Imperfections Model of the FCC-ee with Corrections and Beam-Beam Effects

Master's Thesis Oral Defence
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Outline

I. Introduction and Overview

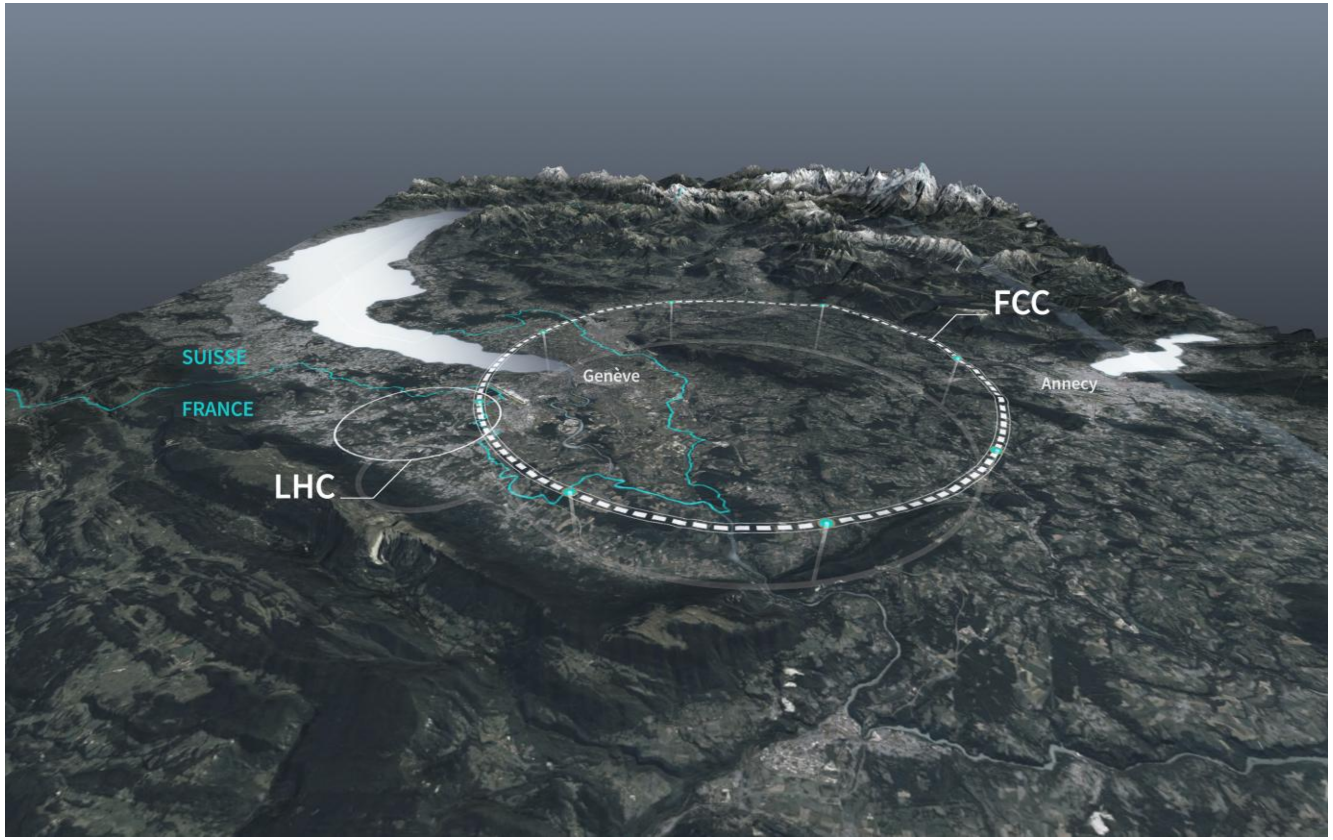
II. Imperfections Model of the FCC-ee

III. Global Orbit and Optics Correction

IV. Impact of Beam-Beam Effects

V. Conclusions

I. Introduction and Overview



Source: CERN Future Circular Collider

The Future Circular electron-positron Collider (FCC-ee):

- Proposed next-generation particle collider of 90.7 km length
- Designed to operate at four energies: 45.6 GeV (Z), 80 GeV (WW), 120 GeV (ZH), and 182.5 GeV ($t\bar{t}$)
- Two optics designs:
  GHC (developed since 2012 by K. Oide et al.)
  LCC (developed since 2022 by P. Raimondi et al.)

2014-2018:
Conceptual Design Study

2021-2025:
Feasibility Study
GHC as the optics baseline

Early 2026:
Optics Evaluation
LCC chosen as the new baseline

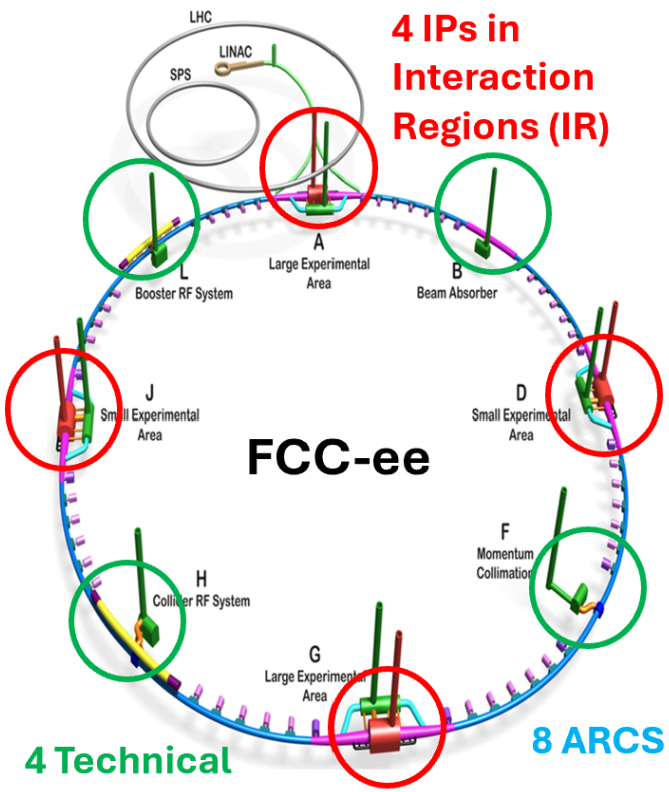
2028:
Expected decision point

Late 2040s:
Start of operations (?)

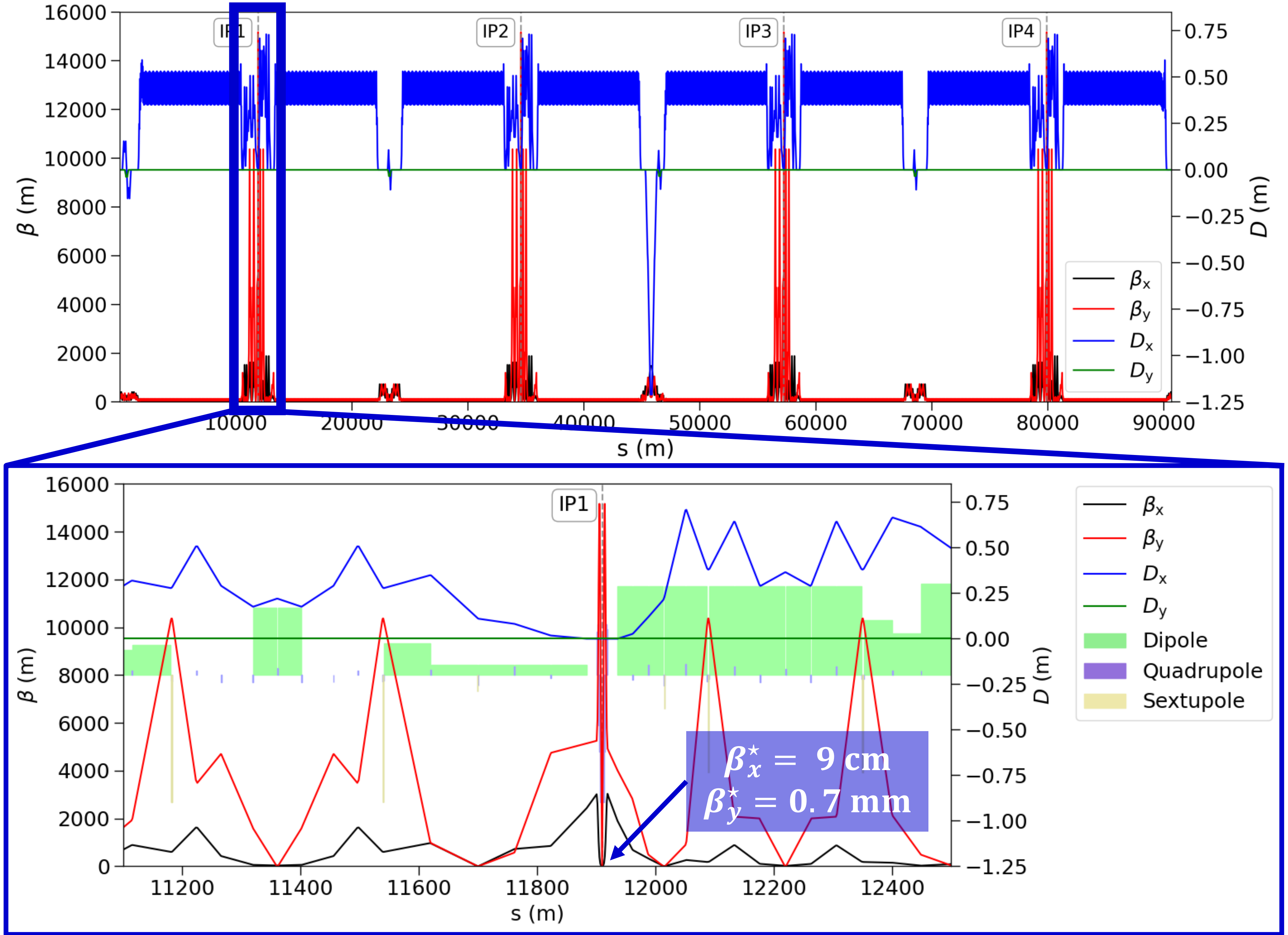
→ **LCC lattice at Z pole**

I. Introduction and Overview

B. LCC Optics



Adapted from FCC Feasibility Study Report.



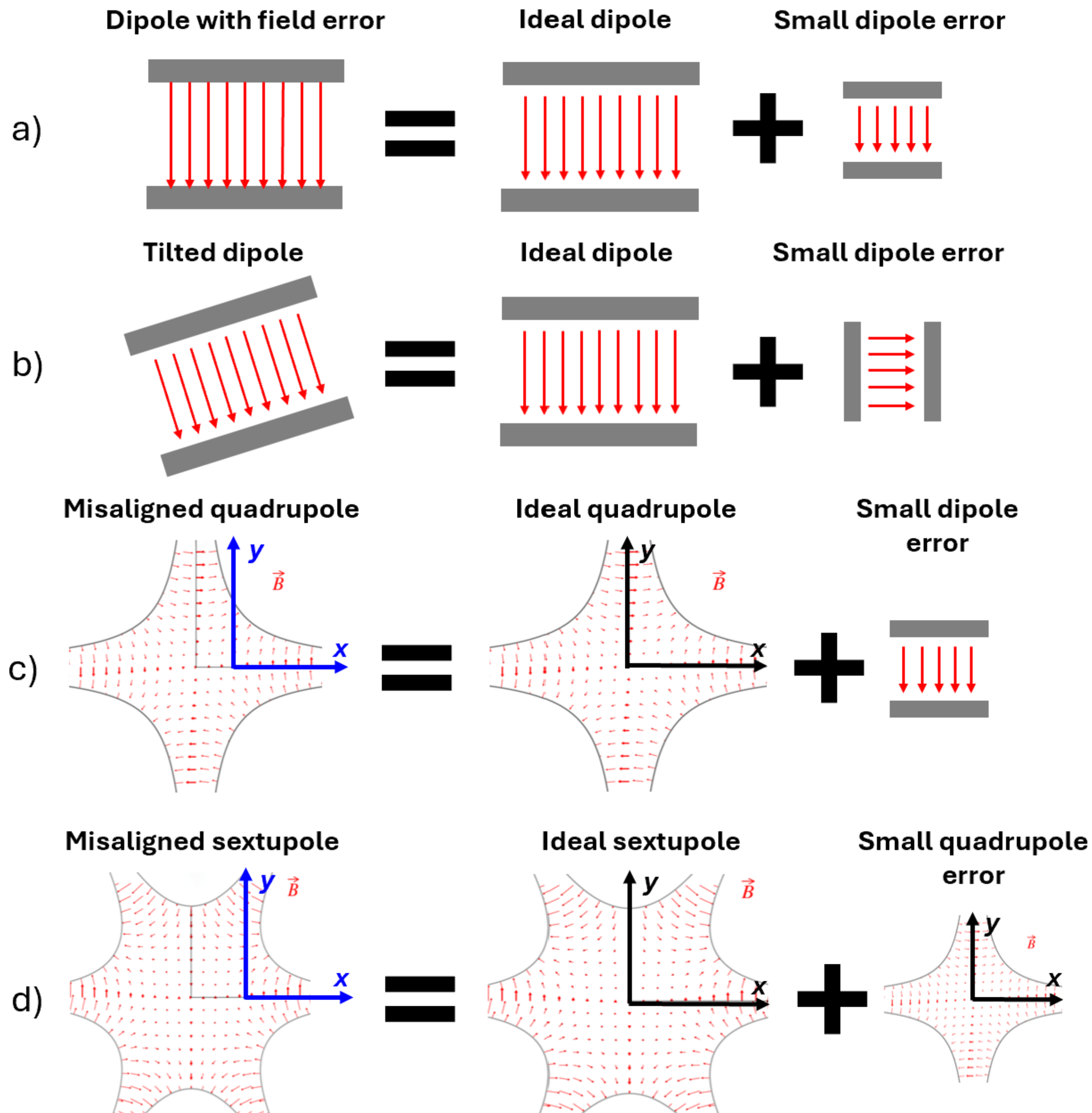
LCC lattice at Z pole

II. Imperfections Model of the FCC-ee

II. Imperfections Model of the FCC-ee — A. Theory

	Field type	Imperfection	Error type	Impact
a)	Dipole	Field error	Dipole	Orbit, trajectory, energy
b)	Dipole	Roll	Dipole	Orbit, trajectory
	Quadrupole	Field error	Quadrupole	Tune, optics
c)	Quadrupole	Offset	Dipole	Orbit, trajectory
	Quadrupole	Roll	Skew quadrupole	Coupling
	Sextupole	Field error	Sextupole	Chromaticity
d)	Sextupole	Horizontal offset	Quadrupole	Tune, optics
	Sextupole	Vertical offset	Skew quadrupole	Coupling

J. Wenninger, “Linear Imperfections”, CAS 2020.



Adapted from J. Wenninger.

Orbit Distortion

$$u(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)} \oint \theta(\bar{s}) \sqrt{\beta(\bar{s})} \cos\left(\pi Q - |\mu(s) - \mu(\bar{s})|\right)\, d\bar{s}$$

Kicks θ at locations $\bar{s}$

u – Transverse coordinate (x or y)
s – Longitudinal coordinate
$\bar{s}$ – Location of the error

Optics Distortion

- **Tune shift**

$$\Delta Q = \frac{1}{4\pi} \oint \delta K(\bar{s}) \beta(\bar{s})\, d\bar{s}$$

Field errors δK at locations $\bar{s}$

- **Beta-beating**

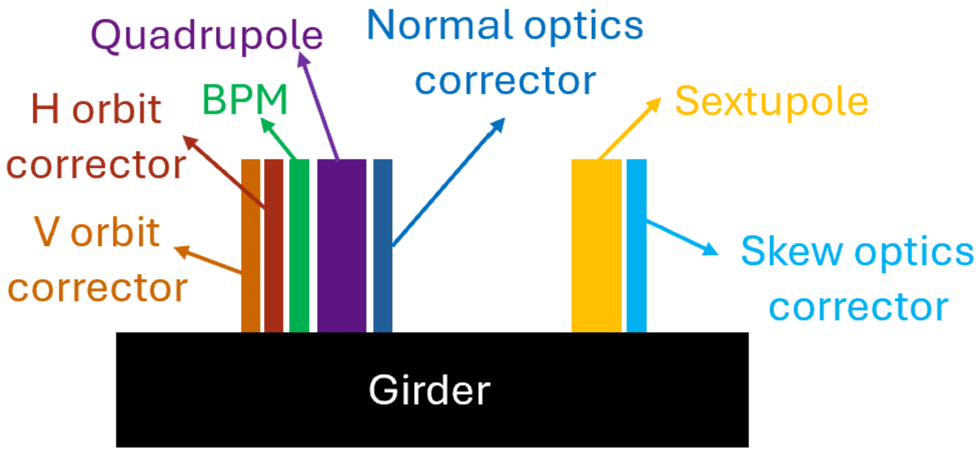
$$\frac{\Delta\beta(s)}{\beta(s)} = \frac{1}{2\sin(2\pi Q)} \oint \beta(\bar{s})\, \delta K(\bar{s}) \cos(2|\mu(s) - \mu(\bar{s})| - 2\pi Q)\, d\bar{s}$$

II. Imperfections Model of the FCC-ee

B. Modelling Imperfections

Element type	# Elements
Dipole	2436
Quadrupole	2684
Sextupole	1952
BPM	2219
H orbit corrector	2259
V orbit corrector	2259
Normal optics corrector	2668
Skew optics corrector	1950

Relevant LCC lattice elements

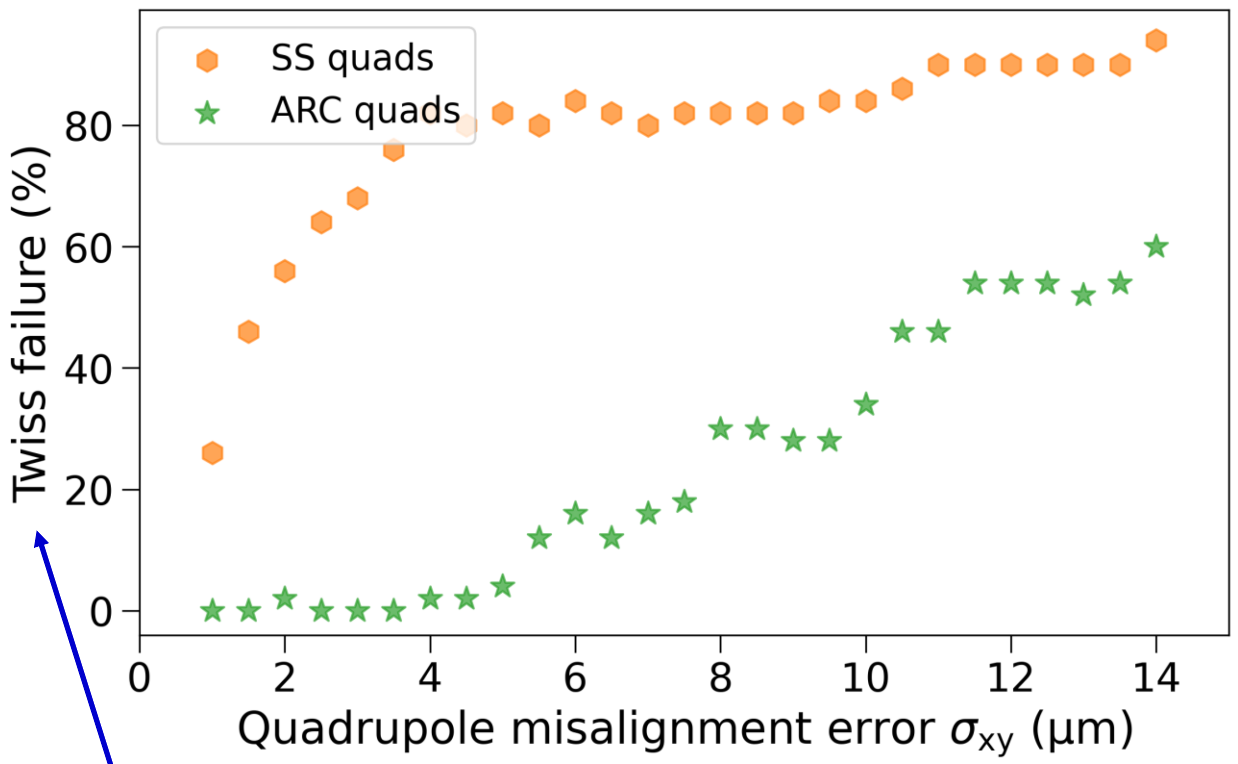


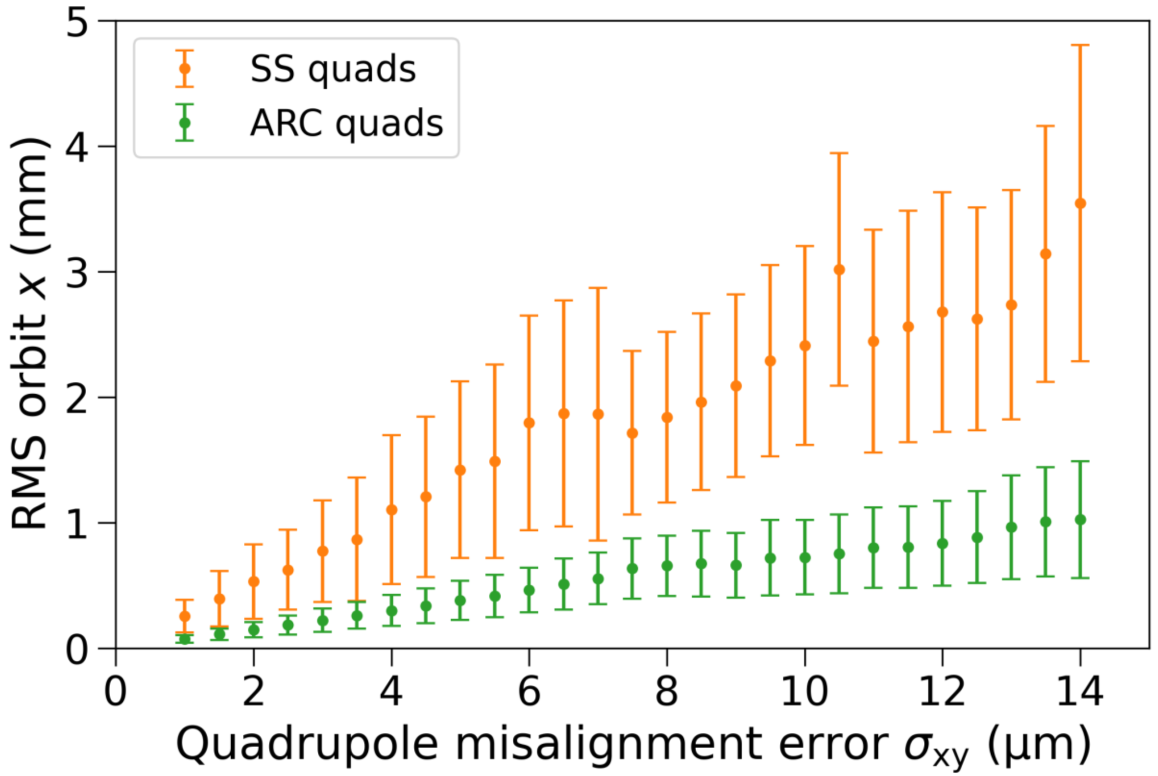
Schematic of an arc section (not to scale)

How to apply errors to the lattice elements (in Xsuite)

- Regex-based filtering system to access the desired element
- Imperfections are applied randomly based on a Gaussian distribution truncated at 2.5σ → different “seeds”
- Imperfections include:
  - Transverse and longitudinal misalignments (x, y, s)
  - Rotations around the longitudinal axis (s-rotation)
  - Field errors
- Appropriate error tolerances need to be chosen for ARC and SS (straight section) elements

Imperfections leading to **orbit** perturbations: Quadrupole misalignments

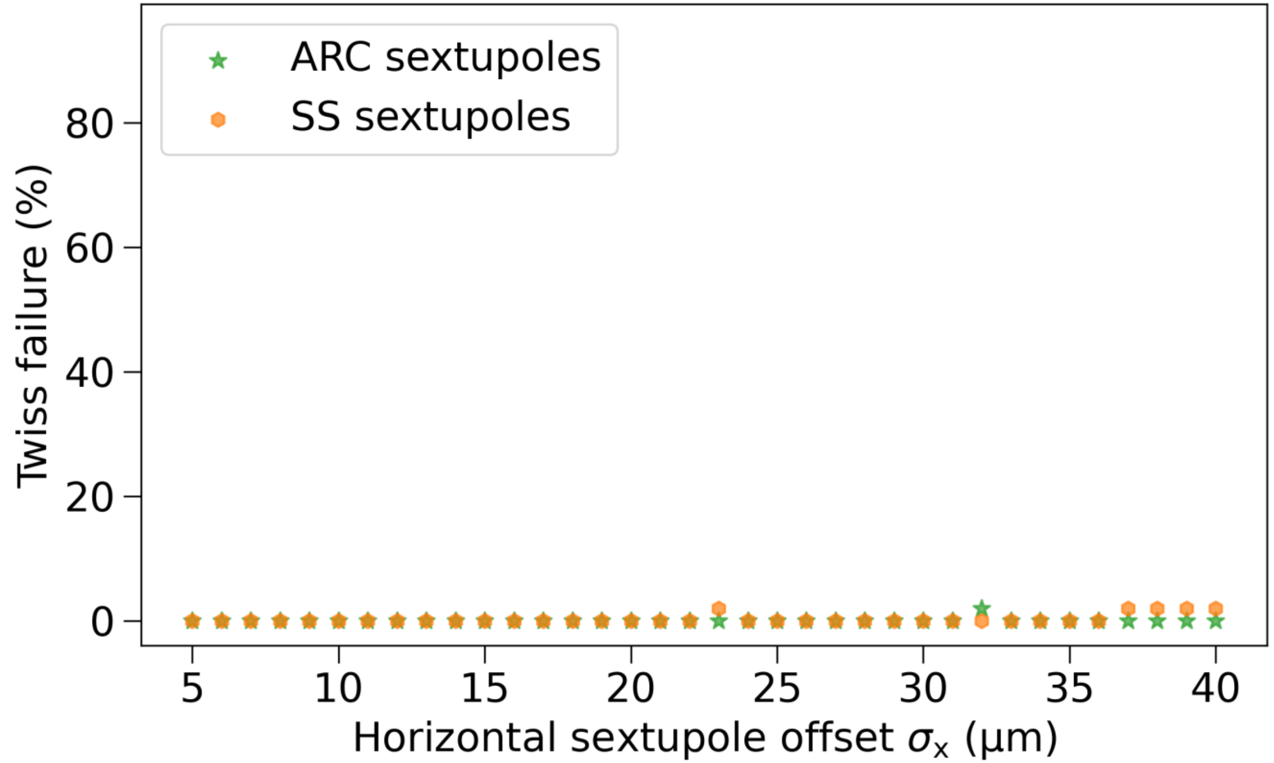


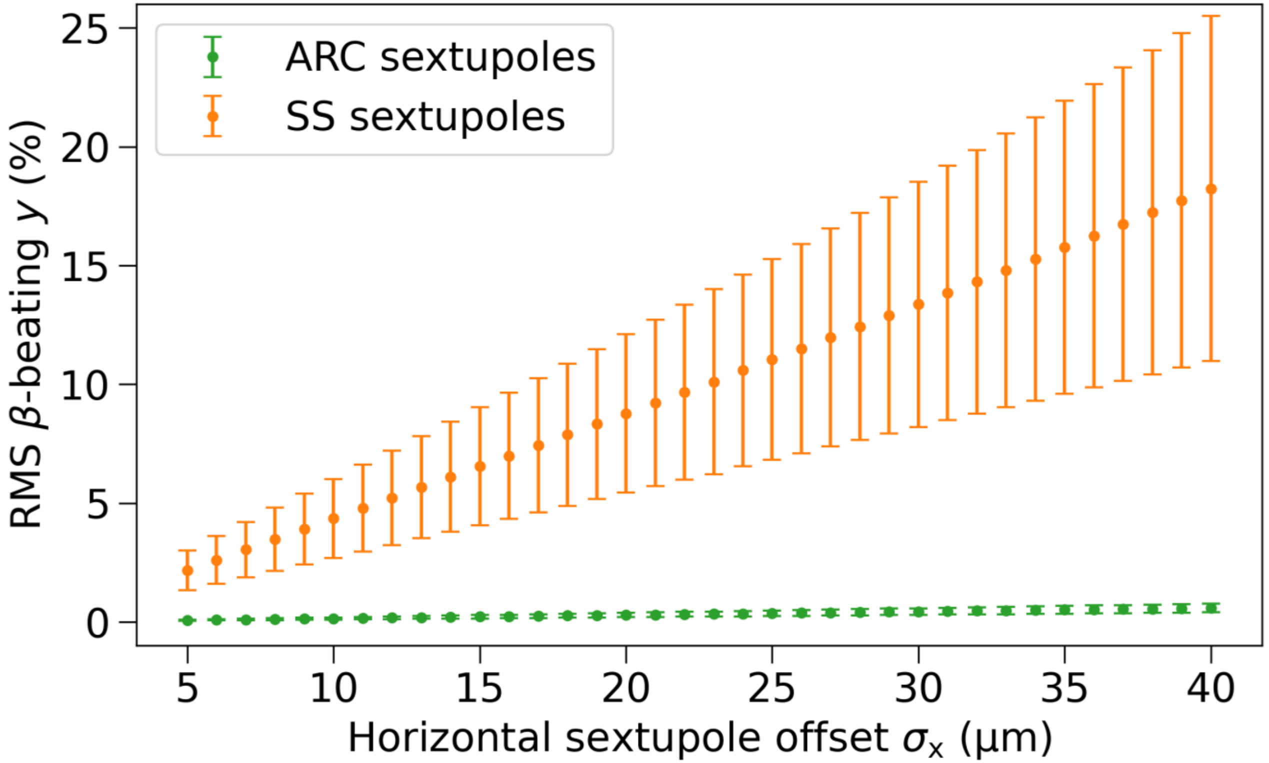


Twiss failure:
Twiss method fails as Twiss matrix becomes unstable and no closed orbit is found

1) Twiss failure more likely for SS quadrupole misalignments
2) Orbit seems more sensitive towards SS quadrupoles → Tighter alignment tolerances

Imperfections affecting the **optics** parameters (such as the β-function): Sextupole misalignments





1) Closed orbit is virtually unaffected
2) Horizontal offset of SS sextupoles produces more pronounced β-beating effects than ARC sextupoles

D. Error Tolerances

Section	Element	Misalignment			Rotation	Field error (rel.)
		σ_x (μm)	σ_y (μm)	σ_s (μm)	σ_θ (μrad)	
ARC	Dipole	1000	1000	500	1000	k_0: 1×10^{-3}
ARC	Quadrupole	50	50	100	50	k_1: 2×10^{-4}
ARC	Sextupole	50	50	100	50	k_2: 2×10^{-4}
ARC	Girder	150	150	500	150	–
STRAIGHT SECTION	IR Dipole	1000	1000	100	1000	k_0: 1×10^{-3}
STRAIGHT SECTION	TR Dipole	1000	1000	500	1000	k_0: 1×10^{-3}
STRAIGHT SECTION	FD Quadrupole	10	10	100	10	k_1: 1×10^{-5}
STRAIGHT SECTION	FF Quadrupole (excl. FD)	30	30	100	30	k_1: 1×10^{-4}
STRAIGHT SECTION	TR Quadrupole	100	100	100	100	k_1: 2×10^{-4}
STRAIGHT SECTION	FF Sextupole (incl. CS)	30	30	100	30	k_2: 1×10^{-4}
–	BPM	10	10	–	10	–

IR = Interaction Region
FF = Final Focussing
TR = Technical Region
CS = Crab Sextupole
FD = Final Doublet
BPM = Beam Position Monitor

III. Global Orbit and Optics Correction

Aim: Compensating for the unintended distortions caused by lattice imperfections by using dedicated corrector magnets

$$\Delta u_i = R_{ij}\,\Delta\theta_j$$

Orbit variation measured at BPM i

Response matrix (RM)

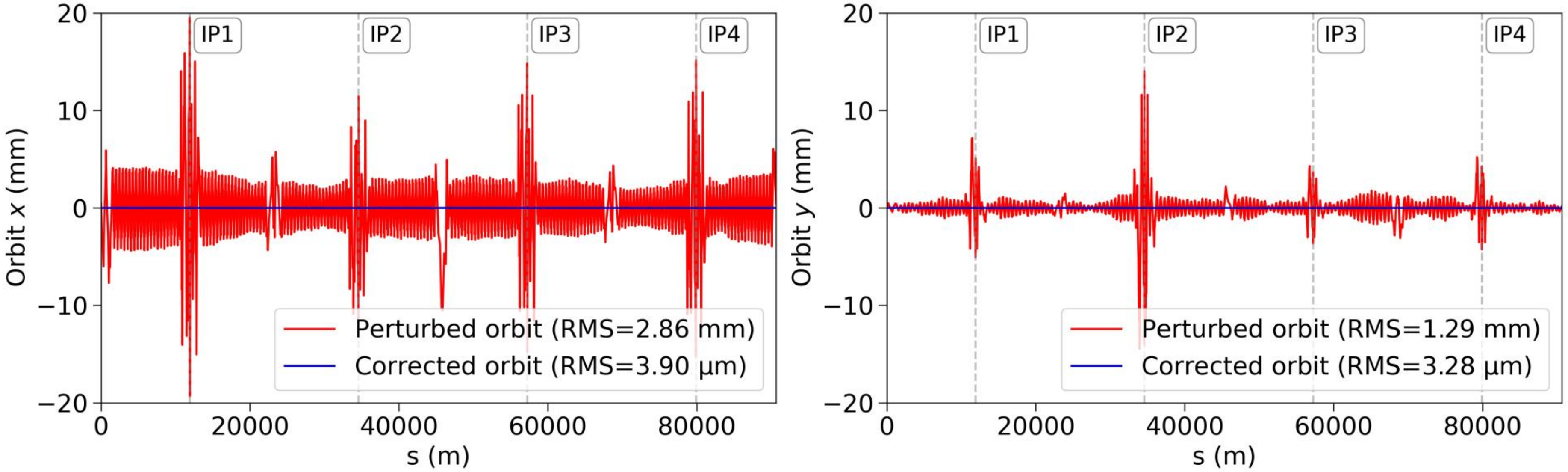
Kick applied by corrector j

Pseudo-inversion of RM using SVD: $\Delta\vec{\theta} = R^{\dagger}\,\Delta\vec{u}$

Orbit correction steps

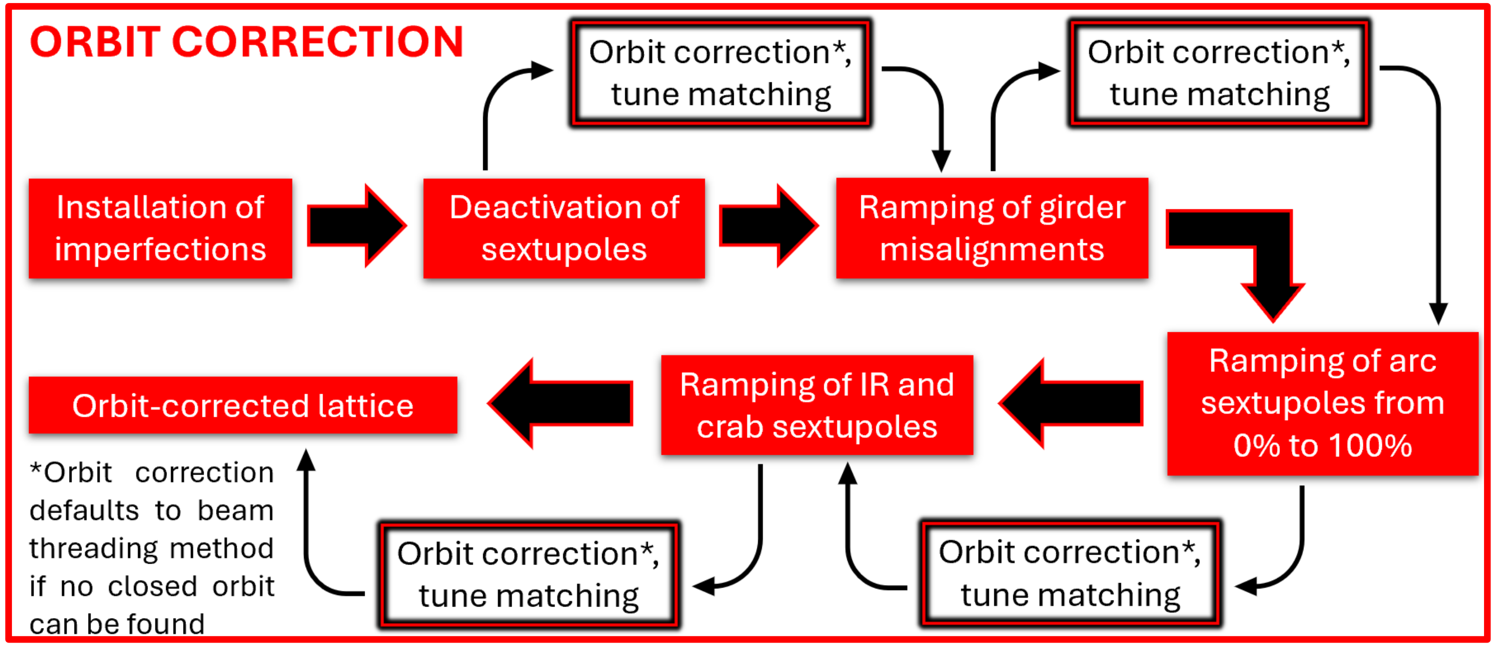
1. Construction of orbit RM
2. Pseudo-inversion via singular value decomposition (SVD)
3. Application of the required corrector strengths to modify the beam trajectory

Lattice with arc quadrupole misalignments before (red) and after (blue) orbit correction

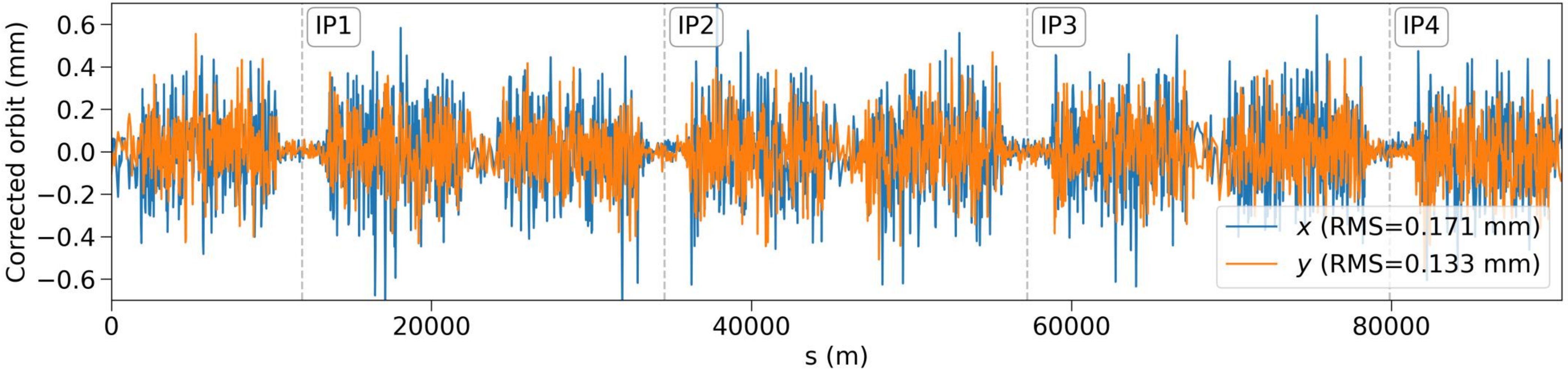


III. Global Orbit and Optics Correction

A. Orbit Correction



Transverse orbits after correction of a lattice with imperfections (no closed orbit exists before correction)



III. Global Orbit and Optics Correction

B. Optics Correction

OPTICS CORRECTION

Choice of observables:

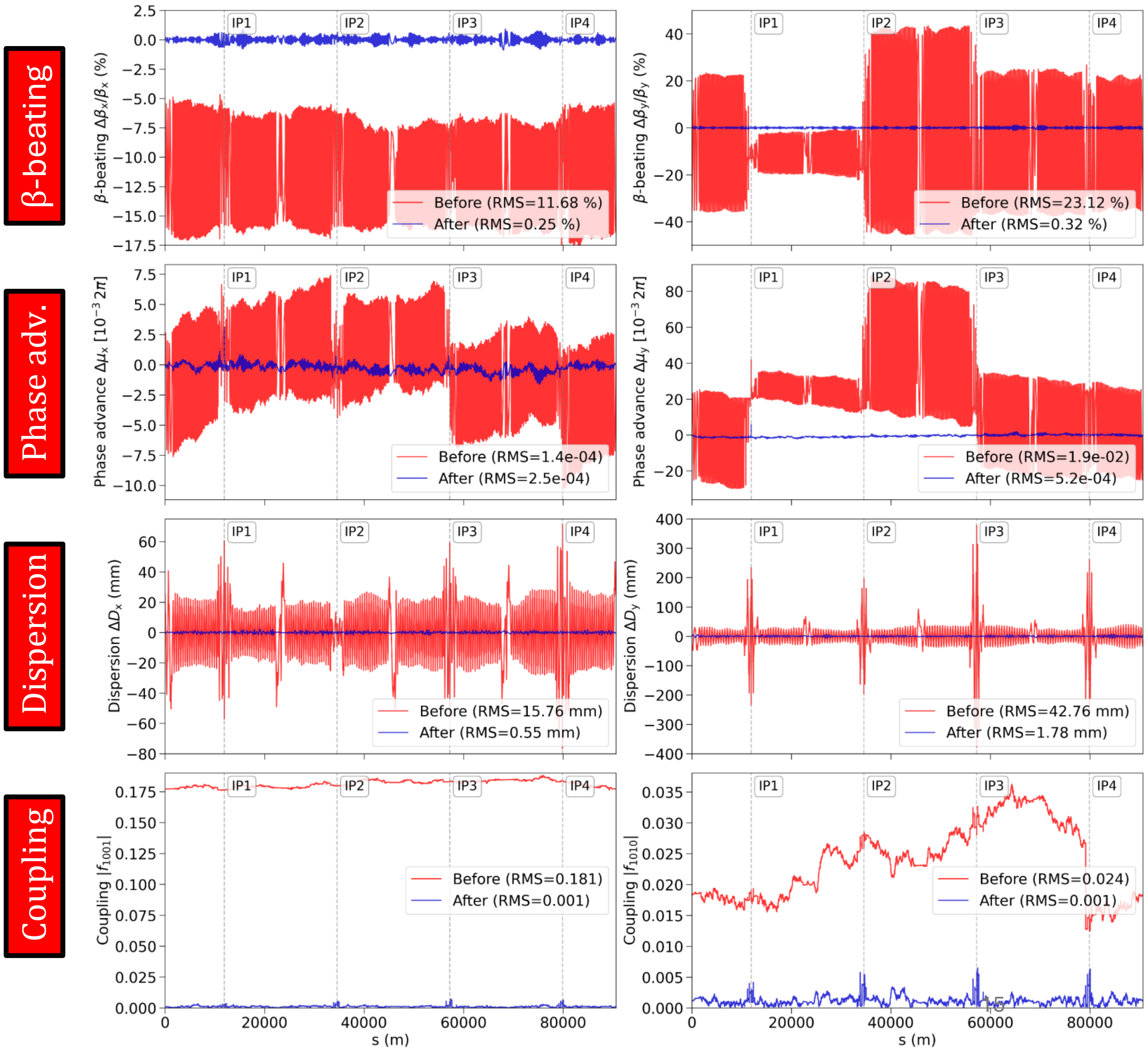
- $\mu_x,\ \mu_y,\ D_x$ for normal correctors
- $\Re\{f_{1001}\},\ \Im\{f_{1001}\},\ \Re\{f_{1010}\},\ \Im\{f_{1010}\},\ D_y$ for skew correctors

↓

Creation of response matrix for normal and skew correctors → Pseudo-inversion of RM using SVD

↓

Determination of corrector kicks → Tune and chromaticity matching



III. Global Orbit and Optics Correction

C. Statistical Analysis of Corrected Lattices

Tune and Chromaticity

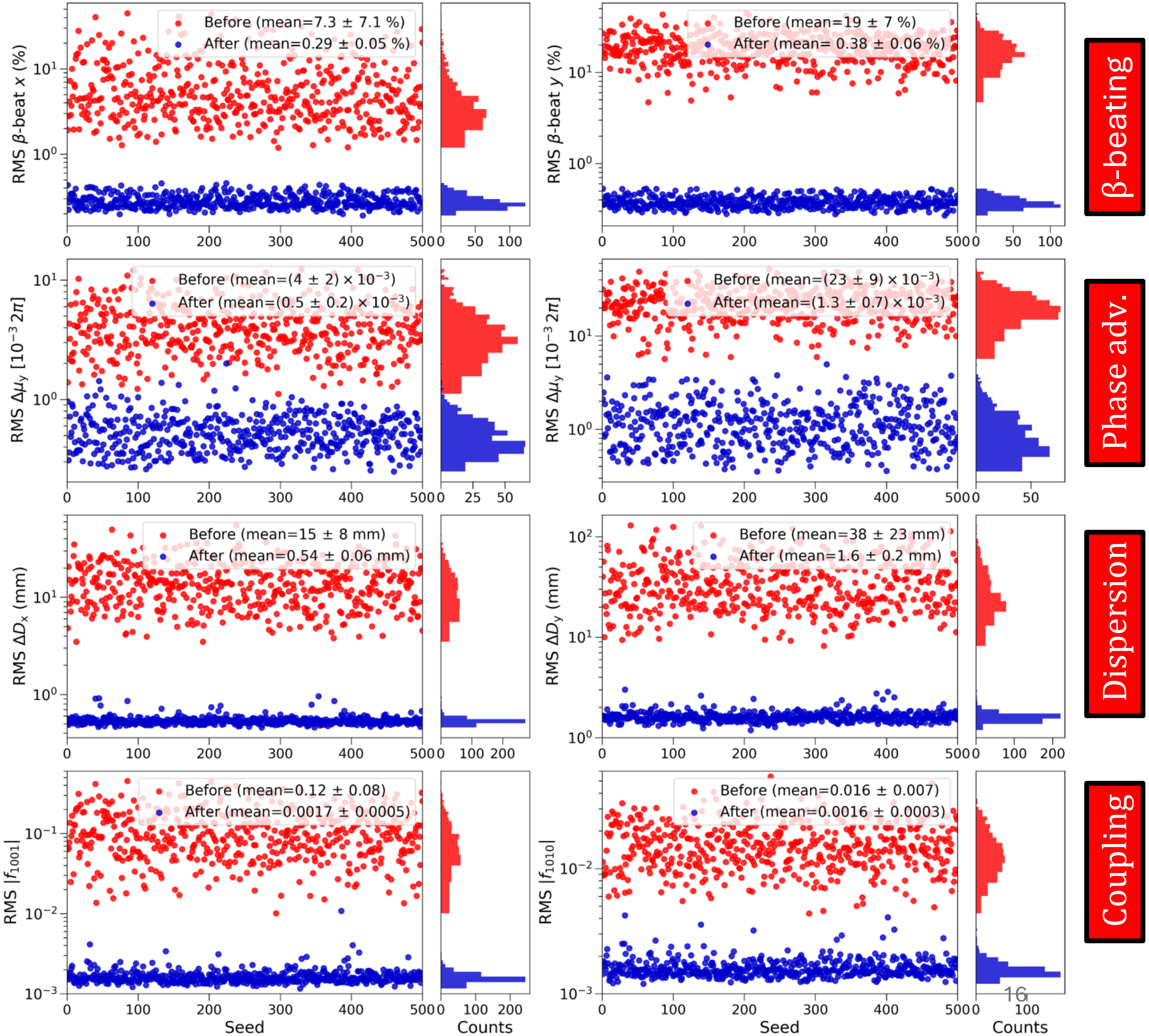
Reference tunes:

$$(Q_x, Q_y) = (194.16, 170.20)$$

Average tune deviation after correction: 10^{-6}

Average chromaticity deviation after correction: 10^{-4}

The correction routine successfully mitigates the optics distortions across the set of generated lattices.



IV. Impact of Beam-Beam Effects

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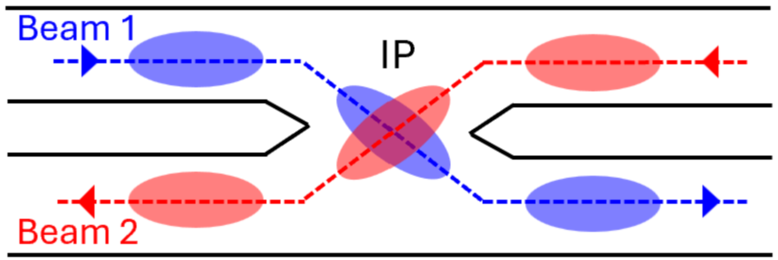
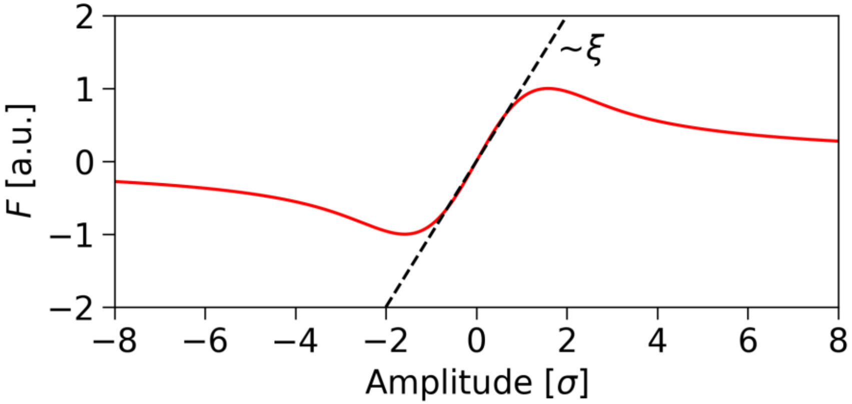
A. Optics Distortions

Collision of two beams at IP



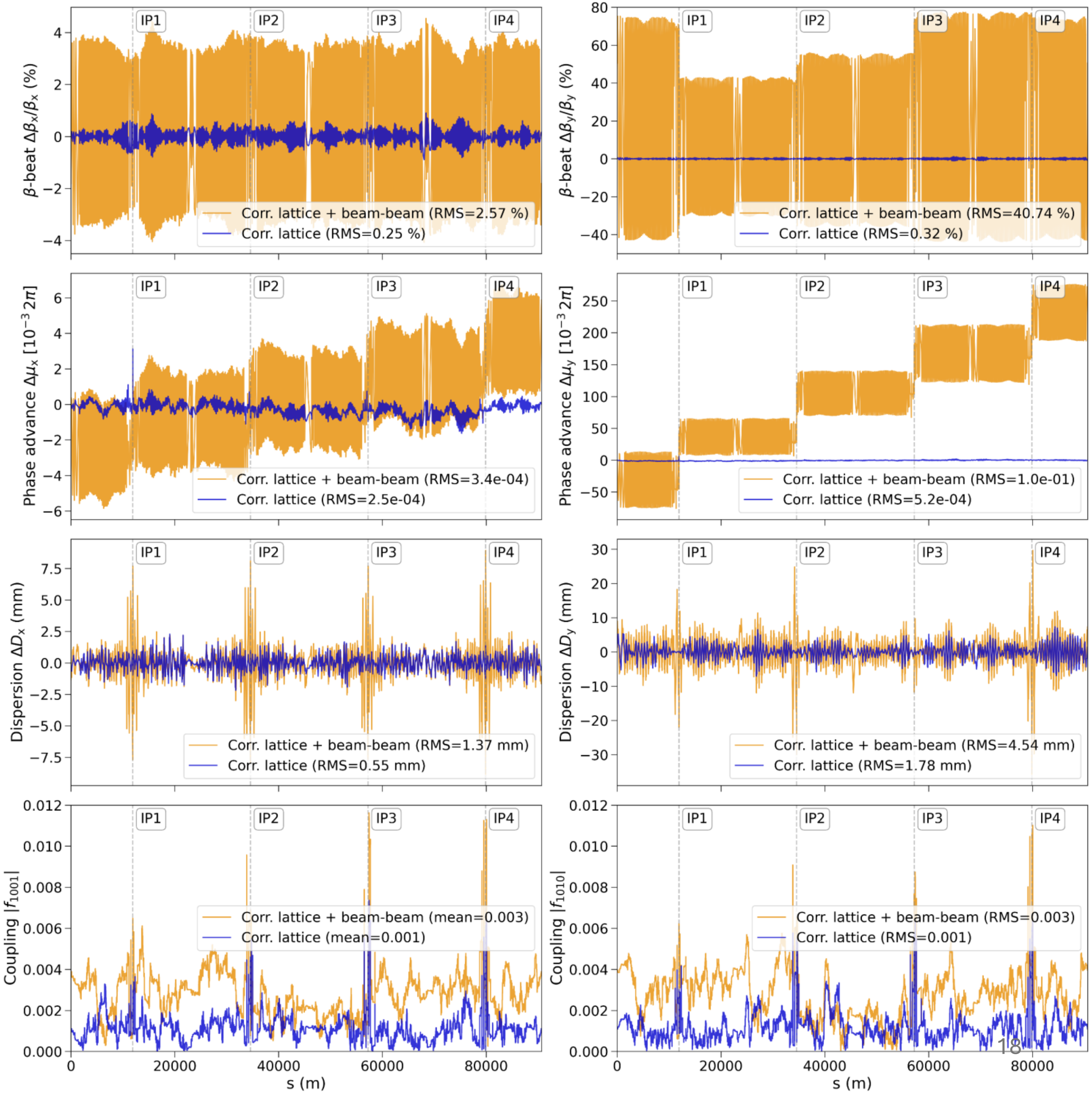
Beam-beam force



Beam-beam parameter

$$\xi_x = 0.0014 \text{ and } \xi_y = 0.09$$

Beam-beam induced optics distortions



β-beating

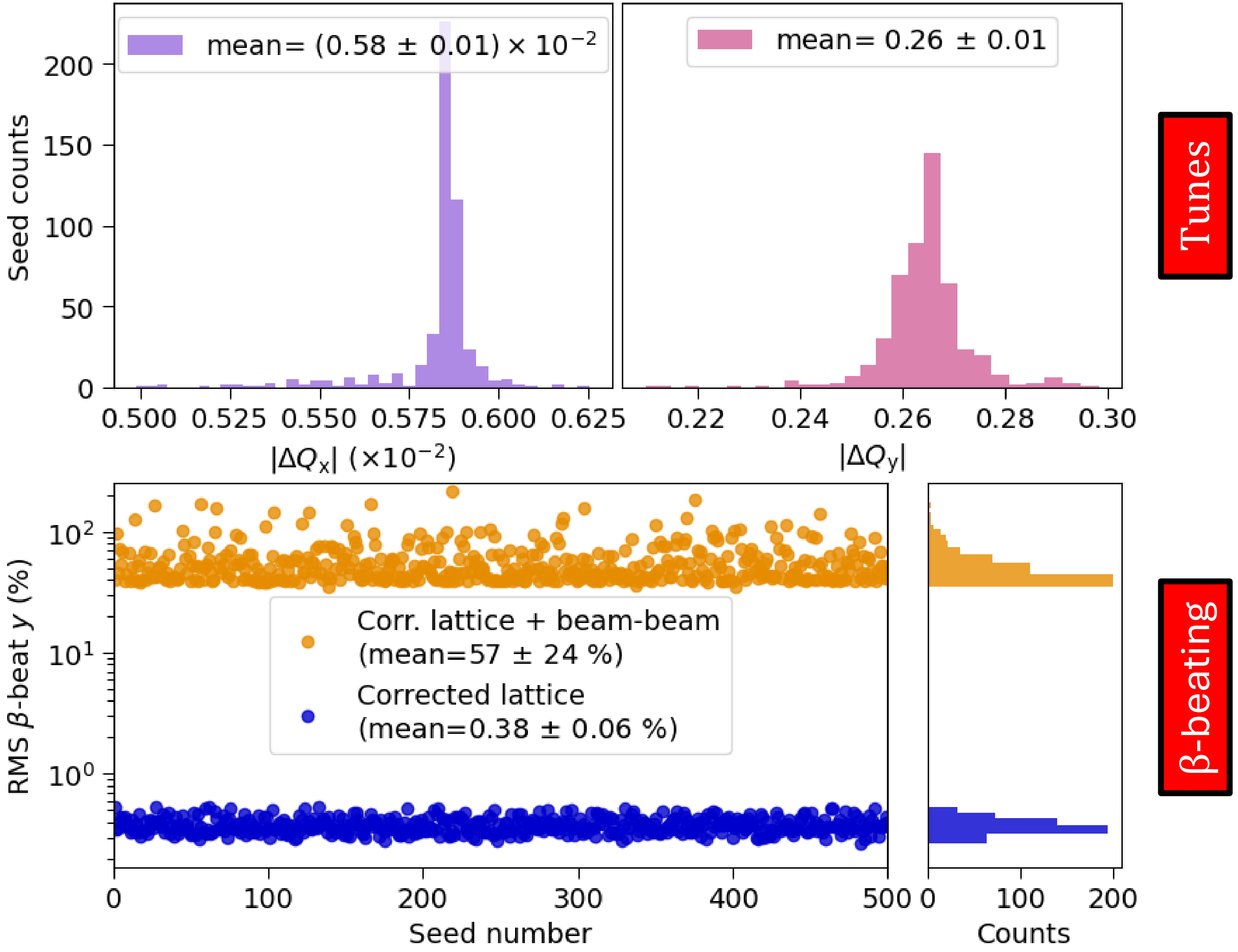
Phase adv.

Dispersion

Coupling

Statistical analysis of 500 corrected lattices for selected optics parameters

Reference tunes: $(Q_x, Q_y) = (194.16, 170.20)$



IV. Impact of Beam-Beam Effects

B. Performance Evaluation

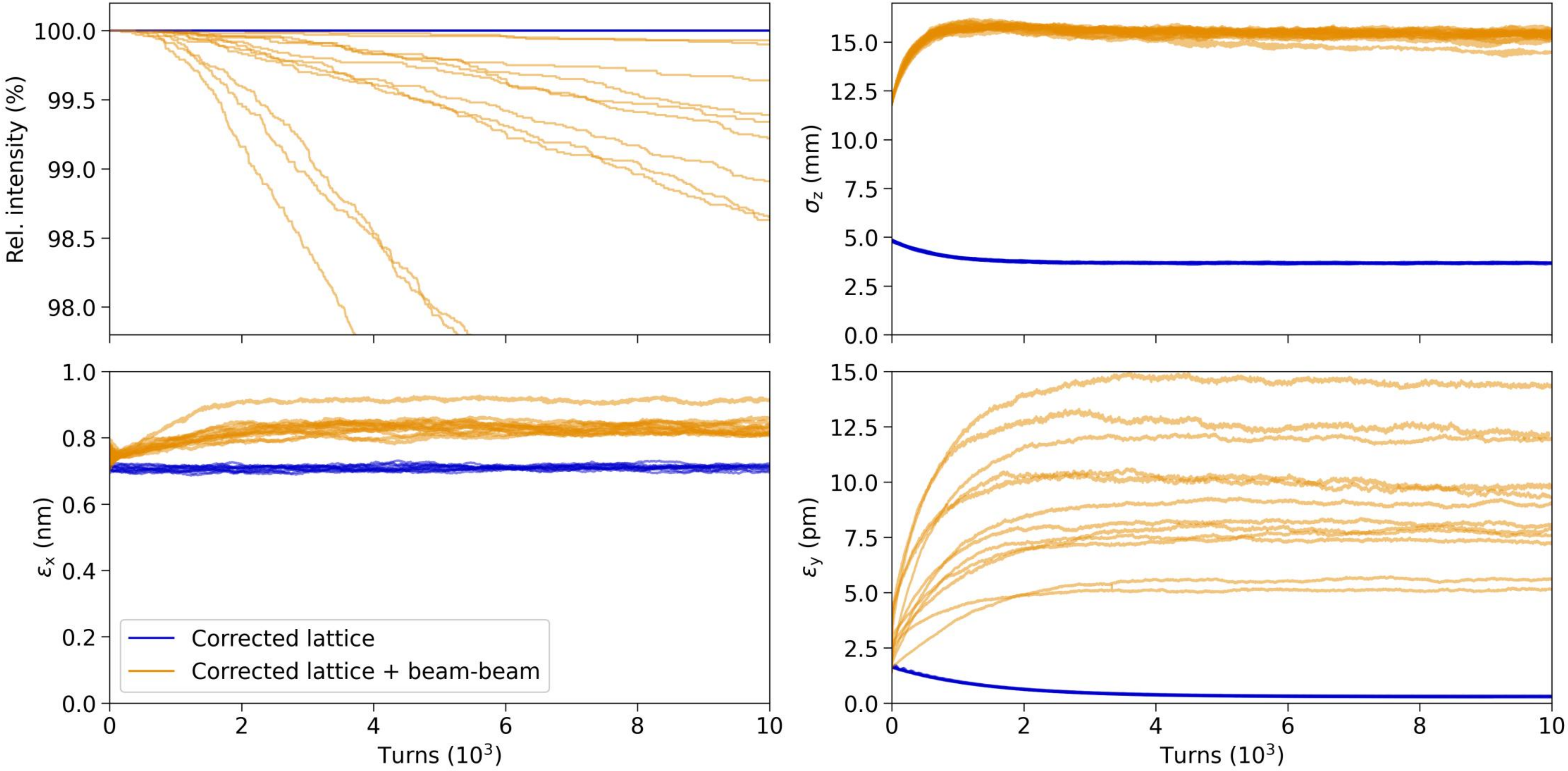
- Tracking 10^4 particles for 10^4 turns in the corrected lattices
- Artificial aperture at 20σ in both transverse planes, RF bucket length as the limit in the longitudinal plane
- Synchrotron radiation with quantum excitations, Beamstrahlung, and tapering are included
- Beam-beam interactions are modelled using the "quasi-strong-strong" scheme

Relative intensity

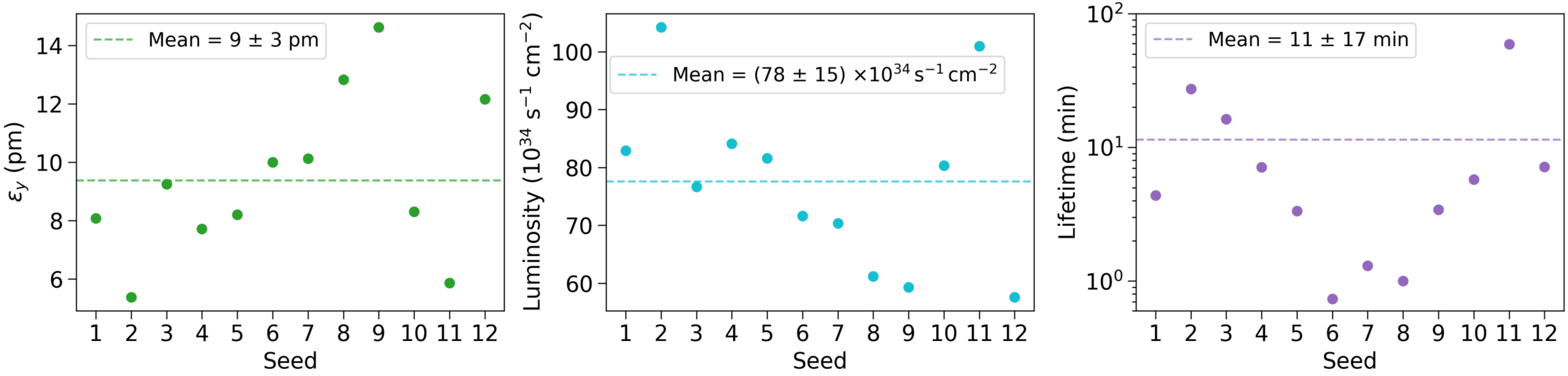
Bunch length

Horizontal emittance

Vertical emittance

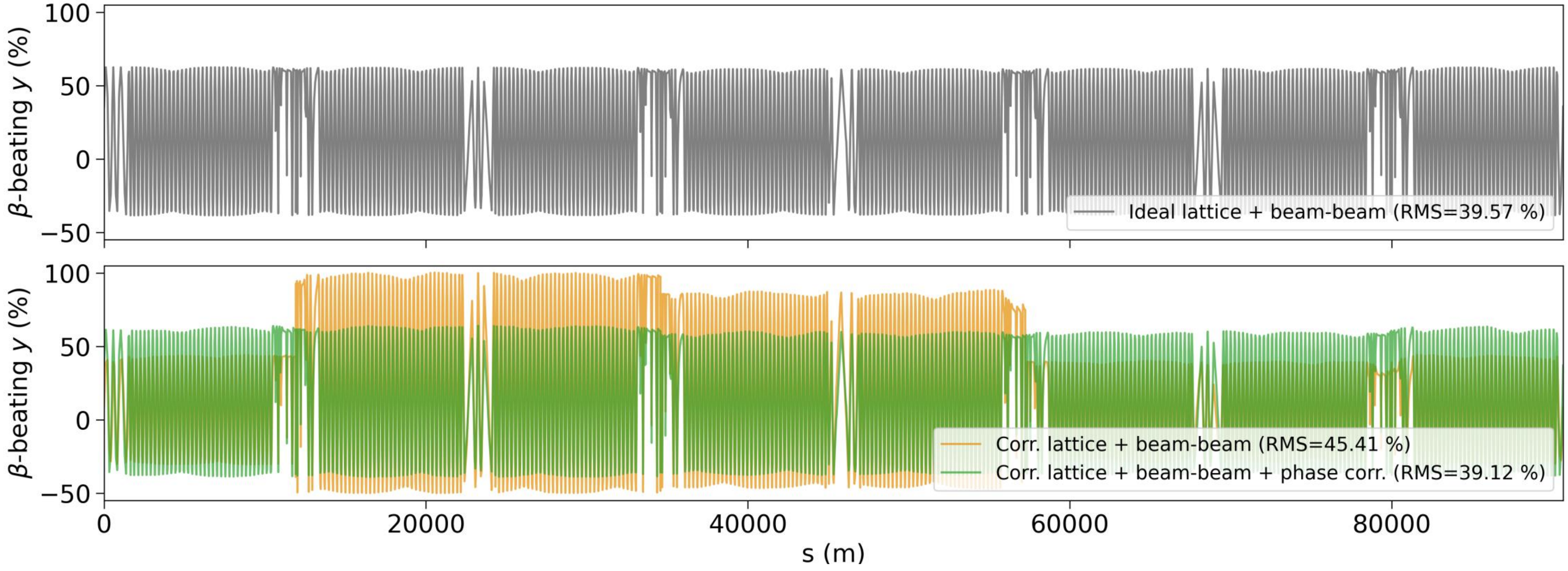


Principal metrics of collider performance: **Luminosity** and **Beam Lifetime**

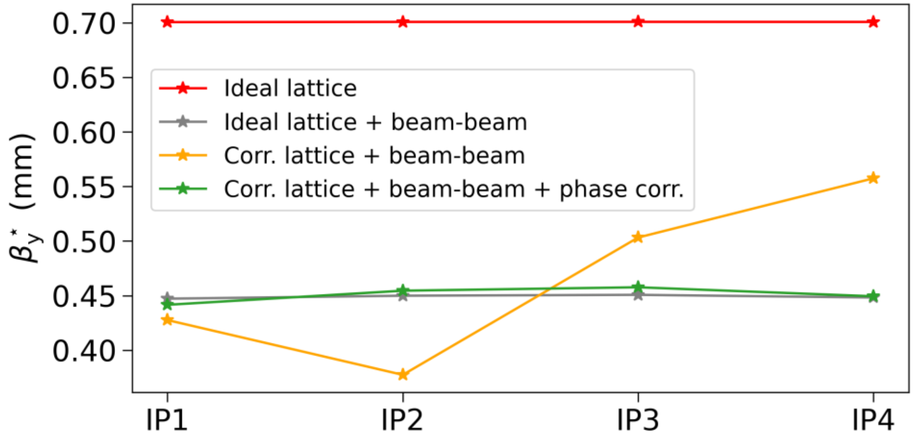


Tracking simulations demonstrate the large distortions induced by beam-beam effects in lattices with corrected imperfections and the resulting degradation in collider performance.

Aim: Addressing the phase advance distortions between the IPs with a phase-matching routine



Phase-Matching:
9 individually powered trim quadrupoles in the dispersion suppression regions are used to match the inter-IP phase advance back to the design value



The “dynamic squeeze” of the β-function caused by beam-beam kicks enhances the luminosity

V. Conclusions

V. Conclusions

- Development of an **imperfections model** of the FCC-ee in Xsuite for the LCC lattice at the Z pole

  ➡ Generic implementation of this model allows for it to be extended to all the other FCC-ee energies as well as other accelerators

- Design of a **global correction scheme** that restores the **orbit** and **optics** parameters to levels compatible with FCC-ee operation

  ➡ Generation of **lattices with corrected imperfections**, which have enabled **polarisation studies** (Y. Wu) and the development of **IP tuning knobs** (T. Prebibaj), presented at IPAC 2026

- The generated lattices with corrected imperfections provide the basis for a quantitative assessment of the machine's behaviour under **beam-beam interactions**

  ➡ The beam-beam induced distortions (beta-beating, tune shift, etc.) lead to a **degradation in collider performance** (luminosity & lifetime), analysed via tracking simulations

  ➡ As a first compensation scheme, **phase-matching** is implemented to restore the β-function at the IPs and re-establish the collider's four-fold symmetry

- The data and code scripts developed during this project and the generated lattices with corrected imperfections are available in the following Github repository: https://github.com/Sievert-L/FCCee_Project_2026

Thank you!

Backup

Outlook

- Further refinement of the imperfections model, including systematic errors, missing BPMs, etc.
- Implementation of updated / more relaxed error tolerances
- Further optimisation of the correction algorithms and comparison of SVD with MICADO
- Development of additional beam-beam compensation methods to recover the nominal collider performance
- Extension towards the LCC lattices at higher beam energies

Parameter Table for GHC & LCC Lattices

Optics Mode	GHC Z	GHC $t\bar{t}$	LCC Z	LCC $t\bar{t}$
Beam energy E (GeV)	45.6	182.5	45.6	182.5
Circumference C (m)	90658.525		90644.816	
Arc-cell setup	90°/90° long	90°/90° short	52°/45°	99°/77°
Momentum compaction factor α_c (10^{-6})	28.5	7.3	28.6	9.5
Energy Loss per turn W_0 (GeV)	0.039	10.01	0.035	9.01
Beam Intensity N (10^{12} particles)	2400	9.405	2424	9.435
Bunch Intensity N_b (10^{11} particles)	2.02	1.85	2.02	2.20
Number of Bunches	12000	51	12000	43
Horizontal β-function at IP β_x^* (cm)	9	90	9	90
Vertical β-function at IP β_y^* (mm)	0.7	14	0.7	14
Horizontal emittance ε_x (nm)	0.74	1.74	0.70	2.10
Target vertical emittance in collision ε_y (pm)	1.48	1.75	1.40	2.11
Transverse tune Q_x/Q_y	214.16 / 214.20	394.19 / 390.27	194.16 / 170.20	346.19 / 262.27
Chromaticity Q'_x/Q'_y	12 / 5	0 / 0	12 / 5	0 / 0
Harmonic number h at 400 MHz	121200		121200	
Total RF voltage of 400 / 800 MHz (GV)	0.09	2.1 / 8.9	0.09	2.0 / 8.1
Synchrotron tune Q_s	0.031	0.086	0.031	0.111
RF momentum acceptance (%)	1	2	1	2
Bunch length σ_z (non coll./coll.) (mm)	5.4 / 16.3	1.9 / 2.7	5.1 / 16.7	1.9 / 2.8
Rel. mom. spread σ_p (non coll./coll.) (10^{-3})	0.40 / 1.29	1.58 / 2.21	0.39 / 1.34	1.52 / 2.33
Longitudinal damping time τ_z (turns)	1159	18	1297	20
Crab-waist ratio (%)	55	40	55	40
Beam-beam parameter ξ_x/ξ_y (10^{-3})	1.5 / 80	61.2 / 108.9	1.4 / 80	67.1 / 120
Luminosity $\mathscr{L}$ ($10^{34}\,s^{-1}/cm^2$)	151	1.54	150	1.45

Parameter table for the GHC and LCC lattices at Z and $t\bar{t}$ mode.
Source: Optics Comparison Report

Twiss Failure

Twiss method = Xsuite's method to compute the Twiss parameters from the one-turn transfer matrix and derive a range of important beam and lattice properties (using the finite difference method around the closed orbit)

Twiss failure may be due to one of the following:

1) Numerical instabilities in Xsuite (finite different method fails)

2) One-turn matrix violates the stability criterion due to unstable optics: $|\mathrm{Tr}(\mathbf{M})| = |2\cos\mu| \leq 2$

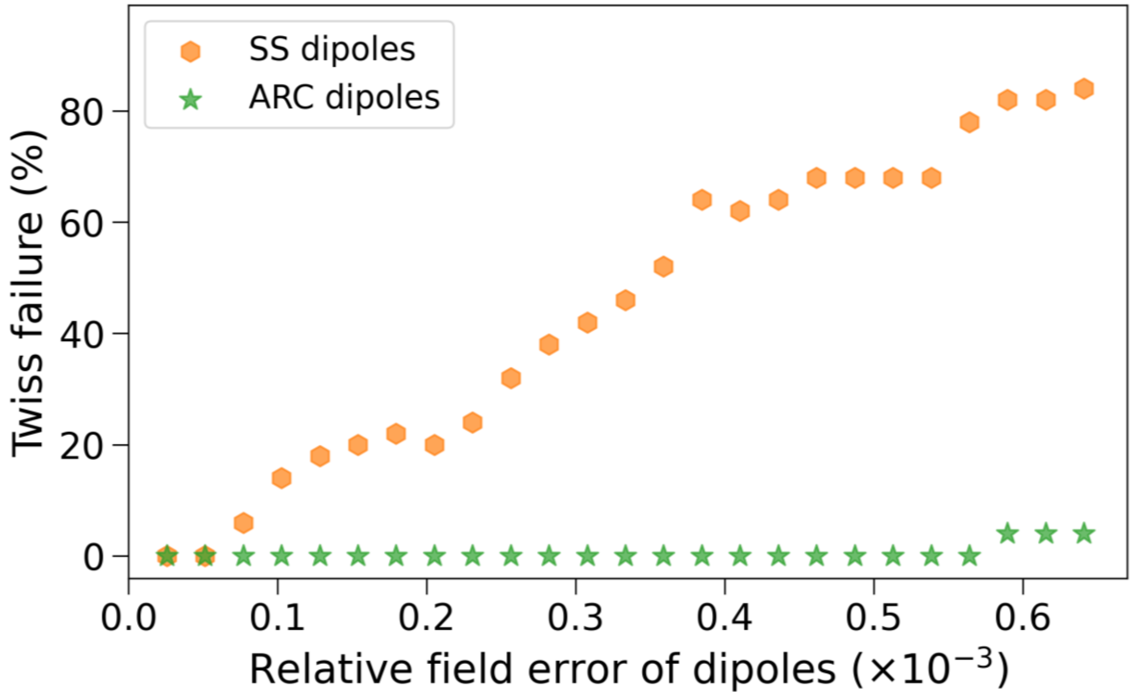
where $$\mathbf{M} = \begin{bmatrix} \cos\mu + \alpha_0 \sin\mu & \beta_0 \sin\mu \\ -\gamma_0 \sin\mu & \cos\mu - \alpha_0 \sin\mu \end{bmatrix}$$

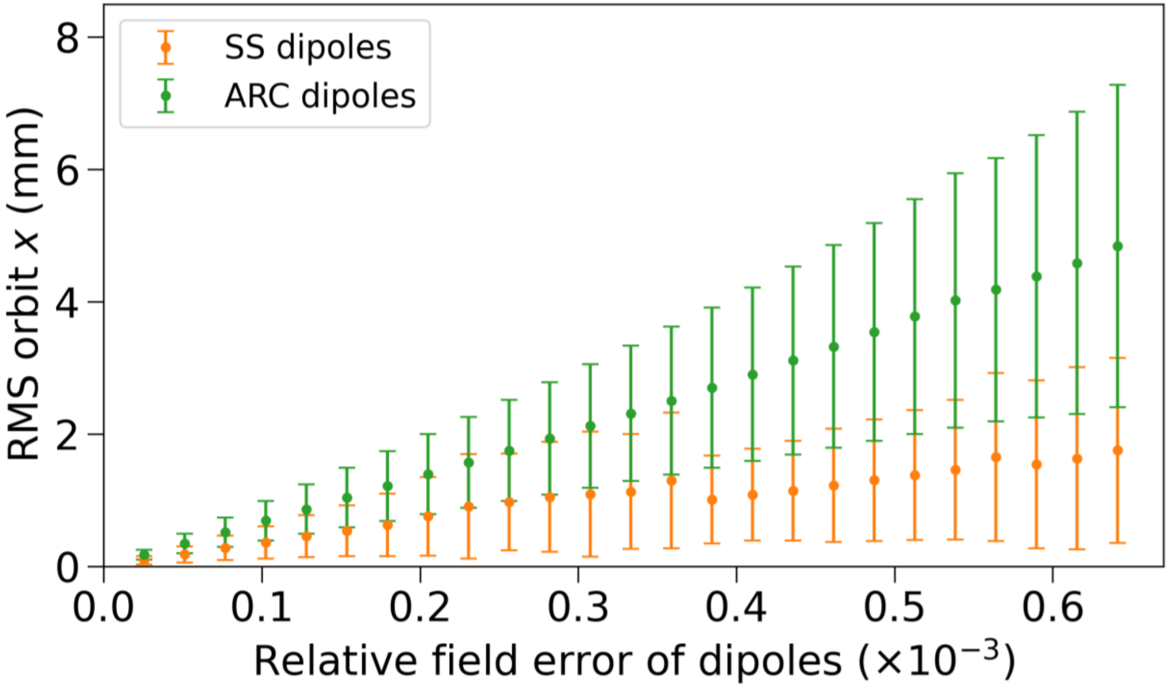
Main tricks used to prevent the method from becoming numerically unstable:

- Using 4D instead of 6D Twiss
- Disabling radiation and tapering
- Cycling of line to dispersion-free RF cavity
- Setting chromatic properties = False
- Introducing very small, non-zero equilibrium emittances

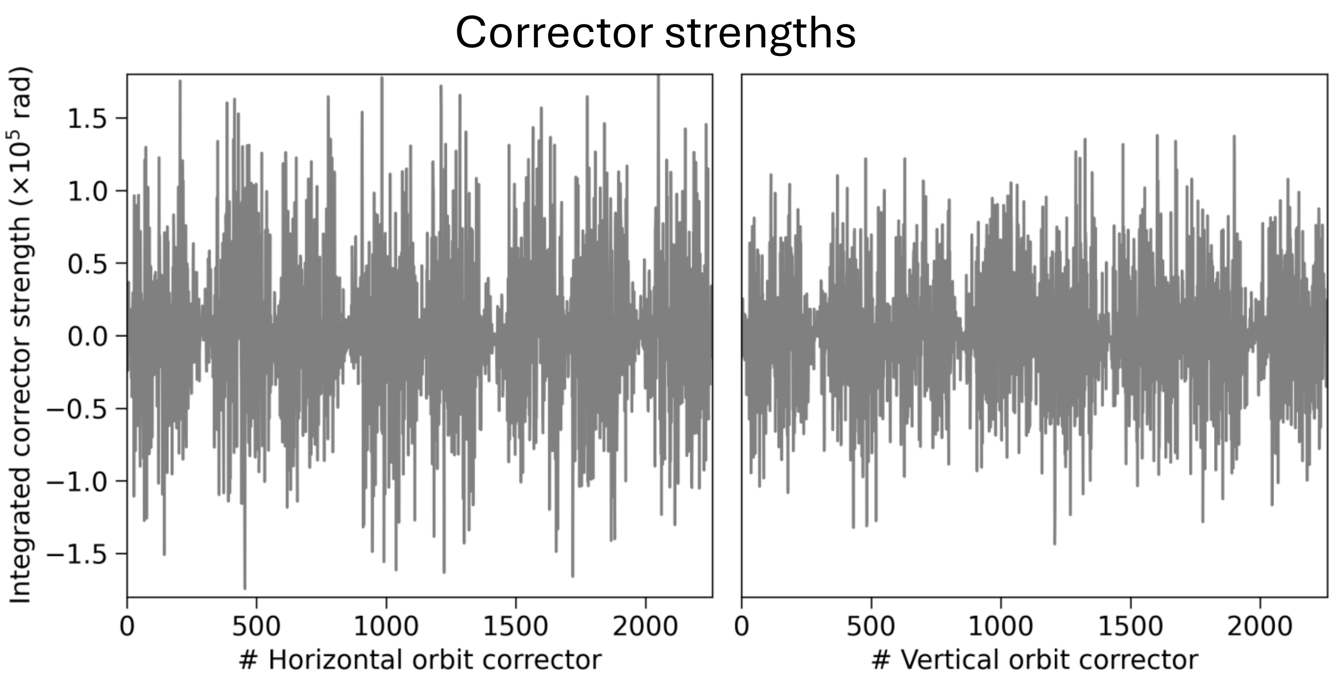
Sensitivity Analysis (Orbit)

Imperfections leading to **orbit** perturbations: Dipolar field errors





Orbit Correction



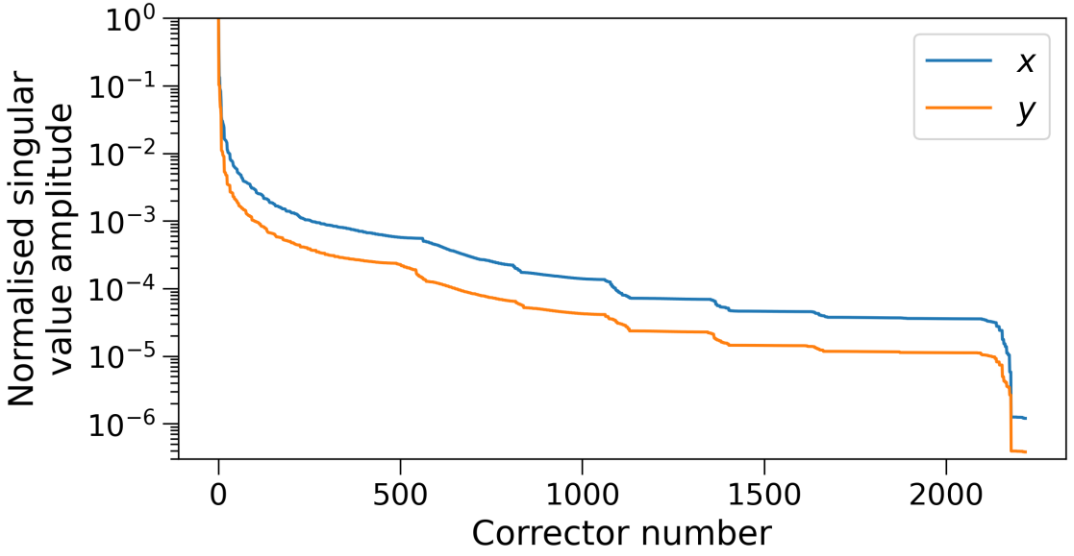
Tricks used to make the orbit correction more stable:

- No radiation / tapering
- 4D Twiss
- Lattice cycled to RF cavity
- Chromatic properties disabled until sextupoles are fully ramped up
- Optimisation of sextupole ramping increments and threading steps

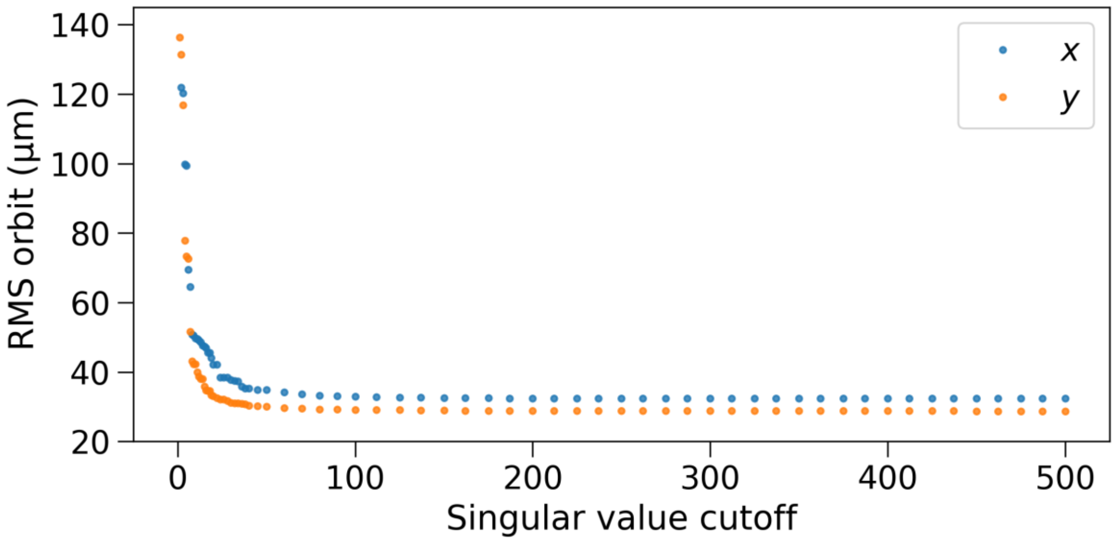
Details on the steps of the correction routine:

- Success of the correction routine is strongly seed-dependent: studies indicate that failing cases can primarily be attributed to girder misalignments
- Despite optimising various measures targeting the stability of the procedure, no satisfactory improvement is achieved
- Singling out the girder misalignments and ramping them up separately bypasses these difficulties and allows for the successful generation of a large number of lattices → Seed success rate: **89 %**
- Without girder misalignment ramping: smaller seed success rate, but **very similar correction levels** achieved

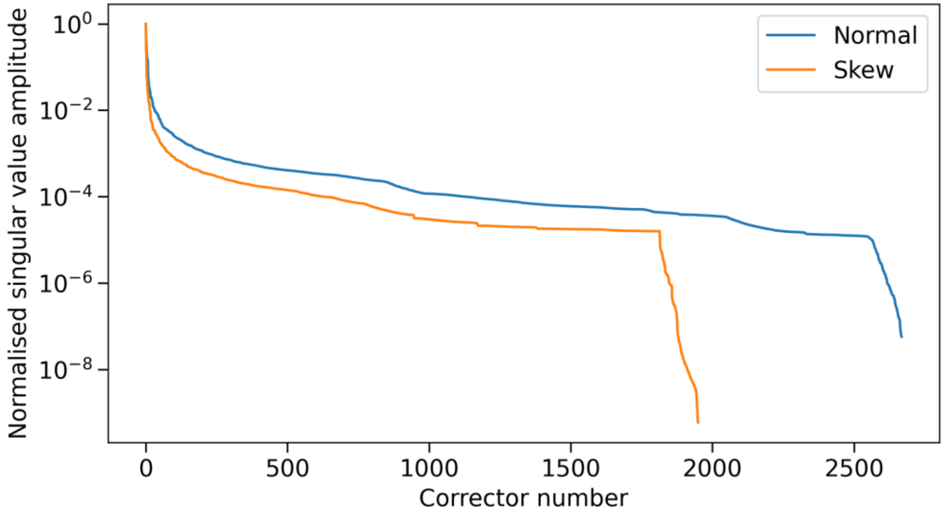
Singular Value Decomposition



Singular value spectrum of the orbit response matrix for the horizontal and vertical correctors.

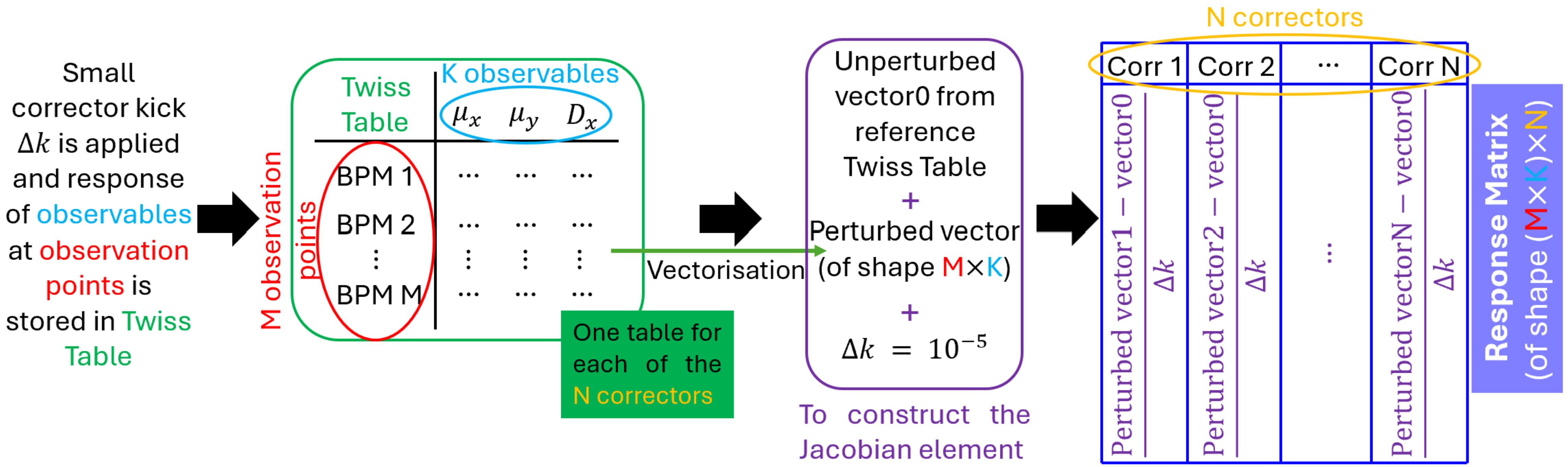


SVD of the orbit response matrix. Optimisation of the required number of singular values by analysing the corrected RMS orbit.



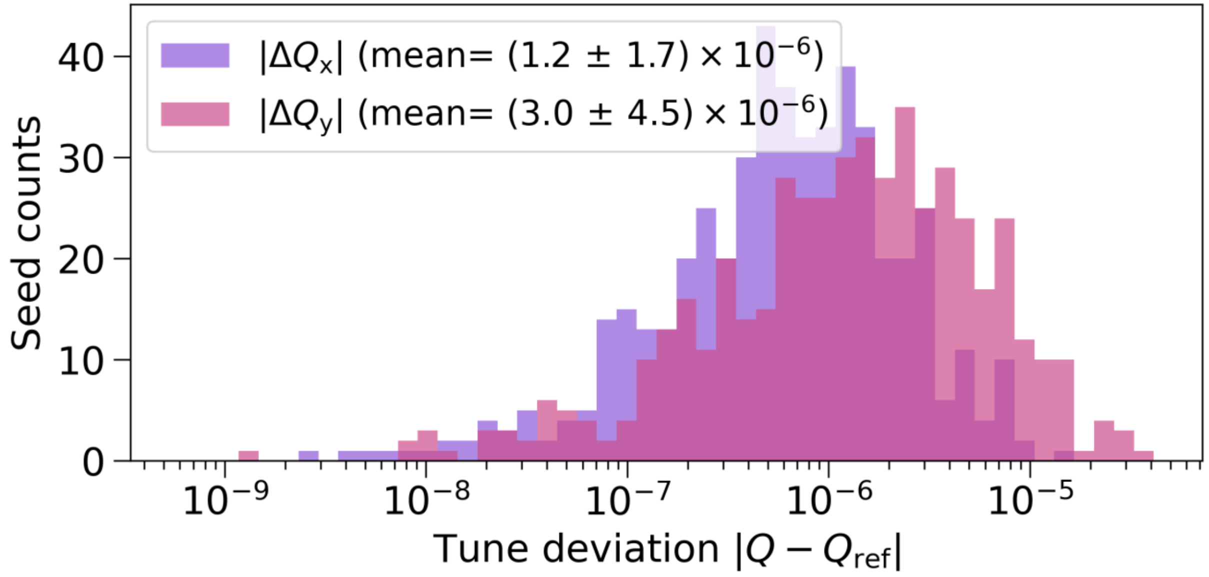
Singular value spectrum of the optics response matrix for the normal and skew correctors.

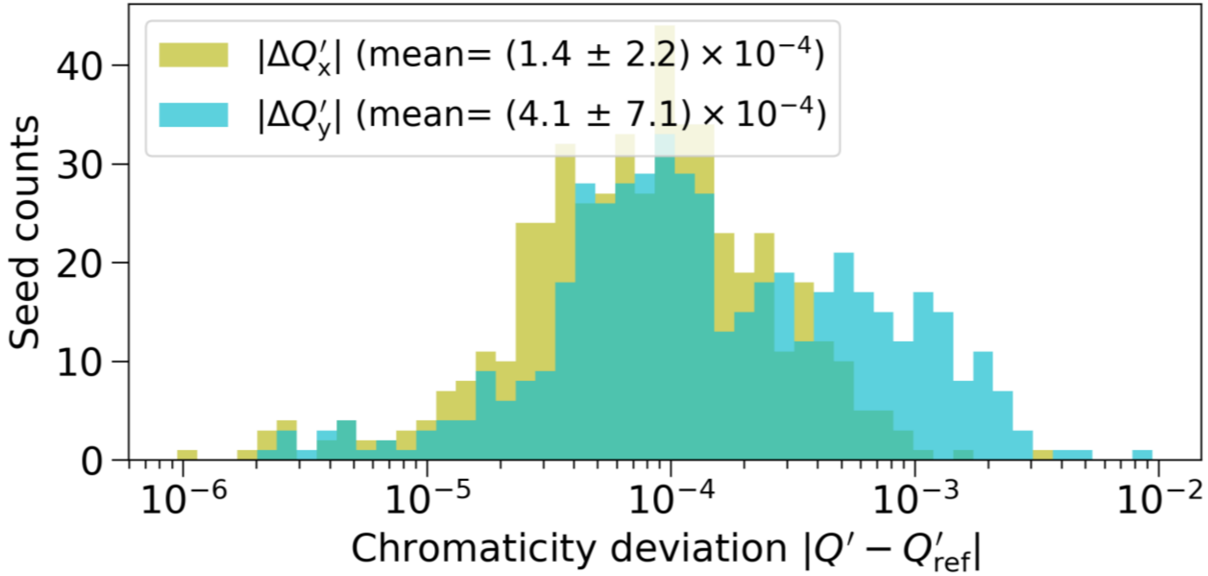
Response Matrix Construction



Construction of the optics response matrix for normal correctors. The response matrix for the skew optics correctors is built in the same way, but with a different set of observables.

Statistical Analysis of Corrected Lattices: Tune and Chromaticity

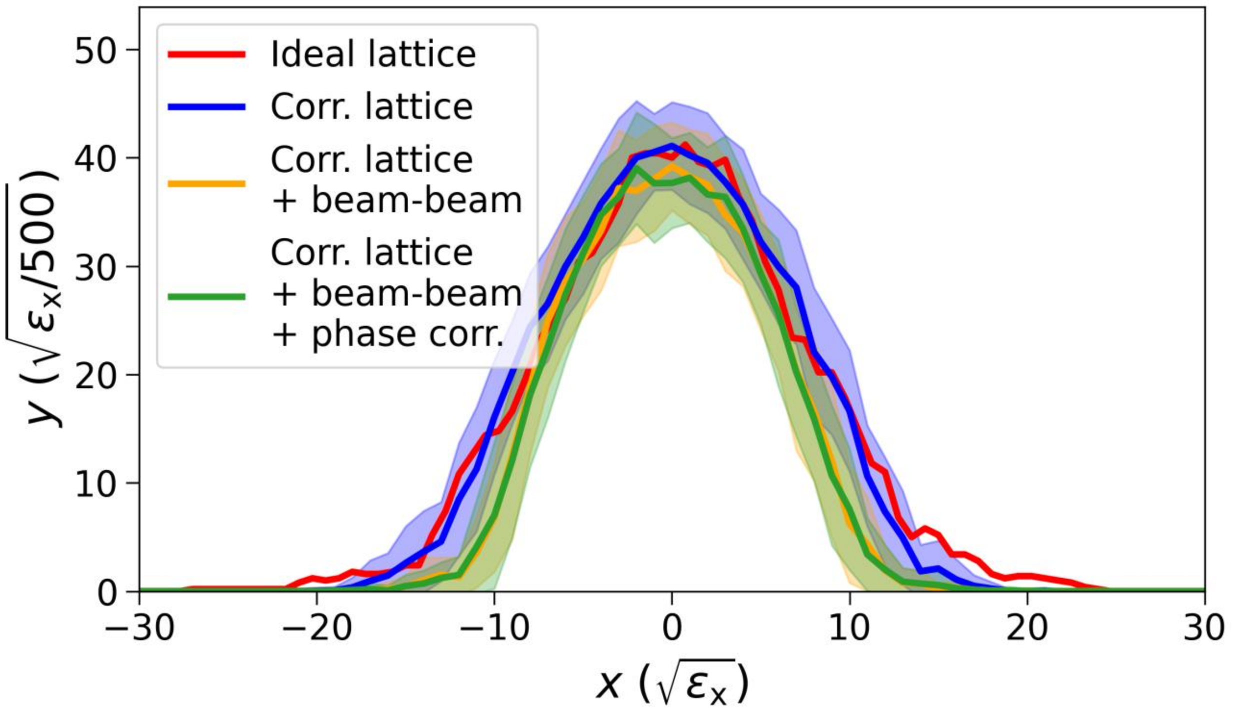




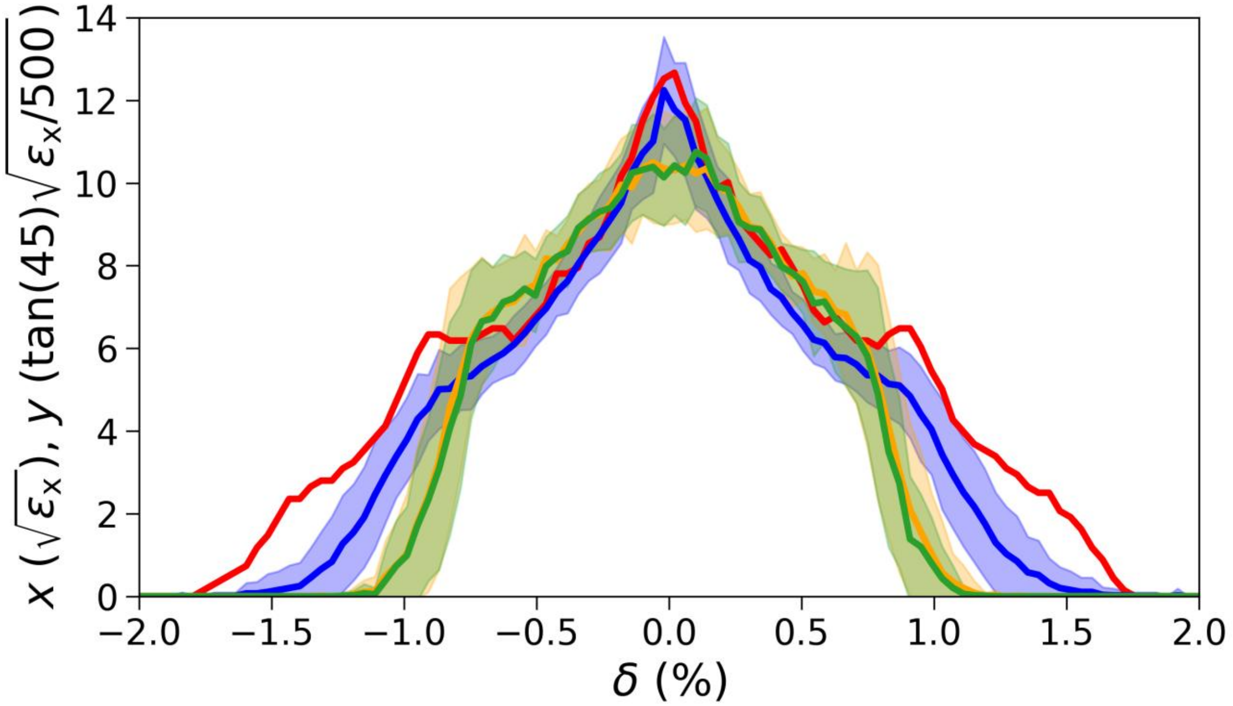
Phase-Matching Routine: DA and MA

Key indicators of beam stability: **Dynamic Aperture** and **Momentum Acceptance**

Dynamic Aperture



Momentum Acceptance



The phase-advance correction does not cause the DA and MA to degrade within the statistical uncertainty of the scans.