Spin density matrix and Quantum Observables of a two-qubit system at Colliders

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Quantum Entanglement @ LHC

$> 5\sigma$ deviations

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)

Nature 633 (2024) 542
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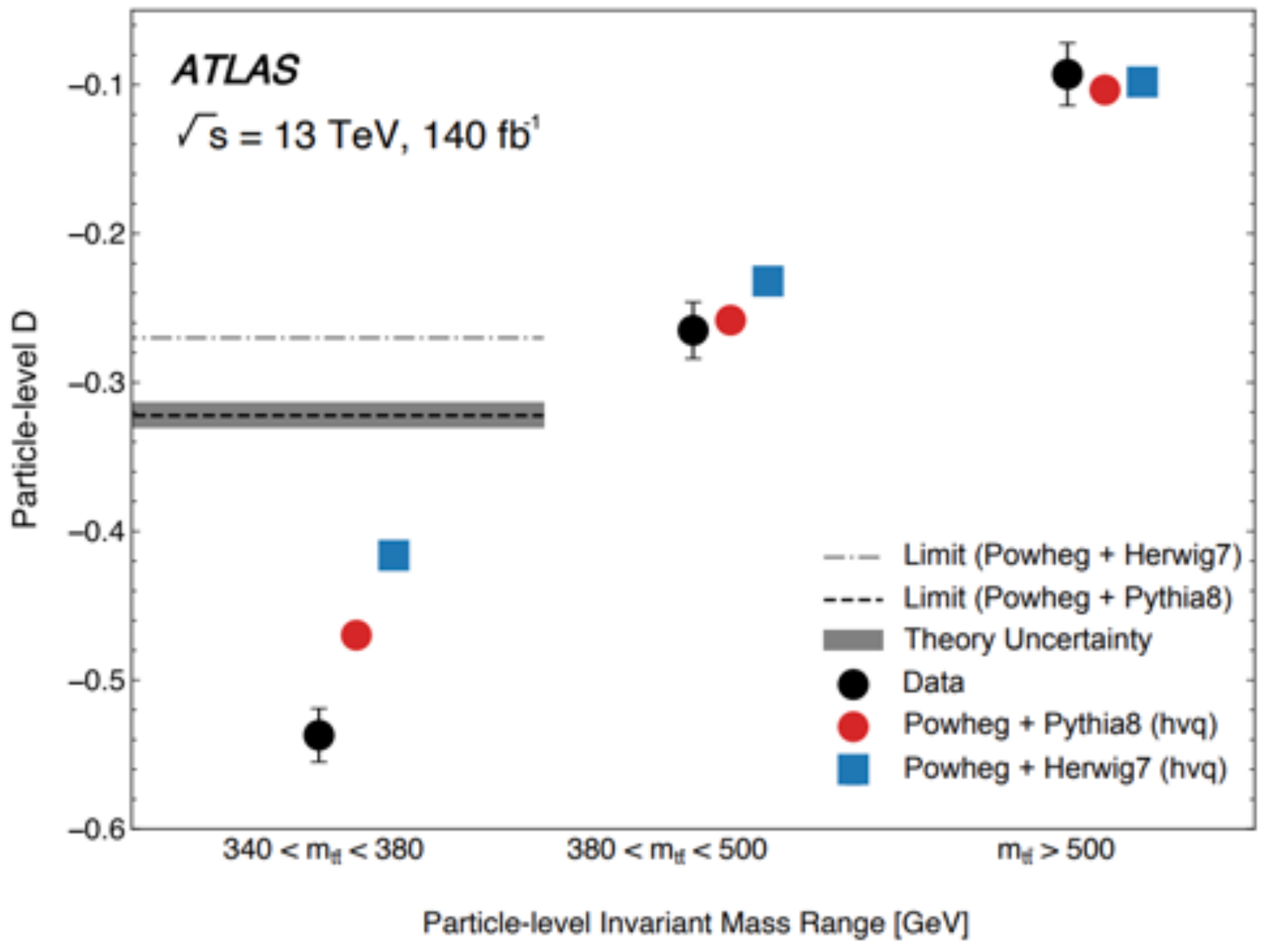
CERN-EP-2023-230
October 3, 2024

Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

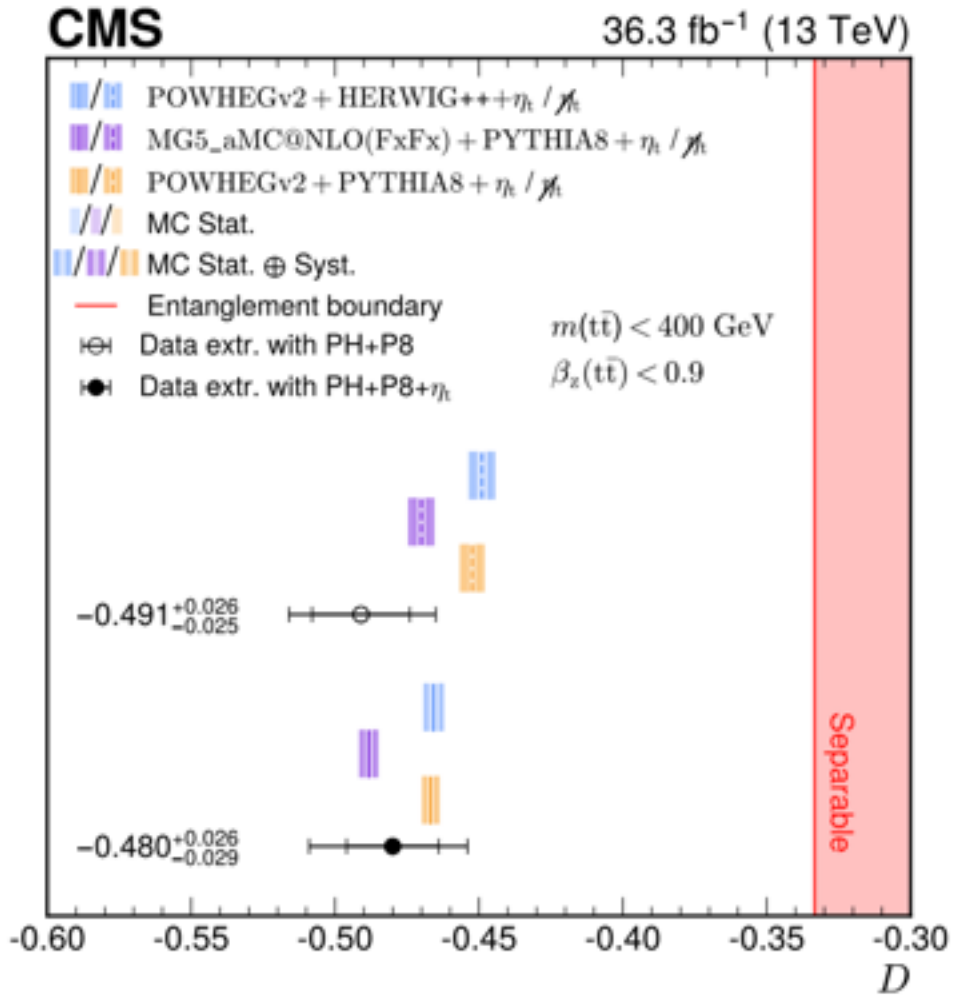


EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-EP-2024-137
2024/10/30
CMS-TOP-23-001

Observation of quantum entanglement in top quark pair production in proton-proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration*

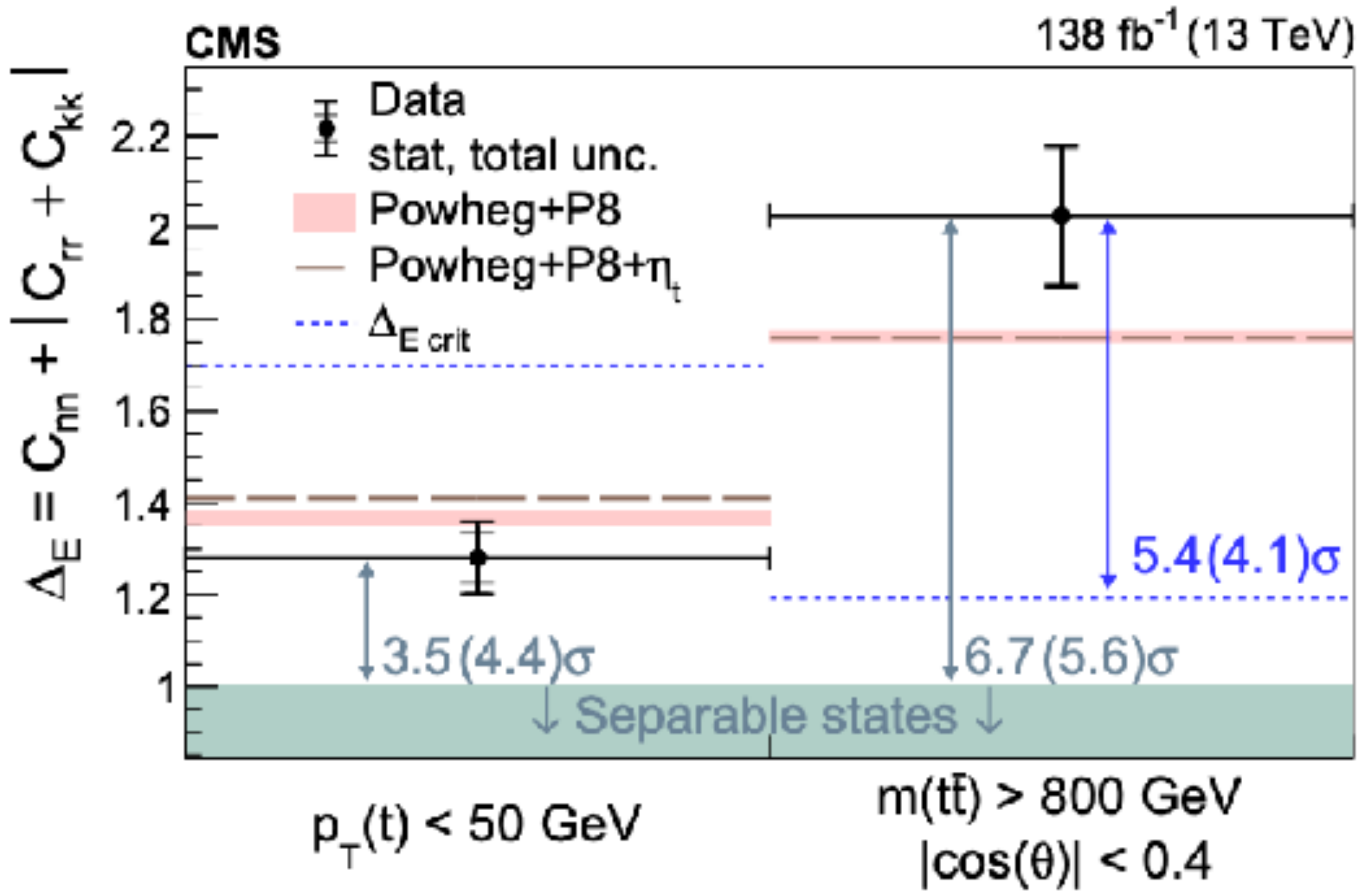


EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-EP-2024-231
2025/01/09
CMS-TOP-23-007

Measurements of polarization and spin correlation and observation of entanglement in top quark pairs using lepton+jets events from proton-proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration*



Quantum Entanglement @TH

- Emergence of symmetries

  - Maximal Entanglement Weinberg angle $\theta_w \approx \dfrac{\pi}{6}$ [Cervera-Lierta, Latorre, Rojo, and Rotolli '17]

  - Entanglement suppression

    SU(4) Spin-flavor symmetry [Beane, Kaplan, Klco and Savage '18]

    CKM/PMNS flavor pattern [Thaler, Trifinopoulos '24]

    SO(8) symmetry in THDM [Carena, Low, Wagner, and Xiao '24]

- Non-perturbative QCD effects [Von Kuk, Lee, Johannes, and Sun '25]

Quantum Entanglement @PH

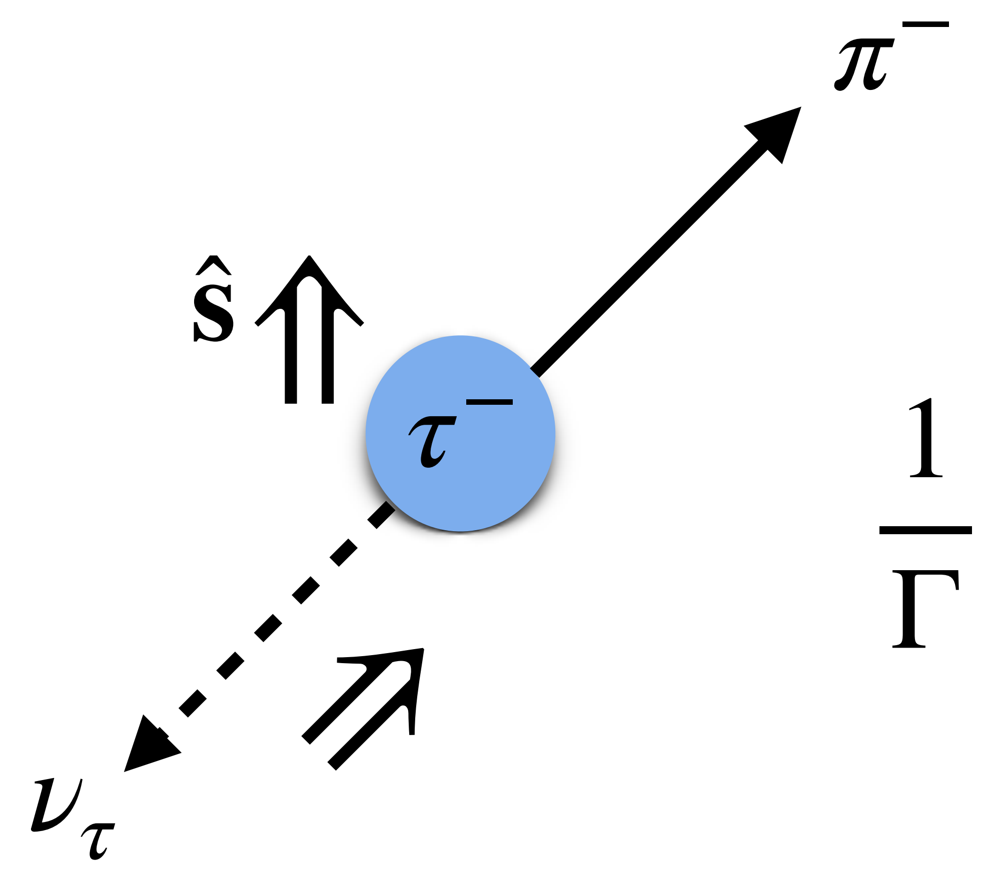
- We use spin correlations as a diagnostic tool to reveal underlying quantum correlations, notably entanglement, in high-energy processes involving heavy-particle production and decay.

  - Top quark pair production [Afik, Nova '20] [Fabbrichesi, Floreanini, Panizzo '21] [Severi, Boschi, Maltoni, Sioli '22] [Aguilar-Saavedra, Casas '22] [Han Low, Barr '23] [Aguilar-Saavedra '23, '24] [Dong, Gonçalves, Kong, Navarro '23] [Maltoni, Severi, Tentori, Vryonidou '24]
  - Tau pair production [Ehatäht, Fabbrichesi, Marzola and Veelken, '23] [Altakach, Lamba, Maltoni, Mawatari, and Sakurai '22] [Fabbrichesi, Marzola '24] [Zhang, Zhou, Liu, Li, Hsu, Han'25]
  - Higgs decays [Barr '21] [Saavedra, Bernal, Casas, Moreno '23] [Bernal, Caban, Rembielinski '23], [Ma, Li '23] [Aguilar-Saavedra '24] [Morales '24] [Sullivan '24] [Del Gratta et al '24] [Gonçalves, Kaladharan, Krauss, Navarro '25] [Gonçalves, Kaladharan, Navarro '25].
  - Vector boson production [Aoude, Madge, Maltoni, Mantani '22] [Fabbrichesi, Floreanini, Gabrielli, Marzola '22] [Barr, Caban, Rembielinski '23] [Morales '23] [Ashby-Pickering, Barr, Wierzchucka '23] [Grossi, Pellliccioli, Vicini '24]

Quantum Tomography

- Polarimeter at collider = particle decay

Ex) $\tau^- \rightarrow \pi^- + \nu$ (at τ^-rest frame)



$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} = \frac{1 + (\hat{\mathbf{p}}_{\pi^-} \cdot \hat{\mathbf{s}})}{2}$$

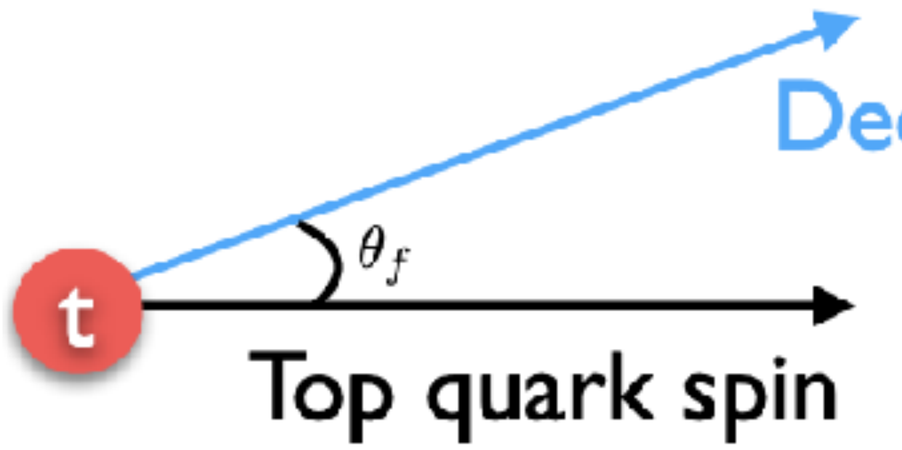
In general,

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\Omega} = \frac{1 + \omega_f(\hat{\mathbf{p}}_f \cdot \hat{\mathbf{s}})}{2}$$

Spin analyzing power

$(\omega_f \in [-1,1])$

For top quark, $\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \omega_f \cos\theta_f)$



	$l^+, \bar{d}$	b	$\bar{\nu}, u$
ω_f	$\boxed{1}$	-0.4	-0.3

Quantum Tomography

- One qubit characterized by 3 parameters

$$\rho = \frac{I_2 + B_i\sigma_i}{2} \qquad B_i = \langle\sigma_i\rangle = tr(\sigma_i\rho)$$

- Two qubit characterized by 15 parameters

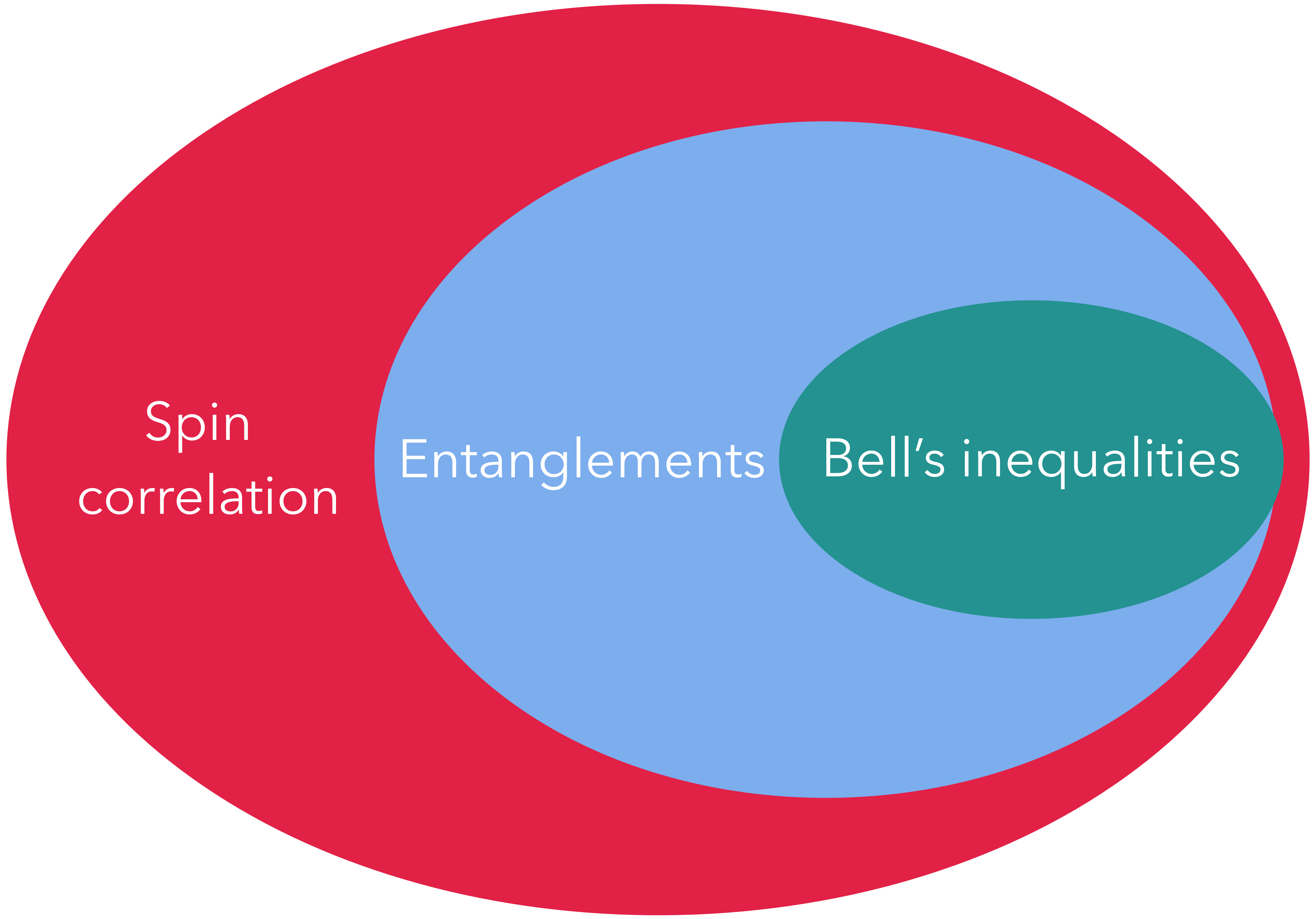
$$\rho = \frac{I_2 \otimes I_2 + B_i\sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

$B_i = \langle\sigma_i \otimes I_2\rangle$

$\bar{B}_i = \langle I_2 \otimes \sigma_i\rangle$

Polarizations

$C_{ij} = \langle\sigma_i \otimes \sigma_j\rangle$ Spin correlations



- Symmetry of theory is very useful.
- Ex) P and CP invariance of QCD

$\rightarrow B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$,

in , $t\bar{t}$ production.

Density Matrix Formalism

$$\rho = |\psi\rangle\langle\psi| \text{ (pure state)}, \qquad \rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \text{ (mixed state)} \qquad \left(p_i \geq 0, \quad \sum_i p_i = 1\right)$$

- ρ are Hermitian, $Tr = 1$, and positive semidefinite.

$$\rho = \rho^\dagger, \; Tr[\rho] = 1, \rho \geq 0$$

- **Purity**, $Tr[\rho^2]$ is 1 for **pure state** and $\dfrac{1}{D}$ for **completely mixed state**,

$$\frac{1}{D} \leq Tr[\rho^2] \leq 1, \qquad (D \text{ is dimension of the Hilbert space})$$

- We can compute expectation value of $\hat{\mathcal{O}}$ as

$$\langle\hat{\mathcal{O}}\rangle = Tr[\hat{\mathcal{O}}\rho]$$

Quantum Entanglement

- A quantum state of two subsystems A and B is **separable** when its density matrix ρ can be expressed as a convex sum

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i \qquad \left(p_i \geq 0, \quad \sum_i p_i = 1 \right)$$

- If the state is not separable, it is named **entangled**

  - **Peres-Horodecki criterion**

  - **Concurrence**

  - $D = \frac{1}{3} Tr[C]$

  - **Clauser-Horne-Shimony-Holt (CHSH) Inequality**

Peres Horodecki Criterion

[Peres '96; Horodecki '97]

- The **Peres-Horodecki criterion** provides a necessary and sufficient condition for entanglement in two-qubit systems and qubit-qutrit systems:

Take the **partial transpose** (transpose indices associated only to Bob (or Alice))

$$\rho^{T2} = \sum_i p_i \rho_A^i \otimes (\rho_B^i)^T$$

For a separable system, ρ^{T2} is non-negative

$\rightarrow$ If ρ^{T2} has at least one negative eigenvalue, the system is entangled.

Peres Horodecki Criterion

[Peres '96; Horodecki '97]

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We consider the entanglement quantifier of negativity $\mathcal{N}(\rho)$ as

$$\mathcal{N}(\rho) = \sum_k \frac{|\lambda_k| - \lambda_k}{2} = \sum_{\lambda_k<0} |\lambda_k|$$

with λ_k being the eigenvalues of the ρ^{T2}.

Concurrence Criterion

- **Concurrence**

[Wooters '98]

For a separable state, the concurrence $C(\rho) = 0$ and for an entangled state, the concurrence is $C(\rho) > 0$ and $C(\rho) = 1$ is the maximally entangled state.

$$C(\rho) = \inf_{\{p_n, |\psi_n\rangle\}} \sum_n p_n C(|\psi_n\rangle)$$

For a two qubit system, explicit formula is given

$$C(\rho) = \max\left\{0,\ \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right\}$$

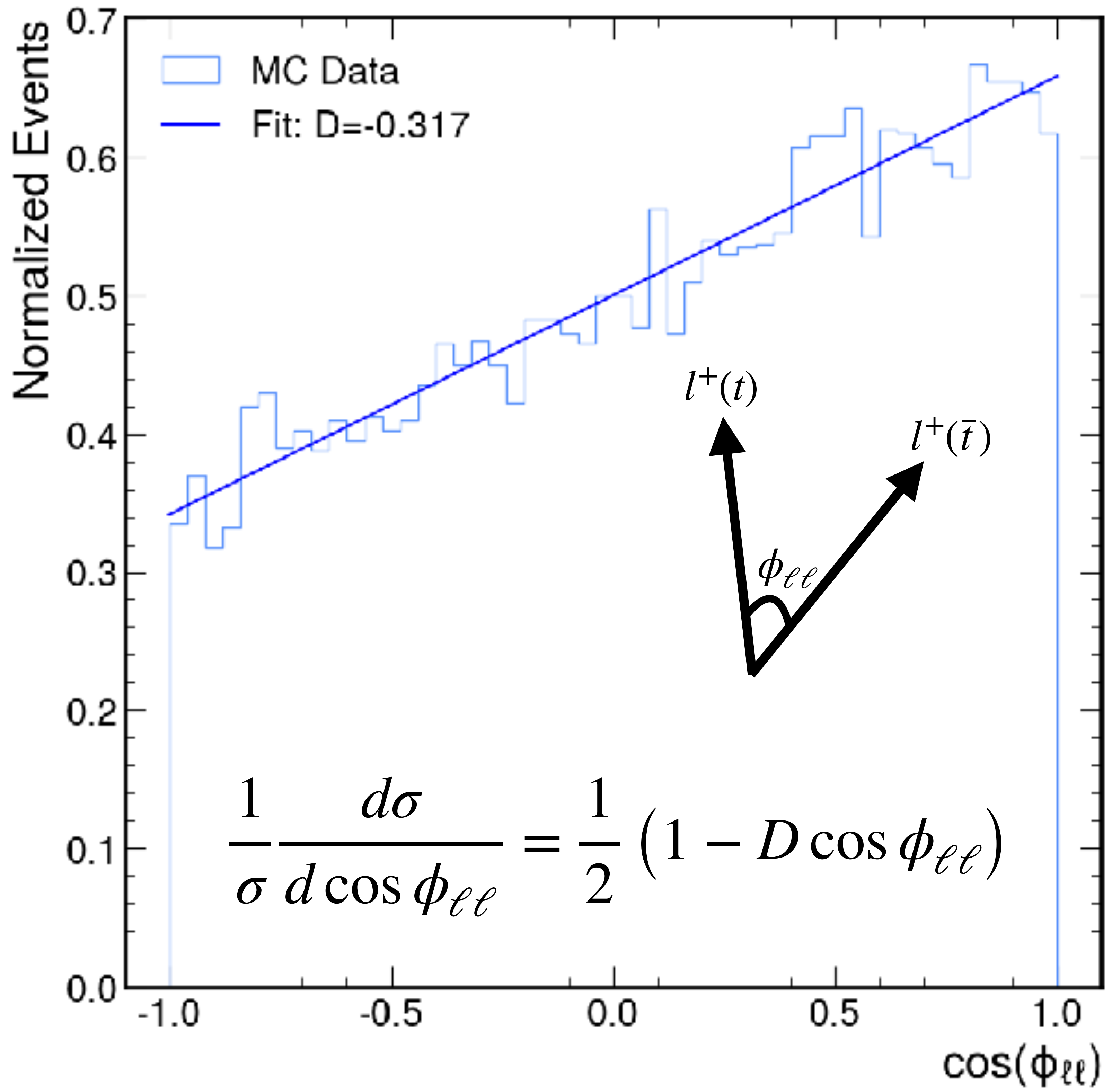
where λ_i are the eigenvalues of the matrix $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$, here $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

Entanglement marker D

[Afik, Nova '21]

- Note that this is only a **sufficient condition** for quantum entanglement adopted by the ATLAS and CMS Collaborations near the $t\bar{t}$ threshold at the LHC

$$D \equiv \frac{1}{3}Tr[C] < -\frac{1}{3}$$



We can measure it by leptons from the top

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{4\pi^2}\left(1 + \alpha_1 \vec{B}_1 \cdot \hat{\ell}_1 + \alpha_2 \vec{B}_2 \cdot \hat{\ell}_2 + \alpha_1\alpha_2 C_{ij}\hat{\ell}_{1i}\hat{\ell}_{2i}\right)$$

The spin density matrix of a two-qubit system which describes production of a pair of top-antitop quarks:

$$\rho = \frac{I_2 \otimes I_2 + B_i\sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

Bell's inequality violation

[Clauser, Horne, Shimony, Holt '69]

- **Clauser-Horne-Shimony-Holt (CHSH) inequality**

$$\left|\vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2)\right| > 2$$

[Horodecki, Horodecki, Horodecki '95]

- Augmented by the Horodecki condition

$$m_{12} \equiv m_1 + m_2 > 1$$

where $m_1 \geq m_2 \geq m_3$ are the three eigenvalues of the following matrix

$$M = CC^T$$

with C being the spin correlation matrix denoted by C_{ij}

Quantum Magic

[Gottesman '98]

- Gottesman–Knill theorem
  Stabilizer circuit can be **perfectly simulated** in polynomial time on a probabilistic **classical computer**.

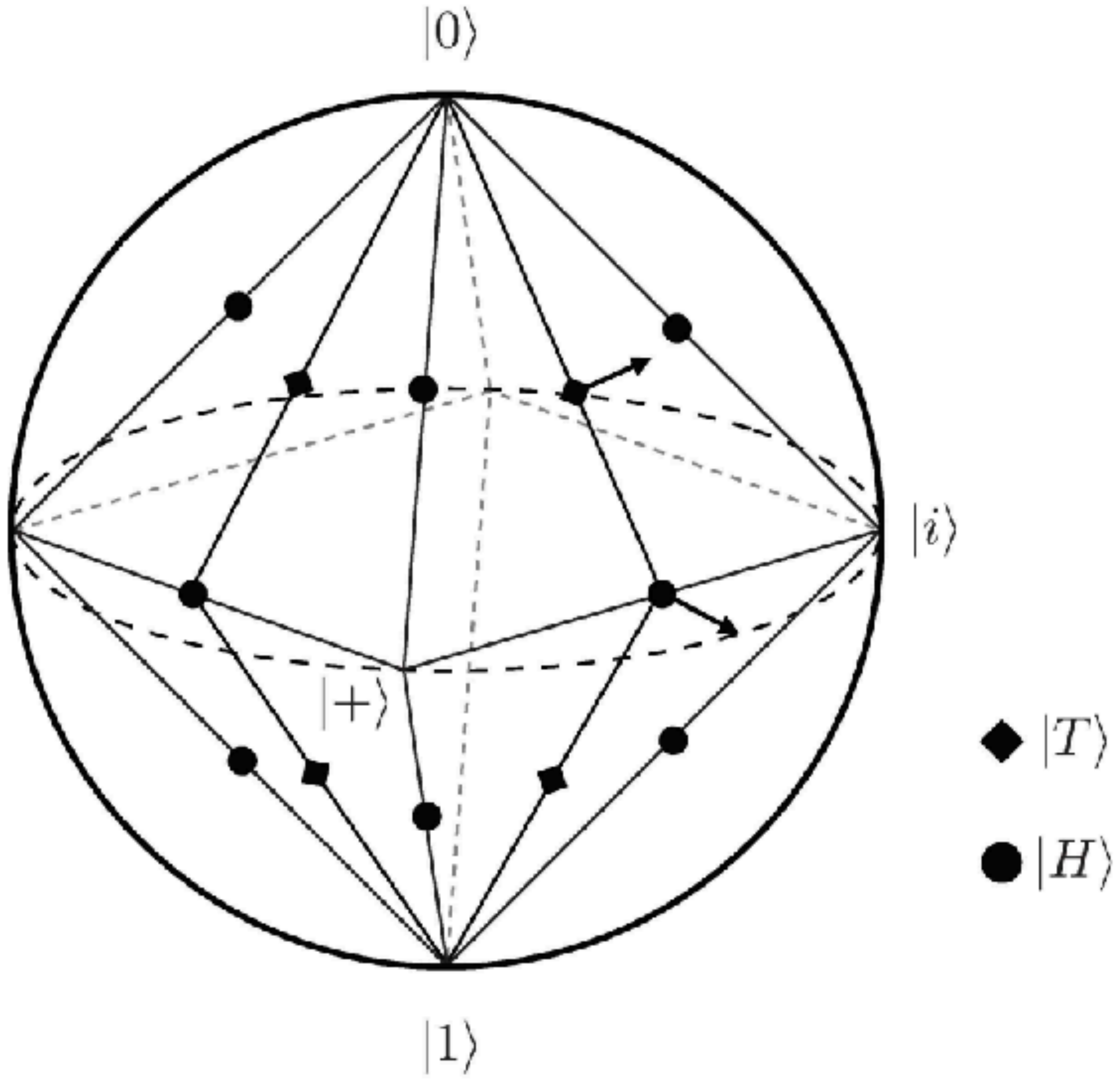
  1. Preparation of qubits in computational-basis states.
  2. Clifford gates (generated by the Hadamard gate, CNOT gate, and phase gate S).
  3. Measurements in the computational basis.

- A magic quantifies the ***non-stabiliser*** resource required for universal quantum computation. The stabilizer Rényi entropy is commonly used. [Leone, Oliviero, and Hamma '21]

$$M_2[\rho] = -\log_2 \left[\frac{1 + \sum_{i=1}^{3} (B_i)^4 + \sum_{j=1}^{3} (\overline{B}_j)^4 + \sum_{i,j=1}^{3} (C_{ij})^4}{1 + \sum_{i=1}^{3} (B_i)^2 + \sum_{j=1}^{3} (\overline{B}_j)^2 + \sum_{i,j=1}^{3} (C_{ij})^2} \right]$$

Quantum Magic

[Gottesman '98]



Credit: García-Álvarez, Ferraro, Ferrini, 1911.12346

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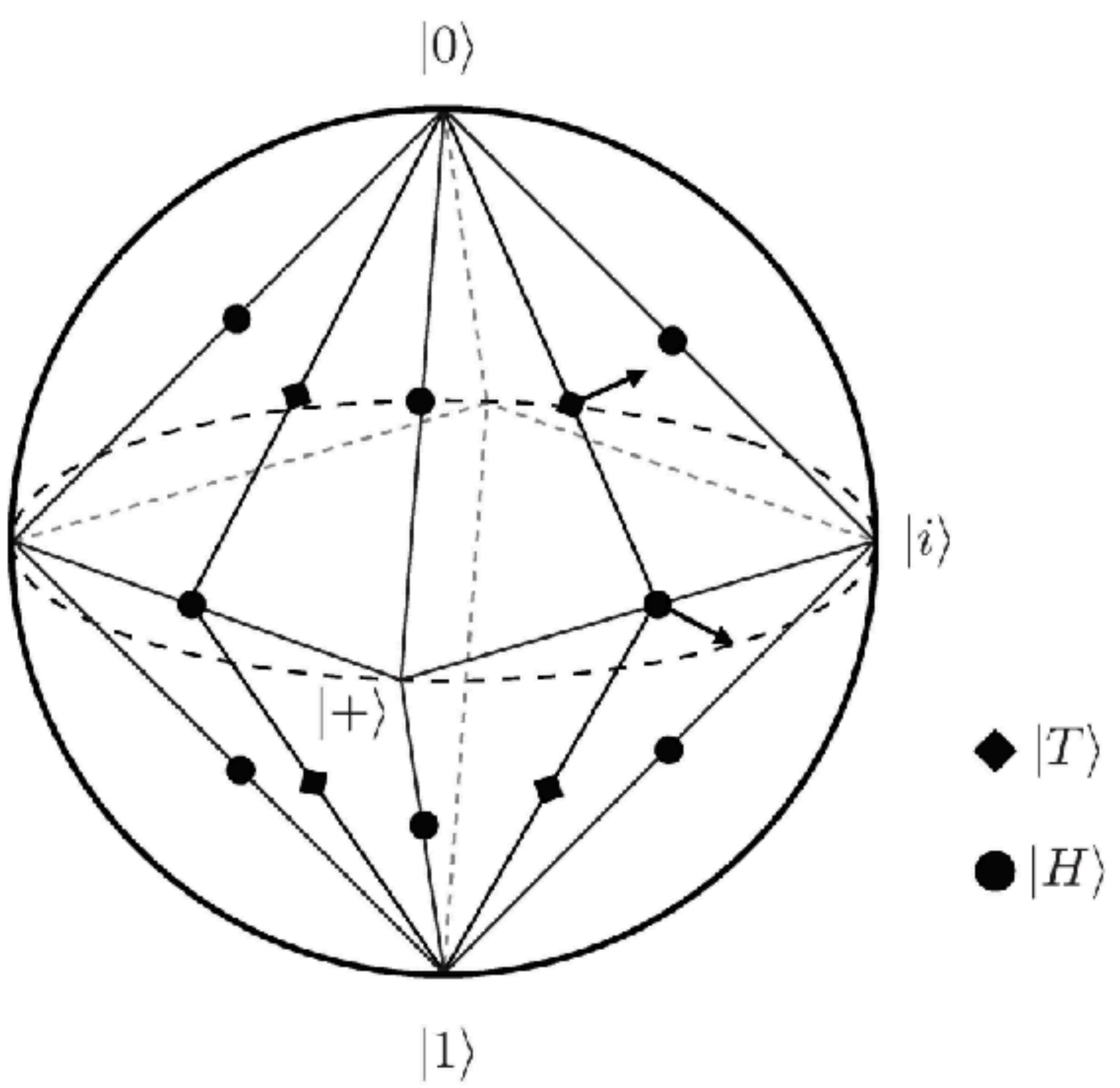
Quantum Magic



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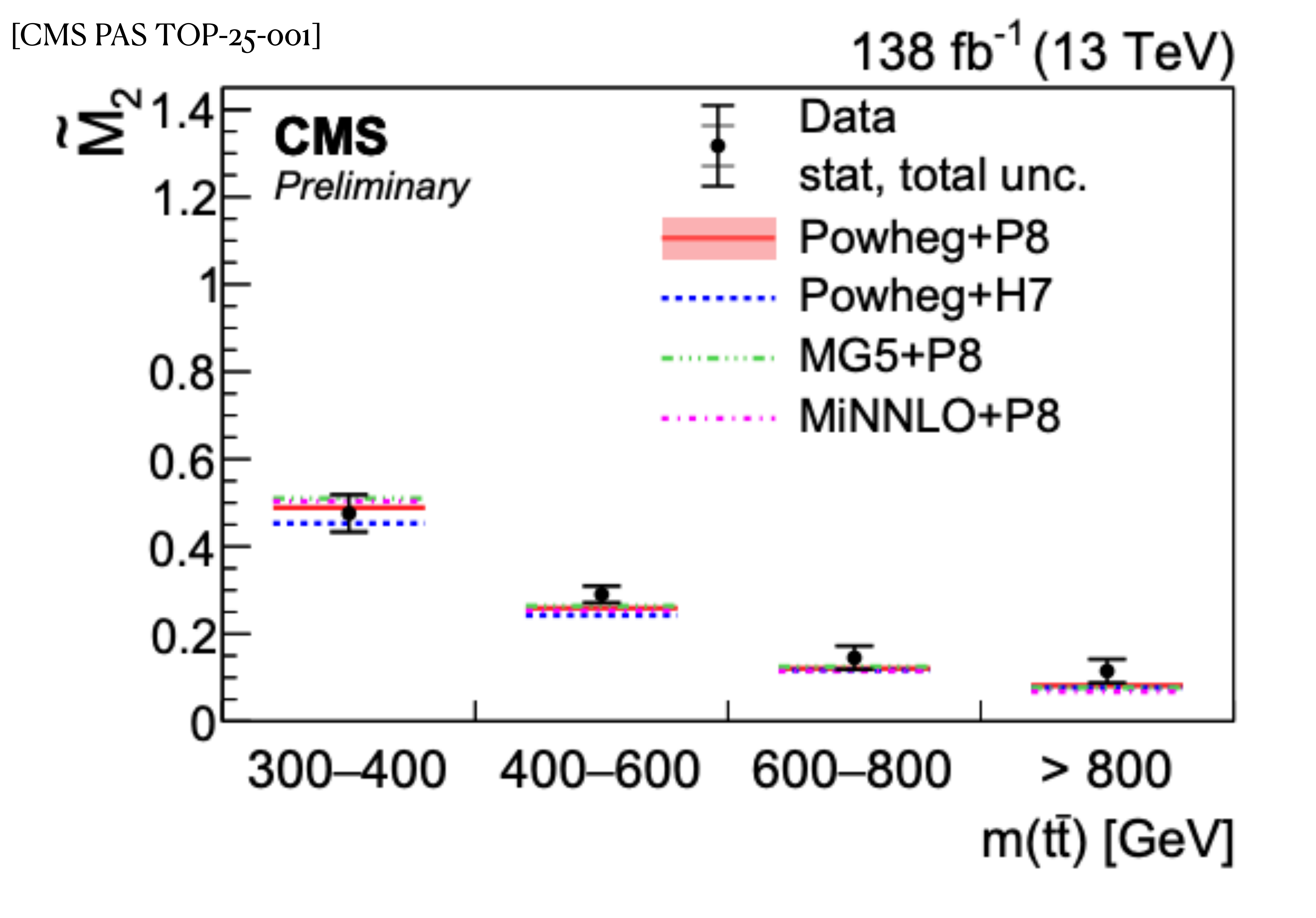

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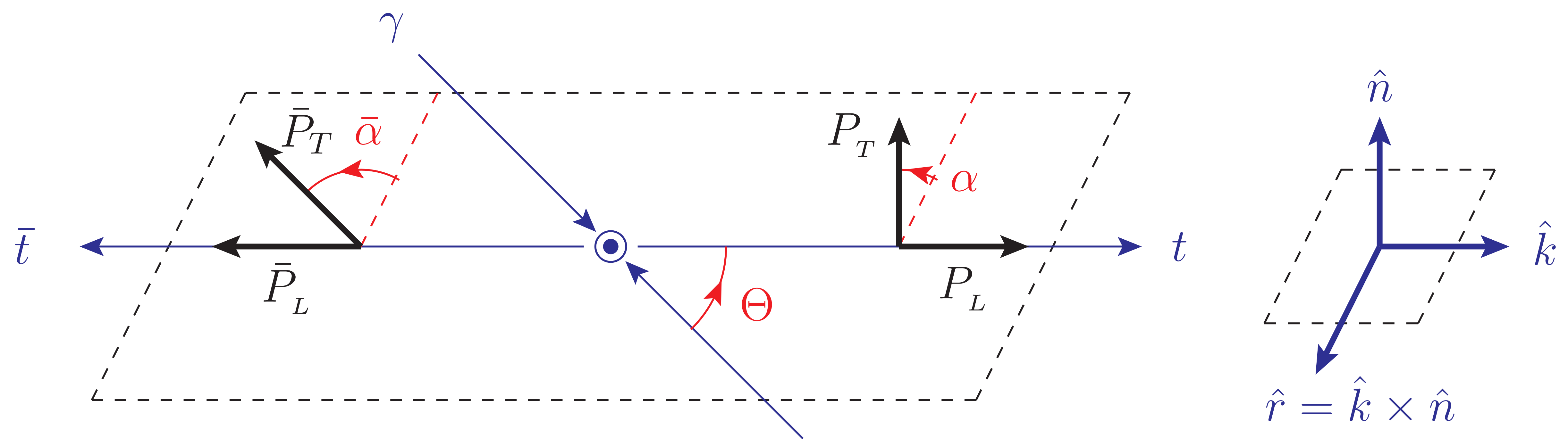
Spin density matrix of a two-qubit system



- Let us consider a process

$$\gamma(k_1, \lambda_1) + \gamma(k_2, \lambda_2) \;\to\; t(p, \sigma) + \bar{t}(\bar{p}, \bar{\sigma})$$

- The helicity amplitude is given by

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} \equiv \mathcal{A}_C \langle \sigma\bar{\sigma}; \lambda_1\lambda_2 \rangle = \frac{8\pi\alpha Q_t^2}{1-\beta^2\cos^2\Theta} \langle \sigma\bar{\sigma}; \lambda_1\lambda_2 \rangle$$

where $\beta^2 = 1 - 4M_t^2/\hat{s}$.

- The reduced amplitude transform under P, CP, and CP$\tilde{\text{T}}$:

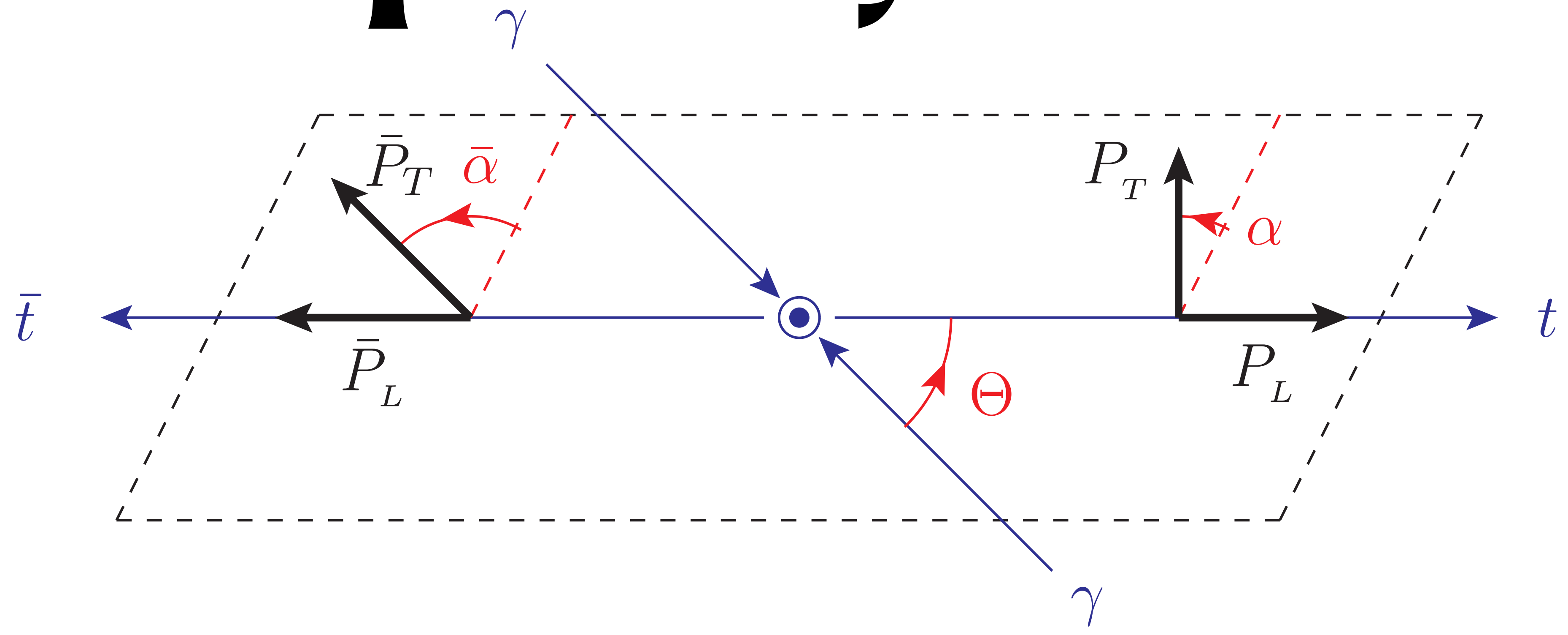
$$\langle \sigma\bar{\sigma}; \lambda_1, \lambda_2 \rangle \;\xleftrightarrow{\text{P}}\; -(-1)^{(\sigma-\bar{\sigma})/2} \langle -\sigma, -\bar{\sigma}; -\lambda_1, -\lambda_2 \rangle \,,$$

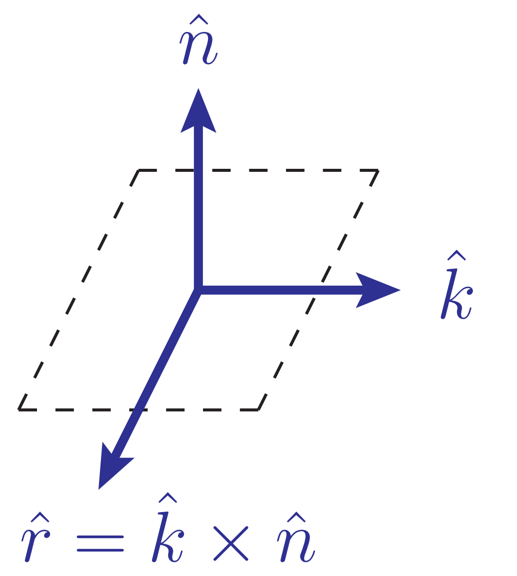
$$\langle \sigma\bar{\sigma}; \lambda_1, \lambda_2 \rangle \;\xleftrightarrow{\text{CP}}\; -(-1)^{(\sigma-\bar{\sigma})/2} \langle -\bar{\sigma}, -\sigma; -\lambda_2, -\lambda_1 \rangle \,,$$

$$\langle \sigma\bar{\sigma}; \lambda_1, \lambda_2 \rangle \;\xleftrightarrow{\text{CP}\tilde{\text{T}}}\; -(-1)^{(\sigma-\bar{\sigma})/2} \langle -\bar{\sigma}, -\sigma; -\lambda_2, -\lambda_1 \rangle^* \,,$$

[**DWK**, S.Y. Choi, J.S. Lee, and C. Park 26']

Spin density matrix of a two-qubit system





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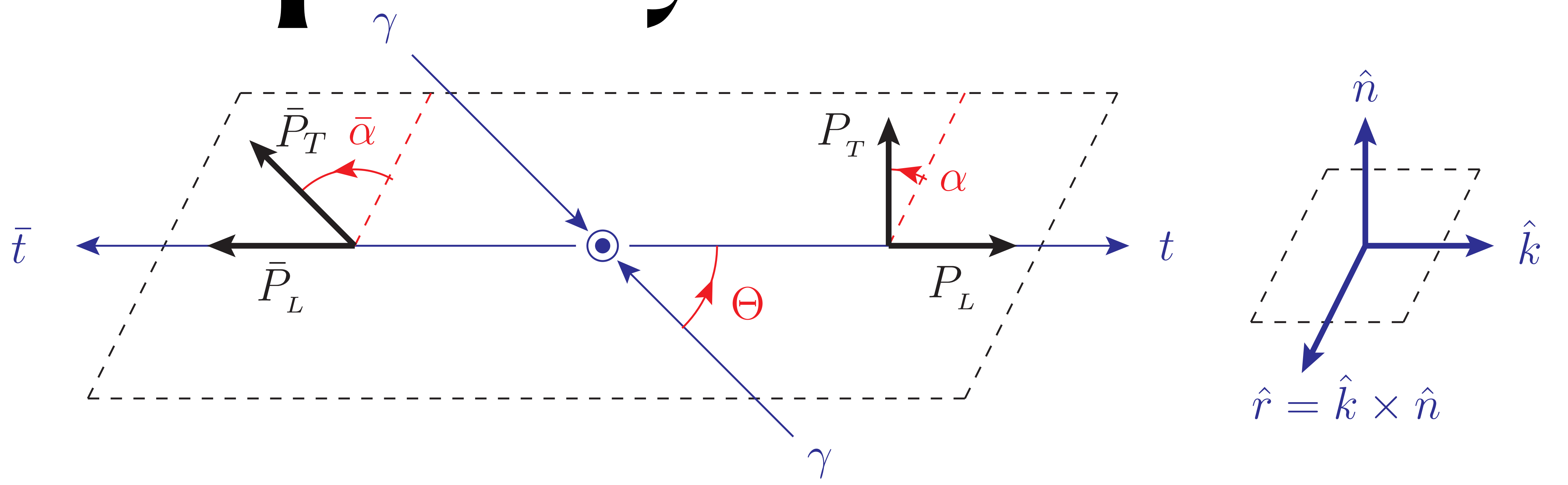
where $\beta^2 = 1 - 4M_t^2/\hat{s}$.

- The reduced amplitude :

$(\lambda_1\lambda_2)\downarrow\ (\sigma\bar{\sigma})\rightarrow$	$\langle ++\rangle$	$\langle --\rangle$	$\langle -+\rangle$	$\langle +-\rangle$
$\langle ++\rangle$	$\frac{2M_t}{\sqrt{\hat{s}}}\left[1+\beta\right]$	$\frac{2M_t}{\sqrt{\hat{s}}}\left[1-\beta\right]$	0	0
$\langle --\rangle$	$\frac{2M_t}{\sqrt{\hat{s}}}\left[-1+\beta\right]$	$\frac{2M_t}{\sqrt{\hat{s}}}\left[-1-\beta\right]$	0	0
$\langle -+\rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}}\beta\,\sin^2\Theta$	$\frac{2M_t}{\sqrt{\hat{s}}}\beta\,\sin^2\Theta$	$-\beta\sin\Theta\,(\cos\Theta+1)$	$-\beta\sin\Theta\,(\cos\Theta-1)$
$\langle +-\rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}}\beta\,\sin^2\Theta$	$\frac{2M_t}{\sqrt{\hat{s}}}\beta\,\sin^2\Theta$	$-\beta\sin\Theta\,(\cos\Theta-1)$	$-\beta\sin\Theta\,(\cos\Theta+1)$

[**DWK**, S.Y. Choi, J.S. Lee, and C. Park 26']

Spin density matrix of a two-qubit system



- The helicity amplitude can be rewritten as

$$\mathcal{M} = \mathcal{A}_C \begin{pmatrix} \langle ++; \lambda_1\lambda_2\rangle & \langle +-; \lambda_1\lambda_2\rangle \\ \langle -+; \lambda_1\lambda_2\rangle & \langle --; \lambda_1\lambda_2\rangle \end{pmatrix} \equiv \mathcal{A}_C\, \mathbf{M}$$

- The polarization-weighted amplitude squared can be obtained by

[Bernreuther, Heisler and Si '15]

$$\overline{|\mathcal{M}|^2} = \frac{1}{4} \sum_{\lambda_1,\lambda_2=\pm} \mathrm{Tr}\left[\mathcal{M}\bar{\rho}_t^T \mathcal{M}^\dagger \rho_t\right] = \frac{1}{4} |\mathcal{A}_C|^2 \sum_{\lambda_1,\lambda_2=\pm} \mathrm{Tr}\left[\mathbf{M}\bar{\rho}_t^T \mathbf{M}^\dagger \rho_t\right] \equiv \frac{1}{4} |\mathcal{A}_C|^2\, \widehat{\Sigma}$$

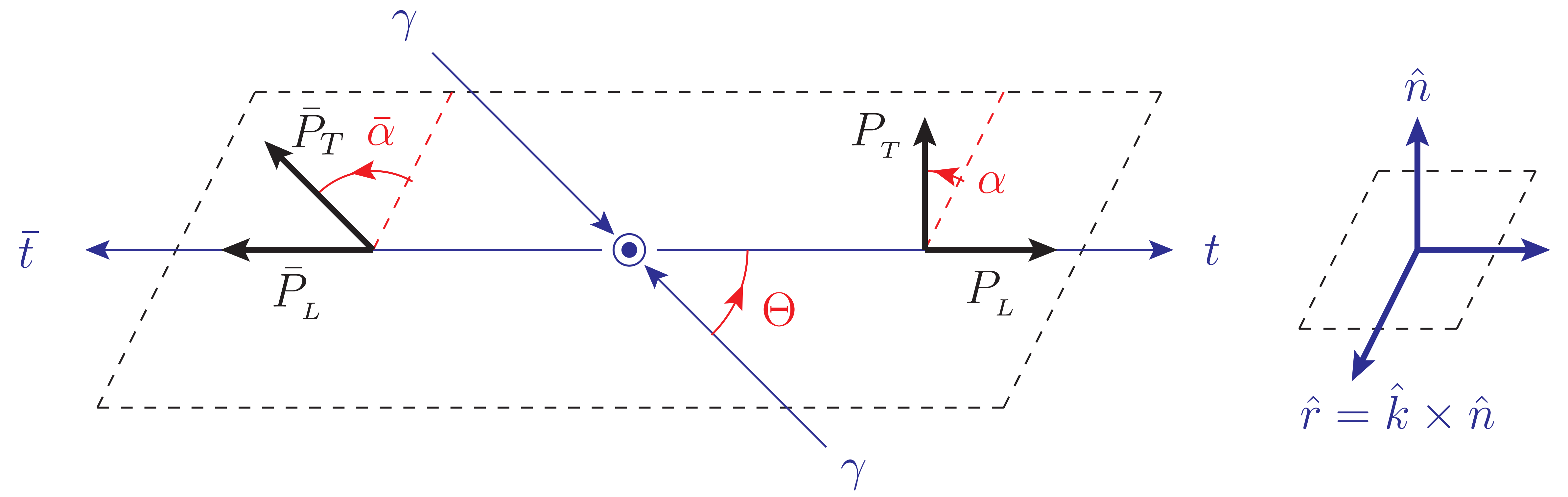
with the polarization density matrices for top and anti-top.

$$\rho_t = \frac{1}{2}\begin{pmatrix} 1+P_L & P_T\,\mathrm{e}^{-i\alpha} \\ P_T\,\mathrm{e}^{i\alpha} & 1-P_L \end{pmatrix}, \quad \bar{\rho}_t = \frac{1}{2}\begin{pmatrix} 1+\bar{P}_L & -\bar{P}_T\,\mathrm{e}^{i\bar{\alpha}} \\ -\bar{P}_T\,\mathrm{e}^{-i\bar{\alpha}} & 1-\bar{P}_L \end{pmatrix}$$

Spin density matrix of a two-qubit system

- The helicity amplitude can be rewritten as

$$\mathcal{M} = \mathcal{A}_C \begin{pmatrix} \langle ++;\lambda_1\lambda_2\rangle & \langle +-;\lambda_1\lambda_2\rangle \\ \langle -+;\lambda_1\lambda_2\rangle & \langle --;\lambda_1\lambda_2\rangle \end{pmatrix} \equiv \mathcal{A}_C\,\mathbf{M}$$



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with th

$$\begin{aligned}
\widehat{\Sigma} = \; & (\widehat{C}_1[+++] + \widehat{C}_3[+++]) \\
+ \; & P_n(\widehat{C}_6[++-] + \widehat{C}_8[+-+]) + P_r(-\widehat{C}_5[-++] - \widehat{C}_7[---]) + P_k(\widehat{C}_2[---] + \widehat{C}_4[-++]) \\
+ \; & \bar{P}_n(\widehat{C}_6[++-] - \widehat{C}_8[+-+]) + \bar{P}_r(-\widehat{C}_5[-++] + \widehat{C}_7[---]) + \bar{P}_k(-\widehat{C}_2[---] + \widehat{C}_4[-++]) \\
+ \; & P_n\bar{P}_n(\widehat{C}_{13}[+++] - \widehat{C}_{15}[+++]) + P_n\bar{P}_r(-\widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-]) + P_n\bar{P}_k(-\widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-]) \\
+ \; & P_r\bar{P}_n(\widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-]) + P_r\bar{P}_r(\widehat{C}_{13}[+++] + \widehat{C}_{15}[+++]) + P_r\bar{P}_k(\widehat{C}_9[+--] + \widehat{C}_{11}[+++]) \\
+ \; & P_k\bar{P}_n(\widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-]) + P_k\bar{P}_r(-\widehat{C}_9[+--] + \widehat{C}_{11}[+++]) + P_k\bar{P}_k(-\widehat{C}_1[+++] + \widehat{C}_3[+++]),
\end{aligned}$$

Spin density matrix of a two-qubit system

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$\bar{t}$

$$\widehat{C}_1[+++] = \frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm}\left[|\langle ++;\lambda_1\lambda_2\rangle|^2 + |\langle --;\lambda_1\lambda_2\rangle|^2\right],$$

$$\widehat{C}_2[---] = \frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm}\left[|\langle ++;\lambda_1\lambda_2\rangle|^2 - |\langle --;\lambda_1\lambda_2\rangle|^2\right],$$

$$\widehat{C}_3[+++] = \frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm}\left[|\langle -+;\lambda_1\lambda_2\rangle|^2 + |\langle +-;\lambda_1\lambda_2\rangle|^2\right],$$

$$\widehat{C}_4[-++] = -\frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm}\left[|\langle -+;\lambda_1\lambda_2\rangle|^2 - |\langle +-;\lambda_1\lambda_2\rangle|^2\right],$$

$$\widehat{C}_5[-++] = \frac{1}{4}\,\Re\mathrm{e}\sum_{\lambda_1,\lambda_2=\pm}\big(\langle ++;\lambda_1\lambda_2\rangle - \langle --;\lambda_1\lambda_2\rangle\big)\big(\langle -+;\lambda_1\lambda_2\rangle - \langle +-;\lambda_1\lambda_2\rangle\big)^*,$$

$$\widehat{C}_6[++-] = -\frac{1}{4}\,\Im\mathrm{m}\sum_{\lambda_1,\lambda_2=\pm}\big(\langle ++;\lambda_1\lambda_2\rangle - \langle --;\lambda_1\lambda_2\rangle\big)\big(\langle -+;\lambda_1\lambda_2\rangle + \langle +-;\lambda_1\lambda_2\rangle\big)^*,$$

$$\widehat{C}_7[---] = \frac{1}{4}\,\Re\mathrm{e}\sum_{\lambda_1,\lambda_2=\pm}\big(\langle ++;\lambda_1\lambda_2\rangle + \langle --;\lambda_1\lambda_2\rangle\big)\big(\langle -+;\lambda_1\lambda_2\rangle + \langle +-;\lambda_1\lambda_2\rangle\big)^*,$$

$$\widehat{C}_8[+-+] = -\frac{1}{4}\,\Im\mathrm{m}\sum\big(\langle ++;\lambda_1\lambda_2\rangle + \langle --;\lambda_1\lambda_2\rangle\big)\big(\langle -+;\lambda_1\lambda_2\rangle - \langle +-;\lambda_1\lambda_2\rangle\big)^*,$$

$$\widehat{C}_9[+--] = \frac{1}{4}\,\Re\mathrm{e}\sum_{\lambda_1,\lambda_2=\pm}\big(\langle ++;\lambda_1\lambda_2\rangle + \langle --;\lambda_1\lambda_2\rangle\big)\big(\langle -+;\lambda_1\lambda_2\rangle - \langle +-;\lambda_1\lambda_2\rangle\big)^*,$$

$$\widehat{C}_{10}[--+] = -\frac{1}{4}\,\Im\mathrm{m}\sum_{\lambda_1,\lambda_2=\pm}\big(\langle ++;\lambda_1\lambda_2\rangle + \langle --;\lambda_1\lambda_2\rangle\big)\big(\langle -+;\lambda_1\lambda_2\rangle + \langle +-;\lambda_1\lambda_2\rangle\big)^*,$$

$$\widehat{C}_{11}[+++] = \frac{1}{4}\,\Re\mathrm{e}\sum_{\lambda_1,\lambda_2=\pm}\big(\langle ++;\lambda_1\lambda_2\rangle - \langle --;\lambda_1\lambda_2\rangle\big)\big(\langle -+;\lambda_1\lambda_2\rangle + \langle +-;\lambda_1\lambda_2\rangle\big)^*,$$

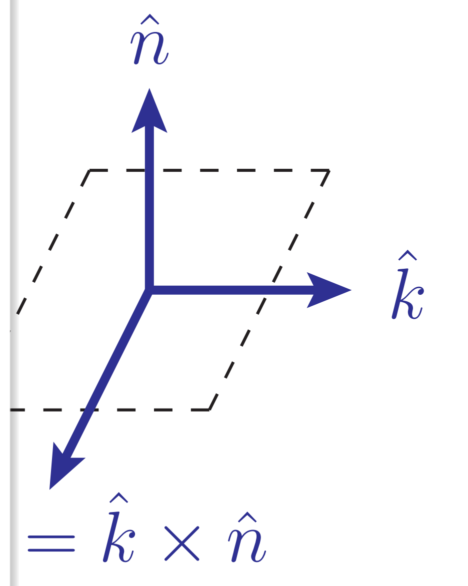
$$\widehat{C}_{12}[-+-] = -\frac{1}{4}\,\Im\mathrm{m}\sum_{\lambda_1,\lambda_2=\pm}\big(\langle ++;\lambda_1\lambda_2\rangle - \langle --;\lambda_1\lambda_2\rangle\big)\big(\langle -+;\lambda_1\lambda_2\rangle - \langle +-;\lambda_1\lambda_2\rangle\big)^*,$$

$$\widehat{C}_{13}[+++] = -\frac{1}{2}\,\Re\mathrm{e}\sum_{\lambda_1,\lambda_2=\pm}\left[\langle ++;\lambda_1\lambda_2\rangle\langle --;\lambda_1\lambda_2\rangle^*\right],$$

$$\widehat{C}_{14}[--+] = \frac{1}{2}\,\Im\mathrm{m}\sum_{\lambda_1,\lambda_2=\pm}\left[\langle ++;\lambda_1\lambda_2\rangle\langle --;\lambda_1\lambda_2\rangle^*\right],$$

$$\widehat{C}_{15}[+++] = -\frac{1}{2}\,\Re\mathrm{e}\sum_{\lambda_1,\lambda_2=\pm}\left[\langle -+;\lambda_1\lambda_2\rangle\langle +-;\lambda_1\lambda_2\rangle^*\right],$$

$$\widehat{C}_{16}[-+-] = -\frac{1}{2}\,\Im\mathrm{m}\sum_{\lambda_1,\lambda_2=\pm}\left[\langle -+;\lambda_1\lambda_2\rangle\langle +-;\lambda_1\lambda_2\rangle^*\right].$$



- The p … are … isler and Si '15]

$\sum$ $,\lambda_2=$

with th

$$\begin{aligned}
\widehat{\Sigma} \;=\; & \big(\widehat{C}_1[+++] + \widehat{C}_3[+++]\big) \\
+\; & P_n\big(\widehat{C}_6[++-] + \widehat{C}_8[+-+]\big) + P_r\big(-\widehat{C}_5[-++] - \widehat{C}_7[---]\big) + P_k\big(\widehat{C}_2[---] + \widehat{C}_4[-++]\big) \\
+\; & \bar{P}_n\big(\widehat{C}_6[++-] - \widehat{C}_8[+-+]\big) + \bar{P}_r\big(-\widehat{C}_5[-++] + \widehat{C}_7[---]\big) + \bar{P}_k\big(-\widehat{C}_2[---] + \widehat{C}_4[-++]\big) \\
+\; & P_n\bar{P}_n\big(\widehat{C}_{13}[+++] - \widehat{C}_{15}[+++]\big) + P_n\bar{P}_r\big(-\widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-]\big) + P_n\bar{P}_k\big(-\widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-]\big) \\
+\; & P_r\bar{P}_n\big(\widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-]\big) + P_r\bar{P}_r\big(\widehat{C}_{13}[+++] + \widehat{C}_{15}[+++]\big) + P_r\bar{P}_k\big(\widehat{C}_9[+--] + \widehat{C}_{11}[+++]\big) \\
+\; & P_k\bar{P}_n\big(\widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-]\big) + P_k\bar{P}_r\big(-\widehat{C}_9[+--] + \widehat{C}_{11}[+++]\big) + P_k\bar{P}_k\big(-\widehat{C}_1[+++] + \widehat{C}_3[+++]\big),
\end{aligned}$$

Spin density matrix of a two-qubit system

- The spin density matrix in $\{n, r, k\}$ basis,

$$\rho = \frac{1}{4}\left[\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{\widehat{A}}\sum_{i=n,r,k} \widehat{B}_i^+ \left(\sigma_i \otimes \mathbf{1}_2\right) + \frac{1}{\widehat{A}}\sum_{j=n,r,k} \widehat{B}_j^- \left(\mathbf{1}_2 \otimes \sigma_j\right) + \frac{1}{\widehat{A}}\sum_{i,j=n,r,k} \widehat{C}_{ij}\left(\sigma_i \otimes \sigma_j\right)\right]$$

- $\hat{B}_i^+$ and $\hat{B}_i^-$ are polarization of top and anti-top

$$\begin{aligned}\widehat{B}_i^+ &= \left(\widehat{C}_6[++-] + \widehat{C}_8[+-+], -\widehat{C}_5[-++] - \widehat{C}_7[---], \widehat{C}_2[---] + \widehat{C}_4[-++]\right),\\ \widehat{B}_j^- &= \left(\widehat{C}_6[++-] - \widehat{C}_8[+-+], -\widehat{C}_5[-++] + \widehat{C}_7[---], -\widehat{C}_2[---] + \widehat{C}_4[-++]\right),\end{aligned}$$

- $\hat{C}_{ij}$ are spin correlation matrix

$$\widehat{C}_{ij} = \begin{pmatrix} \widehat{C}_{13}[+++] - \widehat{C}_{15}[+++] & -\widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-] & -\widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-] \\ \widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-] & \widehat{C}_{13}[+++] + \widehat{C}_{15}[+++] & \widehat{C}_9[+--] + \widehat{C}_{11}[+++] \\ \widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-] & -\widehat{C}_9[+--] + \widehat{C}_{11}[+++] & -\widehat{C}_1[+++] + \widehat{C}_3[+++] \end{pmatrix}$$

- $\hat{A}$ is normalization factor

$$\widehat{A} = \widehat{C}_1[+++] + \widehat{C}_3[+++] \longrightarrow \frac{d\sigma}{d\Omega} = \frac{\alpha^2\hat{\beta}}{32\pi\hat{s}}\hat{A}$$

[**DWK**, S.Y. Choi, J.S. Lee, and C. Park 26']

Spin density matrix of a two-qubit system

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$$\rho = \frac{1}{4}\left[\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{\widehat{A}} \sum_{i=n,r,k} \widehat{B}_i^+ \left(\sigma_i \otimes \mathbf{1}_2\right) + \frac{1}{\widehat{A}} \sum_{j=n,r,k} \widehat{B}_j^- \left(\mathbf{1}_2 \otimes \sigma_j\right) + \frac{1}{\widehat{A}} \sum_{i,j=n,r,k} \widehat{C}_{ij} \left(\sigma_i \otimes \sigma_j\right)\right]$$

Polarization coeffcient	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$	$\widehat{C}_6$	$\widehat{C}_7$	$\widehat{C}_8$
[P, CP, CP$\widetilde{\mathrm{T}}$] Parities	$[+\,+\,+]$	$[-\,-\,-]$	$[+\,+\,+]$	$[-\,+\,+]$	$[-\,+\,+]$	$[+\,+\,-]$	$[-\,-\,-]$	$[+\,-\,+]$
Physical quantity	$\frac{\widehat{A}-\widehat{C}_{kk}}{2}$	$\frac{\widehat{B}_k^+ - \widehat{B}_k^-}{2}$	$\frac{\widehat{A}+\widehat{C}_{kk}}{2}$	$\frac{\widehat{B}_k^+ + \widehat{B}_k^-}{2}$	$-\frac{\widehat{B}_r^+ + \widehat{B}_r^-}{2}$	$\frac{\widehat{B}_n^+ + \widehat{B}_n^-}{2}$	$-\frac{\widehat{B}_r^+ - \widehat{B}_r^-}{2}$	$\frac{\widehat{B}_n^+ - \widehat{B}_n^-}{2}$
Polarization coeffcient	$\widehat{C}_9$	$\widehat{C}_{10}$	$\widehat{C}_{11}$	$\widehat{C}_{12}$	$\widehat{C}_{13}$	$\widehat{C}_{14}$	$\widehat{C}_{15}$	$\widehat{C}_{16}$
[P, CP, CP$\widetilde{\mathrm{T}}$] Parities	$[+\,-\,-]$	$[-\,-\,+]$	$[+\,+\,+]$	$[-\,+\,-]$	$[+\,+\,+]$	$[-\,-\,+]$	$[+\,+\,+]$	$[-\,+\,-]$
Physical quantity	$\frac{\widehat{C}_{rk}-\widehat{C}_{kr}}{2}$	$\frac{\widehat{C}_{kn}-\widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{rk}+\widehat{C}_{kr}}{2}$	$-\frac{\widehat{C}_{kn}+\widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{nn}+\widehat{C}_{rr}}{2}$	$\frac{\widehat{C}_{rn}-\widehat{C}_{nr}}{2}$	$\frac{\widehat{C}_{rr}-\widehat{C}_{nn}}{2}$	$-\frac{\widehat{C}_{rn}+\widehat{C}_{nr}}{2}$

Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathscr{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathscr{A}\langle\sigma\bar{\sigma};\lambda_1\lambda_2\rangle\delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2} = \mathscr{A}\begin{pmatrix}\langle ++;\lambda_1\lambda_2\rangle & 0 \\ 0 & \langle --;\lambda_1\lambda_2\rangle\end{pmatrix}$$

Polarization coeffcient	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$	$\widehat{C}_6$	$\widehat{C}_7$	$\widehat{C}_8$
[P, CP, CP$\widetilde{\mathrm{T}}$] Parities	$[++\,+]$	$[--\,-]$	$[++\,+]$	$[-+\,+]$	$[-+\,+]$	$[++\,-]$	$[--\,-]$	$[+-\,+]$
Physical quantity	$\frac{\widehat{A}-\widehat{C}_{kk}}{2}$	$\frac{\widehat{B}^+_k-\widehat{B}^-_k}{2}$	$\frac{\widehat{A}+\widehat{C}_{kk}}{2}$	$\frac{\widehat{B}^+_k+\widehat{B}^-_k}{2}$	$-\frac{\widehat{B}^+_r+\widehat{B}^-_r}{2}$	$\frac{\widehat{B}^+_n+\widehat{B}^-_n}{2}$	$-\frac{\widehat{B}^+_r-\widehat{B}^-_r}{2}$	$\frac{\widehat{B}^+_n-\widehat{B}^-_n}{2}$
Polarization coeffcient	$\widehat{C}_9$	$\widehat{C}_{10}$	$\widehat{C}_{11}$	$\widehat{C}_{12}$	$\widehat{C}_{13}$	$\widehat{C}_{14}$	$\widehat{C}_{15}$	$\widehat{C}_{16}$
[P, CP, CP$\widetilde{\mathrm{T}}$] Parities	$[+-\,-]$	$[--\,+]$	$[++\,+]$	$[-+\,-]$	$[++\,+]$	$[--\,+]$	$[++\,+]$	$[-+\,-]$
Physical quantity	$\frac{\widehat{C}_{rk}-\widehat{C}_{kr}}{2}$	$\frac{\widehat{C}_{kn}-\widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{rk}+\widehat{C}_{kr}}{2}$	$-\frac{\widehat{C}_{kn}+\widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{nn}+\widehat{C}_{rr}}{2}$	$\frac{\widehat{C}_{rn}-\widehat{C}_{nr}}{2}$	$\frac{\widehat{C}_{rr}-\widehat{C}_{nn}}{2}$	$-\frac{\widehat{C}_{rn}+\widehat{C}_{nr}}{2}$

Spin density matrix of a two-qubit system

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$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathcal{A}\langle\sigma\bar{\sigma};\lambda_1\lambda_2\rangle\delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2} = \mathcal{A}\begin{pmatrix}\langle ++;\lambda_1\lambda_2\rangle & 0 \\ 0 & \langle --;\lambda_1\lambda_2\rangle\end{pmatrix}$$

$$\begin{aligned}\widehat{B}_i^+ &= \left(\widehat{C}_6[++-]+\widehat{C}_8[+-+], -\widehat{C}_5[-++]-\widehat{C}_7[---], \widehat{C}_2[---]+\widehat{C}_4[-++]\right), \\ \widehat{B}_j^- &= \left(\widehat{C}_6[++-]-\widehat{C}_8[+-+], -\widehat{C}_5[-++]+\widehat{C}_7[---], -\widehat{C}_2[---]+\widehat{C}_4[-++]\right),\end{aligned}$$

$$\widehat{C}_{ij} = \begin{pmatrix}\widehat{C}_{13}[+++]-\widehat{C}_{15}[+++] & -\widehat{C}_{14}[--+]-\widehat{C}_{16}[-+-] & -\widehat{C}_{10}[--+]-\widehat{C}_{12}[-+-] \\ \widehat{C}_{14}[--+]-\widehat{C}_{16}[-+-] & \widehat{C}_{13}[+++]+\widehat{C}_{15}[+++] & \widehat{C}_9[+--]+\widehat{C}_{11}[+++] \\ \widehat{C}_{10}[--+]-\widehat{C}_{12}[-+-] & -\widehat{C}_9[+--]+\widehat{C}_{11}[+++] & -\widehat{C}_1[+++]+\widehat{C}_3[+++]\end{pmatrix}$$

$$B_k^+ = -B_k^- = C_2/C_1 \equiv B$$

$$C_n n = C_r r = C_{13}/C_1 \equiv C$$

$$C_{nr} = -C_{rn} = -C_{14}/C_1 \equiv D$$

$$C_{kk} = -1$$

$$\rho = \frac{1}{2}\begin{pmatrix}0 & 0 & 0 & 0 \\ 0 & 1+B & C+iD & 0 \\ 0 & C-iD & 1-B & 0 \\ 0 & 0 & 0 & 0\end{pmatrix}$$

Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathscr{A}\langle\sigma\bar{\sigma};\lambda_1\lambda_2\rangle\delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2} = \mathscr{A}\begin{pmatrix}\langle ++;\lambda_1\lambda_2\rangle & 0 \\ 0 & \langle --;\lambda_1\lambda_2\rangle\end{pmatrix}$$

- Let us compute quantum observables for $\rho = \frac{1}{2}\begin{pmatrix}0 & 0 & 0 & 0 \\ 0 & 1+B & C+iD & 0 \\ 0 & C-iD & 1-B & 0 \\ 0 & 0 & 0 & 0\end{pmatrix}$

$$Tr[\rho^2] = \frac{1+B^2+C^2+D^2}{2} \leq 1$$

$$\mathscr{C}[\rho^2] = \sqrt{C^2+D^2}$$

$$m_{12} = 1 + C^2 + D^2 = 1 + \mathscr{C}^2$$

[**DWK**, S.Y. Choi, J.S. Lee, and C. Park 26']

Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathcal{A}\langle\sigma\bar{\sigma};\lambda_1\lambda_2\rangle\delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2} = \mathcal{A}\begin{pmatrix}\langle ++;\lambda_1\lambda_2\rangle & 0 \\ 0 & \langle --;\lambda_1\lambda_2\rangle\end{pmatrix}$$

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$$m_{12} = 1 + C^2 + D^2 = 1 + \mathscr{C}^2$$

- For a single scalar exchange, $B = 0$, $C^2 + D^2 = 1$

Maximal Entanglement

Violation of CHSH inequality

Spin density matrix of a two-qubit system

- Consider scalar exchanges

$$\mathcal{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}} = \mathcal{A}\langle\sigma\bar{\sigma};\lambda_1\lambda_2\rangle\delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2} = \mathcal{A}\begin{pmatrix}\langle ++;\lambda_1\lambda_2\rangle & 0 \\ 0 & \langle --;\lambda_1\lambda_2\rangle\end{pmatrix}$$

- Let us compute quantum observables for $\rho = \frac{1}{2}\begin{pmatrix}0 & 0 & 0 & 0 \\ 0 & 1+B & C+iD & 0 \\ 0 & C-iD & 1-B & 0 \\ 0 & 0 & 0 & 0\end{pmatrix}$

$$Tr[\rho^2] = \frac{1+B^2+C^2+D^2}{2} \leq 1$$

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- Let us compute quantum observables for $\rho = \frac{1}{2}\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 1+B & C+iD & 0\\ 0 & C-iD & 1-B & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$

- For a single scalar exchange, $B = 0$, $C^2 + D^2 = 1$

$$C_{ij}^{\mathrm{CP-even}} = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{pmatrix} \qquad C_{ij}^{\mathrm{CP-odd}} = \begin{pmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{pmatrix} \qquad C_{ij}^{\mathrm{mixed}} = \begin{pmatrix}\cos 2\delta & \sin 2\delta & 0\\ \sin 2\delta & \cos 2\delta & 0\\ 0 & 0 & -1\end{pmatrix}$$

Spin density matrix of a two-qubit system

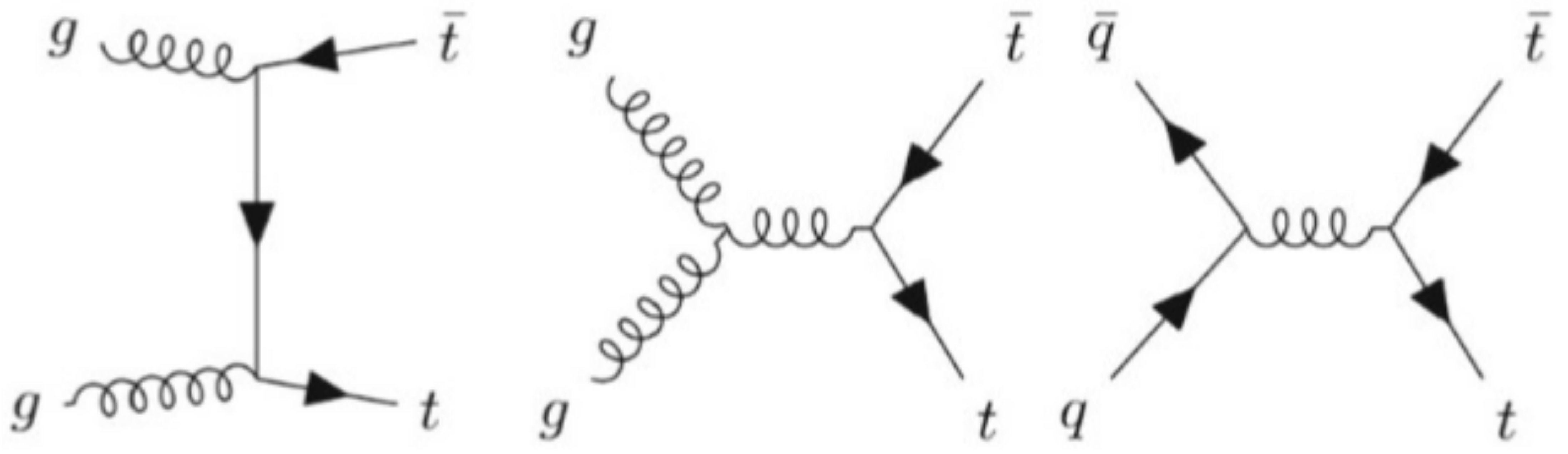
- Let us consider ttbar process,
- Leading order QCD amplitude is P, CP, and CP$\tilde{\mathrm{T}}$ invariant.

Only [+++] coefficient survive!

Polarization coeffcient	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$	$\widehat{C}_6$	$\widehat{C}_7$	$\widehat{C}_8$
[P, CP, CP$\tilde{\mathrm{T}}$] Parities	$[++\,+]$	$[--\,-]$	$[++\,+]$	$[-+\,+]$	$[-+\,+]$	$[++\,-]$	$[--\,-]$	$[+-\,+]$
Physical quantity	$\frac{\widehat{A}-\widehat{C}_{kk}}{2}$	$\frac{\widehat{B}^+_k-\widehat{B}^-_k}{2}$	$\frac{\widehat{A}+\widehat{C}_{kk}}{2}$	$\frac{\widehat{B}^+_k+\widehat{B}^-_k}{2}$	$-\frac{\widehat{B}^+_r+\widehat{B}^-_r}{2}$	$\frac{\widehat{B}^+_n+\widehat{B}^-_n}{2}$	$-\frac{\widehat{B}^+_r-\widehat{B}^-_r}{2}$	$\frac{\widehat{B}^+_n-\widehat{B}^-_n}{2}$

Polarization coeffcient	$\widehat{C}_9$	$\widehat{C}_{10}$	$\widehat{C}_{11}$	$\widehat{C}_{12}$	$\widehat{C}_{13}$	$\widehat{C}_{14}$	$\widehat{C}_{15}$	$\widehat{C}_{16}$
[P, CP, CP$\tilde{\mathrm{T}}$] Parities	$[+-\,-]$	$[--\,+]$	$[++\,+]$	$[-+\,-]$	$[++\,+]$	$[--\,+]$	$[++\,+]$	$[-+\,-]$
Physical quantity	$\frac{\widehat{C}_{rk}-\widehat{C}_{kr}}{2}$	$\frac{\widehat{C}_{kn}-\widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{rk}+\widehat{C}_{kr}}{2}$	$-\frac{\widehat{C}_{kn}+\widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{nn}+\widehat{C}_{rr}}{2}$	$\frac{\widehat{C}_{rn}-\widehat{C}_{nr}}{2}$	$\frac{\widehat{C}_{rr}-\widehat{C}_{nn}}{2}$	$-\frac{\widehat{C}_{rn}+\widehat{C}_{nr}}{2}$

Top quark pairs @ LHC



- Spin-density matrix

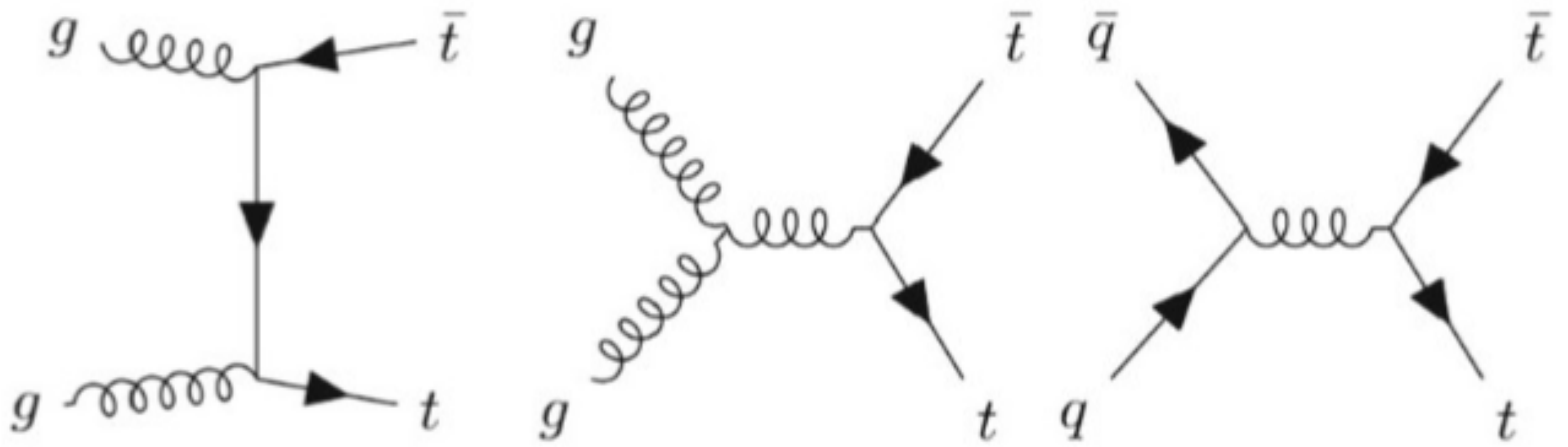
$$\mathscr{R}_{\sigma\sigma'\bar{\sigma}\bar{\sigma}'}(\hat{\beta},\theta) = \frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm 1} \mathscr{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}}(\theta)\, \mathscr{M}^*_{\lambda_1\lambda_2;\sigma'\bar{\sigma}'}(\theta)$$

- Fano decompositioin

$$\mathscr{R}(\hat{\beta},\theta) = \tilde{A}\; I_2 \otimes I_2 + \tilde{B}_i^{+}\,\sigma_i \otimes I_2 + \tilde{B}_i^{-}\, I_2 \otimes \sigma_i + \tilde{C}_{ij}\,\sigma_i \otimes \sigma_j$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\hat{\beta}}{32\pi\hat{s}}\tilde{A}$$

Top quark pairs @ LHC



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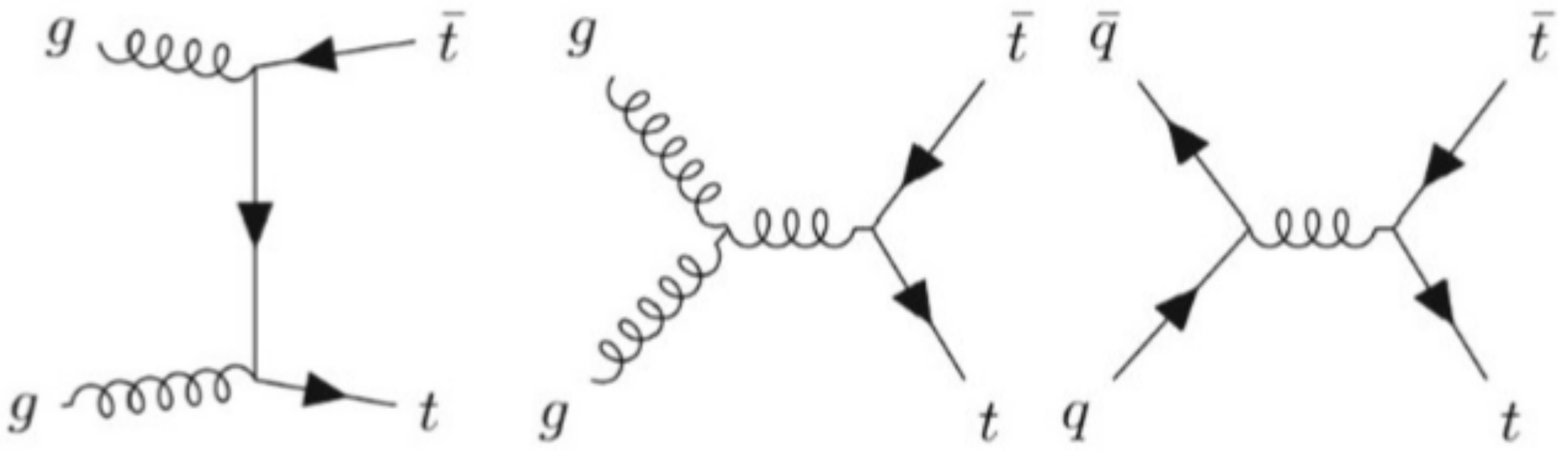
$$\mathscr{R}(\hat{\beta},\theta) = \tilde{A}\; I_2 \otimes I_2 + \tilde{B}_i^+\, \sigma_i \otimes I_2 + \tilde{B}_i^-\, I_2 \otimes \sigma_i + \tilde{C}_{ij}\, \sigma_i \otimes \sigma_j$$

$$\rho = \frac{\mathscr{R}}{Tr(\mathscr{R})} = \frac{I_2 \otimes I_2 + B_i^+ \sigma_i \otimes I_2 + \bar{B}_i^- I_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

Top quark pairs @ LHC

[Afik, Nova '20]
[Severi, Boschi, Maltoni, Sioli '21]
[Saavedra, Casas '22]



Only [+++] coefficient survive!

$C_{rk} = C_{kr}$ also noted!

$$\widehat{C}_{ij} = \begin{pmatrix} \boxed{\widehat{C}_{13}[+++] - \widehat{C}_{15}[+++]} & -\widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-] & -\widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-] \\ \widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-] & \boxed{\widehat{C}_{13}[+++] + \widehat{C}_{15}[+++]} & \widehat{C}_{9}[+--] + \boxed{\widehat{C}_{11}[+++]} \\ \widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-] & -\widehat{C}_{9}[+--] + \boxed{\widehat{C}_{11}[+++]} & \boxed{-\widehat{C}_{1}[+++] + \widehat{C}_{3}[+++]} \end{pmatrix}$$

Top quark pairs @ LHC

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$$\widehat{C}_{ij} = \begin{pmatrix} \boxed{\widehat{C}_{13}[+++] - \widehat{C}_{15}[+++]} & -\widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-] & -\widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-] \\ \widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-] & \boxed{\widehat{C}_{13}[+++] + \widehat{C}_{15}[+++]} & \widehat{C}_{9}[+--] + \boxed{\widehat{C}_{11}[+++]} \\ \widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-] & -\widehat{C}_{9}[+--] + \boxed{\widehat{C}_{11}[+++]} & \boxed{-\widehat{C}_{1}[+++] + \widehat{C}_{3}[+++]} \end{pmatrix}$$

- $gg \to t\bar{t}$

$$F_g(z) = \frac{7 + 9\beta^2 z^2}{192(1-\beta^2 z^2)^2},$$
$$\tilde{A}^{gg} = F_g(z)(1 + 2\beta^2(1-z^2) - \beta^4(2 - 2z^2 + z^4)),$$
$$\tilde{C}^{gg}_{nn} = -F_g(z)(1 - 2\beta^2 + \beta^4(2 - 2z^2 + z^4)),$$
$$\tilde{C}^{gg}_{kk} = -F_g(z)(1 - 2z^2(1-z^2)\beta^2 - \beta^4(2 - 2z^2 + z^4)),$$
$$\tilde{C}^{gg}_{rr} = -F_g(z)(1 - \beta^2(2-\beta^2)(2 - 2z^2 + z^4)),$$
$$\tilde{C}^{gg}_{rk} = \tilde{C}^{gg}_{kr} = F_g(z)2z(1-z^2)^{3/2}\beta^2\sqrt{1-\beta^2},$$

- $q\bar{q} \to t\bar{t}$

$$F_q = \frac{1}{18},$$
$$\tilde{A}^{q\bar{q}} = F_q(2 - \beta^2(1-z^2)),$$
$$\tilde{C}^{q\bar{q}}_{nn} = -F_q\beta^2(1-z^2),$$
$$\tilde{C}^{q\bar{q}}_{kk} = F_q(2z^2 + \beta^2(1-z^2)),$$
$$\tilde{C}^{q\bar{q}}_{rr} = F_q(2-\beta^2)(1-z^2),$$
$$\tilde{C}^{q\bar{q}}_{rk} = \tilde{C}^{q\bar{q}}_{kr} = F_q 2z\sqrt{(1-\beta^2)(1-z^2)},$$

Only [+++] coefficient survive!

$C_{rk} = C_{kr}$ also noted!

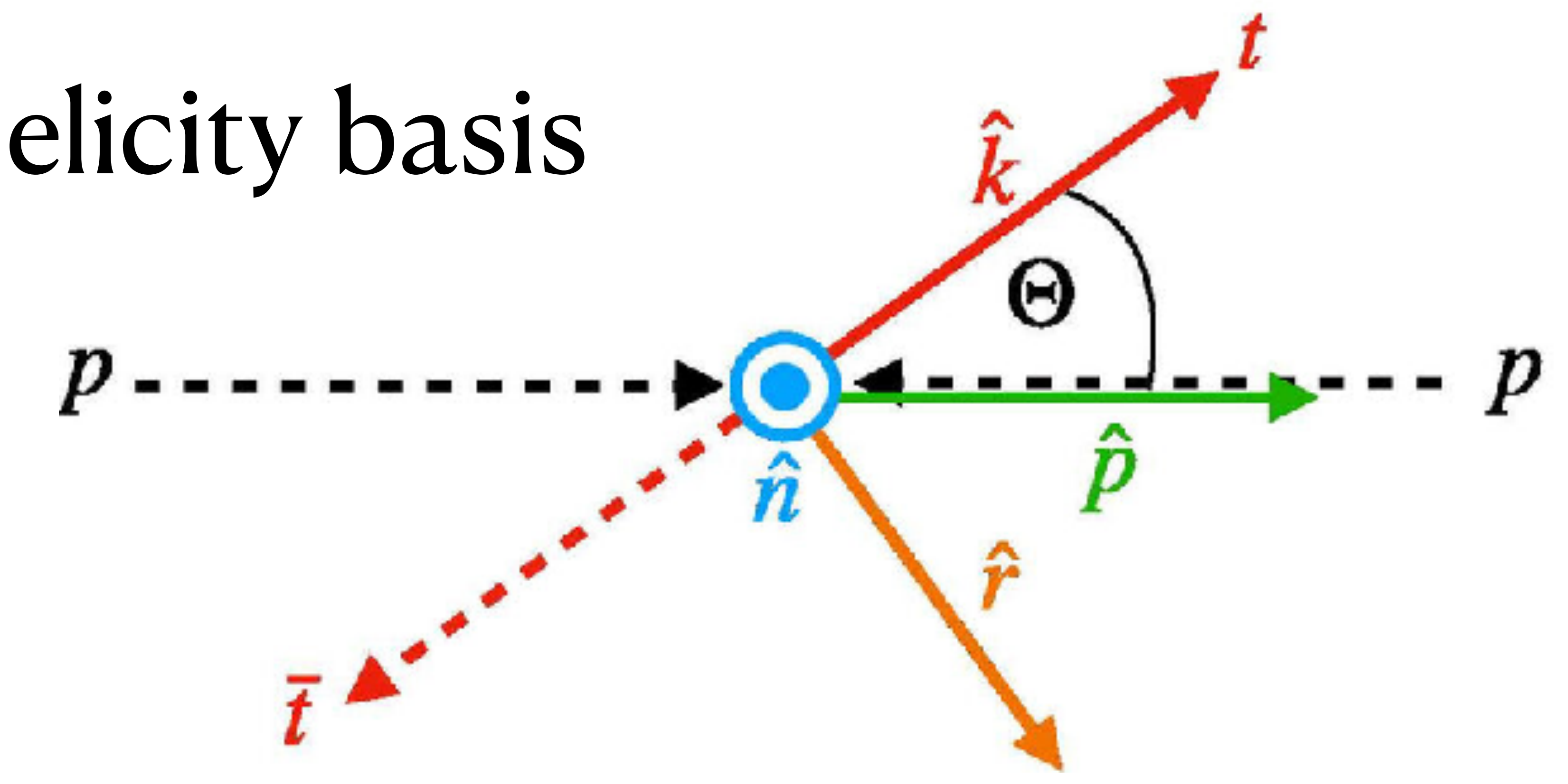
$z = \cos\theta$

Top quark pairs @ LHC

- Angular averaged spin-density matrix

$$R = \frac{1}{4\pi}\int d\Omega \mathscr{R}(\hat{\beta}, z), \qquad \rho = \frac{R}{Tr(R)}$$

- Helicity basis

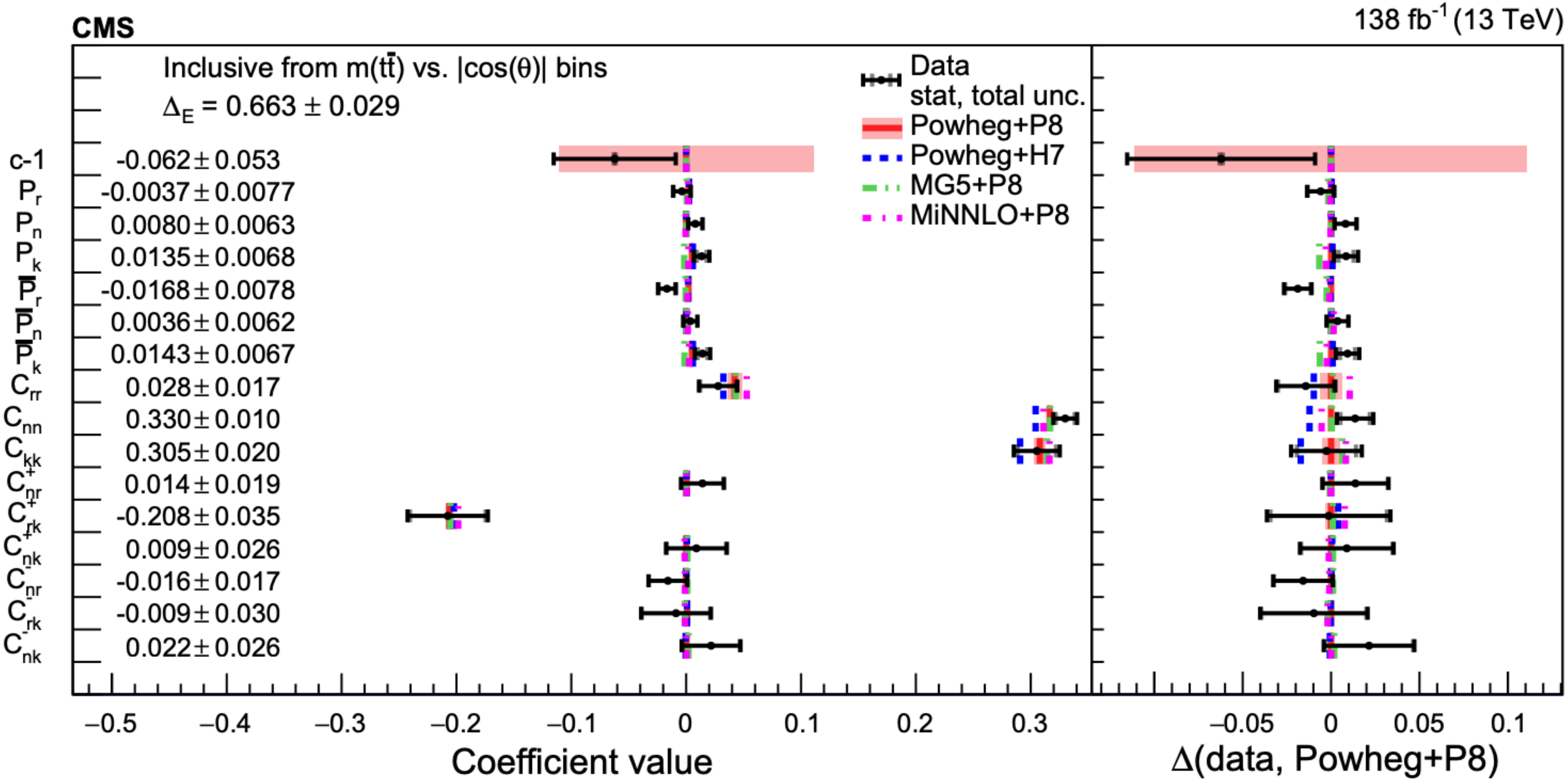


$$\beta = \sqrt{1 - \frac{4m_t^2}{s}} \qquad z = \cos\theta$$

[Afik, Nova '20]
[Severi, Boschi, Maltoni, Sioli '21]
[Saavedra, Casas '22]

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{4\pi^2}\left(1 + \alpha_1 \vec{B}_1 \cdot \hat{\ell}_1 + \alpha_2 \vec{B}_2 \cdot \hat{\ell}_2 + \alpha_1\alpha_2 C_{ij} \hat{\ell}_{1i} \hat{\ell}_{2i}\right)$$

Top quark pairs @ LHC



Peres-Horodecki criterion

$$\rho^{\mathrm{T}_2} = \frac{1}{4}\begin{bmatrix} 1 + B_3^+ + B_3^- + C_{33} & B_1^- + C_{31} + i(B_2^- + C_{32}) & B_1^+ + C_{13} - i(B_2^+ + C_{23}) & C_{11} + C_{22} + i(C_{12} - C_{21}) \\ B_1^- + C_{31} - i(B_2^- + C_{32}) & 1 + B_3^+ - B_3^- - C_{33} & C_{11} - C_{22} - i(C_{12} + C_{21}) & B_1^+ - C_{13} - i(B_2^+ + C_{23}) \\ B_1^+ + C_{13} + i(B_2^+ + C_{23}) & C_{11} - C_{22} + i(C_{21} + C_{12}) & 1 - B_3^+ + B_3^- - C_{33} & B_1^- - C_{31} + i(B_2^- - C_{32}) \\ C_{11} + C_{22} + i(C_{21} - C_{12}) & B_1^+ - C_{13} + i(B_2^+ - C_{23}) & B_1^- - C_{31} - i(B_2^- - C_{32}) & 1 - B_3^+ - B_3^- + C_{33} \end{bmatrix} < 0$$

Peres-Horodecki criterion

$$\rho^{\mathrm{T}_2} = \frac{1}{4}\begin{bmatrix} 1+B_3^+ + B_3^- + C_{33} & B_1^- + C_{31} + i(B_2^- + C_{32}) & B_1^+ + C_{13} - i(B_2^+ + C_{23}) & C_{11} + C_{22} + i(C_{12} - C_{21}) \\ B_1^- + C_{31} - i(B_2^- + C_{32}) & 1 + B_3^+ - B_3^- - C_{33} & C_{11} - C_{22} - i(C_{12} + C_{21}) & B_1^+ - C_{13} - i(B_2^+ + C_{23}) \\ B_1^+ + C_{13} + i(B_2^+ + C_{23}) & C_{11} - C_{22} + i(C_{21} + C_{12}) & 1 - B_3^+ + B_3^- - C_{33} & B_1^- - C_{31} + i(B_2^- - C_{32}) \\ C_{11} + C_{22} + i(C_{21} - C_{12}) & B_1^+ - C_{13} + i(B_2^+ - C_{23}) & B_1^- - C_{31} - i(B_2^- - C_{32}) & 1 - B_3^+ - B_3^- + C_{33} \end{bmatrix} < 0$$

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$D^{(1)} = 1/3(+C_{kk} + C_{rr} + C_{nn}),$$
$$D^{(k)} = 1/3(+C_{kk} - C_{rr} - C_{nn}),$$
$$D^{(r)} = 1/3(-C_{kk} + C_{rr} - C_{nn}),$$
$$D^{(n)} = 1/3(-C_{kk} - C_{rr} + C_{nn}).$$

$$D_{\min} < -1/3 \qquad \text{(entanglement)}$$

Peres-Horodecki criterion

$$\rho^{T_2} = \frac{1}{4}\begin{bmatrix} 1+B_3^+ + B_3^- + C_{33} & B_1^- + C_{31} + i(B_2^- + C_{32}) & B_1^+ + C_{13} - i(B_2^+ + C_{23}) & C_{11} + C_{22} + i(C_{12} - C_{21}) \\ B_1^- + C_{31} - i(B_2^- + C_{32}) & 1 + B_3^+ - B_3^- - C_{33} & C_{11} - C_{22} - i(C_{12} + C_{21}) & B_1^+ - C_{13} - i(B_2^+ + C_{23}) \\ B_1^+ + C_{13} + i(B_2^+ + C_{23}) & C_{11} - C_{22} + i(C_{21} + C_{12}) & 1 - B_3^+ + B_3^- - C_{33} & B_1^- - C_{31} + i(B_2^- - C_{32}) \\ C_{11} + C_{22} + i(C_{21} - C_{12}) & B_1^+ - C_{13} + i(B_2^+ - C_{23}) & B_1^- - C_{31} - i(B_2^- - C_{32}) & 1 - B_3^+ - B_3^- + C_{33} \end{bmatrix} < 0$$

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$\boxed{D^{(1)} = 1/3(+C_{kk} + C_{rr} + C_{nn}),}$$

$$D^{(k)} = 1/3(+C_{kk} - C_{rr} - C_{nn}),$$

$$D^{(r)} = 1/3(-C_{kk} + C_{rr} - C_{nn}),$$

$$D^{(n)} = 1/3(-C_{kk} - C_{rr} + C_{nn}).$$

$$D = \frac{tr[C]}{3}.$$

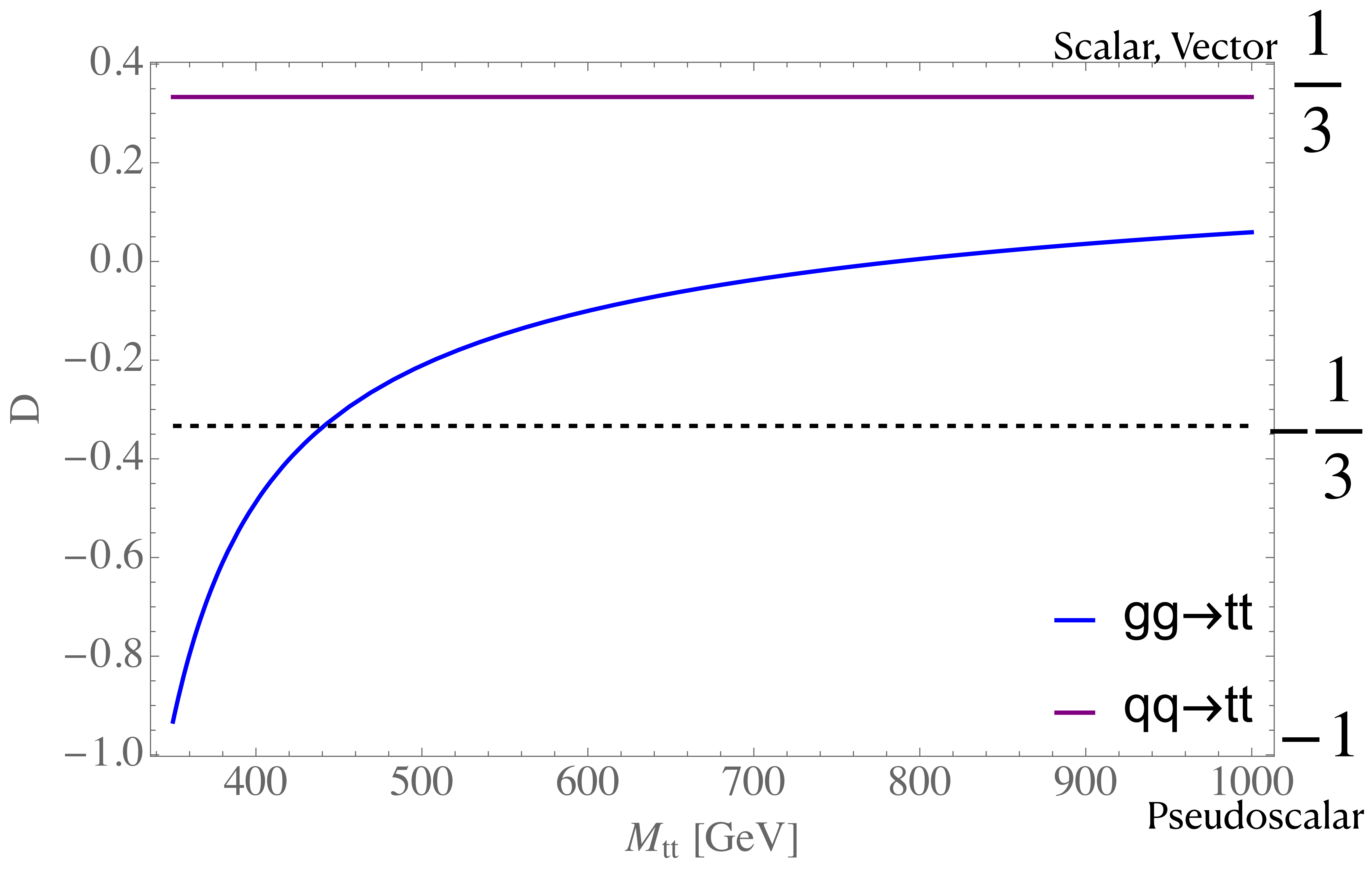
$$D_{\min} < -1/3 \qquad \text{(entanglement)}$$

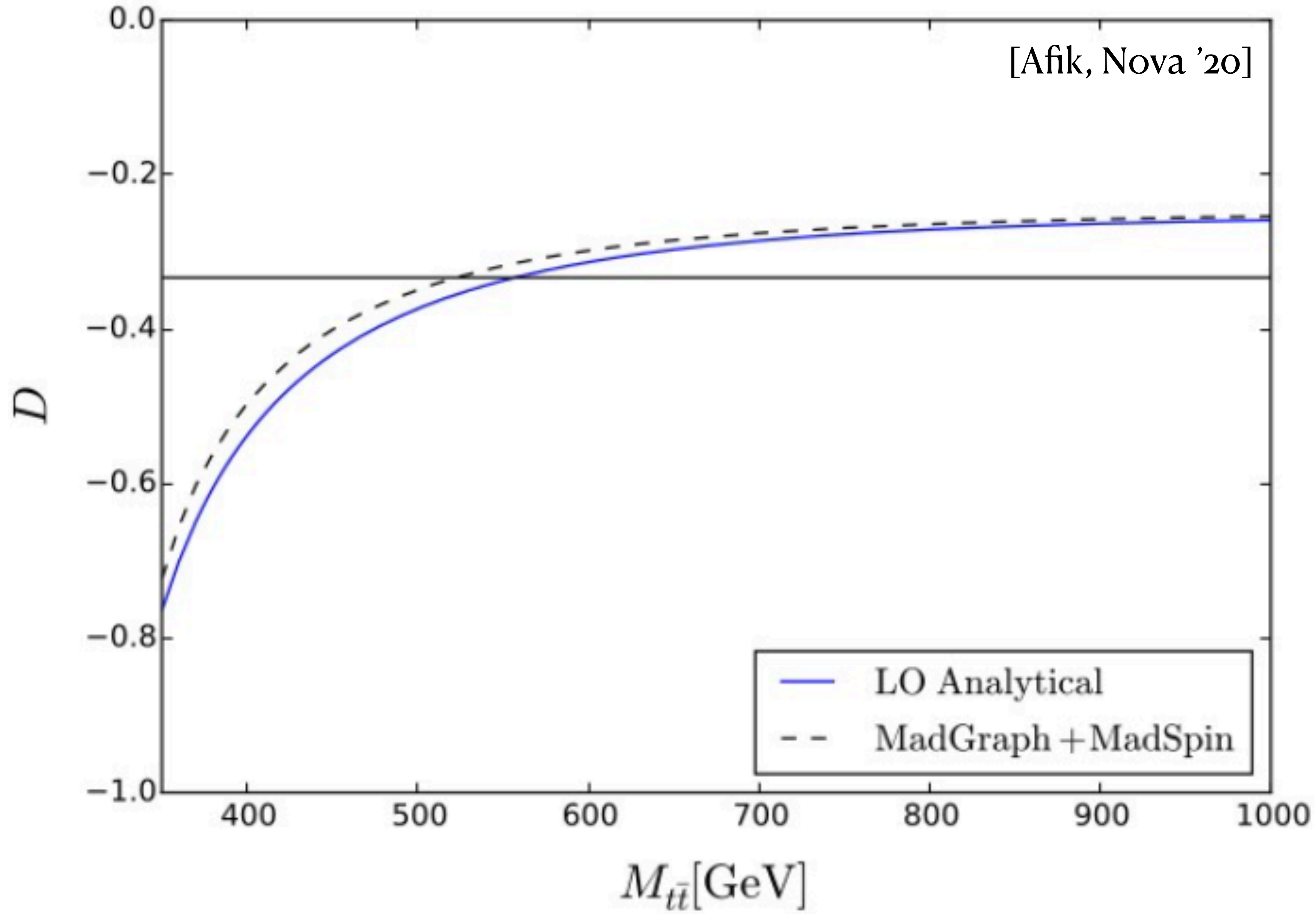
$$\frac{1}{\sigma}\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{4\pi^2}\left(1 + \alpha_1 \vec{B}_1 \cdot \hat{\ell}_1 + \alpha_2 \vec{B}_2 \cdot \hat{\ell}_2 + \alpha_1\alpha_2 C_{ij}\hat{\ell}_{1i}\hat{\ell}_{2i}\right) \longrightarrow \frac{d\sigma}{d\cos\phi} = \frac{1}{2}\left(1 - D\cos\phi\right),$$

Entanglement marker D $\quad D < -1/3$

$$\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\left(1 - D\cos\phi\right), \qquad D = \frac{tr[C]}{3}.$$

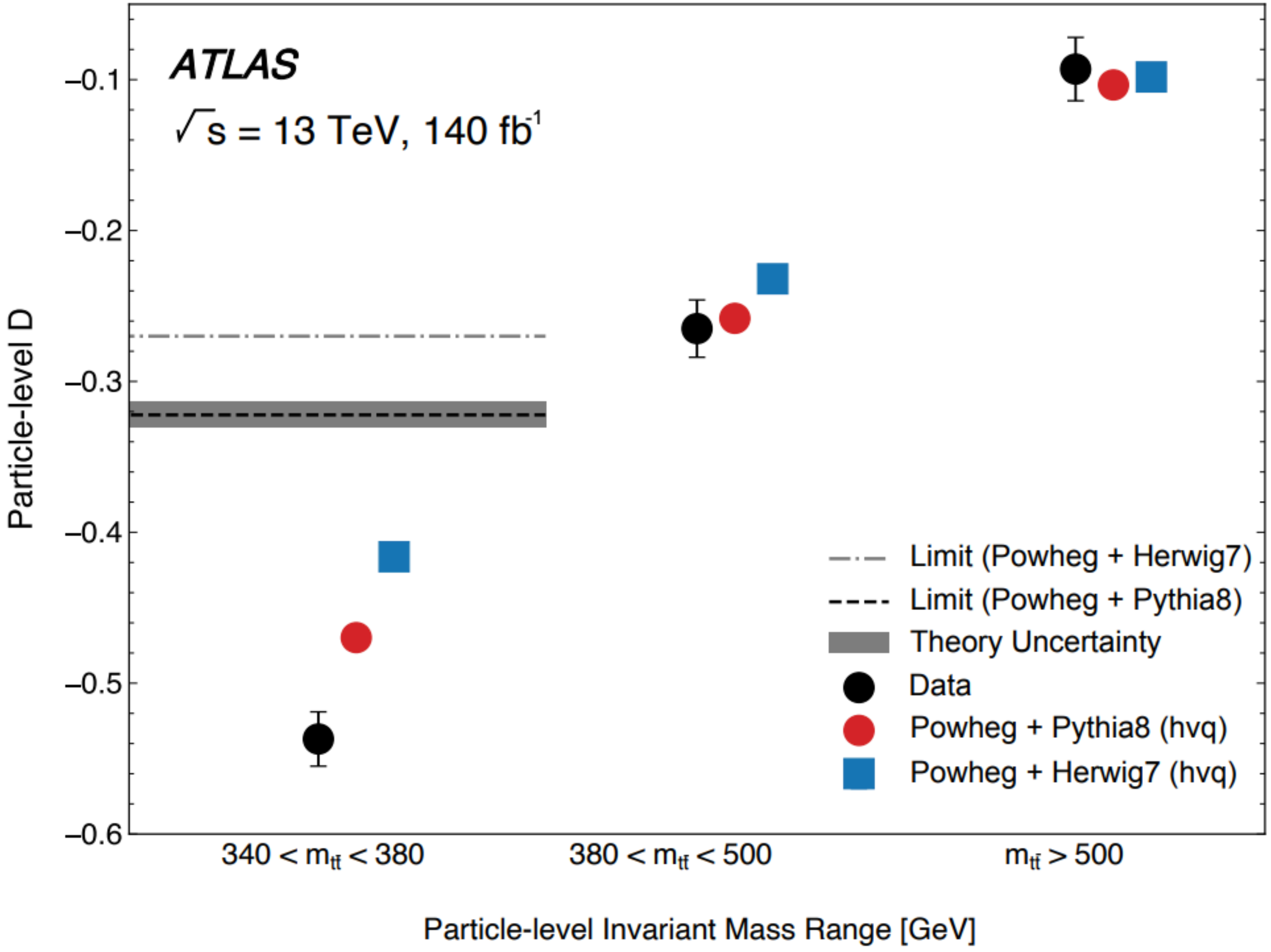
- ϕ is the angle between the leptons measured in the parent top/antitop frames

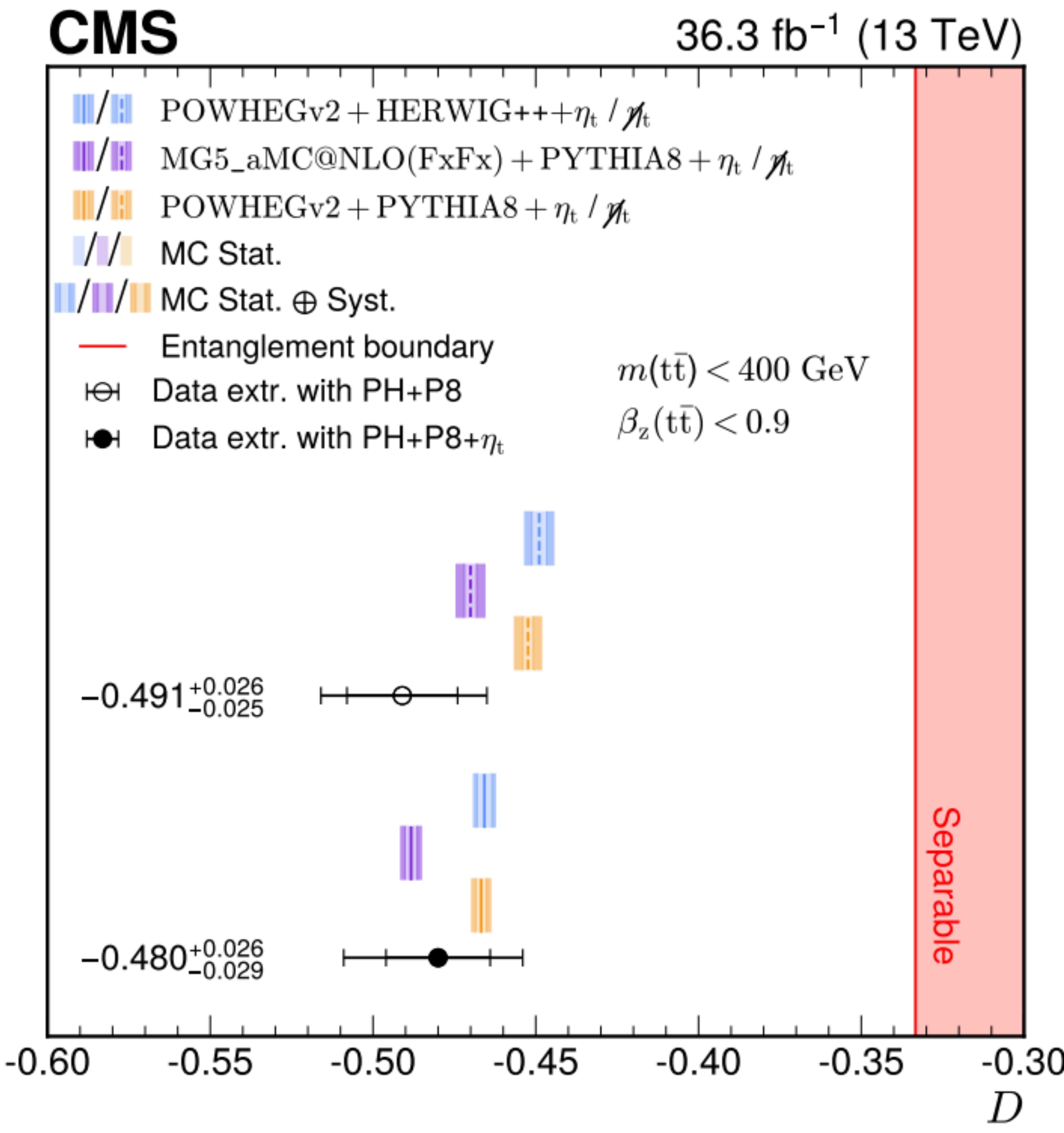




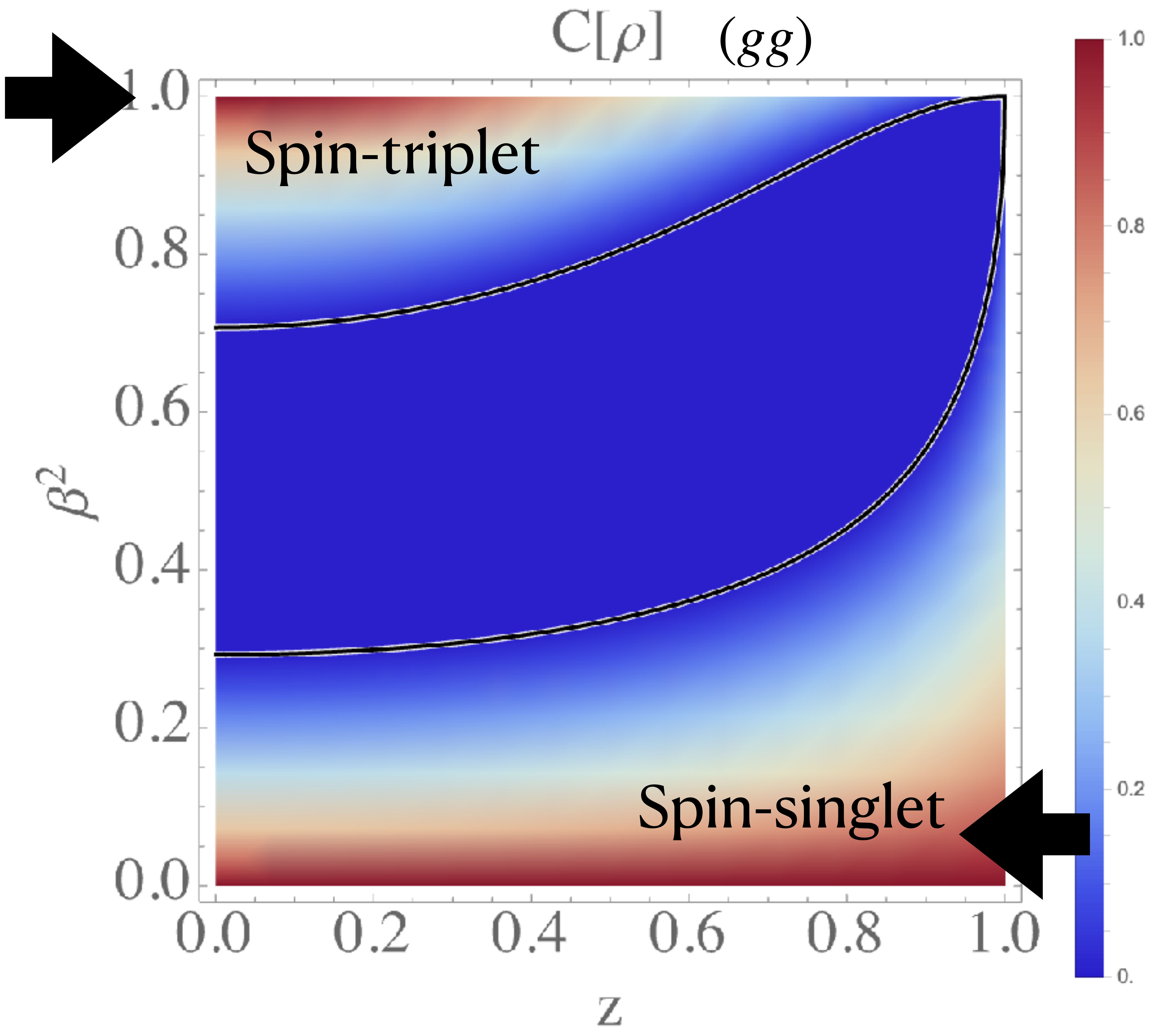
Entanglement marker D

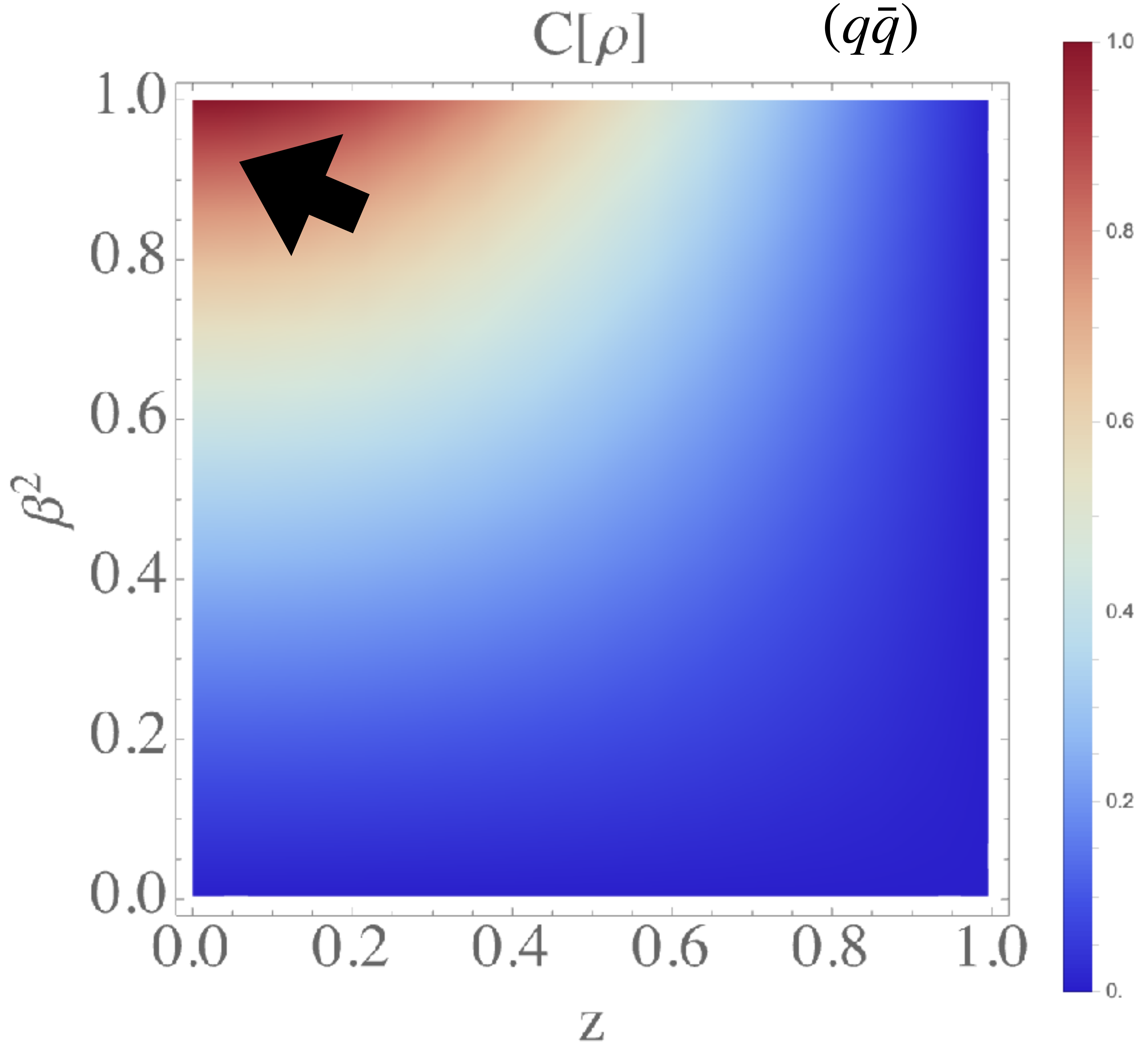
$D < -1/3$





Concurrence





Bell's inequality violation

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$\left|\vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2)\right| > 2$$

$$D^{(1)} = {}^1\!/_3(+C_{kk} + C_{rr} + C_{nn}),$$
$$D^{(k)} = {}^1\!/_3(+C_{kk} - C_{rr} - C_{nn}),$$
$$D^{(r)} = {}^1\!/_3(-C_{kk} + C_{rr} - C_{nn}),$$
$$D^{(n)} = {}^1\!/_3(-C_{kk} - C_{rr} + C_{nn}).$$

$$D_{\min} \equiv \min\{D^{(1)}, D^{(k)}, D^{(r)}, D^{(n)}\},$$

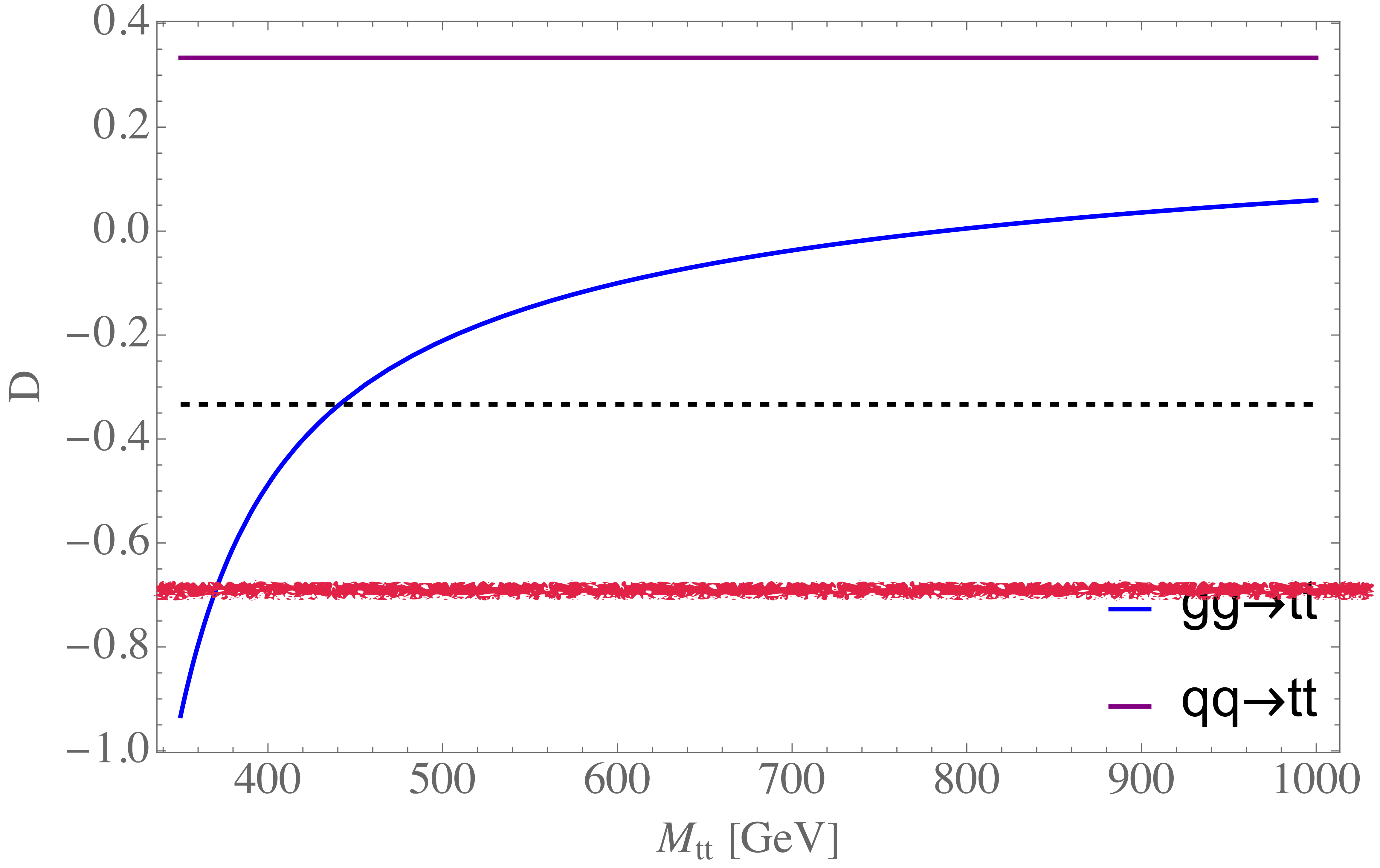
$$D_{\min} < -{}^1\!/_3 \qquad \text{(entanglement)}$$

$$D_{\min} < -\frac{1}{\sqrt{2}} \qquad \text{(Bell violation)}$$

Bell's inequality violation

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$\left|\vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2)\right| > 2$$



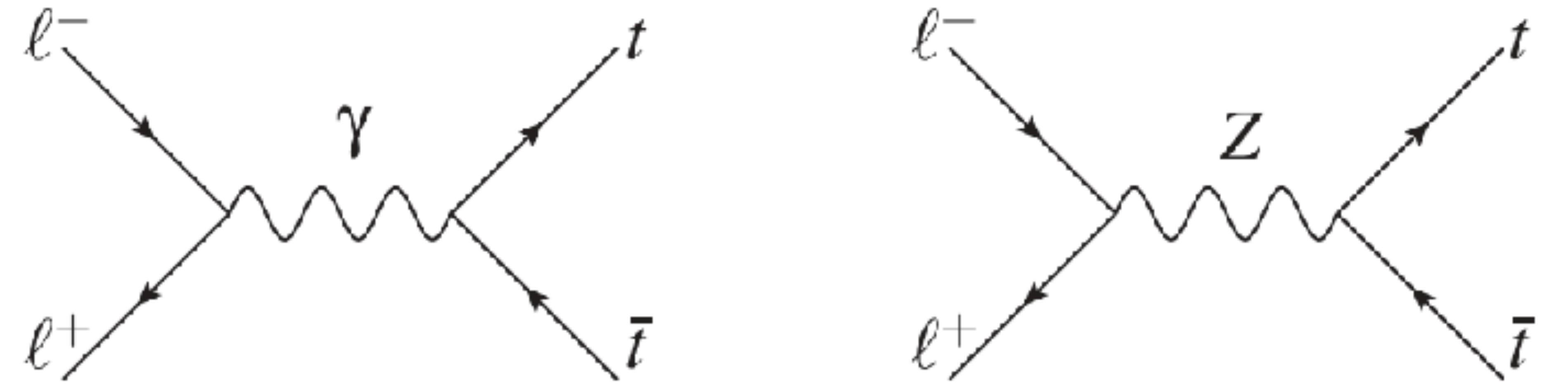
$$D_{\min} < -1/3 \qquad \text{(entanglement)}$$

$$D_{\min} < -\frac{1}{\sqrt{2}} \qquad \text{(Bell violation)}$$

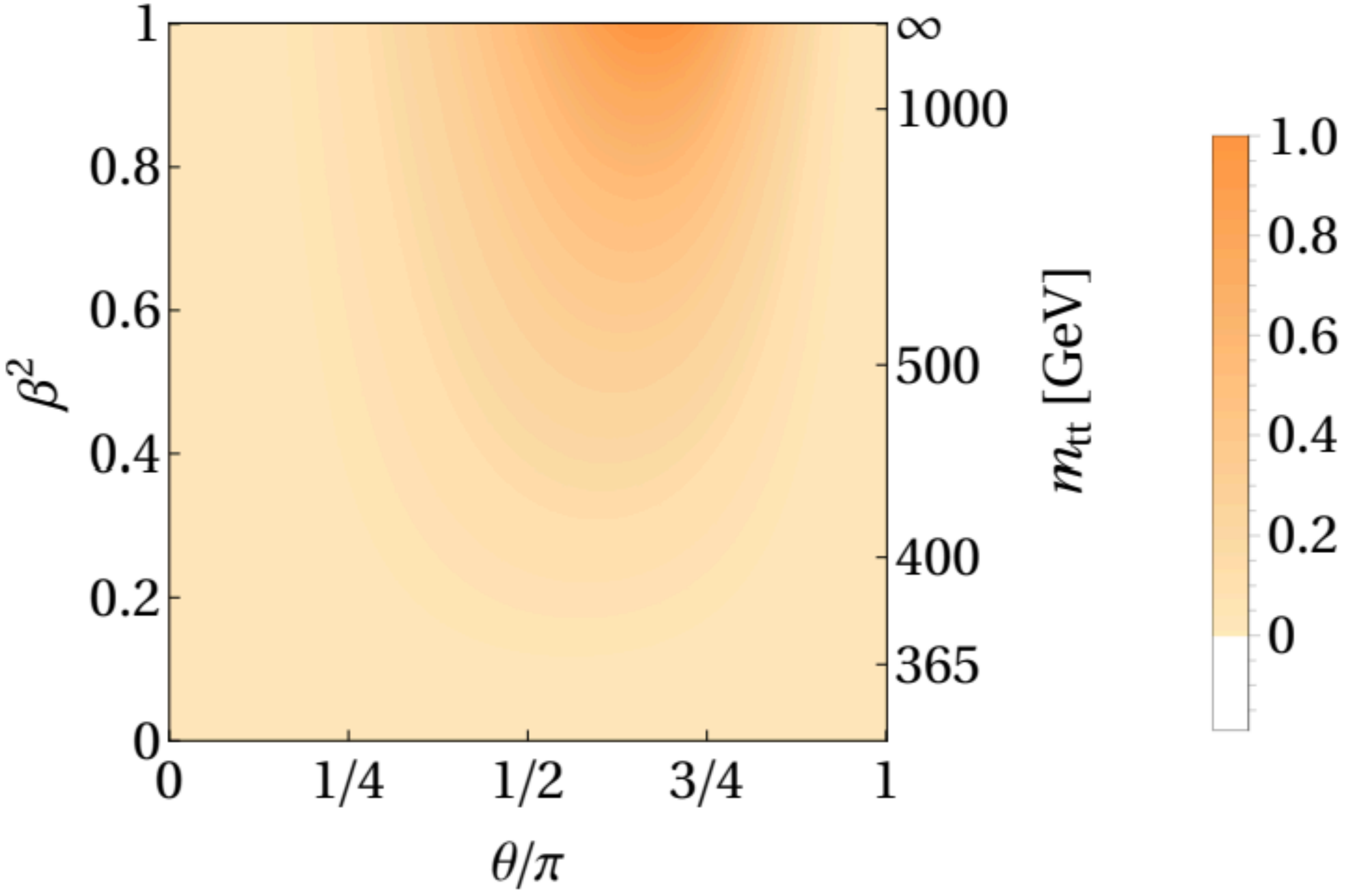
Top quark pairs @ Lepton Collider

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$\widehat{C}_{ij} = \begin{pmatrix} \boxed{\widehat{C}_{13}[+++] - \widehat{C}_{15}[+++]} & -\widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-] & -\widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-] \\ \widehat{C}_{14}[--+] - \widehat{C}_{16}[-+-] & \boxed{\widehat{C}_{13}[+++] + \widehat{C}_{15}[+++]} & \widehat{C}_{9}[+--] + \boxed{\widehat{C}_{11}[+++]} \\ \widehat{C}_{10}[--+] - \widehat{C}_{12}[-+-] & -\widehat{C}_{9}[+--] + \boxed{\widehat{C}_{11}[+++]} & \boxed{-\widehat{C}_{1}[+++] + \widehat{C}_{3}[+++]} \end{pmatrix}$$



Concurrence



$$\begin{cases} A^{[0]} &= F^{[0]} \left(\beta^2 c_\theta^2 - \beta^2 + 2\right), \\ \widetilde{C}^{[0]}_{kk} &= F^{[0]} \left[\beta^2 - \left(\beta^2 - 2\right) c_\theta^2\right], \\ \widetilde{C}^{[0]}_{kr} &= F^{[0]}\, 2\sqrt{1-\beta^2}\, c_\theta s_\theta, \\ \widetilde{C}^{[0]}_{rr} &= F^{[0]}\, s_\theta^2 (2-\beta^2), \\ \widetilde{C}^{[0]}_{nn} &= -F^{[0]}\, \beta^2 s_\theta^2, \end{cases} \qquad \begin{cases} A^{[1]} = 2\, F^{[1]}\, c_\theta, \\ \widetilde{C}^{[1]}_{kk} = 2\, F^{[1]}\, c_\theta, \\ \widetilde{C}^{[1]}_{kr} = F^{[1]} \sqrt{1-\beta^2}\, s_\theta, \\ \widetilde{C}^{[1]}_{rr} = 0, \\ \widetilde{C}^{[1]}_{nn} = 0; \end{cases} \qquad \begin{cases} A^{[2]} = F^{[2]} \left(1 + c_\theta^2\right), \\ \widetilde{C}^{[2]}_{kk} = F^{[2]} \left(1 + c_\theta^2\right), \\ \widetilde{C}^{[2]}_{kr} = 0, \\ \widetilde{C}^{[2]}_{rr} = -F^{[2]}\, s_\theta^2, \\ \widetilde{C}^{[2]}_{nn} = F^{[2]}\, s_\theta^2\,. \end{cases}$$

$$F^{[0]} = 48 e^4 \left(Q_t^2 Q_\ell^2 + 2\,\mathrm{Re}\, \frac{4 Q_t Q_\ell g_{Vt} g_{V\ell} m_t^2}{D_Z} + \frac{16 g_{Vt}^2 m_t^4 (g_{V\ell}^2 + g_{A\ell}^2)}{|D_Z|^2} \right),$$

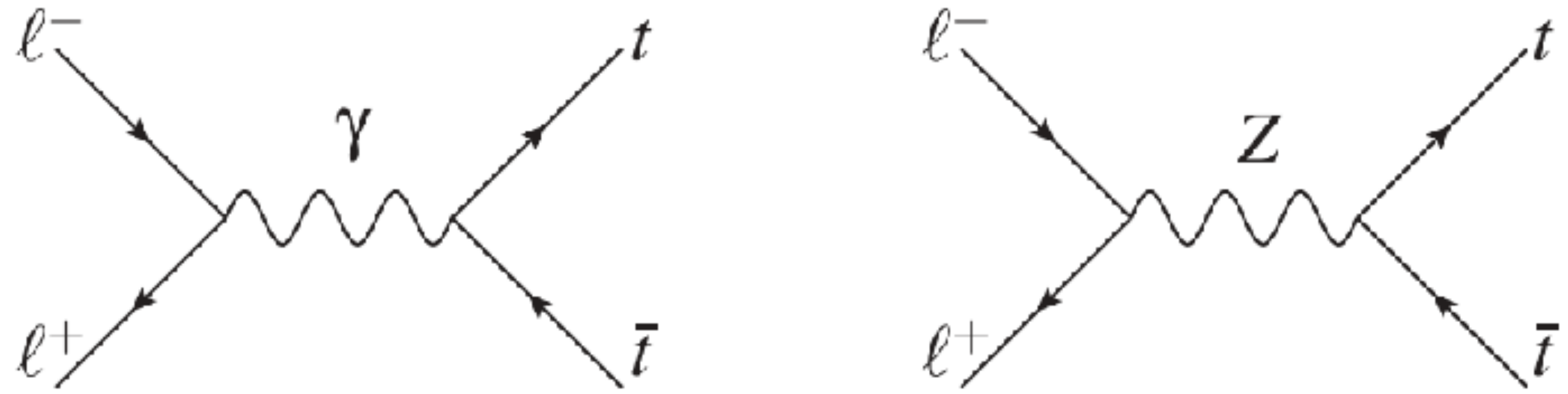
$$F^{[1]} = 192 e^4 g_{At} g_{A\ell} m_t^2 \beta \left(\frac{16 g_{Vt} g_{V\ell} m_t^2}{|D_Z|^2} + 2\,\mathrm{Re}\, \frac{Q_t Q_\ell}{D_Z} \right),$$

$$F^{[2]} = \frac{768 e^4 g_{At}^2 m_t^4 \beta^2 (g_{V\ell}^2 + g_{A\ell}^2)}{|D_Z|^2},$$

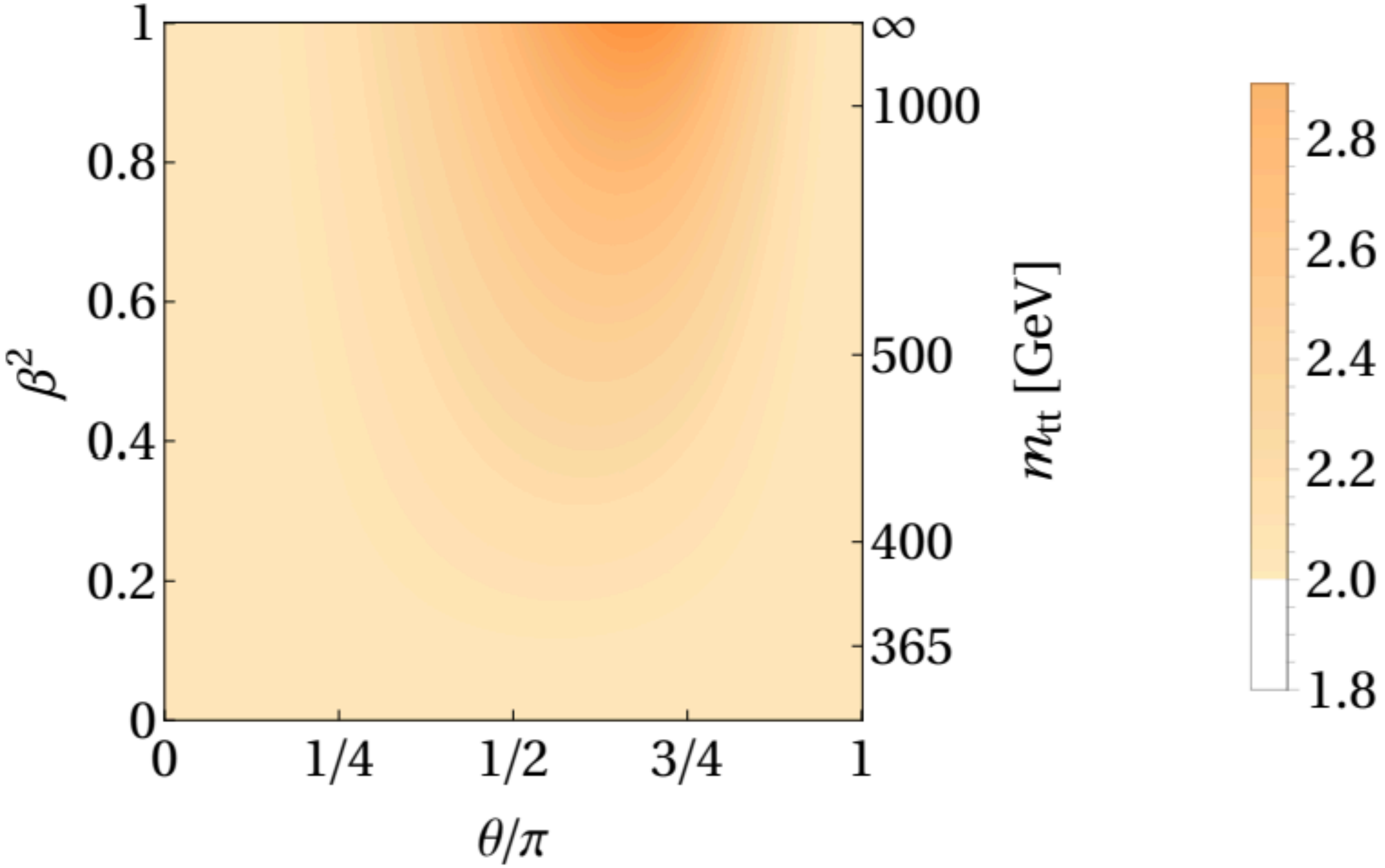
$$D_Z = c_W^2 s_W^2 (4 m_t^2 - (1-\beta^2)\, m_Z^2)$$

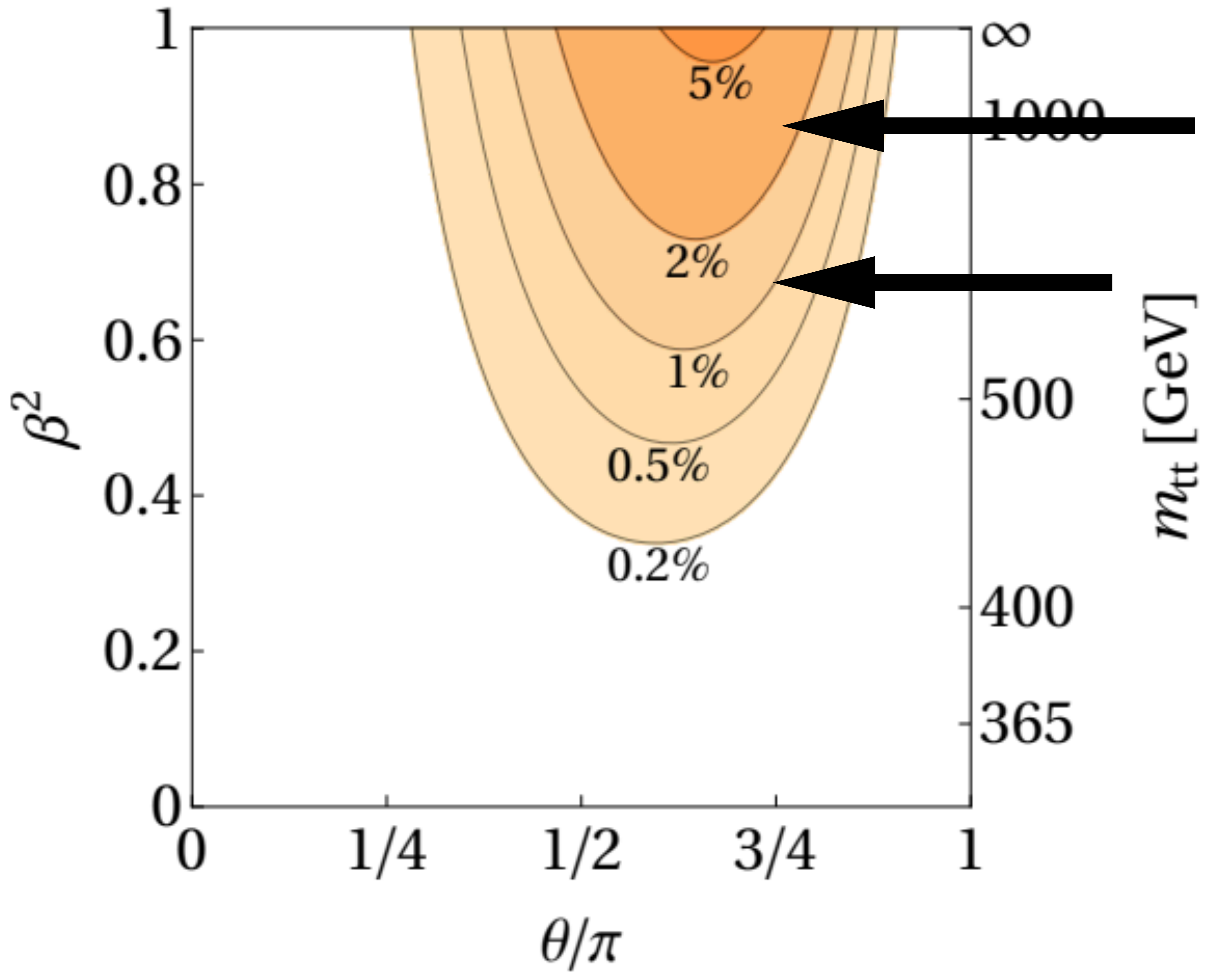
Top quark pairs @ Lepton Collider

[Maltoni, Severi, Tentori, Vryonidoud '24]



Bell's inequality violation

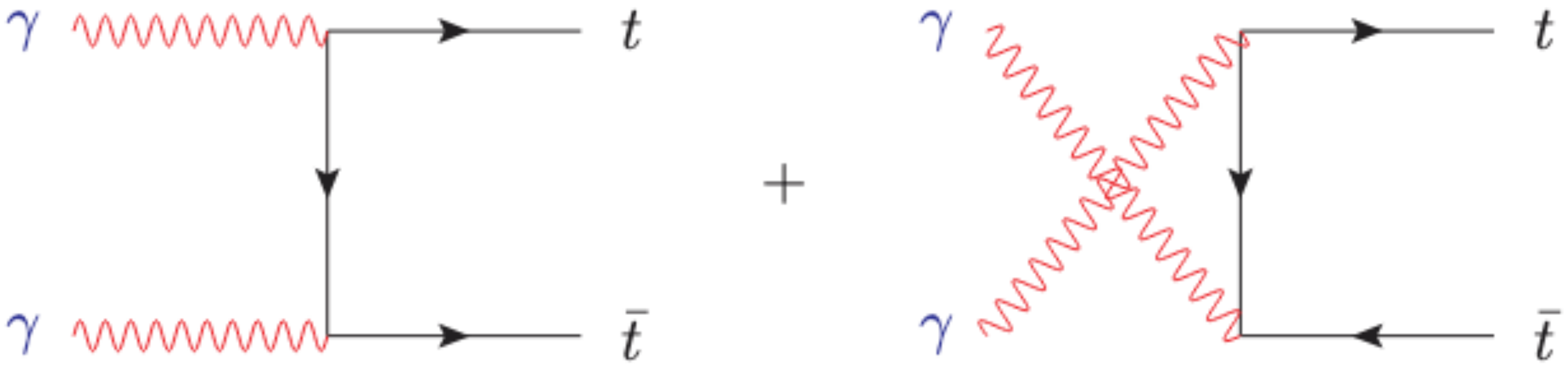




[Han, Low, Su '25]
[**DWK**, S.Y. Choi, J.S. Lee, and C. Park 26']

Top quark pairs @ Photon Collider

- We can also consider the process $\gamma\gamma \to t\bar{t}$.
  The process is very similar to the process $gg \to t\bar{t}$.

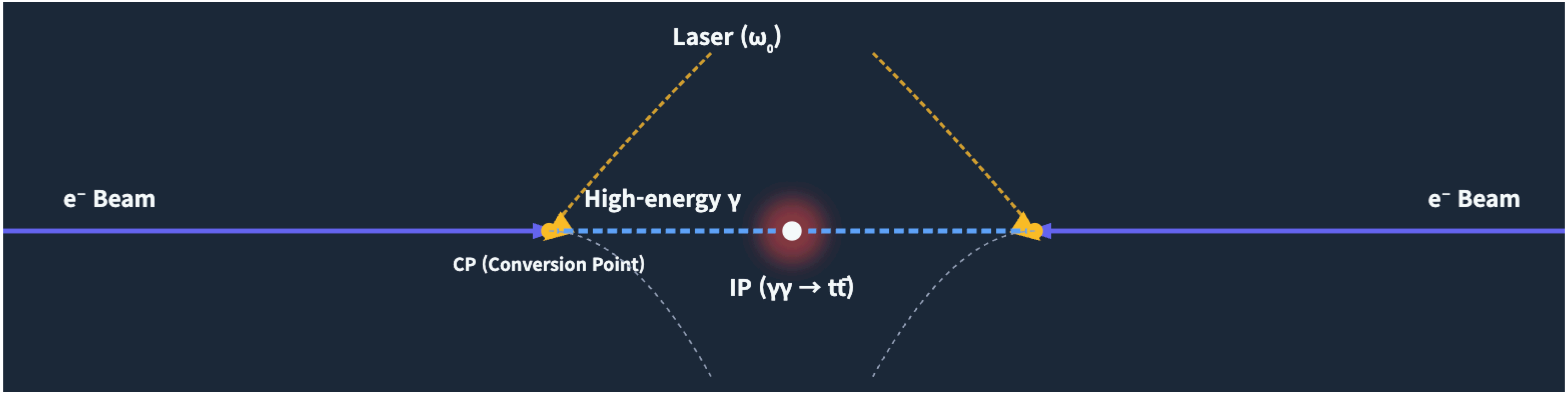


- Also independent with the process $l^+l^- \to t\bar{t}$.

$$
\begin{aligned}
F_\gamma &= \frac{64\pi^2\alpha^2 Q_t^4}{(1-\beta^2 z^2)^2}, \qquad \tilde{B}_i^\pm = 0,\\
\tilde{A}^{\gamma\gamma} &= F_\gamma(1+2\beta^2(1-z^2)-\beta^4(2-2z^2+z^4)),\\
\tilde{C}_{nn}^{\gamma\gamma} &= F_\gamma(1+2\beta^2(1-z^2)+\beta^4(2-2z^2+z^4)),\\
\tilde{C}_{kk}^{\gamma\gamma} &= -F_\gamma(1-2z^2(1-z^2)\beta^2-\beta^4(2-2z^2+z^4)),\\
\tilde{C}_{rr}^{\gamma\gamma} &= -F_\gamma(1-\beta^2(2-\beta^2)(2-2z^2+z^4)),\\
\tilde{C}_{rk}^{\gamma\gamma} &= \tilde{C}_{kr}^{\gamma\gamma} = F_\gamma 2z(1-z^2)^{3/2}\beta^2\sqrt{1-\beta^2},\\
\tilde{C}_{nr}^{\gamma\gamma} &= \tilde{C}_{rn}^{\gamma\gamma} = \tilde{C}_{nk}^{\gamma\gamma} = \tilde{C}_{kn}^{\gamma\gamma} = 0,
\end{aligned}
$$

Top quark pairs @ Photon Collider

[Ginzburg, Kotkin, Serbo, Telnov '83]



- It is well established that at a future linear e^+e^- collider, the Compton backscattering of the laser light with the $e^\pm$ beam can provide very high-energy photons.
- Especially, top-quark pair production is possible with large rates in (un)polarized photon-photon fusion.

[**DWK**, S.Y. Choi, J.S. Lee, and C. Park 26']

Top quark pairs @ Photon Collider

[Ginzburg, Kotkin, Serbo, Telnov '83]

- The $\gamma\gamma$ luminosity of the two colliding photons with helicity (λ_1, λ_2) is

$$\frac{1}{\mathcal{L}_{ee}} \frac{\mathrm{d}\mathcal{L}_{\gamma\gamma}^{\lambda_1\lambda_2}}{\mathrm{d}\tau} = \left(\langle 00 \rangle_\tau + \lambda_1 \langle 20 \rangle_\tau + \lambda_2 \langle 02 \rangle_\tau + \lambda_1 \lambda_2 \langle 22 \rangle_\tau \right) \left(\frac{2\pi\alpha^2}{\sigma_c^{\gamma_1} \, x \, M_e^2} \right) \left(\frac{2\pi\alpha^2}{\sigma_c^{\gamma_2} \, x \, M_e^2} \right)$$

with $\tau = \hat{s}/s$.

$$\sigma_c^{\gamma_1} = \sigma_c^0 + P_e P_c \sigma_c^1 ; \quad \sigma_c^{\gamma_2} = \sigma_c^0 + \tilde{P}_e \tilde{P}_c \sigma_c^1 .$$

$$\langle ij \rangle_\tau \equiv \int_{\tau/y_{\rm max}}^{y_{\rm max}} \frac{\mathrm{d}y}{y} f_i(y) \tilde{f}_j(\tau/y) ,$$

$$\sigma_c^0 = \frac{2\pi\alpha^2}{x M_e^2} \left[\left(1 - \frac{4}{x} - \frac{8}{x^2} \right) \log(x+1) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(x+1)^2} \right] ,$$

$$\sigma_c^1 = \frac{2\pi\alpha^2}{x M_e^2} \left[\left(1 + \frac{2}{x} \right) \log(x+1) - \frac{5}{2} + \frac{1}{x+1} - \frac{1}{2(x+1)^2} \right] .$$

$$f_0(y) = \frac{1}{1-y} + (1-y) - 4r(1-r) - r x (2r-1)(2-y) P_e P_c ;$$

$$f_2(y) = r x \left[1 + (1-y)(2r-1)^2 \right] P_e - (2r-1) \left(\frac{1}{1-y} + 1 - y \right) P_c ;$$

$$\tilde{f}_0(\tau/y) = \frac{1}{1-\tau/y} + (1-\tau/y) - 4\tilde{r}(1-\tilde{r}) - \tilde{r} x (2\tilde{r}-1)(2-\tau/y) \tilde{P}_e \tilde{P}_c ,$$

$$\tilde{f}_2(\tau/y) = \tilde{r} x \left[1 + (1-\tau/y)(2\tilde{r}-1)^2 \right] \tilde{P}_e - (2\tilde{r}-1) \left(\frac{1}{1-\tau/y} + 1 - \tau/y \right) \tilde{P}_c ,$$

x is machine parameter

$$x = \frac{4 E_b \omega_0}{M_e^2} = 15.3 \left(\frac{E_b}{\rm TeV} \right) \left(\frac{\omega_0}{\rm eV} \right) , \; = 4.8$$

y is fraction of the e^- to photon

$$r = \frac{y}{x(1-y)} , \quad \tilde{r} = \frac{\tau/y}{x(1-\tau/y)} .$$

Top quark pairs @ Photon Collider

- The $\gamma\gamma$ luminosity weighted polarization coefficients

$$\widehat{C}_i = \sum_{\lambda_1,\lambda_2=\pm} c_i^{\lambda_1\lambda_2}\,, \quad \longrightarrow \quad \widehat{C}_i^w \equiv \sum_{\lambda_1,\lambda_2=\pm} w^{\lambda_1\lambda_2}\, c_i^{\lambda_1\lambda_2}\,,$$

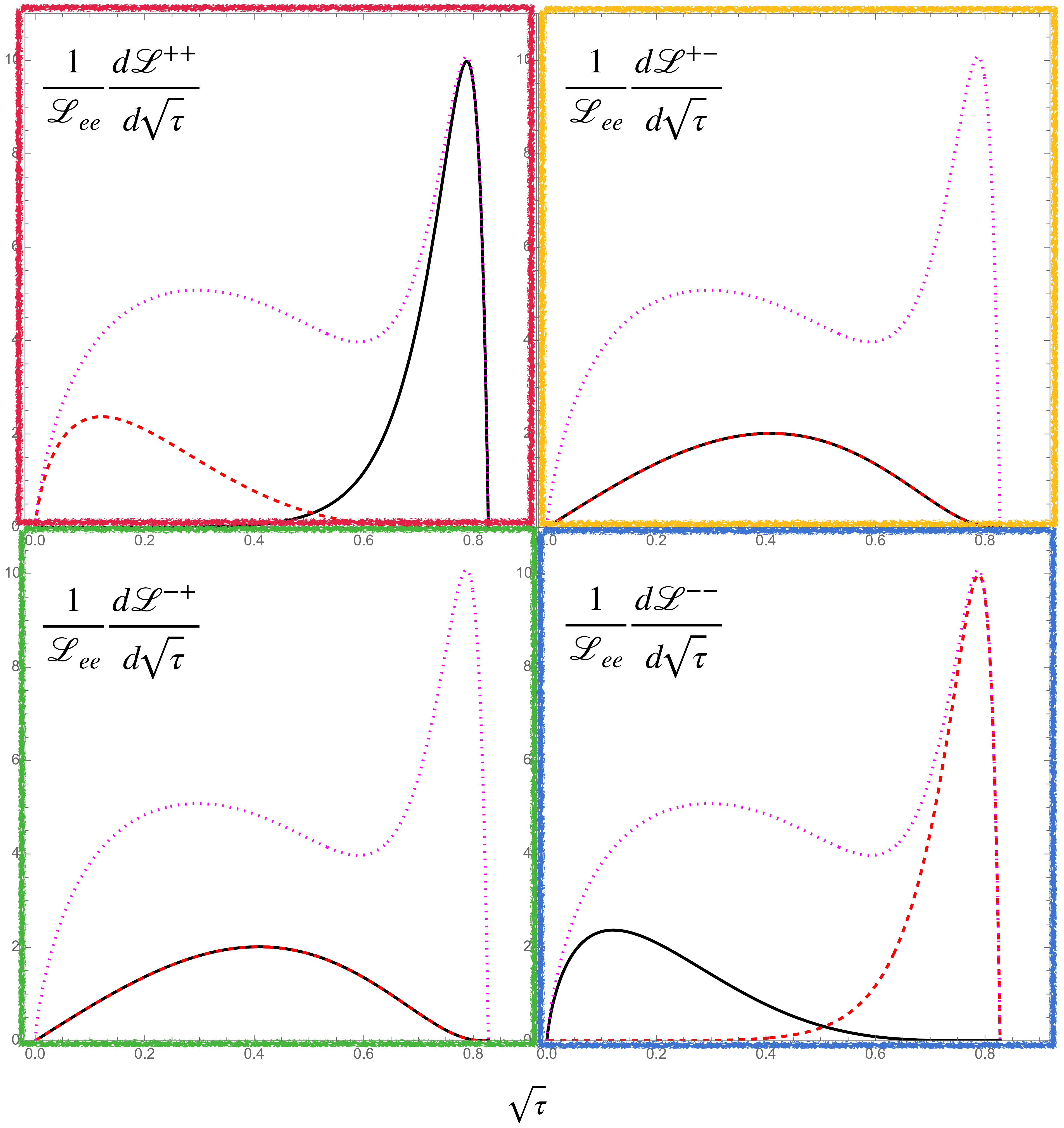
$$w^{\lambda_1\lambda_2} \equiv \frac{L^{\lambda_1\lambda_2}}{\sum_{\lambda_1,\lambda_2=\pm} L^{\lambda_1\lambda_2}}\,, \qquad L^{\lambda_1\lambda_2} \equiv \frac{1}{\mathcal{L}_{ee}} \frac{\mathrm{d}\mathcal{L}_{\gamma\gamma}^{\lambda_1\lambda_2}}{\mathrm{d}\sqrt{\tau}}\,.$$

$$\frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm} L^{\lambda_1\lambda_2} = \frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm} \frac{1}{\mathcal{L}_{ee}} \frac{\mathrm{d}\mathcal{L}_{\gamma\gamma}^{\lambda_1\lambda_2}}{\mathrm{d}\sqrt{\tau}} \xrightarrow{P_e P_c \,=\, \tilde{P}_e \tilde{P}_c \,=\, 0} \frac{1}{\mathcal{L}_{ee}} \frac{\mathrm{d}\mathcal{L}_{\gamma\gamma}^{\mathrm{unp}}}{\mathrm{d}\sqrt{\tau}}\,.$$

[**DWK**, S.Y. Choi, J.S. Lee, and C. Park 26']

Top quark pairs @ Photon Collider

[Ginzburg, Kotkin, Serbo, Telnov '83]

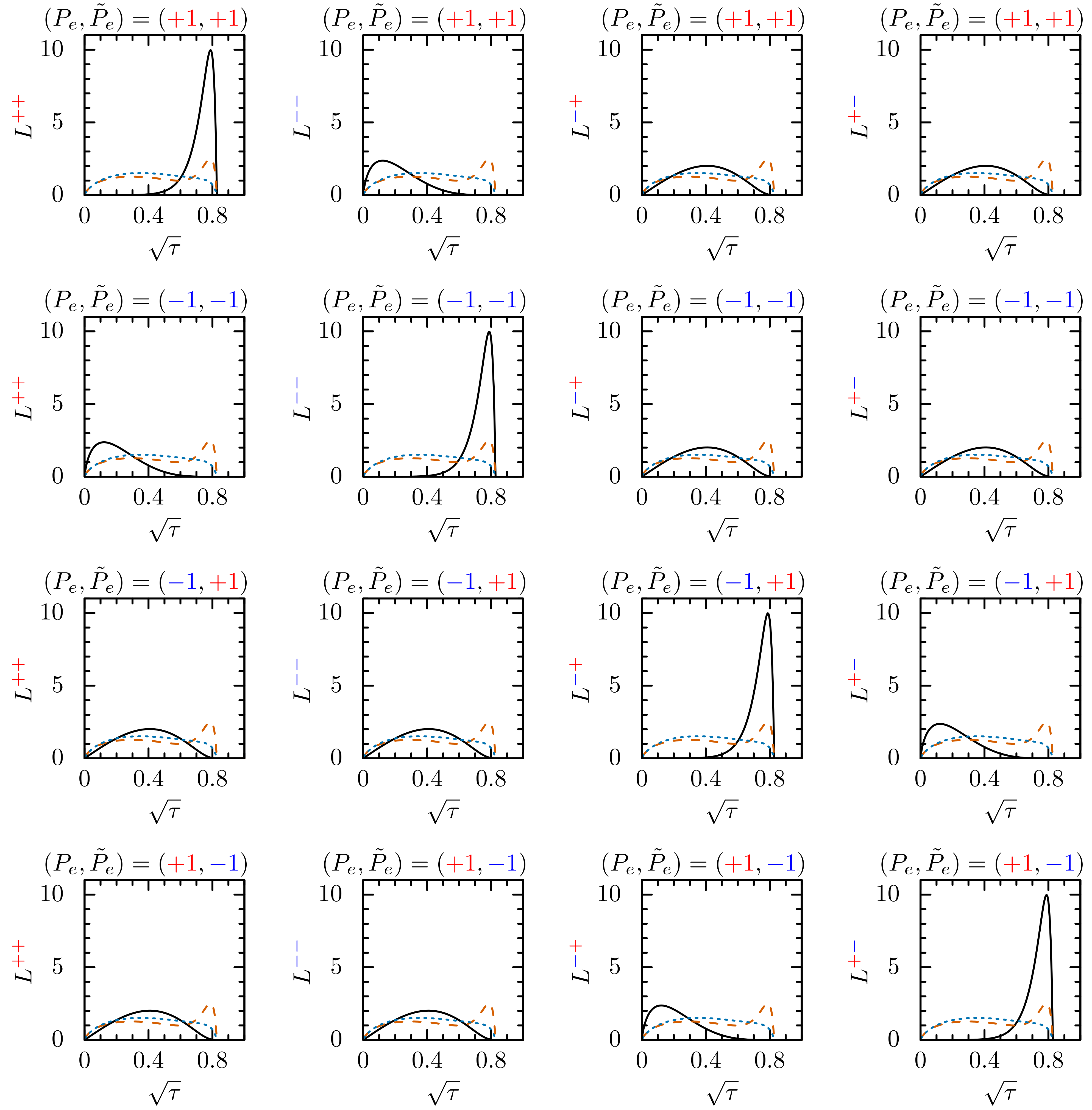


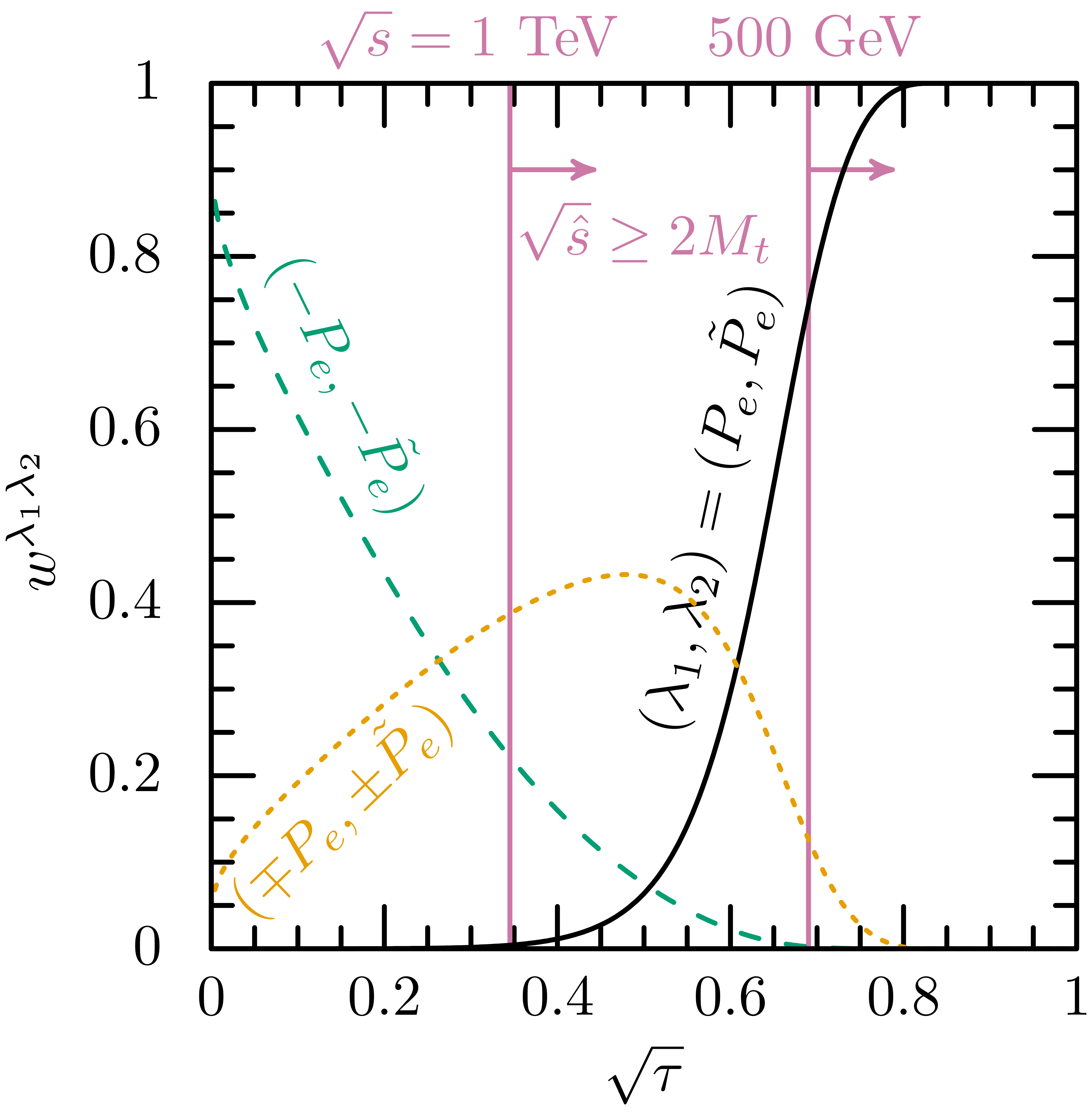
- For the polarized laser beams we can almost extract corresponding polarization of initial photon.

$(\lambda_1\lambda_2)\downarrow$ $(\sigma\bar{\sigma})\rightarrow$	$\langle ++\rangle$	$\langle --\rangle$	$\langle -+\rangle$	$\langle +-\rangle$
$\langle ++\rangle$	$\frac{2M_t}{\sqrt{\hat{s}}}\left[1+\beta\right]$	$\frac{2M_t}{\sqrt{\hat{s}}}\left[1-\beta\right]$	0	0
$\langle --\rangle$	$\frac{2M_t}{\sqrt{\hat{s}}}\left[-1+\beta\right]$	$\frac{2M_t}{\sqrt{\hat{s}}}\left[-1-\beta\right]$	0	0
$\langle -+\rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}}\beta\sin^2\Theta$	$\frac{2M_t}{\sqrt{\hat{s}}}\beta\sin^2\Theta$	$-\beta\sin\Theta\,(\cos\Theta+1)$	$-\beta\sin\Theta\,(\cos\Theta-1)$
$\langle +-\rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}}\beta\sin^2\Theta$	$\frac{2M_t}{\sqrt{\hat{s}}}\beta\sin^2\Theta$	$-\beta\sin\Theta\,(\cos\Theta-1)$	$-\beta\sin\Theta\,(\cos\Theta+1)$

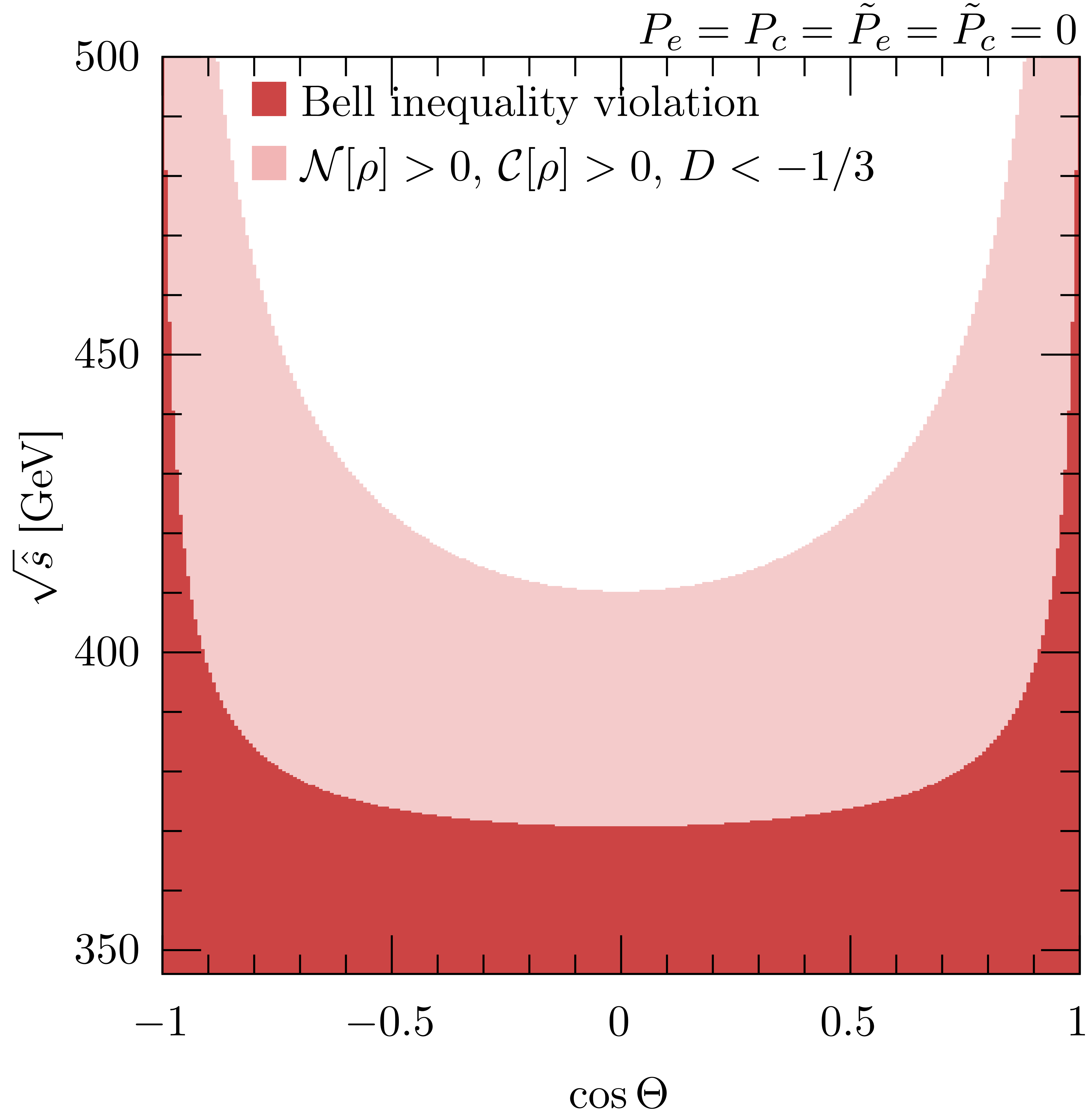
Top quark pairs @ Photon Collider

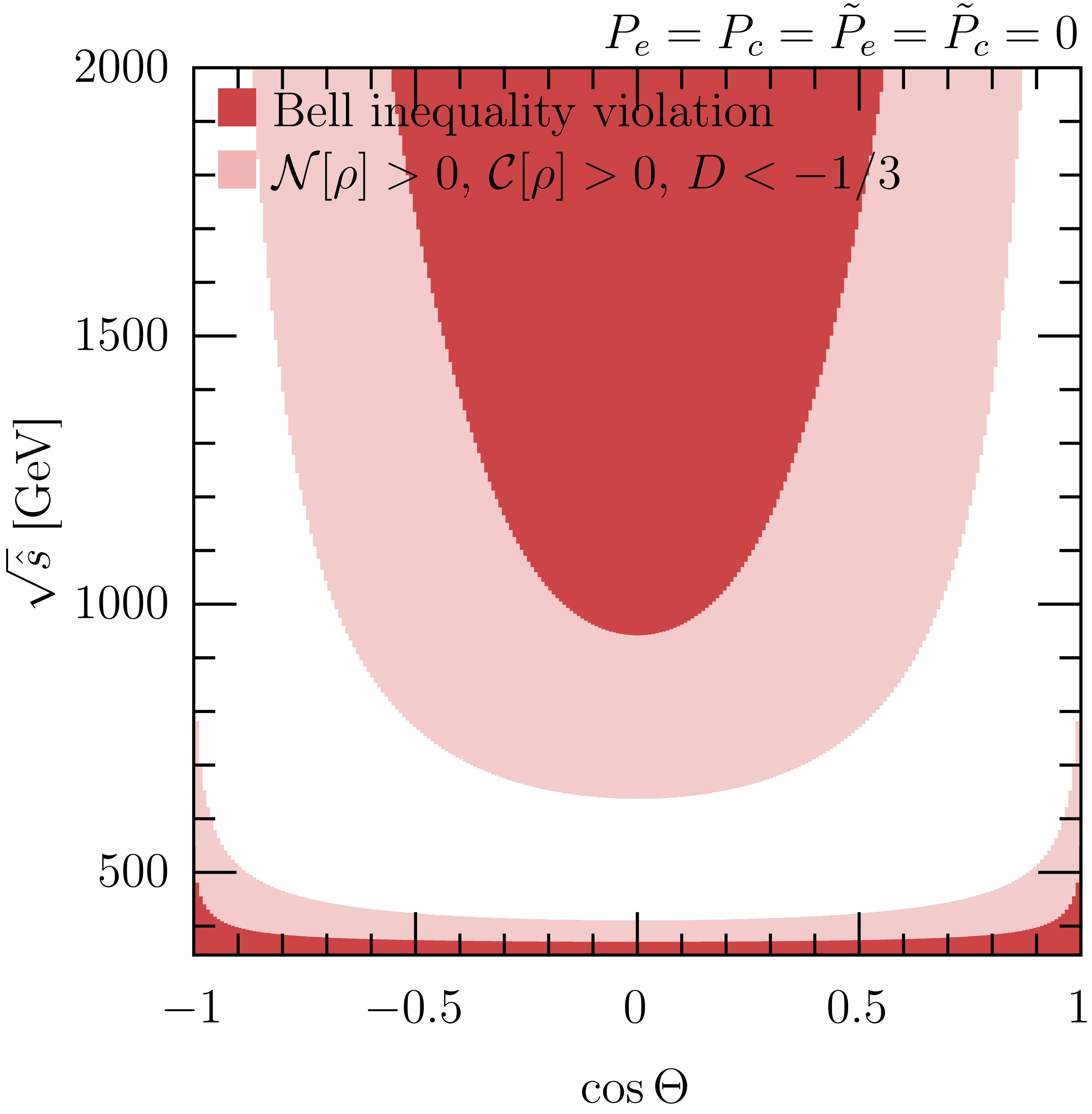
- The $\gamma\gamma$ luminosity of the two colliding photons with helicity (λ_1, λ_2) for $P_e P_c = \tilde{P}_e \tilde{P}_c = -1$





Top quark pairs @ Photon Collider





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Top quark pairs @ Photon Collider

- For perfectly polarized colliding photons,

$(\lambda_1\lambda_2)\downarrow\ (\sigma\bar{\sigma})\rightarrow$	$\langle ++\rangle$	$\langle --\rangle$	$\langle -+\rangle$	$\langle +-\rangle$
$\langle ++\rangle$	$\frac{2M_t}{\sqrt{\hat{s}}}\,[1+\beta]$	$\frac{2M_t}{\sqrt{\hat{s}}}\,[1-\beta]$	0	0

$$\rho = \frac{1}{2(1+\beta^2)}\left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & (1+\beta)^2 & -(1-\beta^2) & 0 \\ \hline 0 & -(1-\beta^2) & (1-\beta)^2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \; ; \quad \rho^{T_2} = \frac{1}{2(1+\beta^2)}\left(\begin{array}{cc|cc} 0 & 0 & 0 & -(1-\beta^2) \\ 0 & (1+\beta)^2 & 0 & 0 \\ \hline 0 & 0 & (1-\beta)^2 & 0 \\ -(1-\beta^2) & 0 & 0 & 0 \end{array}\right)$$

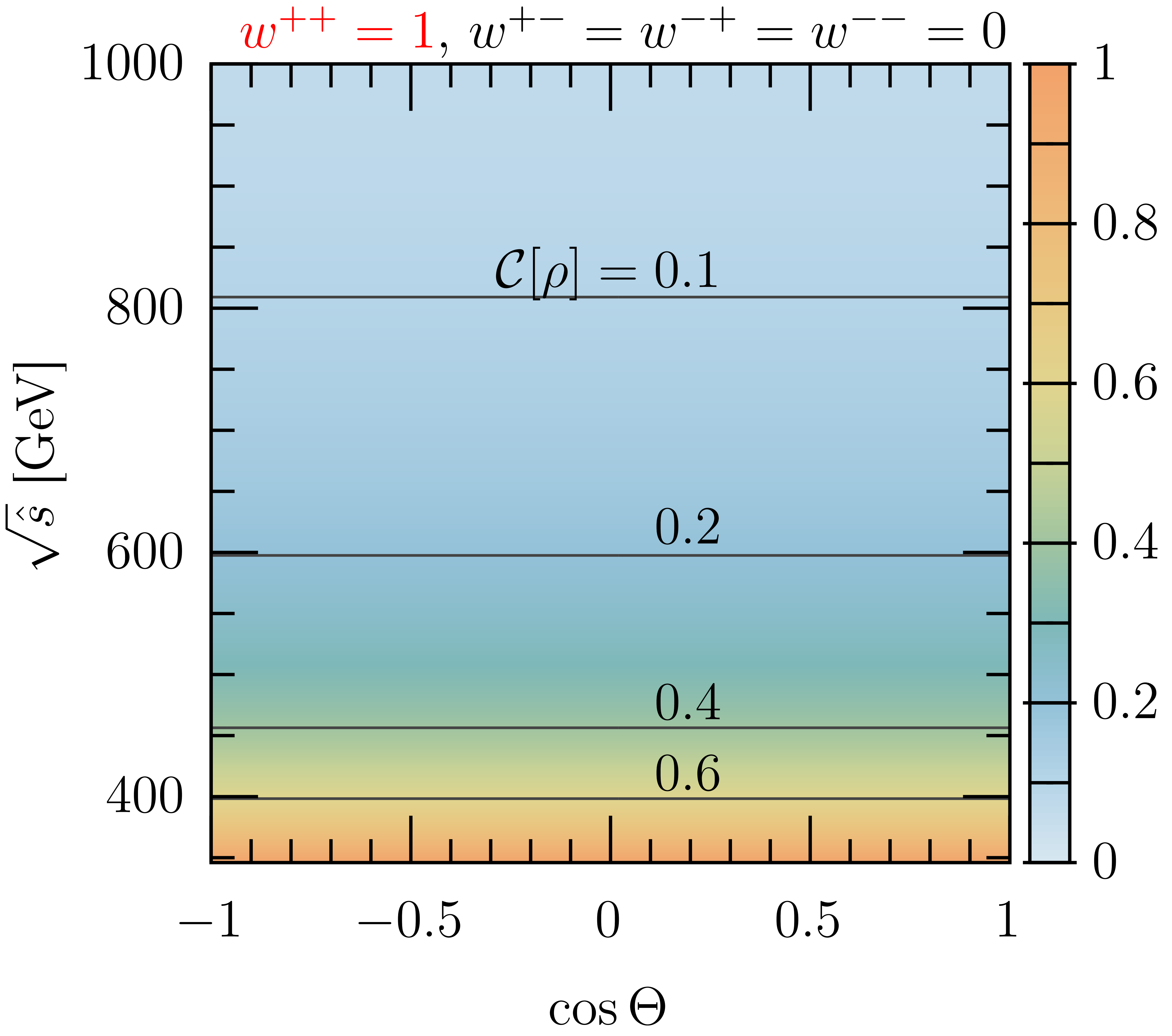
$$C[\rho] = \frac{1-\beta^2}{1+\beta^2} > 0 \qquad\qquad m_{12} = 1 + \frac{(1-\beta^2)^2}{(1+\beta^2)^2} > 1$$

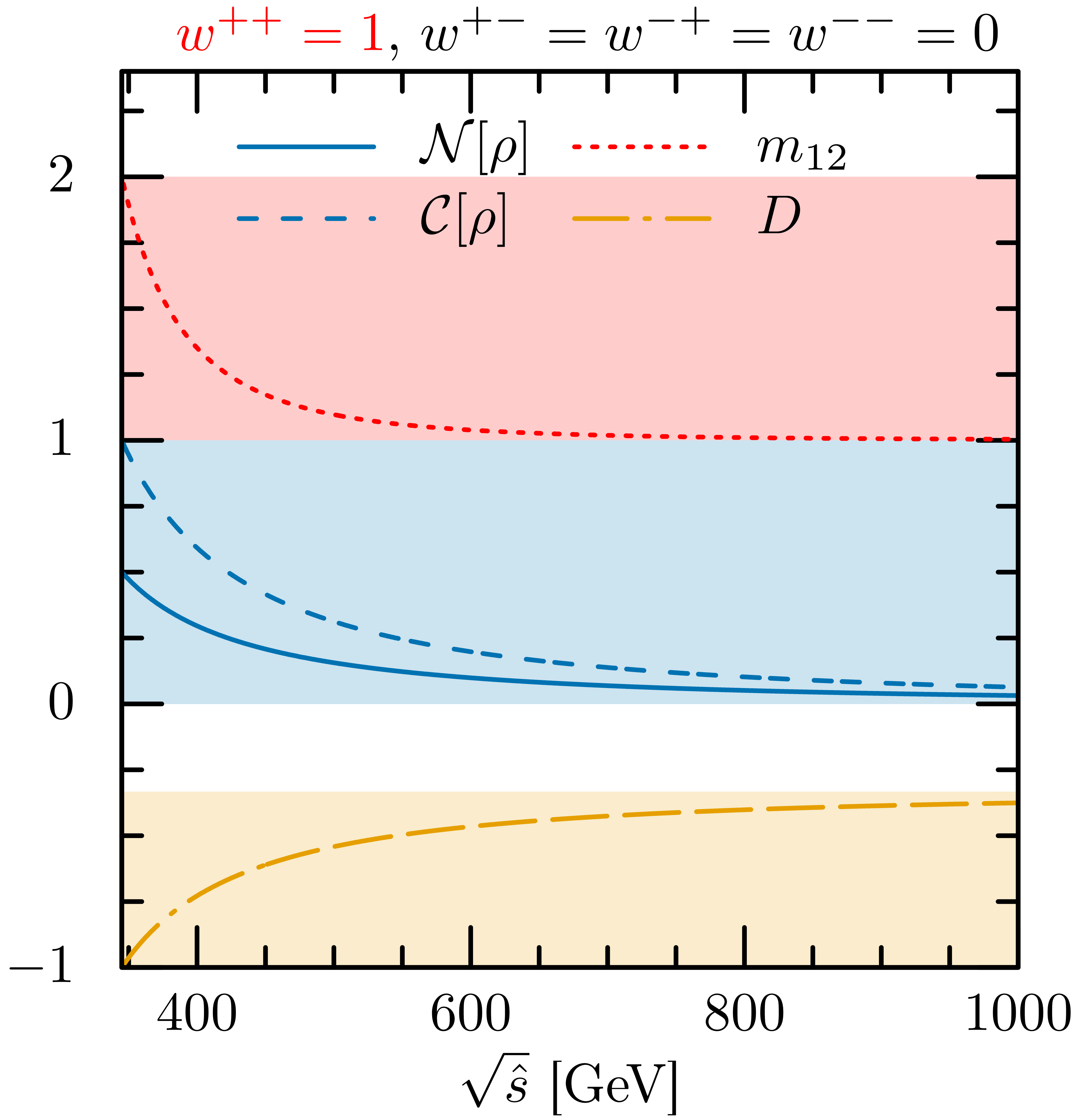
Entire region is Entangled and violate CHSH inequality

Top quark pairs @ Photon Collider

- For perfectly polarized colliding photons,

$(\lambda_1\lambda_2)\downarrow\ (\sigma\bar\sigma)\rightarrow$	$\langle ++\rangle$	$\langle --\rangle$	$\langle -+\rangle$	$\langle +-\rangle$
$\langle ++\rangle$	$\frac{2M_t}{\sqrt{\hat s}}\,[1+\beta]$	$\frac{2M_t}{\sqrt{\hat s}}\,[1-\beta]$	0	0





Top quark pairs @ Photon Collider

- For perfectly polarized colliding photons,

$(\lambda_1\lambda_2)\downarrow\ (\sigma\bar{\sigma})\rightarrow$	$\langle ++\rangle$	$\langle --\rangle$	$\langle -+\rangle$	$\langle +-\rangle$
$\langle +-\rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}}\beta\sin^2\Theta$	$\frac{2M_t}{\sqrt{\hat{s}}}\beta\sin^2\Theta$	$-\beta\sin\Theta\,(\cos\Theta-1)$	$-\beta\sin\Theta\,(\cos\Theta+1)$

$$\mathscr{C}[\rho]^{(+-/-+)} = \frac{(1-z^2)\beta^2}{2-(1-z^2)\beta^2} > 0 \qquad m_{12}^{(+-/-+)} = 1 + \frac{(1-z^2)^2\beta^4}{(2-(1-z^2)\beta^2)^2}$$

Entire region is Entangled and violate CHSH inequality

[**DWK**, S.Y. Choi, J.S. Lee, and C. Park 26']

Top quark pairs @ Photon Collider

- For perfectly polarized colliding photons,

$(\lambda_1\lambda_2)\downarrow$ $(\sigma\bar{\sigma})\rightarrow$	$\langle ++\rangle$	$\langle --\rangle$	$\langle -+\rangle$	$\langle +-\rangle$
$\langle +-\rangle$	$-\frac{2M_t}{\sqrt{\hat{s}}}\beta\,\sin^2\Theta$	$\frac{2M_t}{\sqrt{\hat{s}}}\beta\,\sin^2\Theta$	$-\beta\sin\Theta\,(\cos\Theta-1)$	$-\beta\sin\Theta\,(\cos\Theta+1)$

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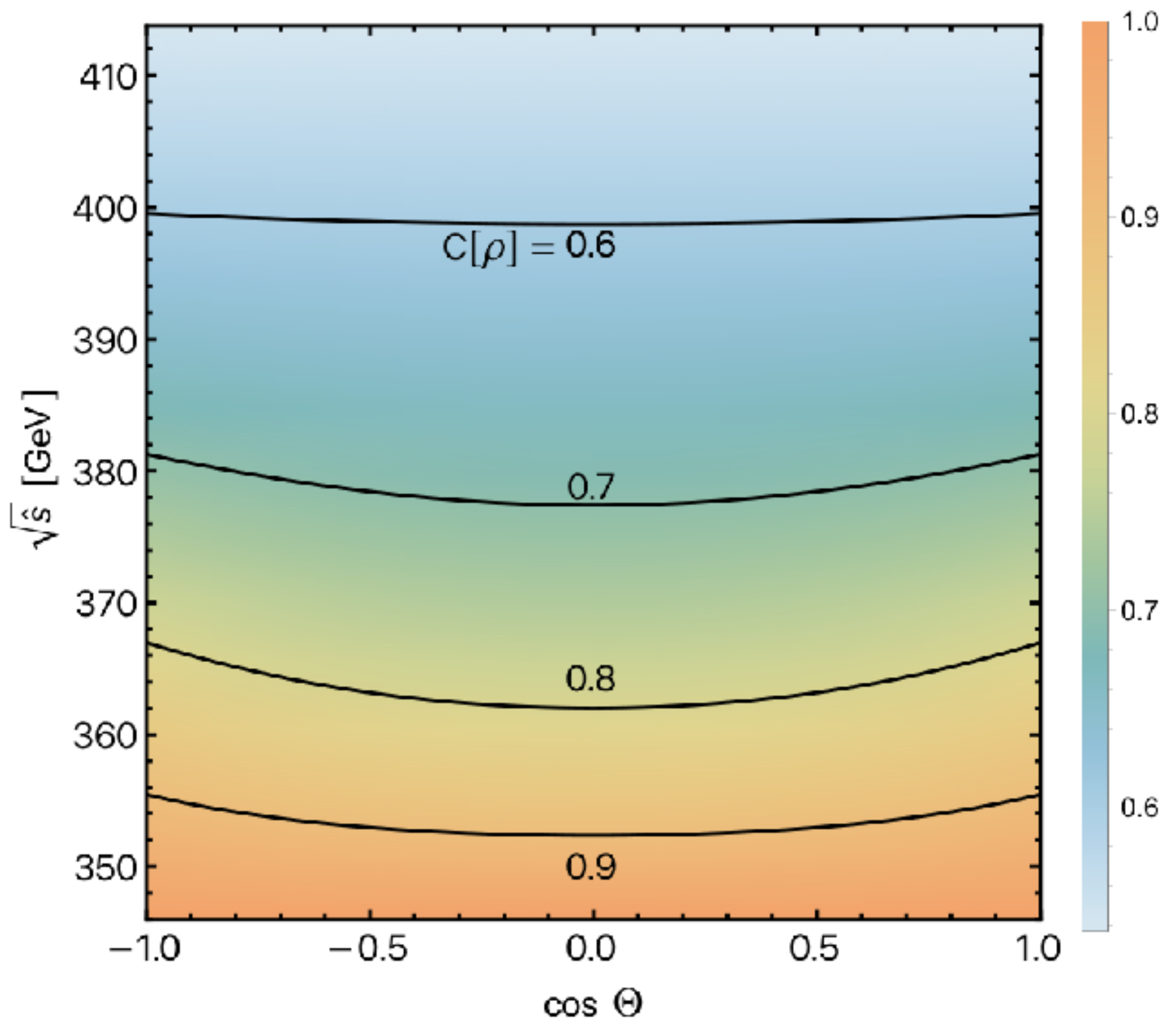
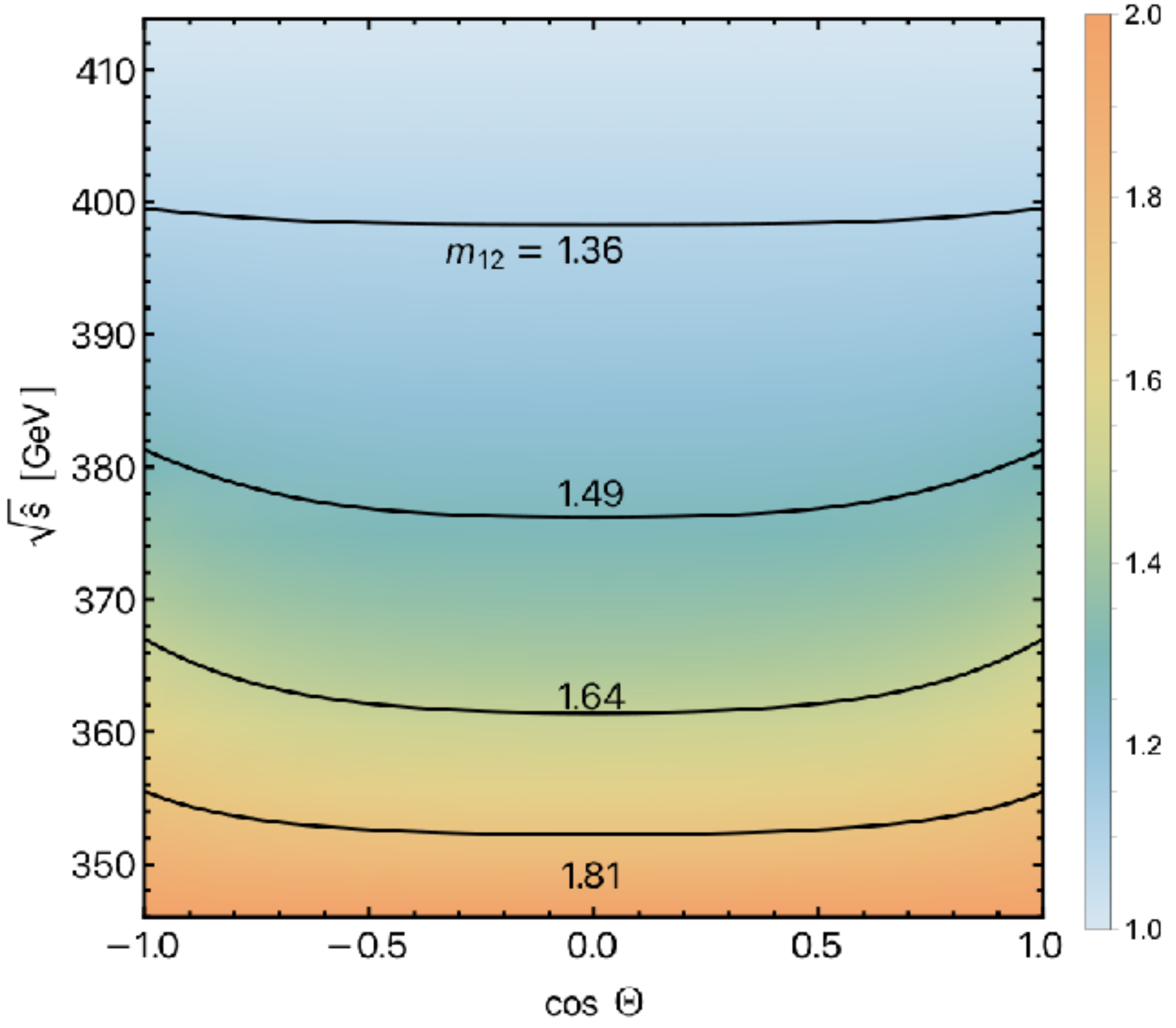
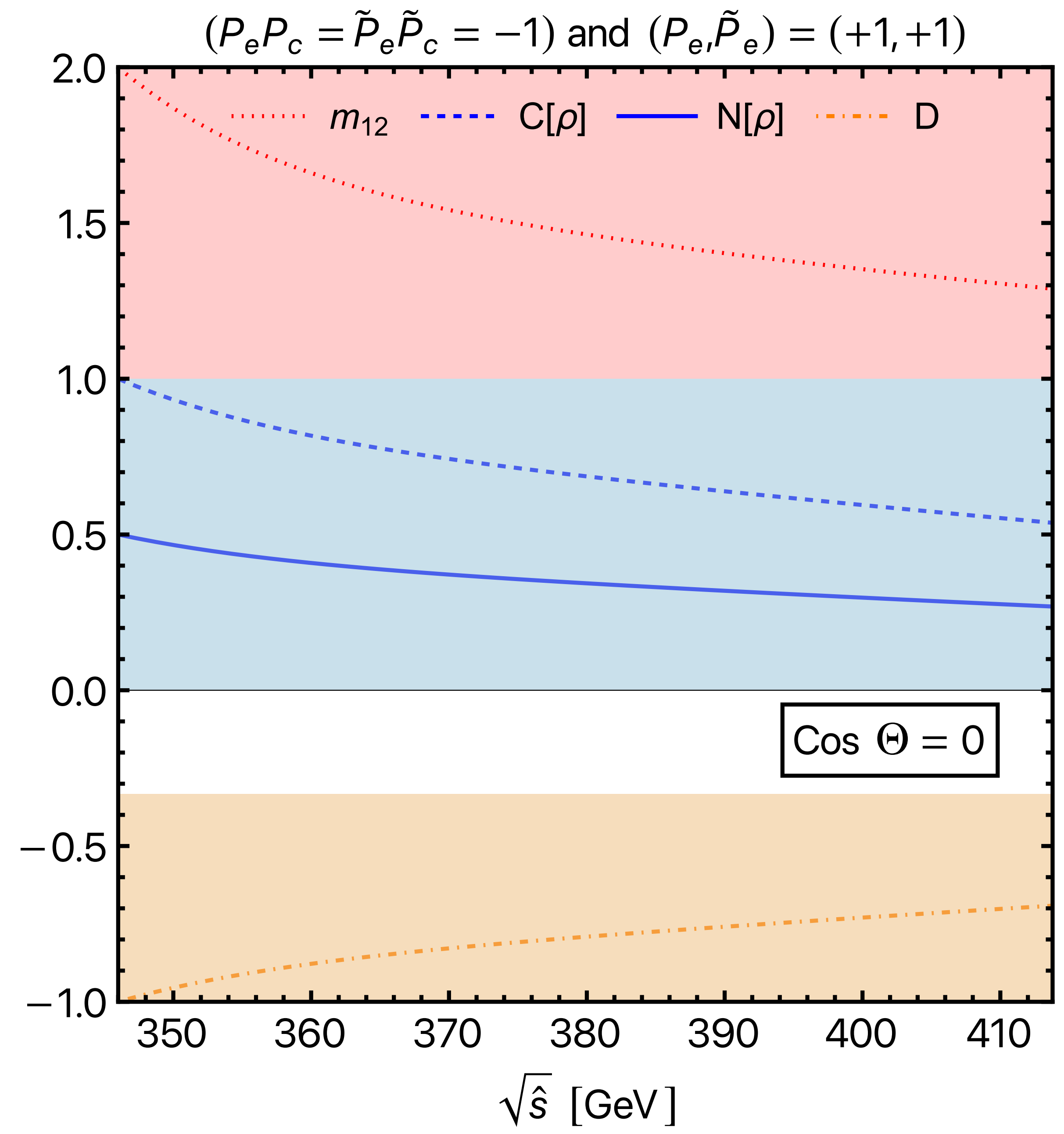
Top quark pairs @ Photon Collider

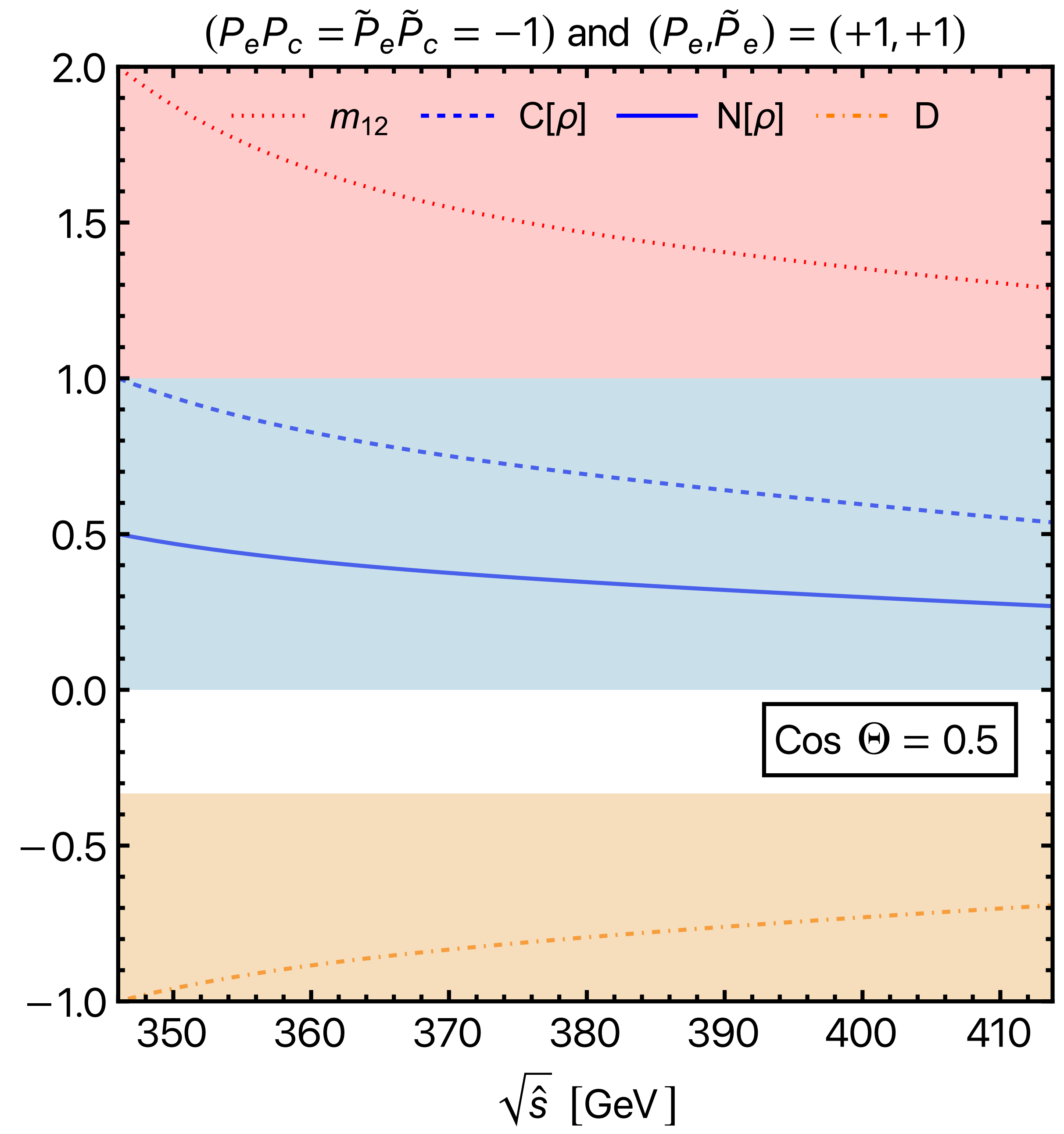
- Polarized colliding photons, $\sqrt{\hat{s}} = 500\,\text{GeV}$

$$C[\rho] \approx \frac{1-\beta^2}{1+\beta^2} > 0 \qquad m_{12} \approx 1 + \frac{(1-\beta^2)^2}{(1+\beta^2)^2} > 1$$









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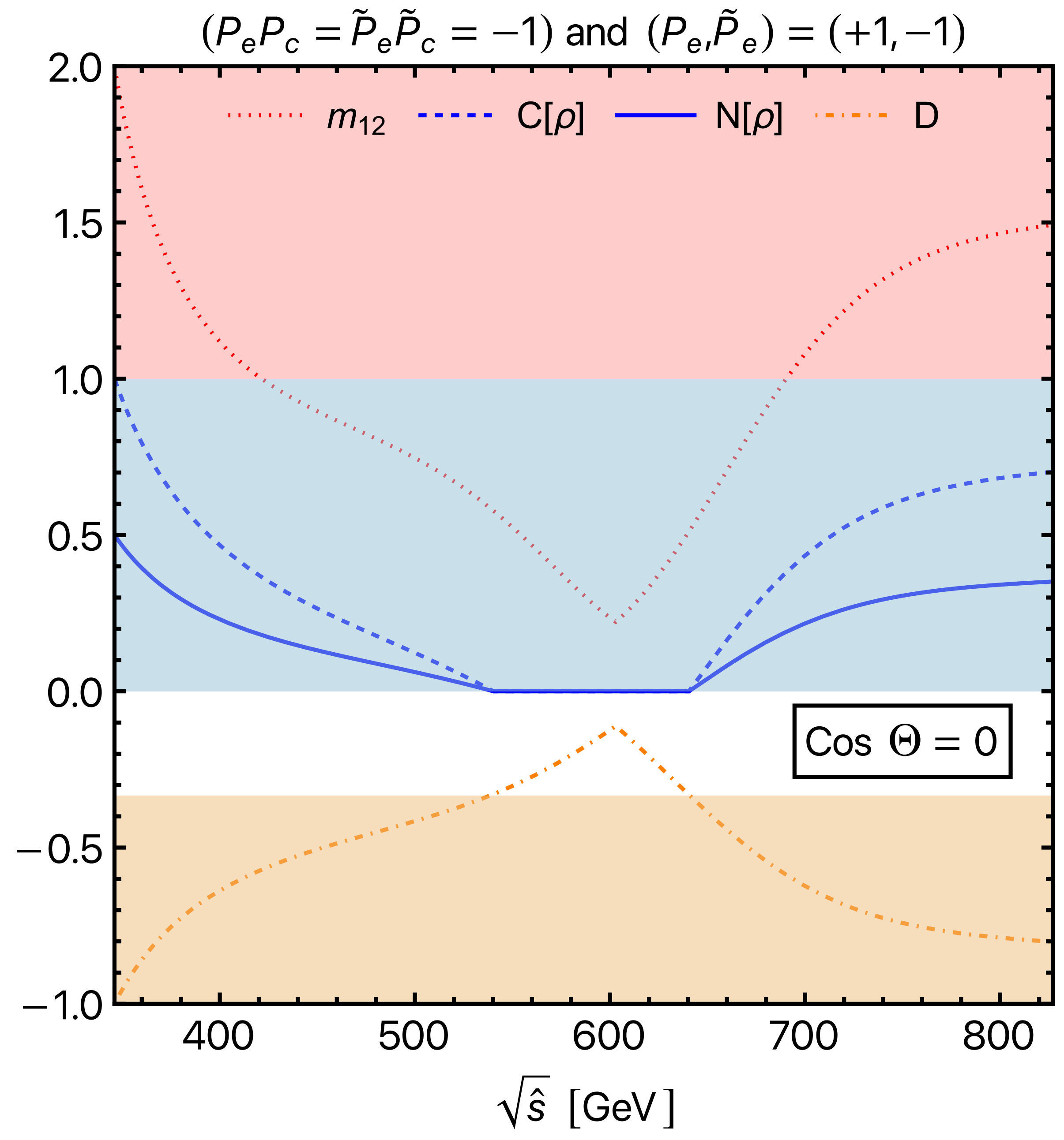
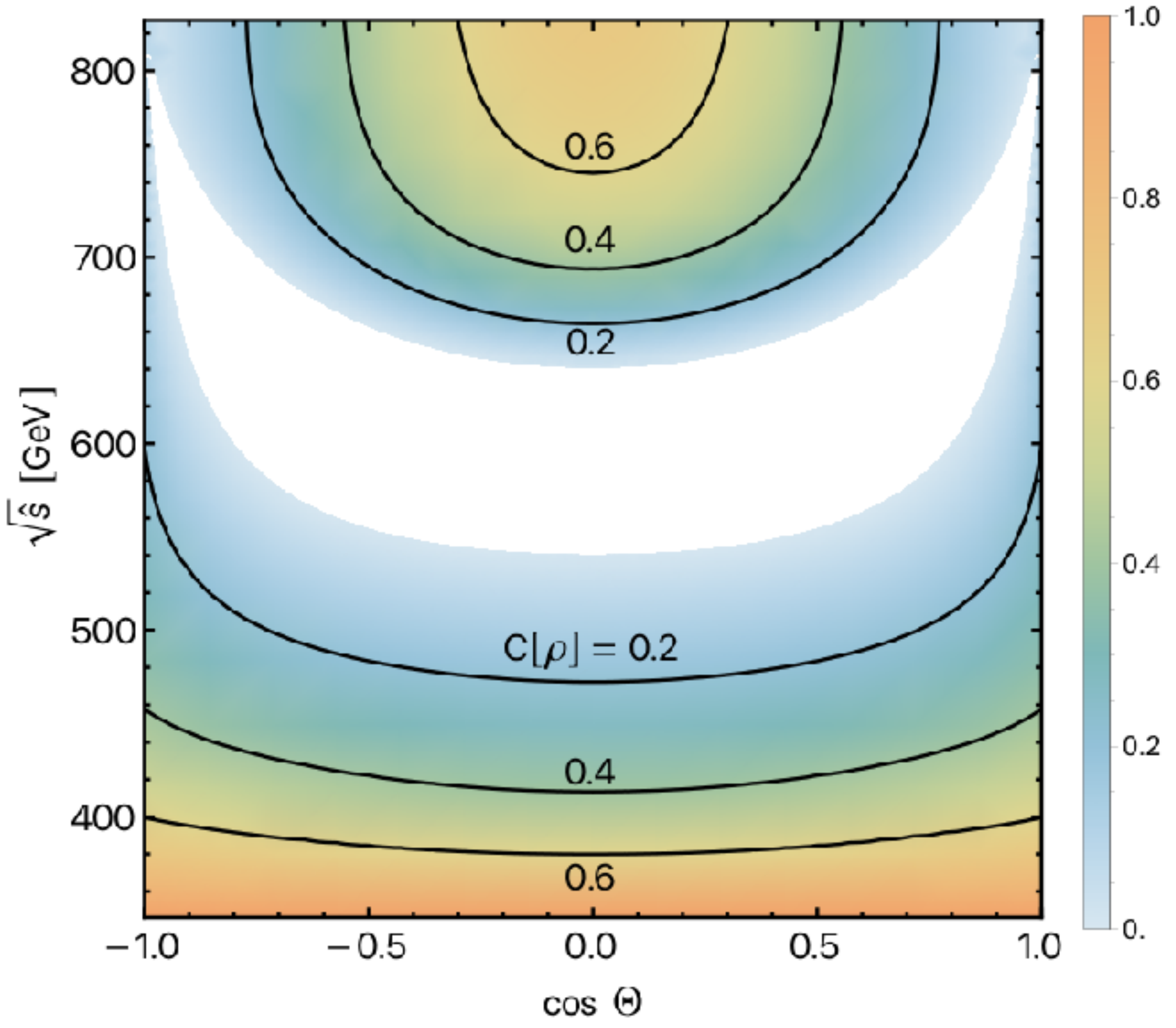
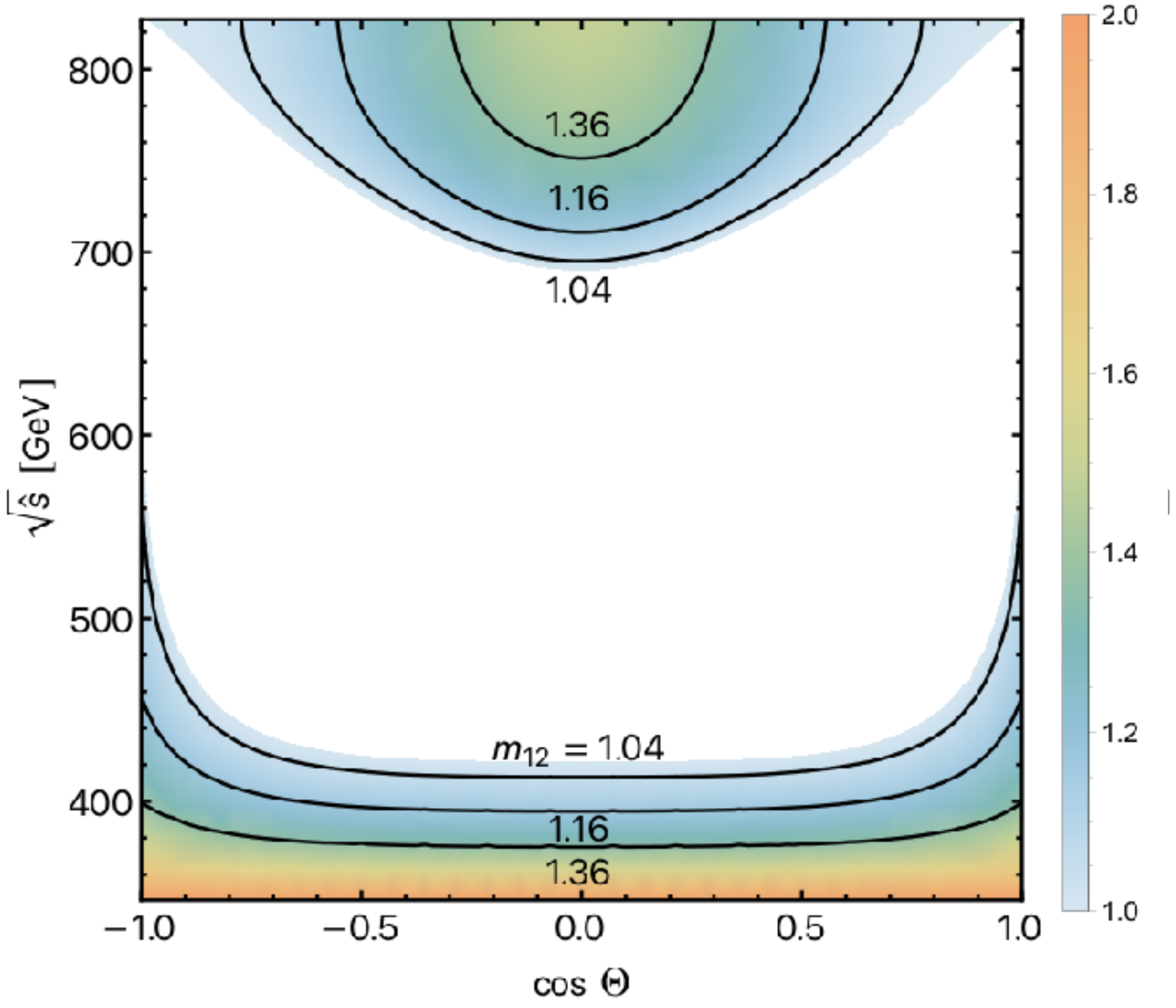
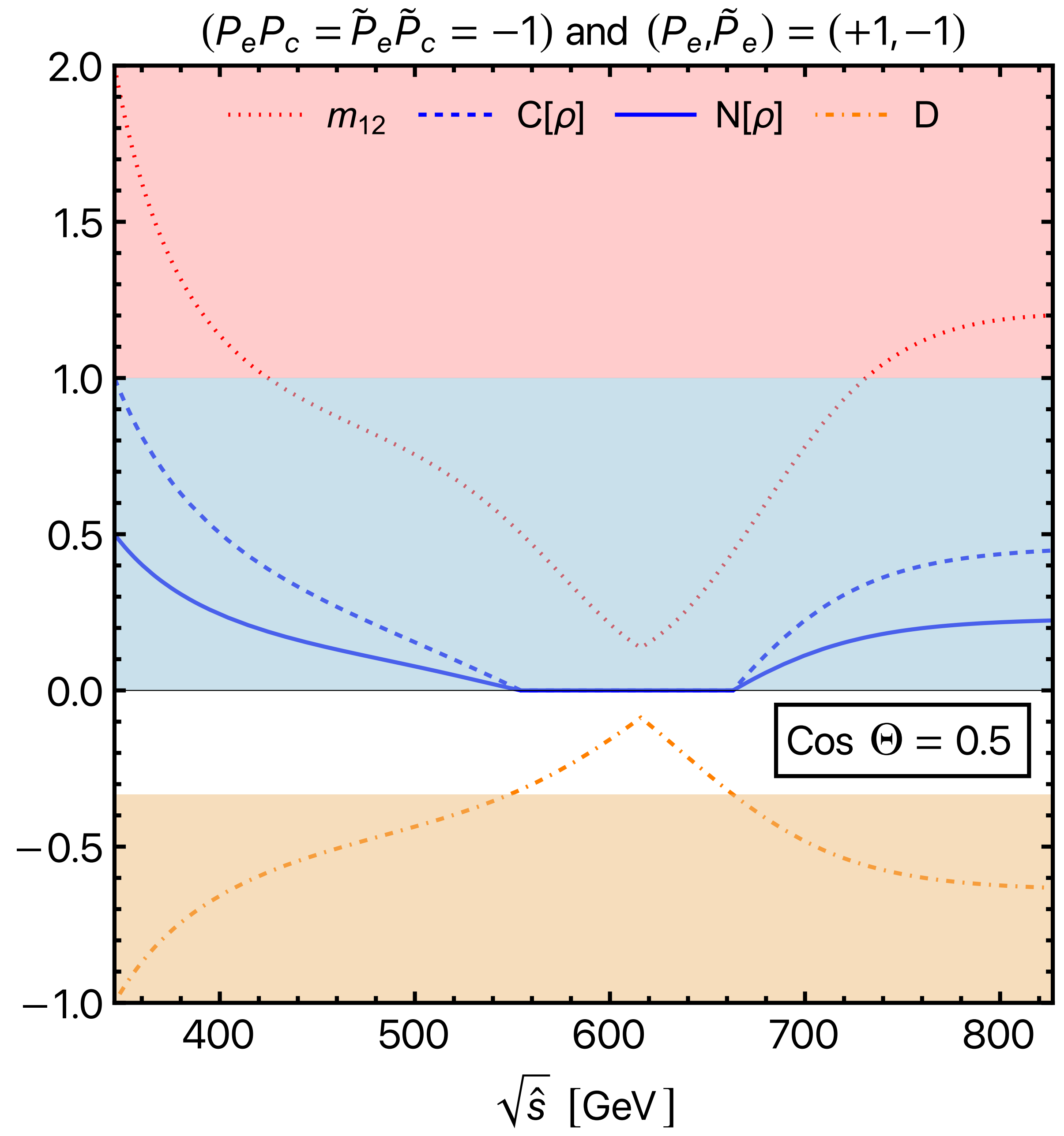
Top quark pairs @ Photon Collider

- Polarized colliding photons, $\sqrt{\hat{s}} = 1$ TeV









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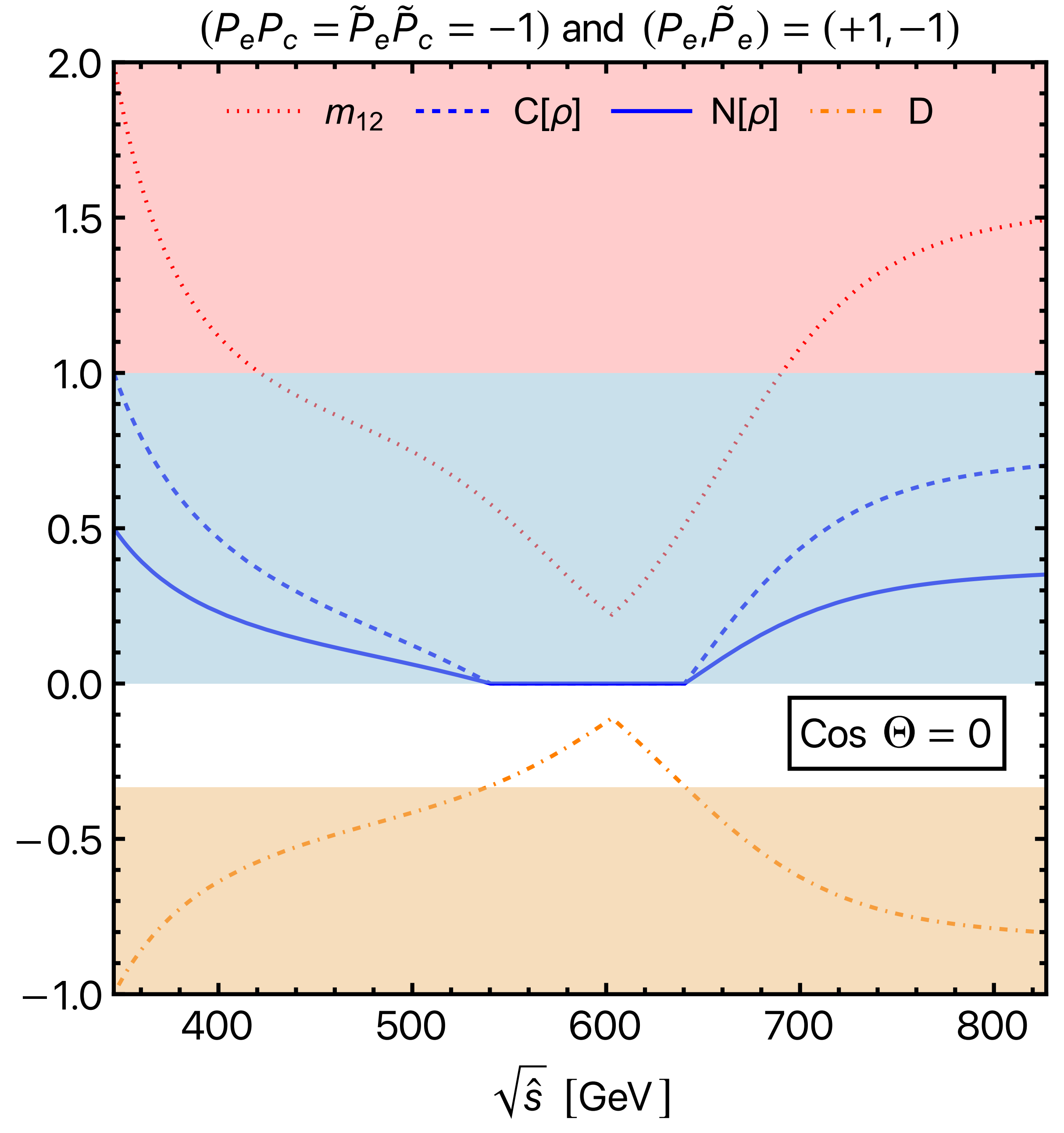
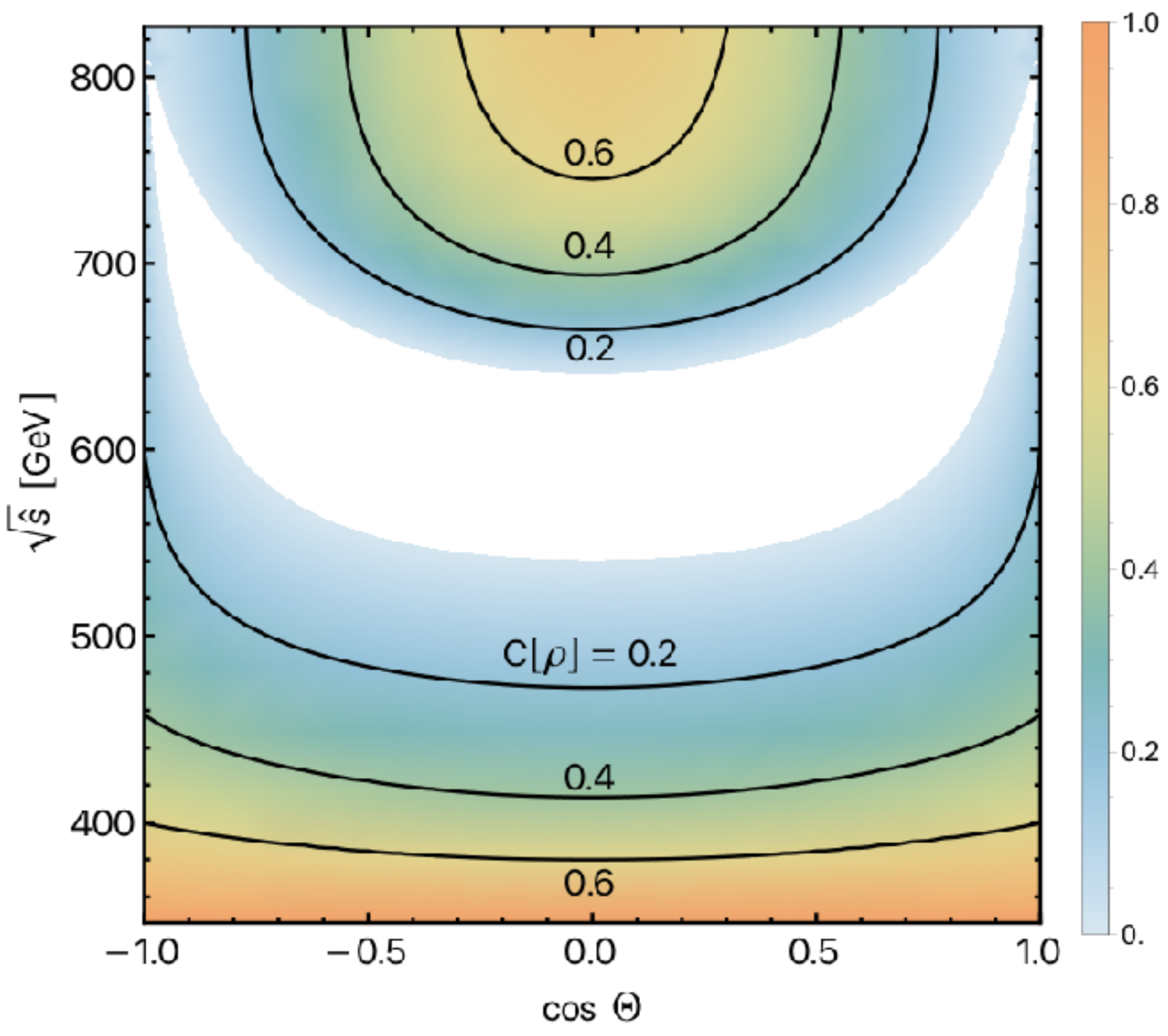
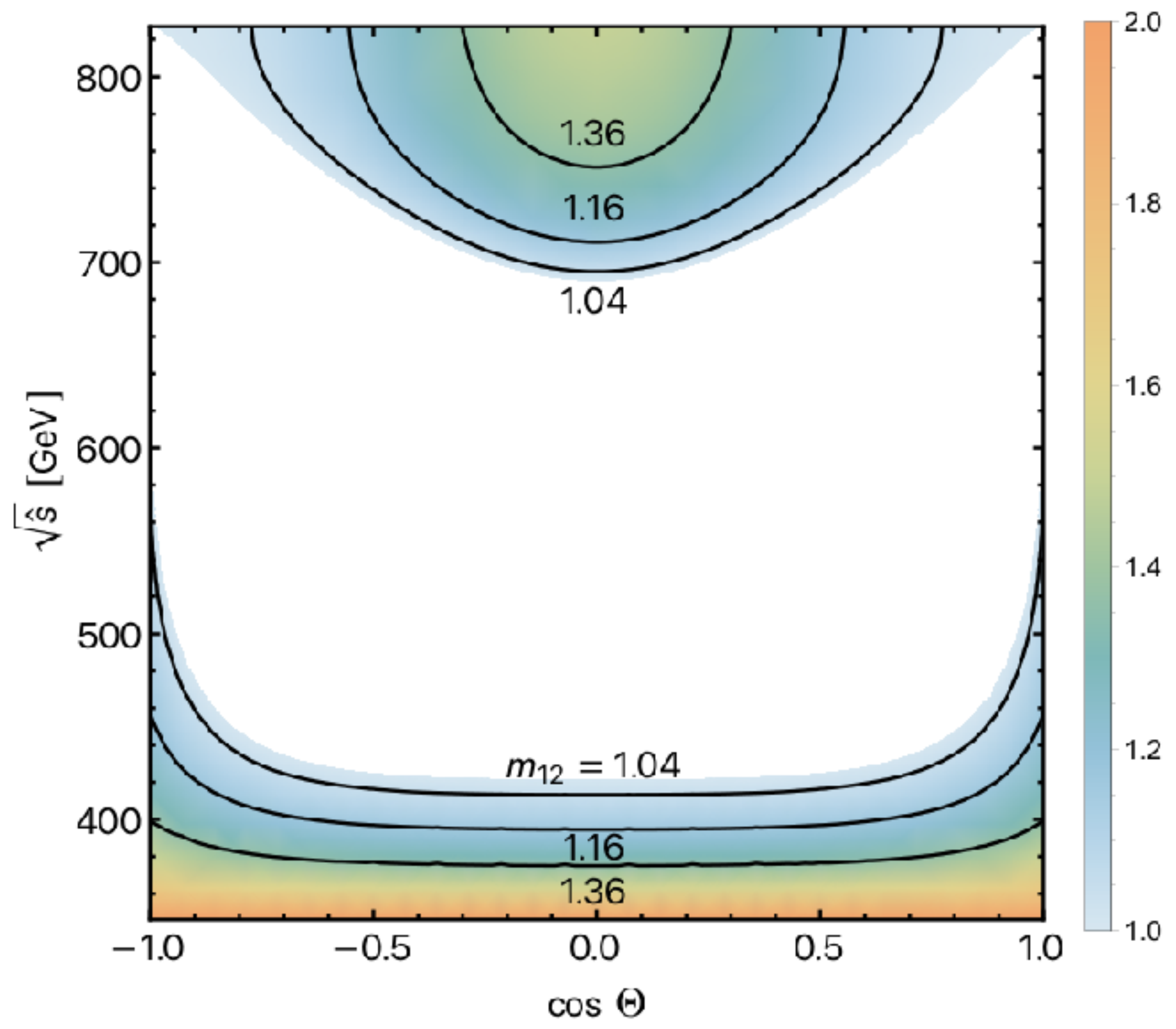
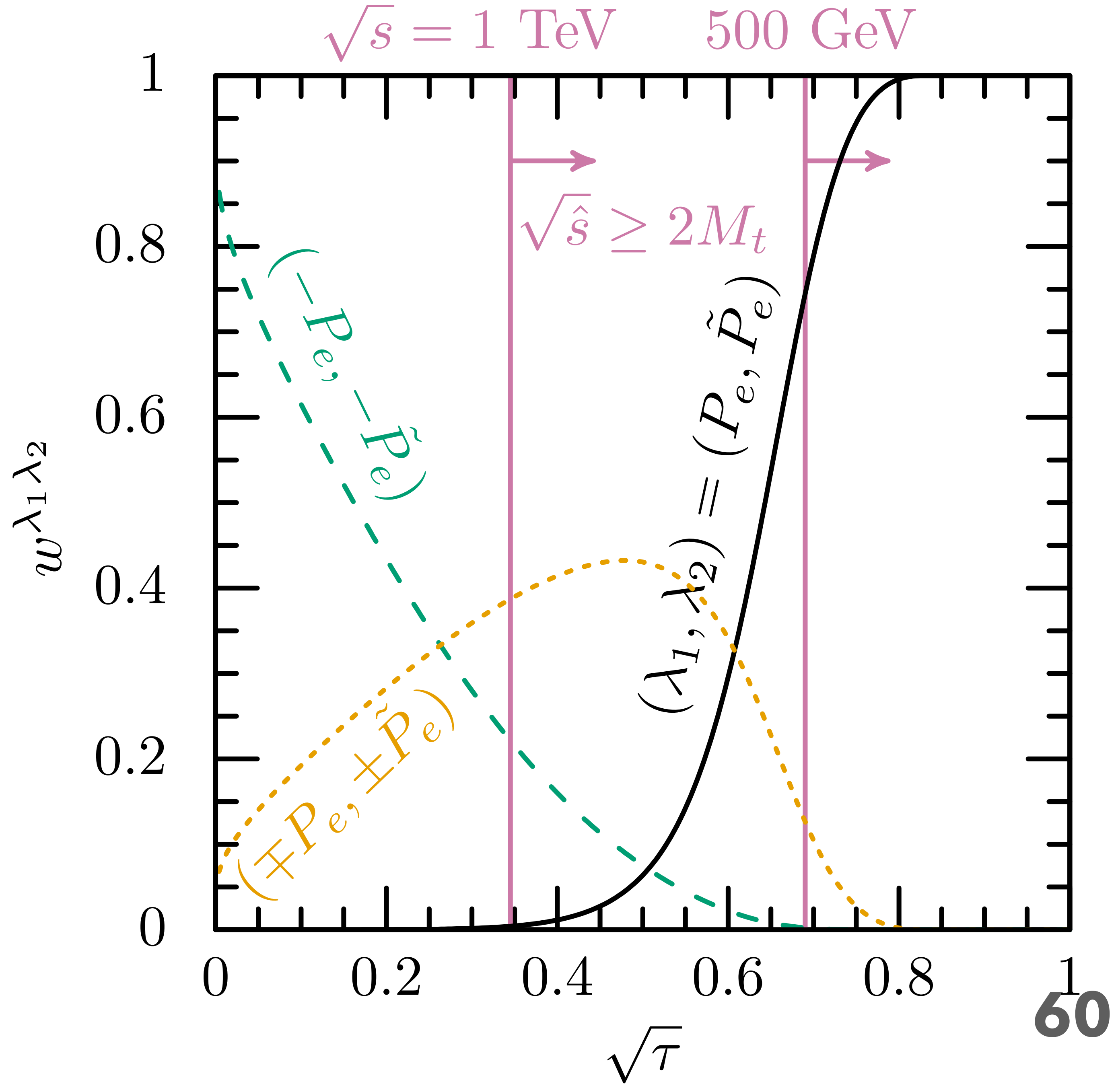
Top quark pairs @ Photon Collider

- Polarized colliding photons, $\sqrt{\hat{s}} = 1$ TeV









Top quark pairs @ Photon Collider

Conclusion

- The **quantum observables**, constructed by Polarization and Spin correlations have become a powerful tools for BSM searches and precision measurements.
- We present a **general formalism** to construct the spin density matrix of a two qubit system at colliders from the **amplitude level**, taking account of **P**, **CP**, and **CPT** properties.

Polarization coeffcient	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$	$\widehat{C}_6$	$\widehat{C}_7$	$\widehat{C}_8$
[P, CP, CP$\widetilde{\mathrm{T}}$] Parities	$[+++]$	$[---]$	$[+++]$	$[-++]$	$[-++]$	$[++-]$	$[---]$	$[+-+]$
Physical quantity	$\frac{\widehat{A}-\widehat{C}_{kk}}{2}$	$\frac{\widehat{B}_k^+-\widehat{B}_k^-}{2}$	$\frac{\widehat{A}+\widehat{C}_{kk}}{2}$	$\frac{\widehat{B}_k^++\widehat{B}_k^-}{2}$	$-\frac{\widehat{B}_r^++\widehat{B}_r^-}{2}$	$\frac{\widehat{B}_n^++\widehat{B}_n^-}{2}$	$-\frac{\widehat{B}_r^+-\widehat{B}_r^-}{2}$	$\frac{\widehat{B}_n^+-\widehat{B}_n^-}{2}$
Polarization coeffcient	$\widehat{C}_9$	$\widehat{C}_{10}$	$\widehat{C}_{11}$	$\widehat{C}_{12}$	$\widehat{C}_{13}$	$\widehat{C}_{14}$	$\widehat{C}_{15}$	$\widehat{C}_{16}$
[P, CP, CP$\widetilde{\mathrm{T}}$] Parities	$[+--]$	$[--+]$	$[+++]$	$[-+-]$	$[+++]$	$[--+]$	$[+++]$	$[-+-]$
Physical quantity	$\frac{\widehat{C}_{rk}-\widehat{C}_{kr}}{2}$	$\frac{\widehat{C}_{kn}-\widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{rk}+\widehat{C}_{kr}}{2}$	$-\frac{\widehat{C}_{kn}+\widehat{C}_{nk}}{2}$	$\frac{\widehat{C}_{nn}+\widehat{C}_{rr}}{2}$	$\frac{\widehat{C}_{rn}-\widehat{C}_{nr}}{2}$	$\frac{\widehat{C}_{rr}-\widehat{C}_{nn}}{2}$	$-\frac{\widehat{C}_{rn}+\widehat{C}_{nr}}{2}$

Conclusion

- The **quantum observables**, constructed by Polarization and Spin correlations have become a powerful tools for BSM searches and precision measurements.
- We present a **general formalism** to construct the spin density matrix of a two qubit system at colliders from the **amplitude level**, taking account of **P**, **CP**, and **CPT** properties.
- We show that a Photon collider with polarization control, we could also check **Bell's inequality violation** at high energy collider!
- $\tau^+\tau^-$, ZZ, W^+W^-, ... also suitable subject to study further.