

2.1 Freeze-out mechanism

Assume that at high temperatures ($x \ll 1$)

$$\frac{\Gamma_{\text{ann}}^{\text{eq}}}{H} > 1$$

$\Rightarrow$ DM is in thermal equilibrium: $Y_{\text{DM}} \approx Y_{\text{DM}}^{\text{eq}}$

Once DM becomes non-rel. ($x > 1$), $Y_{\text{DM}}^{\text{eq}}$ becomes exponentially suppressed

$$\lim_{x \rightarrow \infty} Y_{\text{DM}}^{\text{eq}} = 0$$

$\Rightarrow \Gamma_{\text{ann}}^{\text{eq}}$ is also exp. suppressed:

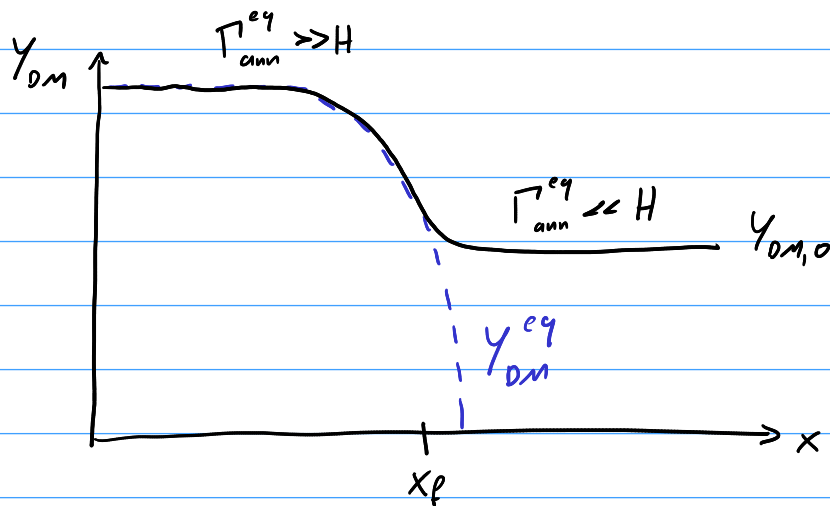
$$\lim_{x \rightarrow \infty} \frac{\Gamma_{\text{ann}}^{\text{eq}}}{H} = 0$$

$\Rightarrow$ Thermal equil. cannot be maintained

Freeze-out temperature T_f : $\left. \frac{\Gamma_{\text{ann}}^{\text{eq}}}{H} \right|_{T=T_f} = 1$

$$\Rightarrow n_{\text{DM}}^{\text{eq}} \Big|_{T=T_f} = \frac{H}{\langle \sigma v \rangle} \Big|_{T=T_f} = \sqrt{\frac{8\pi^3}{30}} \sqrt{g} \frac{T_f^2}{M_p \langle \sigma v \rangle}$$

$$\Rightarrow Y_{\text{DM}}^{\text{eq}} \Big|_{T=T_f} = \sqrt{\frac{45}{g_* \pi}} \frac{1}{M_p T_f \langle \sigma v \rangle} = \sqrt{\frac{45}{g_* \pi}} \frac{x_f}{M_p m_{\text{DM}} \langle \sigma v \rangle}$$



Freeze-out
mechanism

After freeze-out $Y_{DM} \approx \text{const}$

$$\Rightarrow Y_{DM}|_{T=T_0} = Y_{DM}^{eq}|_{T=T_f}$$

$$\Rightarrow S_{DM}|_{T=T_0} = m_{DM} \cdot \underset{\substack{\uparrow \\ \text{present-day entropy density}}}{s_0} \cdot Y_{DM}|_{T=T_0} = \sqrt{\frac{45}{g_* \pi}} \frac{x_f s_0}{M_P \langle \sigma v \rangle}$$

x_f depends only mildly on m_{DM} and $\langle \sigma v \rangle$

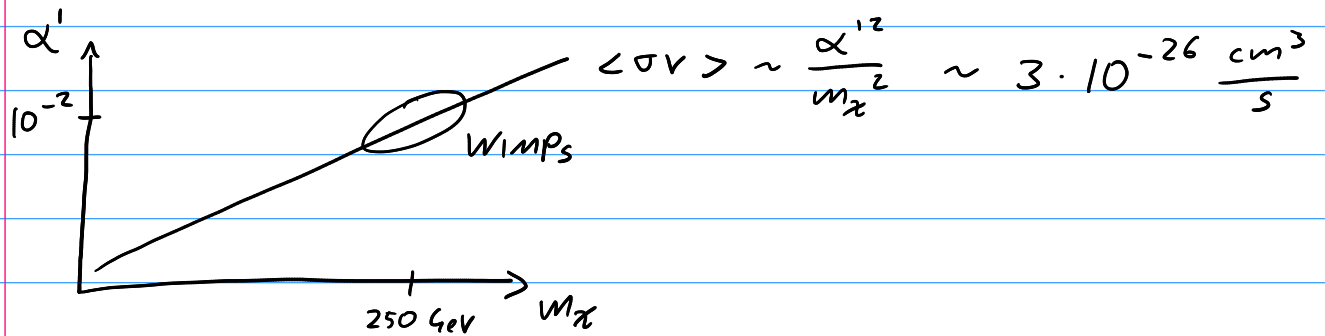
Typical values: $x_f \sim 20 - 30$

$$\Rightarrow S_{DM}|_{T=T_0} \sim \frac{1}{\langle \sigma v \rangle}$$

$$\text{CMB: } \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \approx 3 \cdot 10^{-26} \frac{\text{cm}^3}{\text{s}} \approx 10^{-3} \text{ GeV}^{-2}$$

2.2 Light dark matter

Freeze-out works also for different combinations of mass and couplings $\alpha' = \frac{g'^2}{4\pi}$



Unitarity bound: $\alpha' \lesssim 1 \Rightarrow m_\chi \lesssim 25 \text{ TeV}$

What is the lower bound on m_χ ?

Freeze-out must happen before BBN!

$$T_{\text{BBN}} \sim 100 \text{ keV}, \quad x_f = \frac{m_\chi}{T_f} \sim 30$$

$$T_f > T_{\text{BBN}} \Rightarrow m_\chi \gtrsim 3 \text{ MeV}$$

(almost model-independent)

Possible to have sub-GeV DM for $\alpha' \lesssim 10^{-5}$

How to realize $\alpha' \ll \alpha_w$?

↳ Need new type of interaction!

Example: New gauge boson

Idea: Extend SM gauge group
strong int. electroweak int. new!

$$\underbrace{SU(3)_c \times SU(2)_w \times U(1)_\gamma}_{SM} \times U(1)'$$

↑
gives rise to
dark photon A'

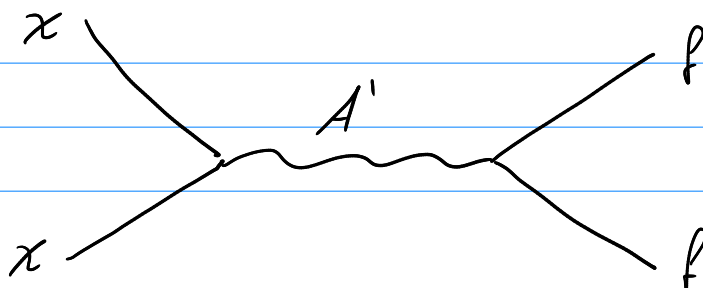
⇒ All particles can carry $U(1)'$ charges

$$\mathcal{L} = g' q_f \bar{f} \gamma^\mu f A'_\mu$$

dark gauge coupling charge fermion dark photon

⇒ Add DM particle χ

$$\mathcal{L} = g' q_\chi \bar{\chi} \gamma^\mu \chi A'_\mu$$



$\Rightarrow$ Gauge coupling g' is free parameter
 $\hookrightarrow$ can vary by orders of magnitude
 $\hookrightarrow$ possible to have $\alpha' = \frac{g'^2}{4\pi} \ll \alpha_w$

$\Rightarrow$ DM stability consequence of dark charge conservation

Problem: Searches for new gauge interactions imply $g' q_{sm} \lesssim 10^{-4}$

If $q_{sm} \sim q_{DM} \sim \mathcal{O}(1)$

$\Rightarrow g' \lesssim 10^{-4} \Rightarrow \alpha' \lesssim 10^{-9} \Rightarrow m_\chi \lesssim 25 \text{ keV}$
 $\hookrightarrow$ incompatible with BBN

Solution: Need $q_{sm} \ll q_{DM}$! Unnatural?

Key idea: Kinetic mixing $\mathcal{L} = \frac{\epsilon}{2} F^{\mu\nu} F'_{\mu\nu}$

$\nearrow$
 field-strength tensor
 $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$



$\Rightarrow$ Dark photon obtains couplings of SM photon

$$\Rightarrow (g' q_{SM})^{\text{eff}} = \underset{\substack{\uparrow \\ \text{direct charge}}}{g' q_{SM}} + \underset{\substack{\uparrow \\ \text{electric charge}}}{\epsilon \cdot e}$$

For $q_{SM} = 0$ and $\epsilon \ll 1$ possible to hide dark photons

$$\Rightarrow \langle \sigma v \rangle \sim \frac{\alpha' \epsilon^2 \alpha_{em}}{m_\chi^2}$$

Example: $\alpha' = 0.1$, $\epsilon = 10^{-4} \Rightarrow m_\chi \sim 10 \text{ MeV}$