

Problem set

Problem 1: Heavy neutrinos

We assume that in addition to the three well-known neutrino species there is a fourth neutrino ν with mass $m_\nu \gg 1 \text{ MeV}$ and interactions

$$\mathcal{L} = \frac{g}{2 \cos \theta_W} \bar{\nu} \gamma_\mu (1 - \gamma^5) \nu Z^\mu, \quad (1)$$

where $g \approx 0.65$ is the weak gauge coupling and $\theta_W \approx 0.50$ is the Weinberg angle. This interaction induces the annihilation processes $\nu \bar{\nu} \rightarrow f \bar{f}$, where f denotes Standard Model fermions. The corresponding cross section is approximately given by

$$\langle \sigma v \rangle = \frac{g^4 m_\nu^2}{16\pi \cos^4 \theta_W (4m_\nu^2 - M_Z^2)^2} \times C, \quad (2)$$

where $M_Z = 91 \text{ GeV}$ is the Z -boson mass and C is a numerical constant that corresponds roughly to the number of final states that are kinematically accessible.

- a) Draw the Feynman diagram for the annihilation process. Why does the cross section become large for $m_\nu \approx m_Z/2$?
- b) Consider the limit $m_\nu \gg M_Z$. Find the value of m_ν that reproduces the observed dark matter relic abundance $\Omega_\nu h^2 \approx 0.12$ via thermal freeze-out.
- c) The previous calculation only includes fermionic final states. Which other final states are neglected? How would including them change the result qualitatively?
- d) Now consider the case $m_\nu \ll M_Z$. Show that a smaller dark matter mass corresponds to a larger dark matter abundance.
- e) To be consistent with observations, $\Omega_\nu h^2$ must not exceed the total amount of DM in the Universe. Use this requirement to obtain a lower bound on m_ν .

This bound is called the Lee-Weinberg bound.

Problem 2: Sterile neutrinos

Sterile neutrinos inherit the interactions of active neutrinos suppressed by a mixing angle $\theta \ll 1$. As a result, any weak interaction process that leads to the production of neutrinos in the early universe (such as $e^- + p \rightarrow \nu_e + n$) can instead produce a sterile neutrino with probability $\sin^2 \theta \approx \theta^2$. Assuming that all particles that participate in sterile neutrino production are relativistic, the production rate can therefore schematically be written as

$$\Gamma_{\text{prod}} \sim G_F^2 \theta^2 T^k, \quad (3)$$

where $G_F = 1.16 \cdot 10^{-5} \text{ GeV}^{-2}$.

- a) Infer the exponent k from dimensional analysis by requiring that the right-hand side of the expression above has the correct mass dimension.
- b) In the lecture we showed that

$$Y_{\text{DM}} = \int_{x_{\text{max}}}^1 \frac{Y_{\text{DM},0}^{\text{eq}}}{x} \frac{\Gamma_{\text{prod}}}{H} dx.$$

Using $Y_N^{\text{eq}} \approx 1$ for $x < 1$ (i.e. $T > m_N$) and $H \approx T^2/M_p$ with $M_p \approx 1.22 \cdot 10^{19} \text{ GeV}$, show that

$$Y_{N,0} = \int_{x_{\text{max}}}^1 \frac{G_F^2 m_N^3 \theta^2 M_p}{x^4} dx, \quad (4)$$

where $x_{\text{max}} = m_N/T_{\text{max}}$ is the largest inverse temperature for which sterile neutrino production is efficient.

- c) In realistic sterile neutrino models, one has $m_N \ll 1 \text{ MeV}$ and $T_{\text{max}} \approx 300 \text{ MeV}$. Use these values to calculate

$$\Omega h^2 = \frac{s_0}{\rho_{c,0}} m_N Y_{N,0} \approx 280 \text{ keV}^{-1} m_N Y_{N,0}. \quad (5)$$

- d) What value of m_N gives $\Omega h^2 = 0.12$ for $\theta = 10^{-5}$?