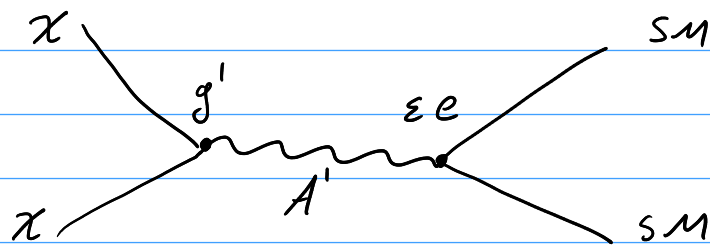


3.1 Freeze-in

Consider



How small can ϵ be?

Central assumption: thermal equil.

$$\Gamma_{\text{ann}}^{\text{eq}} > H \quad \text{for } T > m_X$$

$$\langle \sigma v \rangle \approx \frac{\epsilon^2 \alpha_{\text{em}} \alpha' \pi}{T^2} \quad \text{for } T > m_X$$

$$\Rightarrow \Gamma_{\text{ann}}^{\text{eq}} = \langle \sigma v \rangle n_X \approx \frac{36(3) g_X T^3}{4\pi} \frac{\epsilon^2 \alpha_{\text{em}} \alpha'}{T^2}$$

$$H \approx \frac{T^2}{M_P} \approx \epsilon^2 \alpha_{\text{em}} \alpha' T$$

$$\frac{\Gamma_{\text{ann}}^{\text{eq}}}{H} > 1 \Rightarrow \epsilon^2 \alpha_{\text{em}} \alpha' > \frac{T}{M_P} > \frac{m_X}{M_P}$$

$$\text{For } m_X \sim 10 \text{ MeV} \Rightarrow \epsilon \cdot g' > 10^{-9}$$

For smaller values of $\epsilon \cdot g'$, thermal equil. cannot be established

Recap: Boltzmann eq.:

$$\frac{x}{Y_{\text{eq}}^{\text{eq}}} \frac{dY_{\text{DM}}}{dx} = - \frac{\Gamma_{\text{ann}}^{\text{eq}}}{H} \left(\left(\frac{Y_{\text{DM}}}{Y_{\text{eq}}^{\text{eq}}} \right)^2 - 1 \right)$$

$\Rightarrow$ For $\frac{\Gamma}{H} < 1$, Y_{DM} changes only slowly

Assume that initial DM abundance is small

$$Y_{\text{DM}}(x = x_0) \ll Y_{\text{DM}}^{\text{eq}}$$

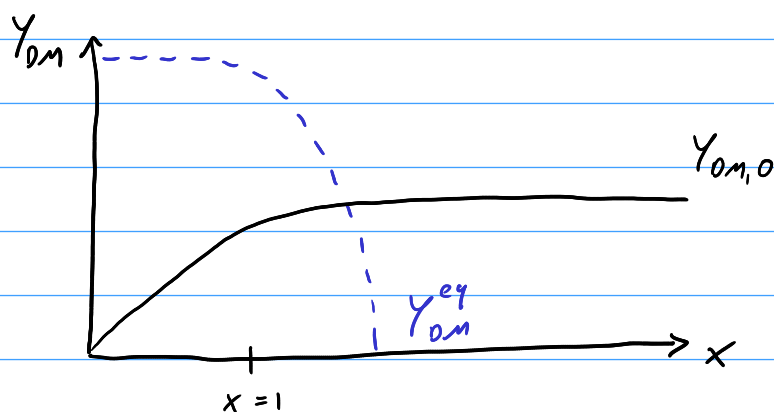
$$\Rightarrow \frac{x}{Y_{\text{eq}}^{\text{eq}}} \frac{dY_{\text{DM}}}{dx} = \frac{\Gamma_{\text{ann}}^{\text{eq}}}{H}$$

$$\Rightarrow Y_{\text{DM}}(x_1) = \int_{x_0}^{x_1} \frac{Y_{\text{DM}}^{\text{eq}}}{x} \frac{\Gamma_{\text{ann}}^{\text{eq}}}{H} dx + Y_{\text{DM}}(x = x_0)$$

- For $x_0 = 0$, $Y_{\text{DM}}(x = x_0) = 0$

- $Y_{\text{DM}}^{\text{eq}}$ and $\Gamma_{\text{ann}}^{\text{eq}}$ become exponentially suppressed for $x > 1$

$$\Rightarrow Y_{\text{DM},0} = \lim_{x_1 \rightarrow \infty} Y_{\text{DM}}(x_1) \approx \int_0^1 \frac{Y_{\text{DM}}^{\text{eq}}}{x} \frac{\Gamma_{\text{ann}}^{\text{eq}}}{H} dx$$



Freeze-in
mechanism

Back to the dark photon model:

$$Y_{DM}^{eq} \sim 1 \quad \text{for } x < 1$$

$$\Gamma_{ann}^{eq} \sim \alpha_{em} \alpha' \varepsilon^2 \frac{m_\chi}{x}$$

$$H \sim \frac{m_\chi^2}{M_P x^2}$$

$$\begin{aligned} \Rightarrow Y_{DM,0} &\approx \int_0^1 \frac{1}{x} \varepsilon^2 \alpha_{em} \alpha' \frac{m_\chi}{x} \frac{M_P x^2}{m_\chi^2} dx \\ &= \varepsilon^2 \alpha_{em} \alpha' \frac{M_P}{m_\chi} \int_0^1 dx \\ &= \varepsilon^2 \alpha_{em} \alpha' \frac{M_P}{m_\chi} \end{aligned}$$

$$\Omega_{DM} h^2 = \frac{m_\chi Y_{DM,0} S_0}{S_c / h^2} \approx 10^{24} \varepsilon^2 \alpha'$$

$$\Rightarrow \Omega_{DM} h^2 = 0.12 \quad \text{achieved for } \varepsilon g' \sim 10^{-12}$$

DM candidates with such small couplings are called "Feebly Interacting Massive Particles" (FIMPs)