QSNET Quiz Questions

Quantum Technologies for Fundamental Physics
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1 Quick-fire Questions

a) What is the best published inaccuracy / uncertainty in the realization of the SI second using the best caesium fountain microwave atomic clocks, e.g. 4×10^{-9} ? [1]

ANSWER: $(\Delta\nu/\nu) = 2 \times 10^{-16}$

b) What is the best published inaccuracy / uncertainty for the systematic frequency offsets in an optical atomic clock e.g. 4×10^{-9} ? [1]

ANSWER: $(\Delta\nu/\nu) = 9 \times 10^{-19}$, [Brewer, Chen, et. al (2019), Phys. Rev. Lett. 123, 033201]

c) The frequency stability of some ultra-stable laser cavities at short timescales is actually **better** than that of state-of-the-art atomic clocks!! Why then, in principle, is an atom or ion a better basis for a frequency standard than an ultra-stable cavity? [2]

ANSWER: The unperturbed frequency of an atomic transition is identical for every atom of the same species. This makes them better ***standards*** than cavities, since the resonant frequency of a cavity depends on its length, which could be anything, and will likely drift over time.

d) Name one advantage of using lab-based atomic clocks for fundamental physics research, compared with spaceborne experiments or high-energy particle accelerators. [1]

ANSWER: Atomic clocks are cheap compared with many HEP experiments, and more easily realised. They are also easier to fix and upgrade compared with experiments in orbit.

e) Name 3 of the atomic / ionic / molecular species that will be used the in QSNET network of atomic clocks. (One bonus point for specifying the correct isotope!) [3 (+3)]

ANSWER: See table of QSNET clocks below.

f) What range of particle rest masses (in eV) is representative of ultralight dark matter? [2]

ANSWER: Approximately 1 eV down to 10^{-22} eV

g) What range of particle rest masses (in eV) is detectable with atomic clock experiments? [2]

ANSWER: Approximately 10^{-14} eV down to 10^{-22} eV

h) What types of dark matter fields (bosonic or fermionic) are implied by these mass ranges? Why? [2]

ANSWER: Bosonic (integer-spin) fields. At these mass ranges, fermionic fields cannot produce the number densities required to account for the observed dark matter density.

i) In the simplest case of coherent ultralight dark matter, what is the functional dependence of ultralight dark matter on time, e.g. $\phi(t) \approx \phi_0 t^2$? [1]

ANSWER: $\phi(t) = \phi_0 \cos(\omega t + \theta)$

j) How does the field amplitude of ultralight dark matter, ϕ_0, scale with its mass (m_ϕ) & density (ρ)? [2]

ANSWER: $\phi_0 \approx \sqrt{\rho}/m_\phi$;

k) What oscillation harmonics are expected from n^{th}-order interactions of the Standard Model fields with a new (ultralight) field? [1]

ANSWER: BY expanding $\cos^n(\theta)$, we see that for odd(/even)-order interactions, you would expect to see odd(/even) harmonics up to the n^{th} harmonic. E.g. for n=5, the 1st, 3rd and 5th harmonics.

2 Terminology and Nomenclature

Using the metrological definitions below, assign the most relevant concept to statements A) to J):

Instability: an estimate of the noise exhibited by measurements of a physical quantity (statistical uncertainty).

Accuracy: an estimate of how well a measurement of a physical quantity can be corrected to the "true" value of that quantity (systematic offset uncertainty).

Realisability: an estimate of the practicality (or in some cases, possibility) of constructing, running, or maintaining an experimental apparatus or sensor to measure a physical quantity.

A) A feature of a data set measured by Allan Deviation, $\sigma_y(\tau)$, and other M-sample variance statistics.

B) There is a high degree of confidence in the averaged value of the transition frequency.

C) The cooling and clock transitions for a given ion species can be easily excited with current technology.

D) All known systematic offsets to the clock's transition frequency are compensated in real time or in post-processing.

E) Optical frequency standards have higher Q-factors than microwave frequency standards.

F) Portable optical atomic clocks are not yet commercially available.

G) A geodetic measurements was conducted to evaluate the frequency shift on the clock due to gravity.

H) The clock transition is based on a transition in a rare, metastable isotope of an actinide-series element.

I) Oscillations in fundamental constants have been excluded at certain power levels for certain frequencies.

J) Optical atomic clocks are better candidates than microwave ones for the future redefinition of SI second.

[5]

ANSWERS:
Instability: A, E, I
Accuracy: B, D, G, J
Realisability: C, F, H

3 Short Answer Questions

a) Outline the advantages of using frequency standards based on optical frequencies compared with using frequency standards based on microwave frequencies. [2]

ANSWER: For any given, fixed frequency perturbation, higher frequencies can be realised with a higher fractional accuracy and stability. E.g., ± 5 Hz on a 1 kHz sinusoid = $\pm$ 0.5%, but ± 5 Hz on a 1 GHz sinusoid = $\pm$ 0.0000005%

b) Conversely, outline some of the advantages of using microwave frequency standards or a hydrogen maser flywheel oscillator rather than optical frequency standards. [2]

ANSWER: The technology is much more mature: there are commercial options, uptimes can reach close to 100% , there are compact / chip-scale options. Also, they can deliver a direct RF output without conversion e.g. 100 MHz. Moreover, they don't need ultra-stable lasers or a femtosecond frequency comb to interrogate them.

c) What are the advantages of a clock based on an optical lattice of neutral atoms compared with a clock based on a singly-charged trapped ion? What are the disadvantages? [4]

ANSWER: Higher stability (faster averaging) due to probing N atoms at a time. However, it is harder to control the local environment for each of the N atoms (consider the frequency shift due to the lattice light). They are also harder to build and run, due to more lasers & loading stages.

d) What additional challenges might be involved in building a frequency standard based on a highly charged ion? What advantages might it offer in the context of searching for BSM physics? [3]

ANSWER: It is hard to predict with high precision the energies of transitions, and it is hard to find HCI species with transitions that can be probed with current laser technology. It takes a lot of work to produce, control, cool, and filter the ionization states. The trap needs an excellent vacuum to achieve a reasonable trapping lifetime. However, if you can overcome all of these challenges, HCI species can have enormous sensitivities to changes in the fine structure constant. The high ionization level also causes the ion to have lower sensitivity to several common sources of environmental noise.

e) What additional challenges might be involved in building a frequency standard based on transitions between rotational or vibrational states in a trapped molecule / ion? What advantages might it offer in the context of searching for BSM physics? [3]

ANSWER: Molecules have higher numbers and densities of energy states, which can produce a lot of peaks in your spectrum. The availability and proximity of so many other states also makes it harder to find a closed cooling cycle: electrons easily transition to unwanted energy states and "go dark". On the plus side, rovibrational transitions in molecules are sensitive to changes in μ but not very sensitive to changes in α, which provides an independent measure of $\Delta\mu$ to the clocks based on hyperfine transitions.

f) Outline one advantage and one disadvantage of using an atomic clock based on a transition between electron energy levels which exhibits a narrow natural linewidth. [2]

ANSWER: Narrow linewidths give greater frequency stability, but it becomes more difficult to excite. E.g., one may need to increase the intensity of the probe laser light to increase the number of photons that can excite the transition, but a higher laser intensity will induce a large AC Stark shift.

g) When might the Allan Deviation be a more suitable measure of instability than the Standard Deviation? What extra information does it provide? [3]

ANSWER: In the presence of noise processes that are not statistically stationary e.g. pink noise, random walk noise, etc.. In these situations, the standard deviation will not converge as sample number increases. The slope of an ADEV curve (and its variants) categorizes the noise on a signal, and indicates which noise types are dominant over which timescales. Not only does this tell you what analysis methods / estimators might be unreliable for your data set, but it can give clues as to where dominant noise sources might be coming from.

h) What is the motivation for attempting to detect changes in fundamental physical "constants" which are dimensionless, rather than dimensionful? [2]

ANSWER: Experimentally, we don't have to worry about whether the test object is changing or whether the reference is changing. (It is debated whether it even makes sense to discuss observing changes in dimensionless variables, because of the need to have a reference against which to observe the change.)

i) How many independent, dimensionless parameters can be generated from the set:

$$\{e, \hbar, c, k_B, G, M_\odot, S_{Au}\}$$

where S_{Au} is the Seebeck coefficient of gold, measured in V/K (volts per kelvin) [4]
ANSWER: Only 2: $C_1 = eS_{Au}/k_B$ and $C_2 = M_\odot^2\sqrt{G/\hbar c}$

This can be solved with brute force by dimensional analysis of each quantity and combining them to form dimensionless parameters OR by using the Buckingham Pi Theorem: N=7 variables, P=5 physical dimensions (charge, time, mass, length, temperature). Number of dimensionless parameters of the system is $M = (N - P) = (7 - 5) = 2$.

4 QSNET clocks

Species	Transition Wavelength	K_α	K_μ	Fractional Inaccuracy (approximate)	Fractional Instability (approximate)
$^{251}Cf^{+17}$	485 nm	-43.5	0.0	1.0×10^{-18} (target)	$2.0 \times 10^{-15}/\sqrt{\tau}$ (target)
$^{251}Cf^{+17}$	618 nm	47.0	0.0	5.0×10^{-19} (target)	$3.0 \times 10^{-15}/\sqrt{\tau}$ (target)
$^{171}Yb^{+}$(E3)	467 nm	-5.95	0.0	3.0×10^{-18}	$1.0 \times 10^{-15}/\sqrt{\tau}$
$^{171}Yb^{+}$(E2)	436 nm	1.00	0.0	3.0×10^{-17}	$1.0 \times 10^{-14}/\sqrt{\tau}$
^{87}Sr	698 nm	0.06	0.0	2.0×10^{-18}	$5.0 \times 10^{-17}/\sqrt{\tau}$
^{133}Cs	32.6 mm	2.83	1.0	2.0×10^{-16}	$2.0 \times 10^{-14}/\sqrt{\tau}$
CaF	17.0 μm	0.00	0.5	8.0×10^{-18} (target)	$1.0 \times 10^{-15}/\sqrt{\tau}$ (target)
N_2^+	2.31 μm	0.00	0.5	4.0×10^{-18} (target)	$1.0 \times 10^{-14}/\sqrt{\tau}$ (target)

a) The caesium primary frequency standard is based on an energy transition between the hyperfine levels of the ground state of ^{133}Cs. Why are hyperfine transitions sensitive to both α and μ? [3]

ANSWER: The gross structure of atomic levels results from the electromagnetic interactions between electrons and the protons in the nucleus, the strength of which is governed by α. The fine structure depends on various other effects (e.g. spin-orbit coupling) which also feature electromagnetic interactions governed by α. Hyperfine structure arises from electron interactions with nuclear spin, which depends on the proton mass through the nuclear magneton. As a result, transitions between hyperfine states feature prominent dependencies both on α and μ.

b) Only considering the "sensitivity" columns, which ratio would you use to search for variations in α? Which ratio would you use to search for variations in μ? [3]

ANSWER: Cf^{15+}/Cf^{17+} for α; anything against Cs for μ (except for CaF and N_2^+) .

c) Now considering all the properties of the clocks given in the table, why might the ratios you gave as answers to question 4b) NOT be the best available for constraining fast variations in α and μ? Which ratios would you use when accounting for **all** properties of the clocks? [3]

ANSWER: Sensitivity must be balanced with instability and inaccuracy: Just because Cf/Sr has 8x the sensitivity of Yb+(E3)/Sr, if the instability ends up being 10x worse, it won't give the best constraints. Conversely, Cs is 2x more sensitive to $\Delta\mu$ than CaF, but maybe the CaF clock could be realised with 5x better instability.

d) Which ratios are least sensitive to changes in α? Which are least sensitive to changes in μ?
Why would we want to include these ratios in a data campaign to constrain variations in α and μ? [3]

ANSWER: CaF / N_2^+ for α; any ratio between the 5 topmost species for μ. We would like these as control systems to see if features of interest show up at the same level on high-sensitivity and low-sensitivity systems. If you see a signal of the same height on systems with different sensitivities, those features aren't due to $\Delta\alpha$ or $\Delta\mu$.

e) The QSNET project is a collaboration between several institutes across the UK. How might the clocks across different QSNET sites be linked and their frequencies compared? What additional noise might be introduced? [2]

ANSWER: Satellite-based time and frequency transfer techniques can used to produce frequency ratios across the sites. Part of the potential future activities of QSNET would be to establish a fibre optic network between the laboratories. Additional noise could be introduced on the links themselves, e.g., seismic effects, temperature effects, or issues with amplifiers along the connection between laboratories for a fibre optic link.

f) What BSM effects could be probed with a geographically distributed network of clocks, that could not be probed using a collection of clocks at one laboratory or institute? [2]

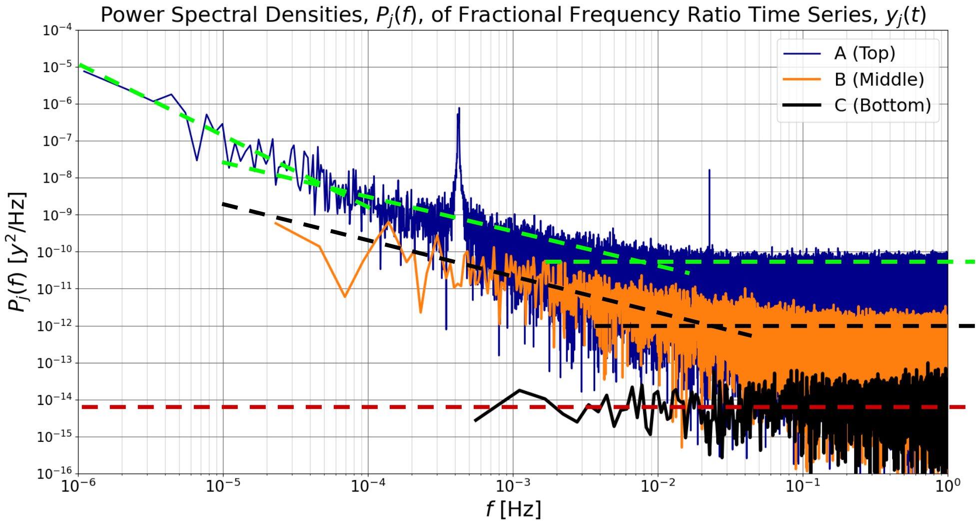
ANSWER: Non-local and transient effects, e.g., the passage of Earth through extended dark matter clusters or domain walls.

5 Noise Estimation

The noise on a signal is often categorized into "colours" based on the power-law behaviour of the signal's power spectral density, $P_y(f)$. The table below illustrates some of the common noise types arising in frequency metrology, with H, W, X, Y, and Z all representing constants of proportionality:

Noise Colour	Frequency Dependence	Comments
White	$P_y(f) \approx H$	Constant power across all frequencies
Blue	$P_y(f) \approx Wf$	Often used for dithering in image processing
Violet	$P_y(f) \approx Xf^2$	Can arise from differentiated white frequency noise
Pink	$P_y(f) \approx Yf^{-1}$	Also known as "flicker" noise
Red	$P_y(f) \approx Zf^{-2}$	Can arise from integrated white frequency noise

Consider the graph below displaying three simulated power spectral densities, $P_j(f)$, such as those that might describe the time fractional frequency ratios, $y_j(t)$, arising from the analysis of data acquired from optical and microwave atomic clocks.



A) White + Pink + Red: $H \approx 1 \times 10^{-10}$; $Y \approx 1 \times 10^{-12}$; $Z \approx 1 \times 10^{-16}$

B) White + Pink: $H \approx 1 \times 10^{-12}$; $Y \approx 1 \times 10^{-14}$

C) White: $H \approx 1 \times 10^{-14}$

a) Estimate the white frequency noise level, H, of dataset C. [1]

ANSWER: $H \approx 1.0 \times 10^{-14}$/Hz

b) Assume that dataset C corresponds to Cf^{+17}/Cf^{+15} frequency ratio data and that no peaks are statistically significant. Having estimated the noise level, to what level of power would you constrain oscillations in α? [Hint: Look at Table 1 for values of K_α] [2]

ANSWER: $1.0 \times 10^{-14}/90.5 = 1.1 \times 10^{-16}$/Hz, but this bound is only applicable over the frequency range [1 Hz, 5.5×10^{-4} Hz]

c) What noise types are present in dataset B, and over what frequency ranges?
Bonus: estimate the coefficients for each noise type [2 (+2)]

ANSWER: Between 1 s to 20 s: white frequency noise of $H \approx 1.0 \times 10^{-12}$/Hz. Between 20 s to 12 hours: pink/flicker noise of $Y \approx 1.0 \times 10^{-13}$/Hz.

d) What noise types are present in dataset A, and over what frequency ranges?
Bonus: estimate the coefficients for each noise type [3(+3)]

ANSWER: Between 1 s to 100 s: white frequency noise of $H \approx 1.0 \times 10^{-10}$/Hz. Between 100 s to 12 hours: pink/flicker noise of $Y \approx 1.0 \times 10^{-12}$/Hz. Between 12 hours to 10 days, red noise of $Z \approx 1.0 \times 10^{-16}$/Hz

e) Besides BSM physics, describe two other plausible physical processes that could cause significant oscillations in atomic clock ratio data. Briefly outline how you would attempt to distinguish signals due to these processes from signals due to BSM phenomena. [2]

ANSWER: Tides, daily temperature cycle, AC fields, mechanical vibrations, etc. Some of these processes could be ruled out by order of magnitude considerations, but some would require independent experiments to determine the size of the effect they would cause. The key point is that if you discover a signal, there's a lot of work ahead of you before you can claim it's dark matter.

f) For a bin-width of $\Delta f = 10.5$ μHz, what would you estimate as the oscillation amplitude of the peak at f = 23 mHz, assuming Fig 2 is a 1-sided spectrum, such that $A(f) = \sqrt{P(f) * \Delta f}$.
How would this change if Fig 2 showed only the positive frequencies of a 2-sided spectrum? [3]

ANSWER: P(23 mHz) $\approx 1.9 \times 10^{-8}$, so A(23 mHz) $\approx \sqrt{1.9 \times 10^{-8}) \times (10.5 \times 10^{-6})} = 4.5 \times 10^{-7}$. For a 2-sided spectrum, powers are split across positive and negative frequencies, so you'd want to double the power first before using the equation (assuming you're looking at a real signal)

g) Estimate the full-width at half-maximum (FWHM) of the broadened peak at $\Delta f = 417$ μHz.
Assuming this peak to result from wave-like cold dark matter, what information could you extract about the properties of the dark matter from the linewidth of the peak?
(Hint: what differentiates ***cold*** dark matter from other types?) [2]

ANSWER: FWHM $= 2\gamma \approx 3.0 \times 10^{-5}$ Hz. A spread in frequencies δf suggests a spread in the kinetic energies of the individual CDM particles that form the coherent wave. So the FWHM provides information about the thermal distribution of the particles.

6 Strontium Lattice Clock

The tables below describe systematic effects in a ^{87}Sr optical lattice clock that perturb the clock transition frequency effects, methods for determining the frequency shifts caused by the effects, and techniques for reducing the uncertainty in these measurements to achieve greater accuracy in the evaluation of the clock transition frequency.

a) Pair up each systematic effect shift with the measurement that could be made to determine the frequency shift it would cause.
(The accompanying image (see Fig. 1) of the strontium optical lattice clock may give some clues as to the frequency shifts and offsets being measured.) [3]

ANSWER: See annotated table

b) Pair up each measurement with an appropriate method for reducing the uncertainty in the measurement of the frequency shift caused by the systematic effect. [3]

ANSWER: See annotated table

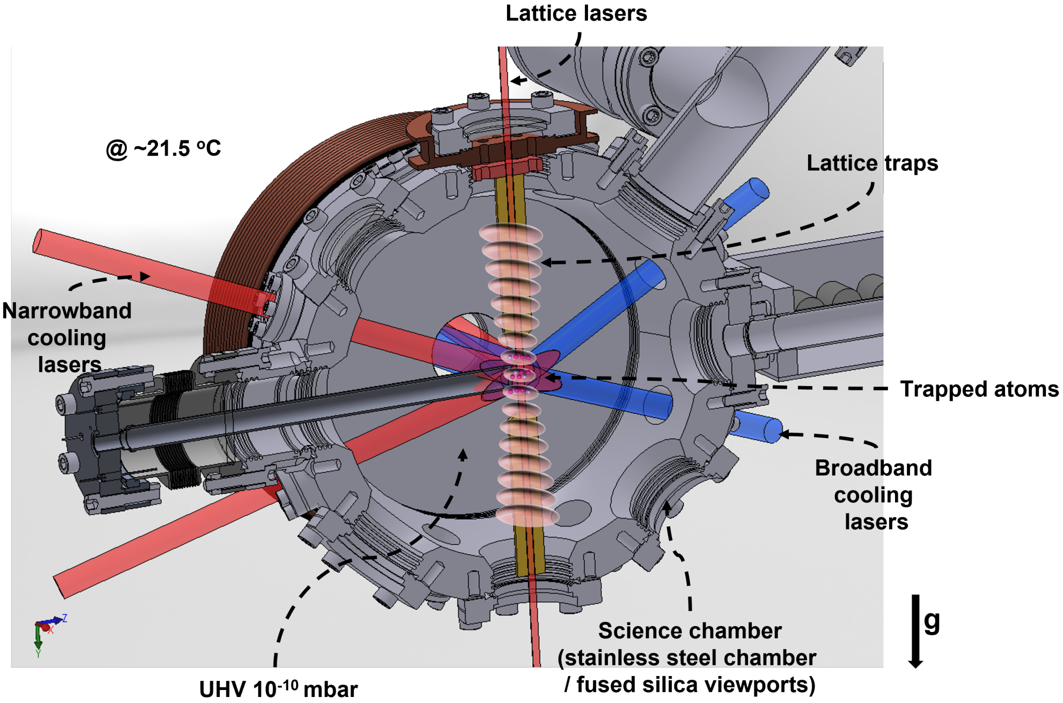


Figure 1: Physics package for NPL-Sr1 optical lattice clock.

Systematic Effect	Measurement	Method for Reducing Measurement Uncertainty
Relativistic redshift [**A**]	Frequency difference between interleaved servos at different lattice depth. [**D**]	Increase knowledge of local geopotential [**A**]
Zeeman Effect [**B**]	Temperature measurement at the location of trapped atoms. [**C**]	Tuned s- p- wave cancellation [**E**]
Blackbody Radiation [**C**]	Frequency difference between interleaved servos containing different atom numbers. [**E**]	Operate the optical lattice trap at a shallow trap depth [**D**]
Lattice shift [**D**]	Differential frequency measurement of two stretched clock states of interest. [**B**]	Increase vacuum quality[**F**]
Cold collisions [**E**]	The average lifetime of the trapped atom cloud [**F**]	Realize in-situ thermometry [**C**]
Background Gas Collisions [**F**]	Height of the atom cloud above a conventionally adopted equipotential [**A**]	Use clock states with lower sensitivity to B-fields; Calibrate B-field sensitivities [**B**]

c) The frequency shift caused by Black-body radiation (BBR) is one of the dominant sources of uncertainty in most optical atomic clocks. Describe how BBR introduces uncertainty into the frequency measurement of the clock transition in an optical lattice clock. [3]

At non-zero temperatures, the photons emitted by blackbody radiation couple the clock state to other intermediate atomic levels in the involved atoms and cause shifts of the clock states or detuning of the clock transition via the AC Stark effect. This can be corrected to a large degree, given the BBR temperature at the location of atoms can be measured accurately, e.g., in-situ thermometry, where the BBR-induced uncertainty will be limited by the BBR temperature-frequency shift coefficient uncertainty. In the situation without in-situ temperature measurement, BBR temperature is estimated based on the components' temperature around the science chamber. In NPL-Sr1, The science chamber is made of stainless steel chamber and fused silica viewports, which possess different emissivities. The temperature gradient across the science chamber leads to variations in estimated BBR temperature due to the uncertainties of emissivity.