

Simulating Microwave SQUID Multiplexers Using Coupled-Mode Theory

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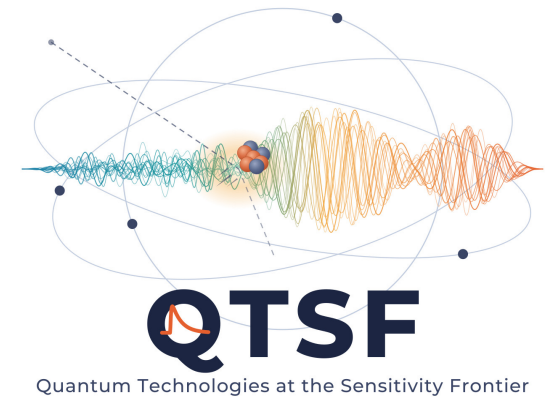
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Many cryogenic detector experiments need thousands of readout channels

Target mass: thresholds rise with mass, so large targets split into many small detectors

e.g. dark matter, CEvNS, $0\nu\beta\beta$

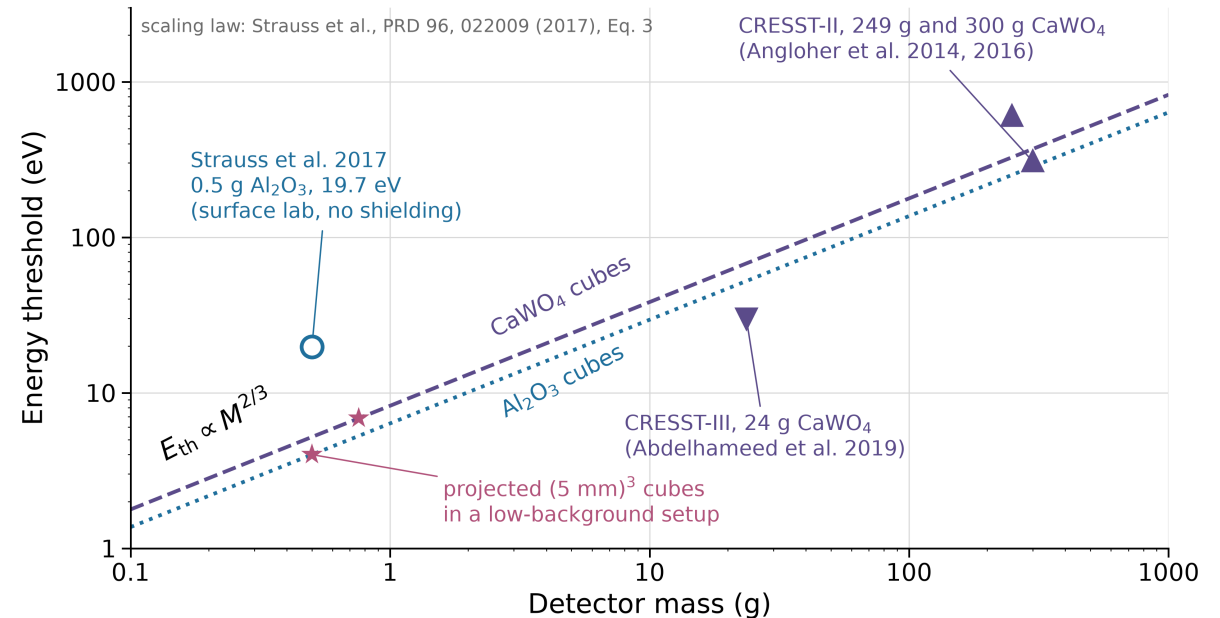
Imaging: cameras need one pixel per point in the image

e.g. CMB, mm/sub-mm astronomy

Count rate: pile-up limits the events each pixel can handle, so high statistics need many pixels

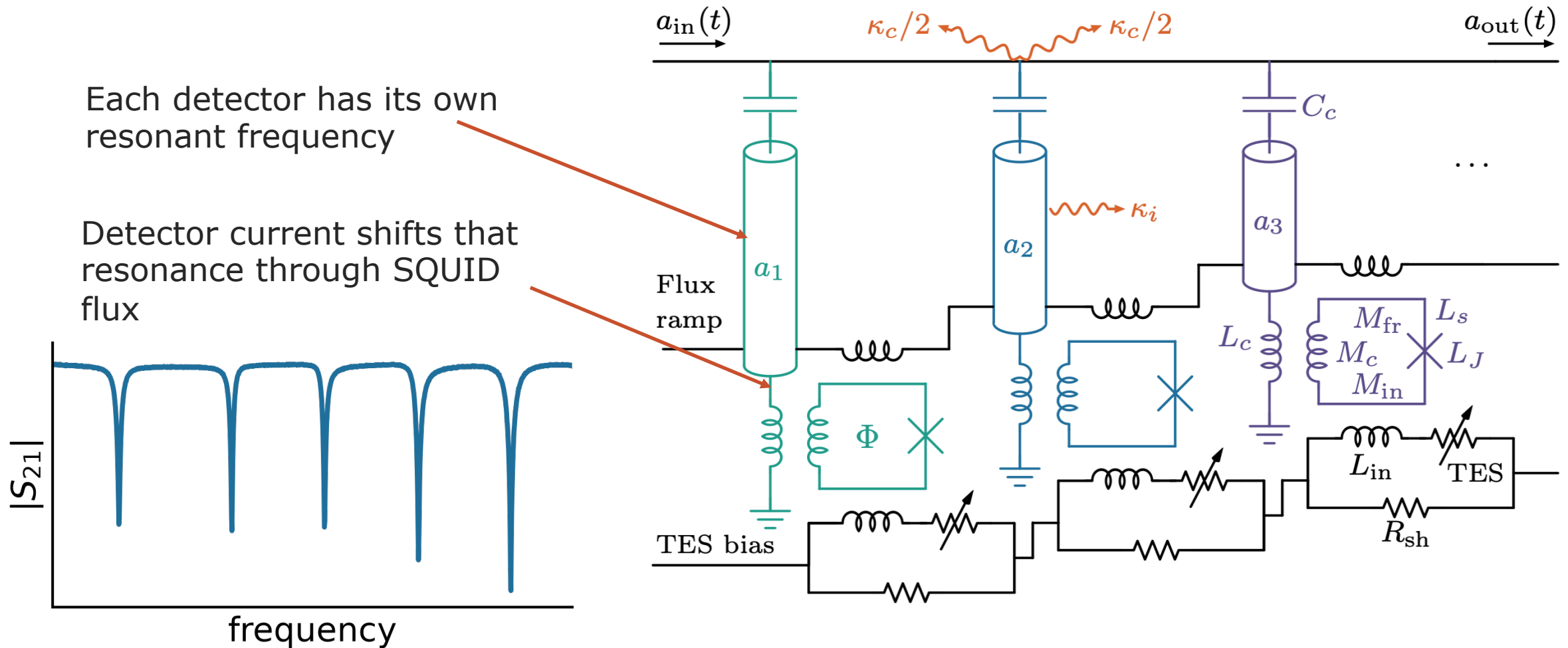
e.g. neutrino mass, X-ray spectroscopy

All three lead to 1000+ channels, too many to wire out individually

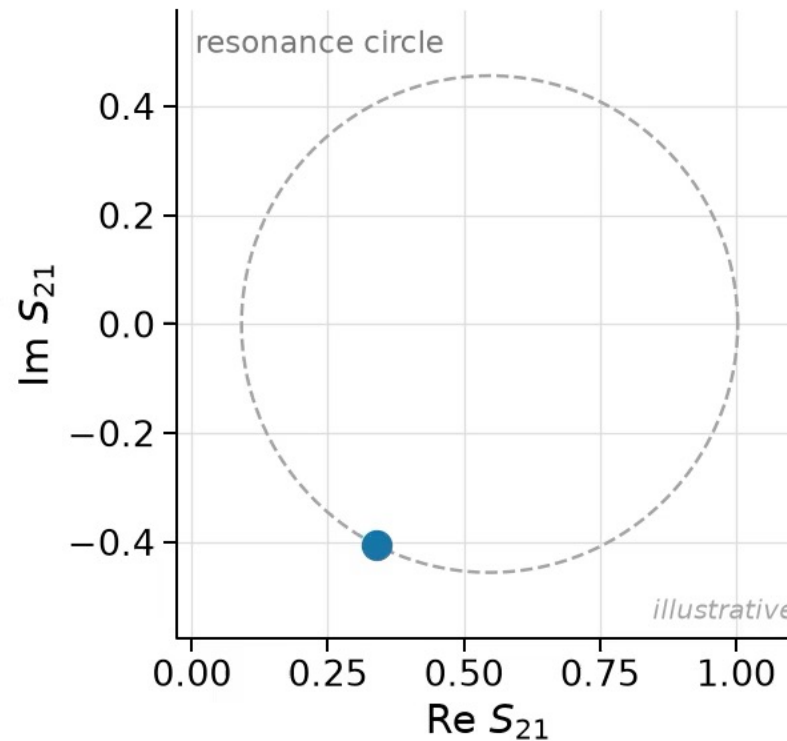
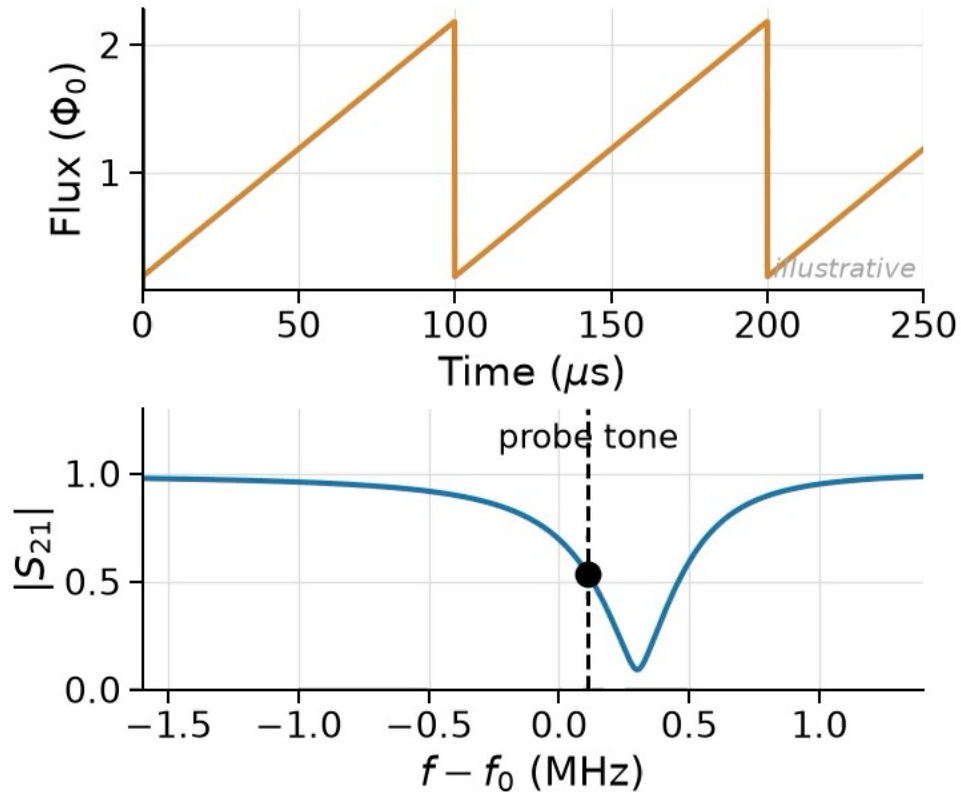


Energy threshold vs. detector mass; scaling law after Strauss et al. (2017), with CRESST data points

Each detector gets its own resonance, tuned by SQUID flux



The flux ramp modulates every channel



$$f_{\text{mod}} = n\Phi_0 \cdot f_{\text{ramp}}$$

Typically, 1-3 Φ_0 per ramp

Cycles through **all flux sensitivities**

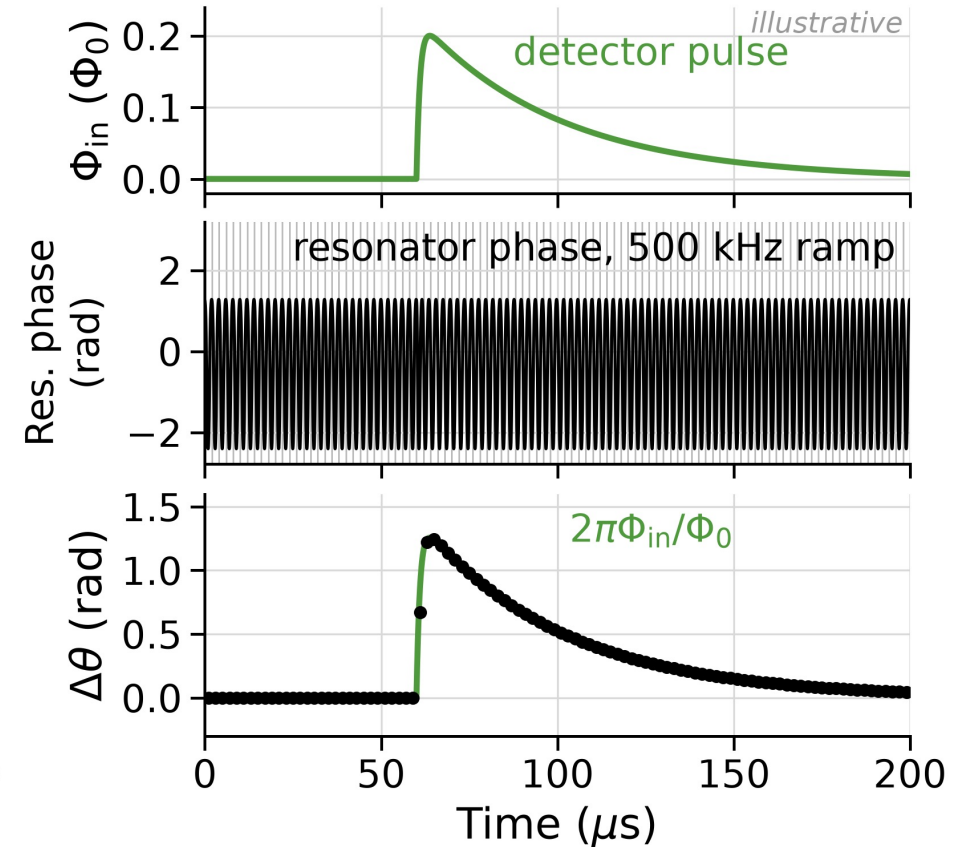
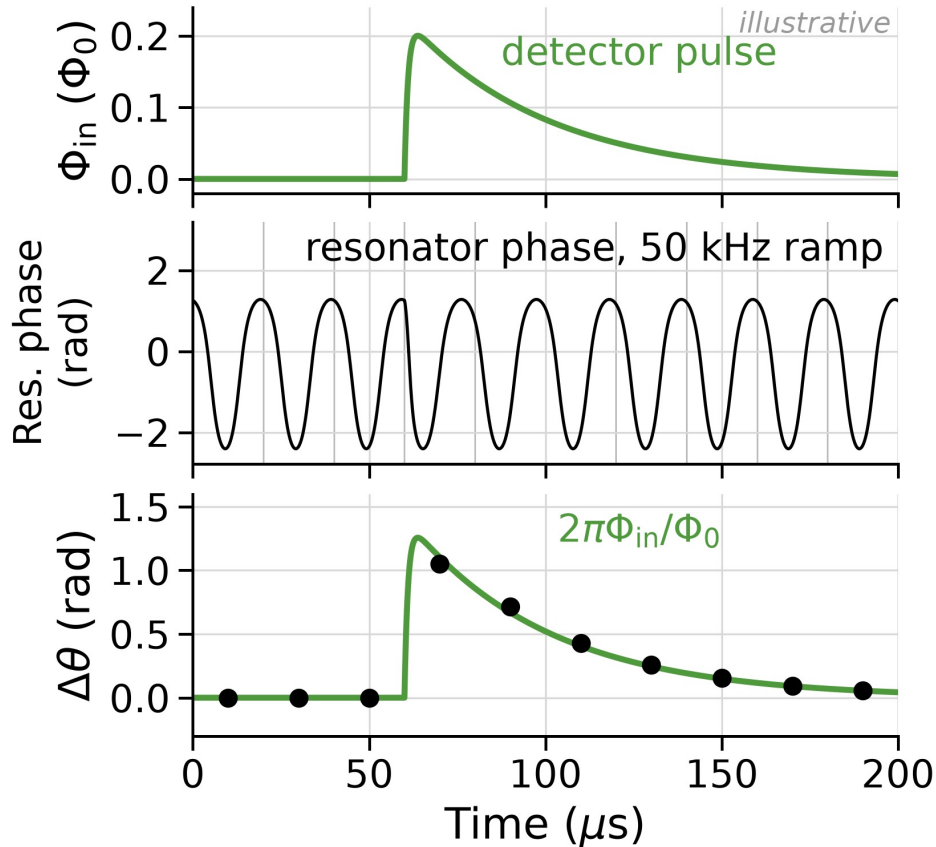
f_{mod} **sets effective sampling speed**

Detector signals shift the phase of that modulation

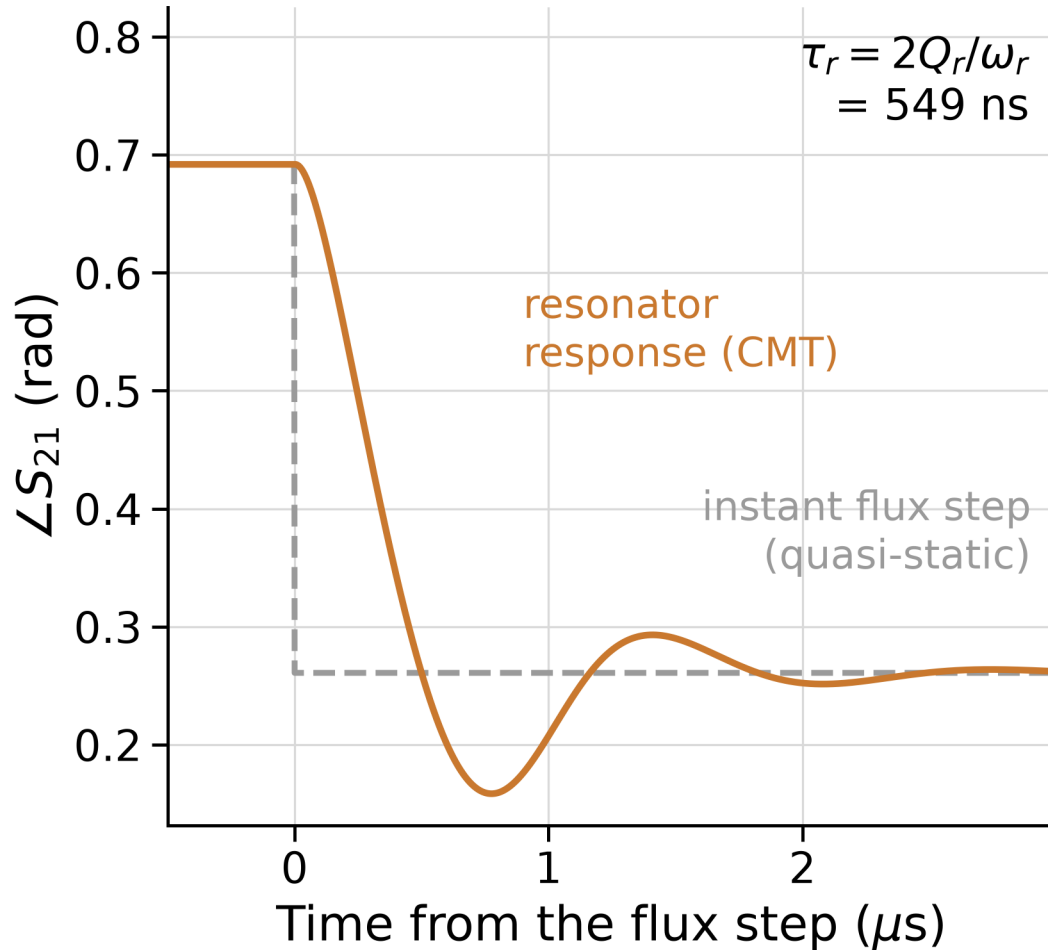
$\Delta\theta$ = phase of modulated signal

Modulation rate = sample rate

Real signals **1/100** the amplitude



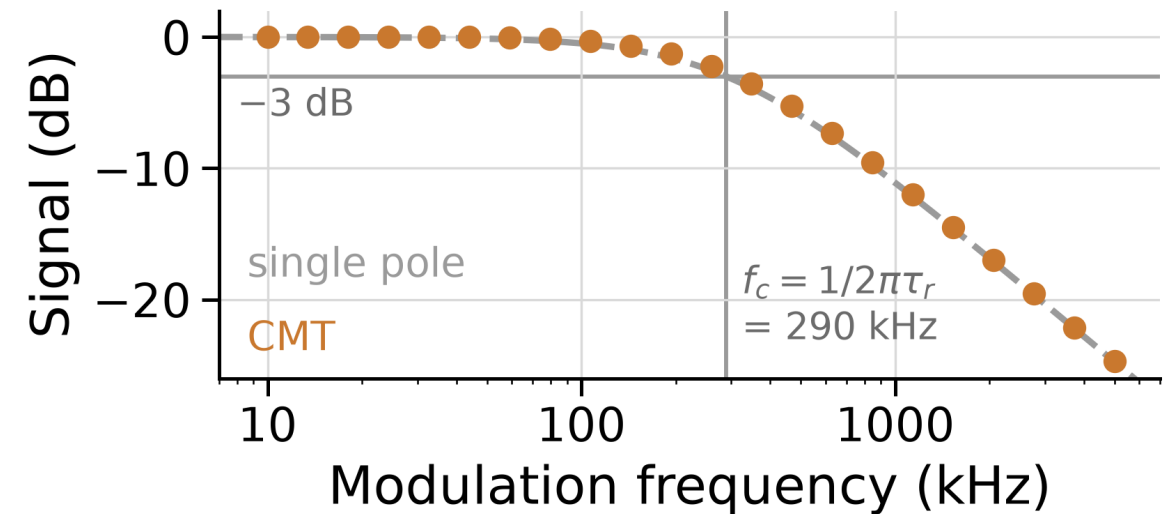
Limit 1: Resonator ring-up



Photons take time to enter/leave:

$$\tau_r = \frac{2Q_r}{\omega_r}$$

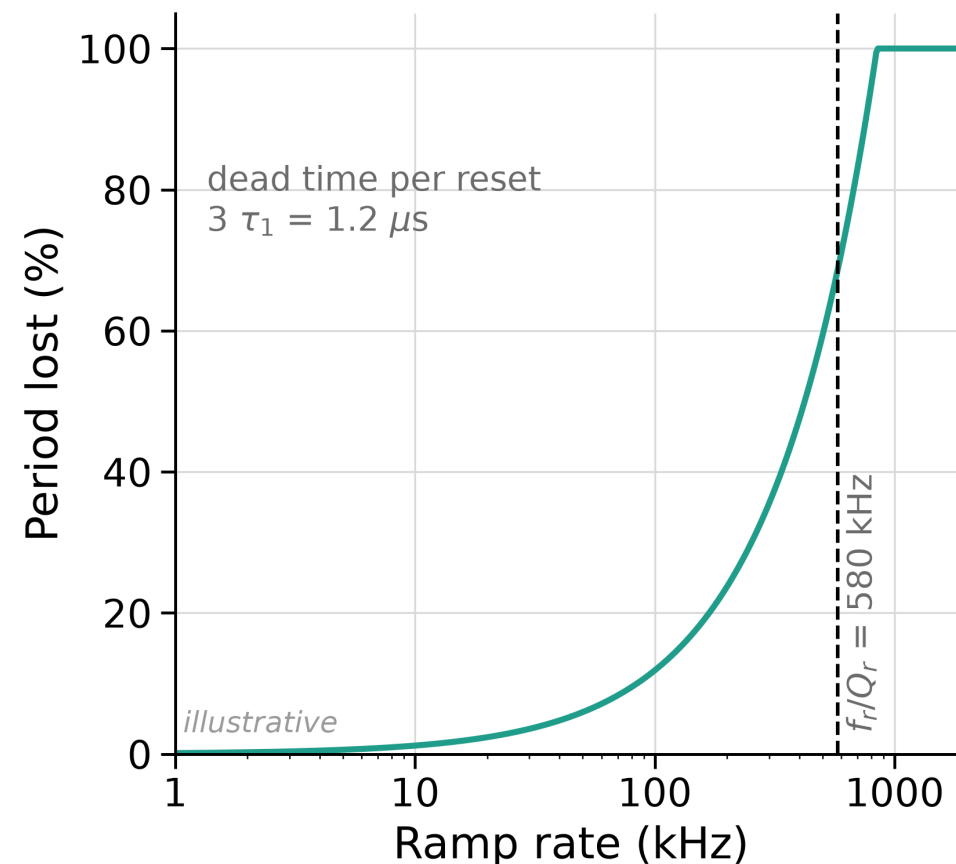
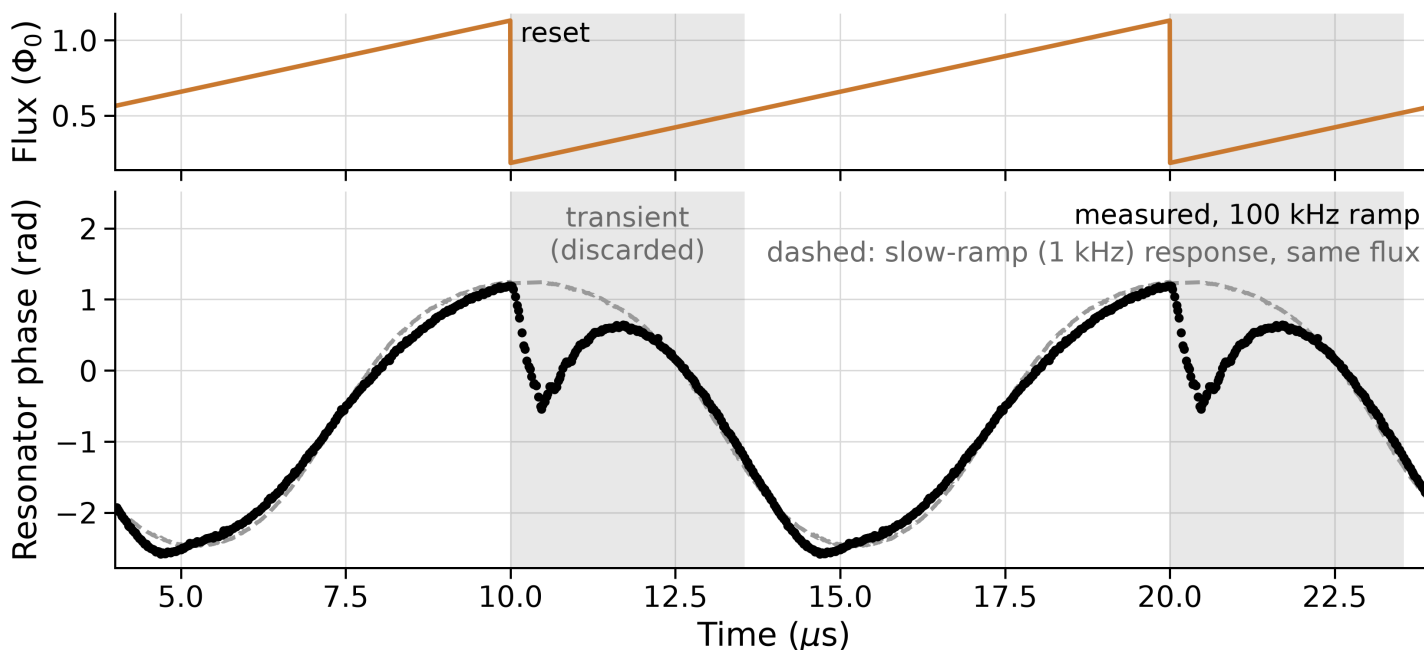
Signals are **attenuated above BW**



Limit 2: Ramp reset transients

Non-ideal ramp causes transients at reset

Typically discarded, so **ramp rate is limited**



Simulation can guide design and readout choices, but no data-tested transient simulator

Approach	Time-domain?	Resonator dynamics?	Flux line?	Compared to measured transients?	Open source?
Full-wave / multiphysics (Elkin, Roth)	No	No	No	No	No
Steady-state (Wegner, Neidig)	No	No	No	No	No
Quasi-static time-domain (e.g., SMuRF, Yu et al.)	Yes	No	No	No	Readout only
First-order dynamic (Schuster)	Yes	Yes	Ideal Butterworth	No	No
This work	Yes	Yes	Yes	Yes	Yes



Equation of Motion from Coupled Mode Theory (CMT)

$$\frac{da_n}{dt} = \underbrace{-i\Delta(t)a_n(t)}_{\text{Detuning}} \underbrace{- \frac{\kappa_n}{2}a_n(t)}_{\text{Loss}} + \underbrace{\sqrt{\frac{\kappa_{c,n}}{2}}a_{\text{in}}(t)}_{\text{Drive}}$$

Detuning

Drive tone offset from resonance

$$\Delta(t) = \omega_d(t) - \omega_r(t)$$

Loss

Feedline coupling plus internal loss

$$\kappa_c = \omega_0 / Q_c \quad \kappa_i = \omega_0 / Q_i$$

Drive

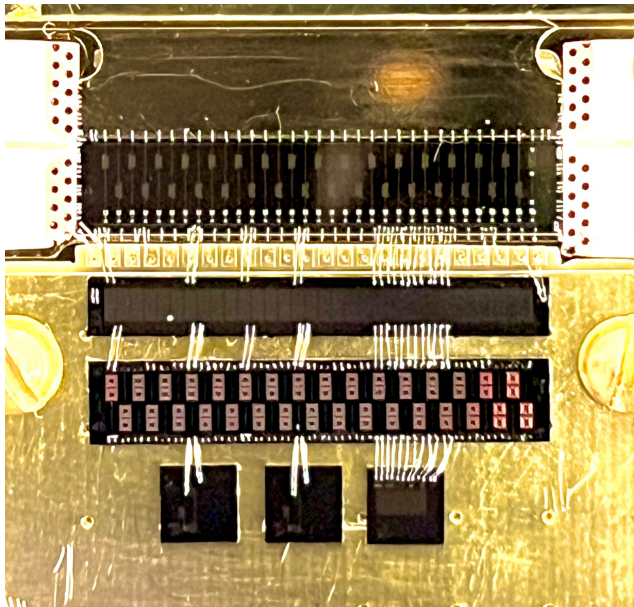
Feedline input via the coupler

$a_{\text{in}}(t)$: incident field

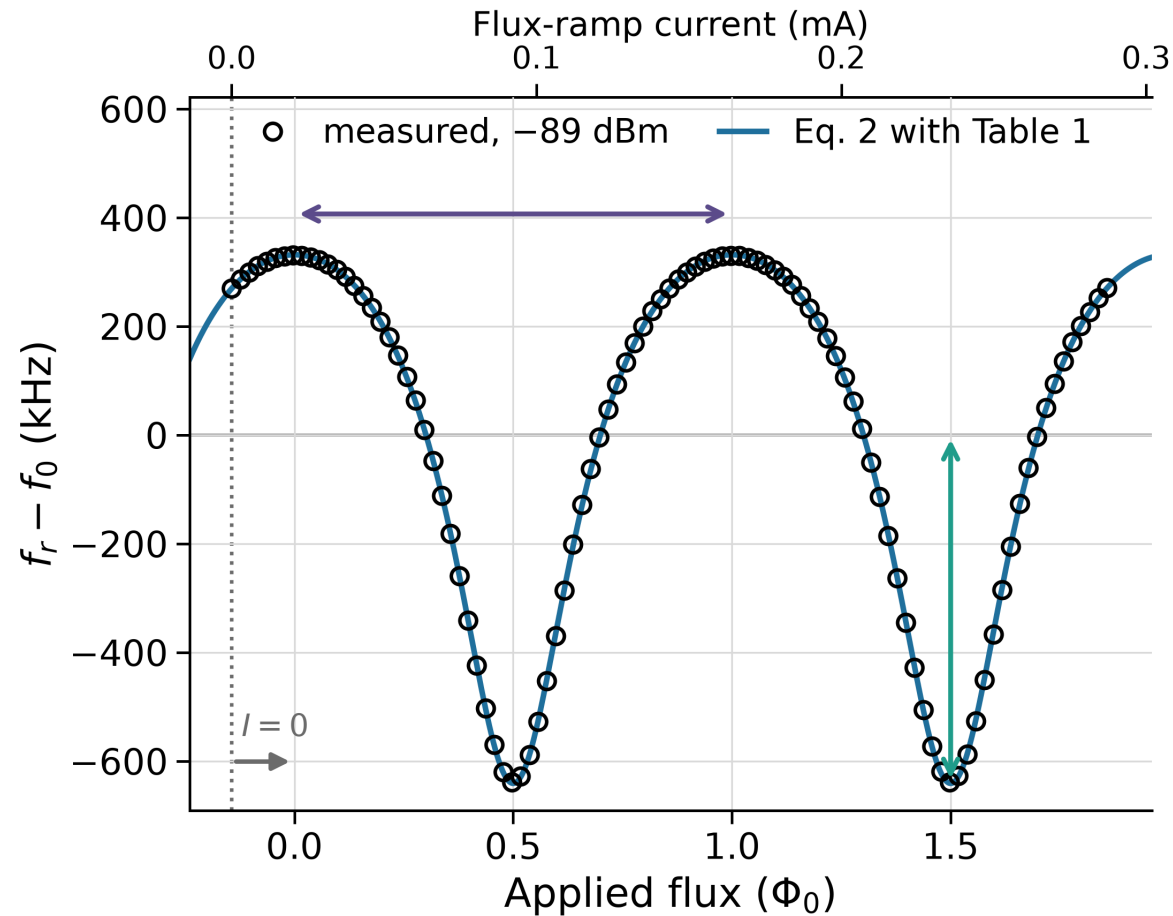
Measured transmission

$$S_{21}(t) = a_{\text{out}}(t) / a_{\text{in}}(t)$$

A VNA sweep and a slow flux scan characterize each channel



32-channel multiplexer
fabricated by NIST



SQUID: flux curve, Eq. 2

$M_{fr} = 14.409$ pH period

$M_c = 3.695$ pH depth

$\lambda = 0.318$ shape

$\Phi(I = 0) = -0.14 \Phi_0$ offset

$f_0 = 5515.689$ MHz bare f_r

Resonator: circle fits

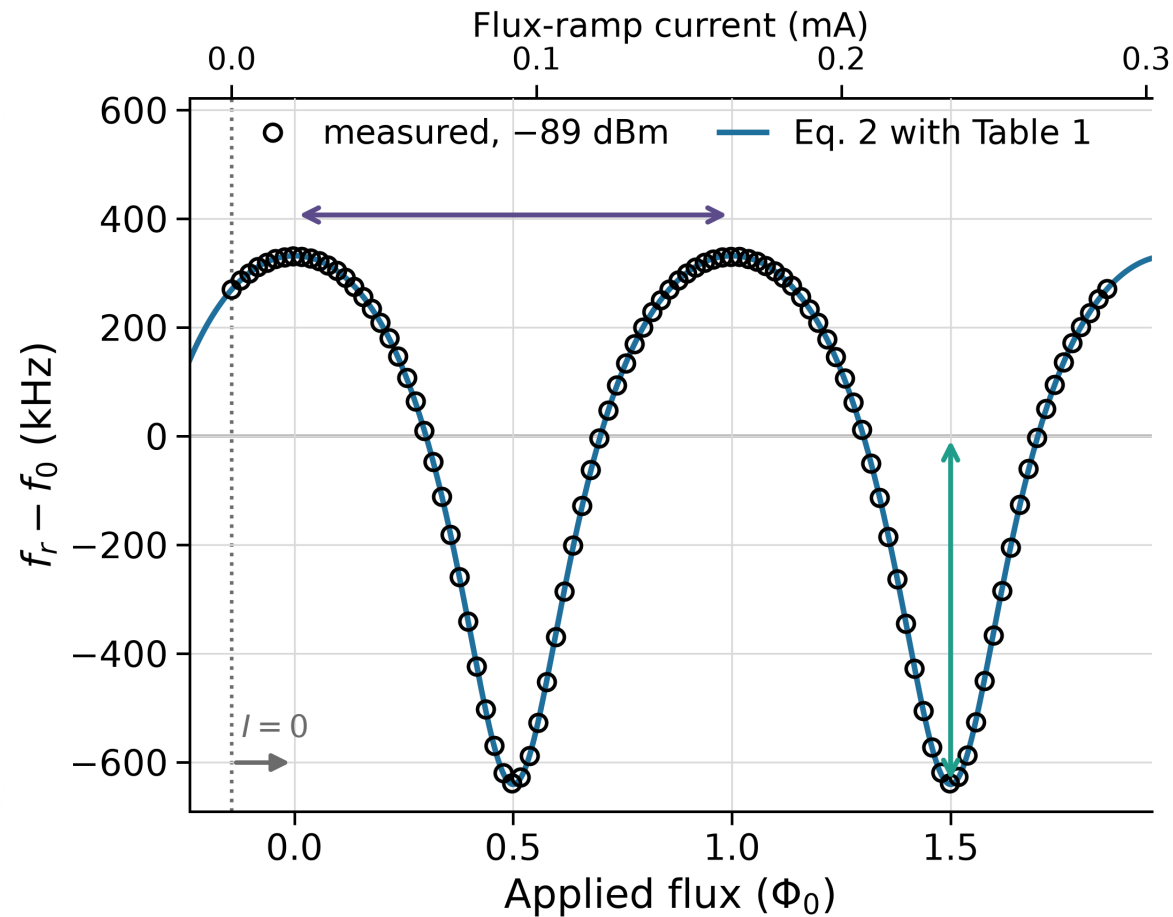
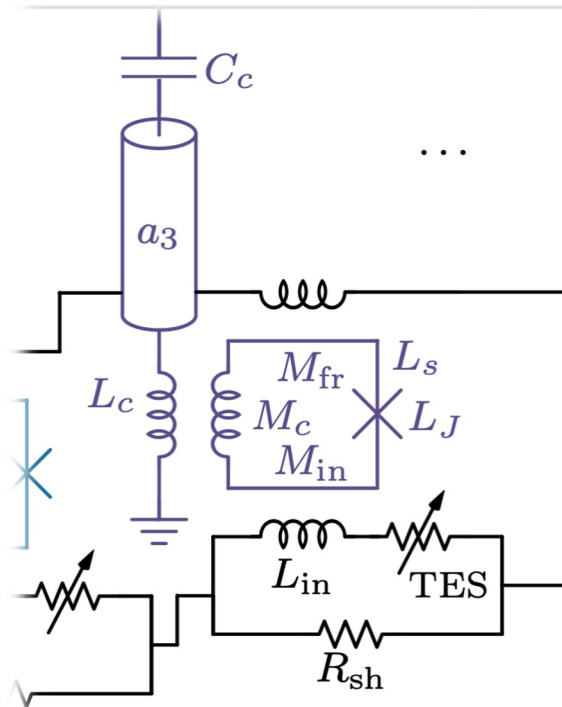
$Q_r = 9515$ loaded

$Q_c = 10475$ coupling

$Q_i = 103803$ internal

$\alpha = -0.086$ rad asymmetry

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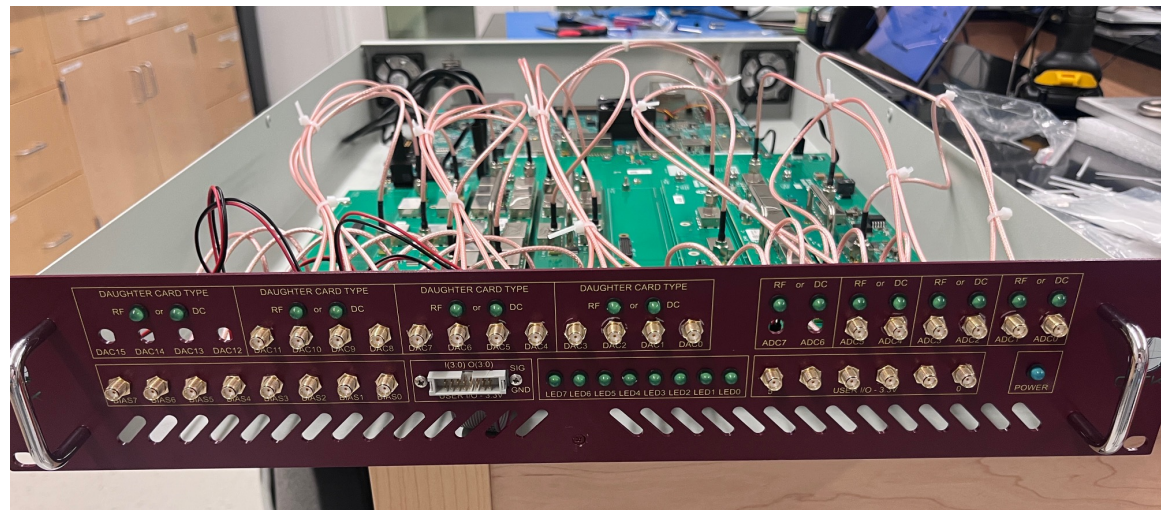
QICK-enabled ZCU216 used for probe tone and flux ramp

QICK on a **ZCU216 RFSoc** drives all inputs

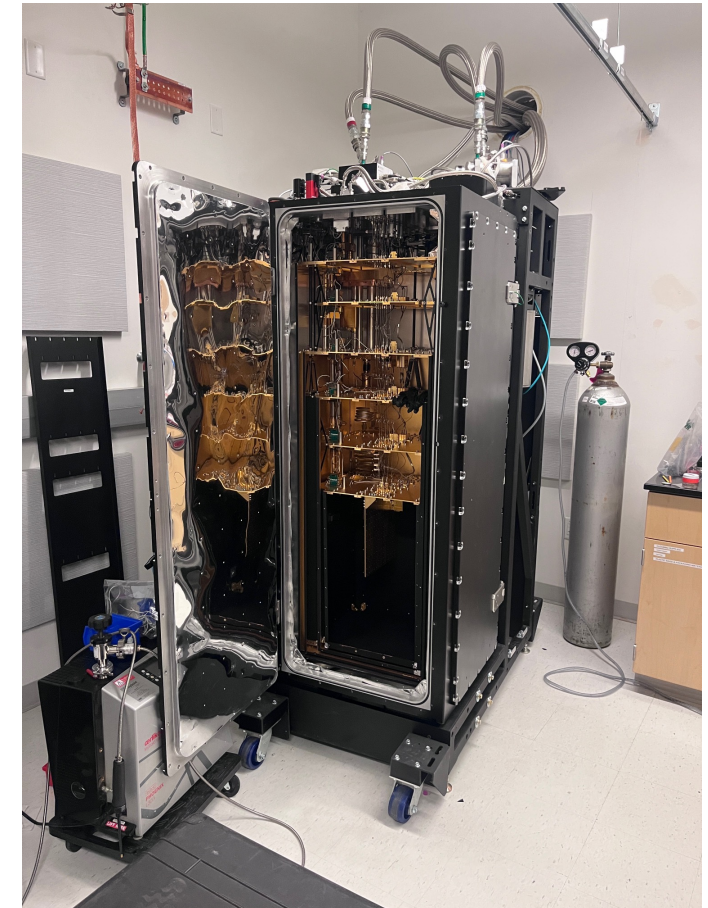
Probe tones: RF balun daughter board

Flux ramps: DC board

One low-pass filter (225 MHz) on the flux-ramp line



QICK box: ZCU216 RFSoc with balun and DC boards



Maybell DR used for these measurements

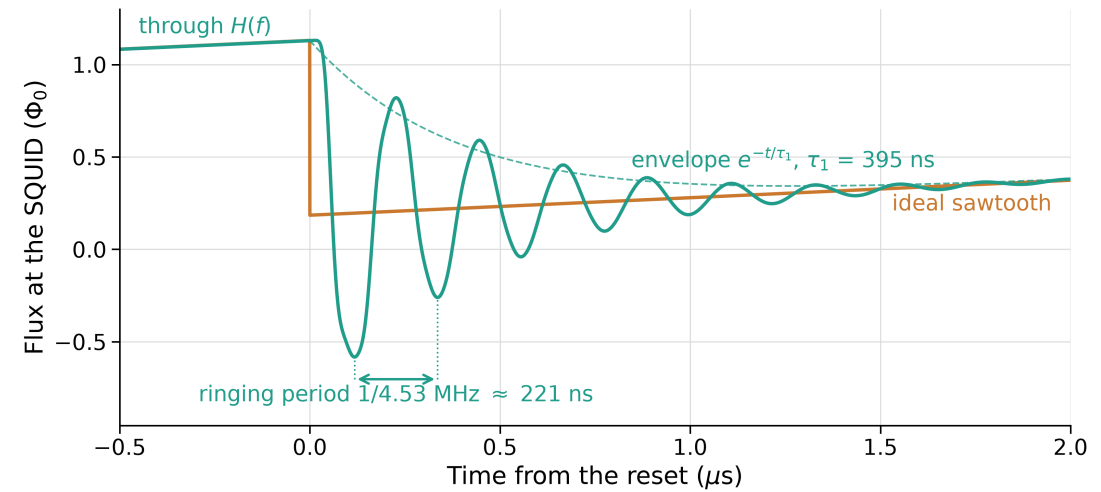
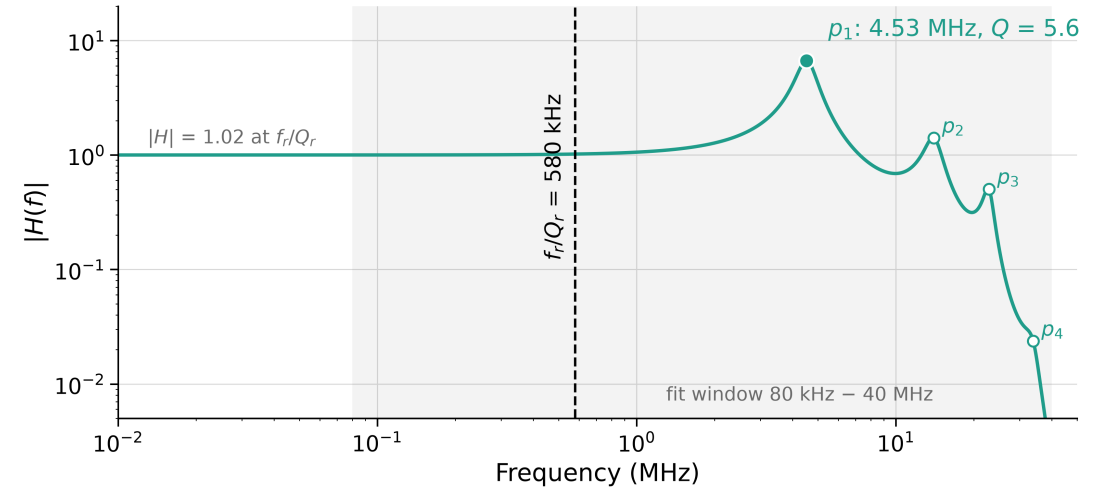
Flux lines often have MHz resonances and roll-off

Measured once with **VNA**, not fitted to any IQ data

Poles fitted to build **transfer function** $H(f)$

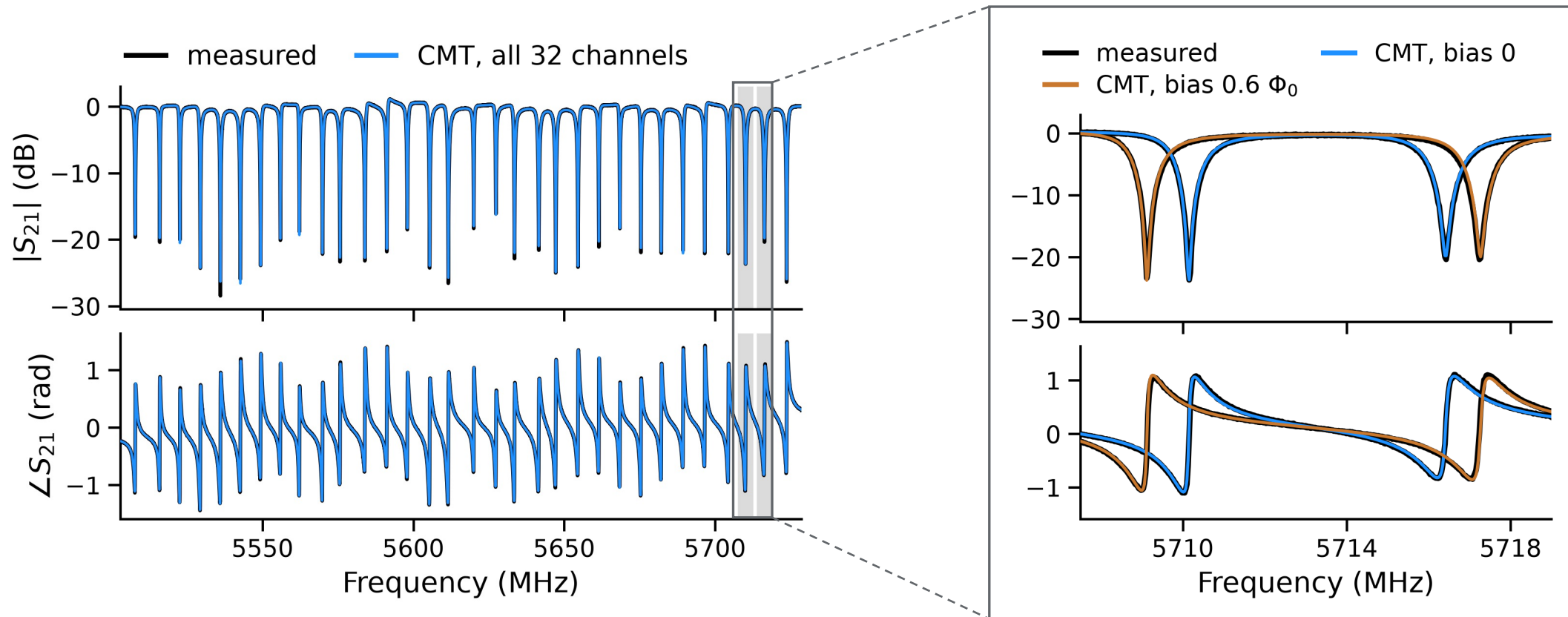
Dominant Resonance: **4.53 MHz**

Ringings seen at ramp reset **sets transient response**



Top: measured $|H(f)|$. Bottom: predicted flux at the SQUID (not measured)

CMT model matches measured S21 on all 32 channels



Full spectrum (32 channels $\times$ 10,000 frequencies): **~ 1.5 min**

Single resonator (1 channel $\times$ 351 frequencies): **0.2 s**

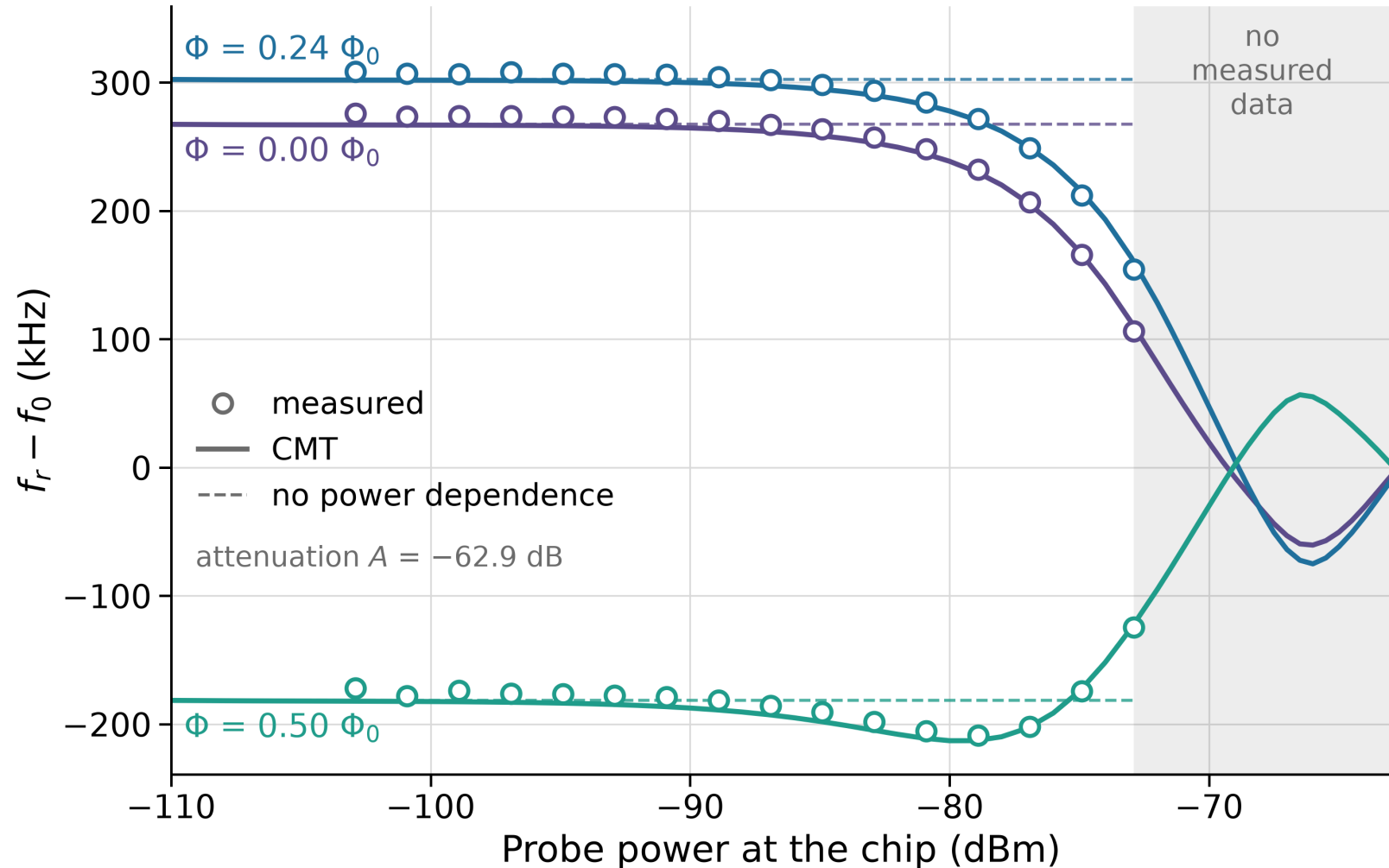
Low-power characterization predicts behavior at operating power

Probe power adds **RF flux** into SQUID, shifting the resonance.

The computes this from **energy stored in the resonator**

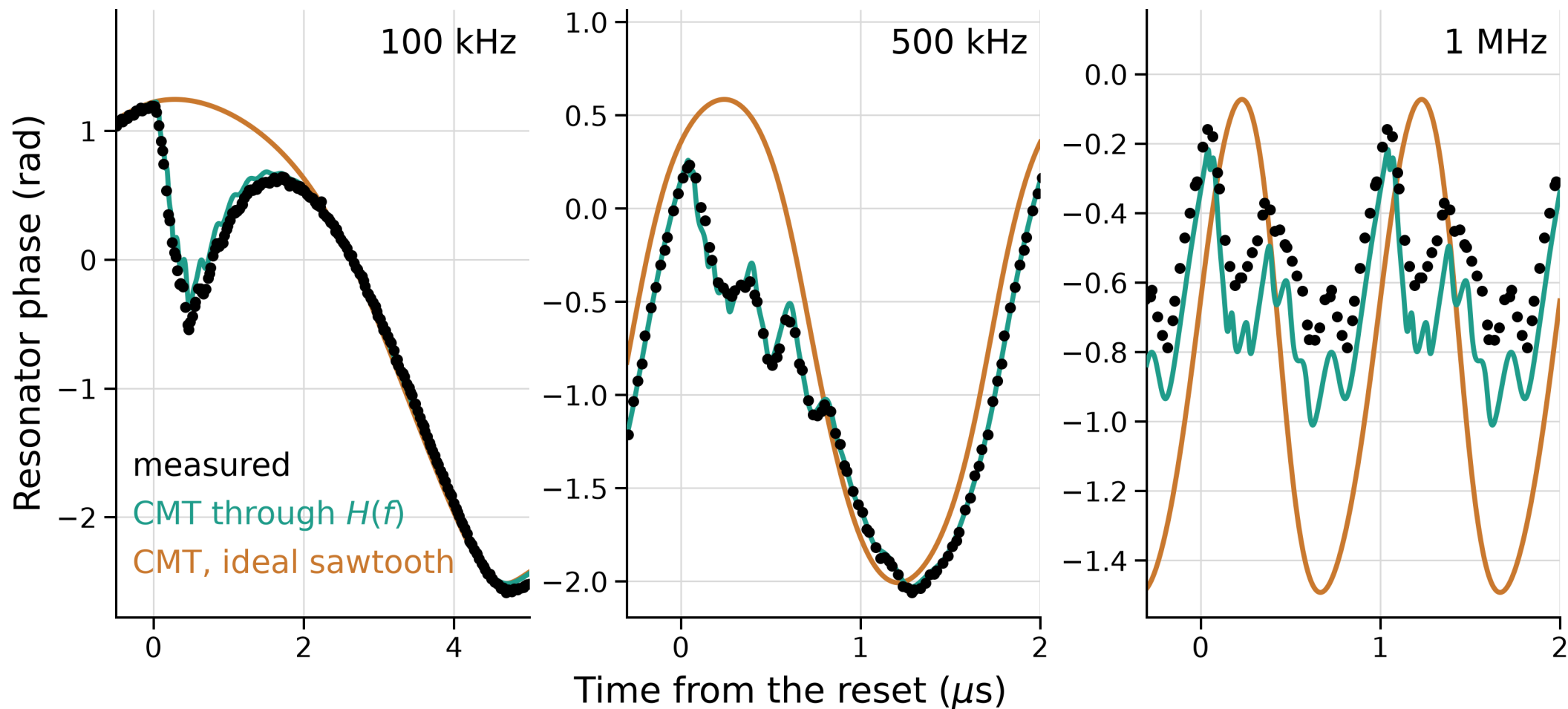
One fitted parameter: A

Power sweep (171 points, 60,021 solves): **~45 s**



Resonance shift vs. probe power at three flux biases; model uses parameters from one low-power sweep

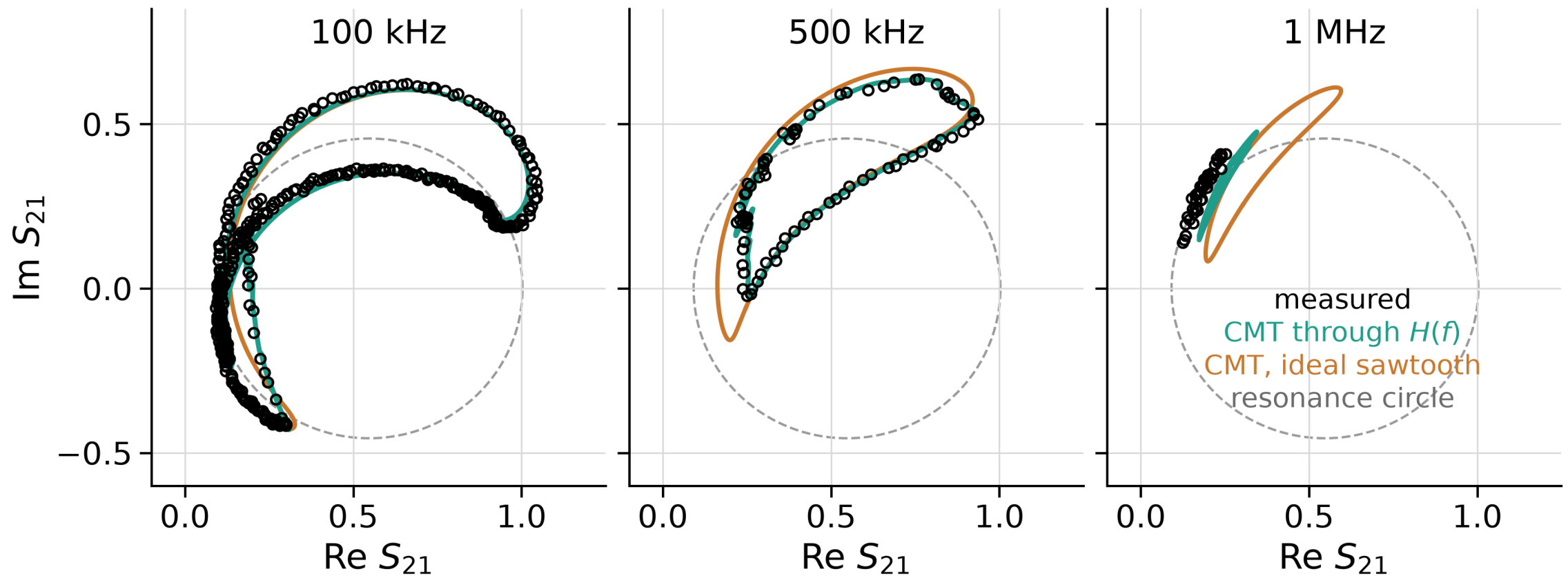
With quasi-static calibration, model predicts resonator dynamics above resonator BW



Measured resonator phase after each ramp reset; calibration held fixed from the 1 kHz record

Simulating 5 μs per trace: **~ 0.5 s**

With quasi-static calibration, model predicts resonator dynamics above resonator BW

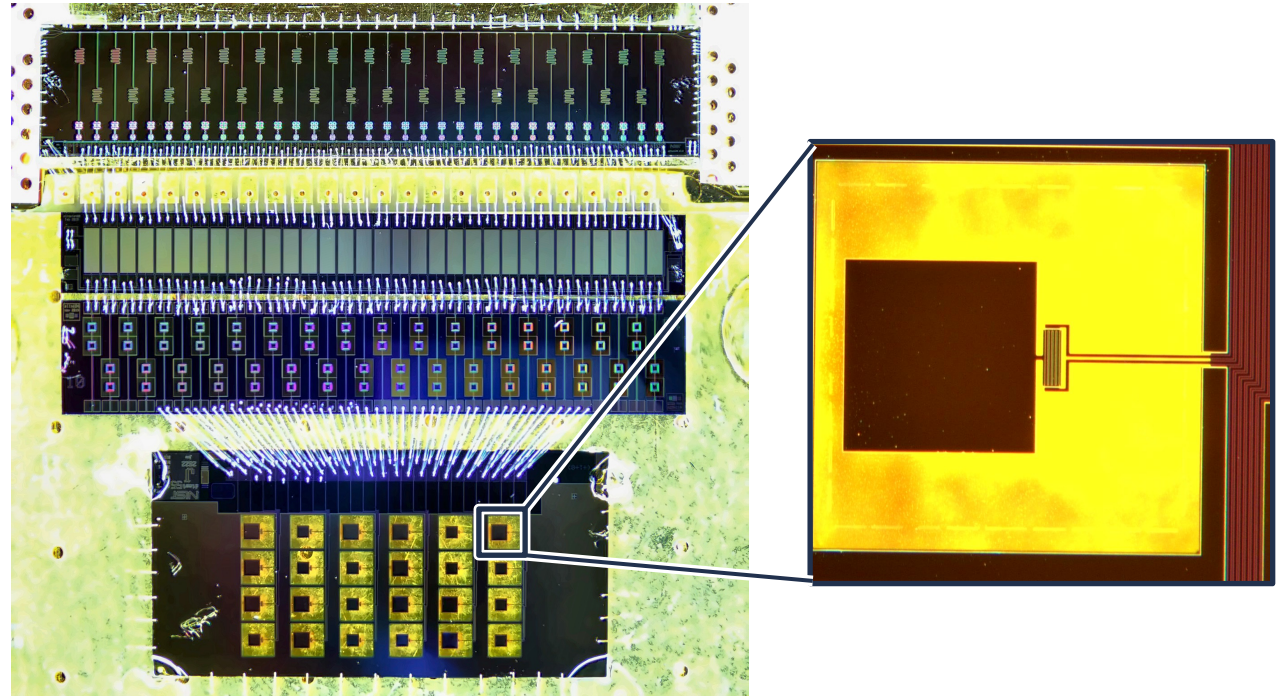


The same records in the complex plane ($\text{Re}/\text{Im } S_{21}$); gray: resonance circle from the VNA fit

Simulating 5 μs per trace: **~ 0.5 s**

Future work

- **TES readout** with NIST uMux: sidecar TES chip from NIST, Argonne TES modular readout
- **Flux-ramp correction** by pre-inverting $H(f)$ to minimize transients
- Incorporating **noise models** into the simulation



Conclusion

1 Validated against data

The CMT model reproduces **steady-state and flux ramp responses** using only VNA characterization.

2 Flux line sets the transient

In this system, the post-reset transient is dominated by the **flux line (4.53 MHz)**, not the resonator bandwidth (580 kHz).

3 Predictable, so designable

Because the transient is predictable, **it can be accounted for in design.**

arXiv paper to be released in the next week, along with open-source code



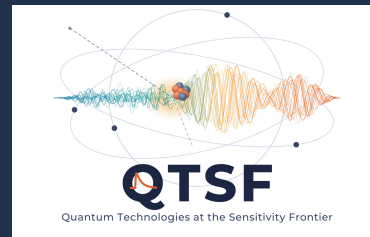
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Questions?

Paper and simulation code to be released in the next week!

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→ <https://qtsf.mines.edu>



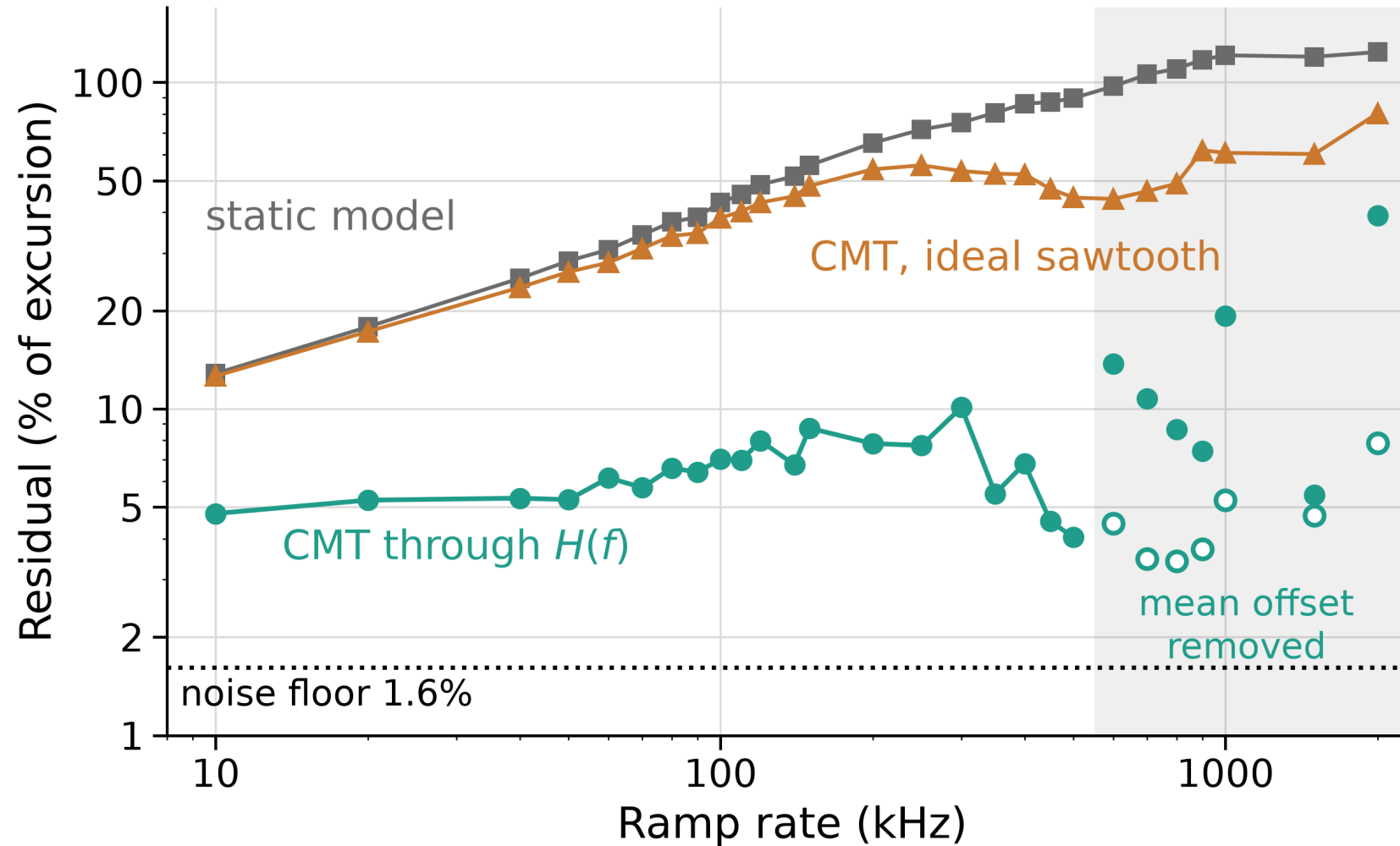
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Supplemental Slides

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Error over different ramp rates



Transfer function VNA measurement

Two-port full **S-matrix measurement** on FR input ports

Extract differential impedance using S-parameters

Fit rational function, assuming a pole-only function

Build transfer function from fitted poles

$$S_{dd} = \frac{1}{2} (S_{11} - S_{12} - S_{21} + S_{22})$$

$$Z_{dd} = 2Z_0 \frac{1 + S_{dd}}{1 - S_{dd}},$$

$$H(s) = \prod_k \left(1 - \frac{s}{p_k} \right)^{-1}$$