QTFP Winter School 2023 - AION Tutorial

Lecturer: Professor Timothy Kovachy (Northwestern University)
Tutors: Anna Marchant (STFC), Kimberly Tkalčec (University of Cambridge),
Samuel Lellouch (University of Birmingham)

Tutorial: 11th January 2023

Tutorial questions

Exercise 1. Pulse fidelity, velocity selection and large-momentum transfer

(i) Consider a two-level atom initially prepared in its ground-state at time $t = 0$, and driven by a quasi-resonant light field. Express the probability to find the atom in its excited-state as a function of the light pulse duration, detuning and Rabi frequency. Sketch/ plot the result as a function of pulse duration for different detunings, and identify the optimal pulse duration to realize a population inversion (called π-time) and the transfer efficiency, referred to as the pulse fidelity. Compare your sketch with other members of the group.

(ii) If the given atom has a non-zero velocity, what is the laser frequency that it sees due to the Doppler effect?

(iii) Consider a thermal atomic cloud at temperature T described by a Maxwell-Boltzmann velocity distribution. Qualitatively explain why a high temperature reduces the efficiency of the population transfer at the cloud scale. By making an appropriate choice of pulse duration, compute the cloud-averaged pulse fidelity. *Hint: To simplify the evaluation of the integral, we will approximate the central peak of the sinc function by a Gaussian of same RMS spread, namely* $\pi^2/4 \times sinc^2[\pi/2 \times \sqrt{1+u^2}] \approx e^{-u^2/2}$.

(iv) Which experimental parameters can be tuned to increase the pulse fidelity? Which other physical effects not accounted for here could also reduce the pulse fidelity? Discuss your answers with other members of the group.

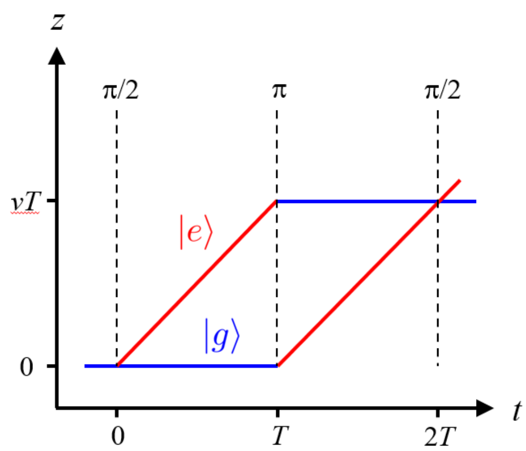
(v) *Bonus question:* In a large-momentum transfer interferometer based on sequential π-pulses, the large momentum separation is imparted by successively applying a large number of π pulses. In practice, they need to come from alternating directions; discuss why. Considering the case of strontium atoms interrogated via the intercombination line transition (wavelength $\lambda = 689$nm, mass $m = 1.44 \times 10^{-25}$kg, electric dipole $d = 7.35 \times 10^{-31}$C.m), calculate which temperatures would be required to maintain a fringe contrast above 5% for a momentum separation of $1000\hbar k$, assuming a laser intensity of $100mW/cm^2$. *Hint: You can assume that the interferometer fringe contrast can be fairly estimated by the product of the individual pulse fidelities of the sequence.*

Exercise 2. Interferometer sensitivity

Consider a Mach-Zehnder interferometer where atoms are subjected to a uniform gravitational acceleration, g. The atomic trajectories of the ground and excited state are shown in blue and red respectively.

(i) If the phase shift of the interferometer to an acceleration g is given by

$$\Delta\phi = kgT^2, \quad (1)$$



where k is the wavevector of the laser, find an expression for the relative sensitivity to gravitational accelerations, $\delta g/g$.

Hint: In this shot noise limited regime, assume $\delta\phi = 1/\sqrt{N}$ where N is the number of atoms per shot.

(ii) Calculate the sensitivity achievable with an interferometer measuring 10^6 atoms with $T = 1.2$ s, operating at $\lambda = 689$ nm. How could this sensitivity be enhanced in an experiment? Discuss your answer with other members of the group.

Tutorial solutions

Exercise 1.

(i) *Refer to AION lecture 1 notes A5 (pg. 7 of the pdf file.)*

The population in the excited states is given by $P = |C_2|^2$.

$$P(\Omega, \delta, t) = \frac{\Omega^2}{\Omega^2 + \delta^2} \sin^2(\sqrt{\Omega^2 + \delta^2} t/2). \quad (2)$$

The π-time is at the peak of the sine wave, such that

$$t_\pi(\Omega, \delta) = \pi/\sqrt{\Omega^2 + \delta^2}. \quad (3)$$

The maximum population occurs when the sin^2 term in Eq. 2 goes to 1, giving

$$P_{max} = \frac{\Omega^2}{\Omega^2 + \delta^2}. \quad (4)$$

The population of the excited state vs. time for some example detunings is shown in Fig. 1.

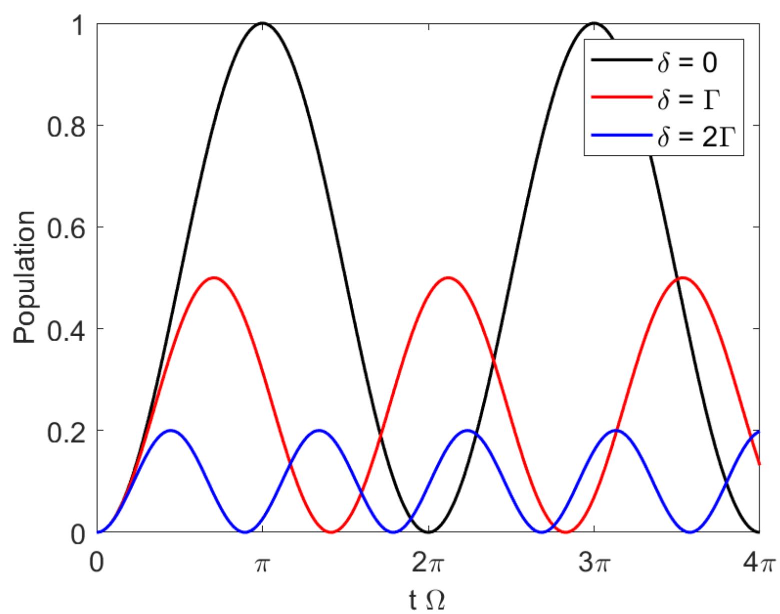


Figure 1: Rabi oscillations for multiple detunings.

(ii) The atoms will see a frequency

$$\omega = \omega_L - \mathbf{k}.\mathbf{v} \quad (5)$$

with ω_L the laser frequency and k the laser wavevector.

(iii) A high temperature means a higher range of velocities, hence detunings, within the cloud. In particular, there will be fast atoms which experience a large Doppler shift which leads to a low pulse fidelity. Moreover, the Rabi oscillations of individual atoms oscillate at different frequencies, resulting in dephasings between them which lead to an overall reduction of the oscillation amplitude after cloud averaging.

The zero velocity class has the highest pulse fidelity and is also the most populated, so it is optimal to choose $t_\pi(\Omega, 0) = \pi/\Omega$ as the pulse duration. With such pulse duration, the fidelity for a detuned atom is given by

$$P(\Omega, \delta, t_\pi(\Omega, 0)) = \frac{\Omega^2}{\Omega^2 + \delta^2} \sin^2(\pi\sqrt{\Omega^2 + \delta^2}/2\Omega) = \frac{\pi^2}{4}\mathrm{sinc}^2(\frac{\pi}{2}\sqrt{1 + \delta^2/\Omega^2}) \tag{6}$$

Averaging this pulse fidelity over the cloud velocity distribution $f(\mathbf{v}) = \frac{1}{\sigma_v^3(2\pi)^{3/2}} e^{-\mathbf{v}^2/2\sigma_v^2}$ (with $\sigma_v = \sqrt{k_B T/m}$) yields

$$F = \int_{\mathbf{v}} \frac{\pi^2}{4}\mathrm{sinc}^2(\frac{\pi}{2}\sqrt{1 + (kv_z)^2/\Omega^2}) f(\mathbf{v}) d^3\mathbf{v}. \tag{7}$$

Integrals over v_x, v_y are trivial, and for the integral over v_z, approximating the central peak of the sinc by a Gaussian gives

$$\begin{aligned} F &= \int_{-\infty}^{\infty} \frac{\pi^2}{4}\mathrm{sinc}^2(\frac{\pi}{2}\sqrt{1 + (kv_z)^2/\Omega^2}) \frac{1}{\sigma_v\sqrt{2\pi}} e^{-v_z^2/2\sigma_v^2} dv_z \\ &\approx \frac{1}{\sigma_v\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(kv_z)^2/2\Omega^2} e^{-v_z^2/2\sigma_v^2} dv_z \\ &\approx \frac{1}{\sqrt{1 + (\sigma_v k/\Omega)^2}} \end{aligned} \tag{8}$$

(iv) From the above result, increasing the fidelity can involve reducing the temperature (hence σ_v) or increasing the laser intensity (hence Rabi frequency) by e.g. increasing the total power or reducing the beam waist.

Other effects neglected here:
- spontaneous emission (loss of atoms);
- finite cloud size and laser spatial intensity distribution: due to this, different atoms in the cloud see different Rabi frequencies, producing dephasings between their Rabi oscillations reducing the efficiency of the population transfer.

(v) Alternating directions allows momentum kicks to accumulate and not cancel each other out. Assuming a first π-pulse gives the atom a momentum kick of $\hbar k$ through absorption of a photon, if the next π pulse was coming from the same direction, it would bring back the atom into the ground state through stimulated emission of a photon k, conferring the momentum $-\hbar k$ to the atom (and thus cancelling out the effect of the first pulse). Alternating directions allows the successive absorptions and stimulated emissions to all impart $+\hbar k$ onto the atoms, overall driving large momentum transfer.

In order to calculate the fidelity of an M pulse sequence, we can assume that $F_{\text{total}} = F^M$. Let's denote $w = \sigma_v k_L/\Omega$. If M is the LMT order, we want $(1 + w^2)^{-M/2} > 0.05$. This gives $1 + w^2 < (0.05)^{-2/M}$, i.e. $T < [(0.05)^{-2/M} - 1]m\Omega^2/k_B k^2$. [Note: For large M, this gives $T < 6m\Omega^2/M k_B k^2$: all the more stringent that M is large, and the intensity is small].

To get a value for the temperature, we use $\Omega = d/\hbar \times \sqrt{2I/c\epsilon_0}$. This gives $T < 27\mu$K

Approximations made here: we neglected other contrast loss sources; we neglected accumulated Doppler shifts due to the increased momentum splitting; we neglected the fidelity of the $\pi/2$ pulses and simply assumed that M pulses were needed to reach $M\hbar k$ of LMT (with no consideration about closing the interferometer).

Exercise 2.

(i)

$$\frac{\delta g}{g} = \frac{\delta\phi}{\Delta\phi} \tag{9}$$

$$\frac{\delta g}{g} = \frac{1}{\sqrt{N}}\frac{1}{kgT^2} \tag{10}$$

The phase noise, $\delta\phi$, is set by the signal-to-noise ratio (SNR) of the detected atom population,

$$\delta\phi = \frac{1}{\text{SNR}} = \frac{1}{\sqrt{N}}. \tag{11}$$

(Given that the standard deviation of the noise goes as $\sqrt{N}$ and the signal is N, we therefore have the SNR $= N/\sqrt{N} = \sqrt{N}$.)

(ii) For 10^6 atoms with a flight time of 1.2 s, operating at $\lambda = 689$ nm,

$$\frac{\delta g}{g} = \frac{1}{\sqrt{N}}\frac{1}{kgT^2} \tag{12}$$

$$= 7.8 \times 10^{-12}. \tag{13}$$

The interferometer sensitivity could be increased by extending the flight time or by transferring more momentum to the atoms using a large momentum transfer (LMT) sequence.