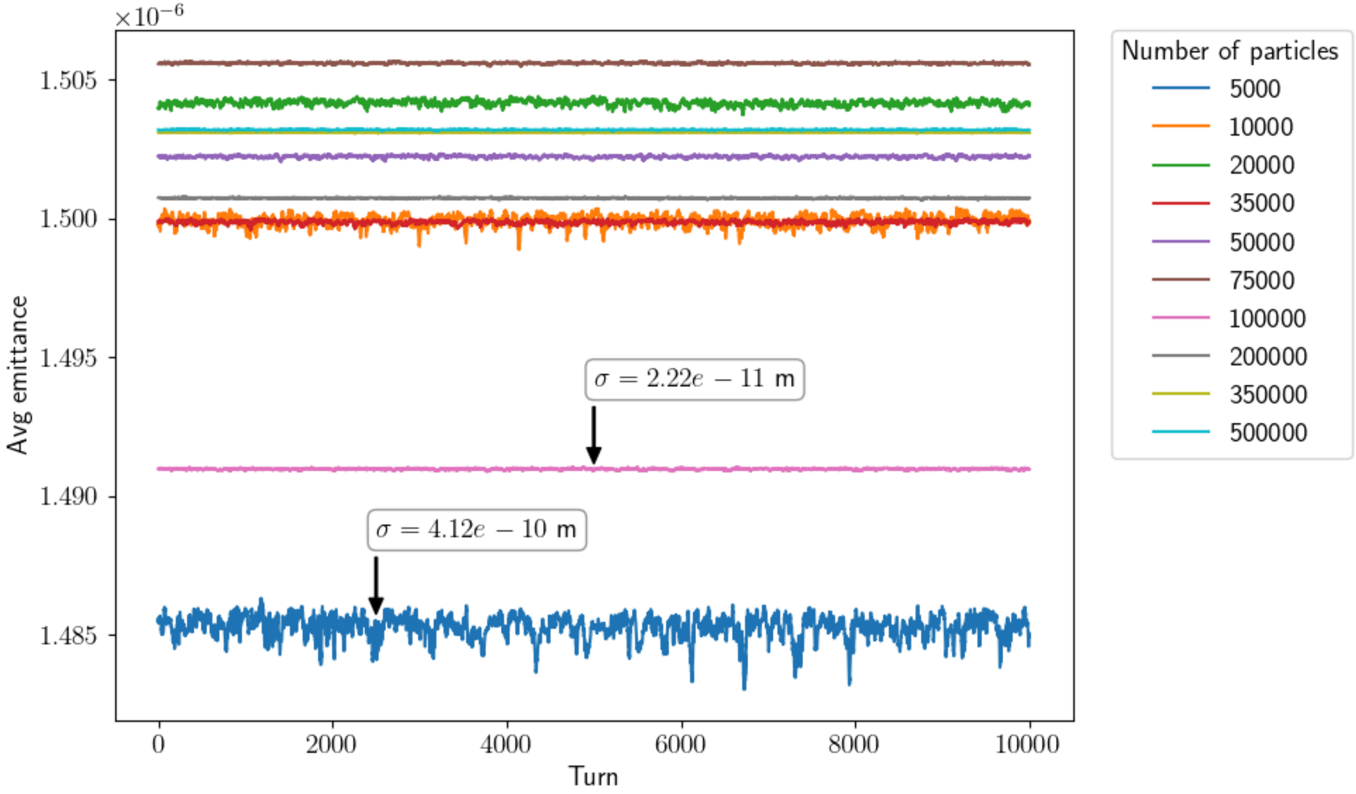
All simulations were done using the libraries from the `Xsuite` stack.

For space charge installation, there are three options:

- **Frozen / Quasi-Frozen**: Assumes a fitted Gaussian distribution.
- **Particle-In-Cell (PIC)**: Solves Poisson's equation on a grid at every space charge installation point.

The latter method needs a minimum amount of simulated particles, so that the numerical approach on the grid is accurate.

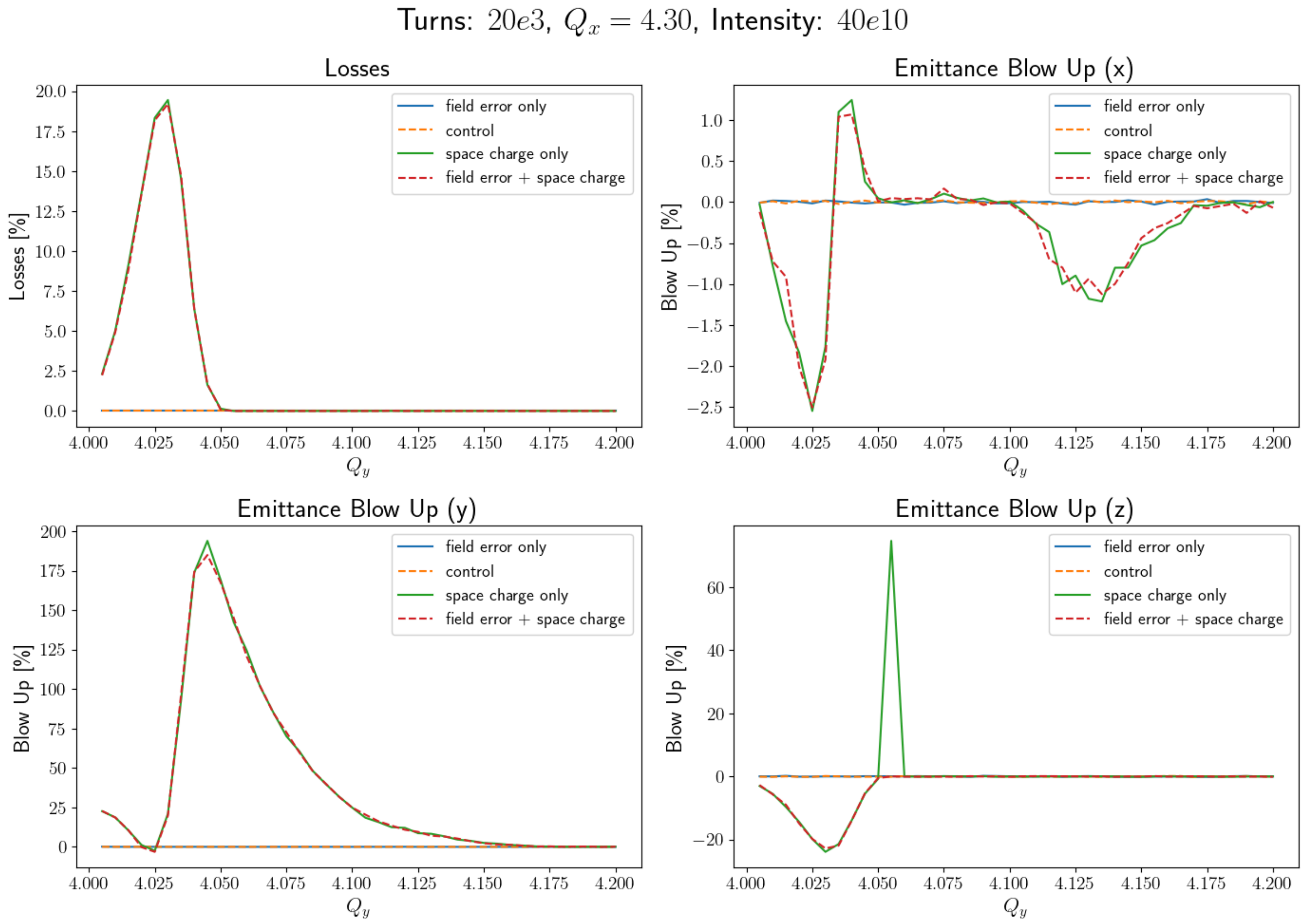
Simulation Modeling Framework

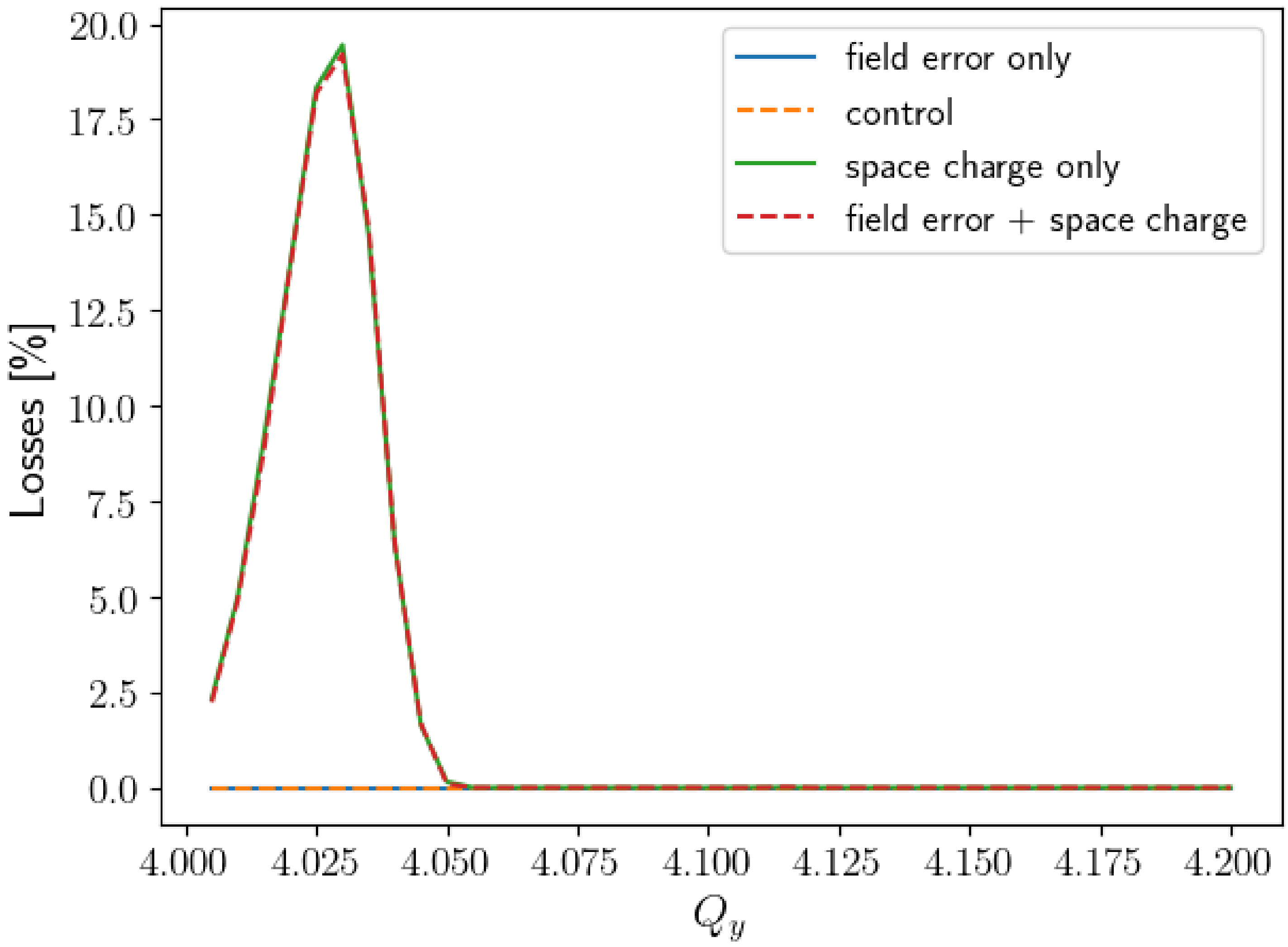


Baseline Scan: Verifying Losses and Blow-up Sources

To establish a simulation baseline and verify that the simulations work as intended with and without errors and effects in the lattice, the following simulations were done:

- **Control**: Expected no losses and no blow-up.
- **Field Error only**: Same as control, as the working point is far from the half-integer resonance.
- **Space Charge only**: Expected losses and emittance blow-up.
- **Field error + Space Charge**: Same as space charge only, field error should not contribute to losses or blow-up.


Turns: $20e3$, $Q_x = 4.30$, Intensity: $40e10$
Losses
Losses [%]
Emittance Blow Up (x)
Emittance Blow Up (y)
Emittance Blow Up (z)
Blow Up [%]
Q_y
field error only
control
space charge only
field error + space charge

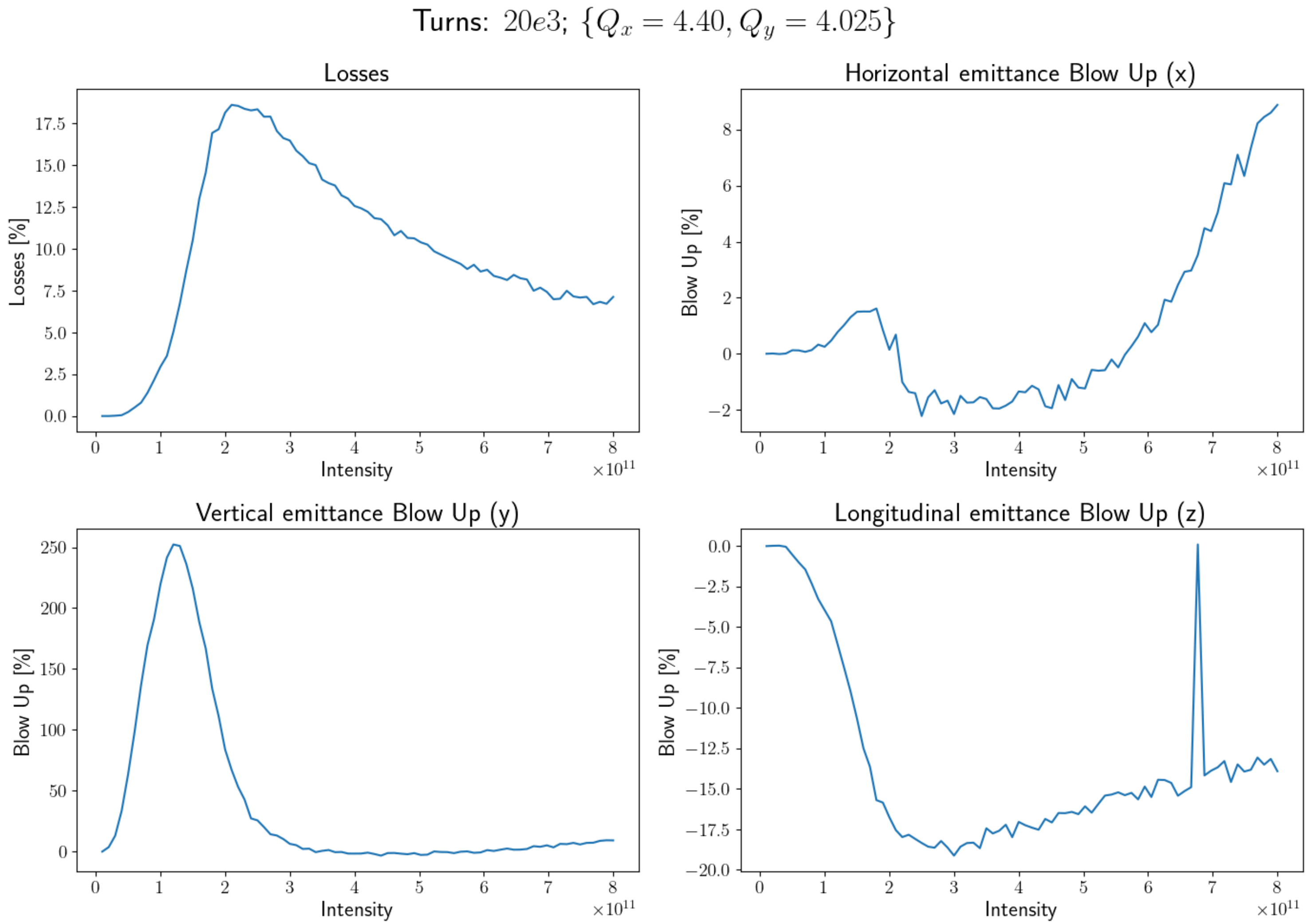


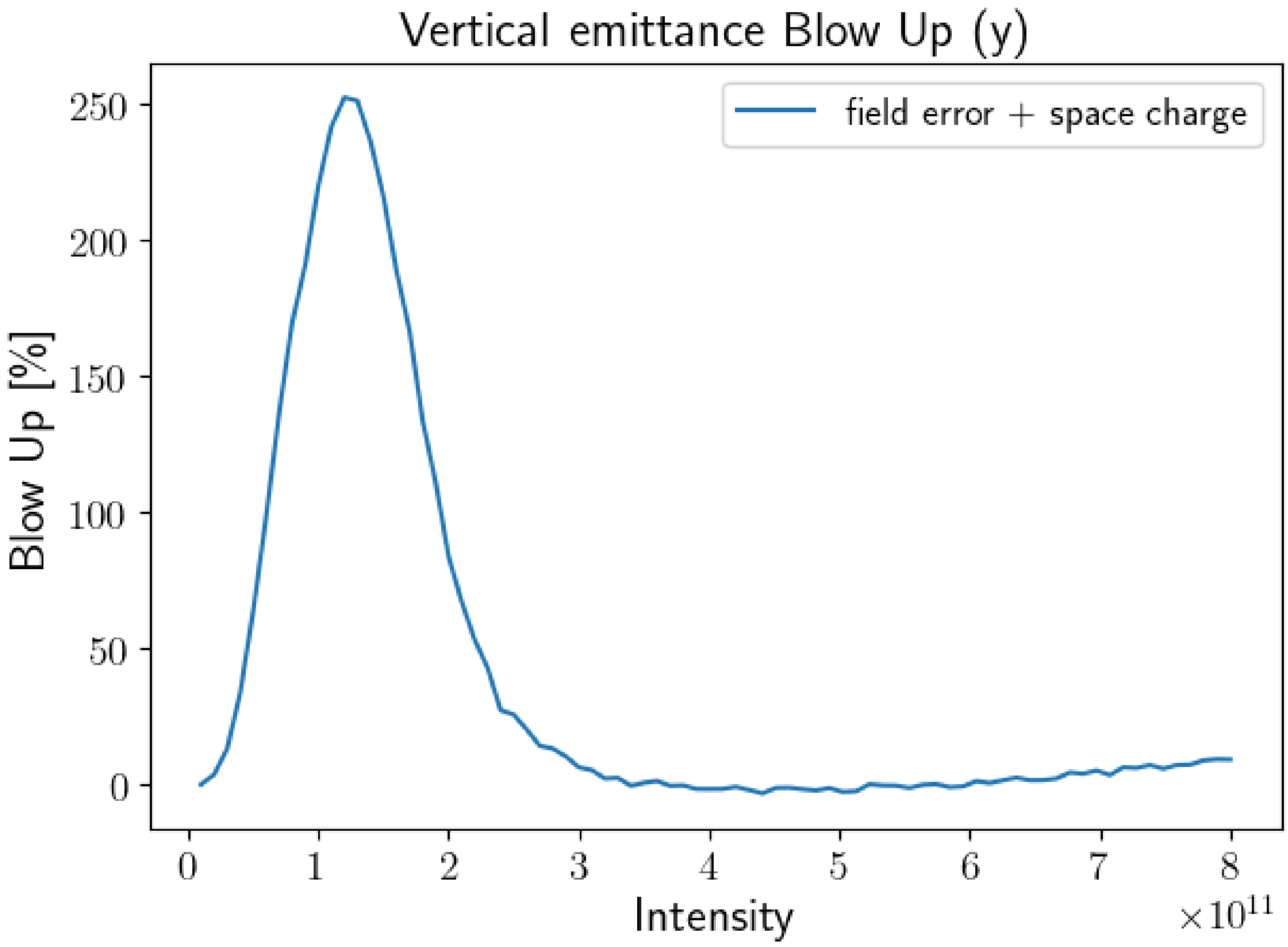
field error only
control
space charge only
field error + space charge
Losses [%]
Q_y
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
4.000
4.025
4.050
4.075
4.100
4.125
4.150
4.175
4.200

Intensity Scan near integer resonance

An intensity scan is performed near the integer resonance. The results are the following:

- **At low intensities**: The tune shifts are small, most of the particles stay close to the resonance, producing high losses and emittance blow-up.
- **At high intensities**: The tune shifts are large, driving the particles away from the resonance. Losses and emittance blow-up are small.


Turns: $20e3$; $\{Q_x = 4.40, Q_y = 4.025\}$
Losses
Losses [%]
Intensity
$\times 10^{11}$
Horizontal emittance Blow Up (x)
Blow Up [%]
Intensity
$\times 10^{11}$
Vertical emittance Blow Up (y)
Blow Up [%]
Intensity
$\times 10^{11}$
Longitudinal emittance Blow Up (z)
Blow Up [%]
Intensity
$\times 10^{11}$

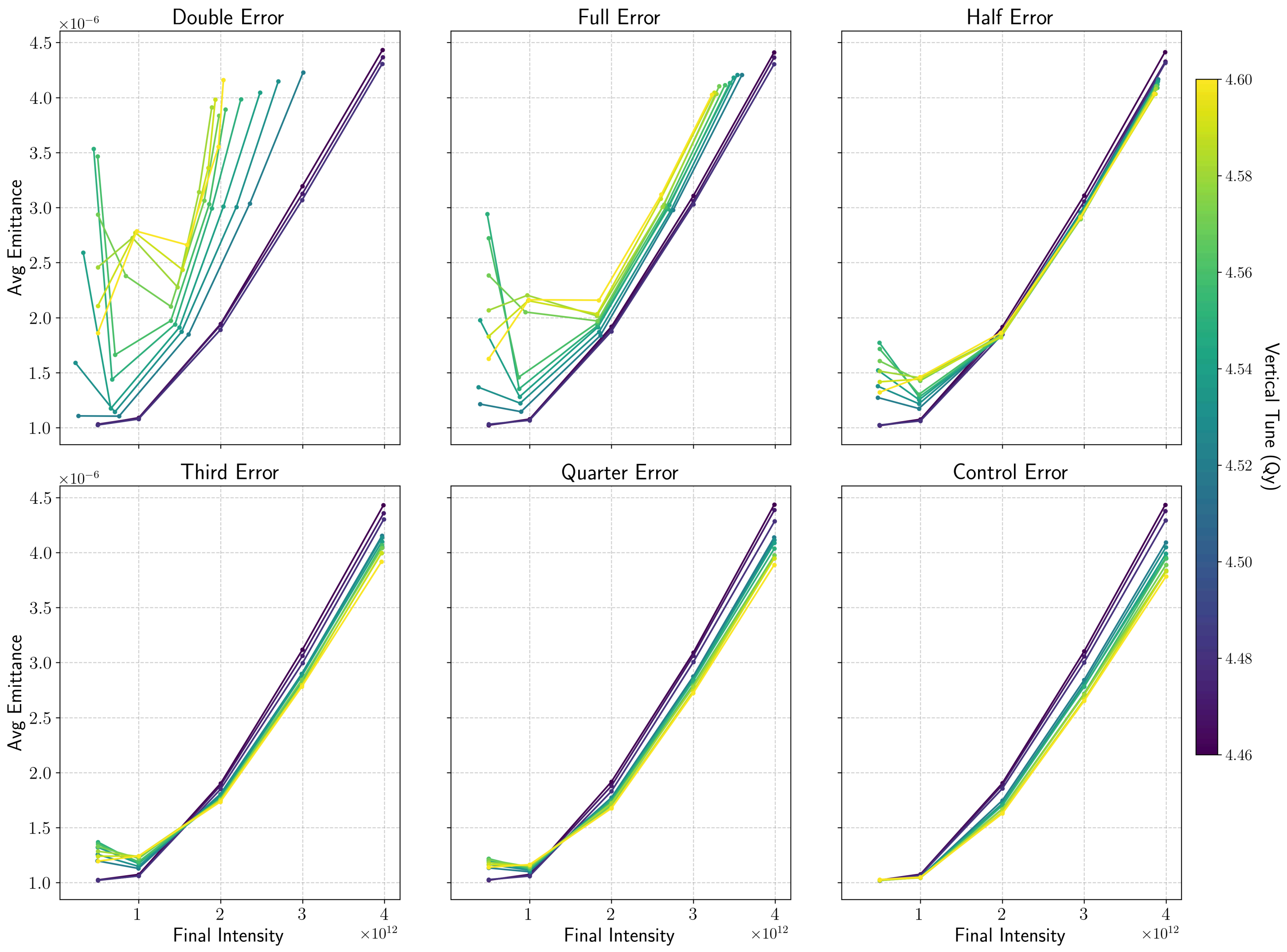


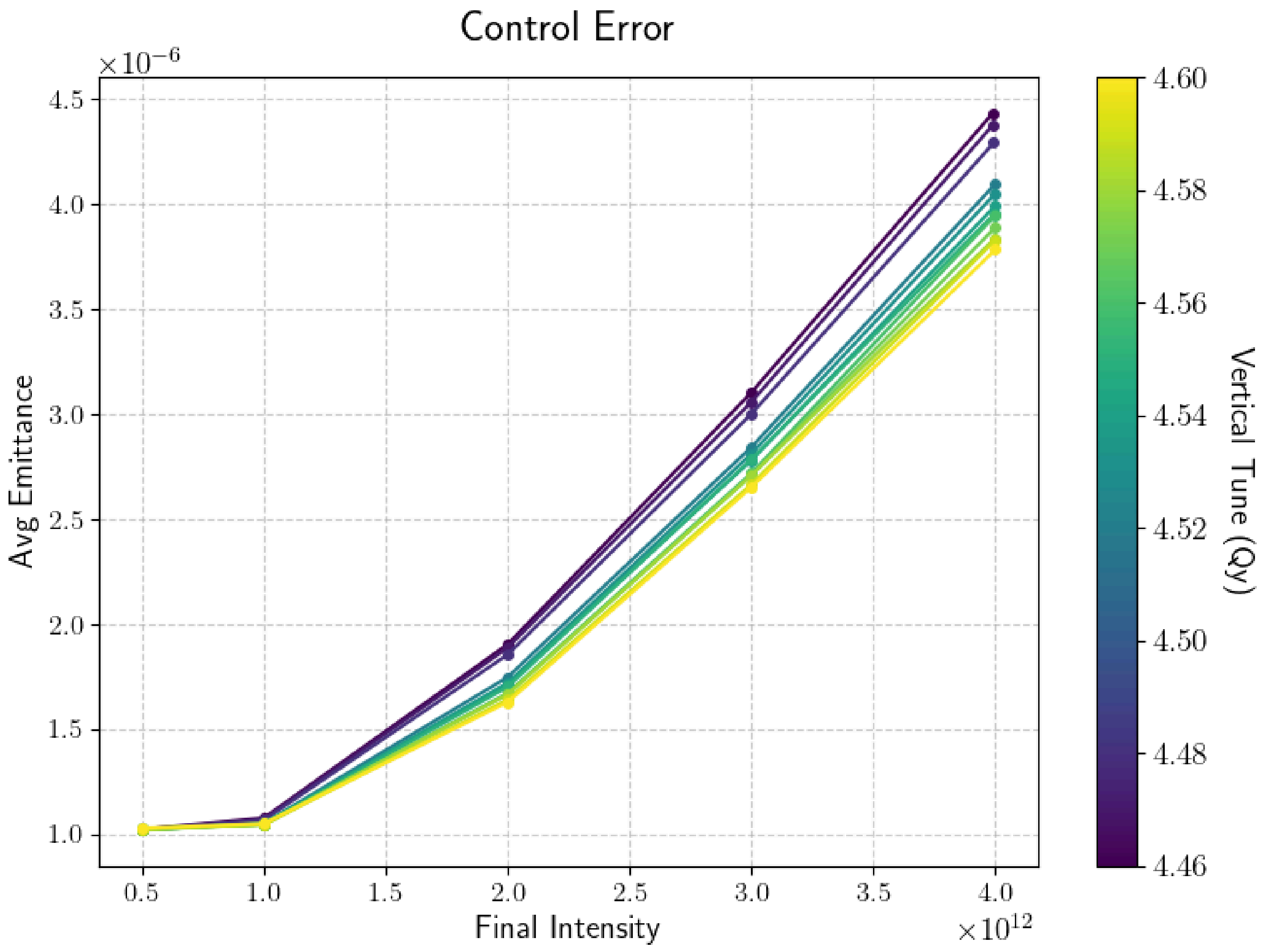
Vertical emittance Blow Up (y)
field error + space charge
Blow Up [%]
0
50
100
150
200
250
0
1
2
3
4
5
6
7
8
Intensity
×10^11

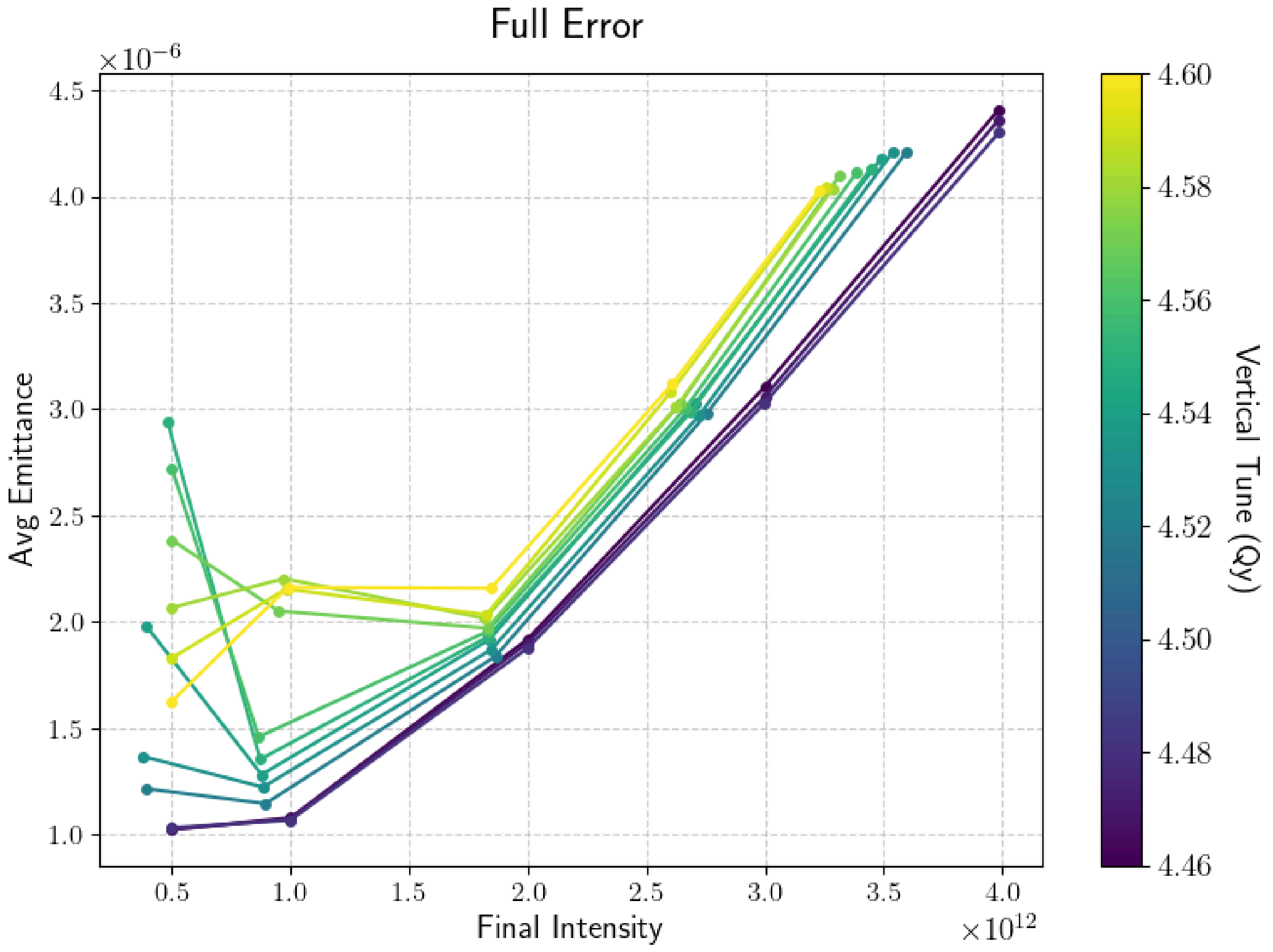
Sensitivity Scan above half integer

Next up, an error sensitivity scan is performed to see how the half-integer resonance holds up when setting the working point above it. We should expect two things:

- When the field error is small, the resonance should not appear excited, therefore, no losses or blow-up should be seen.
- When the field error is large, different intensities should behave as the previous results. At large intensity, the working point close to the resonance should not present large losses or blow-up.


Double Error
Full Error
Half Error
Third Error
Quarter Error
Control Error
Avg Emittance
$\times10^{-6}$
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
Final Intensity
$\times10^{12}$
1
2
3
4
Vertical Tune (Qy)
4.46
4.48
4.50
4.52
4.54
4.56
4.58
4.60


Control Error
$\times 10^{-6}$
Avg Emittance
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Final Intensity
$\times 10^{12}$
Vertical Tune (Qy)
4.60
4.58
4.56
4.54
4.52
4.50
4.48
4.46

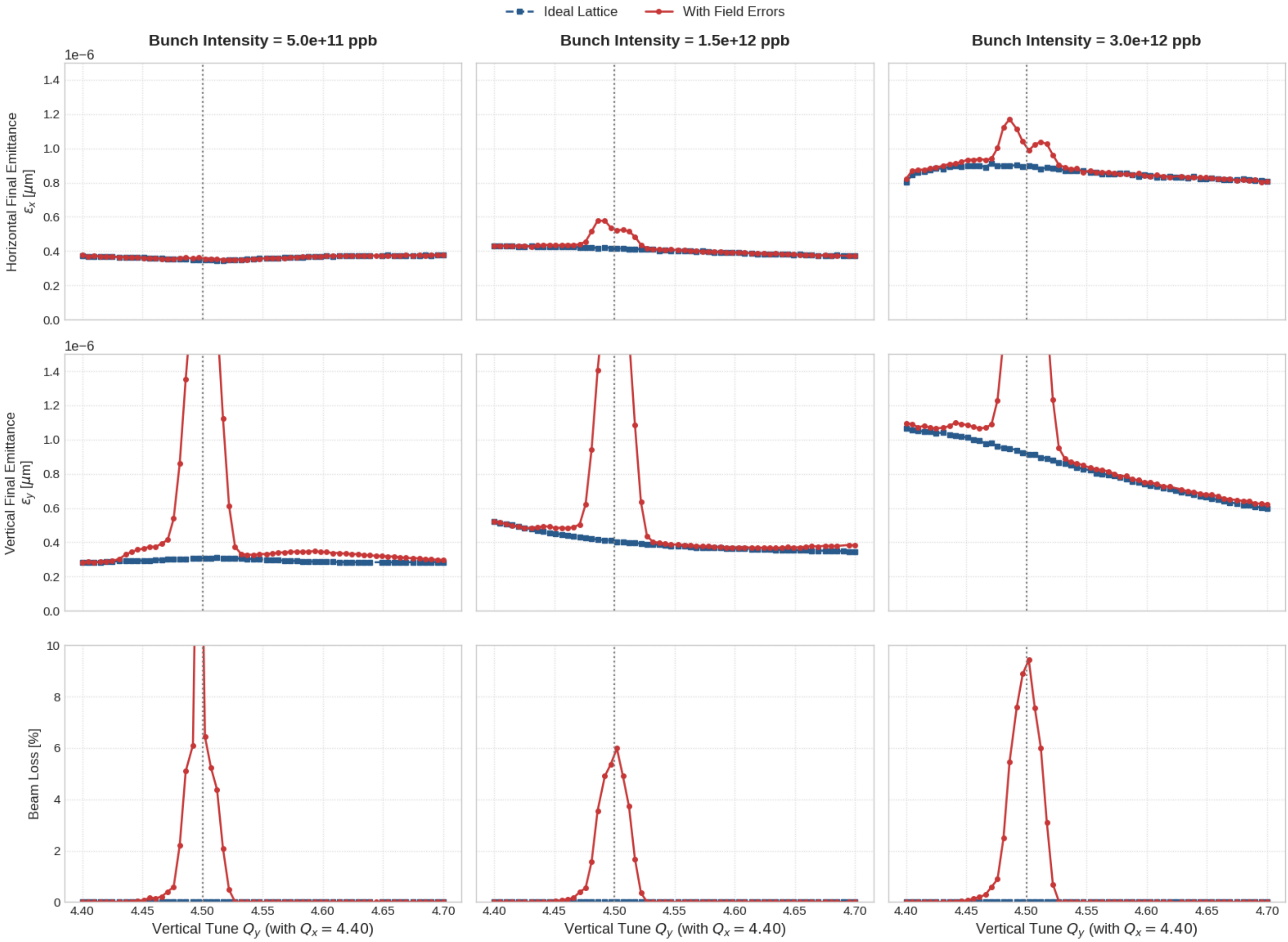


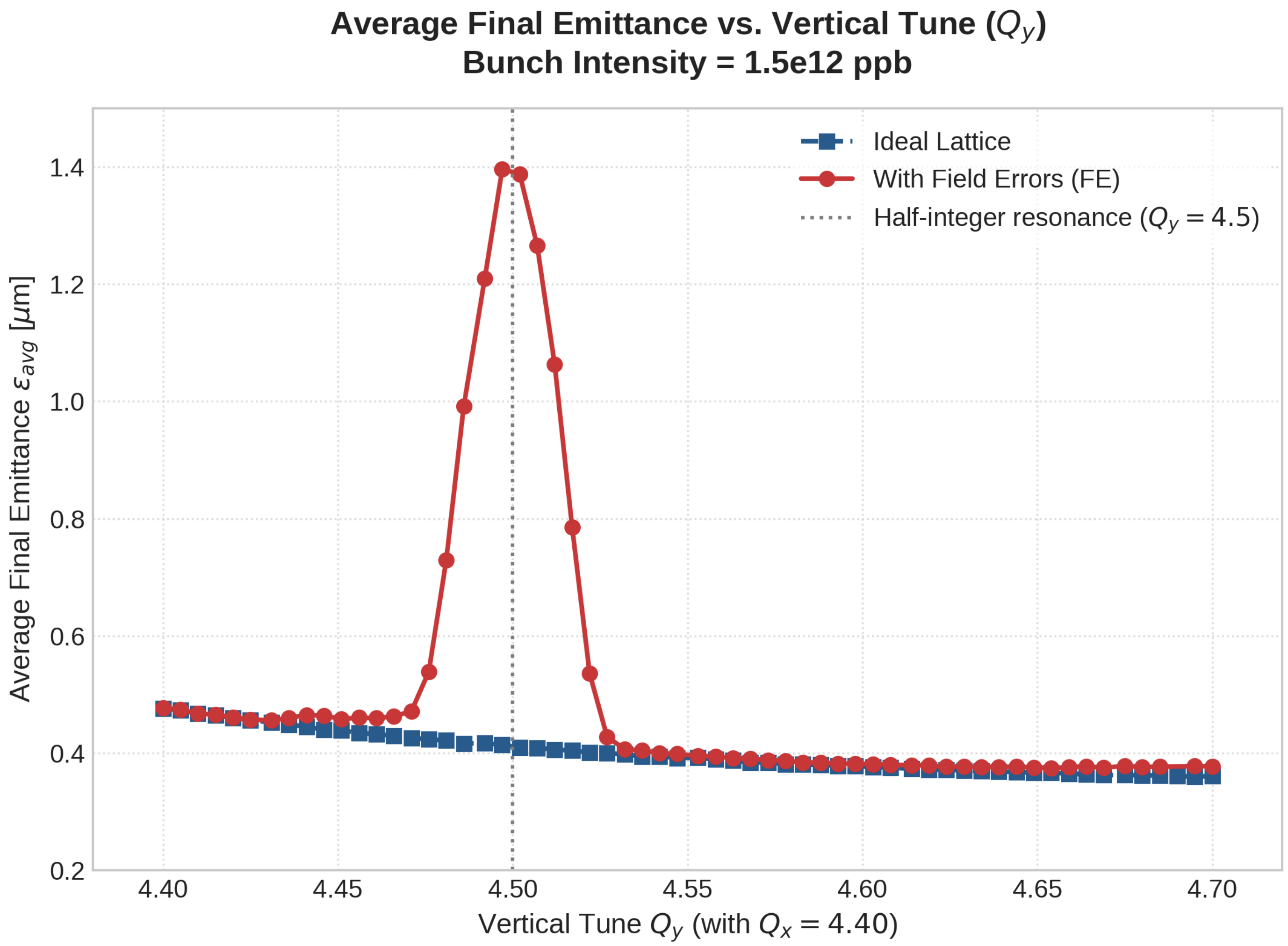
Full Error
×10^−6
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
Avg Emittance
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Final Intensity
×10^12
4.60
4.58
4.56
4.54
4.52
4.50
4.48
4.46
Vertical Tune (Qy)

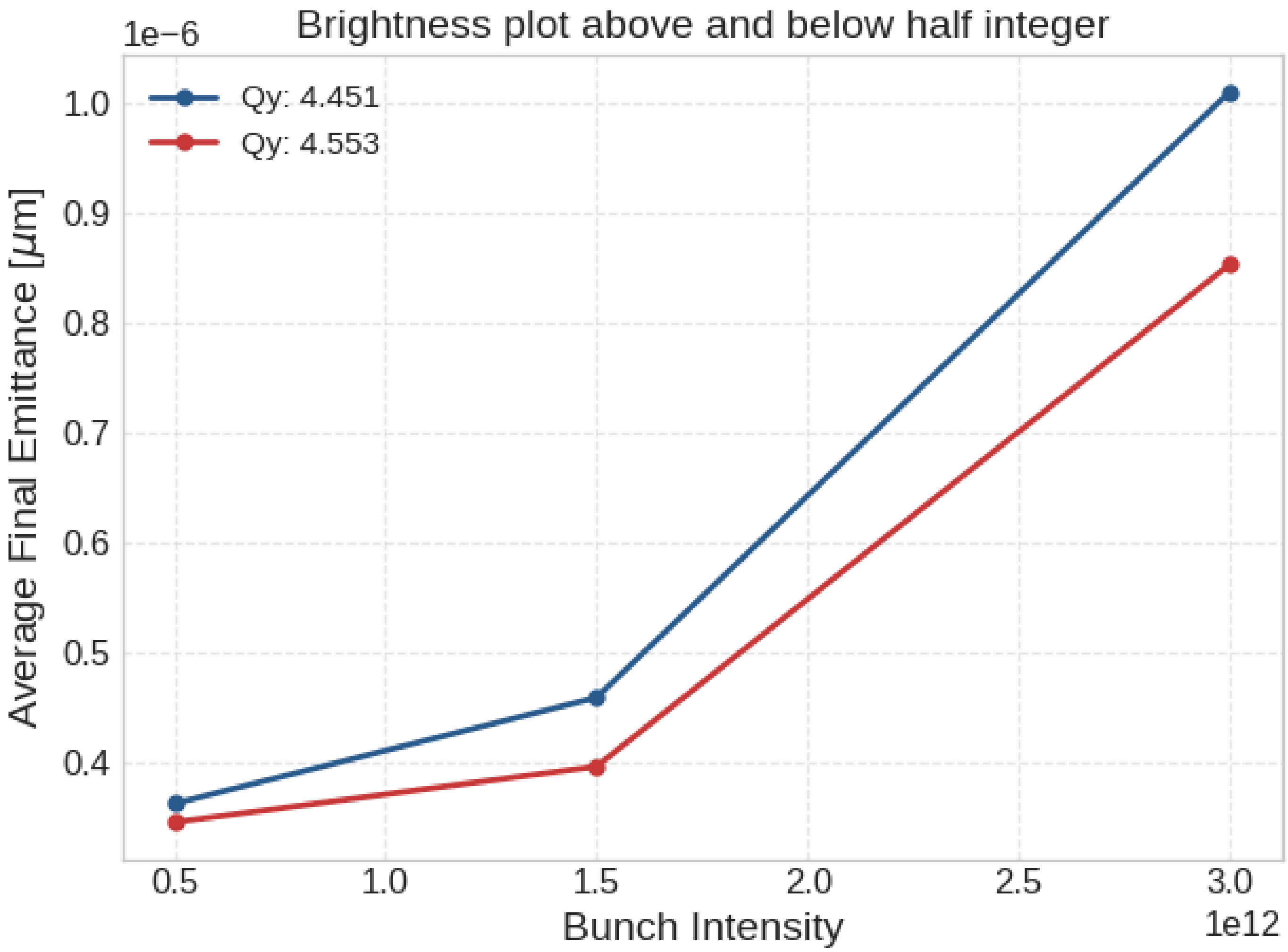
Broad Scan above half-integer

Finally, a broad scan over working points and intensities is performed to see if working above the half-integer resonance hold any advantage over working below it.

This set of simulations include an ideal lattice, and one with field error, includes the initial portion of acceleration, it is done with PIC solver with 100k particles, over 5k turns.


Ideal Lattice
With Field Errors
Bunch Intensity = 5.0e+11 ppb
Bunch Intensity = 1.5e+12 ppb
Bunch Intensity = 3.0e+12 ppb
1e−6
Horizontal Final Emittance
ε_x [μm]
Vertical Final Emittance
ε_y [μm]
Beam Loss [%]
Vertical Tune Q_y (with Q_x = 4.40)


Average Final Emittance vs. Vertical Tune (Q_y)
Bunch Intensity = 1.5e12 ppb
Ideal Lattice
With Field Errors (FE)
Half-integer resonance ($Q_y = 4.5$)
Average Final Emittance ε_{avg} [μm]
Vertical Tune Q_y (with $Q_x = 4.40$)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
4.40
4.45
4.50
4.55
4.60
4.65
4.70


Brightness plot above and below half integer
1e−6
Qy: 4.451
Qy: 4.553
Average Final Emittance [μm]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.5
1.0
1.5
2.0
2.5
3.0
Bunch Intensity
1e12

Conclusions

Main results: A grain of salt

The key findings of the last simulations are the following:

- **At low intensity ($N = 50 \cdot 10^{10}$)**: There seems to be little to no advantage in working above the half-integer resonance in comparison to below it.
- **At medium/high intensities ($N \geq 150 \cdot 10^{10}$)**: Operating above the half-integer resonance ($Q_y \gtrsim 4.55$) yields an advantage in the form of a flatter brightness slope.

Although these results are interesting, they should be taken with a grain of salt…

The rest of the salt

The simulations done at the end lack many of the real machine imperfections and effects. This means that while the results obtained are positive, we should delve deeper with more complicated simulations in order to determine if the advantage holds. Some of the missing effects and imperfections are:

- **Higher order magnetic field errors**
- **Chicane injection and multi-turn injection**

Future Work

More simulations should be carried in order to further study the viability of working above the half-integer resonance.

These future simulations should mainly consider **longer tracking** ($>$ 5k turns) and **dynamic tune ramps**. After these, the missing effects and imperfections from the previous slide should be studied.

Alongside the simulations, runs on the real machine should be done to test the theory, allowing us to compare it to the simulations.

Thank you :)