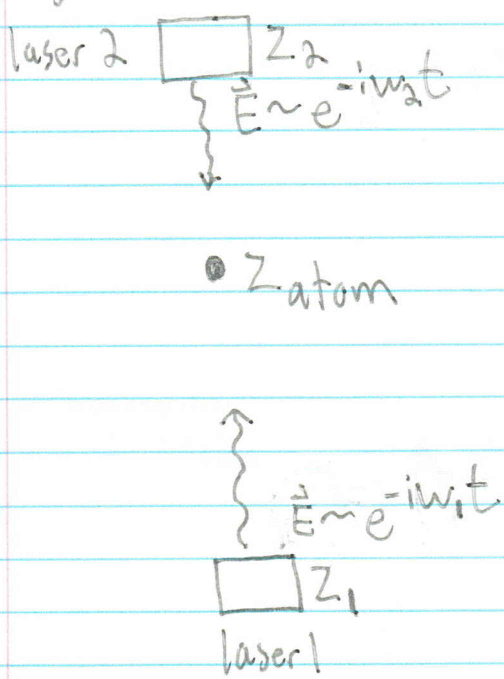
## Relativistic View of the Laser Phase

We can alternatively derive the laser phase from considering the finite travel speed of light. We consider the setup below:




There are two lasers at positions $z_1$ and $z_2$ that drive Bragg (or Raman) transitions. For simplicity, we consider the case in which laser 1 is always kept on, and the timing of the Bragg pulses is determined by when laser 2 is pulsed on. Laser 1 has frequency $\omega_1$, and laser 2 has frequency $\omega_2$. For simplicity, we will assume that $\omega_1 = \omega_2 = \omega$.

For a Bragg pulse to happen at time $t_0$, the light must leave laser 2 at time:

$$t_2 = t_0 - \frac{(z_2 - z_{atom})}{c}$$

This light leaves the laser with phase:

$\phi_2 = -wt_2$, and since phase is constant along a null geodesic (i.e., along the propagation path of a photon), the atom sees phase $\phi_2$ from laser 2. The Bragg transition involes light from laser 1 with phase $\phi_1 = -wt_1$, where the light had to leave laser 1 at time:

$$t_1 = t_0 - \frac{(z_{atom} - z_1)}{c}$$

For a momentum kick $\pm 2\hbar k$, the associated laser phase is thus:

$$\mp w(t_1 - t_2) = \pm w\left(2\frac{z_{atom}}{c} + \frac{(z_1 - z_2)}{c}\right)$$

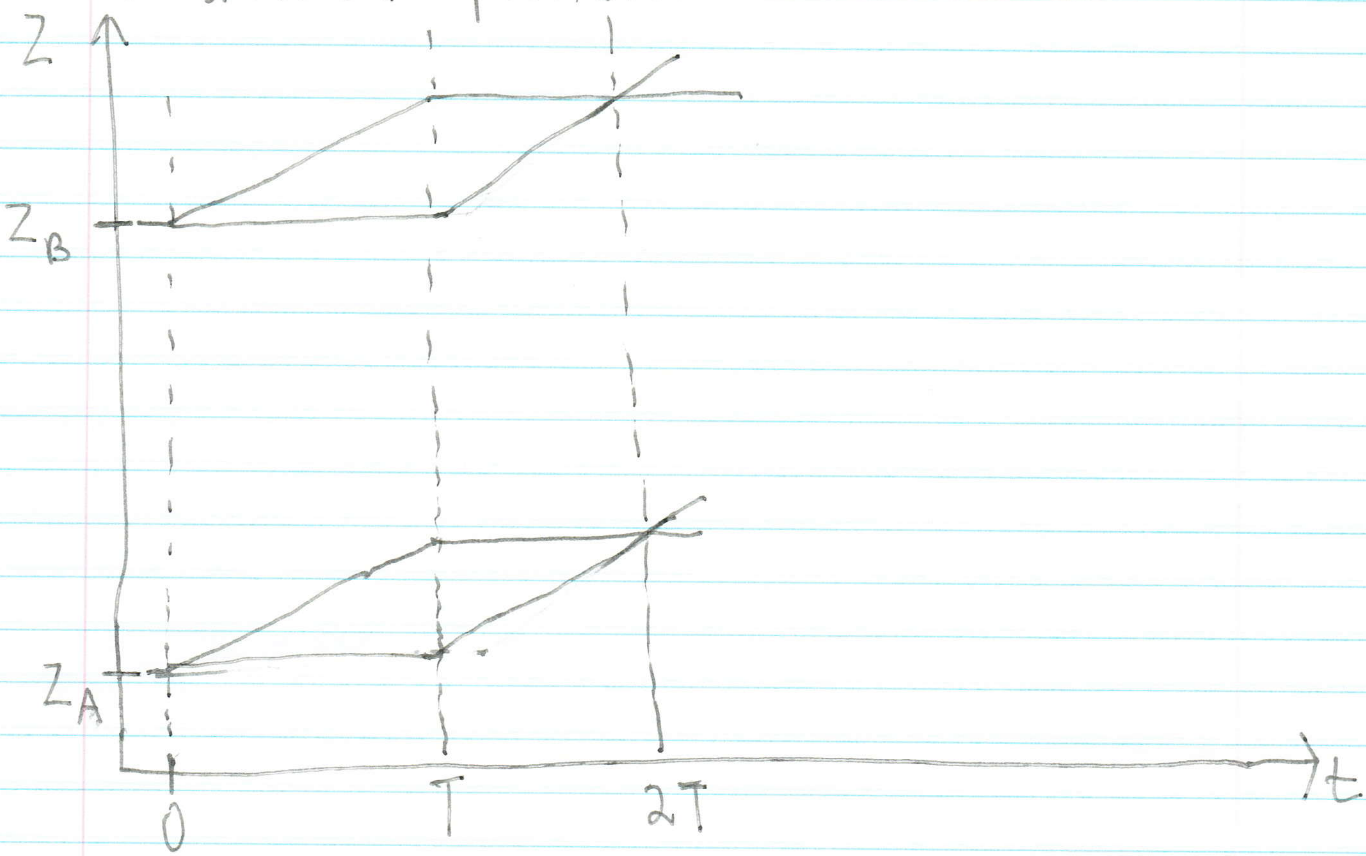
$$= \pm 2kz_{atom} \pm k\frac{(z_1 - z_2)}{c}$$

where $w = ck$.

If we use this expression in our previous example, the contributions from $\pm k(z_1 - z_2)/c$ will cancel in the full interferometer, leaving us with the same $\Delta\phi_{Laser}$ as before.

## Gravity Gradiometry

In many cases, it is useful to measure the difference in phase shifts between interferometers at different positions:




The phase difference can encode information, for instance, about how g varies with height (i.e., gravity gradients):

$$\Delta\phi(z_B) - \Delta\phi(z_A) = -2nk\left[g(z_B) - g(z_A)\right]T^2$$

Moreover, the same laser pulses are used for both interferometers. If there noise in the phase $\phi(t_i)$, this can lead to noise in the phase shift of an individual interferometer. However, this noise will be common to the two interferometers and subtract out in a differential measurement.

It is important that the phase shift not depend on the initial position $z_0$ of the atoms. For constant $k$, this is indeed the case.

But what if $k$ varies by a small amount $\delta k$, for example, for the first pulse only? This could arise from laser frequency noise. Then there is an additional contribution to the phase

$$2n\delta k z_0 = 2n \frac{\delta\omega}{c} z_0$$

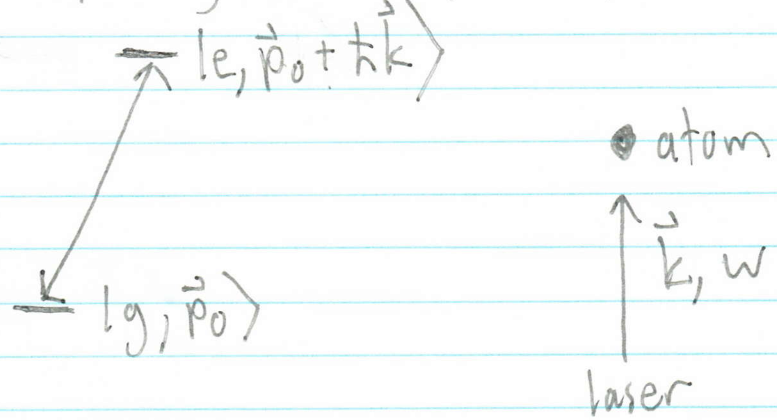
that does depend on $z_0$. There is then a contribution to the differential phase shift that goes as:

$$2n \frac{\delta\omega}{c} (z_B - z_A)$$

For a stable laser with small $\delta\omega$, often this phase shift is negligible. But for long baselines $z_B - z_A$ and/or large $n$, noise in the differential phase $\Delta\phi(z_B) - \Delta\phi(z_A)$ due to $\delta\omega$ fluctuations can become significant.

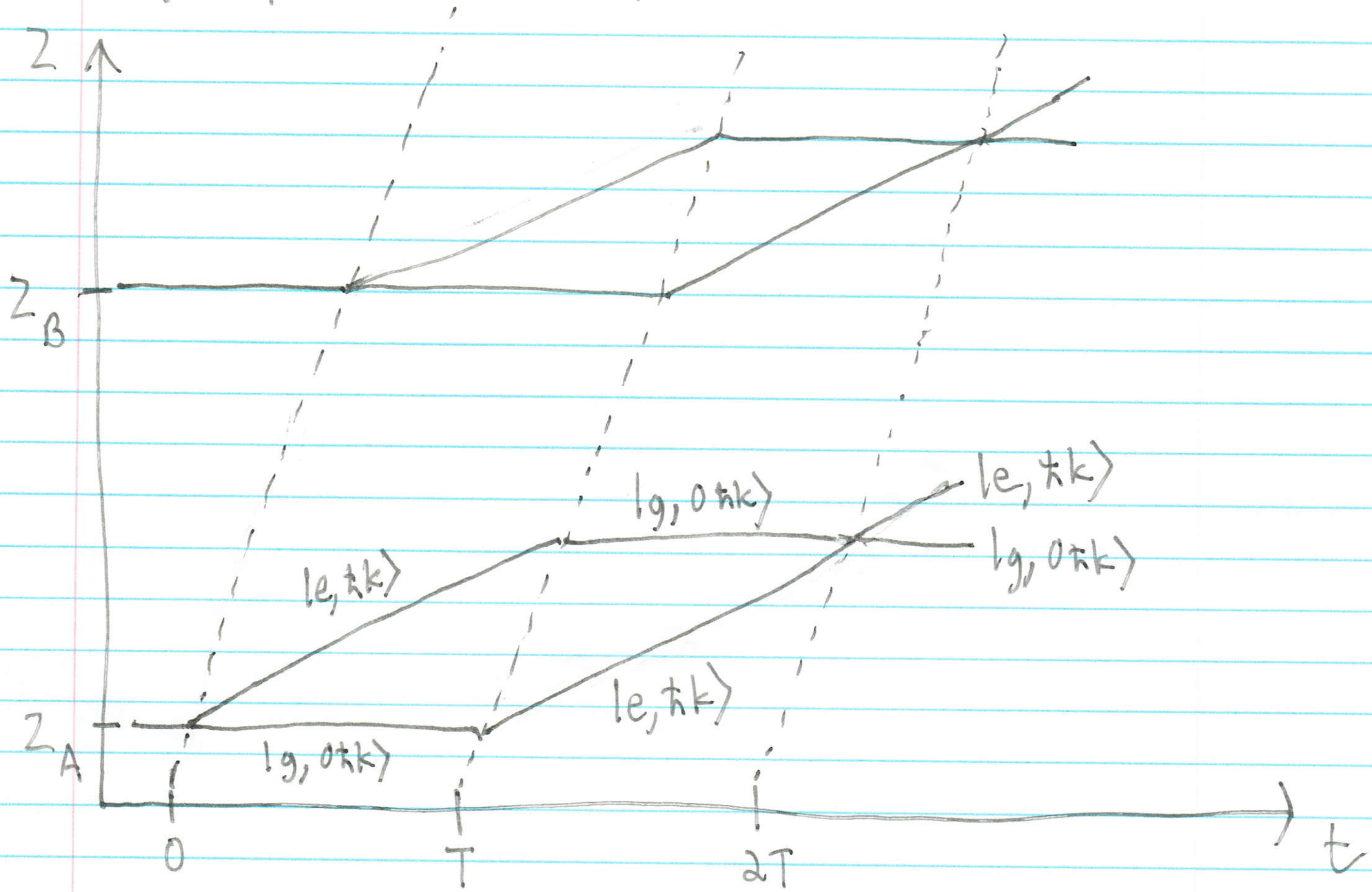
## Single-Photon Atom Optics

Perhaps an even simpler form of atom optics than two-photon Raman or Bragg atom optics involves single-photon transitions, driven on resonance, between a ground and excited state with energy difference corresponding to an optical photon.




For most optically excited transitions, the excited state lifetime is so short (~ns to tens of ns) that long-lived superpositions of $|g\rangle$ and $|e\rangle$ are not possible. But for very weak transitions like those used in optical atomic clocks, the lifetime of $|e\rangle$ can be very long — for example, in excess of 100s for the 698nm clock transition in Sr.

We can consider a gradiometer using single photon atom optics:




The laser, with frequency $\omega$, is pulsed on at times 0, T, and 2T. The two interferometers start at slightly different times due to the finite speed of light as it travels across the baseline. The dashed lines show the photon trajectories, with c exaggerated as unrealistically small for ease of visualization.

Since both interferometers experience exactly the same laser pulses (albeit at slightly delayed times), the two interferometers will experience exactly the same laser phase, in contrast to the the two-photon case. Single-photon interferometry is thus favorable for rejecting laser frequency noise.

For a two-photon interferometer, the sensitivity to gravity came from the laser phase. But for single-photon interferometers, the laser phase does not depend on the trajectories of the atoms. Instead the sensitivity to gravity comes from the falling atomic trajectories influencing the timings at which the atom switches between $|g\rangle$ and $|e\rangle$. To see this, let us say that before the first pulse, the atom has trajectory:

$$z(t) = z_0 - \frac{1}{2} g t^2$$

If the laser is at position $z=0$, then the first pulse reaches the atom at time

$$t_1 = \frac{z_0}{c}$$

The second (mirror) pulse reaches the upper arm at time:

$$t_{2,up} \approx T + \frac{1}{c}\left(z_0 + \frac{\hbar k}{m}T - \frac{1}{2}gT^2\right)$$

Between $t_1$ and $t_{2,up}$, the upper arm receives phase $\phi_{up} = -w_0\left(t_{2,up} - t_1\right)$ due to the excitation energy $\hbar w_0$ of $|e\rangle$.

The second pulse reaches the lower arm at time:

$$t_{2,down} \approx T + \frac{1}{c}\left(z_0 - \frac{1}{2}gT^2\right),$$

transitioning the lower arm to $|e\rangle$. The interferometer concludes when the third pulse reaches the atoms at time

$$t_3 \approx 2T + \frac{1}{c}\left(z_0 + \frac{\hbar k}{m}T - \frac{1}{2}g(2T)^2\right)$$

Between times $t_{2,down}$ and $t_3$, the lower arm accumulates phase:

$$\phi_{down} = -w_0\left(t_3 - t_{2,down}\right)$$

The total gravitational phase shift is:

$$\Delta\phi = \phi_{up} - \phi_{down}$$
$$= -\frac{w_0}{c} g T^2$$

If the laser frequency $w$ is on resonance so that $w = w_0$, then $w_0/c = k$ and this phase shift has the same magnitude as the phase shift for the two-photon case. But the phase shift for the two cases have very different physical origins.