Parametrization *of* Electron Cloud Buildup Models

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CERN

LPAP Activity Meeting

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EPFL

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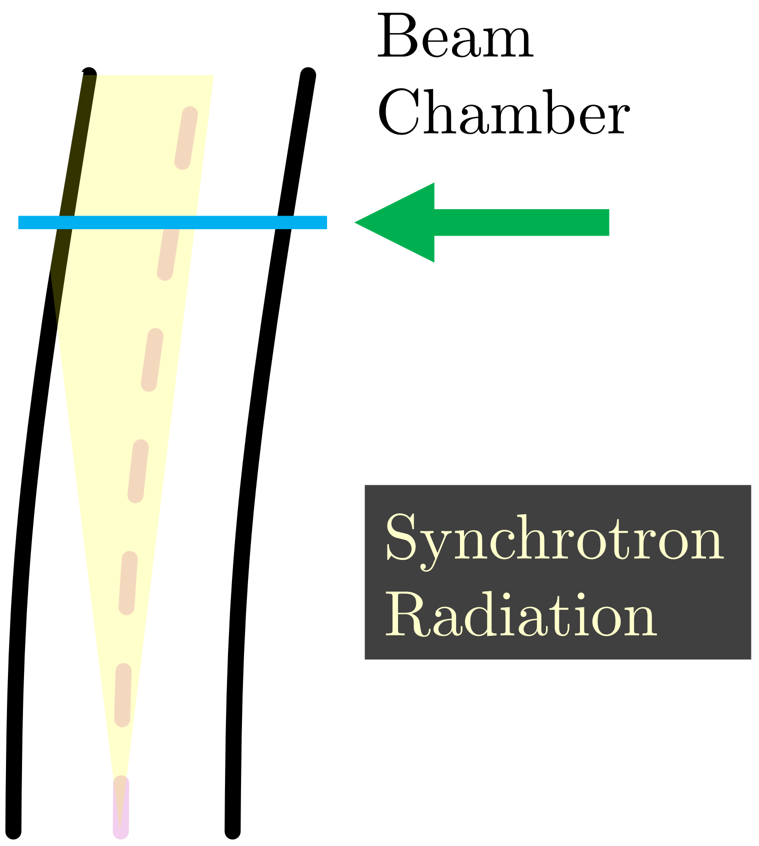
Swiss Accelerator Research and Technology

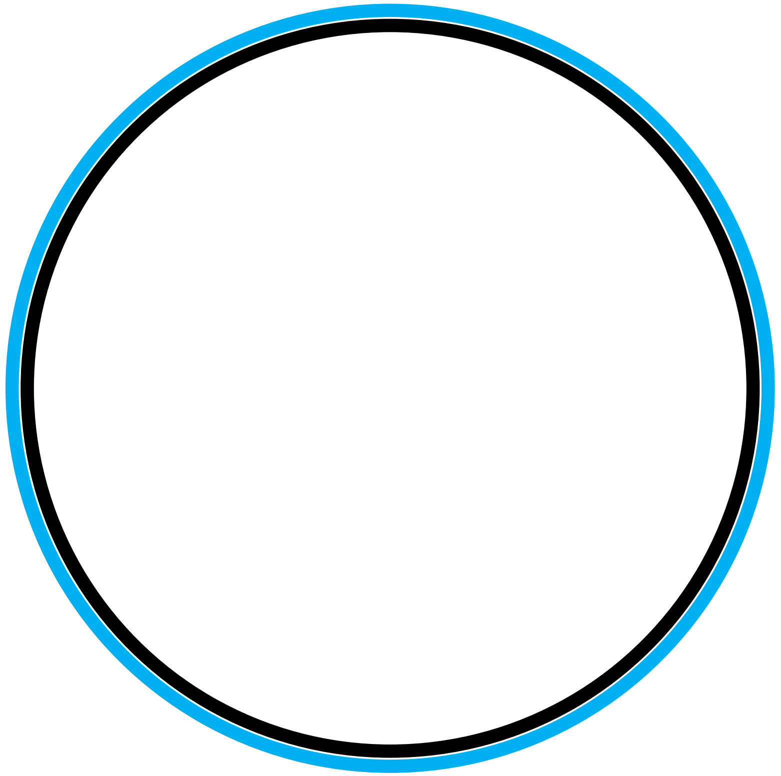
Overview

1. A brief **overview** of electron clouds.
2. Electron cloud buildup **simulations**, using PyECLOUD.
3. **Modelling** electron cloud buildup.
4. **Predicting** model parameter values *a priori*.
5. Observations, **limitations**, and strategies.
6. **Tools** you can use.

A brief overview of electron clouds

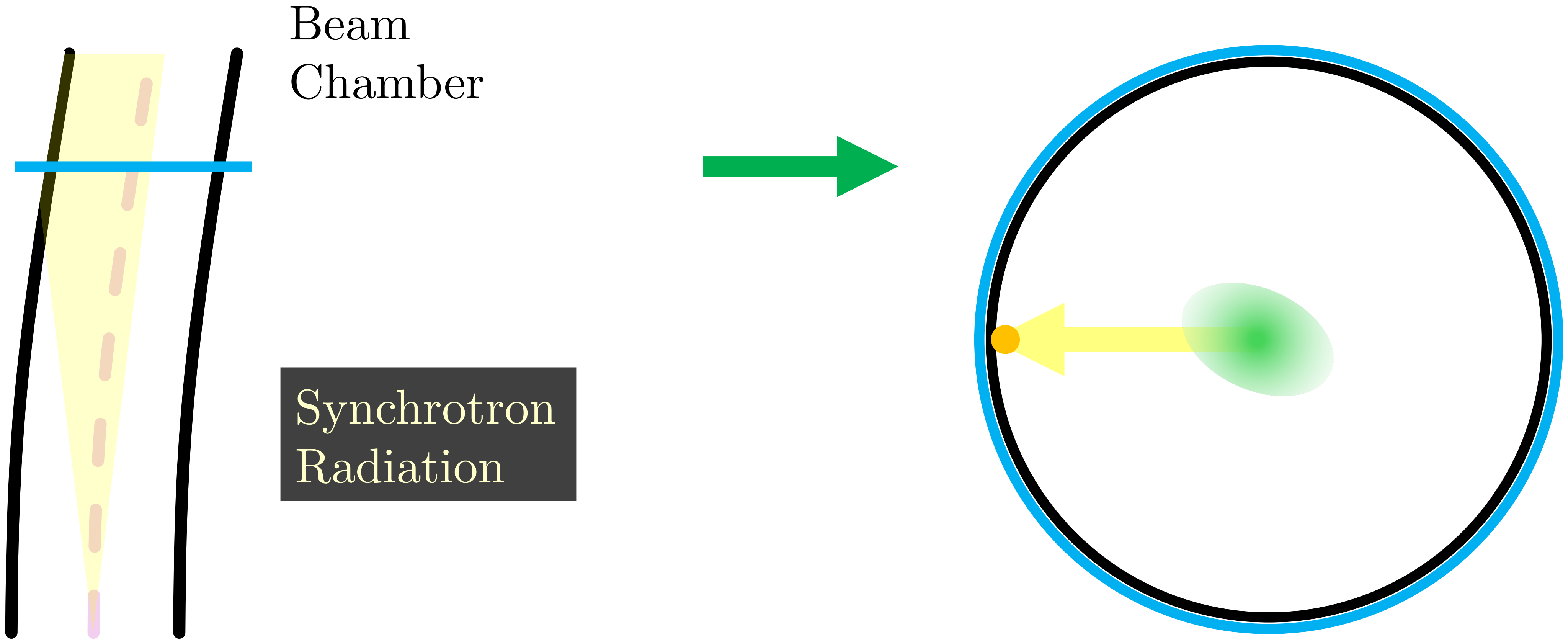
Electron clouds occur when the conditions inside a particle accelerator facilitate the **buildup** of electrons over time.





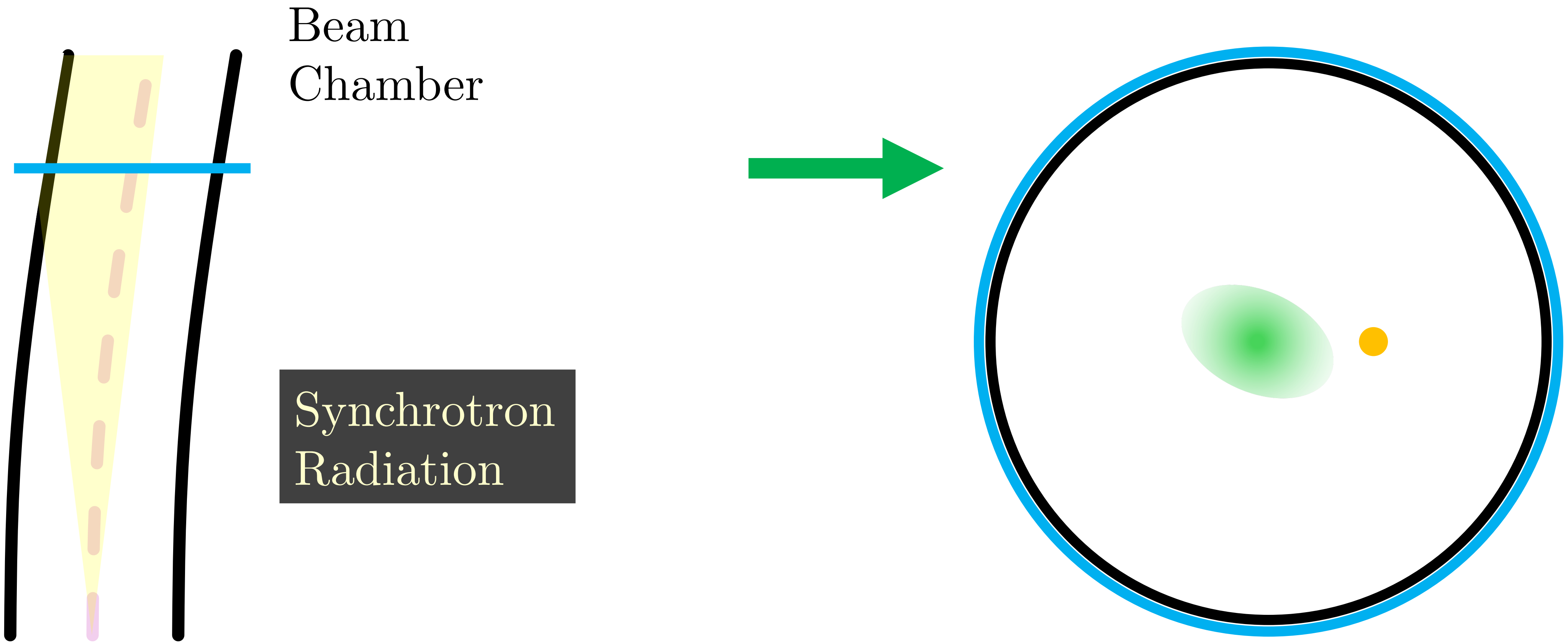
A brief overview of electron clouds

Some amount of initial electrons (from synchrotron radiation or residual gas collisions) is present in the beam chamber.



A brief overview of electron clouds

The electrons are kicked by the electromagnetic force of a passing bunch and hit the beam chamber wall.



A brief overview of electron clouds

The **secondary electron yield** (SEY) curve, a material property, defines secondary electron emission based on incident energy.

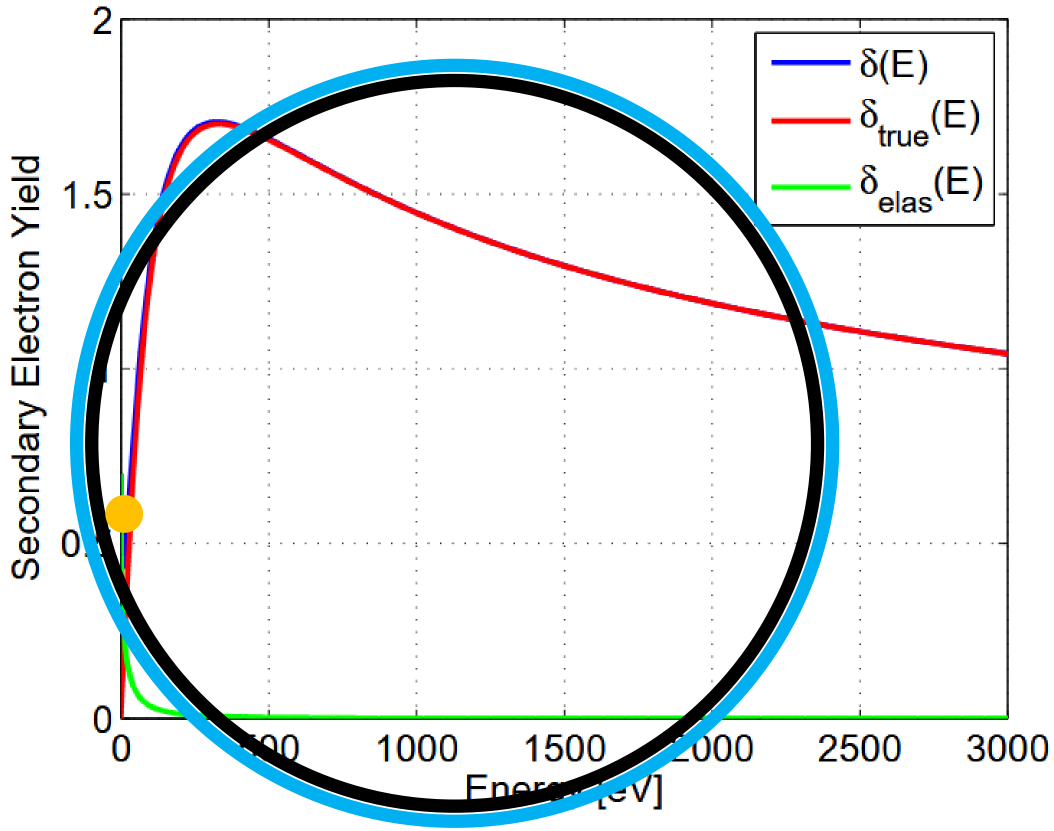


Image: *Giovanni Iadarola*

A brief overview of electron clouds

Some (low energy) electrons are reflected off the wall. Higher energy electrons trigger the emission of true secondaries.

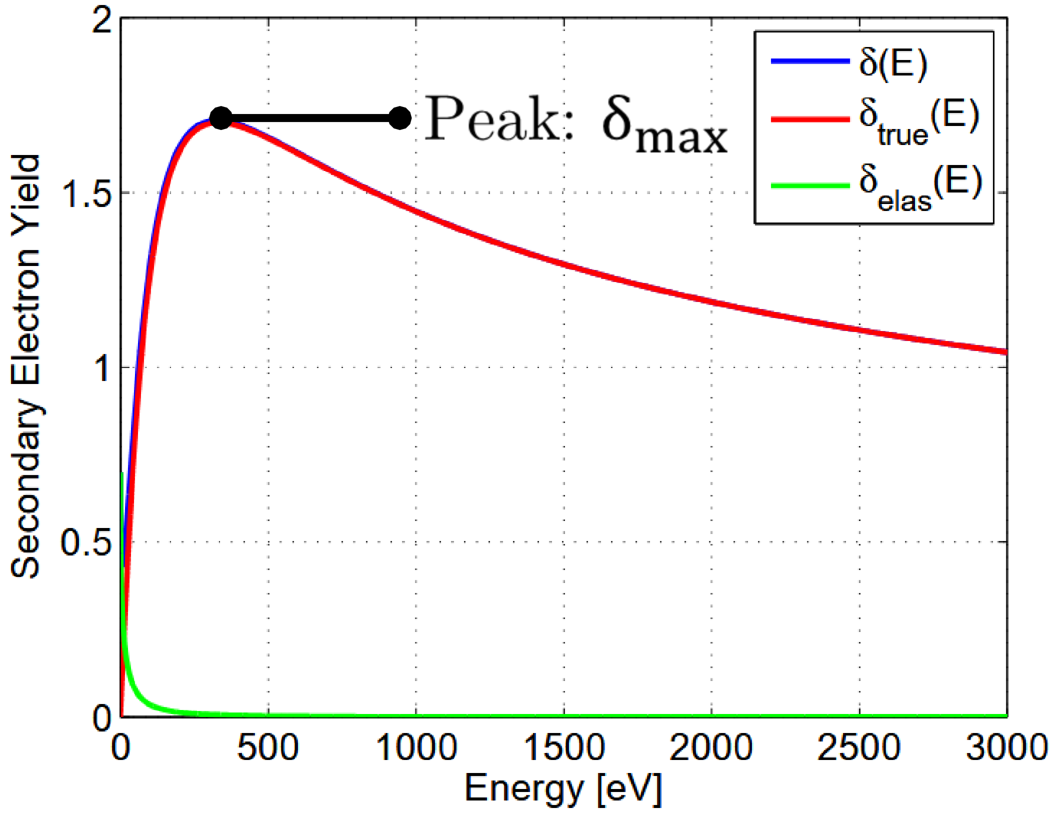
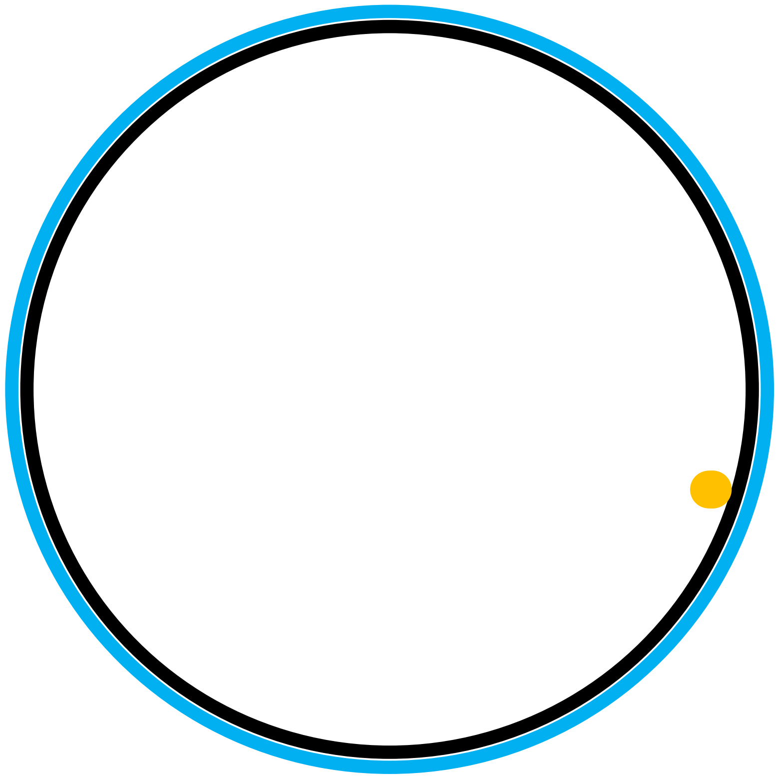


Image: *Giovanni Iadarola*

A brief overview of electron clouds

Extracted electrons that persist until the next bunch passage (t_b) gain energy and repeat the cycle: this is **buildup**.



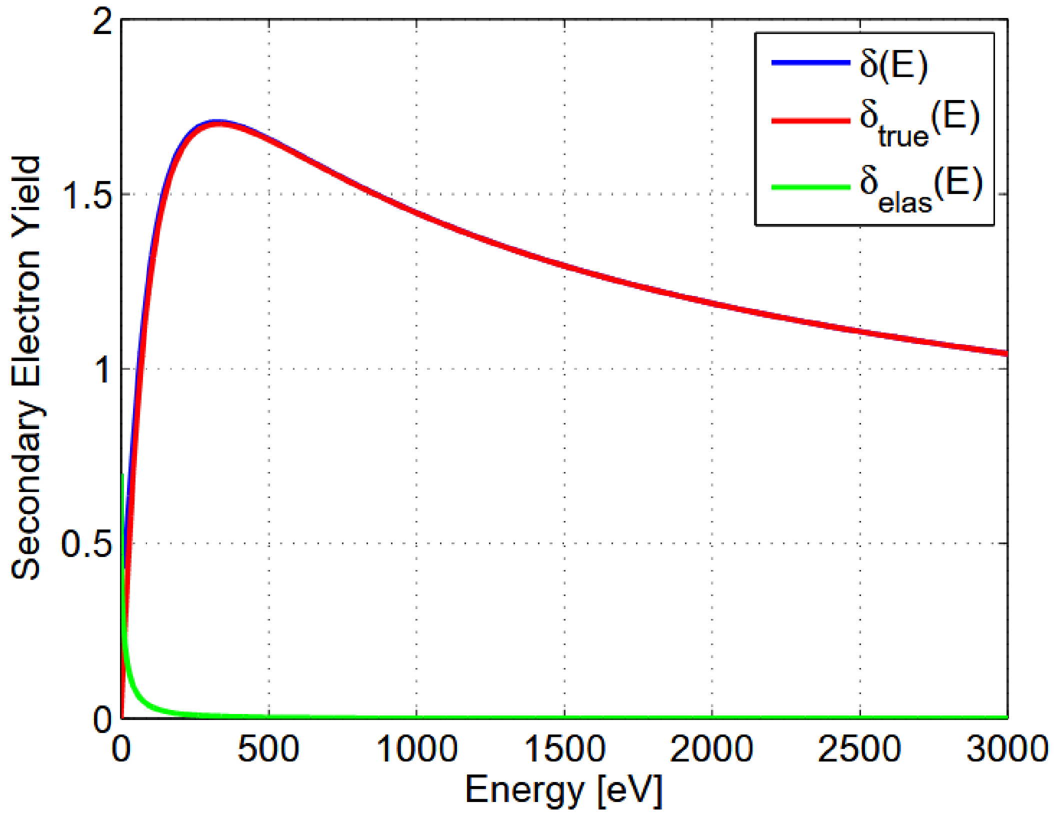
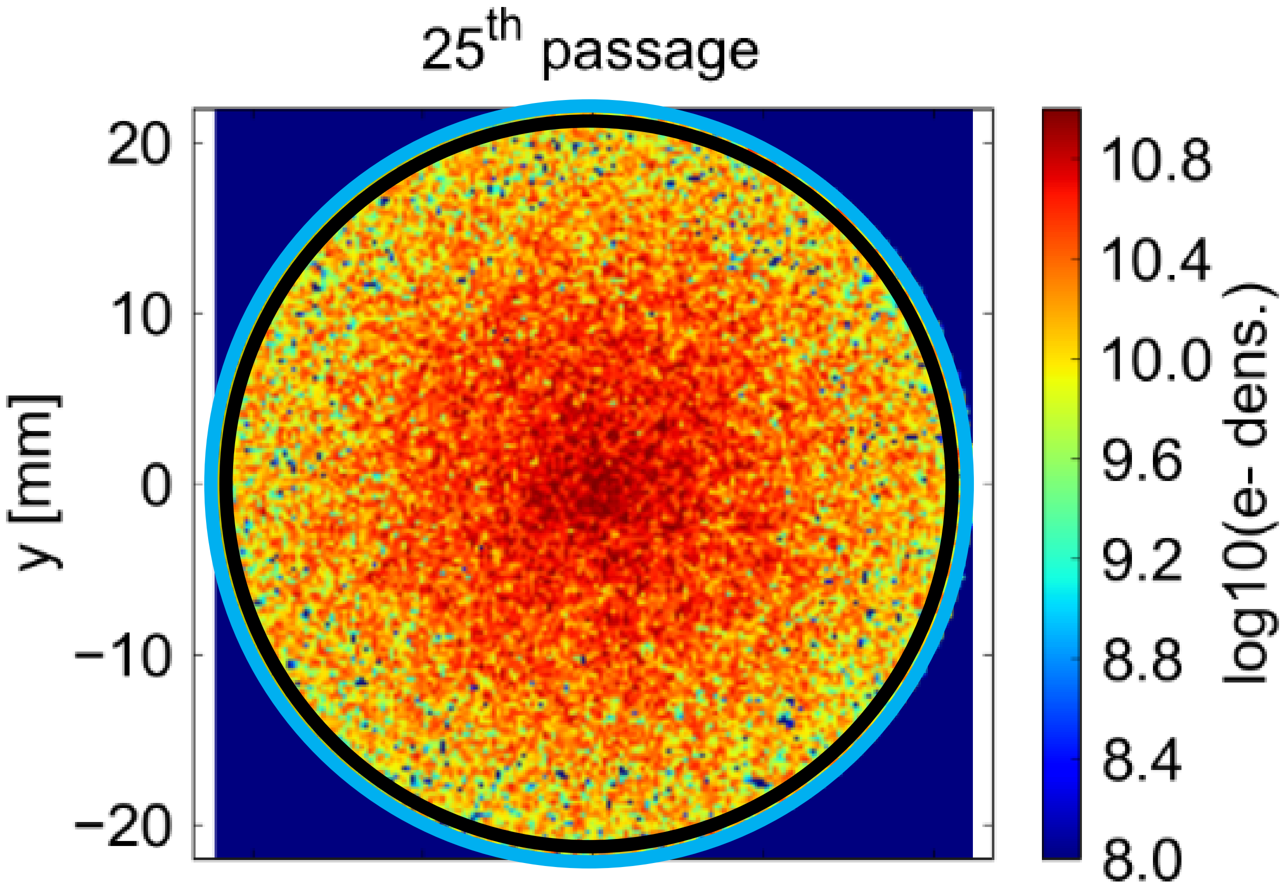
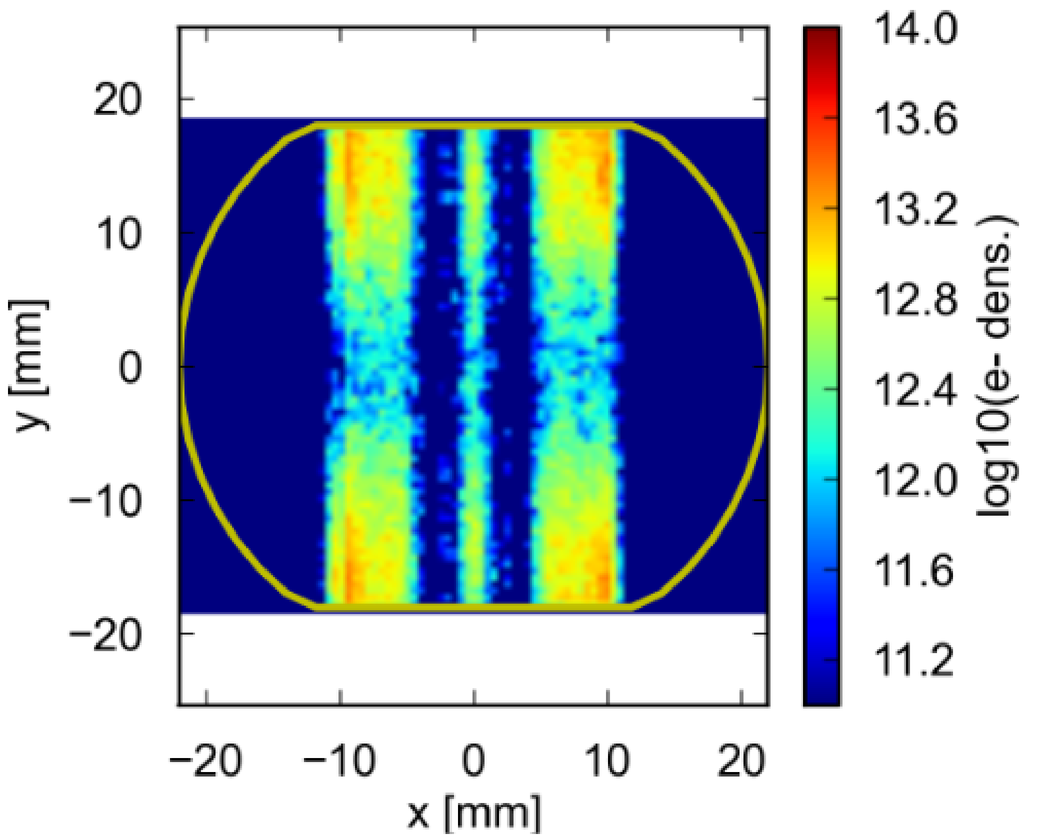


Image: *Giovanni Iadarola*

A brief overview of electron clouds

Magnetic fields influence the shape of the electron cloud.

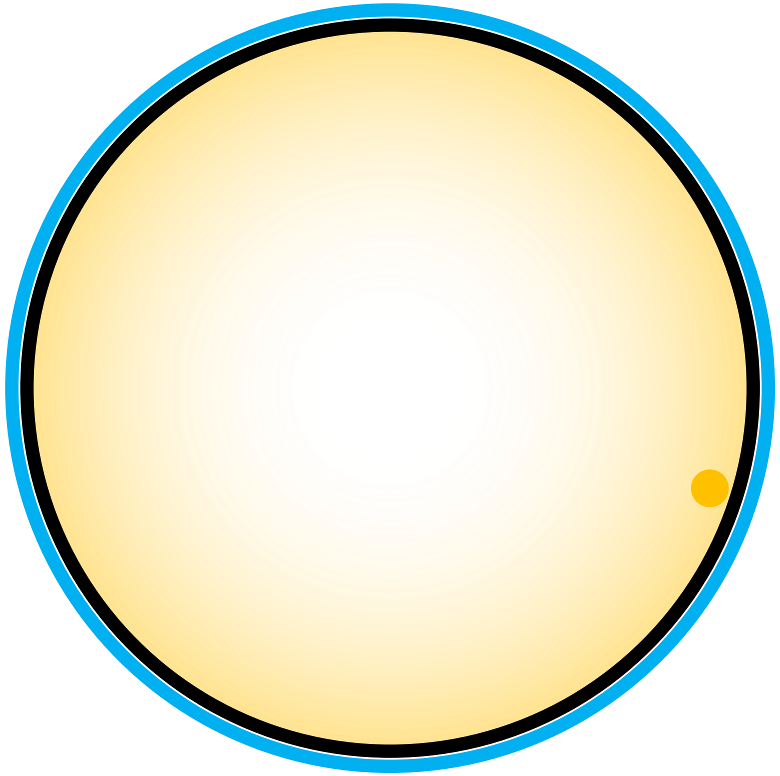




Images: *Giovanni Iadarola*

A brief overview of electron clouds

The cloud **saturates** when its field is strong enough to prevent the extraction of new electrons at the surface.



When? *Depending on...*

1. The bunch intensity.
2. The secondary electron yield curve (material properties).
3. Photoemission from synchrotron radiation.
4. Magnetic field type and strength.

Electron cloud buildup simulations

Bunch
Intensity

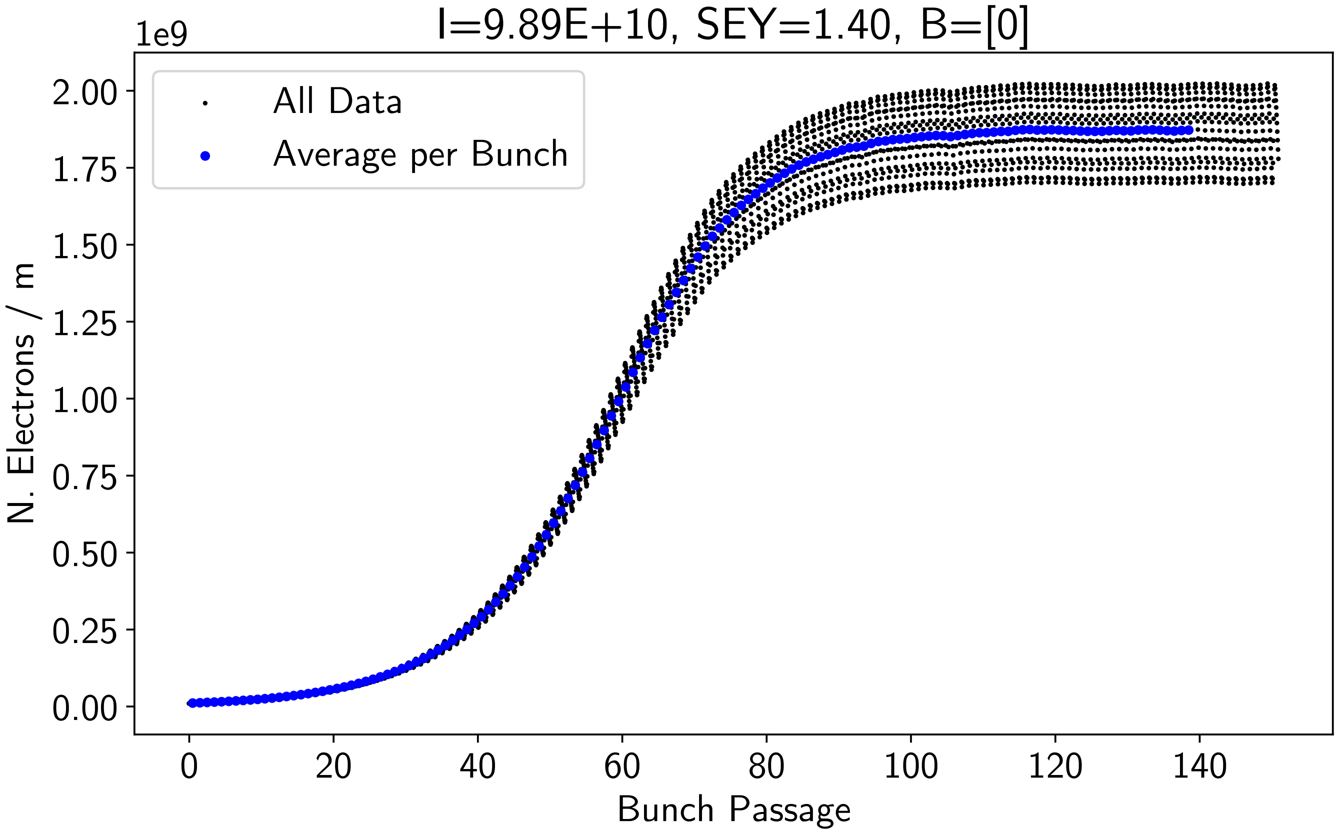
Filling
Scheme

PyECLOUD

Magnets

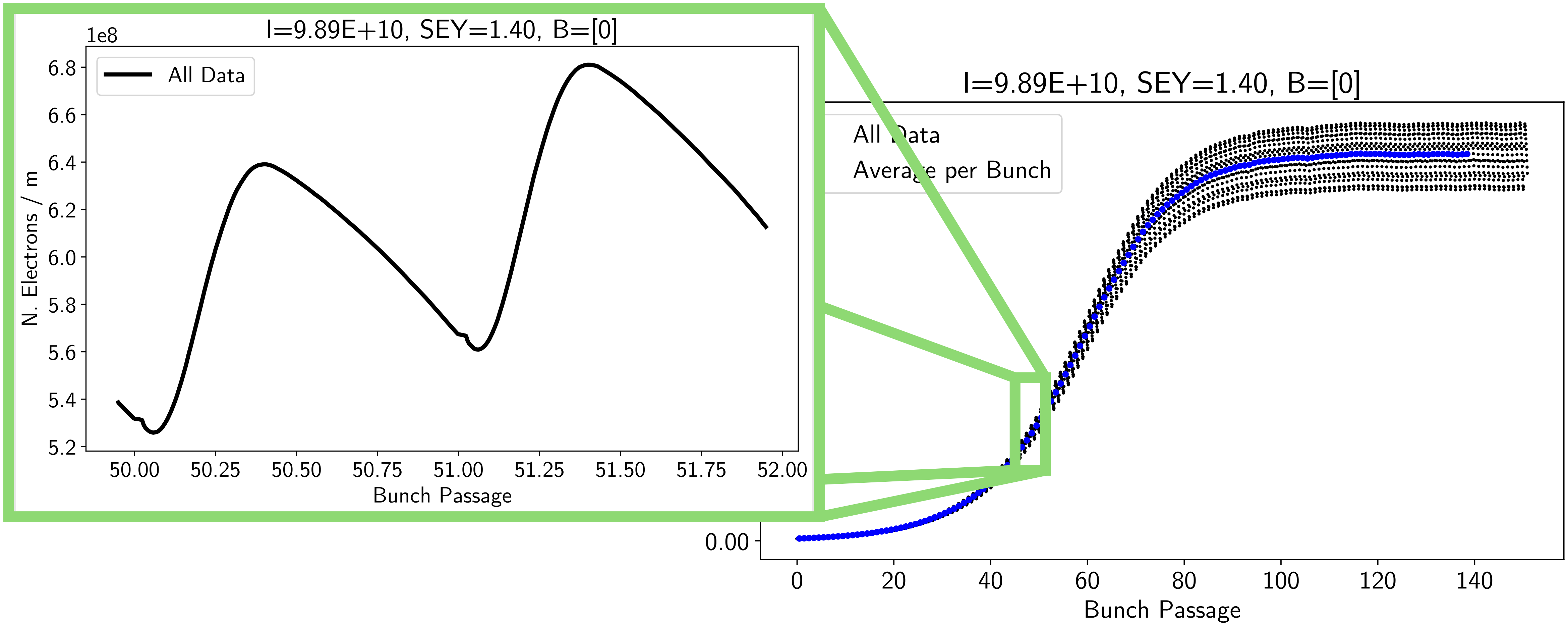
Secondary
Electron Yield

Electron cloud buildup simulations



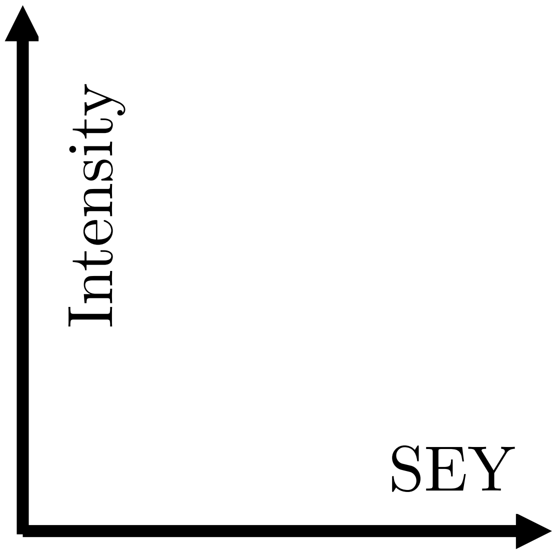
One of the simulation outputs is the electron cloud linear density over time – this is what we will focus on modelling.

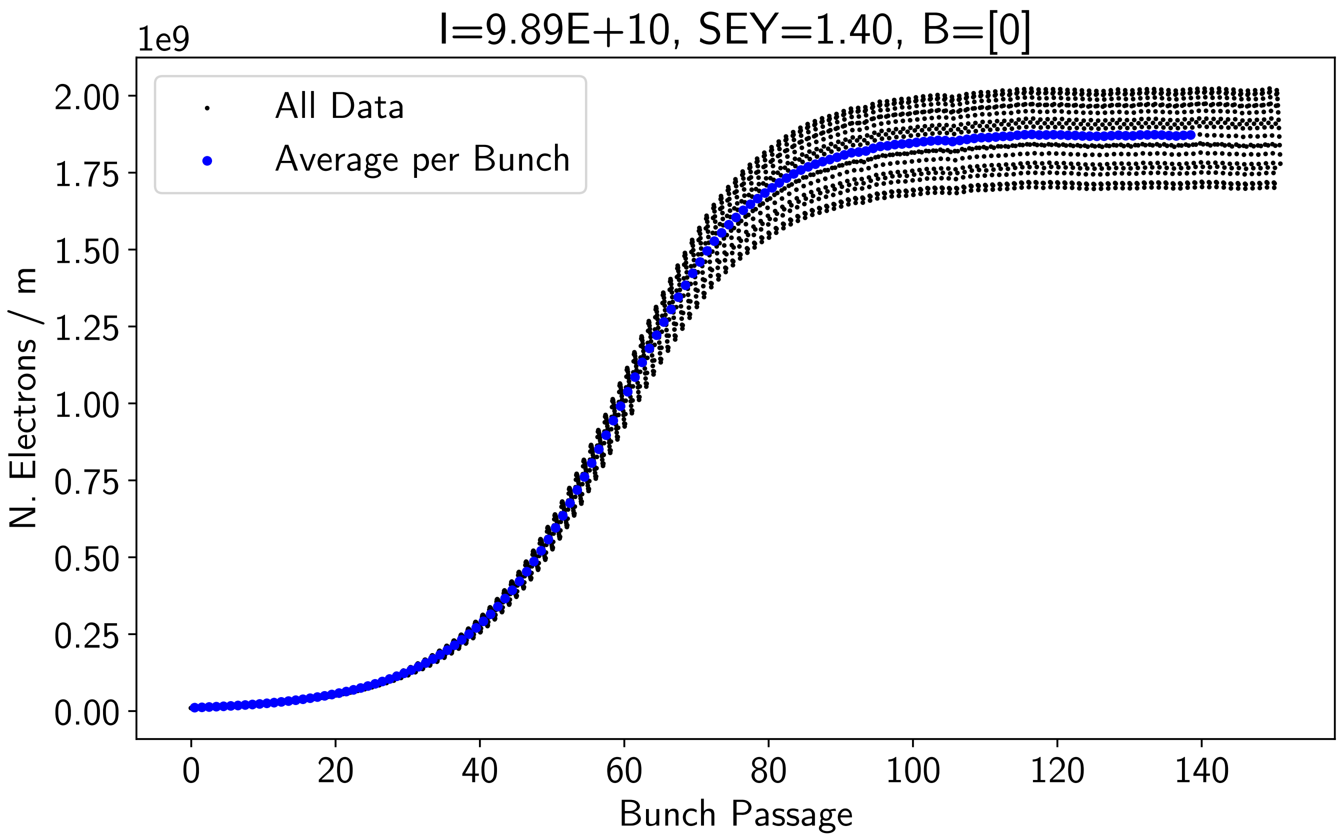
Electron cloud buildup simulations



There is one oscillation per bunch passage.

Electron cloud buildup simulations



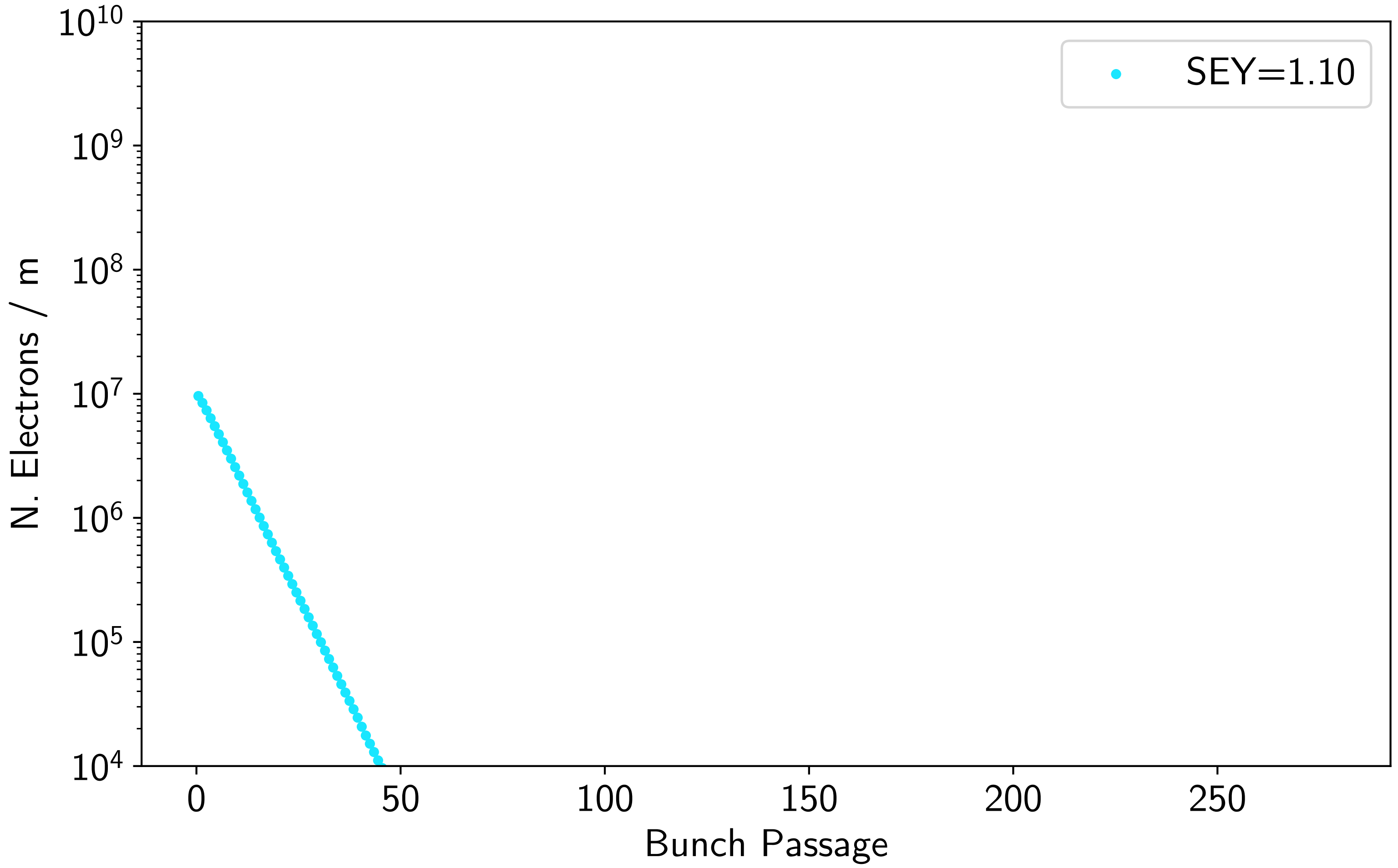


We often perform parameter sweeps to study e-cloud buildup in a new machine configuration.

Sensitivity to parameters: **SEY**

A low SEY value can lead to the electron cloud decaying.

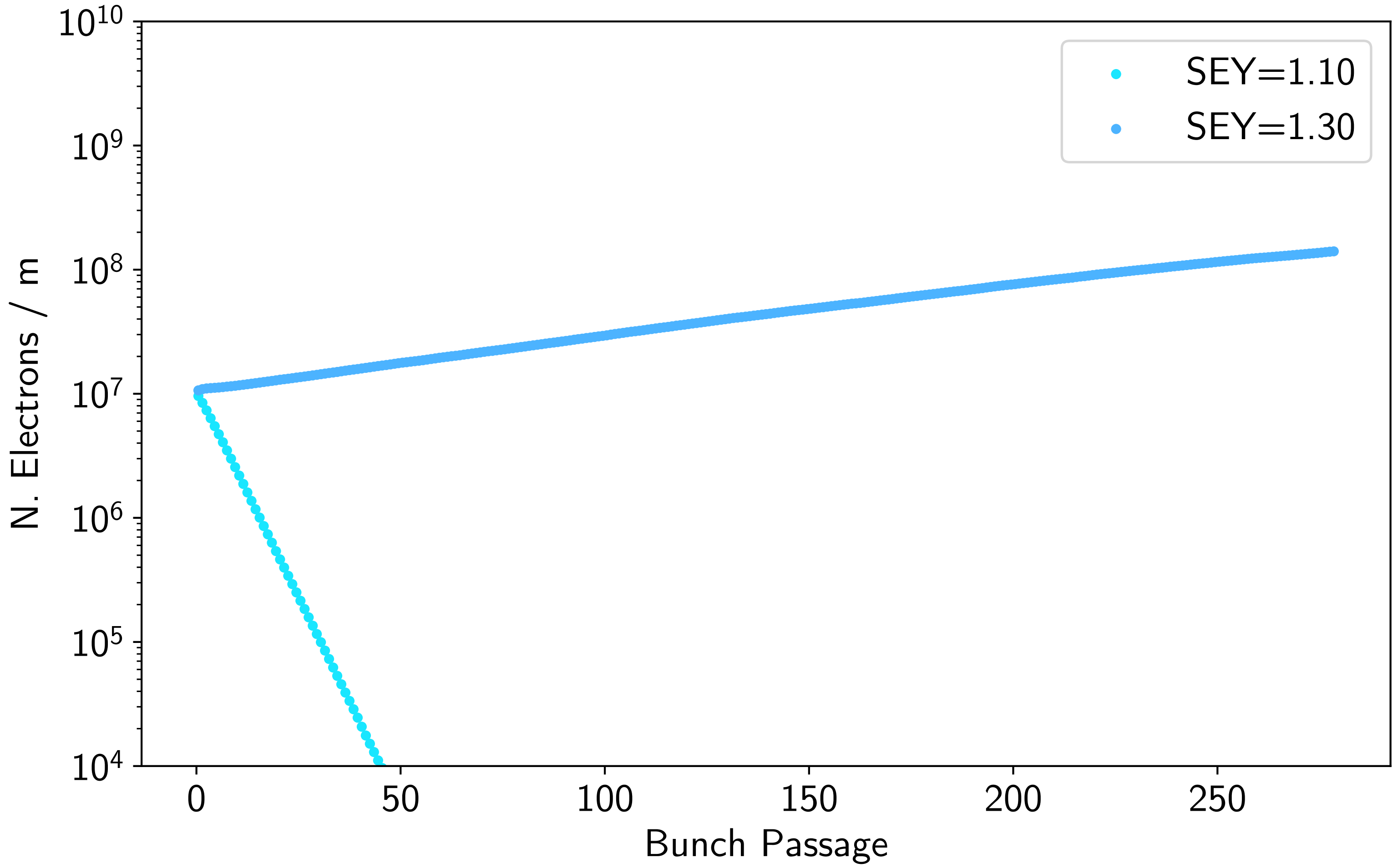
Note, here SEY > 1.0



Intensity:
9.89×10^{10}

Sensitivity to parameters: **SEY**

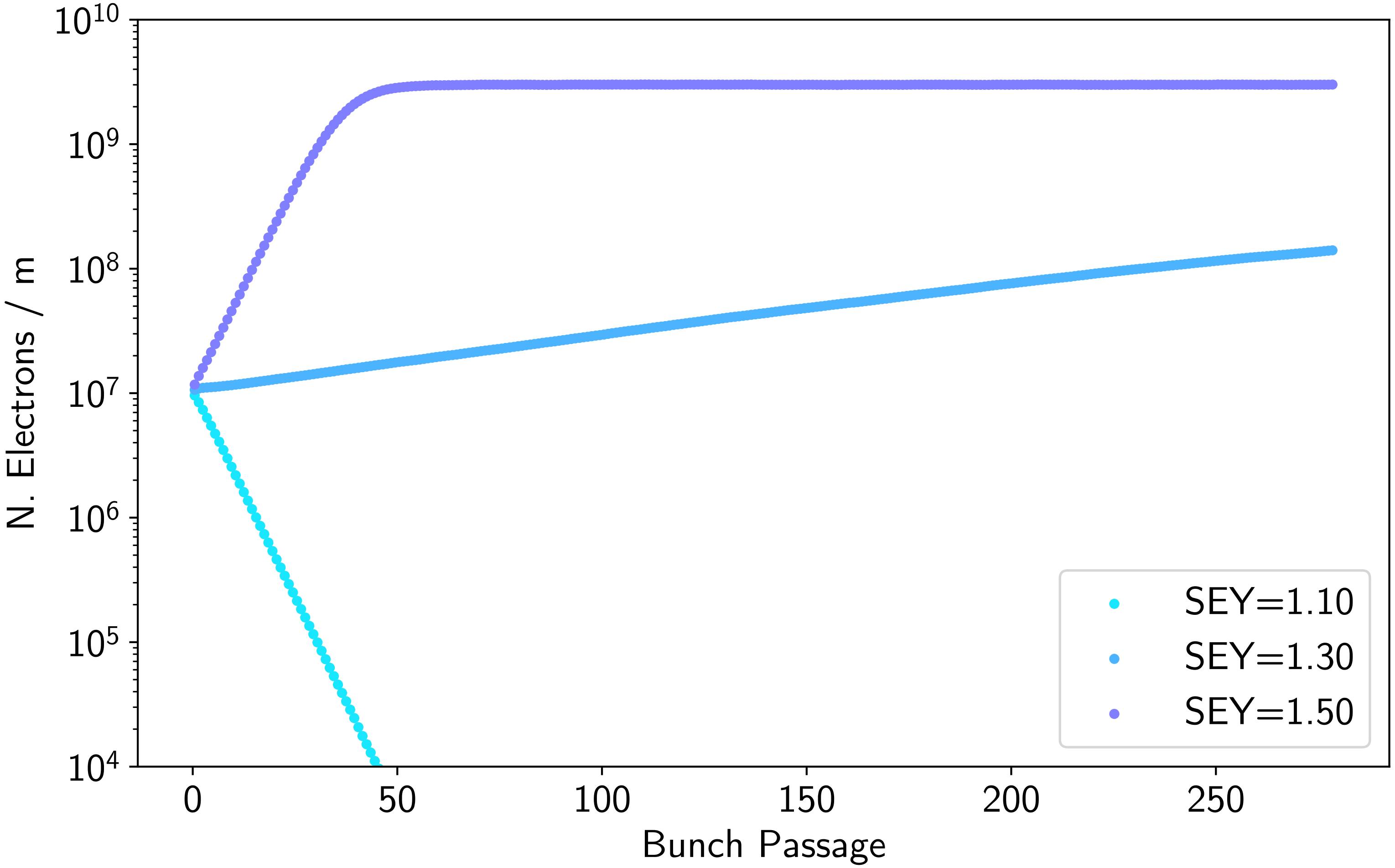
The buildup can be quite slow.



Intensity:
9.89×10^{10}

Sensitivity to parameters: **SEY**

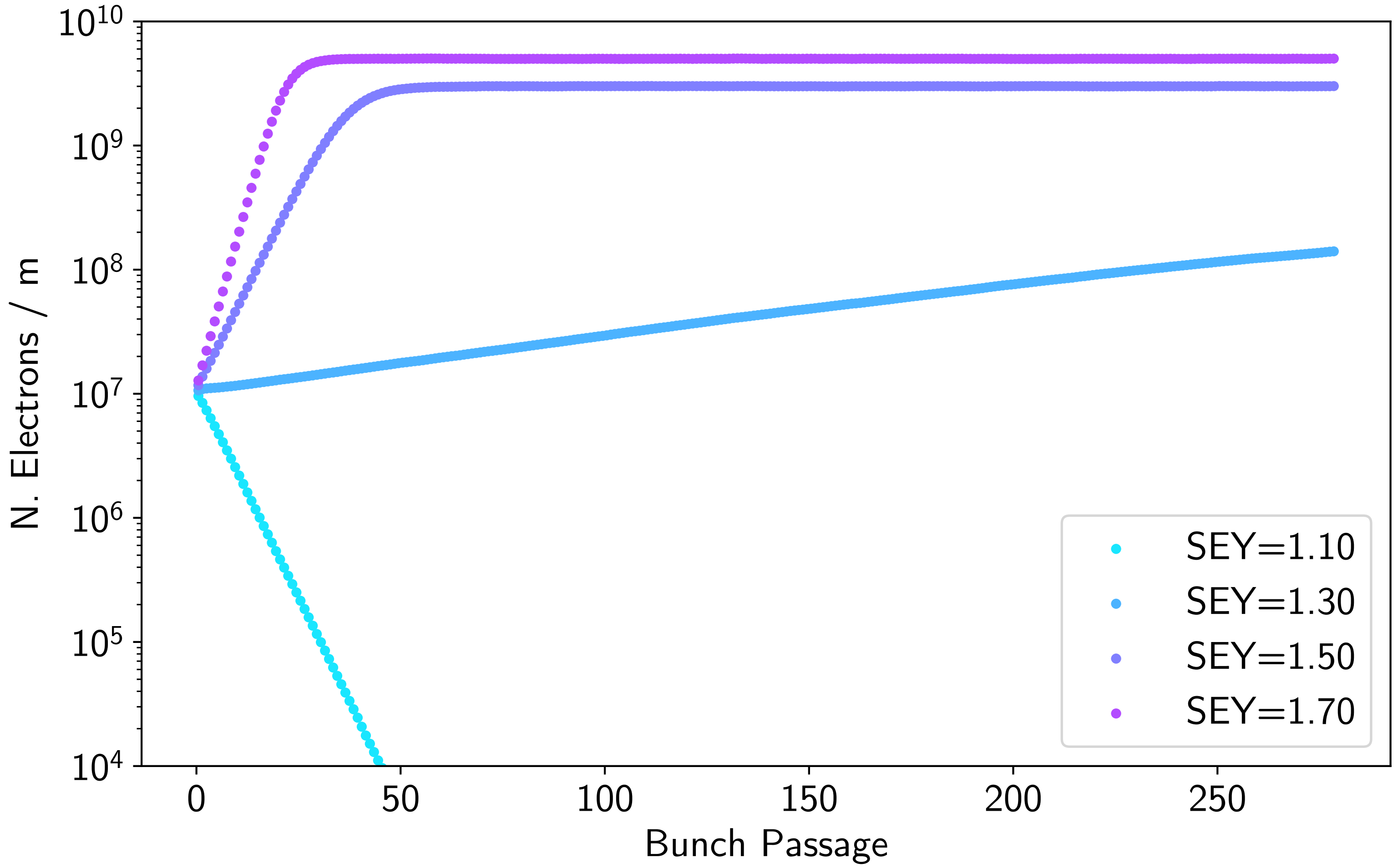
The characteristic shape of a buildup curve looks a lot like a "logistic" or "sigmoid" curve.



Intensity:
9.89×10^{10}

Sensitivity to parameters: **SEY**

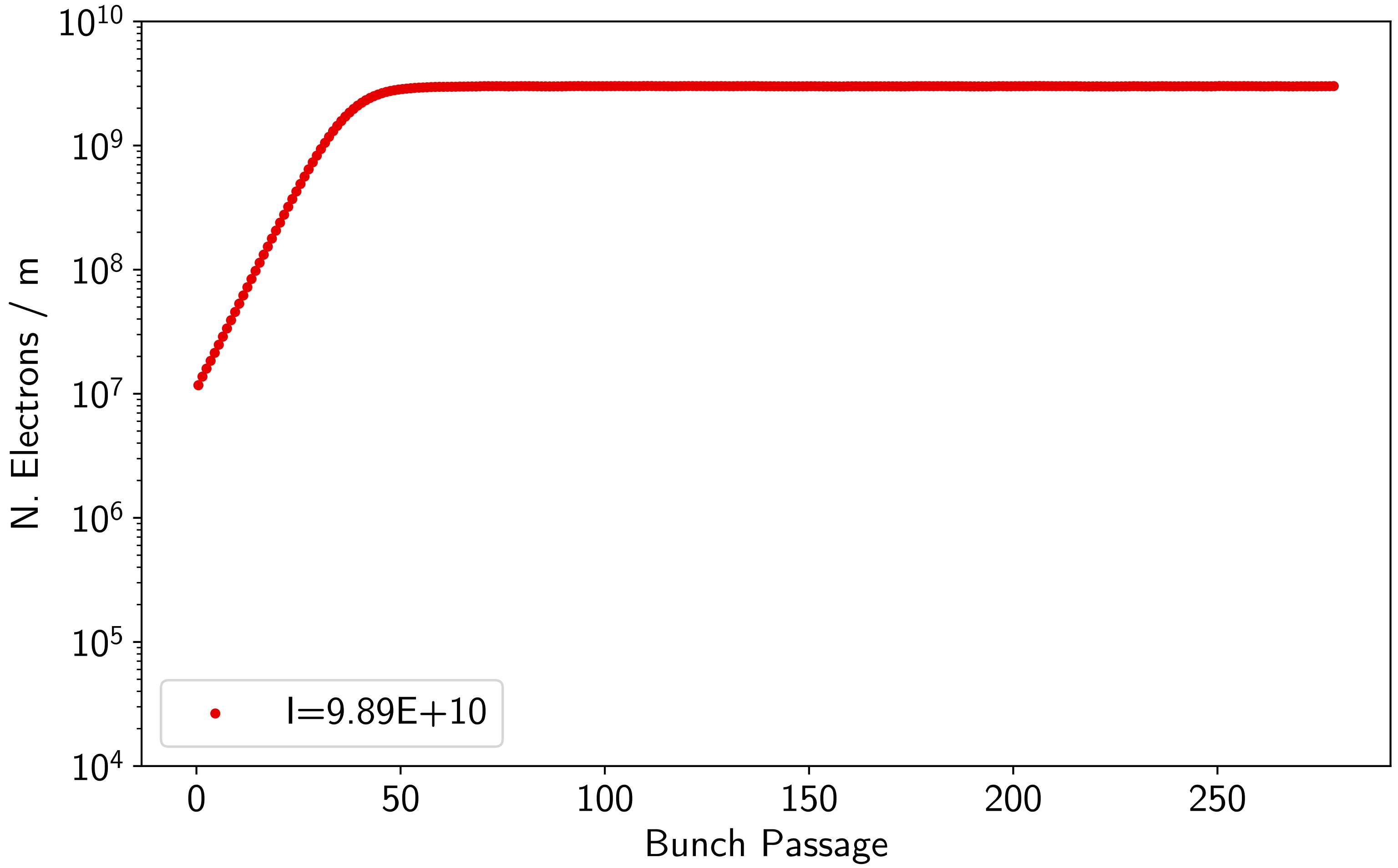
Note that both the saturation **density** and the **time to reach it** change as a function of SEY.



Intensity:
9.89×10^{10}

Sensitivity to parameters: **Intensity**

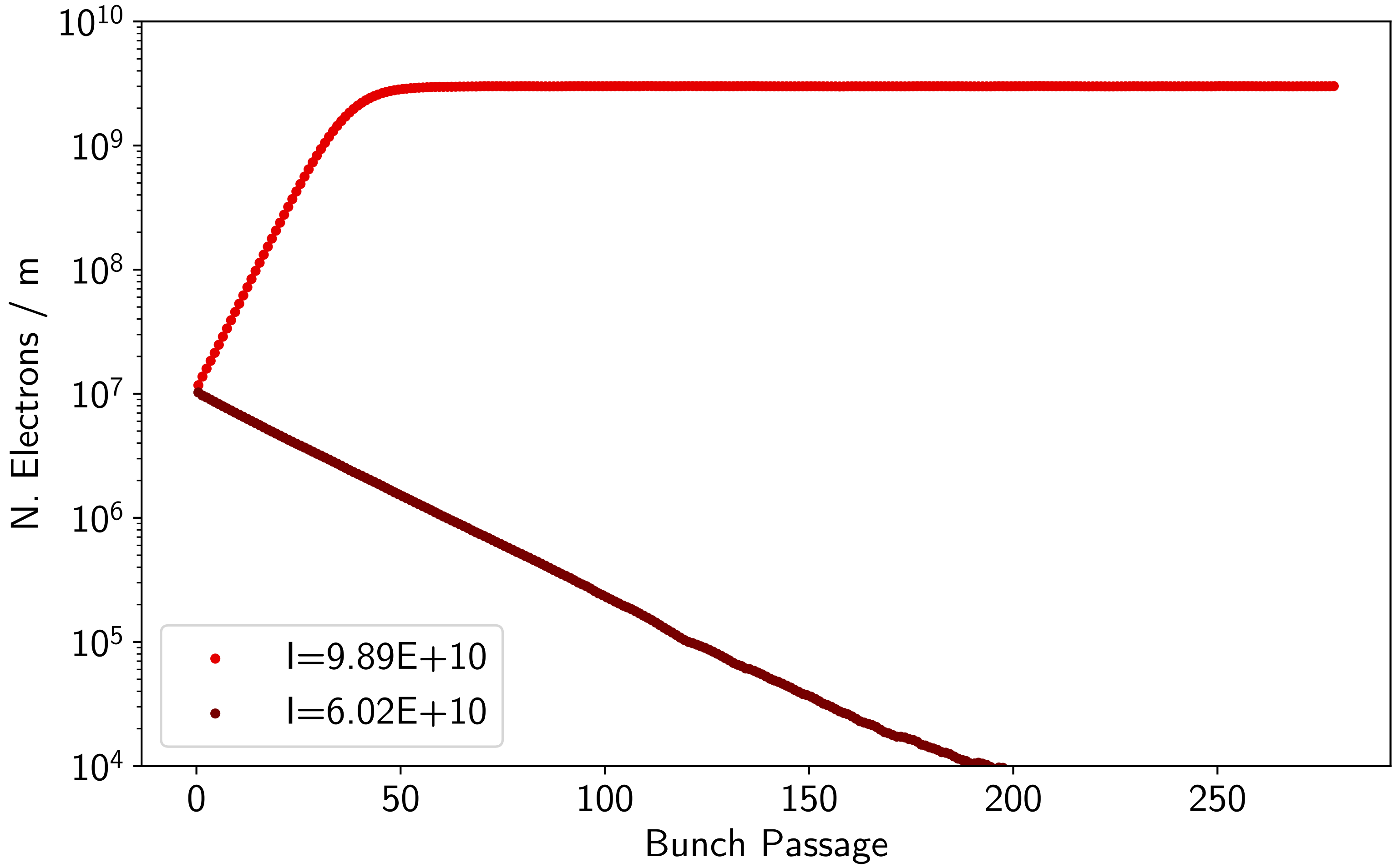
Let's start with one of the curves we already saw.



SEY: 1.5

Sensitivity to parameters: **Intensity**

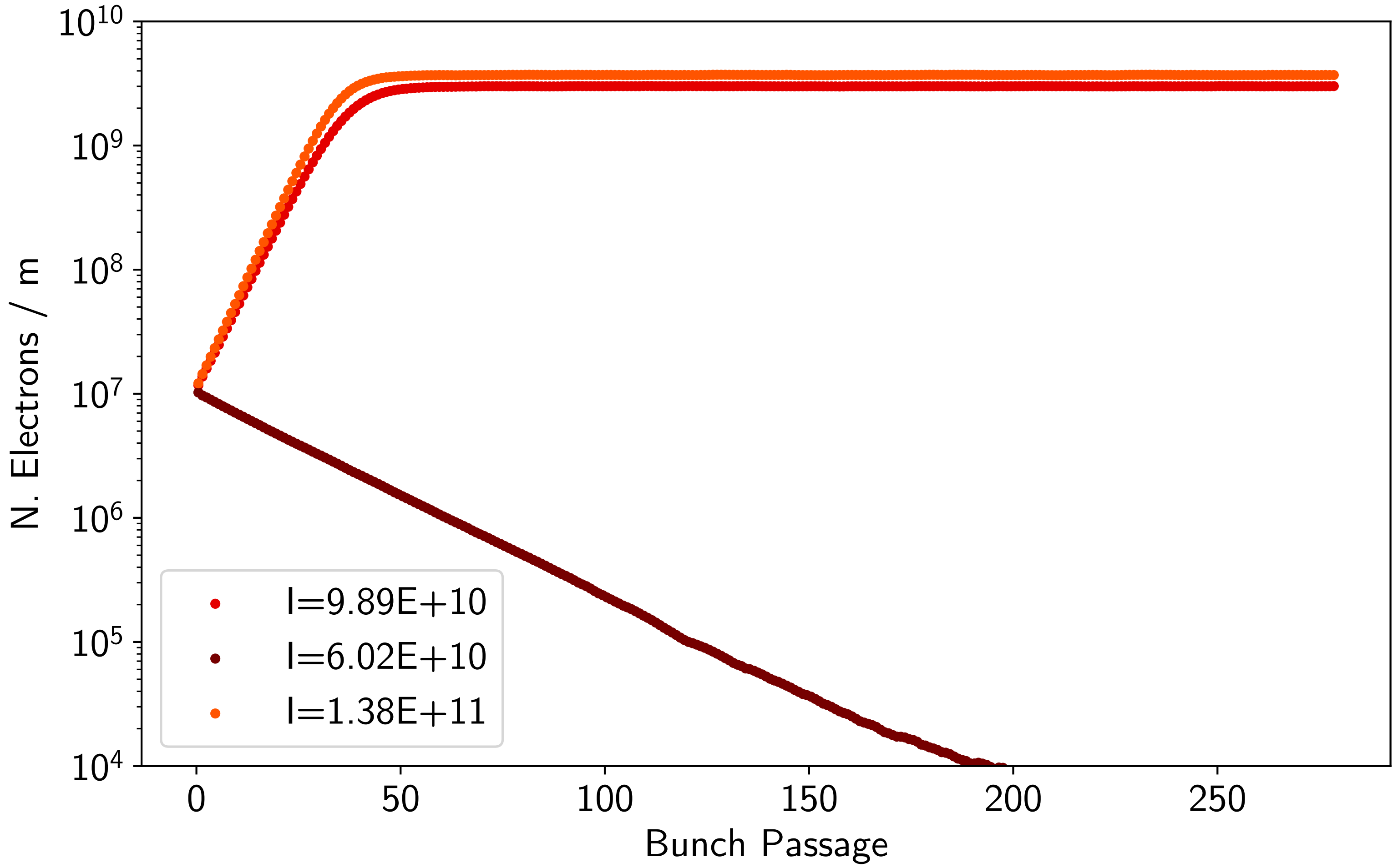
Stepping down in intensity, we again fail to build up.



SEY: 1.5

Sensitivity to parameters: **Intensity**

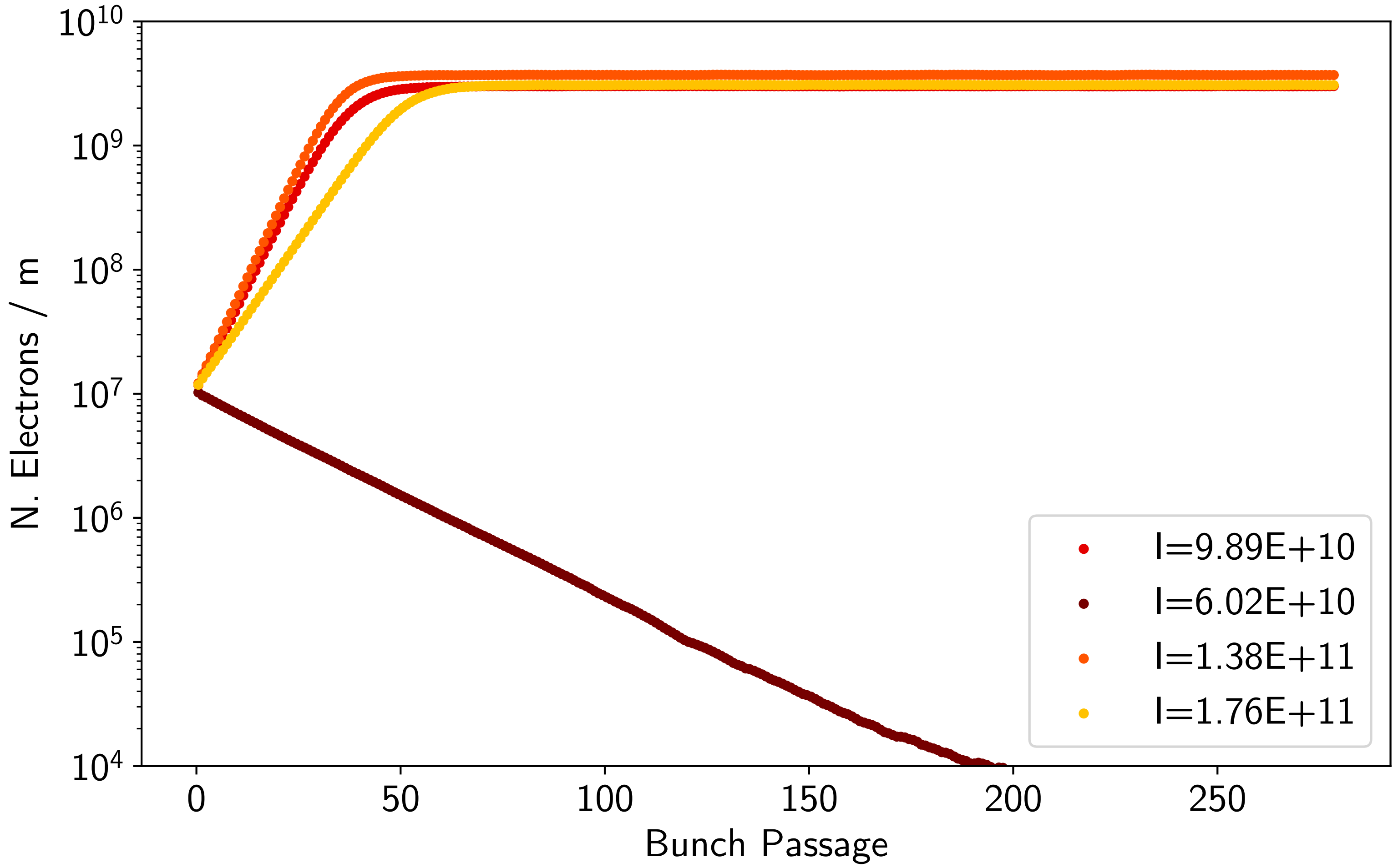
Stepping up, the saturation density increases.



SEY: 1.5

Sensitivity to parameters: **Intensity**

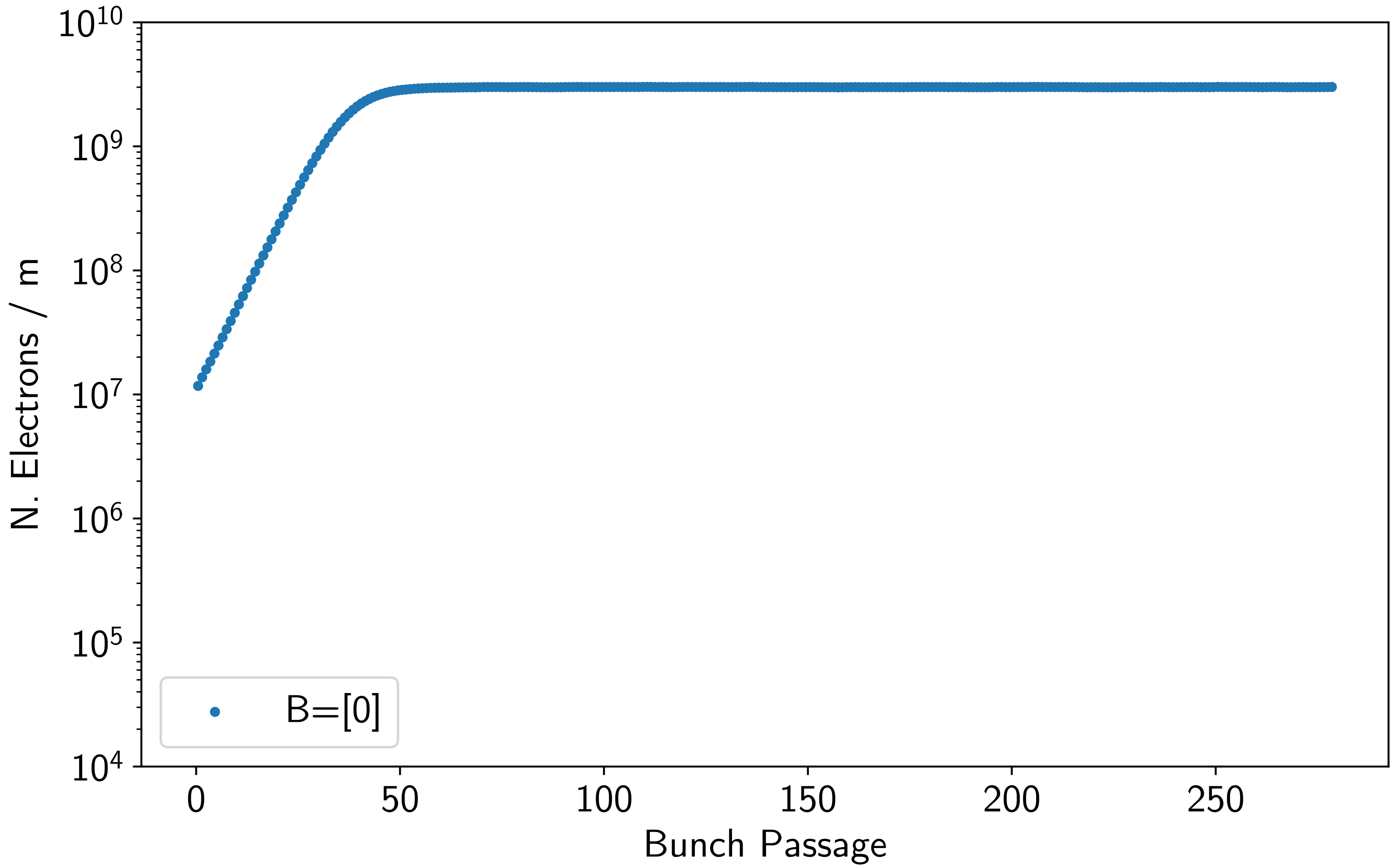
We increase the intensity again – and the saturation density **decreases**! There must exist some *critical intensity*...



SEY: 1.5

Sensitivity to parameters: **Magnets**

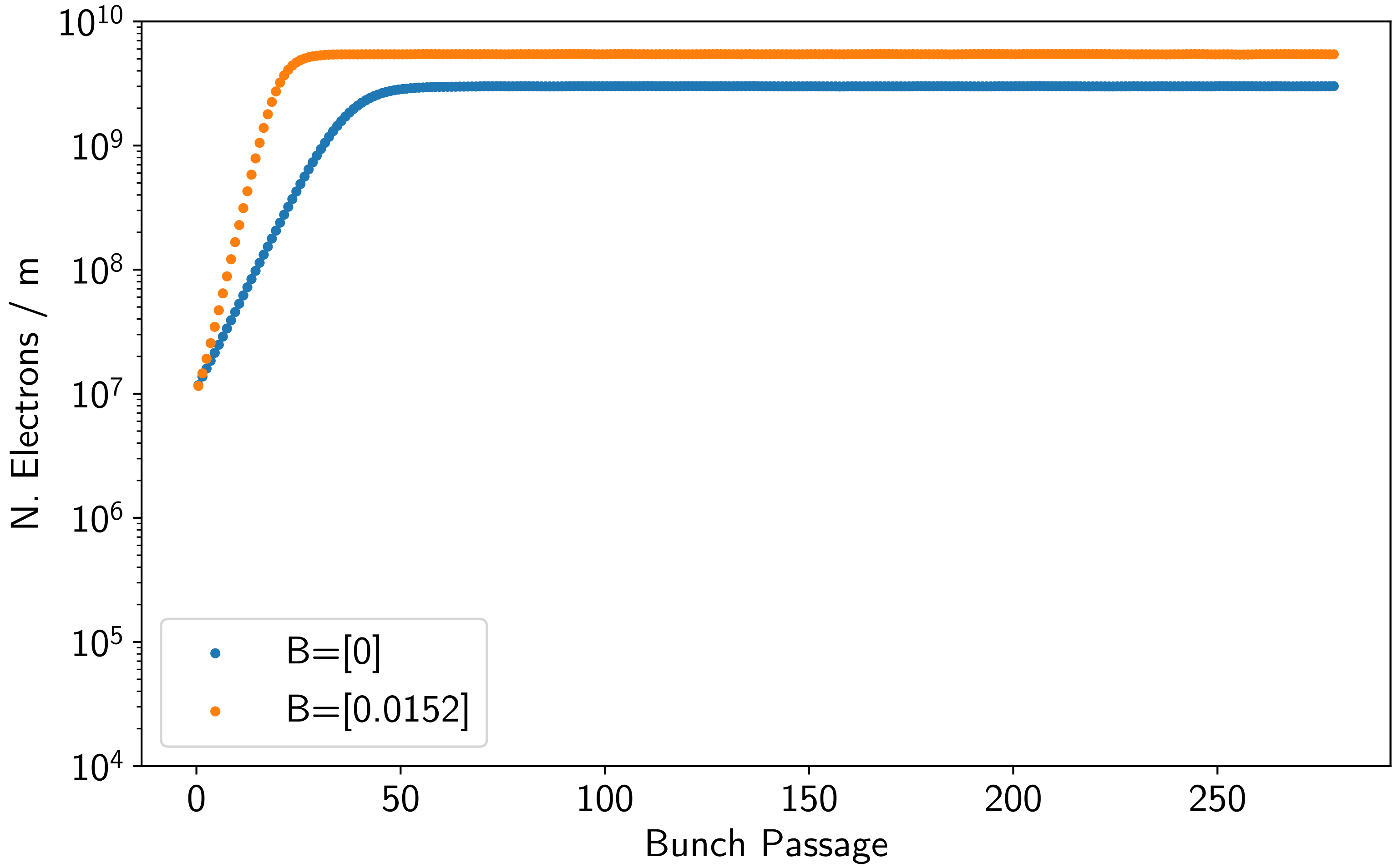
So far, we have only shown plots in drift sections.



Intensity:
9.89×10^{10}
SEY:
1.5

Sensitivity to parameters: **Magnets**

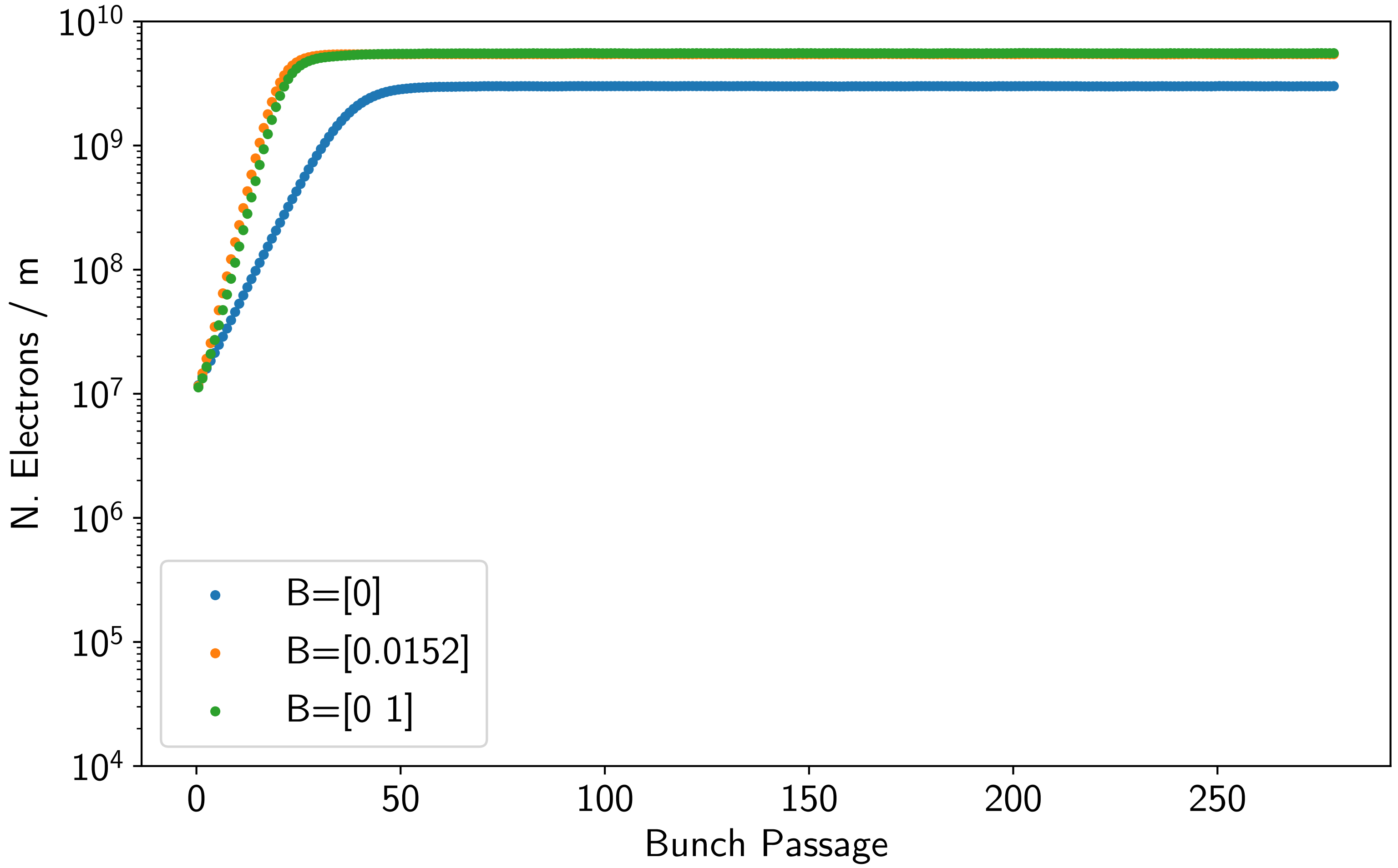
In a dipole, the saturation density increases.



Intensity:
9.89×10^{10}
SEY:
1.5

Sensitivity to parameters: **Magnets**

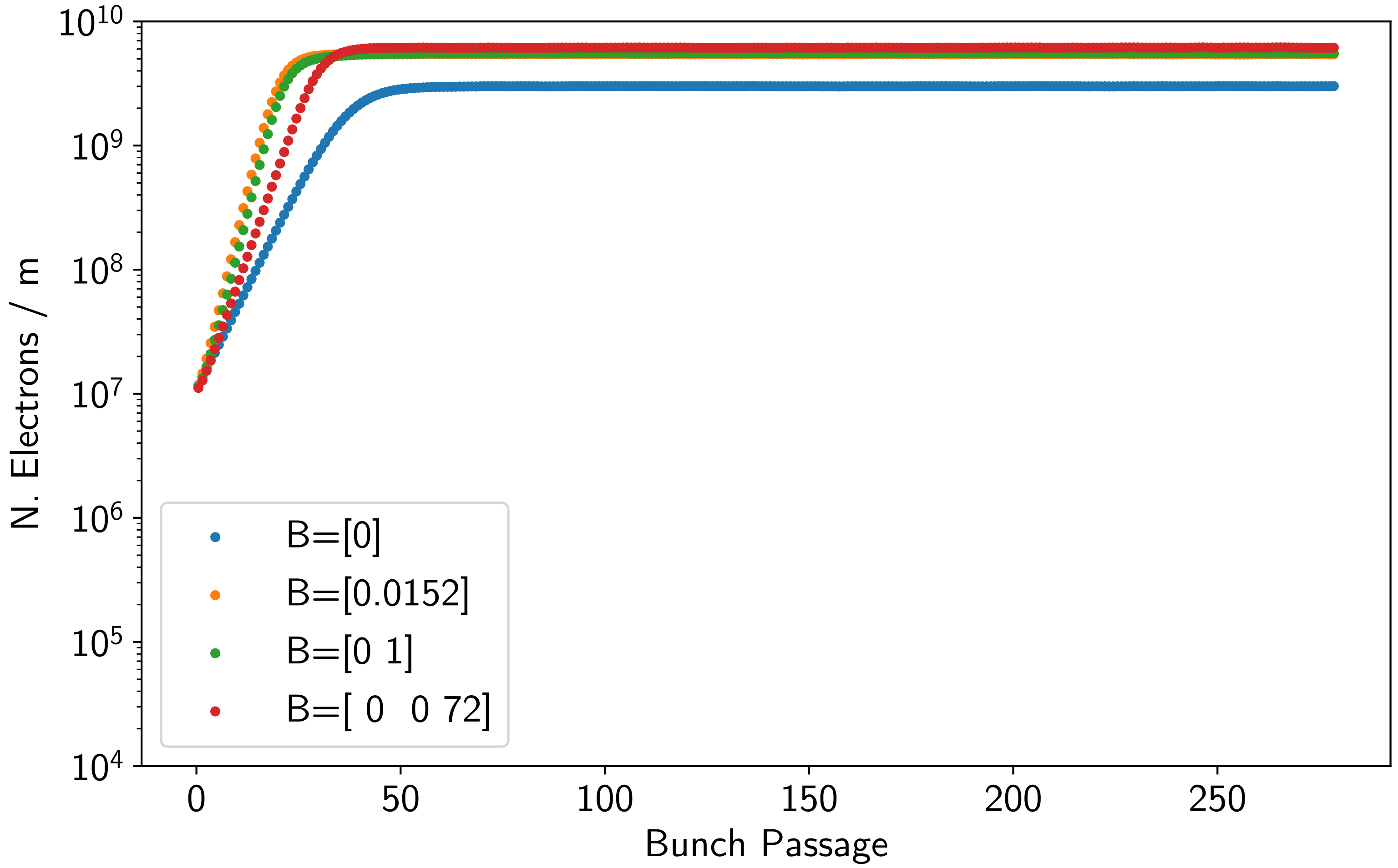
… as in a quadrupole.



Intensity:
9.89×10^{10}
SEY:
1.5

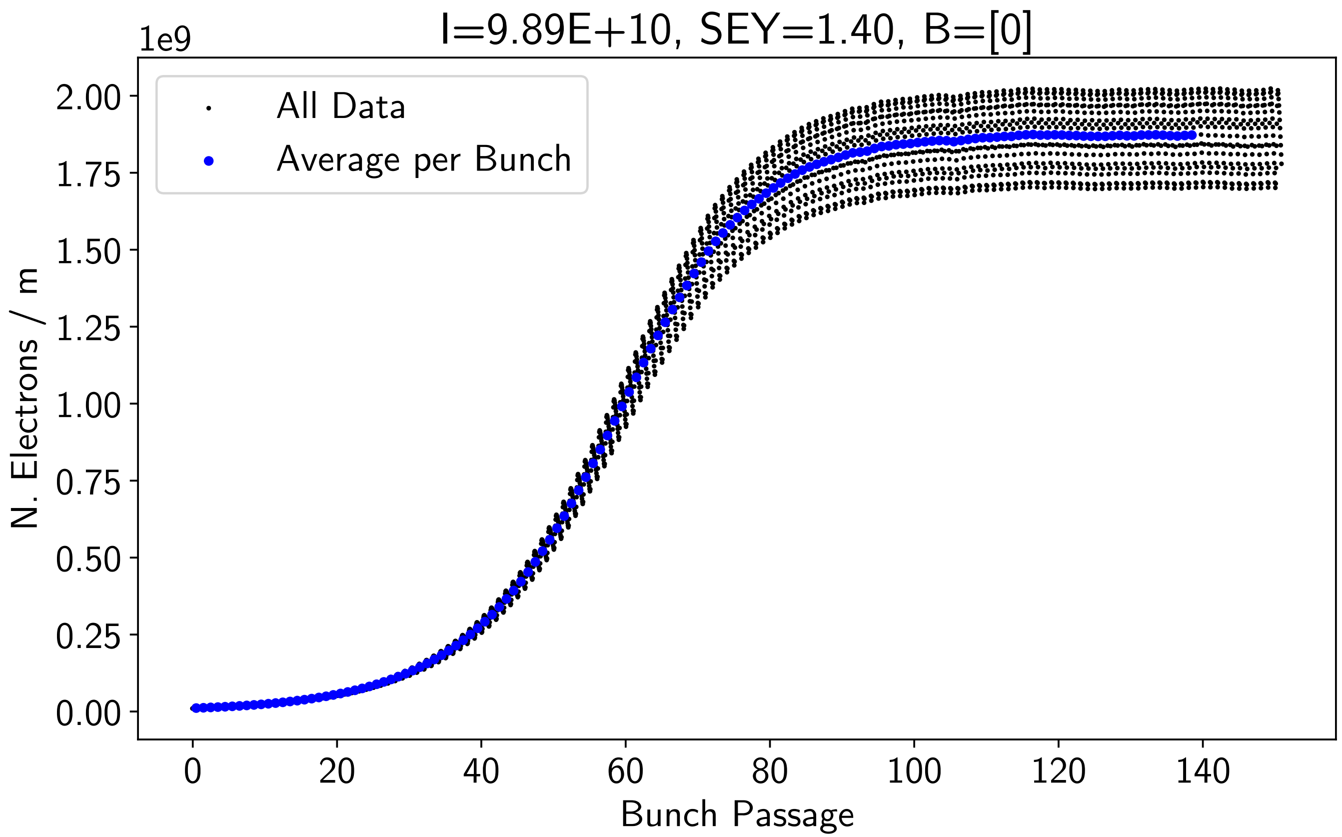
Sensitivity to parameters: **Magnets**

In a sextupole, there is a noticeable effect on the time to reach saturation as well as the saturation density.



Intensity:
9.89×10^{10}
SEY:
1.5

Modelling buildup curves



Desired Features

- Saturation.
- Independent of initial N.
- Increase with SEY.
- Critical (worst) intensity.
- Response to magnets.

Can we reduce the buildup curve data to a (simple) model?
Will such a model capture the trends seen in parameter sweeps?

Modelling buildup curves

There is **existing work** on modelling electron cloud buildup.

I will be expanding on model from "*Electron-Cloud Build-Up: Theory and Data*" by M. A. Furman (2011).

No complete derivation, but here are the core ideas.

$$\dot{N}_e = \dot{N}_p + \left(\dot{N}_{sec} - \dot{N}_{col}\right)$$

Total change in N. of e- → $\dot{N}_e$

Rate of primary e- (photoemission) → $\dot{N}_p$

Rate of secondary e- (SEY) → $\dot{N}_{sec}$

Rate of collisions with walls. → $\dot{N}_{col}$

Modelling buildup curves

$$\dot{N}_e = \dot{N}_p + \left(\dot{N}_{sec} - \dot{N}_{col}\right)$$

We can convert to longitudinal densities and define

$\delta_{eff} \approx \dot{N}_{sec}/\dot{N}_{col}$ to obtain:

$$\dot{\lambda}_e = \dot{n}_p \lambda_b(t) + (\delta_{eff} - 1)\dot{N}_{col}\frac{e}{L}$$

Beam density.

Flux over whole perimeter

The total flux over the perimeter is $\dot{N}_{col}\frac{e}{L} = \frac{\lambda_e}{\Delta t_{tr}}$, where Δt_{tr} is how long it takes an electron, on average, to cross the chamber.

Modelling buildup curves

$$\dot{\lambda_e} = \dot{n}_p \lambda_b(t) + (\delta_{eff} - 1)\frac{\lambda_e(t)}{\Delta t_{tr}}$$

Furman proposes $(\delta_{eff} - 1) = \kappa\left(\frac{\lambda_c - \lambda_e}{\overline{\lambda_b}}\right)$.

- Effective SEY depends on a critical density λ_c.
- The constant κ is of arbitrary sign – we can also decay.
- We normalize by average beam intensity $\overline{\lambda_b}$.

This automatically limits the model to describing **average** density within each bunch passage.

Modelling buildup curves

$$\dot{\lambda_e} = \dot{n}_p \lambda_b(t) + \frac{\kappa}{\overline{\lambda_b} \Delta t_{tr}} \left(\lambda_c - \lambda_e(t)\right) \cdot \lambda_e(t)$$

We switch variables to $x = {}^{t}/_{t_b}$ and $y = {}^{\lambda_e(t)}/_{\overline{\lambda_b}}$.

$$\frac{dy}{dx} = \alpha + \beta(y_c - y) \cdot y$$

- $y_c = {}^{\lambda_c}/_{\overline{\lambda_b}}$
- $\alpha = {}^{\dot{n}_p t_b}/_{Z}$
- $\beta = {}^{\kappa t_b}/_{\Delta t_{tr}}$ *note: nothing to do with betatron motion!*

Modelling buildup curves

$$\frac{dy}{dx} = \alpha + \beta(y_c - y) \cdot y$$

This differential equation has a family of solutions.

$y(0) = 0,\ \alpha > 0$	$y(0) = y_0,\ \alpha = 0$	$y(0) = y_0,\ \alpha > 0$	$y(0) = 0,\ \alpha > 0$	$y(0) = y_0,\ \alpha = 0$	$y(0) = y_0,\ \alpha > 0$	$y(0) = 0,\ \alpha > 0$	$y(0) = y_0,\ \alpha = 0$	$y(0) = y_0,\ \alpha > 0$
$\frac{y_+ y_-\left(e^{\beta(y_+ - y_-)x} - 1\right)}{y_+ - y_- e^{\beta(y_+ - y_-)x}}$ $y_\pm = \frac{y_c}{2} \pm \sqrt{\frac{y_c^2}{4} + \frac{\alpha}{\beta}}$	$\frac{y_0 y_c e^{\beta y_c x}}{y_c + y_0\left(e^{\beta y_c x} - 1\right)}$	$\frac{y_c}{2} + \sqrt{\frac{y_c^2}{4} + \frac{\alpha}{\beta}} \tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha + y_c^2\beta)} - \phi\right)$ $\phi = \tanh^{-1}\left((y_c - 2y_0)\sqrt{\frac{\beta}{4\alpha + y_c^2\beta}}\right)$	$\frac{y_+ y_-\left(e^{\beta(y_+ - y_-)x} - 1\right)}{y_+ - y_- e^{\beta(y_+ - y_-)x}}$ $y_\pm = \frac{y_c}{2} \pm \sqrt{\frac{y_c^2}{4} + \frac{\alpha}{\beta}}$	$\frac{y_0 y_c e^{\beta y_c x}}{y_c + y_0\left(e^{\beta y_c x} - 1\right)}$	$\frac{y_c}{2} + \sqrt{\frac{y_c^2}{4} + \frac{\alpha}{\beta}} \tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha + y_c^2\beta)} - \phi\right)$ $\phi = \tanh^{-1}\left((y_c - 2y_0)\sqrt{\frac{\beta}{4\alpha + y_c^2\beta}}\right)$	$\frac{y_+ y_-\left(e^{\beta(y_+ - y_-)x} - 1\right)}{y_+ - y_- e^{\beta(y_+ - y_-)x}}$ $y_\pm = \frac{y_c}{2} \pm \sqrt{\frac{y_c^2}{4} + \frac{\alpha}{\beta}}$	$\frac{y_0 y_c e^{\beta y_c x}}{y_c + y_0\left(e^{\beta y_c x} - 1\right)}$	$\frac{y_c}{2} + \sqrt{\frac{y_c^2}{4} + \frac{\alpha}{\beta}} \tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha + y_c^2\beta)} - \phi\right)$ $\phi = \tanh^{-1}\left((y_c - 2y_0)\sqrt{\frac{\beta}{4\alpha + y_c^2\beta}}\right)$

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$$\lim_{\alpha \to 0} y(x)$$

$$\lim_{y_0 \to 0} y(x)$$

Modelling buildup curves

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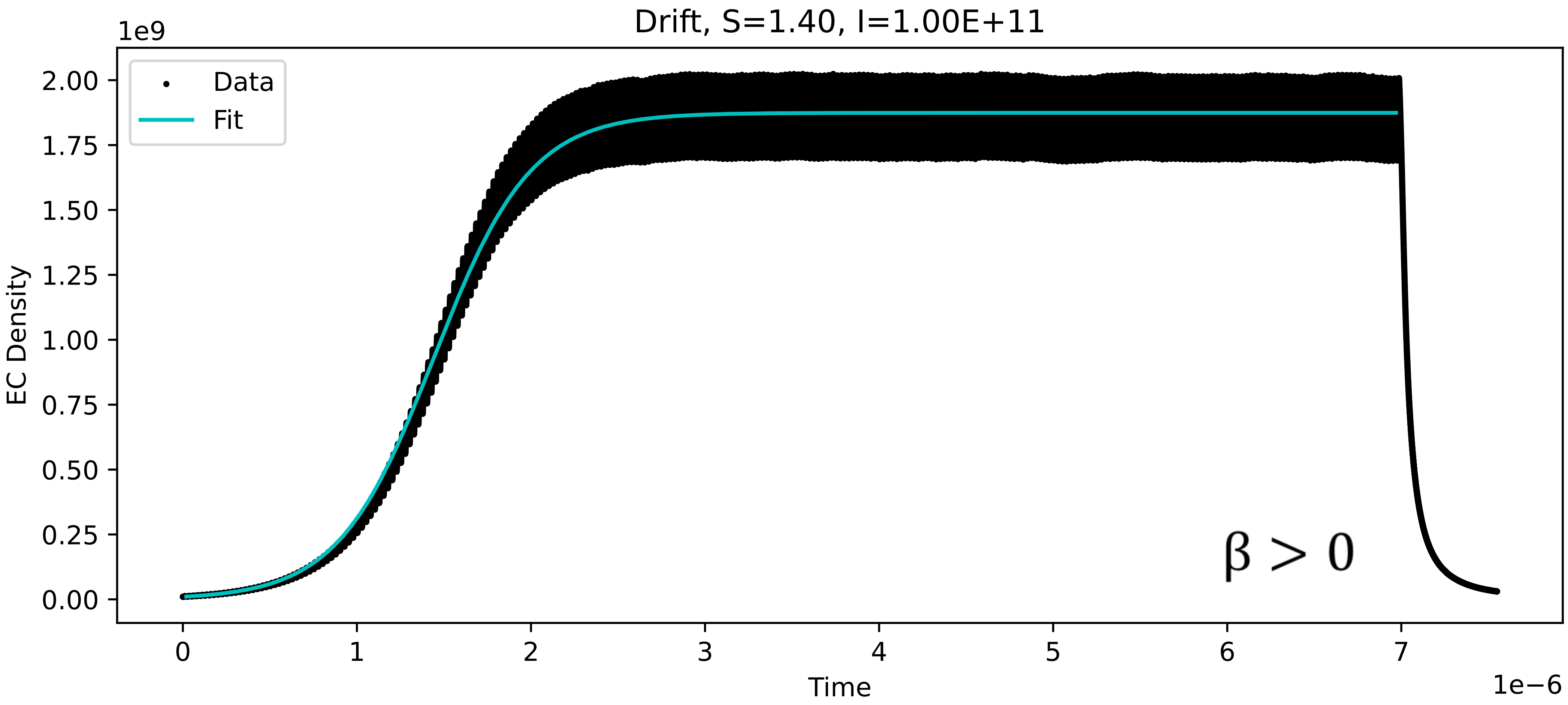
$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=y_c$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=y_c$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=y_c$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$
$y(0)=0,\ \alpha>0$	$y(0)=y_0,\ \alpha=0$	$y(0)=y_0,\ \alpha>0$	$y(0)=0,\ \alpha>0$	$y(0)=y_0,\ \alpha=0$	$y(0)=y_0,\ \alpha>0$	$y(0)=0,\ \alpha>0$	$y(0)=y_0,\ \alpha=0$	$y(0)=y_0,\ \alpha>0$
$\frac{y_+y_-(e^{\beta(y_+-y_-)x}-1)}{y_+-y_-e^{\beta(y_+-y_-)x}}$ $y_\pm=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\frac{y_0y_ce^{\beta y_cx}}{y_c+y_0(e^{\beta y_cx}-1)}$	$\frac{y_c}{2}+\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}\tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha+y_c^2\beta)}-\phi\right)$ $\phi=\tanh^{-1}\left((y_c-2y_0)\sqrt{\frac{\beta}{4\alpha+y_c^2\beta}}\right)$	$\frac{y_+y_-(e^{\beta(y_+-y_-)x}-1)}{y_+-y_-e^{\beta(y_+-y_-)x}}$ $y_\pm=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\frac{y_0y_ce^{\beta y_cx}}{y_c+y_0(e^{\beta y_cx}-1)}$	$\frac{y_c}{2}+\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}\tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha+y_c^2\beta)}-\phi\right)$ $\phi=\tanh^{-1}\left((y_c-2y_0)\sqrt{\frac{\beta}{4\alpha+y_c^2\beta}}\right)$	$\frac{y_+y_-(e^{\beta(y_+-y_-)x}-1)}{y_+-y_-e^{\beta(y_+-y_-)x}}$ $y_\pm=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\frac{y_0y_ce^{\beta y_cx}}{y_c+y_0(e^{\beta y_cx}-1)}$	$\frac{y_c}{2}+\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}\tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha+y_c^2\beta)}-\phi\right)$ $\phi=\tanh^{-1}\left((y_c-2y_0)\sqrt{\frac{\beta}{4\alpha+y_c^2\beta}}\right)$
$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=y_c$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=y_c$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=y_c$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$
$y(0)=0,\ \alpha>0$	$y(0)=y_0,\ \alpha=0$	$y(0)=y_0,\ \alpha>0$	$y(0)=0,\ \alpha>0$	$y(0)=y_0,\ \alpha=0$	$y(0)=y_0,\ \alpha>0$	$y(0)=0,\ \alpha>0$	$y(0)=y_0,\ \alpha=0$	$y(0)=y_0,\ \alpha>0$
$\frac{y_+y_-(e^{\beta(y_+-y_-)x}-1)}{y_+-y_-e^{\beta(y_+-y_-)x}}$ $y_\pm=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\frac{y_0y_ce^{\beta y_cx}}{y_c+y_0(e^{\beta y_cx}-1)}$	$\frac{y_c}{2}+\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}\tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha+y_c^2\beta)}-\phi\right)$ $\phi=\tanh^{-1}\left((y_c-2y_0)\sqrt{\frac{\beta}{4\alpha+y_c^2\beta}}\right)$	$\frac{y_+y_-(e^{\beta(y_+-y_-)x}-1)}{y_+-y_-e^{\beta(y_+-y_-)x}}$ $y_\pm=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\frac{y_0y_ce^{\beta y_cx}}{y_c+y_0(e^{\beta y_cx}-1)}$	$\frac{y_c}{2}+\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}\tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha+y_c^2\beta)}-\phi\right)$ $\phi=\tanh^{-1}\left((y_c-2y_0)\sqrt{\frac{\beta}{4\alpha+y_c^2\beta}}\right)$	$\frac{y_+y_-(e^{\beta(y_+-y_-)x}-1)}{y_+-y_-e^{\beta(y_+-y_-)x}}$ $y_\pm=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\frac{y_0y_ce^{\beta y_cx}}{y_c+y_0(e^{\beta y_cx}-1)}$	$\frac{y_c}{2}+\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}\tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha+y_c^2\beta)}-\phi\right)$ $\phi=\tanh^{-1}\left((y_c-2y_0)\sqrt{\frac{\beta}{4\alpha+y_c^2\beta}}\right)$
$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=y_c$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=y_c$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$	$\lim_{x\to\infty} y(x)=y_c$	$\lim_{x\to\infty} y(x)=\frac{y_c}{2}\pm\sqrt{\frac{y_c^2}{4}+\frac{\alpha}{\beta}}$

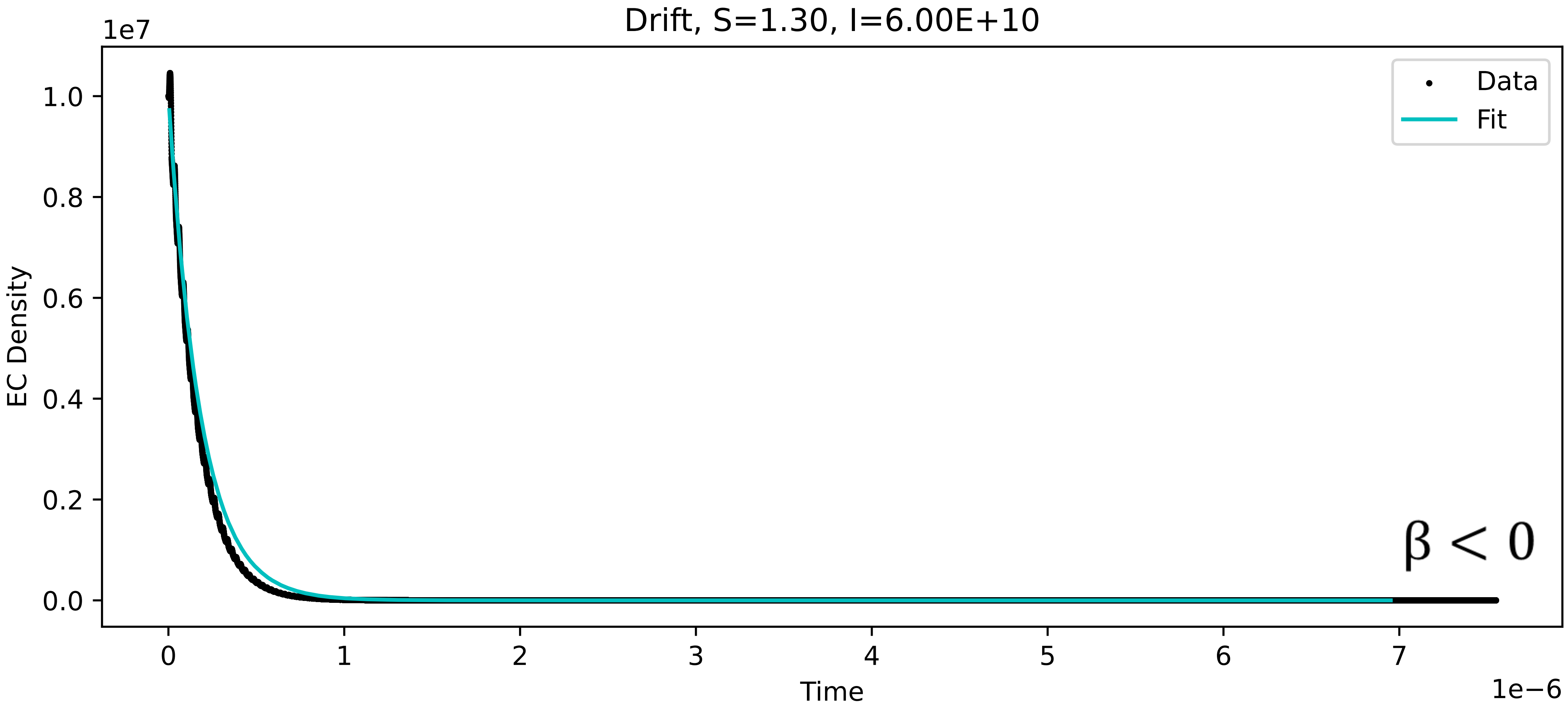
No dependance on initial density!

Modelling buildup curves



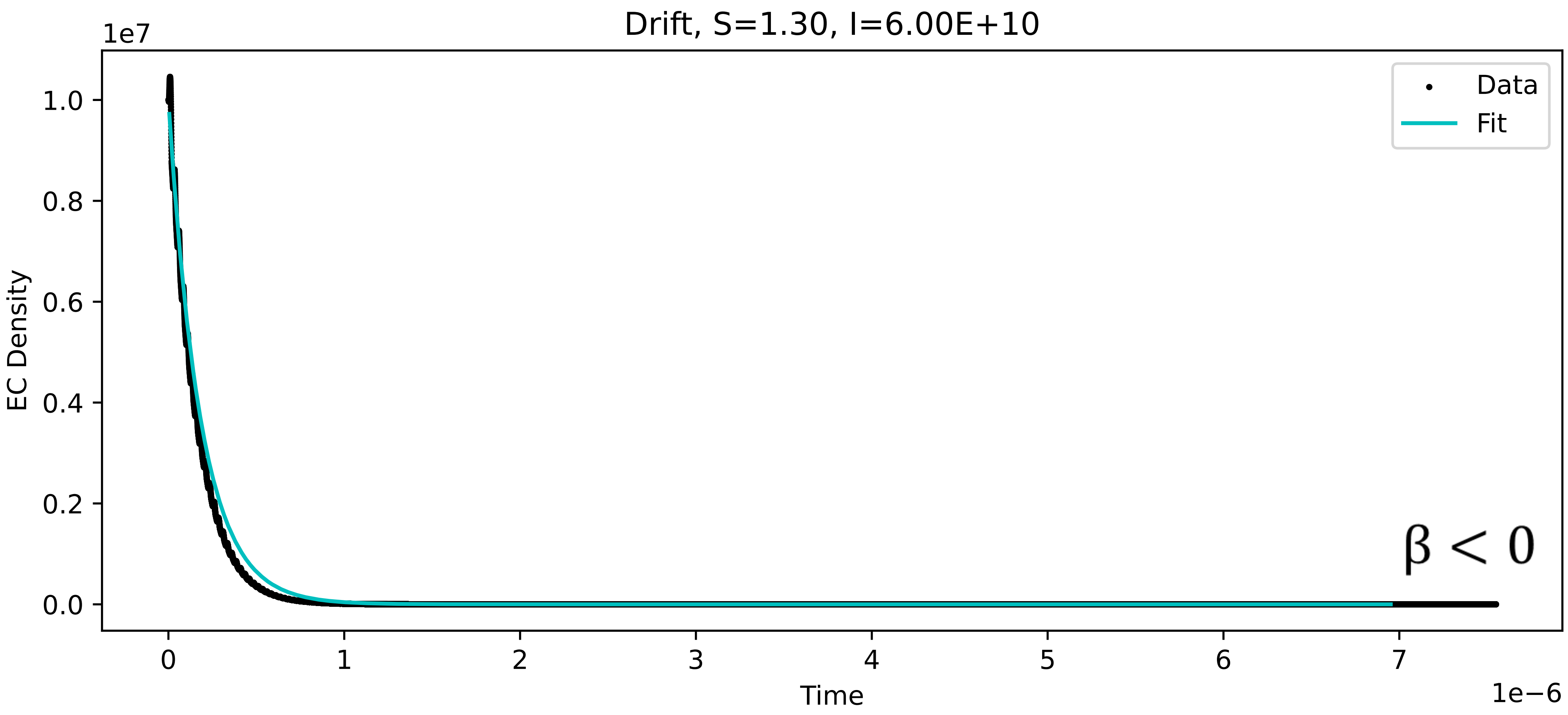
The model captures the average density well.

Modelling buildup curves



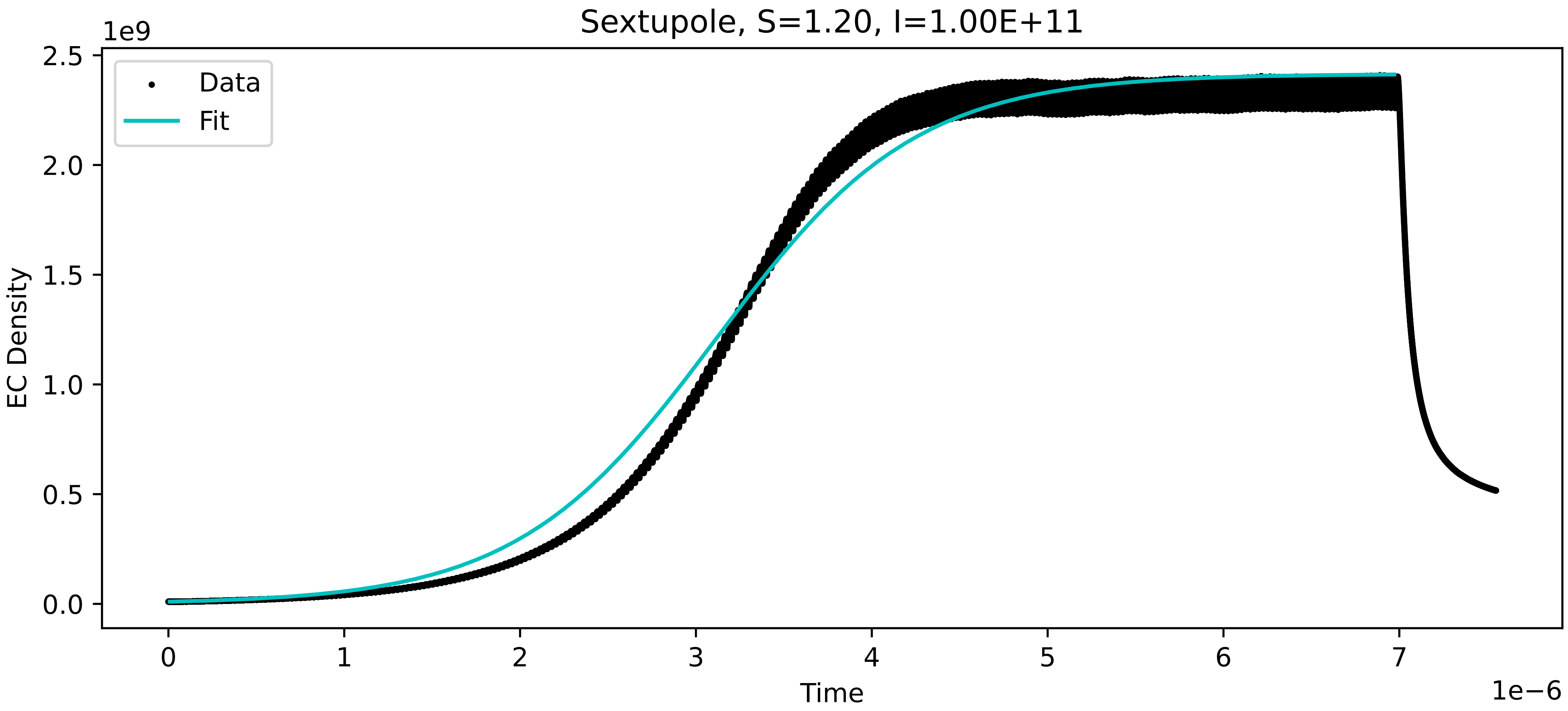
… and it works even when we don't build up.

Modelling buildup curves



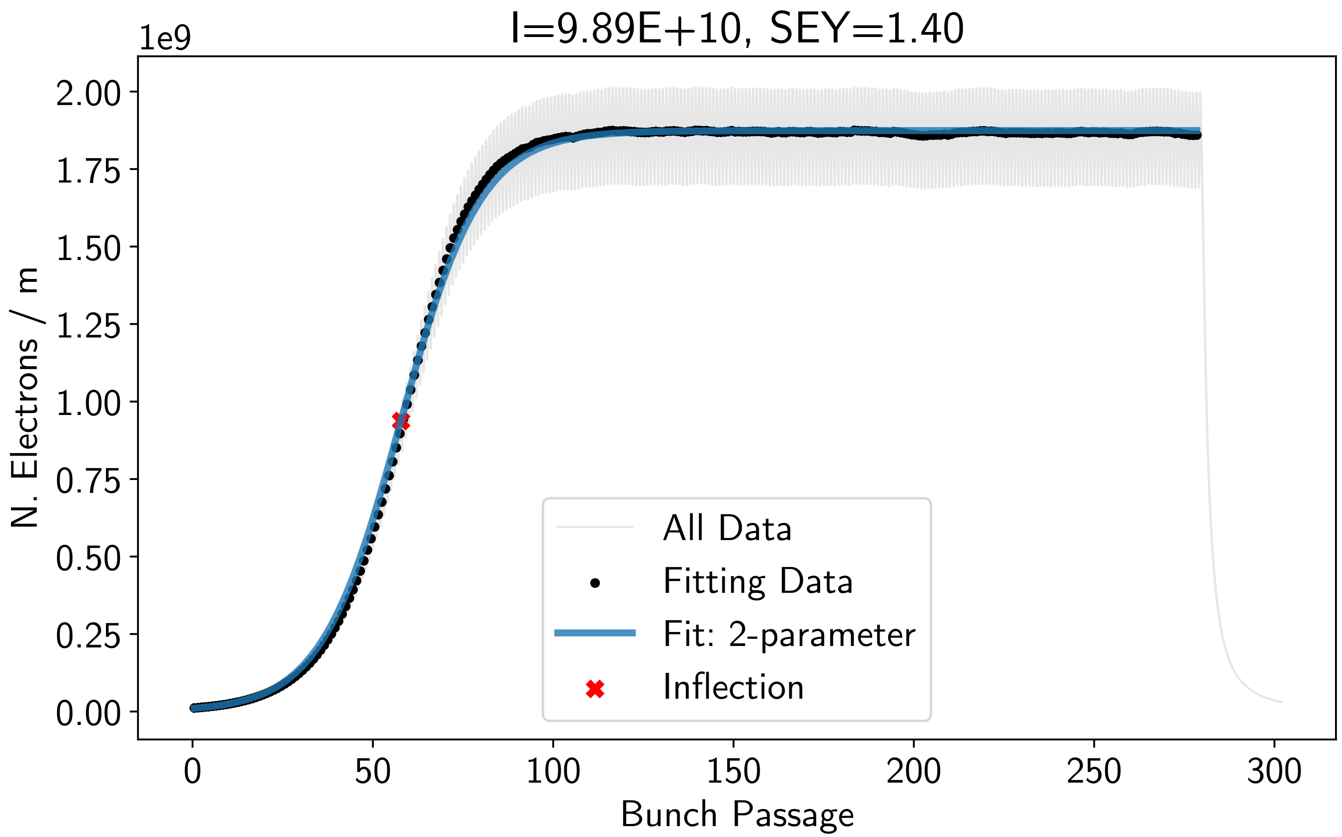
… and it works even when we don't build up.

Modelling buildup curves



For magnets, the curve fit does not work as well (expected).

Modelling buildup curves



Desired Features

- ✅ Saturation
- ✅ Independent of initial N.
- ❓ Increase with SEY.
- ❓ Critical (worst) intensity.
- 🚫 Response to magnets.

To understand dependance on SEY and intensity, we need to turn a parameter sweep into predictive models.

Predictive models

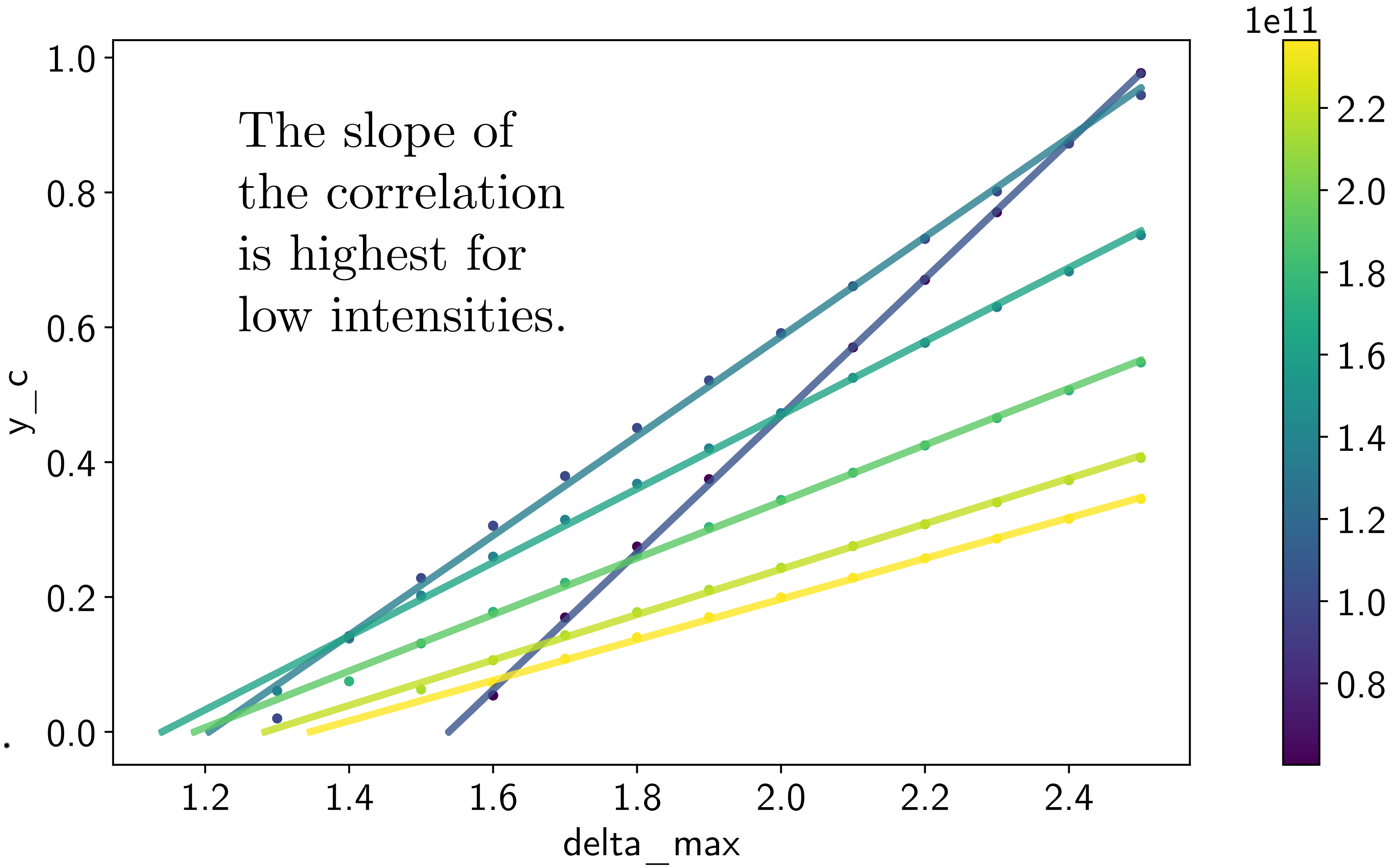
We will start with only the no-photoemission model.

$$y(x) = \frac{y_0 y_c e^{\beta y_c x}}{y_c + y_0(e^{\beta y_c x} - 1)}$$

Let's see how the model performs for the following setup:

- FCC-ee chamber model.
- 25ns bunch spacing.
- Drift section (no magnetic field).
- SEY range: 1.0 to 2.5.
- Intensity range: 10 to 110% of 2.15×10^{11}ppb.

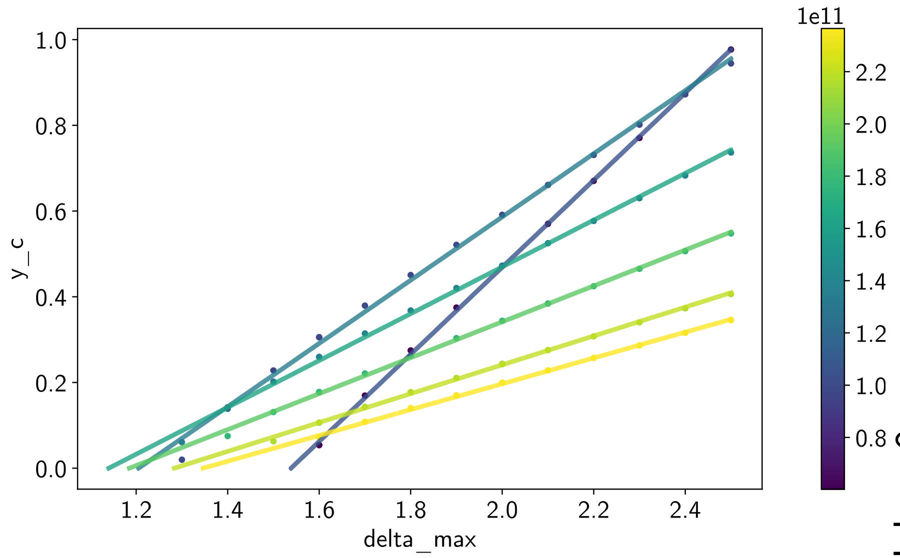
Predictive models: **Critical Intensity**

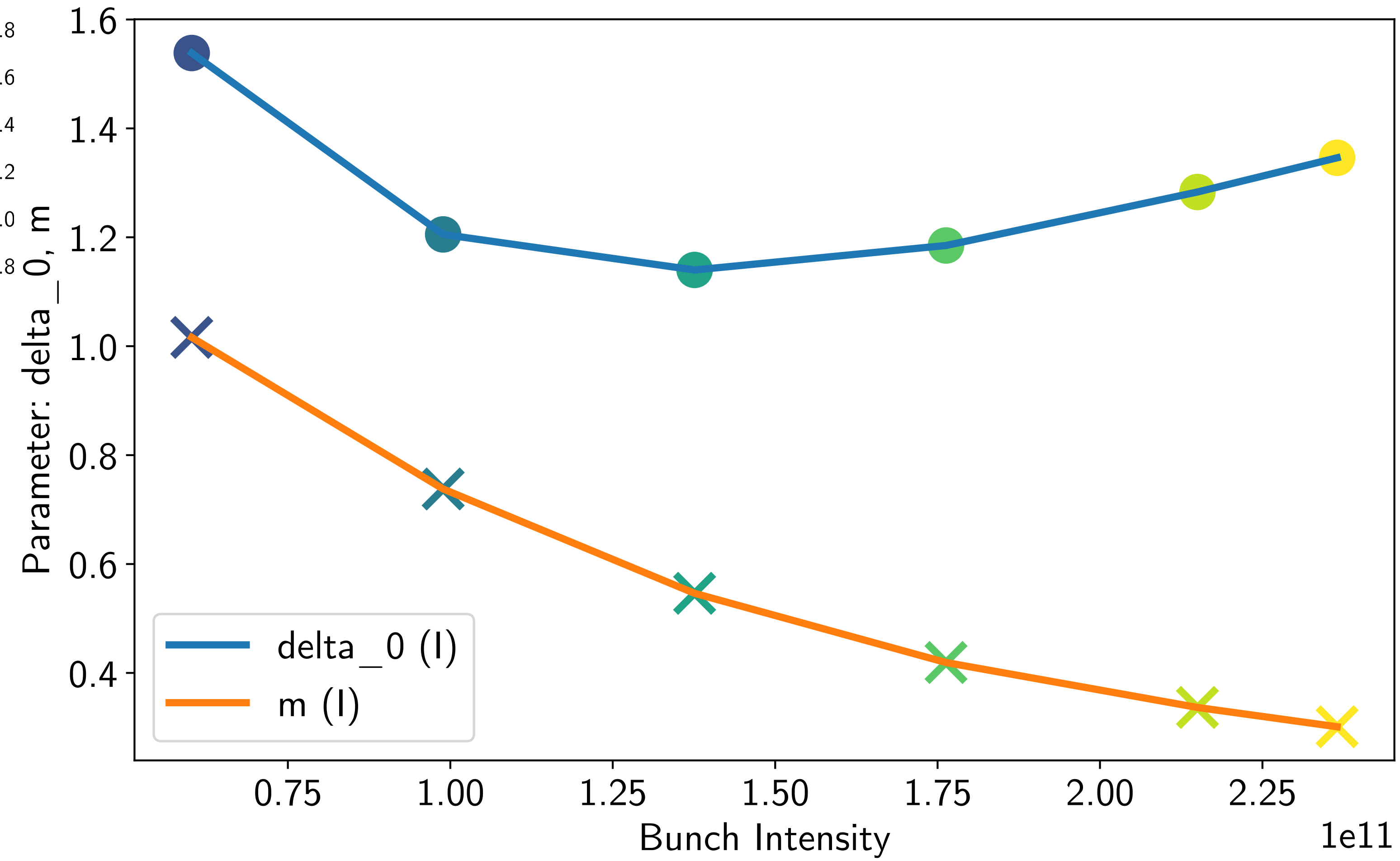


The SEY threshold is where $\mathbf{y_c} = 0$.

$\mathbf{y_c}$ grows **linearly** with SEY.

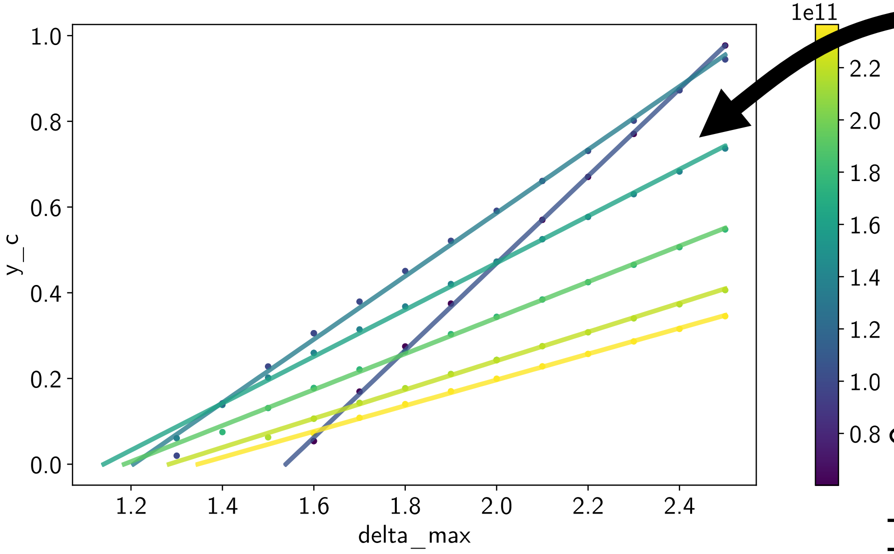
Predictive models: **Critical Intensity**





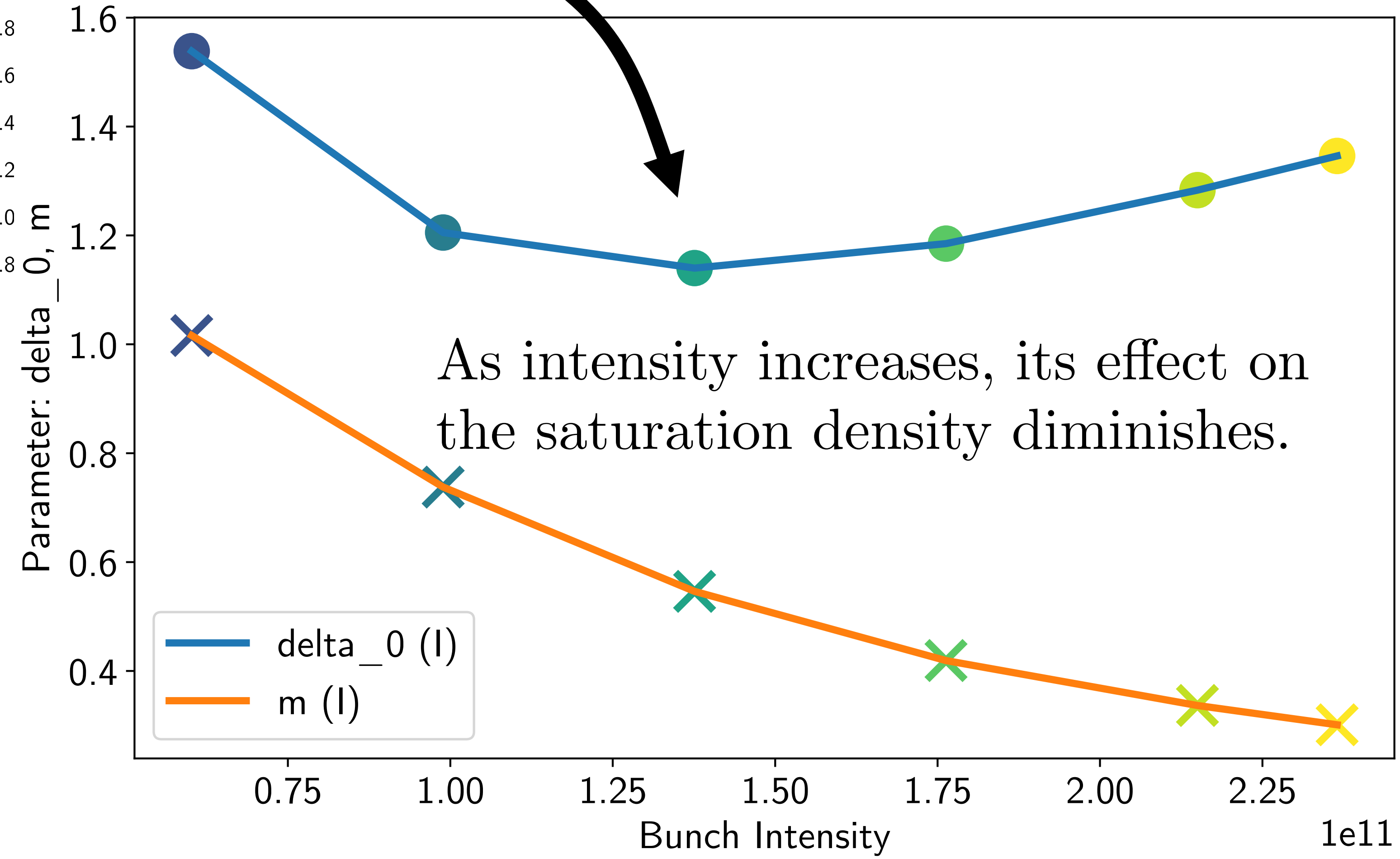
$$y_c(\delta_{max}) = m \cdot (\delta_{max} - \delta_0)$$

Predictive models: **Critical Intensity**

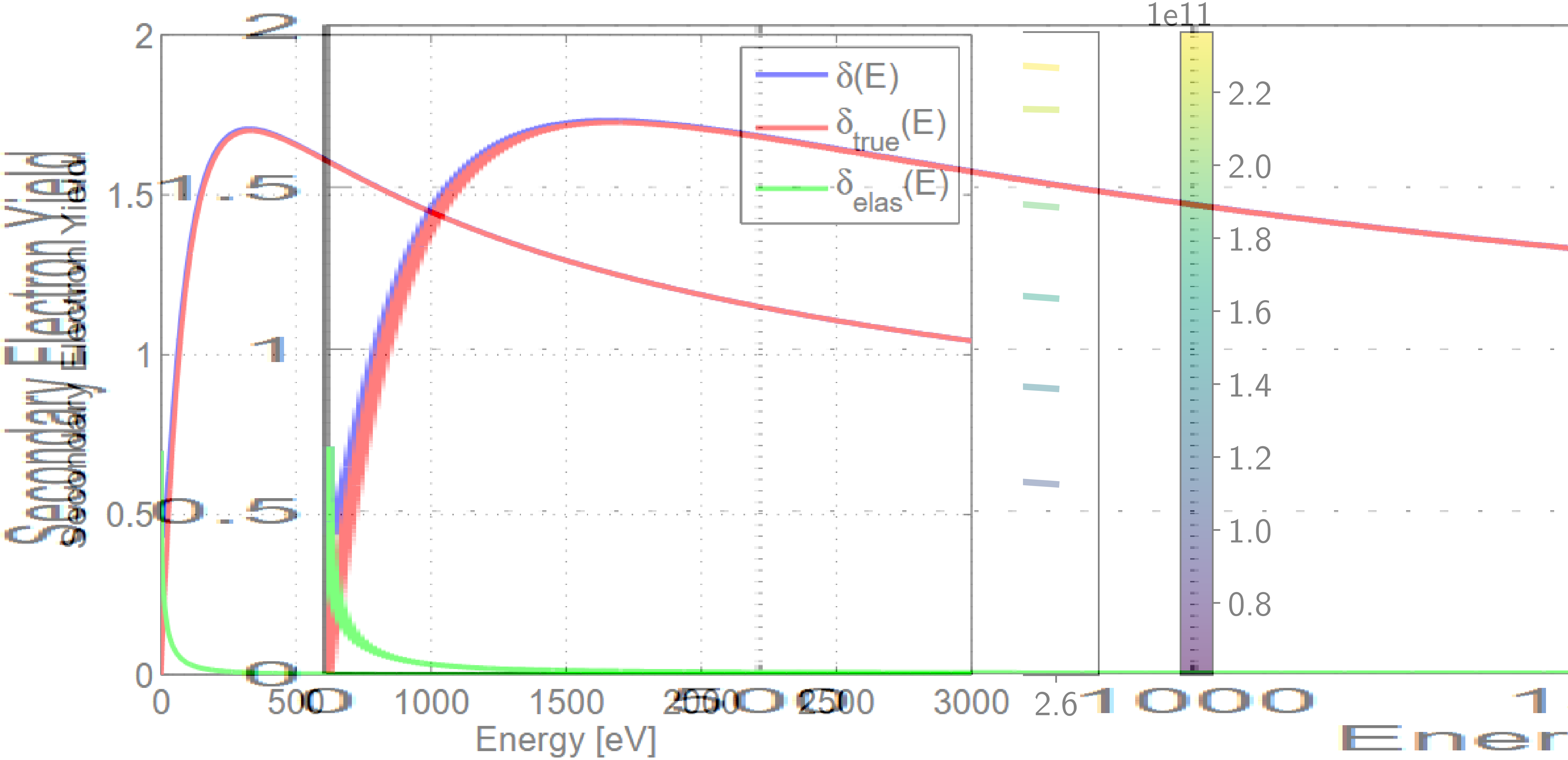


At some **critical intensity**, buildup occurs for the *lowest* SEY.

$$y_c(\delta_{max}) = m \cdot (\delta_{max} - \delta_0)$$



Predictive models: **Beta**



β follows a clear but complex trend.

Aside: the SEY function

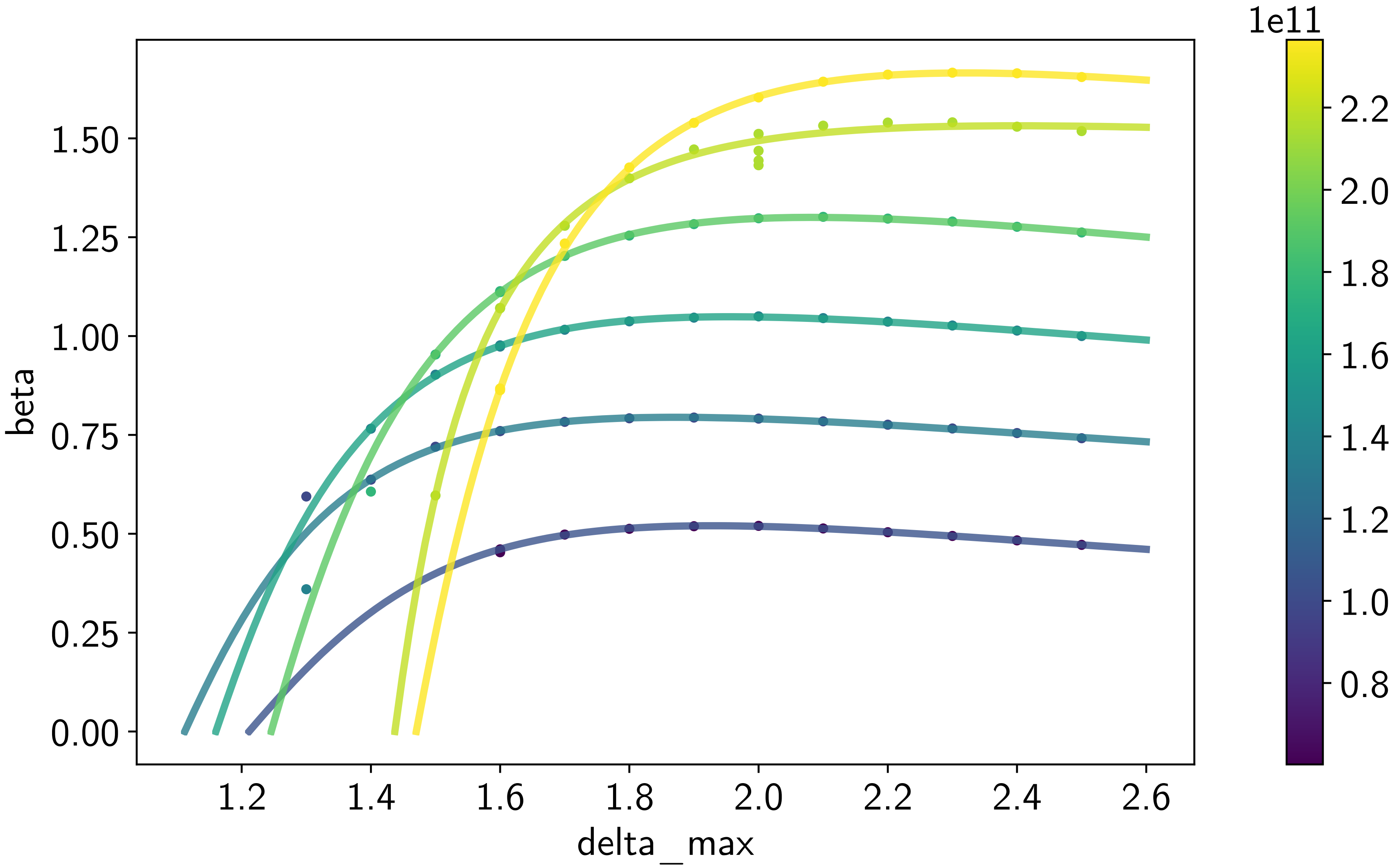
The number of true secondary electrons emitted depends on the energy of the incident electron.

$$\delta_{true}(E) = \frac{\frac{sE}{E_{max}}}{s - 1 + \left(\frac{E}{E_{max}}\right)^{s}}$$

which we can generalize to our model for β:

$$\beta(\delta_{max}) = \frac{ABs(\delta_{max}-\delta_0)}{s - 1 + A(\delta_{max} - \delta_0)^{s}}$$

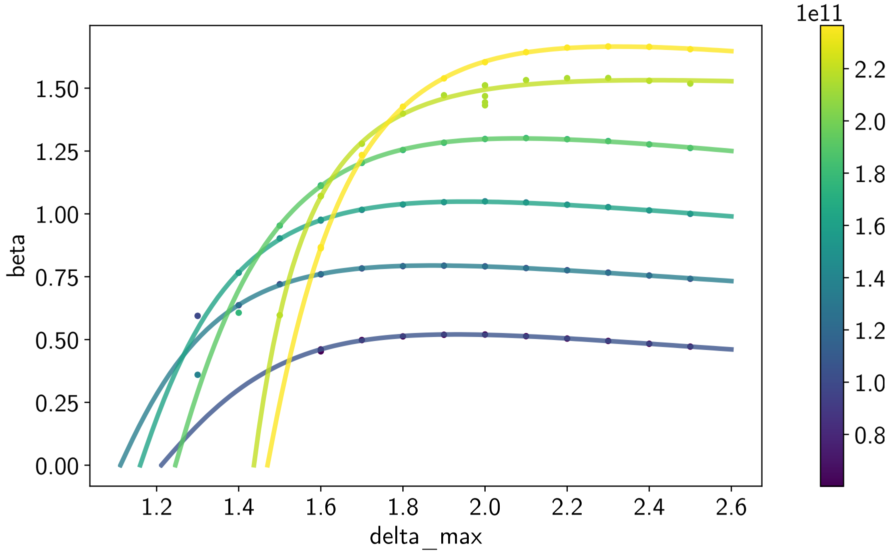
Predictive models: **Beta**



The highest intensity has the highest β.

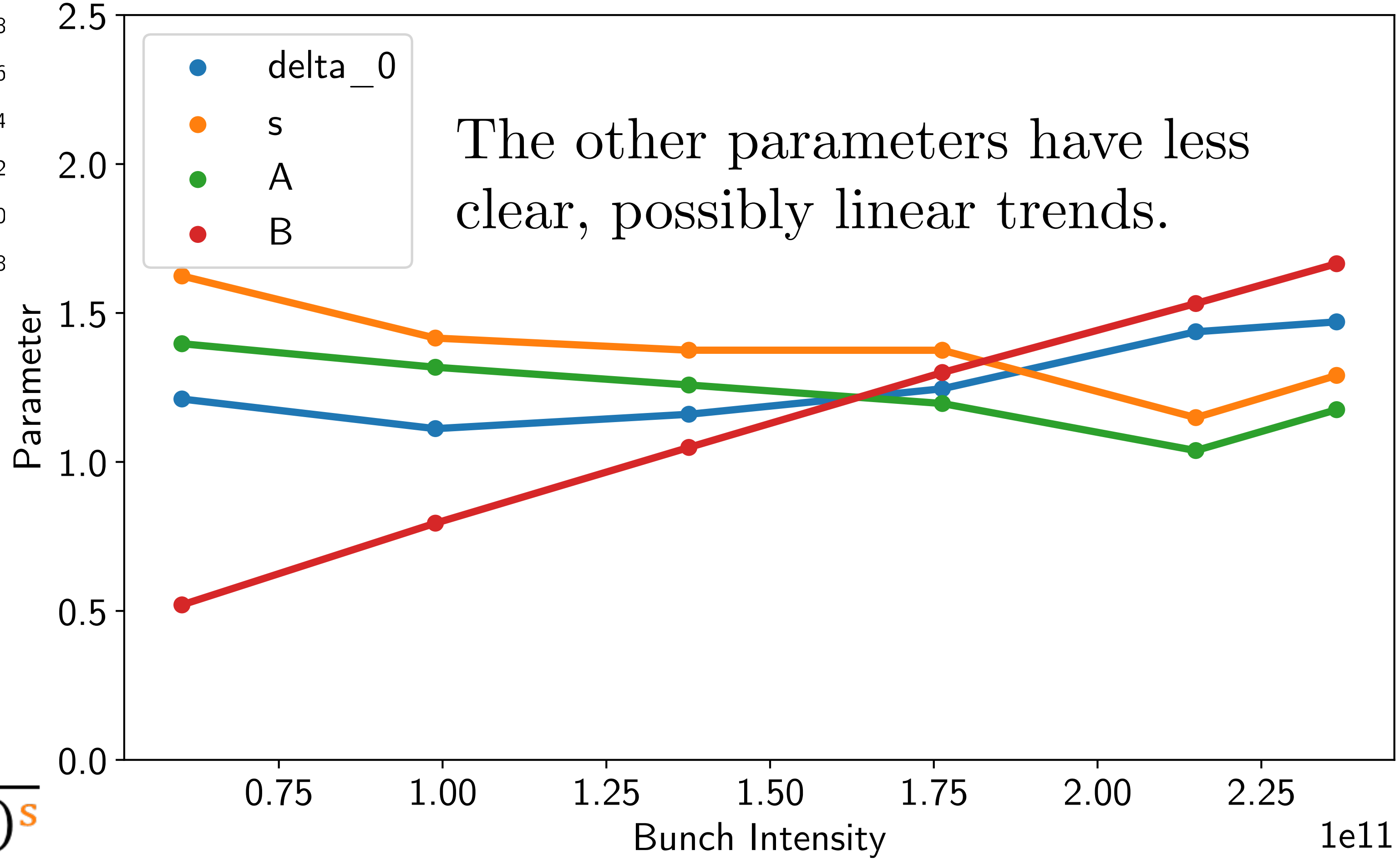
For low intensities, there are not many points in the "steep" region.

Predictive models: **Beta**



The scale factor **B** of the function increases with intensity.

$$\beta(\delta_{max}) = \frac{ABs(\delta_{max} - \delta_0)}{s - 1 + A(\delta_{max} - \delta_0)^s}$$



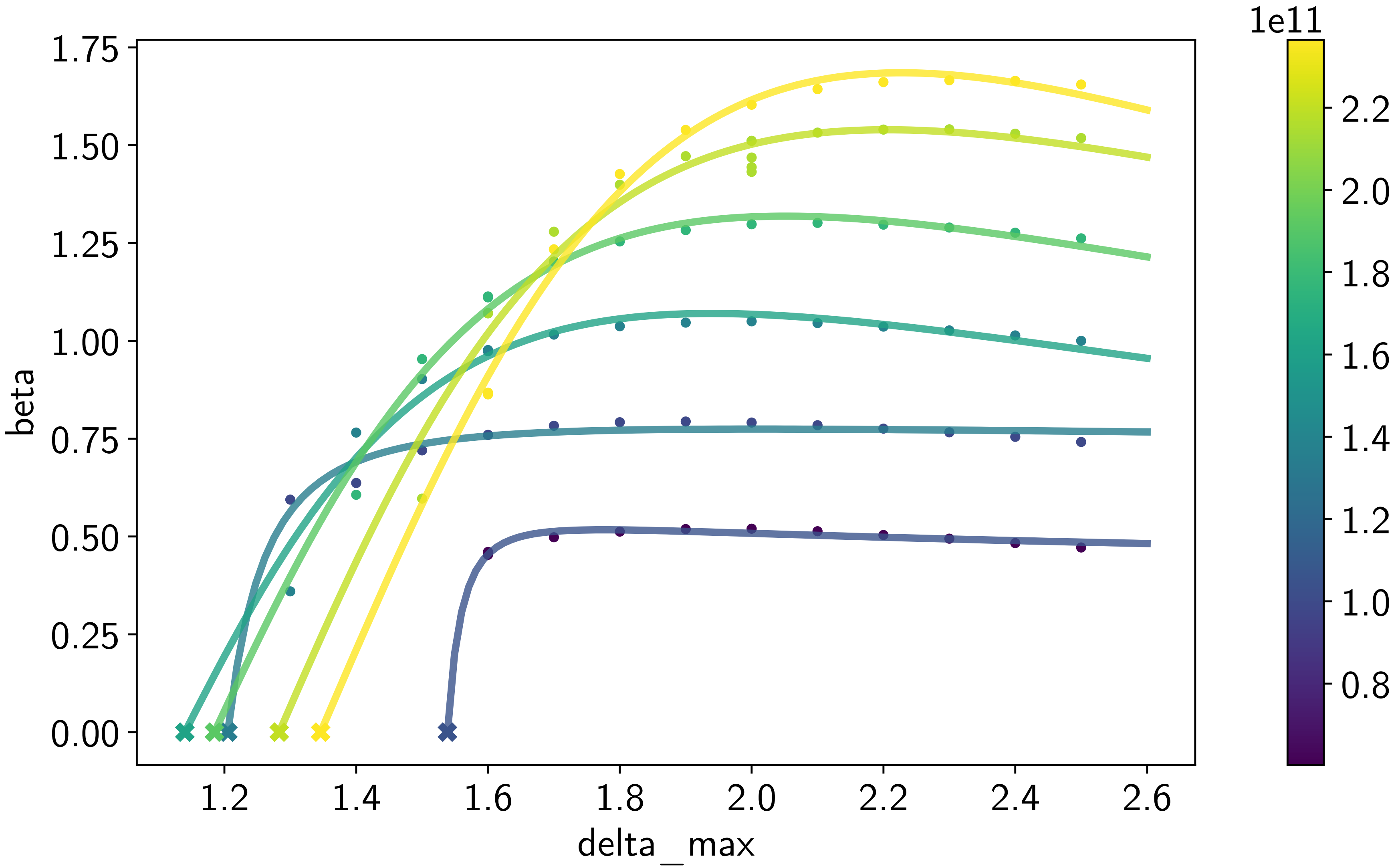
The other parameters have less clear, possibly linear trends.

Summary of predictive models

- y_c as a function of **SEY**:
  - Linear model, two parameters:
    - δ_0 (critical SEY for buildup)
    - m (increase in y_c per unit increase in SEY)

- β as a function of **SEY**:
  - Nonlinear model, four parameters:
    - A (scale factor)
    - B (scale factor)
    - s (exponent)
    - δ_0 (critical SEY for buildup)

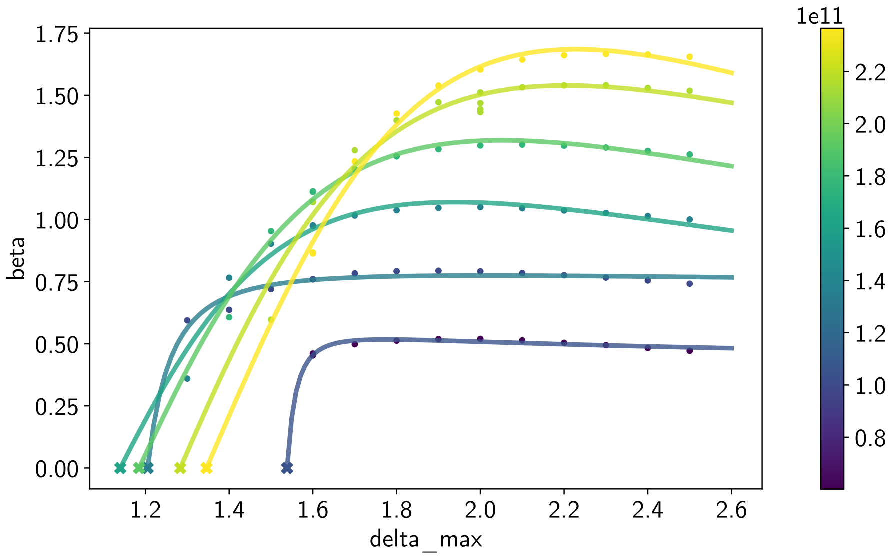
Predictive models: **Beta**



We can include the data point for δ_0 in the fitting data.

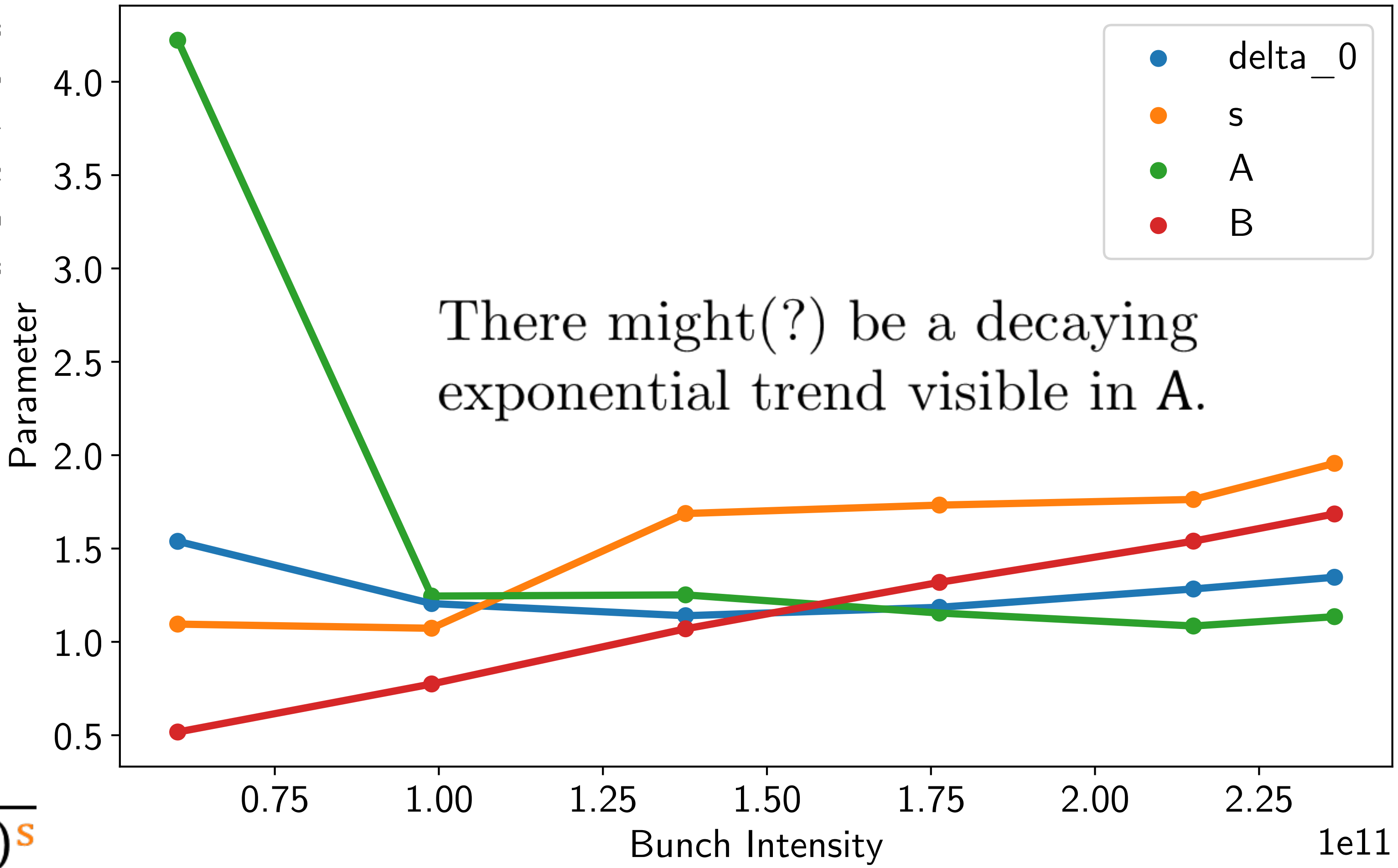
Visually, the quality of some fits appears to degrade.

Predictive models: **Beta**



We largely still see the same behavior of the parameters. However, now we only really have three *new* parameters.

$$\beta(\delta_{max}) = \frac{ABs(\delta_{max} - \delta_0)}{s - 1 + A(\delta_{max} - \delta_0)^s}$$



Counting parameters

To predict both β and y_c as a function of **SEY**, we need to know the values of 5 variables for each intensity.

- 2 for $y_c(\delta_{max})$: δ_0 and m.
- 3 for $\beta(\delta_{max})$: A, B, and s.

If a quadratic can approximate the trend for each of these 5 parameters as a function of **intensity**, we count 25 parameters in total for $\beta(\delta_{max}, I)$ and $y_c(\delta_{max}, I) \rightarrow \mathbf{min\ 25}$ simulations.

- A quadratic model can capture *critical-intensity* behavior.
- This is almost certainly an *overestimate* given that some trends appear clearly linear.

Including photoemission

$$y(x) = \frac{y_c}{2} + \sqrt{\frac{y_c^2}{4} + \frac{\alpha}{\beta}} \tanh\left(\frac{x}{2}\sqrt{\beta(4\alpha + y_c^2\beta)} - \phi\right)$$

$$\phi = \tanh^{-1}\left((y_c - 2y_0)\sqrt{\frac{\beta}{4\alpha + y_c^2\beta}}\right)$$

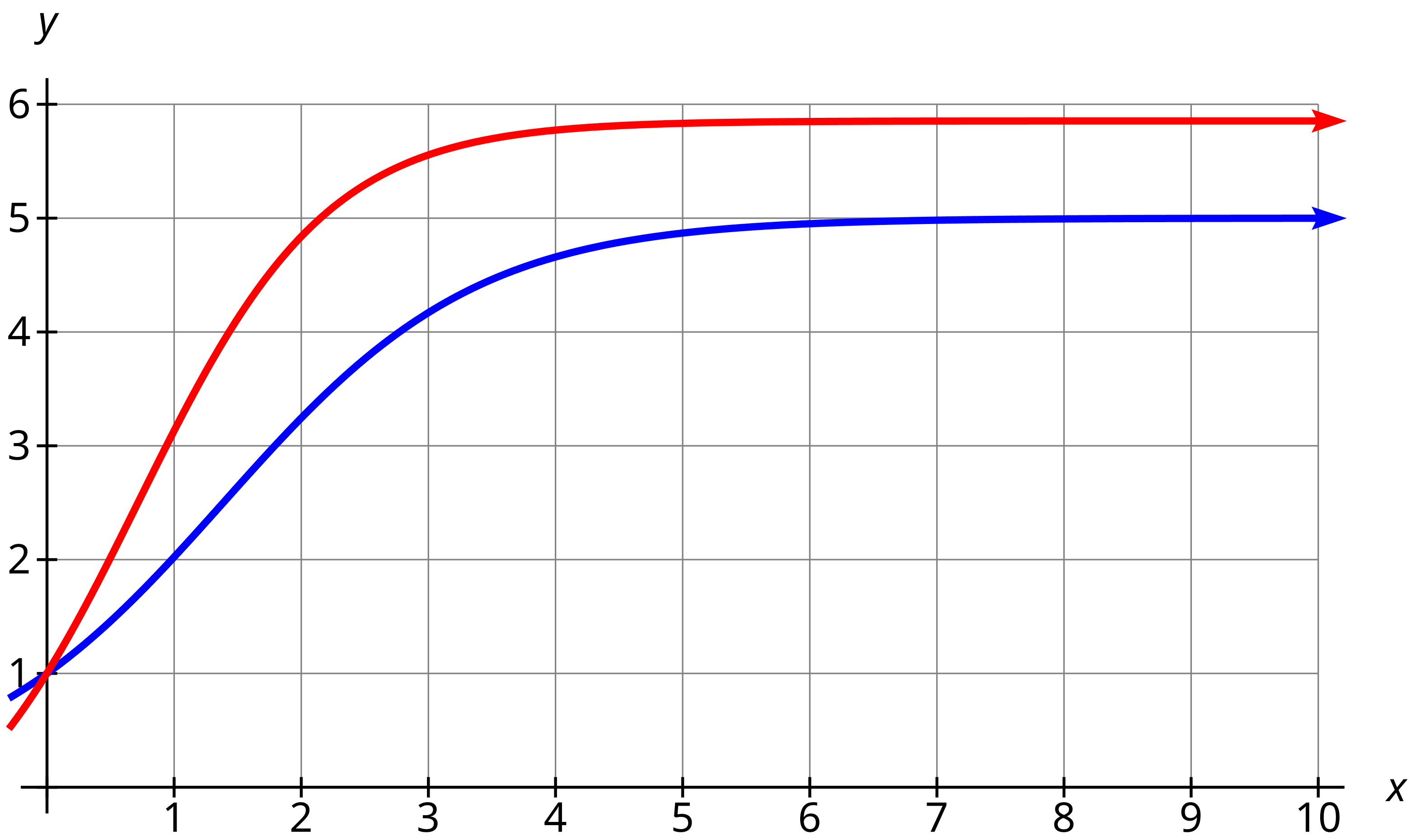
Accounting for photoemission means adding another parameter, α, to the model.

The assumption is that photoemission stays constant.

Including photoemission

Here, $y_0 = 1$, $y_c = 5$, $\beta = 0.2$ for both curves.

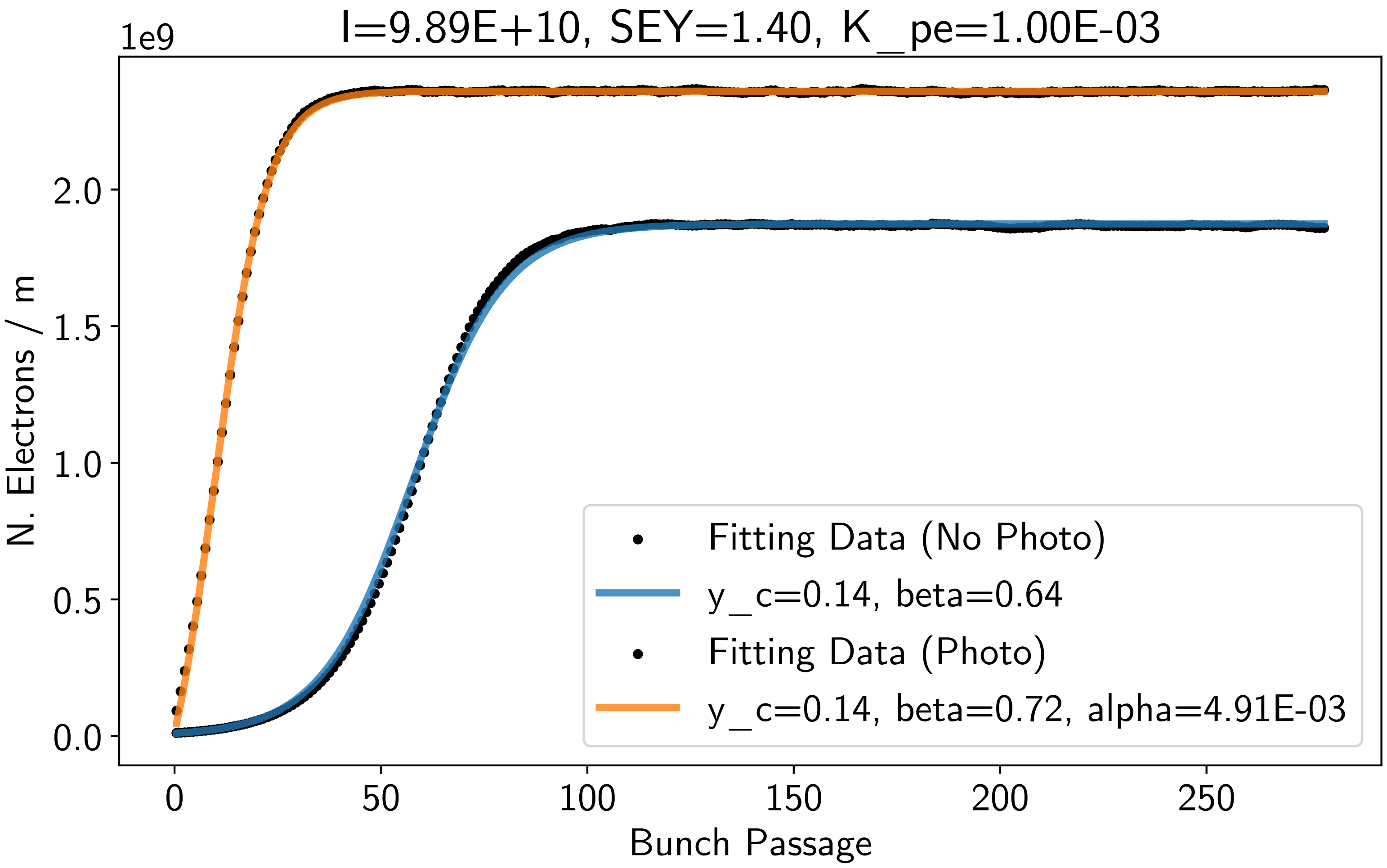
The red curve uses $\alpha = 1$.



For an otherwise identical setup, we expect the only difference between a photoemission and non-photoemission simulation to be α.

Including photoemission

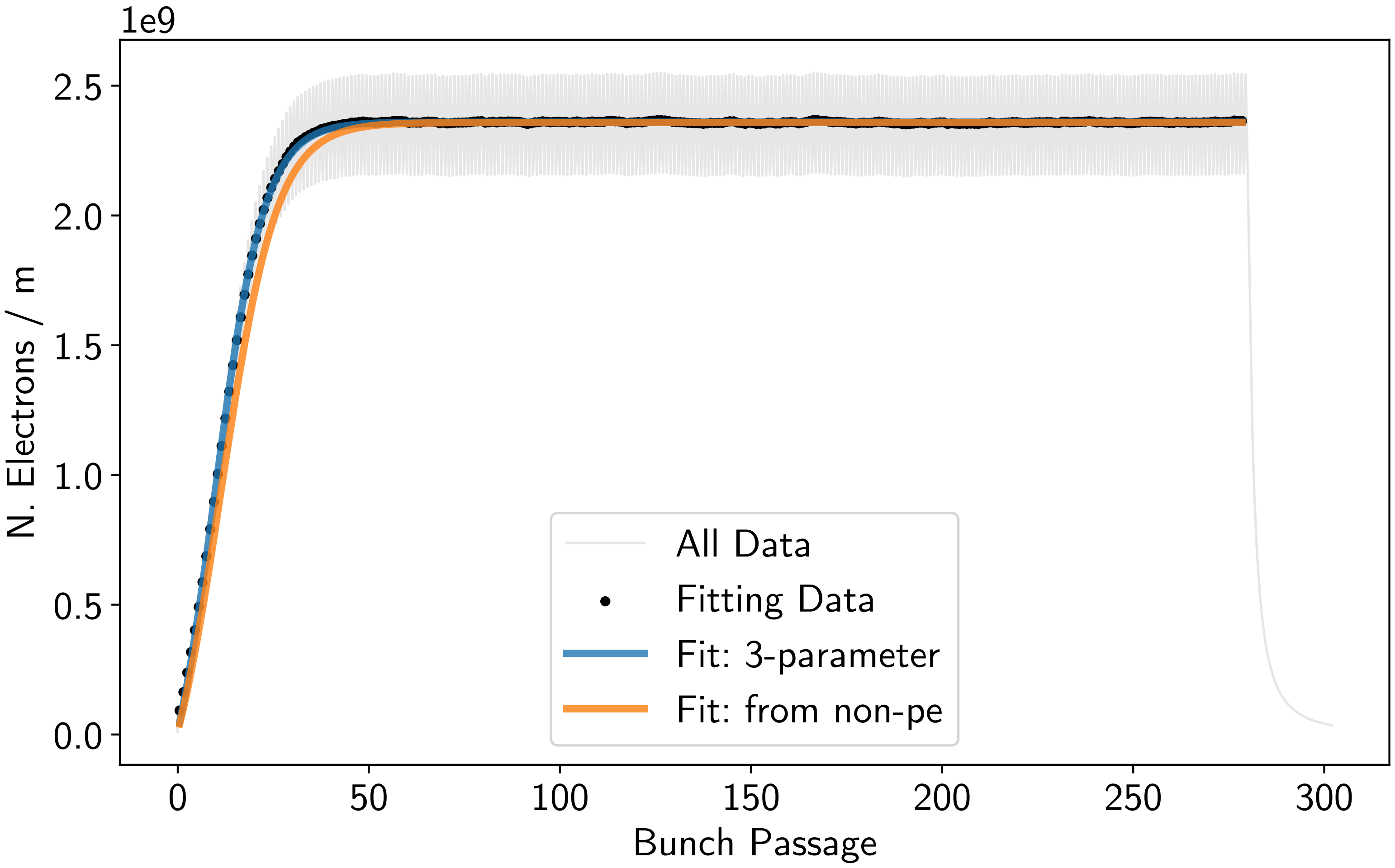
We introduce the simulation parameter $k_{pe,st}$, which defines the number of photoelectrons emitted per beam particle.



Here, we **don't** see the same fit values for β.

Including photoemission

We can force the y_c and β values to be equal, and only find α.



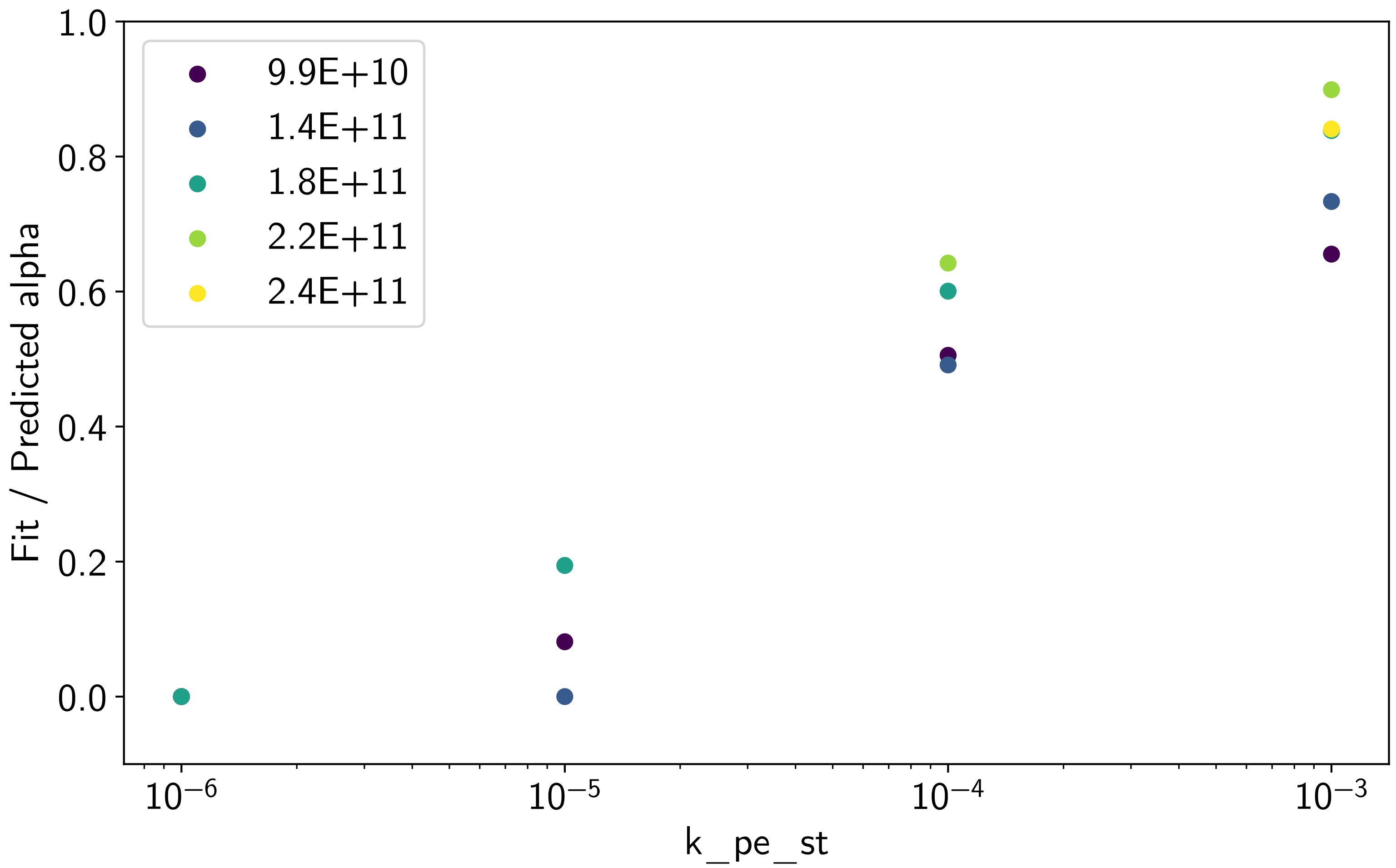
It turns out both functions lie within the data.

However, there is still lots of nuance in the significance of α.

Predicting α

α is defined very similarly to the PyECLOUD parameter $k_{pe,st}$.

It should be the case that $\alpha = ct_b \cdot k_{pe,st}$.

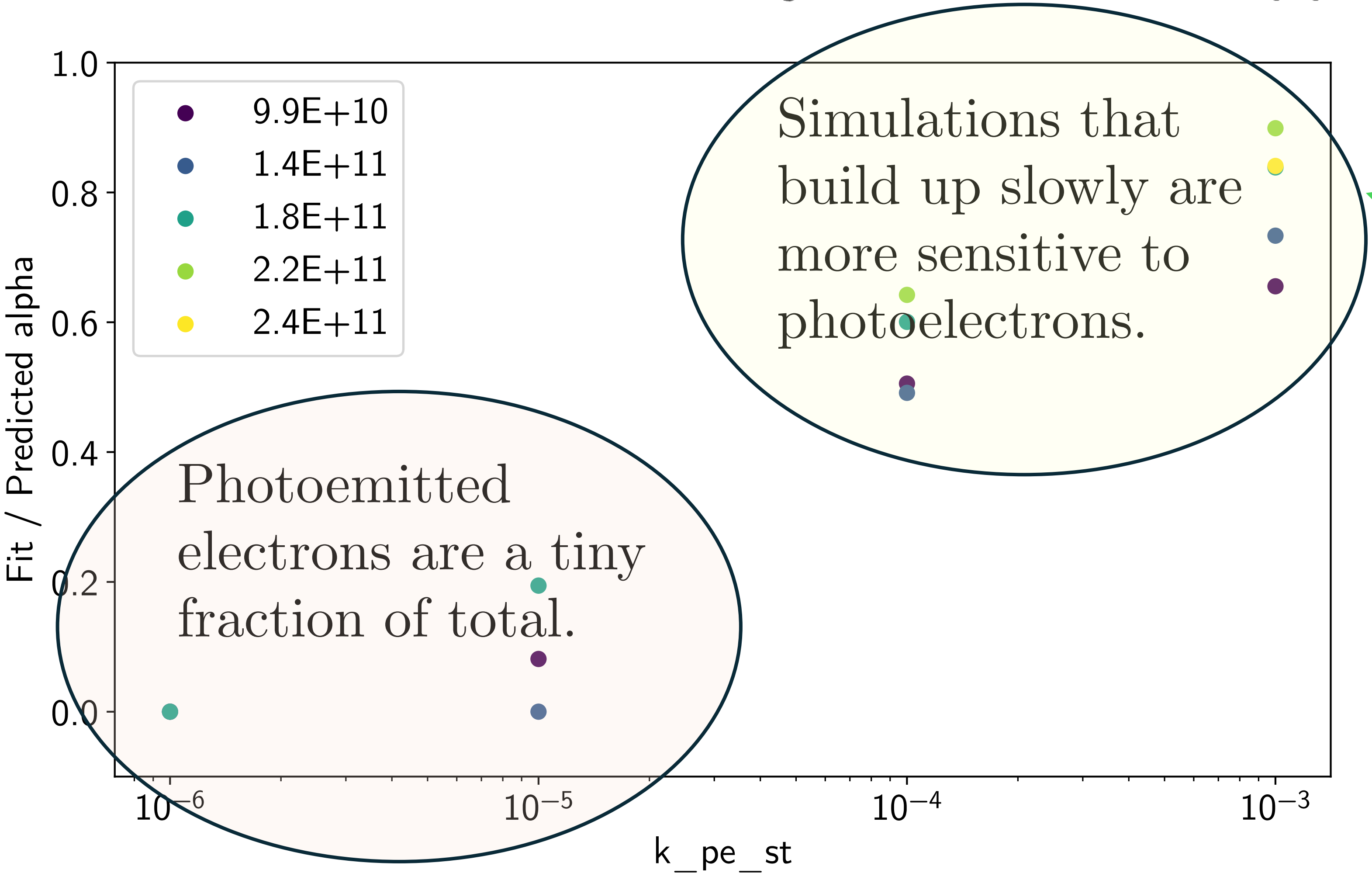


Prediction correct.

Like α = 0.

Predicting α

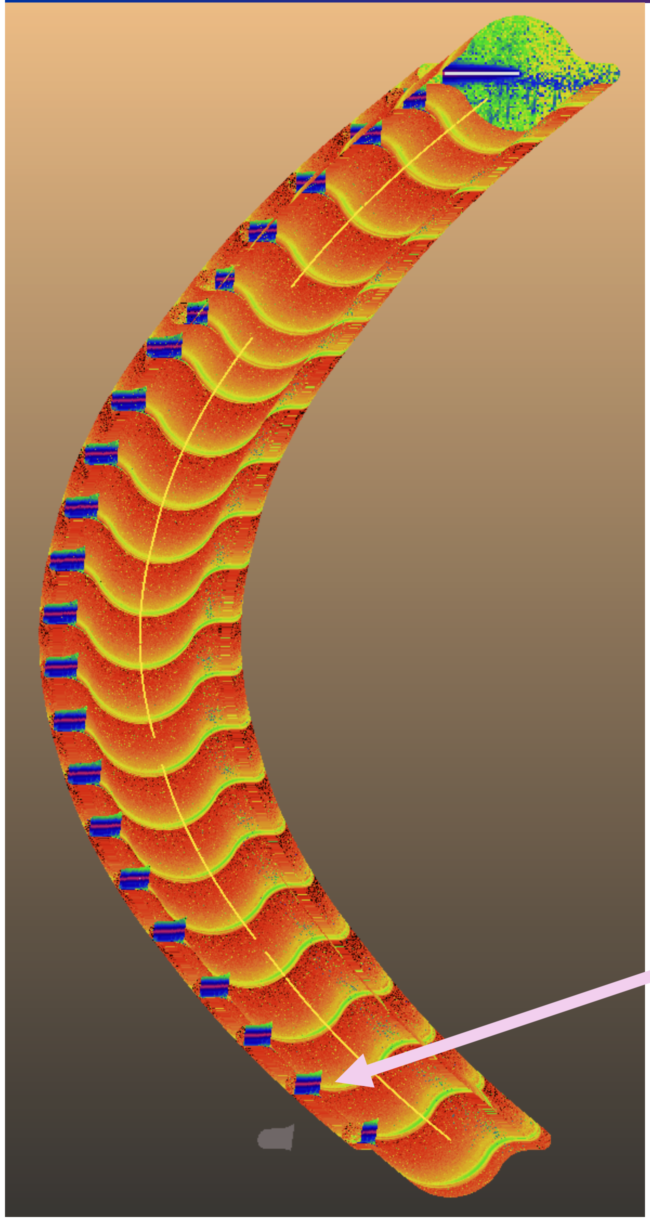
What we "measure" (fit to) is an *average* over a time t_b in which a lot of electron cloud dynamics can happen.



May be another instance of "critical intensity" behavior.

Increases the complexity of predicting simulations with photoemission.

Photoemission distributions



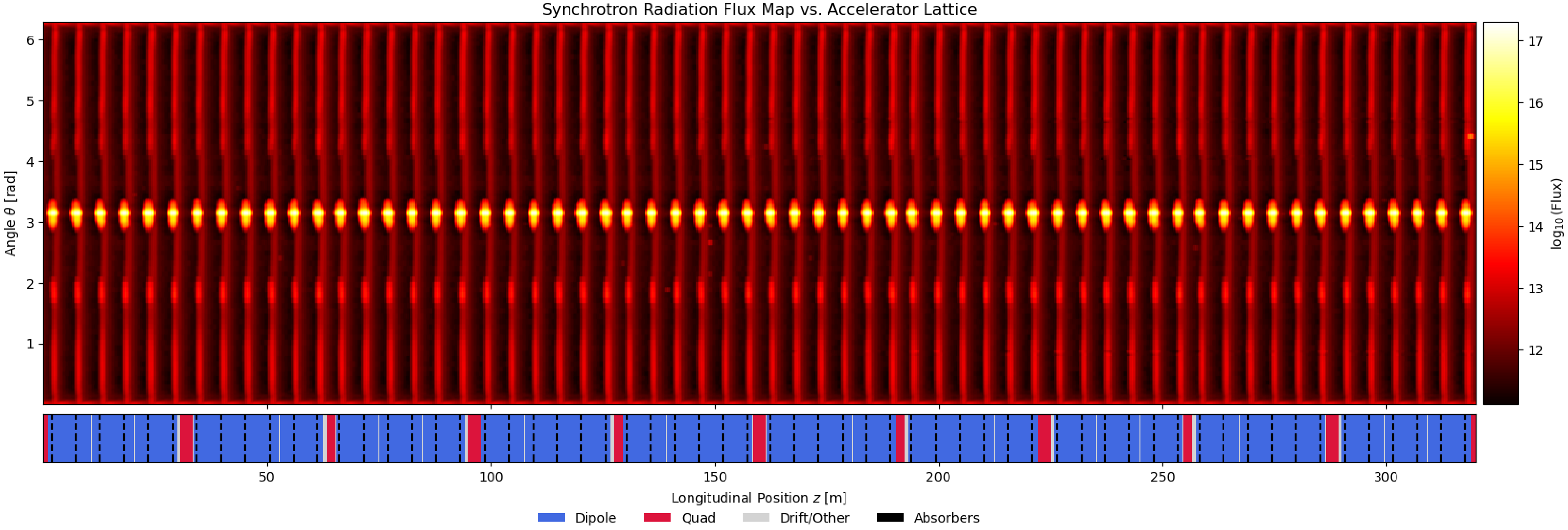
Photoelectrons are not always distributed uniformly – they vary based on synchrotron radiation flux.

In addition to the total amount ($\mathbf{k_{pe,st}}$) of photoelectrons, their **distribution** may matter.

Absorber structures in the FCC-ee lattice.

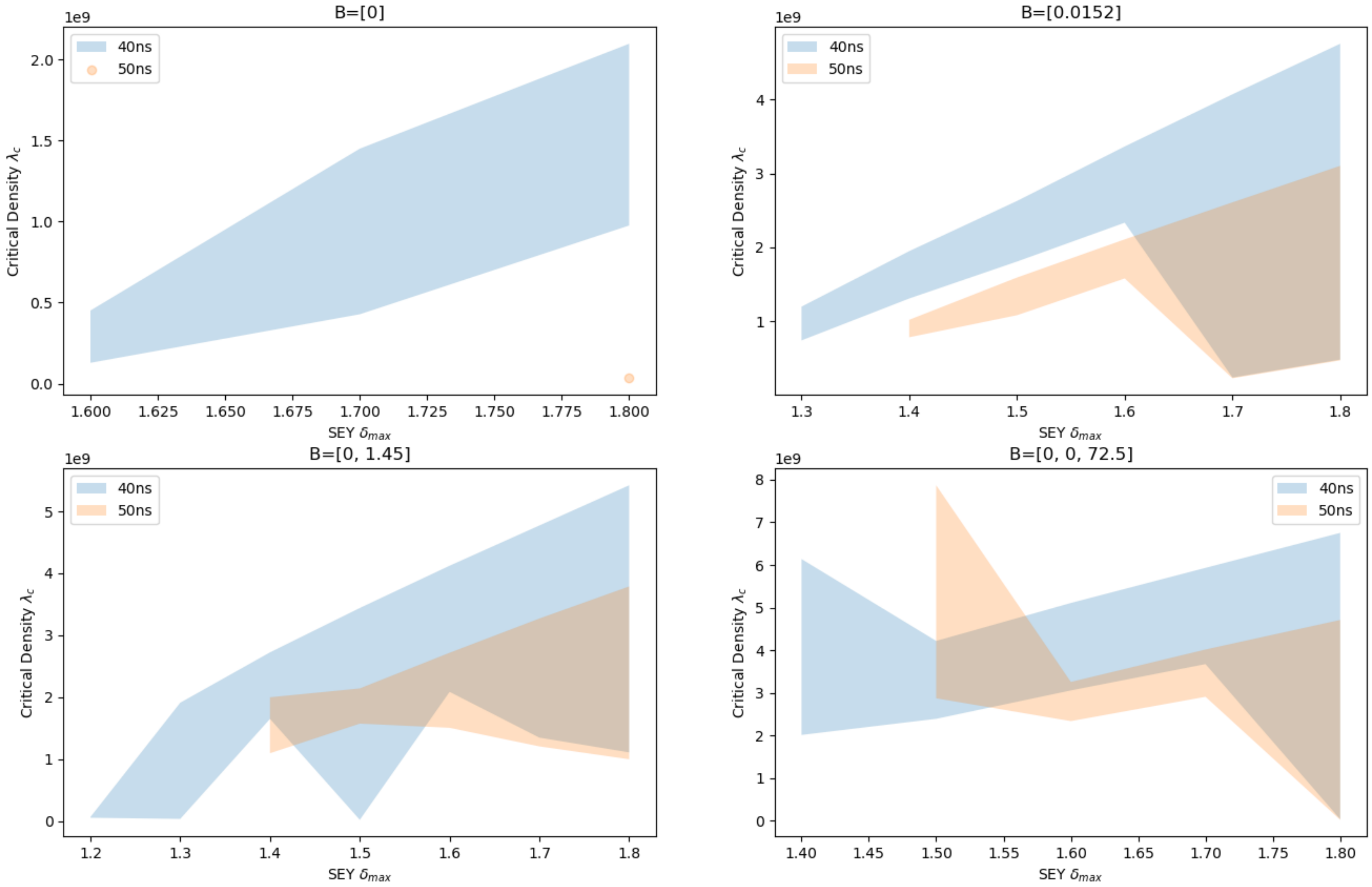
Photoemission distributions

LCC lattice synchrotron radiation flux distributions.

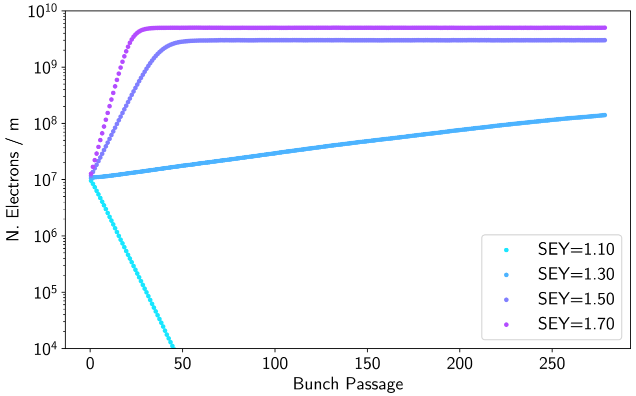


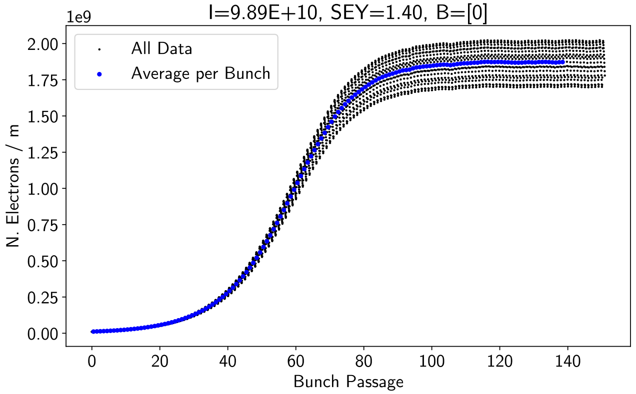
Evaluating Filling Schemes

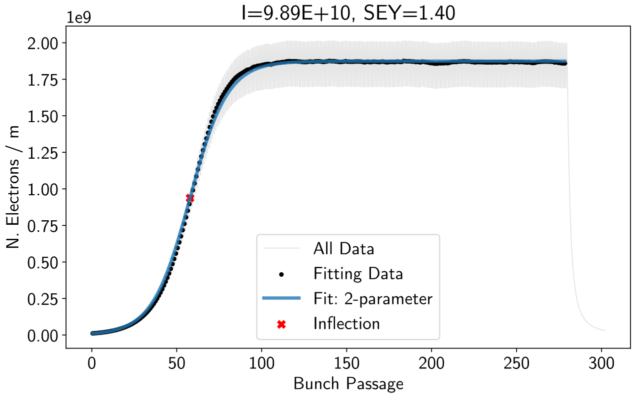
The model easily compares different design options.



In summary





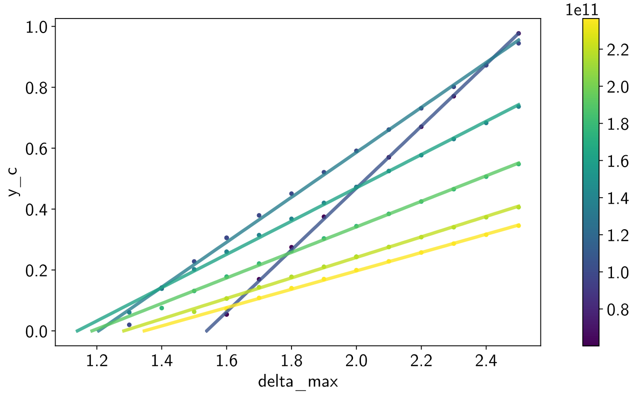


We understand how different parameters can change electron cloud behavior.

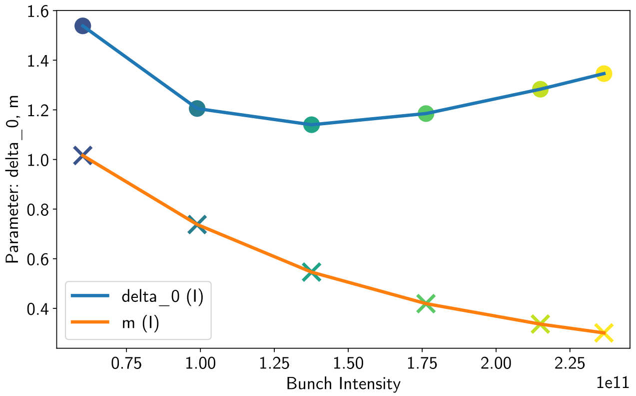
We adopt and expand an existing model for average electron cloud density over time.

We fit this model successfully to simulations.

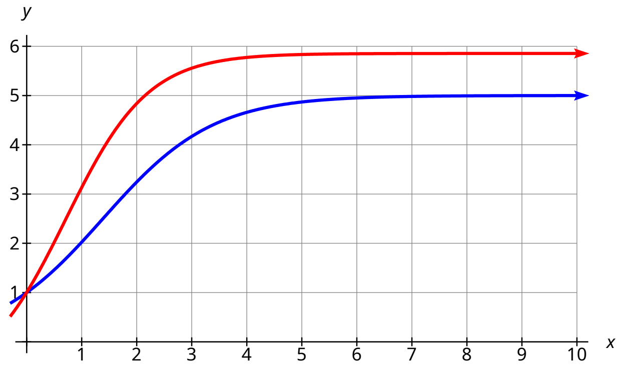
In summary



We can predict our model parameters from simulation inputs...

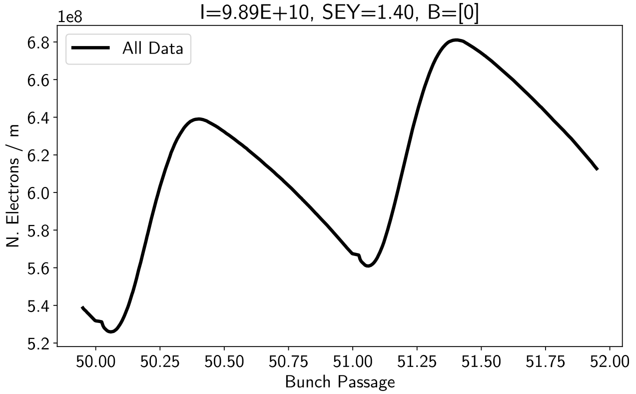


... as a function of both SEY and bunch intensity, finding clear trends.

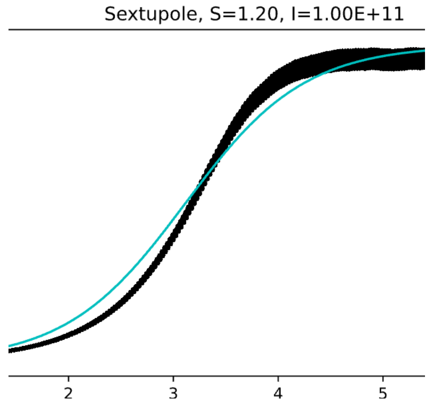


And we understand how photoemission can be added and when it has a strong effect.

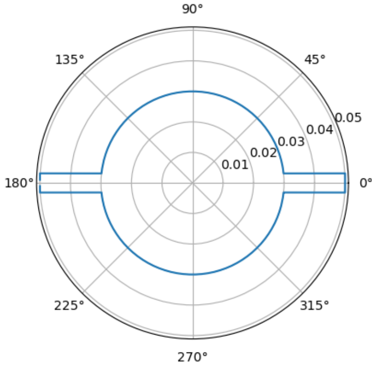
But there are some limitations.



We don't have any information about dynamics at a scale smaller than $\mathbf{t_b}$.



We know the model cannot fully account for magnetic fields *(but a fix is coming soon!)*.



We don't capture any information about the spatial distribution of electrons.

Outlook

This modelling technique provides a simple way to describe electron cloud buildup: (y_c, β, α).

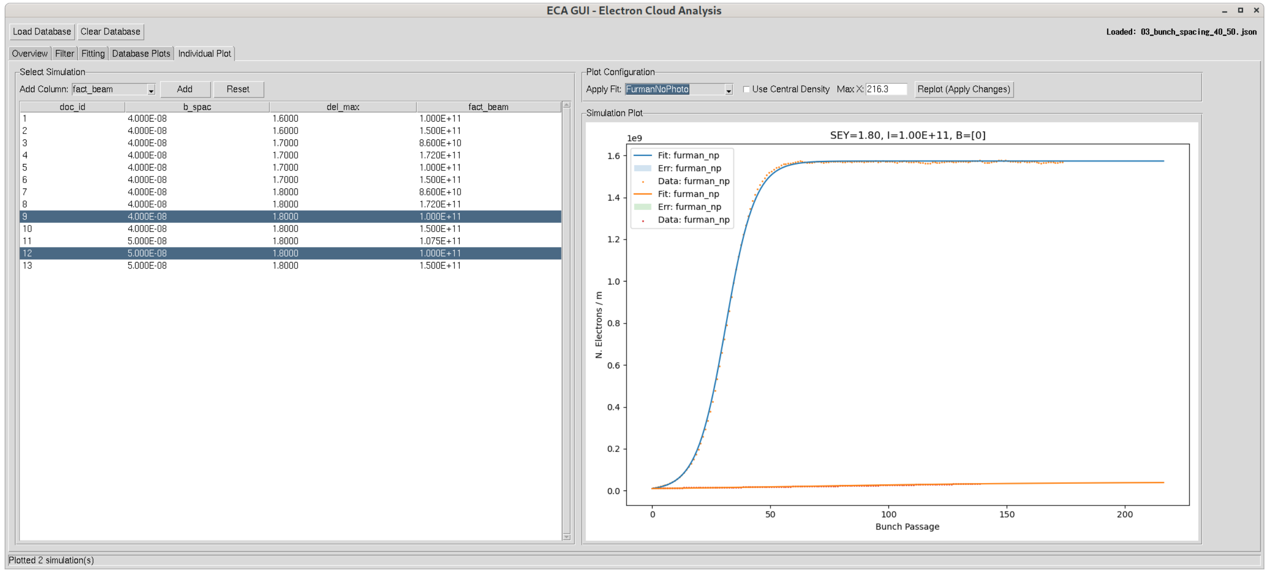
It could allow for a lower simulation load in future studies.

Further mathematical analysis may be able to derive or verify **more robust and detailed** models that shed light on gaps.

Having a simple mathematical formula for electron cloud effects might make it easier to include e-cloud in more complex models.

Tools developed

Developed the **E**lectron **C**loud **A**nalysis package.



Generating, running, curve fitting, and working with PyECLOUD simulations made much easier through a common API.

Transforms hundreds of individual output files into a single database which can easily be queried.

https://github.com/furmada/Electron-Cloud-Analysis

Thank you.