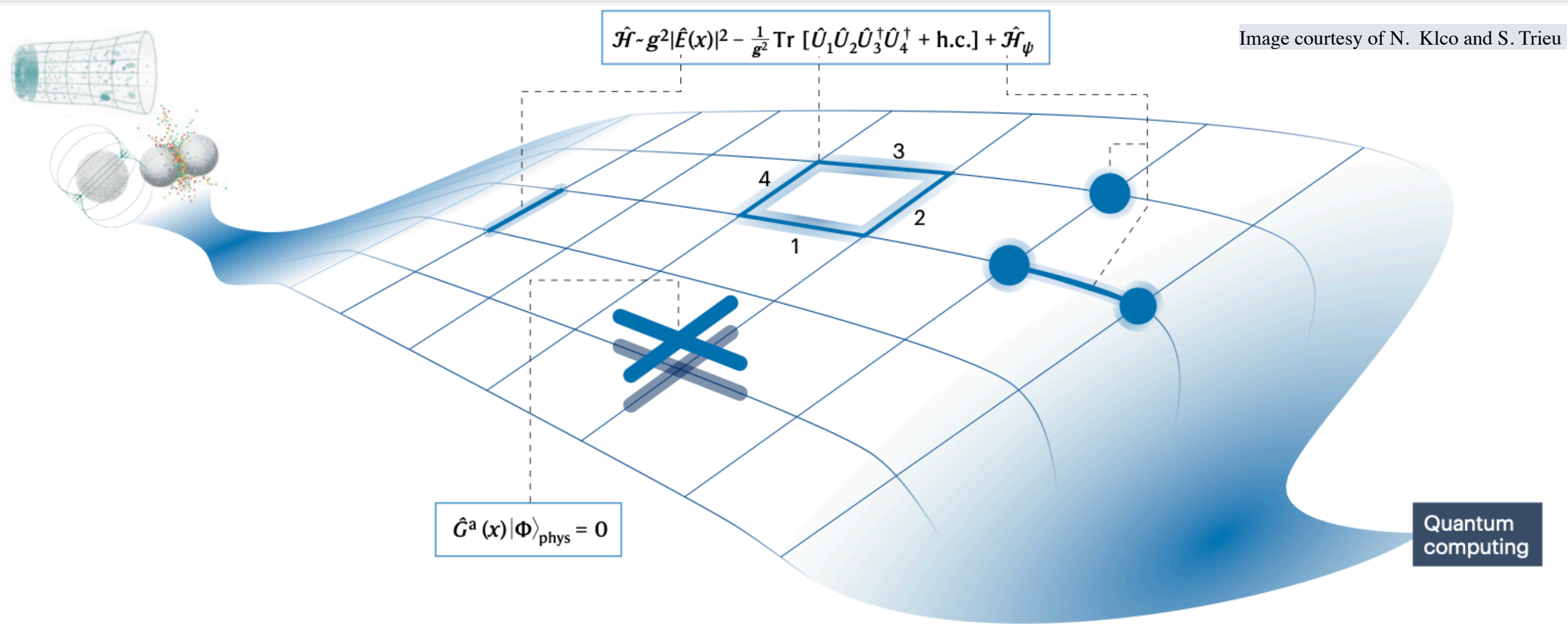


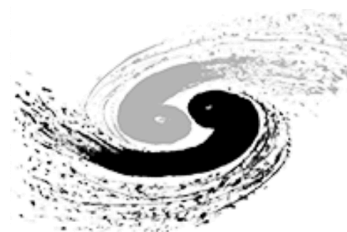


Improved framework for quantum simulation of **bosonic** QFT

— quantum simulations for HEP



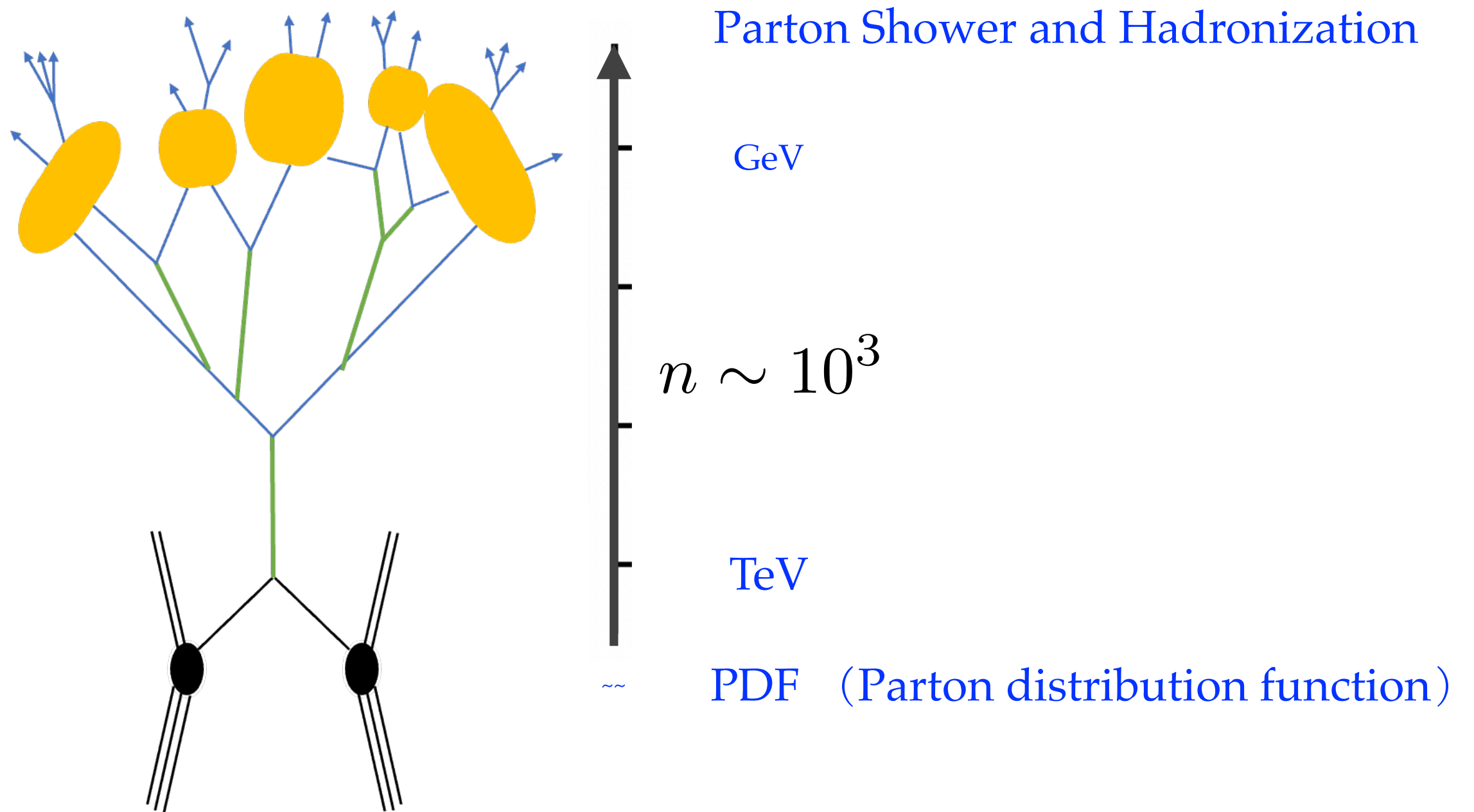
Ying-Ying Li



Institute of High Energy Physics
Chinese Academy of Sciences

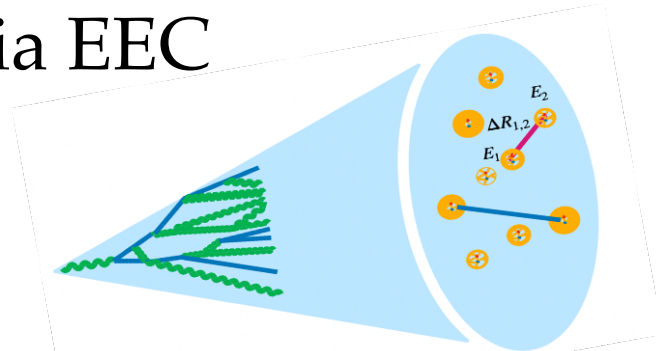
Real-time dynamics of QCD

At large hadron colliders— zero temperature

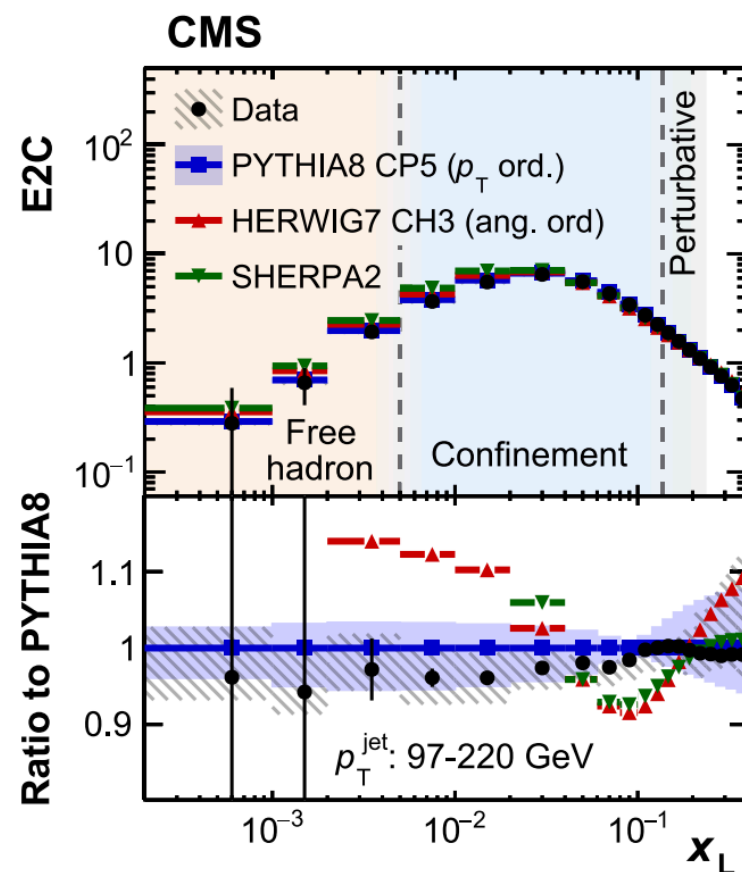


Rich non-perturbative phenomenon

- ❖ Powerful probe of confinement, e.g. via EEC

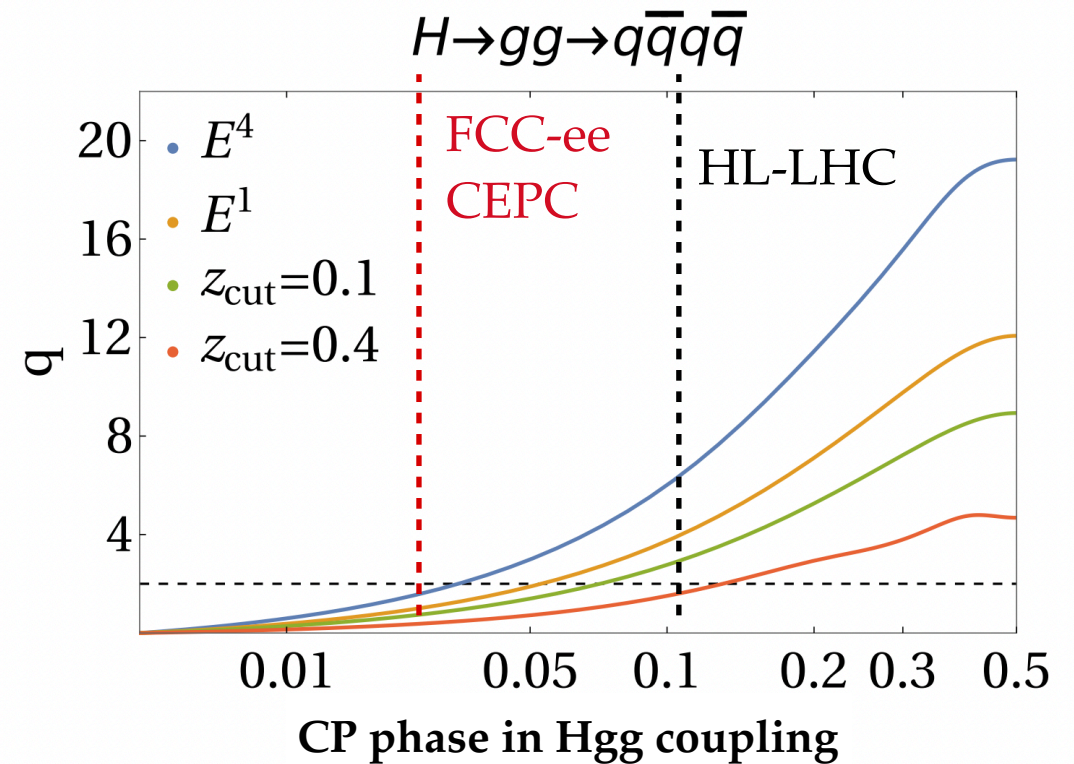
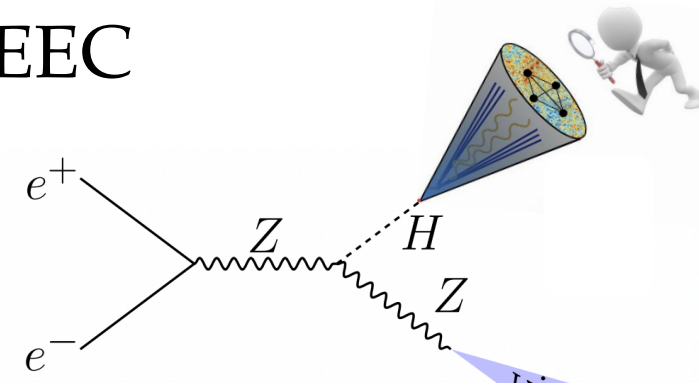


[Phys. Rev. Lett. **133**, 071903]



- ❖ Large uncertainties from Pythia, etc..

- ❖ Precision measurements of SM, e.g. via EEEEC



[Wang, Gao, [YYL](#), Zhu, arXiv: 2601.22248]

- ❖ Results from leading-order approximation

First-principles calculation to fully include correlation information in the evolution of parton to hadron is missing...

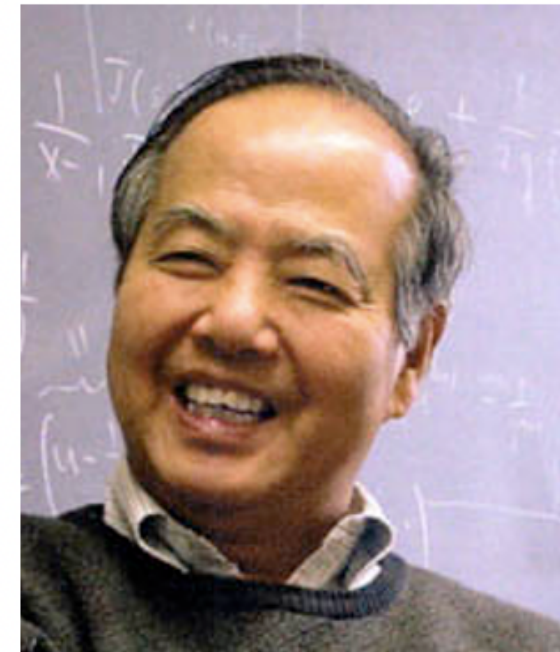
Nuclear Physics A538 (1992) 3c-14c
North-Holland, Amsterdam

**NUCLEAR
PHYSICS A**

**MISSING SYMMETRIES, UNSEEN QUARKS
AND THE PHYSICAL VACUUM**

T. D. Lee

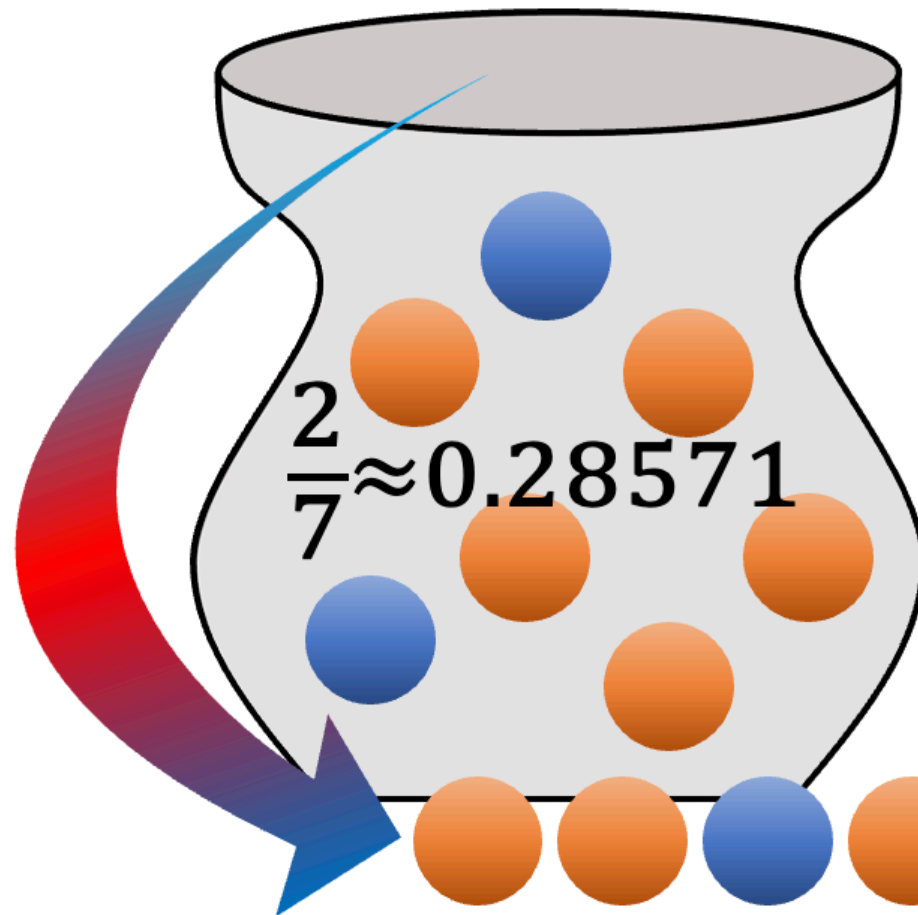
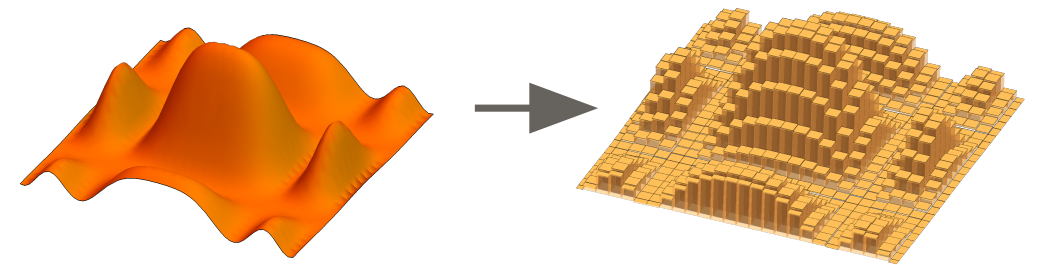
Columbia University, New York, N.Y. 10027



How color confinement happens?
outstanding problem in physics at the end of the
twentieth century

Lattice QCD - classical methods

Euclidean spacetime
(Monte Carlo methods)



$$\frac{1708}{5979} \rightarrow$$

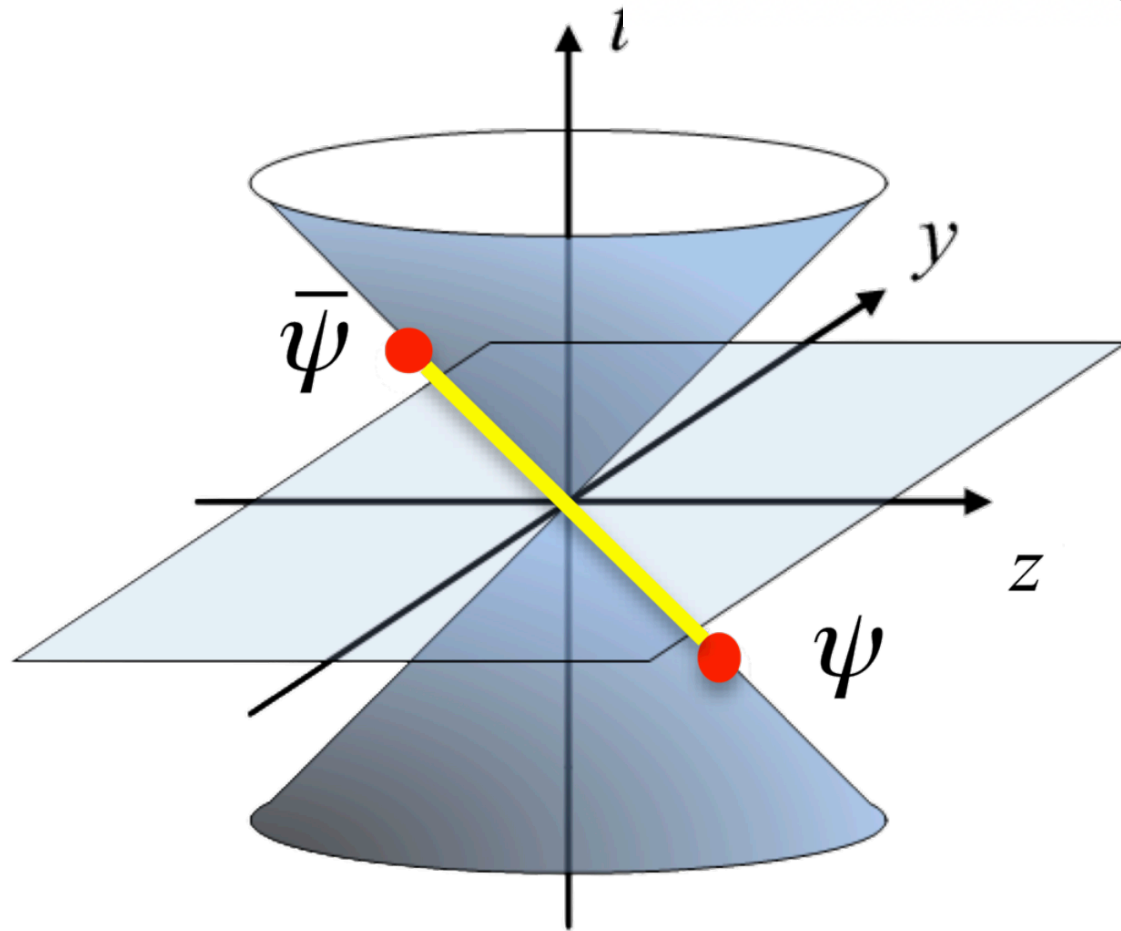
$$\frac{\sum \exp(-S_{\text{blue}})}{\sum \exp(-S_{\text{blue}}) + \sum \exp(-S_{\text{red}})}$$

$$\frac{0}{1} \quad \frac{0}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \dots \quad \frac{1708}{5979} \approx 0.28566$$

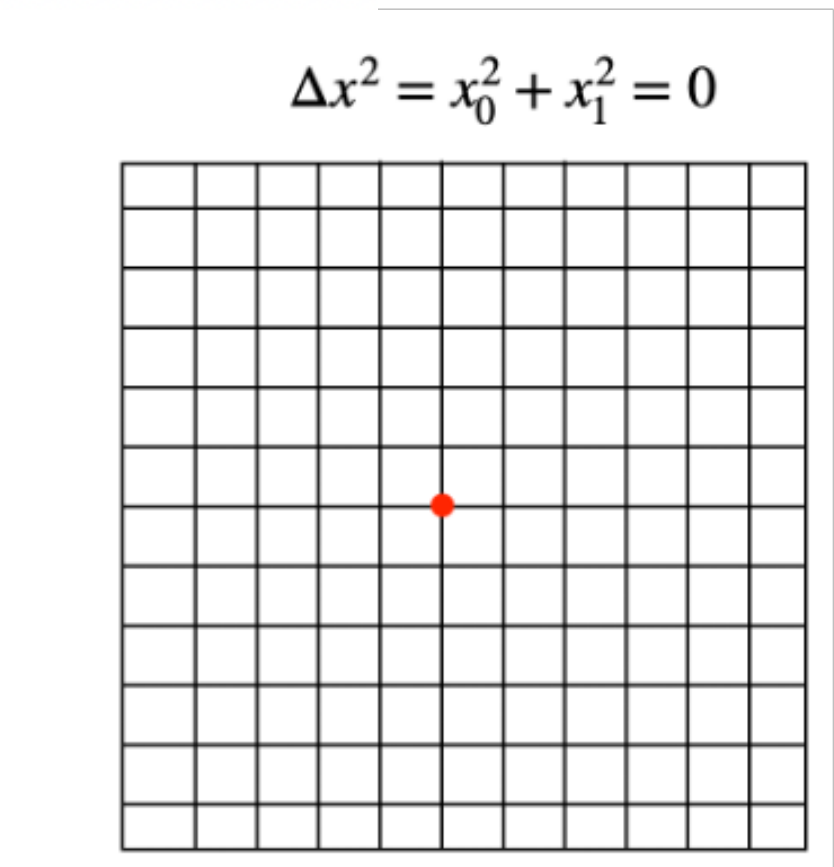
static properties of QCD

Dynamical process (out-of-equilibrium physics) in QCD

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} \int_0^\infty dt r^{d-1} n^i T_{0i}(t, r\vec{n})$$



Minkowski spacetime



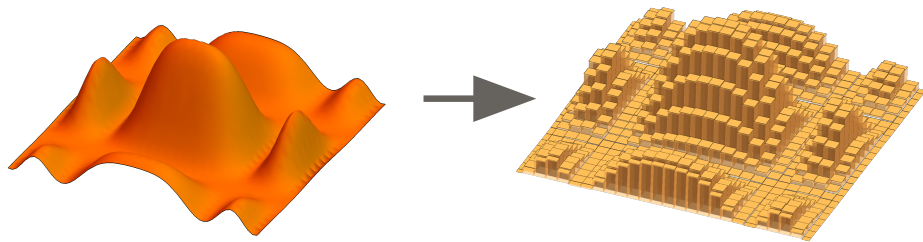
Euclidean spacetime

Classical methods become ineffective

Quantum methods from first principles

Lattice calculations — Minkowski spacetime (real-time)

$$\int \mathcal{D}\phi e^{iS} = \langle x | e^{-iHt} | y \rangle$$



$$\dim H \propto |G|^{N_V}$$

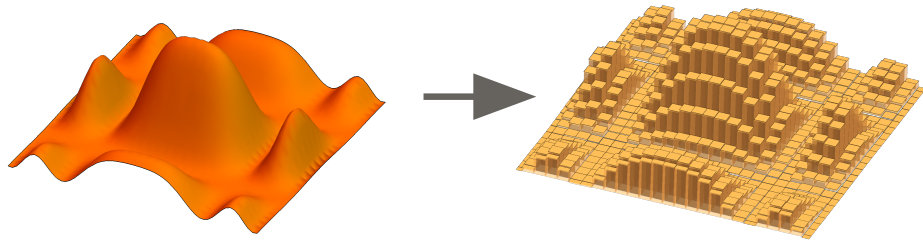
of classical bits: $N_b \propto |G|^{N_V}$
(*CLASSICAL HARD*)

From 1970s Phys. Rev. D 11, 395 (1975) KS Hamiltonian

Quantum methods from first principles

Lattice calculations — Minkowski spacetime (real-time)

$$\int \mathcal{D}\phi e^{iS} = \langle x | e^{-iHt} | y \rangle$$



$$\dim H \propto |G|^{N_V}$$

of classical bits: $N_b \propto |G|^{N_V}$
(*CLASSICAL HARD*)

From 1970s Phys. Rev. D 11, 395 (1975) KS Hamiltonian

❖ Storage

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

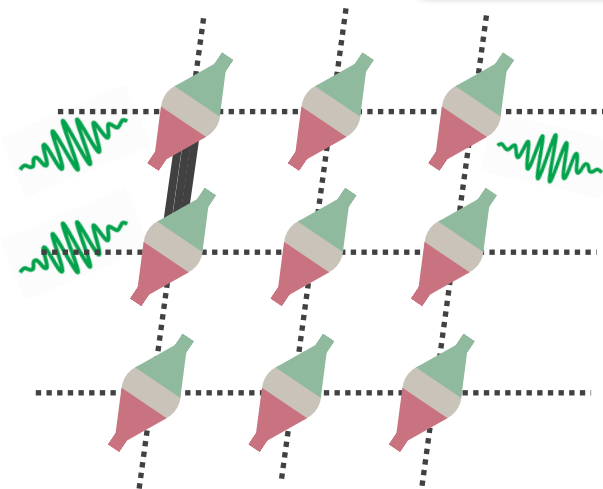
$$\# \text{ of qubits: } N_q \propto N_V \log |G|$$

❖ Operations

Universal Quantum Simulators

Seth Lloyd

Feynman's 1982 conjecture, that quantum computers can be programmed to simulate any local quantum system, is shown to be correct.



$$N(\text{wavy}) \propto N_q^m$$

(*QUANTUM EASY*)

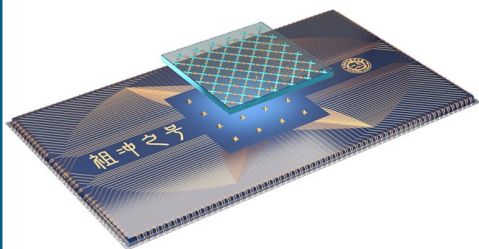
Quantum simulation

Quantum methods from first principles

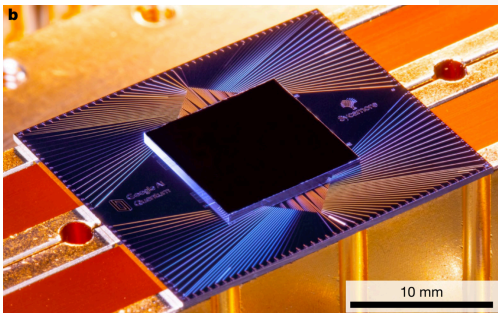


Now - Noisy Intermediate Scale Quantum (NISQ) era
more than 50 well controlled qubits, not error-corrected yet

superconducting processor



176 qubits

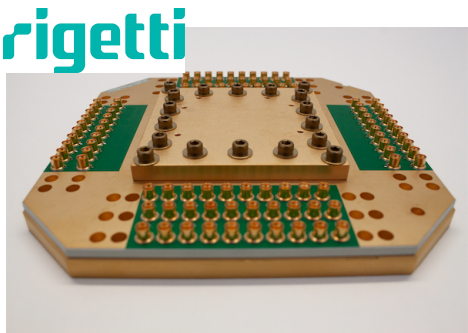


54 qubits



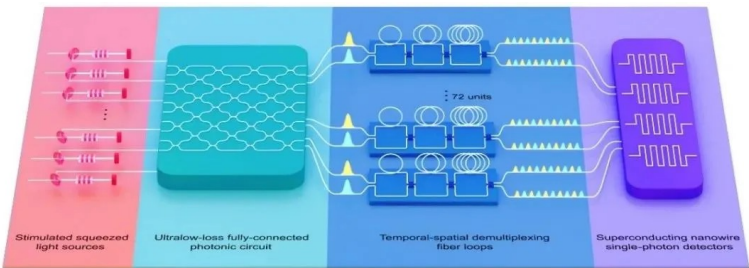
1121 qubits
access to 133 qubits

multi-chip quantum processor



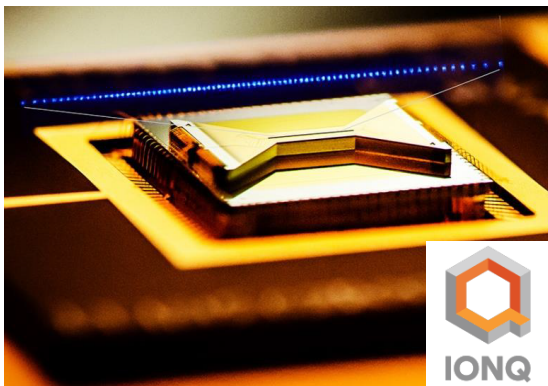
80 qubits

photon qubits



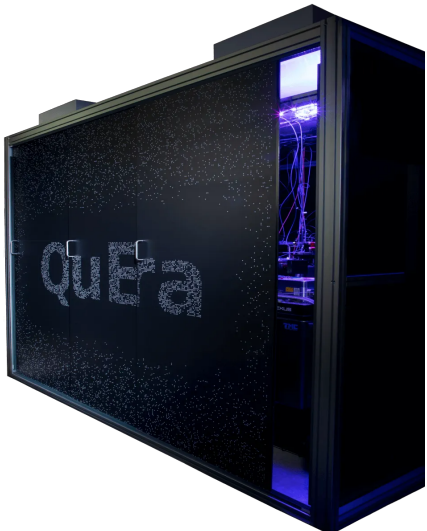
Jiuzhang - 255 qubits

trapped ion qubits



36 qubits

48 logical
qubits



[illegible]

IBM Quantum



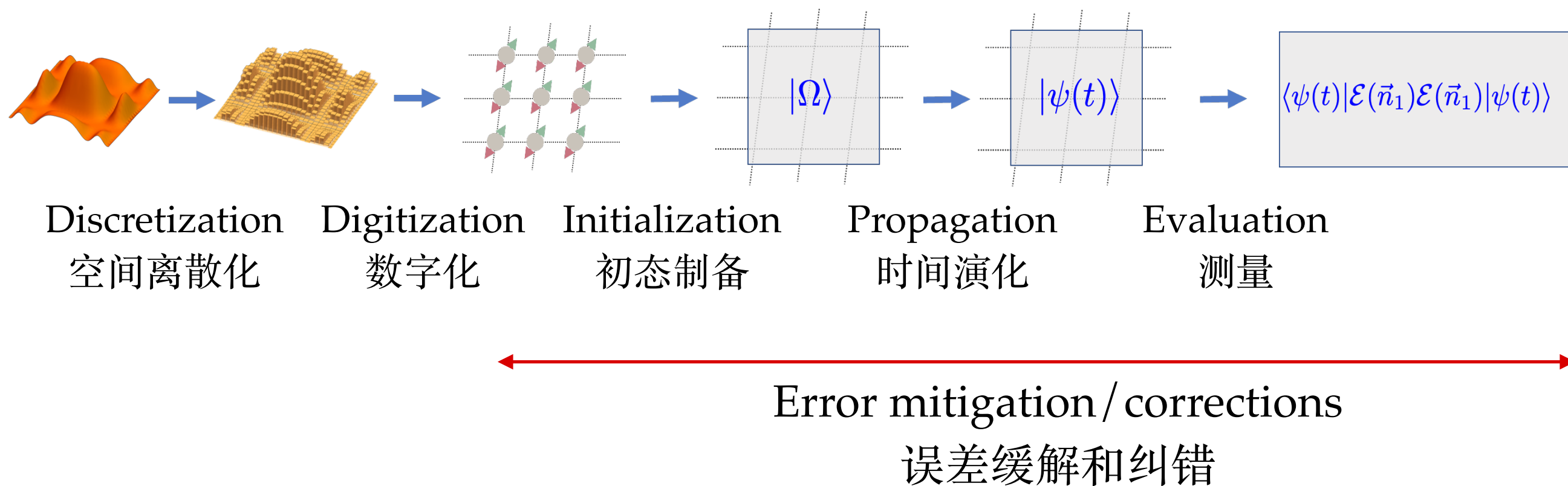
Innovation Roadmap



Dynamics of HEP with quantum computer

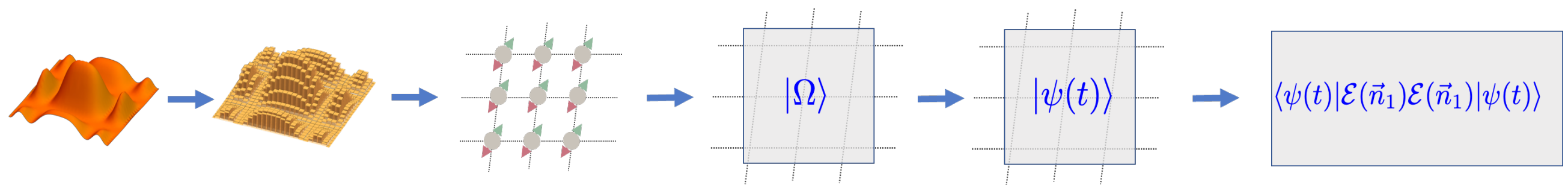
$$\int \mathcal{D}\phi e^{iS} = \langle x | e^{-iHt} | y \rangle$$

[Jordan, Lee, Preskill, 2011] The Theoretical Framework

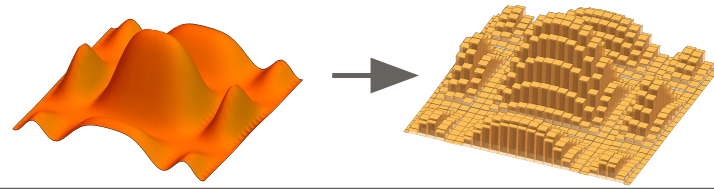


From the theoretical framework to practical computational implementation, numerous **theoretical challenges** and **technical bottlenecks** remain to be overcome

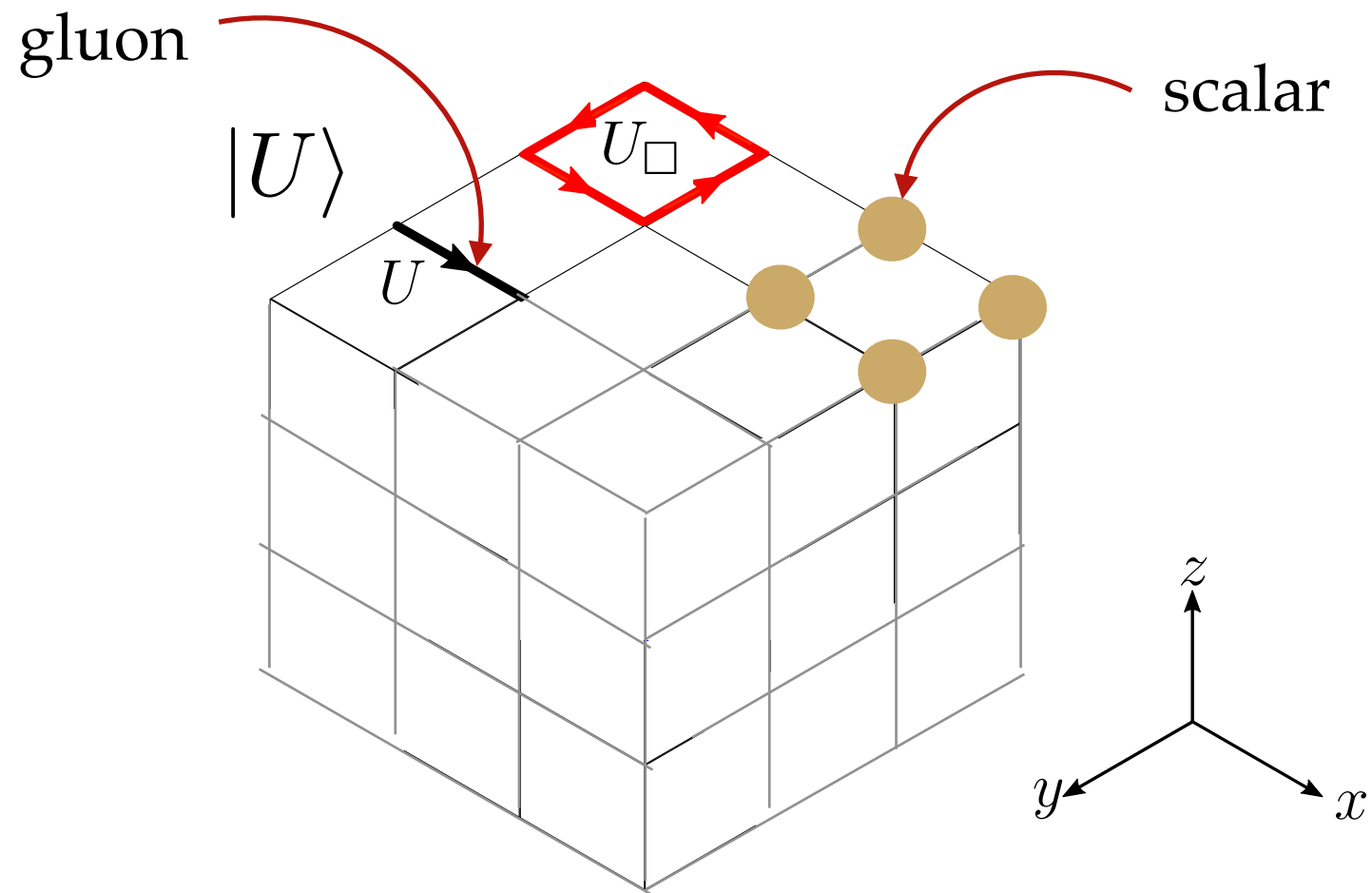
Scalar Field Theory



Discretization



infinities in QFT

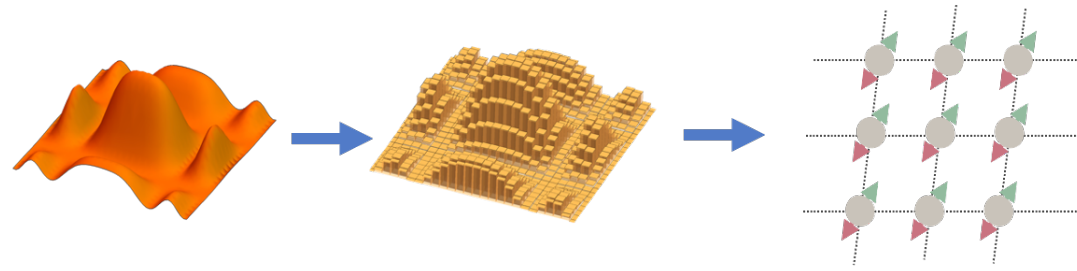


$$[\phi(\mathbf{x}), \pi(\mathbf{y})] = i a^{-d} \delta_{\mathbf{x}, \mathbf{y}} \mathbb{1}$$

$$H = \sum_{\mathbf{x} \in \Omega} a^d \left[\frac{1}{2} \pi(\mathbf{x})^2 + \frac{1}{2} (\nabla_a \phi)^2(\mathbf{x}) + \frac{1}{2} m_0^2 \phi(\mathbf{x})^2 + \frac{\lambda_0}{4!} \phi(\mathbf{x})^4 \right]$$

Digitization

scalar field theory



$$\hat{\phi}(\mathbf{x}) = \left(\frac{\delta p}{2\pi}\right)^{\frac{d}{2}} \sum_{\mathbf{p} \in \tilde{\Lambda}} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} (a_{\mathbf{p}} e^{-i\mathbf{p} \cdot \mathbf{x}} + a_{\mathbf{p}}^{\dagger} e^{i\mathbf{p} \cdot \mathbf{x}})$$

❖ Occupation basis

$$a_{\mathbf{p}}^{\dagger} |\mathbf{p}, n-1\rangle = \sqrt{n} |\mathbf{p}, n\rangle$$

❖ Digitization

$$n \leq N_{\text{cut-off}}$$

$$|\mathbf{p}, n\rangle = |\uparrow\downarrow \cdots \downarrow \cdots \uparrow\uparrow\rangle$$

$n_q = \log_2(N_{\text{cut-off}})$

$\longleftrightarrow$

$$n = \sum_{j=0}^{n_q-1} 2^j b_j$$

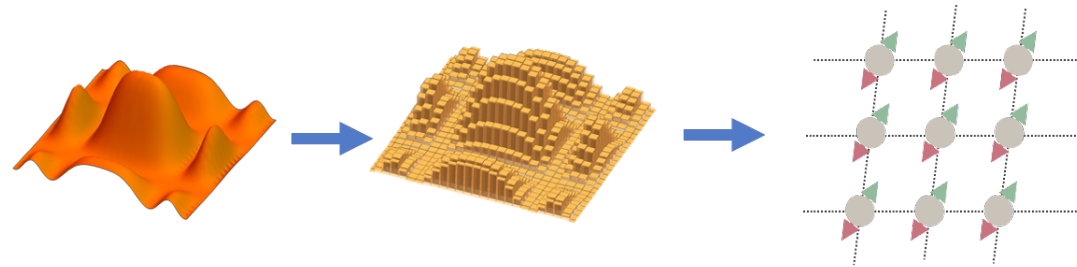
$\downarrow$
 $b_{n_q-1} = 0$

$\downarrow$
 $b_j = 1$

$\downarrow$
 $b_0 = 0$

Digitization

scalar field theory



$$\hat{\phi}(\mathbf{x}) = \left(\frac{\delta p}{2\pi}\right)^{\frac{d}{2}} \sum_{\mathbf{p} \in \tilde{\Lambda}} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} (a_{\mathbf{p}} e^{-i\mathbf{p} \cdot \mathbf{x}} + a_{\mathbf{p}}^{\dagger} e^{i\mathbf{p} \cdot \mathbf{x}})$$

❖ Amplitude basis

$$\hat{\phi}(\mathbf{x}) |\phi(\mathbf{x})\rangle = \phi(\mathbf{x}) |\phi(\mathbf{x})\rangle$$

❖ Digitization $\phi(\mathbf{x}) \in \{-\phi_{\max} + n \delta_{\phi} \mid n = 0, 1, \dots, N_{\phi} - 1\}$, $\delta_{\phi} = \frac{2\phi_{\max}}{N_{\phi} - 1}$

$$n_q = \log_2(N_{\phi})$$

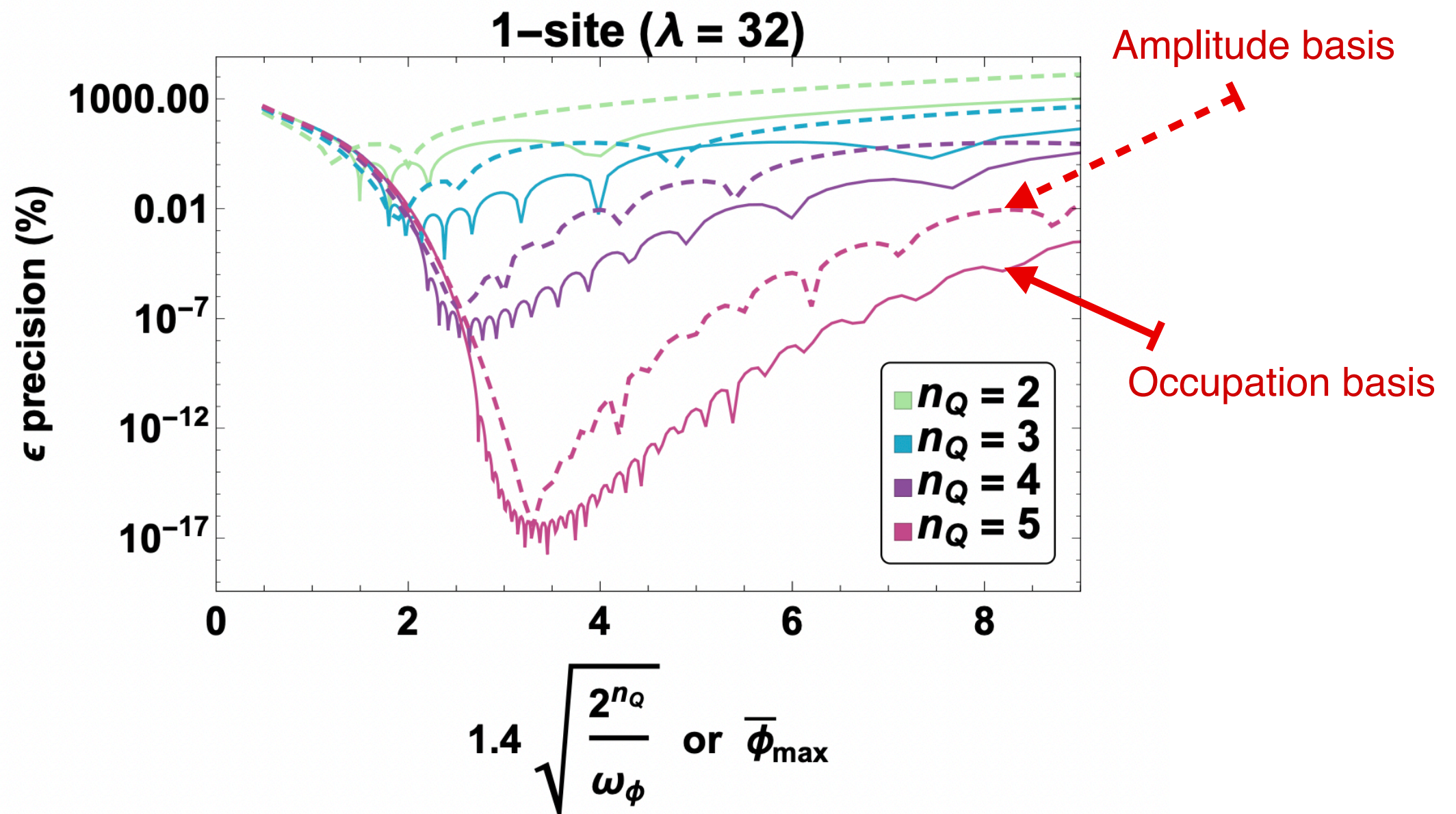
$$|\phi(\mathbf{x})\rangle = |\uparrow\downarrow \cdots \downarrow \cdots \uparrow\uparrow\rangle$$

$$\phi(\mathbf{x}) = \frac{\delta_{\phi}}{2} \sum_{j=0}^{n_q-1} 2^j b_j \quad b_{n_q-1} = 1 \quad b_j = -1 \quad b_0 = 1$$

Digitization

scalar field theory

$$H_{\text{int}} = \frac{\lambda}{s!} \delta x^d \sum_{\mathbf{x} \in \Lambda} \phi(\mathbf{x})^s, \quad s = 4$$



The precision of the ground-state energy (a static property) converges exponentially with the number of states $N_{\text{cut-off}}(N_\phi)$

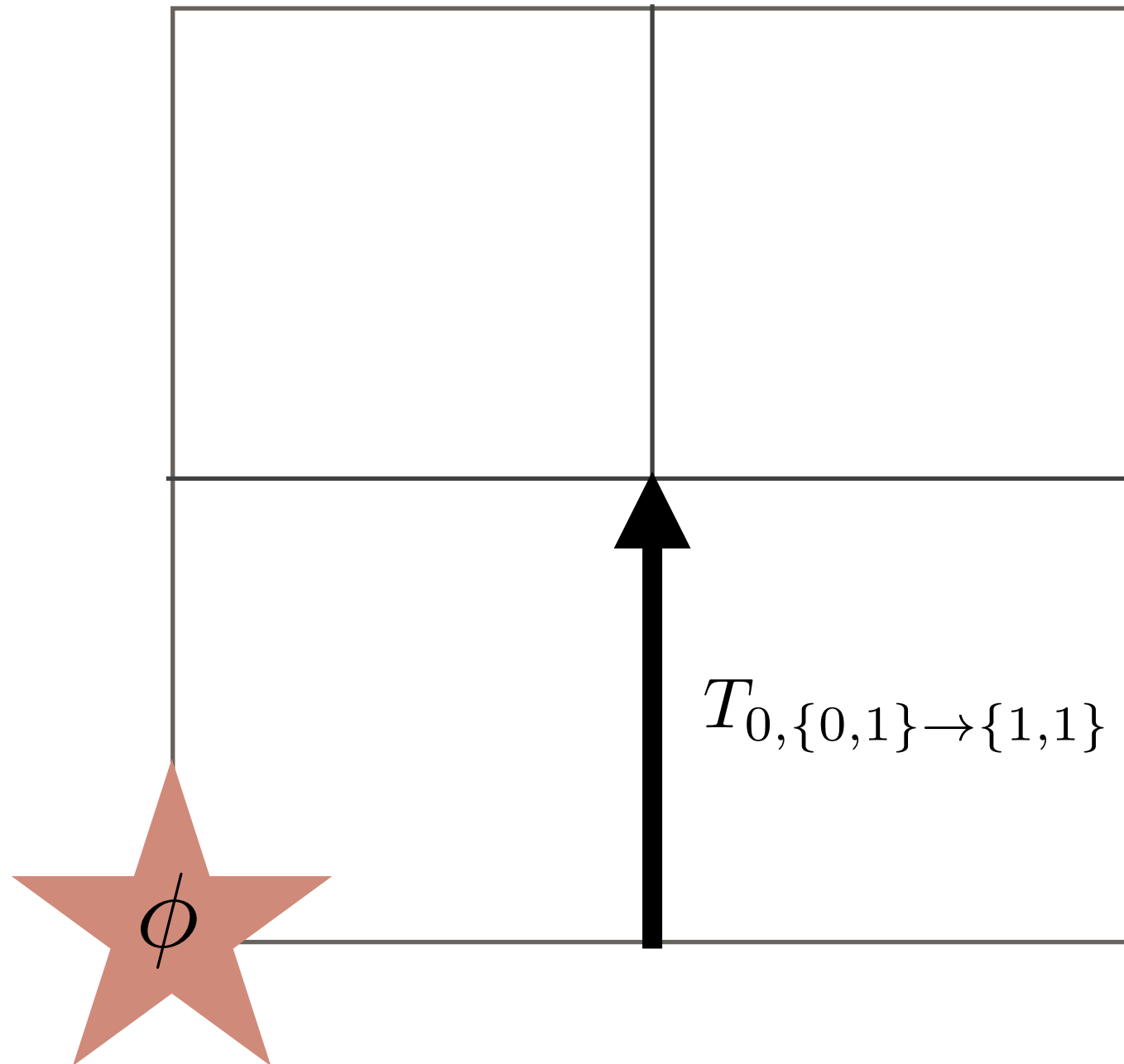
Digitization

scalar field theory



Liang-Qi Zhang

$$H_{\text{int}} = \frac{\lambda}{s!} \delta x^d \sum_{\mathbf{x} \in \Lambda} \phi(\mathbf{x})^s, \quad s = 4$$



source/sink

Dynamical property

energy flow from
position $\{0,1\}$ to
position $\{1, 1\}$

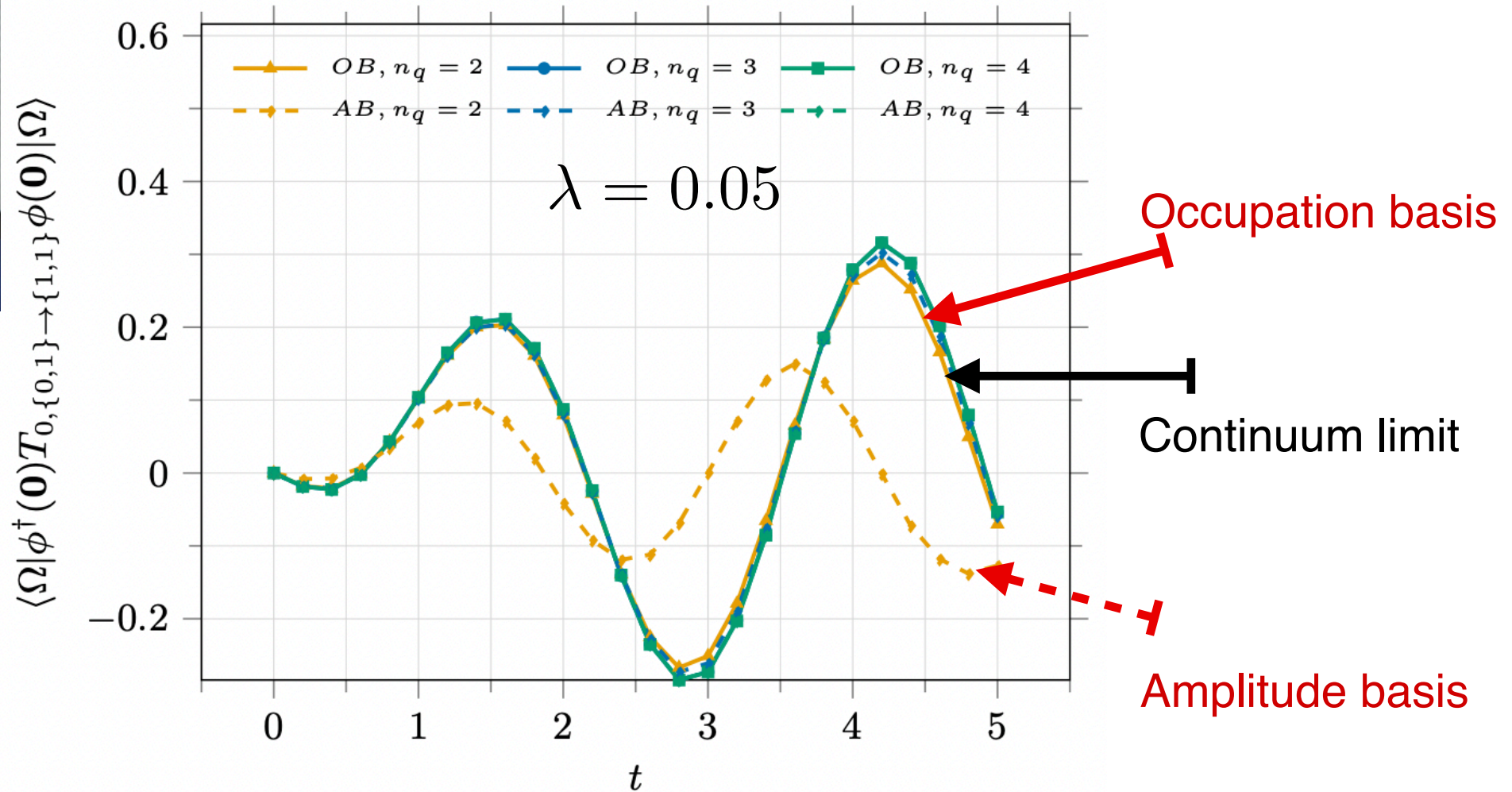
Digitization

scalar field theory



Liang-Qi Zhang

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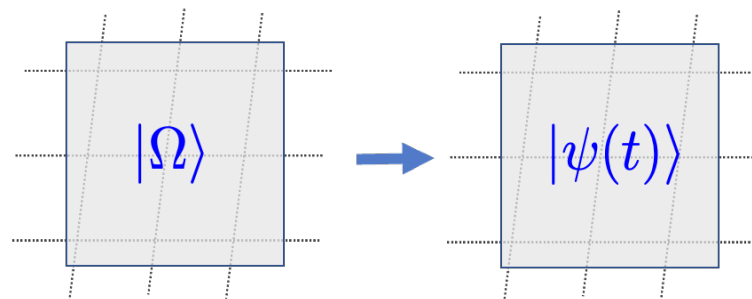


In the occupation basis, the precision of the energy flow (**dynamical property**) in certain parameter region converges faster with the number of states $N_{\text{cut-off}}(N_\phi)$

❖ Occupation basis

Wavepacket prep and
measurements are easy

Propagation is hard



Propagation

scalar field theory

$$H_{\text{int}} = \frac{\lambda}{s!} \delta x^d \sum_{\mathbf{x} \in \Lambda} \phi(\mathbf{x})^s, \quad s = 4$$

$$e^{-iH_{\text{int}}\delta t} = \prod_{\mathbf{x} \in \Lambda} \exp(-i\theta \phi(\mathbf{x})^s)$$



non-diagonal matrix

The CNOT circuit depth for implementing $e^{-iH_{\text{int}}\delta t}(s = 4)$

n_q	1	2	3	4
Non-diag	(64)56	(40128)27696	(2738424)1665752	(132112108)60975028

Pauli string decomposition + Trotterization (with Trotter errors)

Propagation

scalar field theory

$$H_{\text{int}} = \frac{\lambda}{s!} \delta x^d \sum_{\mathbf{x} \in \Lambda} \phi(\mathbf{x})^s, \quad s = 4$$

Single-qubit rotation
& Hermite Transformation

$$e^{-iH_{\text{int}}\delta t} = \prod_{\mathbf{x} \in \Lambda} \exp(-i\theta \phi(\mathbf{x})^s) = \prod_{\mathbf{x} \in \Lambda} P(\mathbf{x}) \exp(-i\theta \tilde{\phi}(\mathbf{x})^s) P^\dagger(\mathbf{x})$$

non-diagonal matrix
diagonal matrix

The CNOT circuit depth for implementing $e^{-iH_{\text{int}}\delta t}(s = 4)$

n_q	1	2	3	4
Non-diag	(64)56	(40128)27696	(2738424)1665752	(132112108)60975028

Propagation

scalar field theory

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non-diagonal matrix

diagonal matrix

no additional Trotter errors arise
once it is diagonalized

The CNOT circuit depth for implementing $e^{-iH_{\text{int}}\delta t}(s = 4)$

n_q	1	2	3	4
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Propagation

scalar field theory

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non-diagonal matrix
diagonal matrix

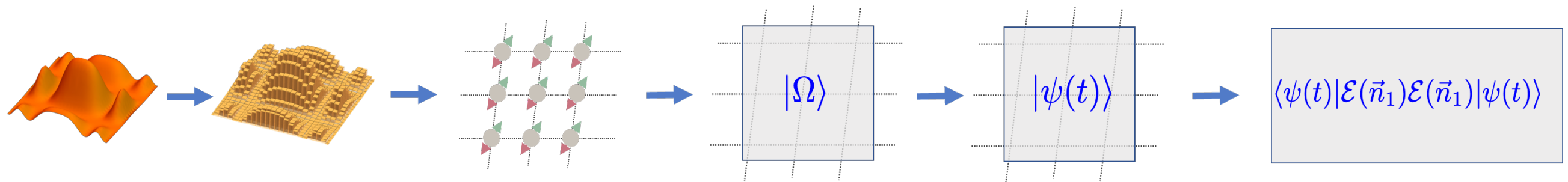
no additional Trotter errors arise
once it is diagonalized

The CNOT circuit depth for implementing $e^{-iH_{\text{int}}\delta t} (s = 4)$

n_q	1	2	3	4
Non-diag	(64)56	(40128)27696	(2738424)1665752	(132112108)60975028
Diag	(64)56	(1656)916	(29552)15704	(439952)132660

Achieving **exponential improvements** in the complexity for implementing the unitary time evolution operator with quantum circuits in the occupation basis.

Abelian Gauge Theories



Digitization

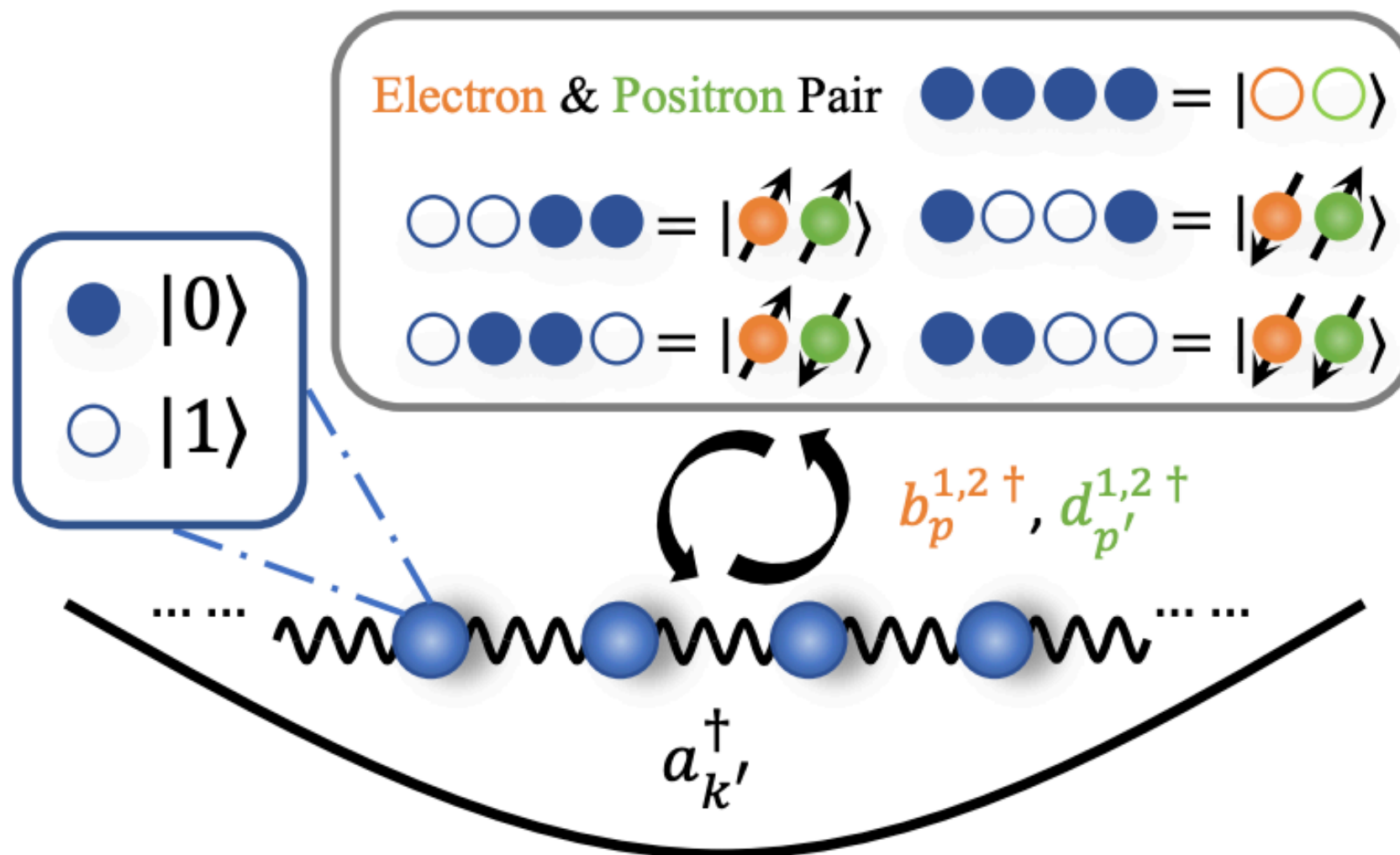
Abelian gauge theory + matter

Hybrid realization

[Davoudi, Linke, Pagano, arXiv:2104.09346]

[Li, Xu, Dong, Huang, **YYL**, Lin, Shu, arXiv: 2607.09844]

$$H_{\text{int}} \supset c_{kij}^r a_k^r \sigma_i^+ \Sigma_{ij}^z \sigma_j^+ + \text{h.c.} \quad \Sigma_{ij}^z \equiv \prod_{i < m < j} \sigma_m^z$$



Digitization

Abelian gauge theory + matter

Hybrid realization

[Davoudi, Linke, Pagano, arXiv:2104.09346]

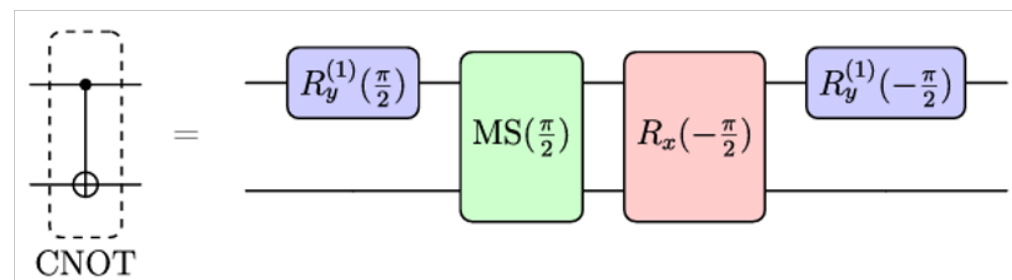
[Li, Xu, Dong, Huang, **YYL**, Lin, Shu, arXiv: 2607.09844]

Gate Sets

Single qubit gate

$$R(\theta, \phi) = e^{-i\frac{\theta}{2}(\cos(\phi)\sum_j \sigma_x^j + \sin(\phi)\sum_j \sigma_y^j)}$$

CNOT gate



Analog gate **A**

$$R_{k,j}^{\sigma^a}(\theta_{k,j}, 0) = e^{-i\theta_{k,j}(a_k^r + a_k^{r\dagger})\sigma_j^y},$$

$$R_{k,j}^{\sigma^a}\left(\theta_{k,j}, \frac{\pi}{2}\right) = e^{\theta_{k,j}(a_k^r - a_k^{r\dagger})\sigma_j^y}$$

Digitization

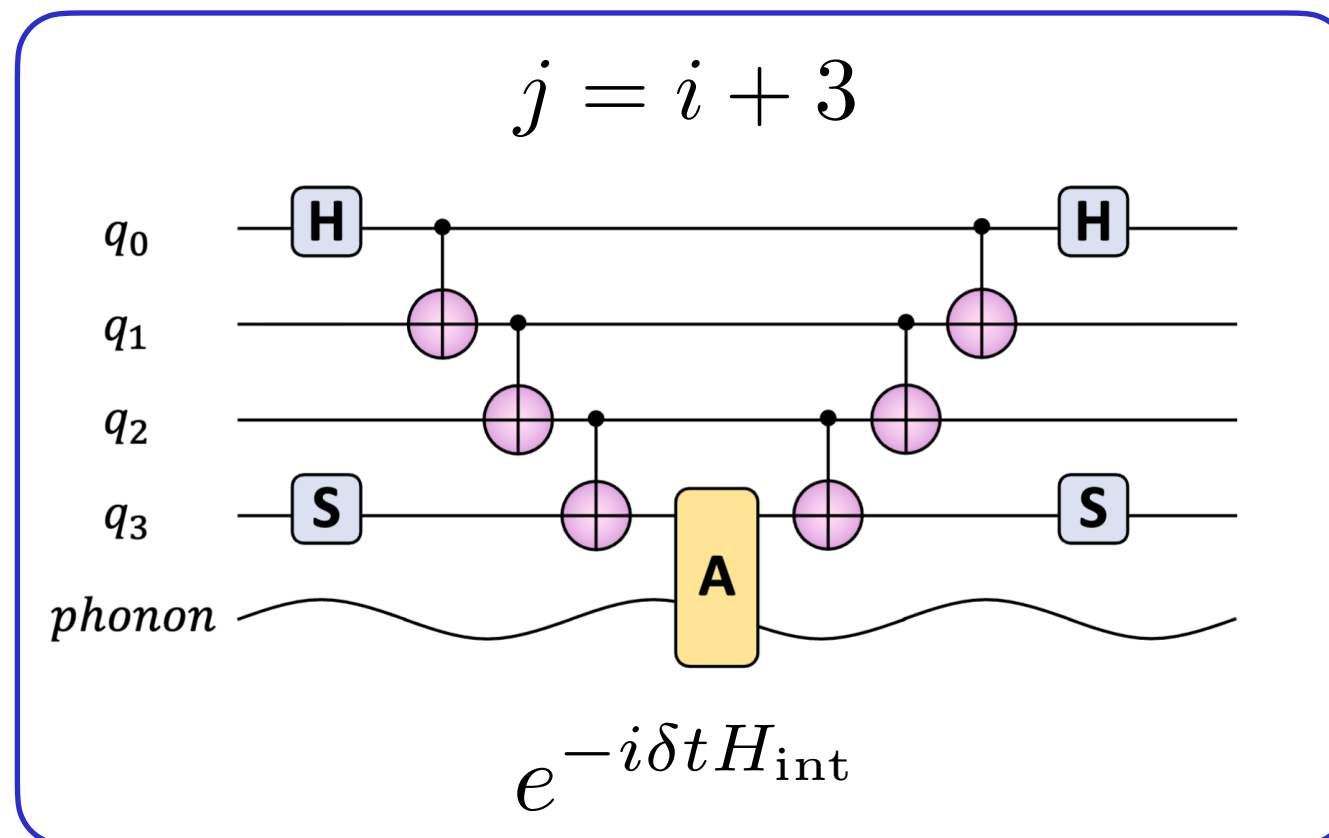
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$$H_{\text{int}} \supset c_{kij}^r a_k^r \sigma_i^+ \Sigma_{ij}^z \sigma_j^+ + \text{h.c.}$$

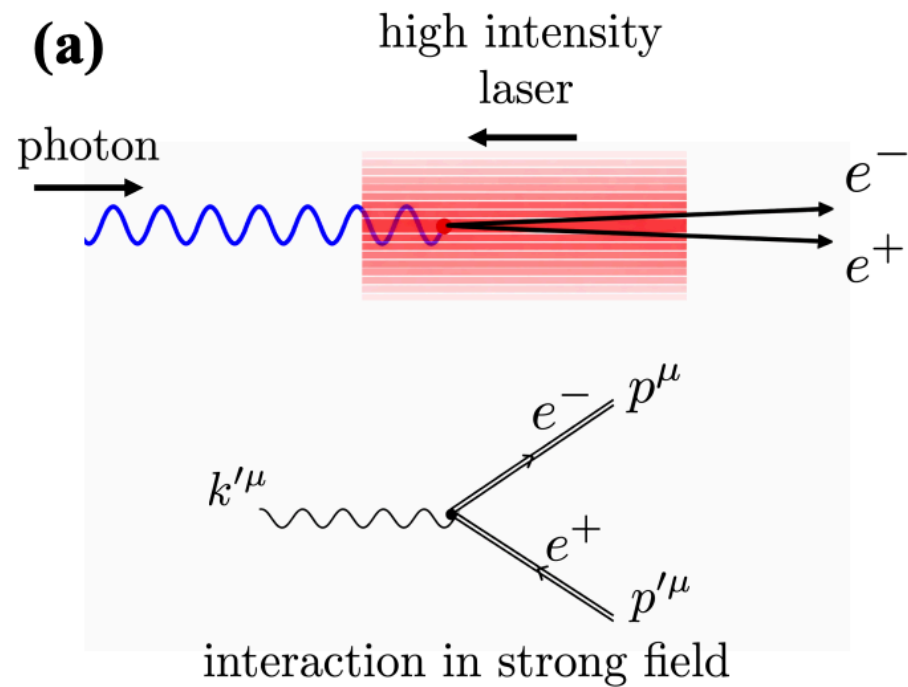


$\mathcal{O}(N_p^3)$ CNOT gates, $\mathcal{O}(N_p^2)$ analog gates

Hardware efficient setup, achieving **polynomial complexity** in both Clifford operations and analog operations

Digitization

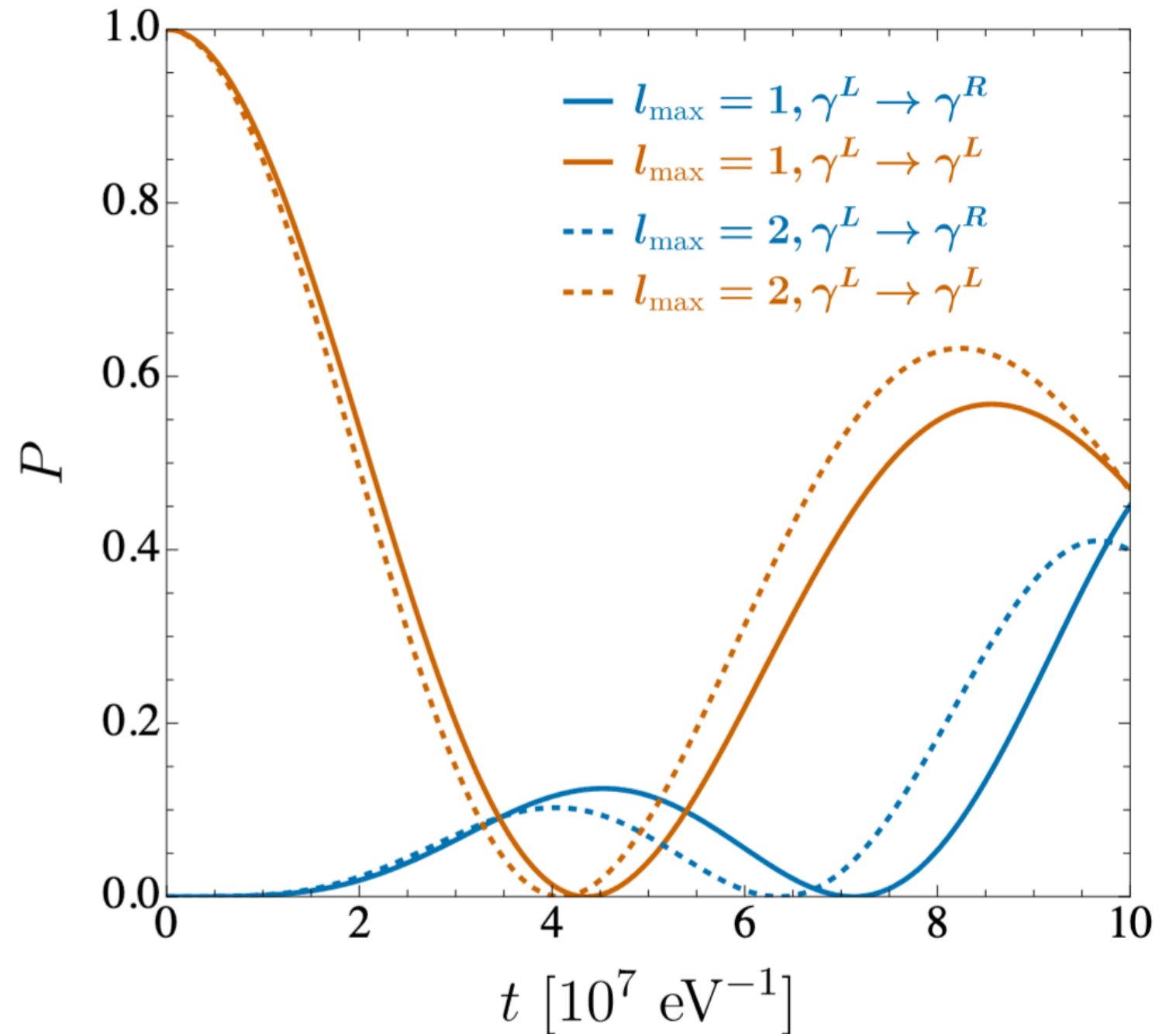
Abelian gauge theory + matter



Hybrid realizations intense-field QED

[Li, Xu, Dong, Huang, **YYL**, Lin, Shu, arXiv: 2607.09844]

$l_{\max}$: maximal number of photons absorbed from the laser

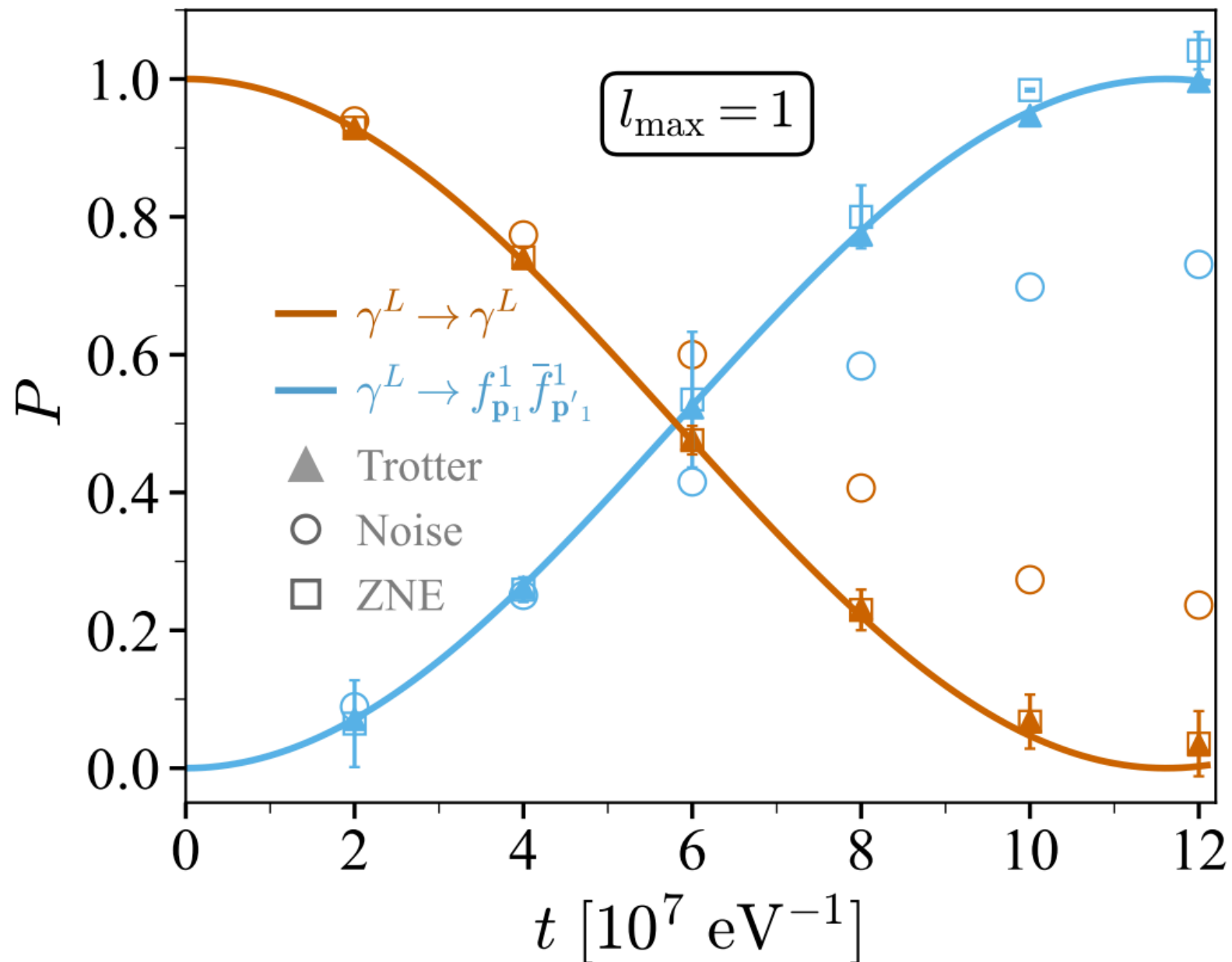


Digitization

Abelian gauge theory + matter

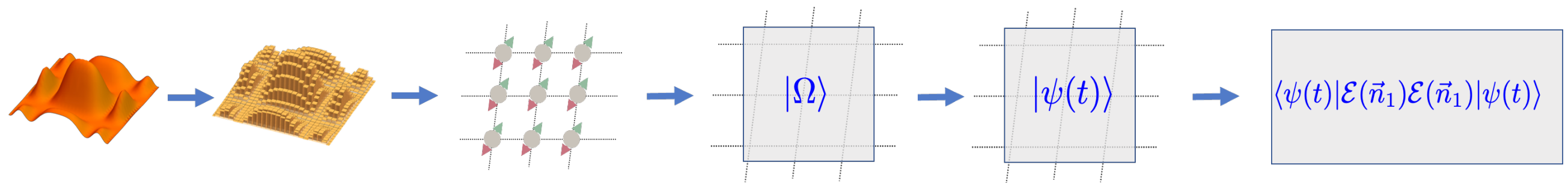
Hybrid realizations intense-field QED

[Li, Xu, Dong, Huang, **YYL**, Lin, Shu, arXiv: 2607.09844]



Realistic for near-term quantum devices

Non-Abelian Gauge Theories

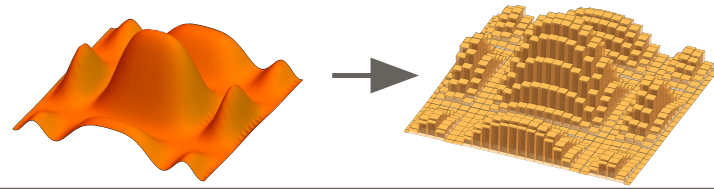


[Carena, Lamm, **YYL**, Liu,
PRL. **129**, 051601]

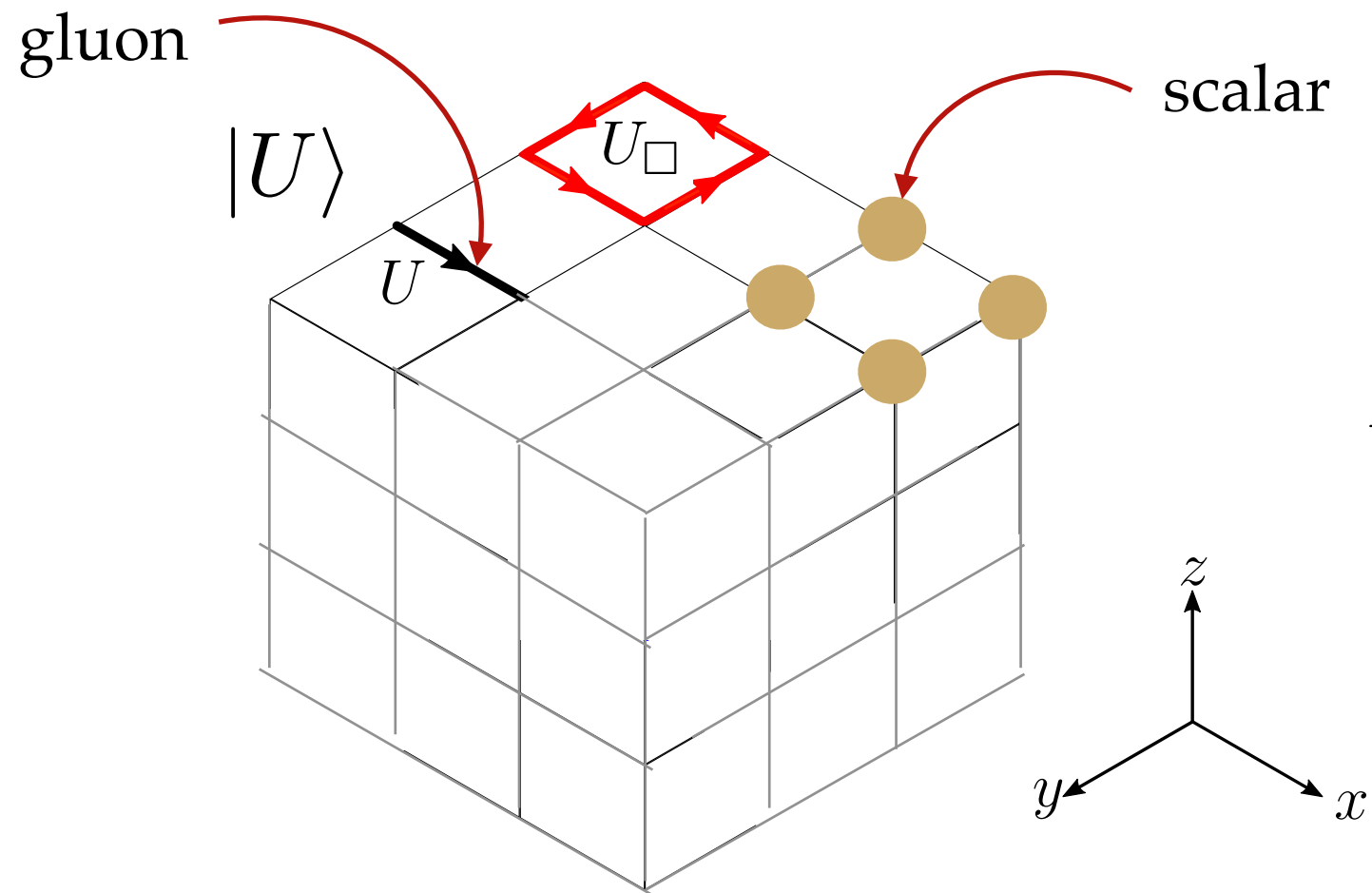
[Lamm, **YYL**, Shu, Wang, Bin,
PRD 110 (2024) 5, 054505]

[Cao, Hou, **YYL**, Liu, Song, Zhang, Zhang,
Zhao, arXiv:2505.06347]

Discretization



infinities in QFT



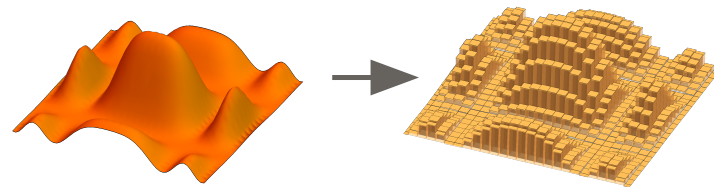
gauge invariant Hamiltonian

KS Hamiltonian [Phys. Rev. D 11, 395 (1975)]

$$H_{KS} = \sum \left(\underbrace{\longrightarrow}_{K_L} + \underbrace{\square}_{U_{\square}} \right)$$

$$H_{KS} \xrightarrow{a \rightarrow 0} H = \int dx (E^2 + B^2)$$

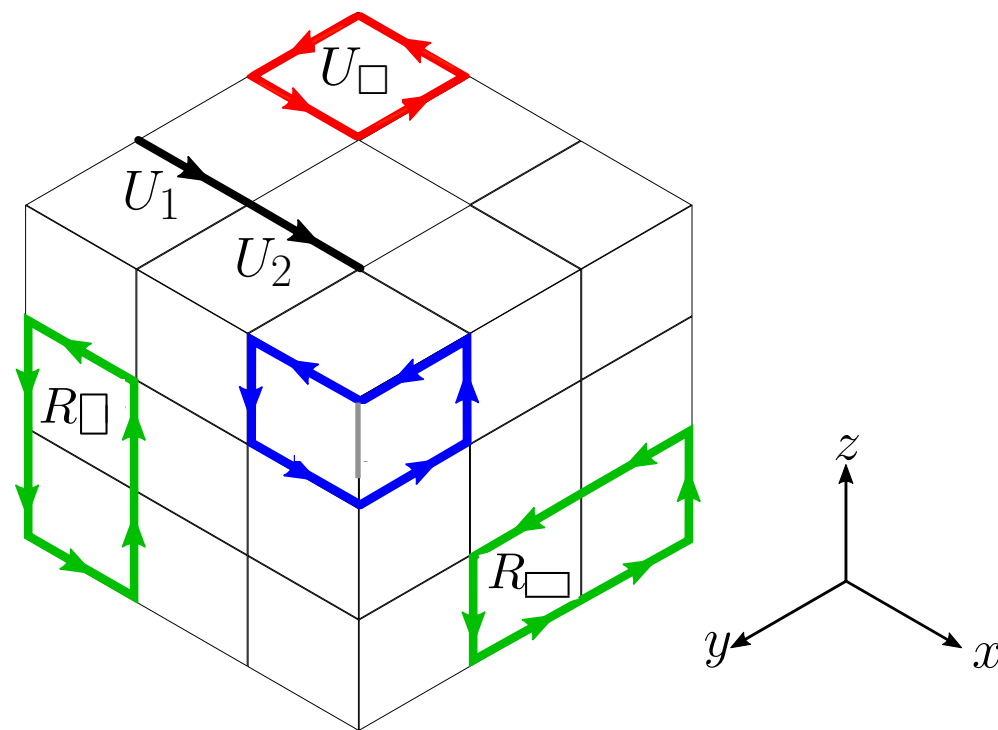
Discretization



infinities in QFT

[Carena, Lamm, [YYL](#), Liu, PRL. **129**, 051601]

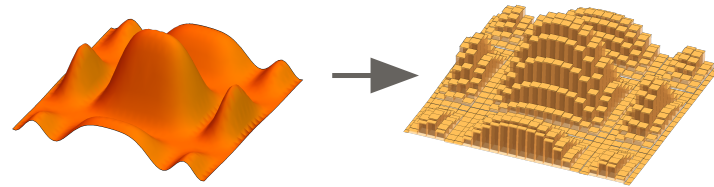
$$K_{2L} = E_i(\mathbf{x})U_i(\mathbf{x})E_i(\mathbf{x} + a\mathbf{i})U_i^\dagger(\mathbf{x})$$



$$H_I = \sum \left(\underbrace{\longrightarrow}_{K_L} + \underbrace{\longrightarrow\longrightarrow}_{K_{2L}} + \underbrace{\square}_{U_\square} + \underbrace{\square}_{R_\square} + \underbrace{\square}_{R_\square} \right)$$

correcting classical a^2 errors

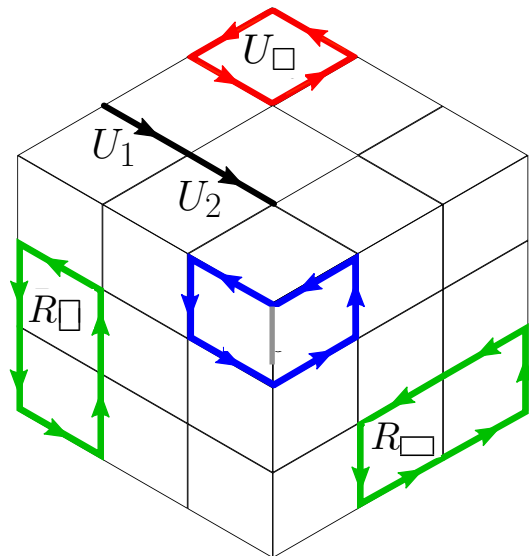
Discretization



infinities in QFT

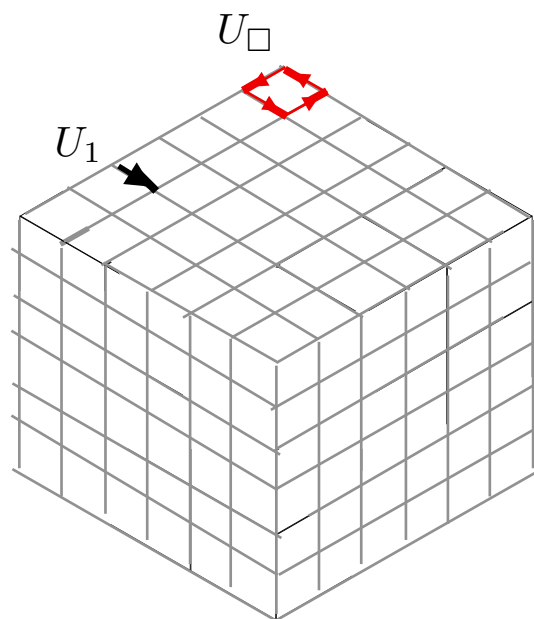
$$K_{2L} = E_i(\mathbf{x})U_i(\mathbf{x})E_i(\mathbf{x} + a\mathbf{i})U_i^\dagger(\mathbf{x})$$

[Carena, Lamm, [YYL](#), Liu, PRL. **129**, 051601]



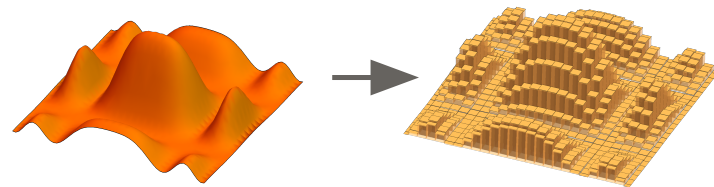
$$| \langle H_{KS}(a) - H \rangle | \sim | \langle H_I(2a) - H \rangle |$$

$$N_q \sim \left(\frac{L}{a} \right)^d$$



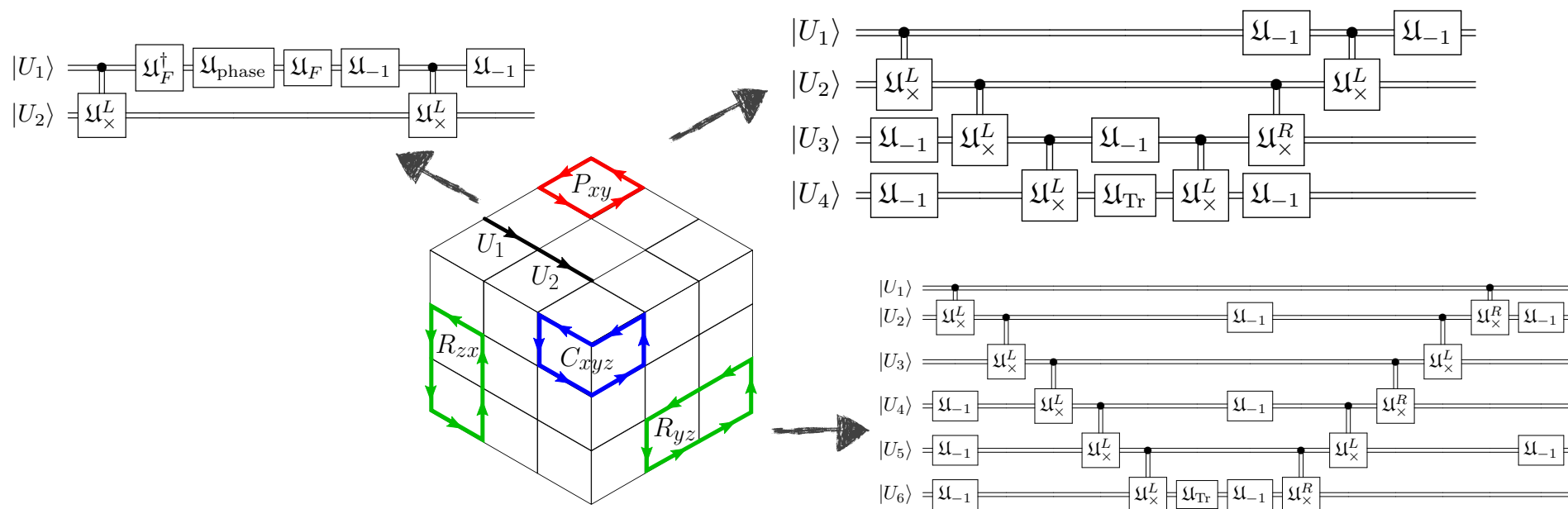
Same error level, larger lattice spacing,
less number of qubits
 needed for improved Hamiltonian

Discretization



infinities in QFT

[Carena, Lamm, [YYL](#), Liu, PRL. **129**, 051601]

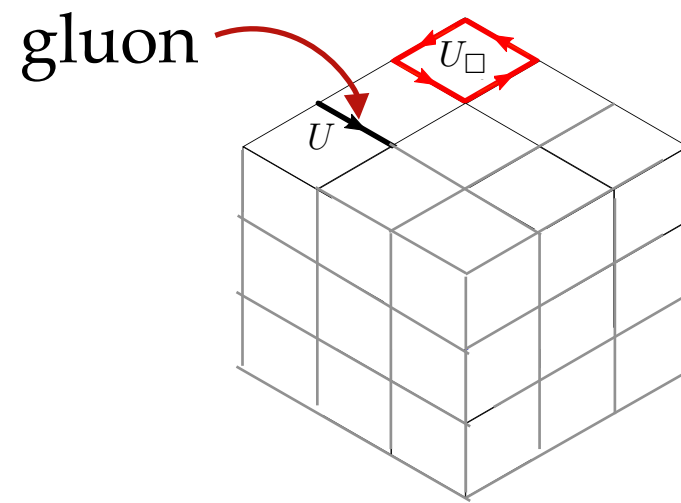


Gate	$N[\hat{K}_{KS} + \hat{V}_{KS}]$	$N[\hat{K}_{2L} + \hat{V}_{rect}]$
$\mathcal{U}_F$	2	2
$\mathcal{U}_{\text{phase}}$	1	1
$\mathcal{U}_{\text{Tr}}$	$\frac{d-1}{2}$	$d-1$
$\mathcal{U}_{-1}$	$3(d-1)$	$2 + 8(d-1)$
$\mathcal{U}_\times$	$6(d-1)$	$4 + 20(d-1)$

- # of Gates per link here for a single trotter is increasing only multiplicatively
- With improved Hamiltonian
- # of links decreases
- # of trotter steps also decreases

Digitization

non-Abelian gauge theory



Formulations & bases: examples

Kogut-Susskind Formulation

- Irrep / angular momentum basis: *Byrnes, Yamamoto, Zohar, Burrell, et al*
- Group-element basis: *Carena, Lamm, YYL, Liu, Zohar, et al.*
- Mixed basis: *Bauer, D'Andrea, Freytsis, Grabowska*
- Large N truncations: *Bauer, Ciavarella*
- Continuous-Variable quantum computing: *Abel, Spannowsky, Williams, et al.*

Gauge magnets/Quantum link models: *Wiese, Chandrasekhara, et al.*

Casimir variables: *Klco, Savage, Stryker, Ciavarella*

Schwinger boson formulations: *Mathur, Anishetty, Raychowdhury, et al*

Loop-string-hadron formulation: *Raychowdhury, Stryker, Davoudi, Kadam, et al.*

Qubit models: *Chandrasekhara, Singh, et al.*

Dual/rotor formulations: *Kaplan, Stryker, Haase, et al.*

q-deformed Kogut-Susskind: *Zache, Zoller et al*

Dual/rotor formulations: *Kaplan, Stryker, Haase, Dellantonio, et al.*

Theory developments and algorithms are still in very early stages, vibrant research directions

Digitization

$$|q\rangle^N \rightarrow |G\rangle$$

infinities in QFT

[Lamm, [YYL](#), Shu, Wang, Bin, PRD 110 (2024) 5, 054505]

$$|U\rangle = \left\{ \begin{pmatrix} \alpha & -\bar{\beta} \\ \beta & \bar{\alpha} \end{pmatrix} : \alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1 \right\} \xrightarrow{\text{red arrow}} |U\rangle = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbf{F}_3, ad - bc \equiv 1 \pmod{3} \right\}$$

3-sphere discrete subgroup

[Lamm, [YYL](#), Shu, Wang, Bin, PRD 110 (2024) 5, 054505]

$$|U\rangle = \left\{ \begin{pmatrix} \alpha & -\bar{\beta} \\ \beta & \bar{\alpha} \end{pmatrix} : \alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1 \right\} \xrightarrow{\text{3-sphere}} |U\rangle = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbf{F}_3, ad - bc \equiv 1 \pmod{3} \right\}$$

3-sphere discrete subgroup

block product encoding: BT

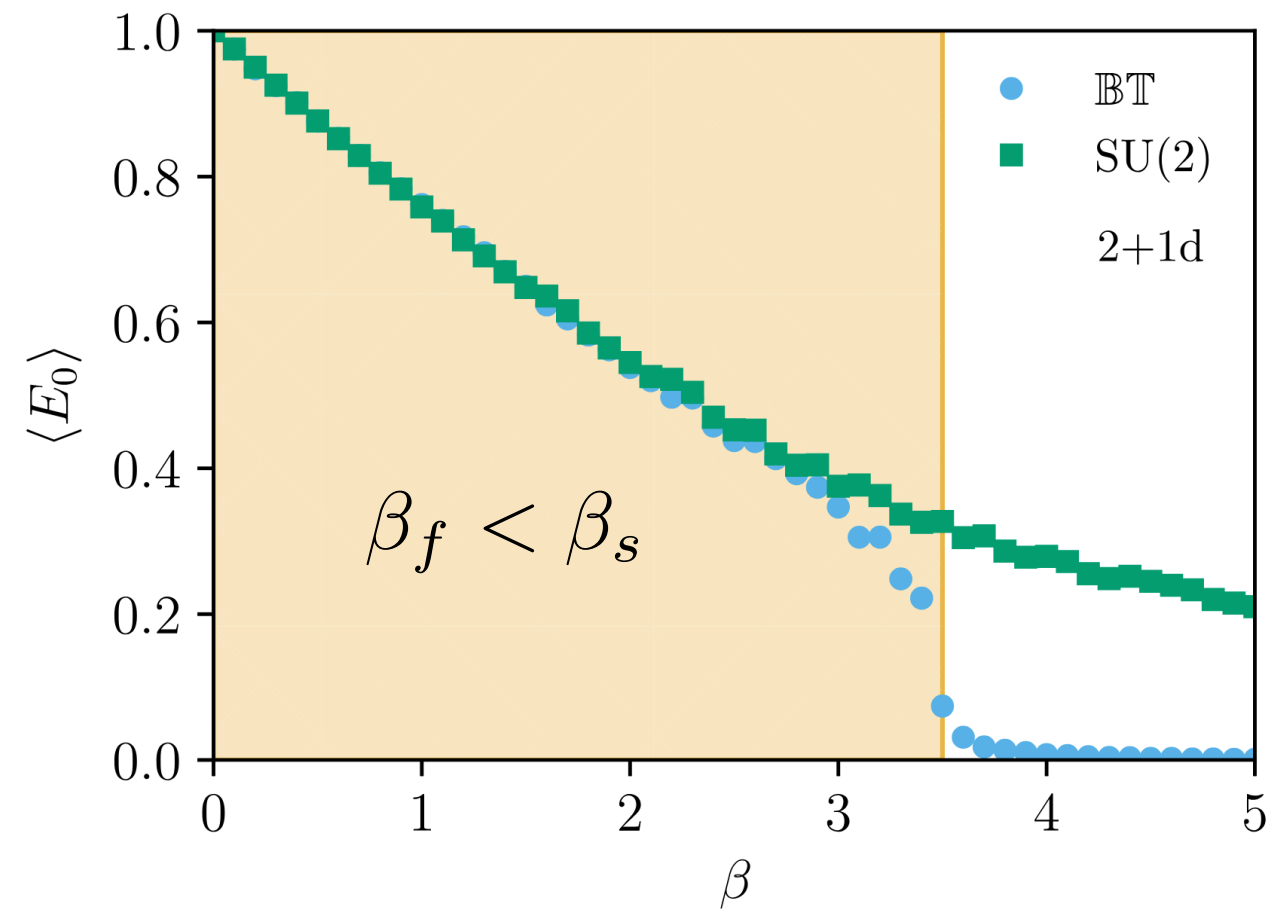
$$|U\rangle = \left| \begin{array}{ccccccccc} \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup \\ \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\ \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown \end{array} \right\rangle$$

block product encoding: BI

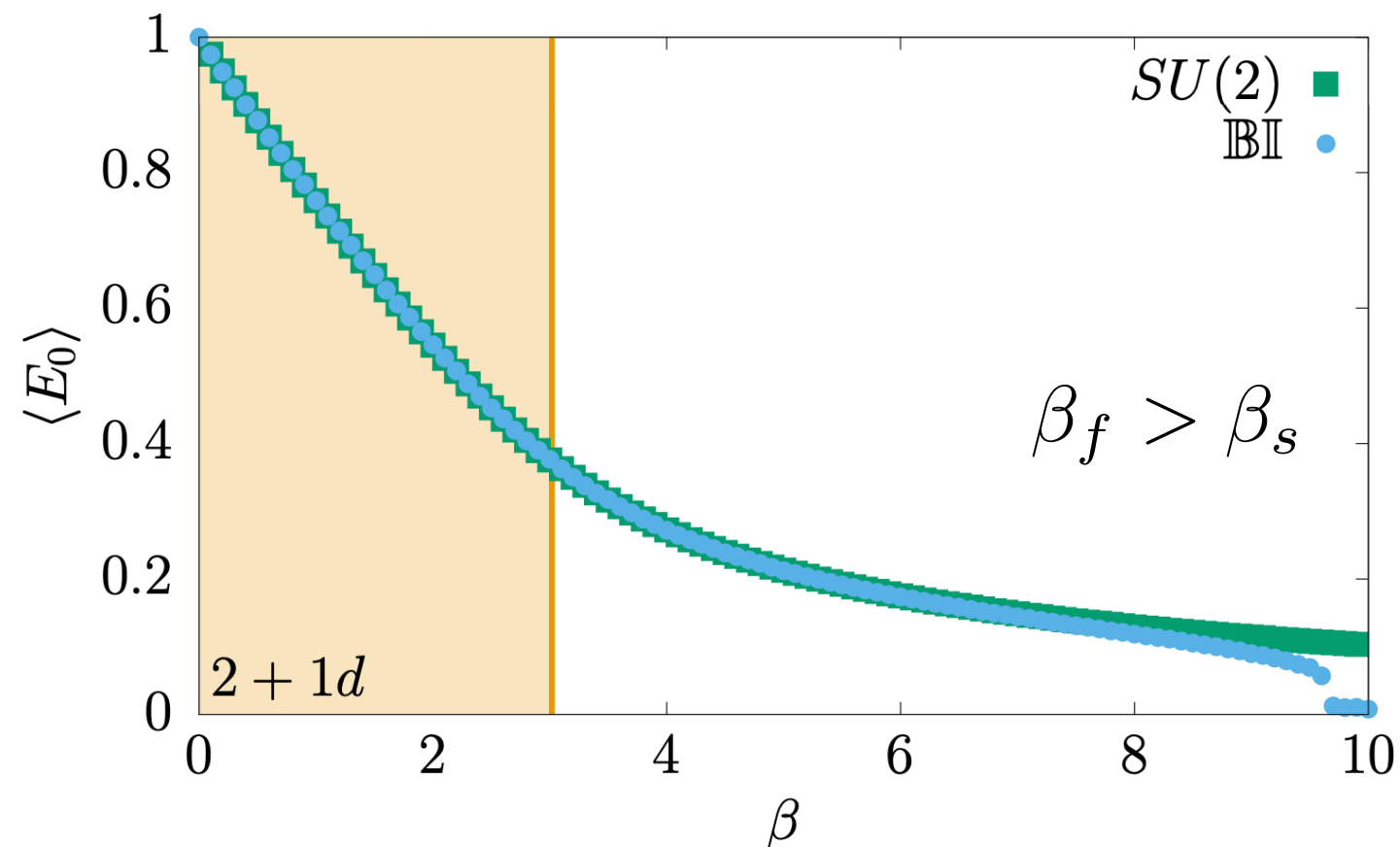
$$|U\rangle = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbf{F}_5, ad - bc \equiv 1 \pmod{5} \right\}$$

$$|U\rangle = \left| \begin{array}{ccccccccccccccc} \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup & \triangleup \\ \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\ \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown & \triangledown \end{array} \right\rangle$$

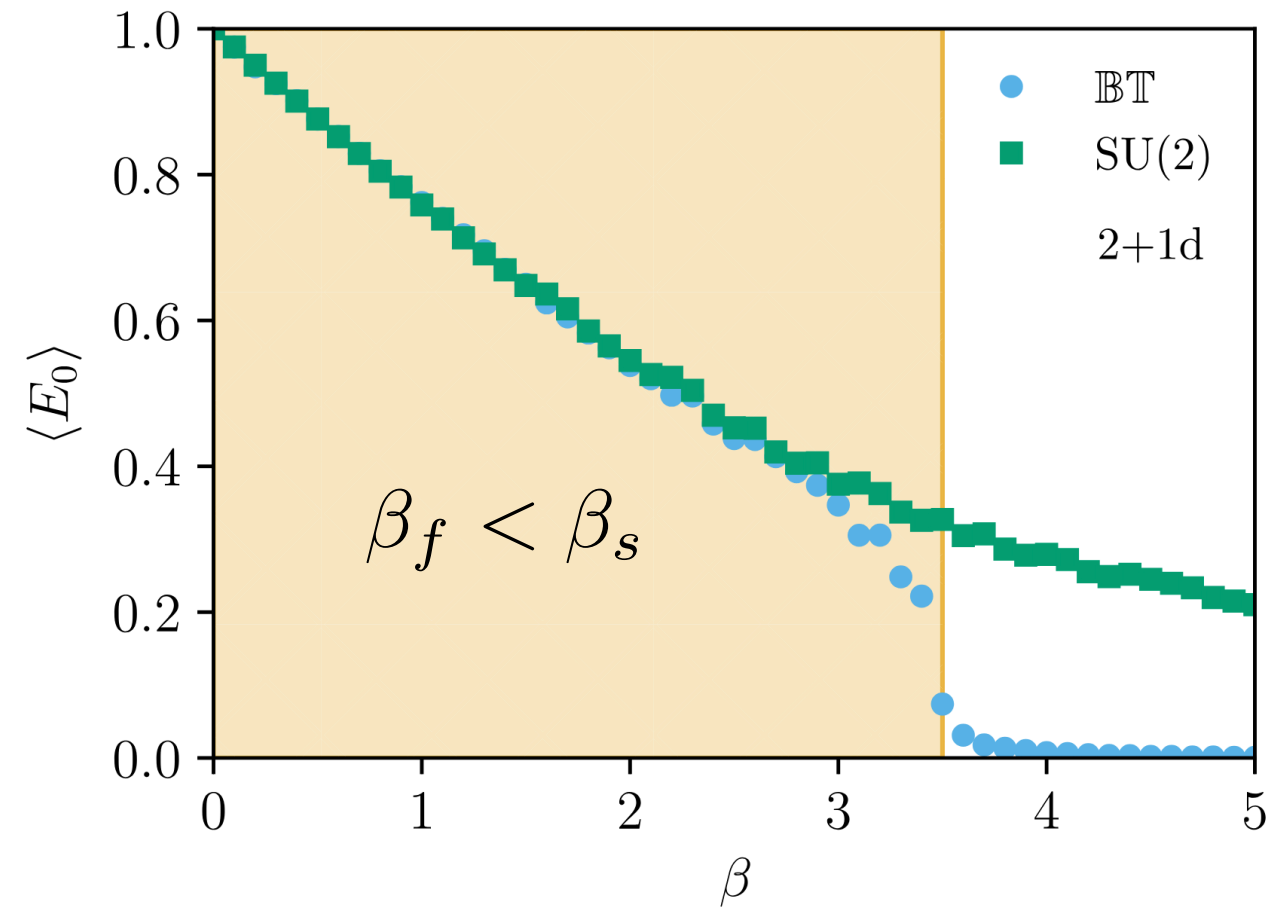
- Block encoding can be **extended to general finite group**, such as S1080 of SU(3)

[Gustafson, Lamm, Lovelace, Mush, PRD **106**, 114501]

- Convergence to predictions of SU(2)

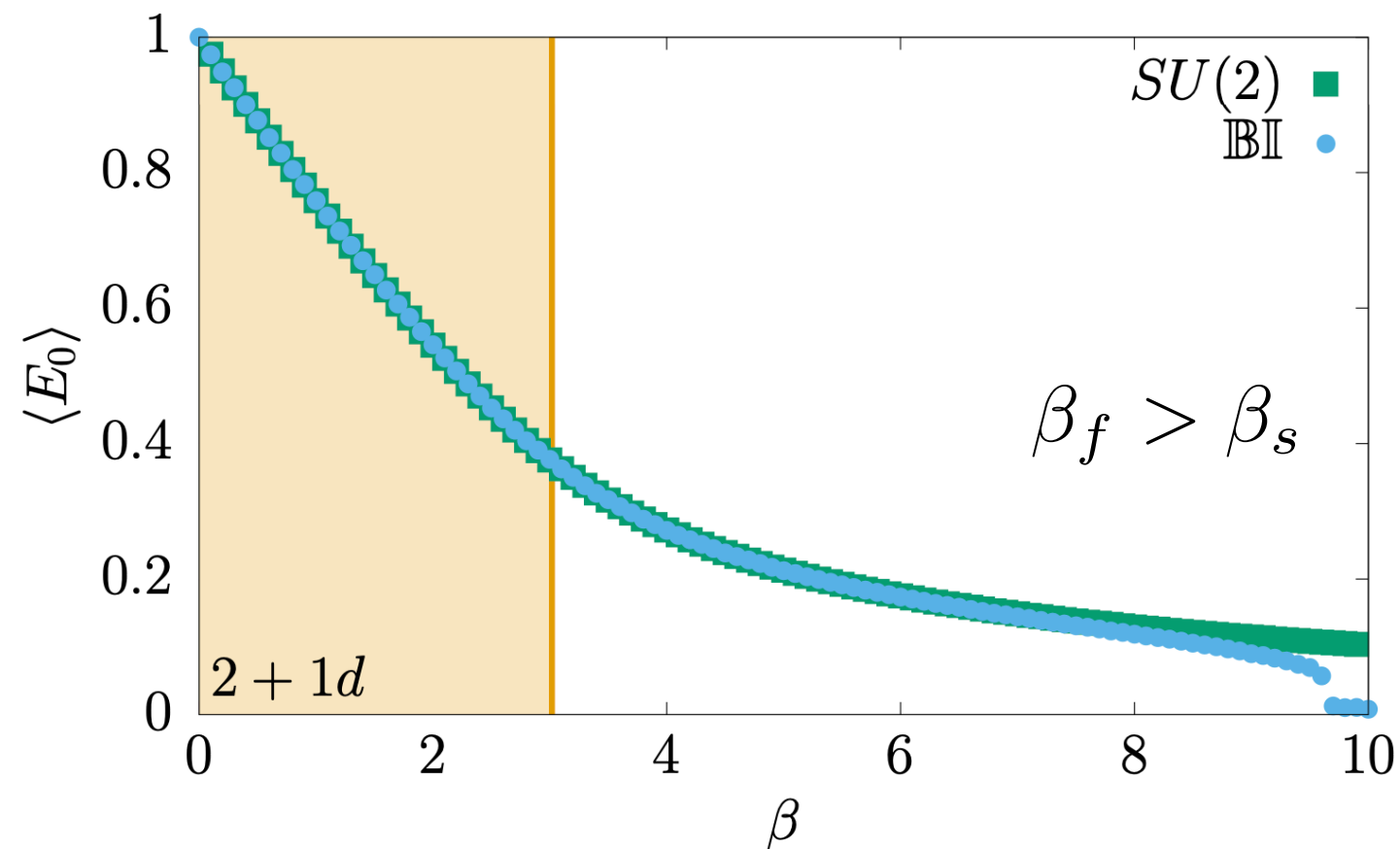
[Lamm, [YYL](#), Shu, Wang, Xu, PRD 110 (2024) 5, 054505]

In the scaling regime:
reduces errors in simulating SU(2)
gauge theories

[Gustafson, Lamm, Lovelace, Mush, PRD **106**, 114501]

Extend the study to SU(3)

- Convergence to predictions of SU(2)

[Lamm, [YYL](#), Shu, Wang, Xu, PRD 110 (2024) 5, 054505]

In the scaling regime:
reduces errors in simulating SU(2)
gauge theories

The Hilbert space is gauge redundant!!!
need to do gauge fixing at some stage...

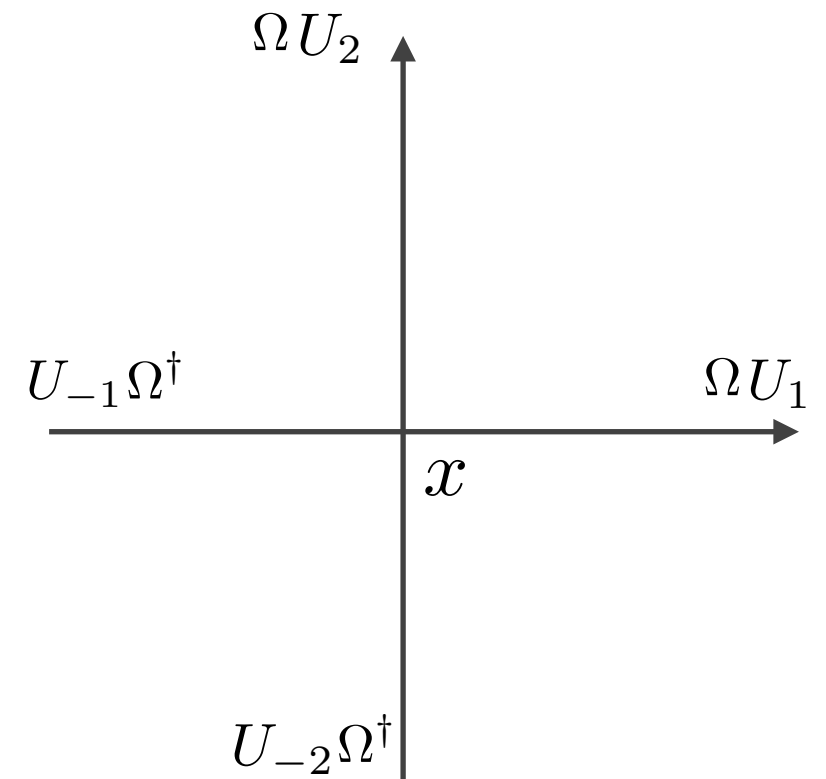
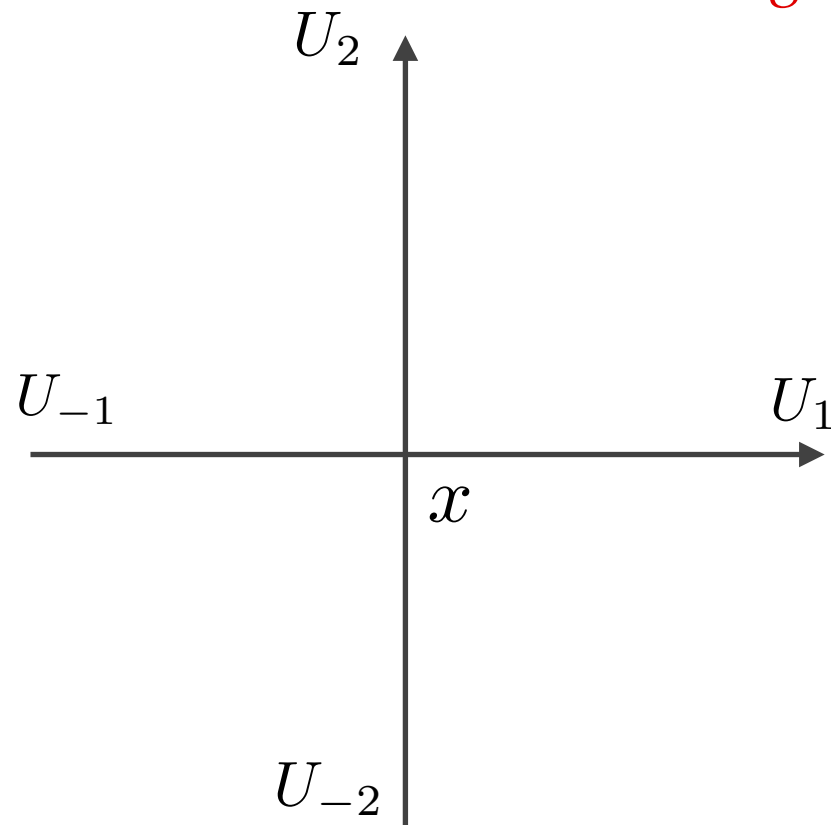
Digitization

$$|q\rangle^N \rightarrow |G\rangle$$

infinities in QFT

$$\hat{\Theta}_\Omega(x) = \exp(i\phi(\Omega)\hat{G}^a(x))$$

$\xrightarrow{\text{gauge transformation}}$



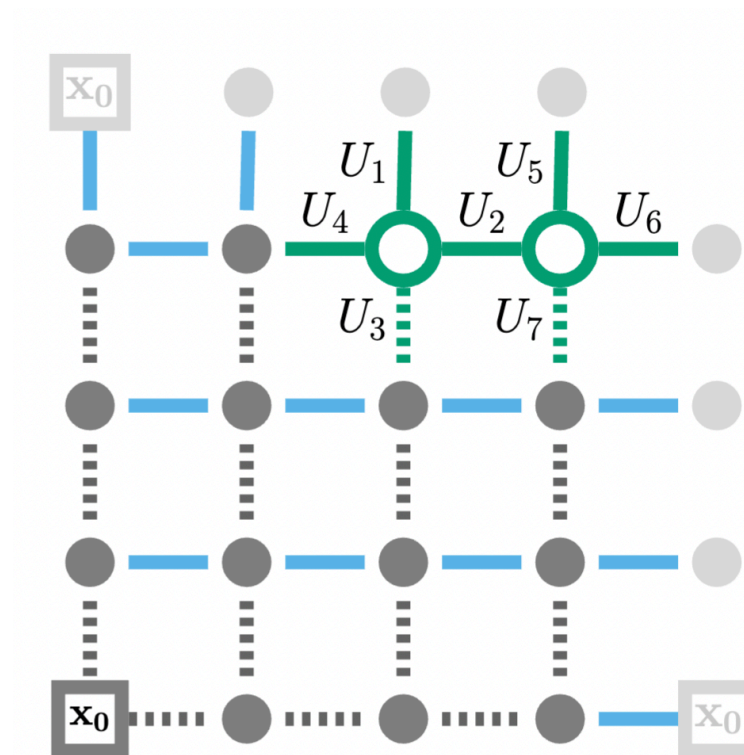
gauge equivalent states

$$\hat{\Theta}_\Omega(x) |U_{-1}U_1U_{-2}U_2\rangle = |U'_{-1}U'_1U'_{-2}U'_2\rangle$$

Digitization

$$|q\rangle^N \rightarrow |G\rangle$$

infinities in QFT



*Gauge redundancy,
keep it in the Hilbert
space, or not keep it*

gauge fixed

$$\hat{\Theta}_\Omega(x) |\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$\hat{G}^a(x) |\psi_{\text{phys}}\rangle = 0 \quad (\text{Gauss's law})$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L - N_V + 1}$$

[see, e.g. Loop-string-hadron, arXiv:2407.19181]

$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L}$$

$$\hat{\Theta}_\Omega(x) |U_1 U_2 U_3 U_4\rangle = |U'_1 U'_2 U'_3 U'_4\rangle$$

gauge redundant

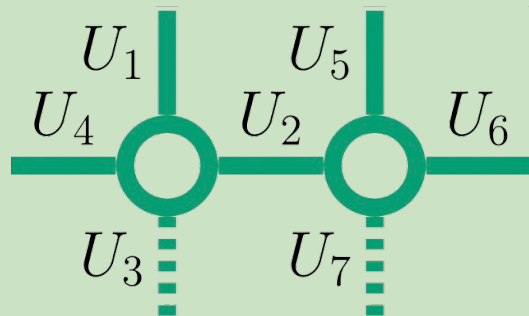
Hamiltonian complexity increases

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L - N_V + 1}$$

$$\hat{\Theta}_\Omega(x) |\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$H_{KS} = \sum (\text{---}\blacktriangleright\text{---} + \text{---}\text{---}\text{---}\text{---})$$

$K_L \qquad U_\square$



kinetic terms for U_3, U_7
depend on other links

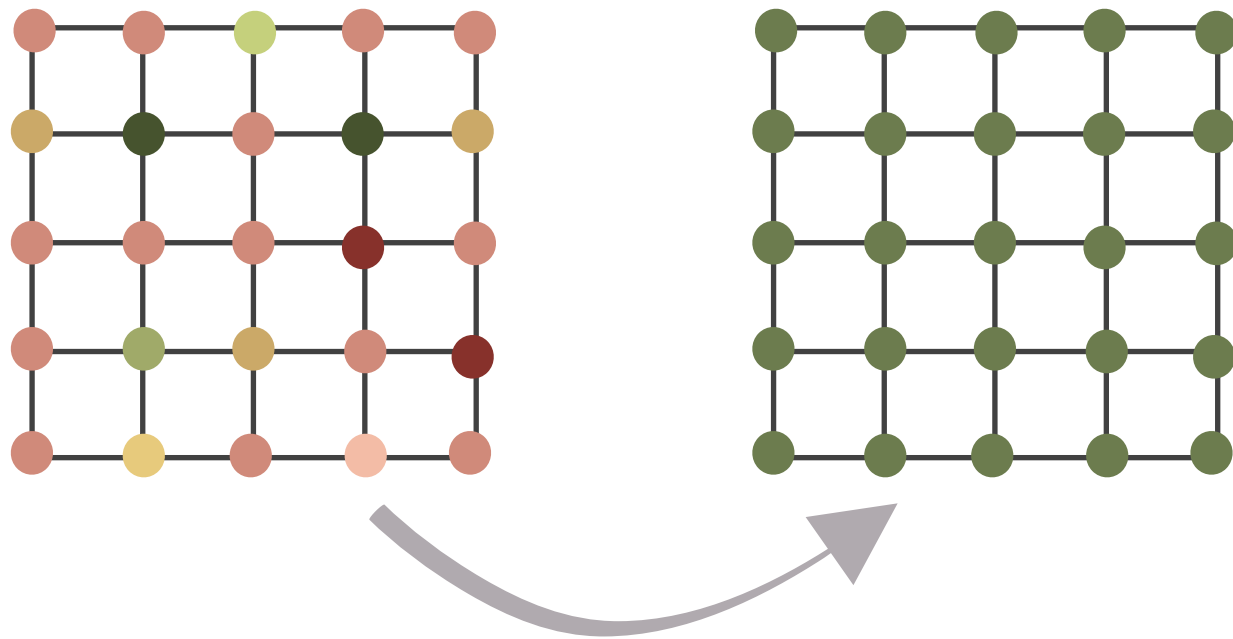
$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L}$$

$$H_{KS} = \sum (\text{---}\blacktriangleright\text{---} + \text{---}\text{---}\text{---}\text{---})$$

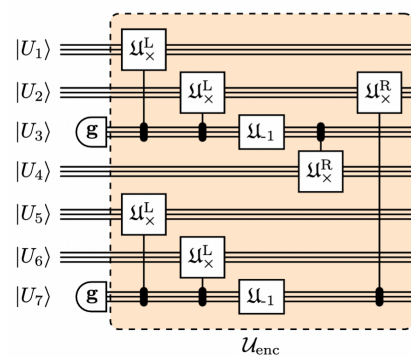
$K_L \qquad U_\square$

When keeping the redundancies...

[Carena, Lamm, **YYL**, Liu, PRD 110 (2024) 5, 054516]



Prepare gauge-
invariant states
(gauge fixing)



Prepare gauge-invariant states using **quantum circuits**

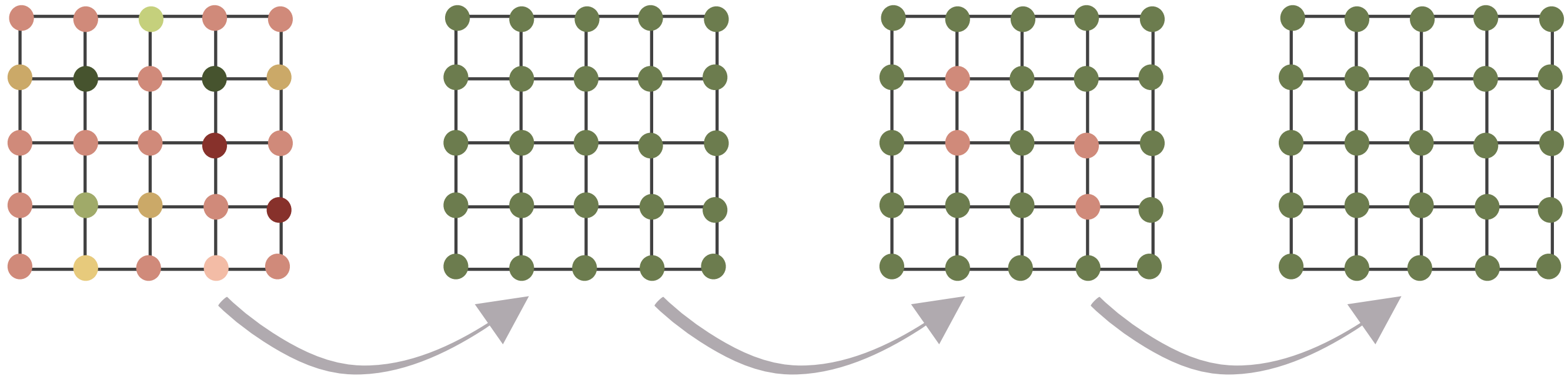
Digitization

$$|q\rangle^N \rightarrow |G\rangle$$

infinities in QFT

When keeping the redundancies...

[Carena, Lamm, **YYL**, Liu, PRD 110 (2024) 5, 054516]

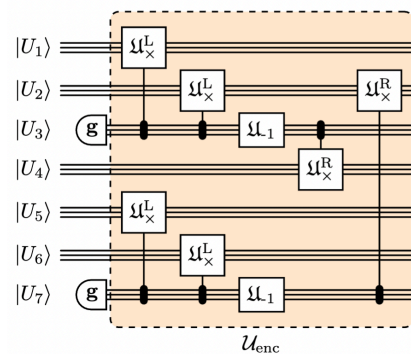


Prepare gauge-invariant states (gauge fixing)

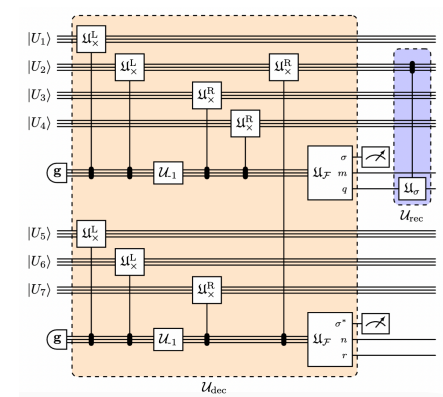
Quantum noises are present in the time evolution

Measure Gauss's Law and recover gauge invariance

KL condition



$$\mathcal{U}(t) = e^{-iH_K st}$$



Utilize the gauge redundancies for quantum error corrections

[Carena, Lamm, [YYL](#), Liu, PRD 110 (2024) 5, 054516]

Fidelity

$\mathcal{H}_{\text{full}} = \mathcal{H}_{\text{inv}}$

$$F_{\text{fixed}} \geq (1 - \epsilon)^{N_L - N_T}$$

$\mathcal{H}_{\text{full}} = \mathcal{H}_{\text{gauge}}$

$$F_{\text{red}} \geq (1 - \epsilon)^{N_L}$$

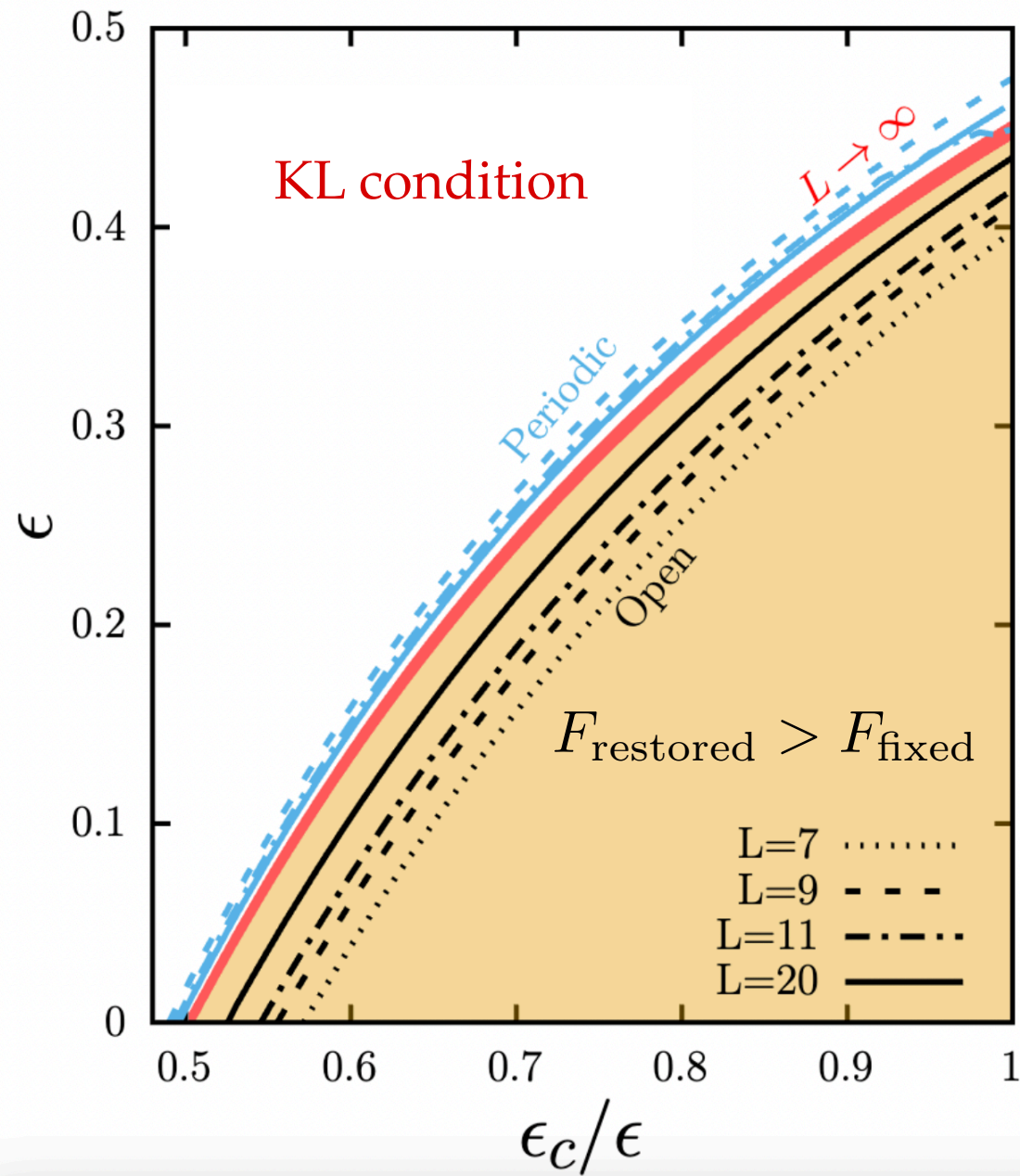
$\mathcal{H}_{\text{full}} = \mathcal{H}_{\text{gauge}}$

restore gauge symmetry

$$F_{\text{restored}} \geq \sum_{n=0}^{N_L} Q_n \epsilon_c^n (1 - \epsilon)^{N_L - n}$$

 $\epsilon < \epsilon_{th}$: redundancy makes the code more error-proof $\epsilon > \epsilon_{th}$: redundancy makes more errors than it can correct

2d lattice

[Carena, Lamm, [YYL](#), Liu, PRD 110 (2024) 5, 054516]

- Below the error threshold, keeping gauge redundancies together with quantum error corrections leads to **higher fidelity**.

Summary

- **Exponential circuit-depth reduction** for scalar QFT time evolution in the occupation basis.
- **Hardware-efficient quantum simulation framework** for QED, targeting intense-field QED.
- **Improved digitization of non-Abelian gauge theories** with scalable encodings for finite groups.
- **Reduced complexity of non-Abelian gauge-theory time evolution** by exploiting gauge redundancy.
- **Enhanced quantum error correction fidelity** through gauge redundancy.

Note: Efficient algorithm to simulation non-Abelian gauge theory is still missing...

“Quantum potential for first-principle calculations!”

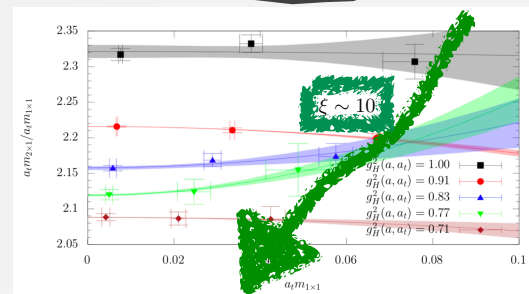
(2030s) narrow down the framework with

- systematic studies of errors from digitization
 - hardware co-design
 - improving algorithms
 - benchmark studies
 - ...
- phase diagrams with spontaneous symmetry breaking, and for improved H
 - qudits for blocking encodings and the redundancies
 - efficient quantum circuits based on qudits
 - HEP case calculations for experiments



$$\begin{aligned} |q\rangle^N &\rightarrow |G\rangle \\ \downarrow \\ \mathcal{U} |G\rangle^L &\rightarrow |\psi_0\rangle \\ \downarrow \\ \mathcal{U} |\psi_0\rangle &\rightarrow |\psi(t)\rangle \\ \downarrow \\ \langle \mathcal{O} \rangle \end{aligned}$$

various
methods



2030s -

S. P. Jordan,
K. S. M. Lee,
J. Preskill



2020 -



2011-

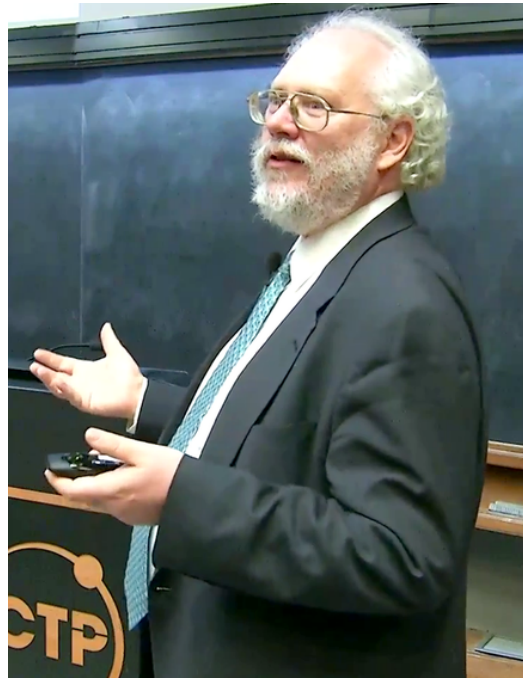
Thank you

LQCD - Quantum methods



1990s - error-correction codes and fault-tolerant methods

Quantum Threshold Theorem



P. Shor



2019 - Google: Quantum Computational Supremacy

The quantum hardware is
producing meaningful results

