

## 3.2 Axions

Consider classical scalar field  $\phi(\vec{x}, t)$

$$\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m_a^2 \phi^2$$

Assume  $\phi$  takes the same initial value every where:

$$\phi(\vec{x}, t_0) = \phi_0$$

Equation of motion in static universe

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} - \frac{\delta \mathcal{L}}{\delta \phi} = 0 \quad \Rightarrow \quad \ddot{\phi} + m_a^2 \phi = 0$$

$$\Rightarrow \phi(x, t) = \phi_0 \cos(m_a t)$$

(harmonic oscillator)

In an expanding universe

$$\ddot{\phi} + 3H\dot{\phi} + m_a^2 \phi = 0$$

↑ Hubble friction

$\Rightarrow$  Damped harmonic oscillator

↳ In an expanding universe, kinetic energy is dissipated (redshifted)

For  $3H > m_a$ : Overdamped  $\Rightarrow \dot{\phi} \approx 0$   
 $\phi \approx \phi_0$

For  $3H < m_a$ : Oscillations possible

Ansatz:  $\phi(t, x) = \phi_a(t) \cos(m_a t)$

$$\Rightarrow -m_a \sin(m_a t) [3H \phi_a(t) + 2 \dot{\phi}_a(t)]$$

$$+ \cos(m_a t) [\ddot{\phi}_a(t) + 3H \dot{\phi}_a(t)] = 0$$

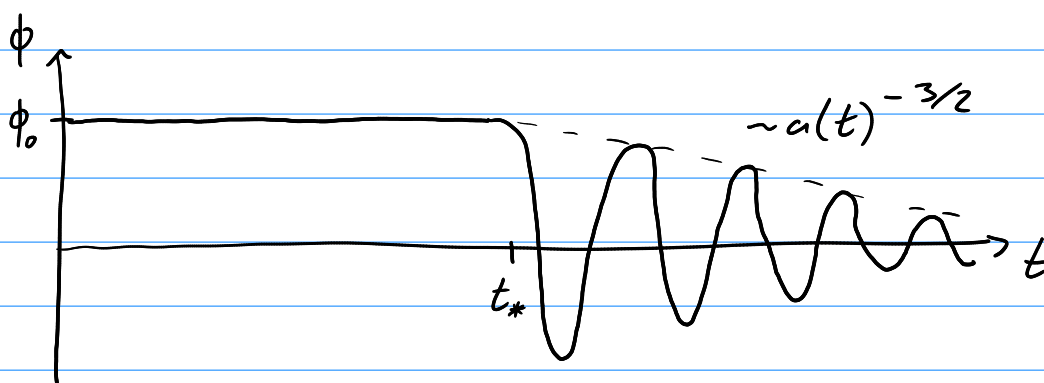
negligible for  $m_a \gg 3H$

$$\Rightarrow 3H \phi_a(t) \approx -2 \dot{\phi}_a(t)$$

$$\Rightarrow -\frac{3}{2} \frac{da}{a} \approx \frac{d\phi_a}{\phi_a} \Rightarrow \phi_a(t) \approx \phi_0 \left( \frac{a(t)}{a(t_*)} \right)^{-3/2}$$

$\uparrow$   
 $3H(t_*) = m_a$

$\Rightarrow$  Oscillations with frequency  $m_a$  and amplitude  $\phi_a(t)$



Energy stored in field given by

$$\rho_a(t) = \frac{1}{2} \dot{\phi}(t)^2 + \frac{1}{2} m_a^2 \phi(t)^2$$

$$\approx m_a^2 \phi_0^2 a(t)^{-3}$$

just like non-relativistic matter

$\Rightarrow$  Axions are produced at rest, even though their mass is tiny compared to  $T$

$\hookrightarrow$  misalignment mechanism

$$\rho_{a,0} = m_a^2 \phi_a(\text{today})^2$$

$$= m_a^2 \phi_0^2 \left( \frac{a(t_*)}{a_0} \right)^3 = m_a^2 \phi_0^2 \frac{S_0}{S_*}$$

$$3 H_* = m_a \quad \Rightarrow \quad T_*^2 \sim m_a$$

$$\Rightarrow S_* \sim m_a^{3/2}$$

$$\Rightarrow S_0 \sim \sqrt{m_a} \phi_0^2$$

$$\Omega_a h^2 = \frac{\rho_{a,0}}{\rho_c} = 0.1 \sqrt{\frac{m_a}{1 \mu\text{eV}}} \left( \frac{\phi_0}{10^{13} \text{ GeV}} \right)^2$$