Digital Quantum Simulation for the Spectroscopy of Schwinger Model

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Based on an on-going collaboration (arXiv:2404.14788 + 2607.xxxxx)
with Masazumi Honda (RIKEN iTHEMS & Saitama U.)

Digital Quantum Simulation for the Spectroscopy of Schwinger Model

[1] Basic : Quantum Advantage and Digital Quantum Simulation

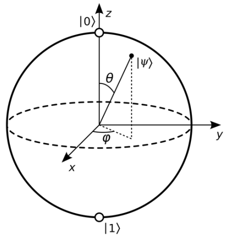
[2] Physics : Schwinger Model, or (1+1)D QED with Topological Term

[3] Q-Algo : Coherent Imaging Spectroscopy

[4] Discussion : Advantage / Drawback, Complexity Analysis

Qubits

Quantum annealers are collection of interacting qubits, in which interaction strength and coupling are tunable.



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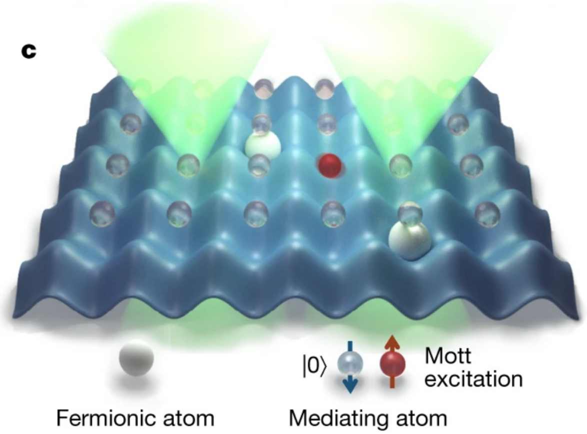
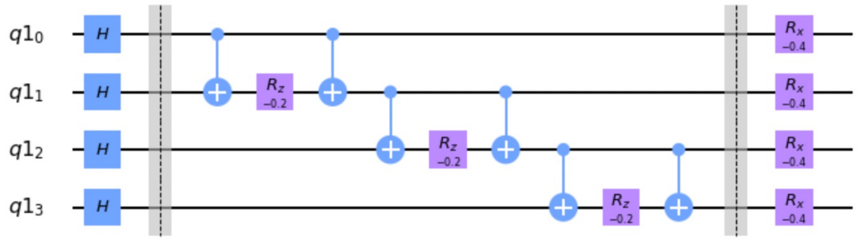


Quantum gates are logical circuit elements acting on (several) qubit(s).

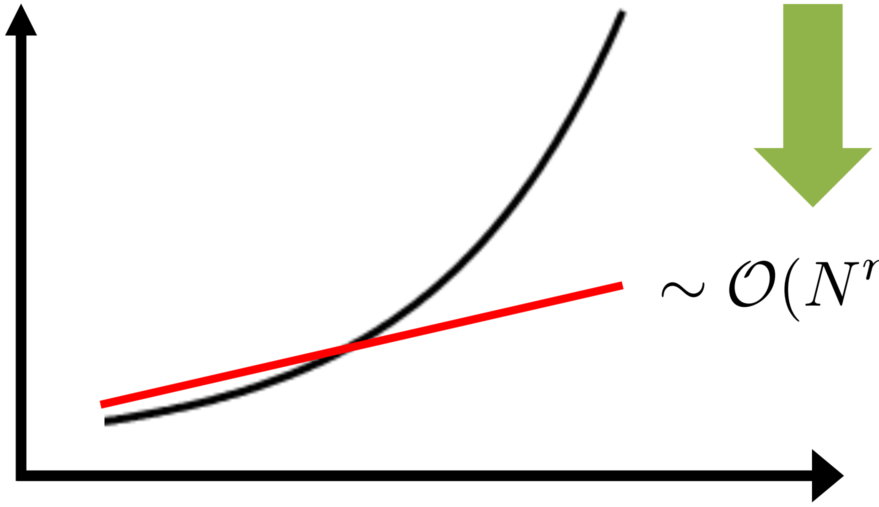


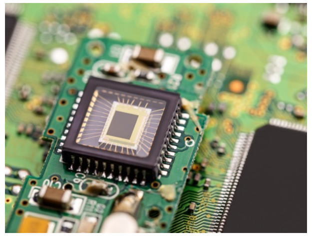
Operator	Gate(s)	Matrix
Pauli-X (X)	X, $\oplus$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)	Y	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)	H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	S	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	T	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)	Z	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

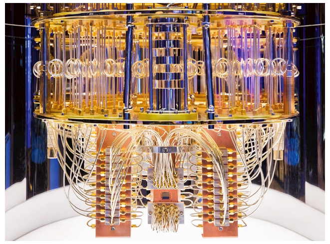
NB Gate-based circuit is versatile. However, in the NISQ (Noisy Intermediate-Scale Quantum) compute era, annealer is also useful.

Quantum Advantage

Quantum Advantage refers to a computational advantage one can take when solving a problem by using quantum algorithm and quantum processor over their classical counterparts.

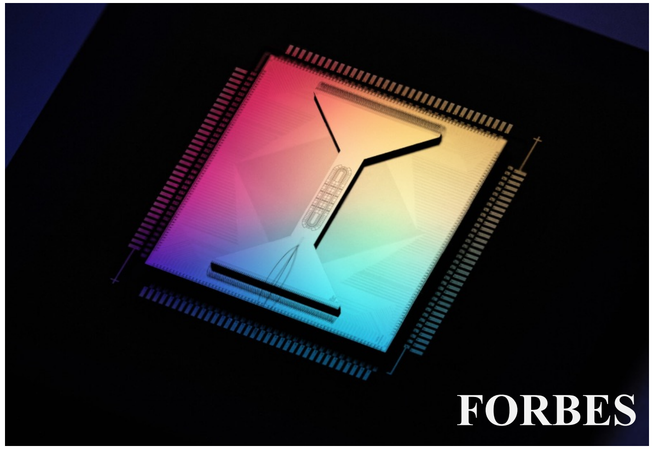




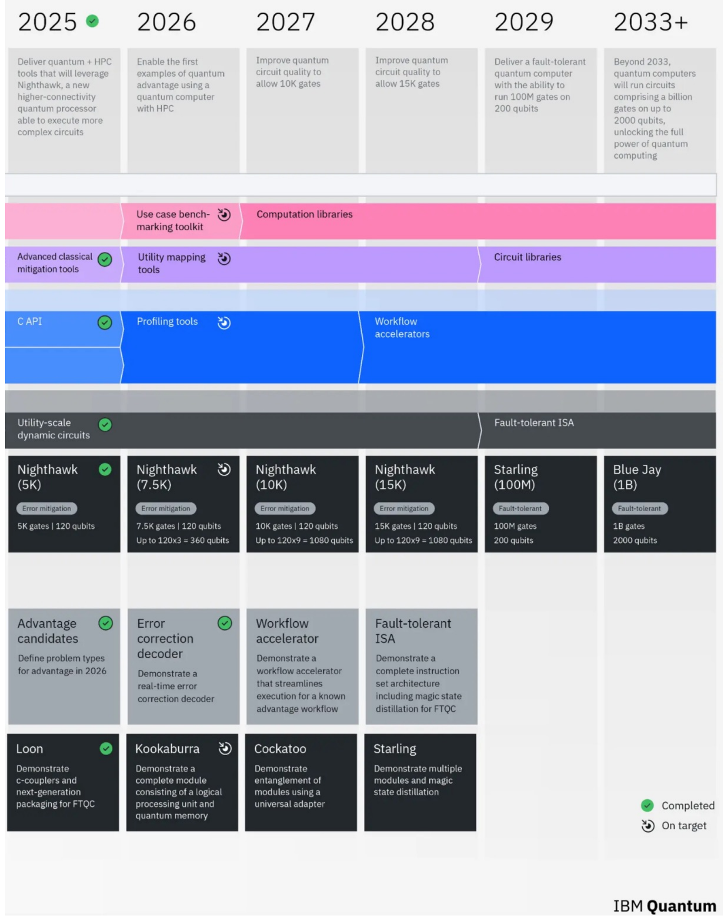


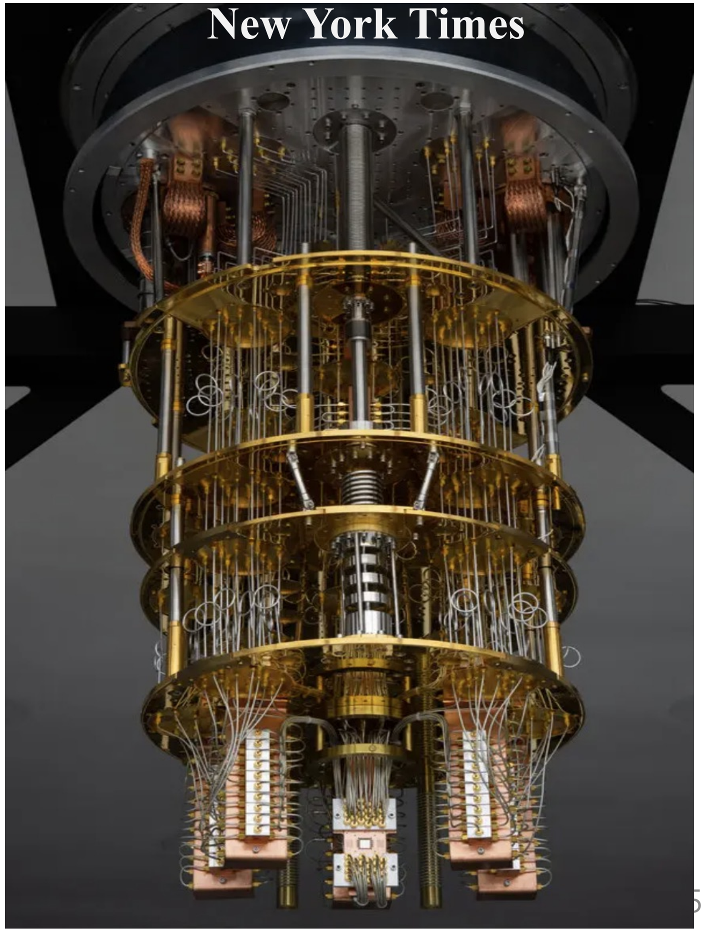
Quantum Advantage

Quantum Simulation is inspired by that the quantum circuit provides a natural embedding of **Hamiltonian time-evolution** on a Quantum Processor.



	2020	2023	2025	2027	2029	2030+
SYSTEMS	QUANTINUUM H1	QUANTINUUM H2	QUANTINUUM HELIOS	QUANTINUUM SOL	QUANTINUUM APOLLO	QUANTINUUM LUMOS
PHYSICAL QUBITS	20	56	98	192	1,000s	1,000,000+ QUBITS
PHYSICAL 2-QUBIT GATE ERROR	1×10^{-3}	1×10^{-3}	$< 5 \times 10^{-4}$	$< 2 \times 10^{-4}$	1×10^{-4}	
LOGICAL QUBITS		> 12	50	~ 100	100s	
LOGICAL ERROR RATES		1×10^{-3}	$< 10^{-4}$	$\sim 10^{-5}$	1×10^{-5} to 1×10^{-10}*	
NVIDIA GPU INTEGRATION			QEC Decoding ~1 PFLOP ~100 us latency	QEC Decoding ~10 PFLOP ~10 us latency	Control + QEC Decoding ~100 PFLOP ~0.5 us latency	





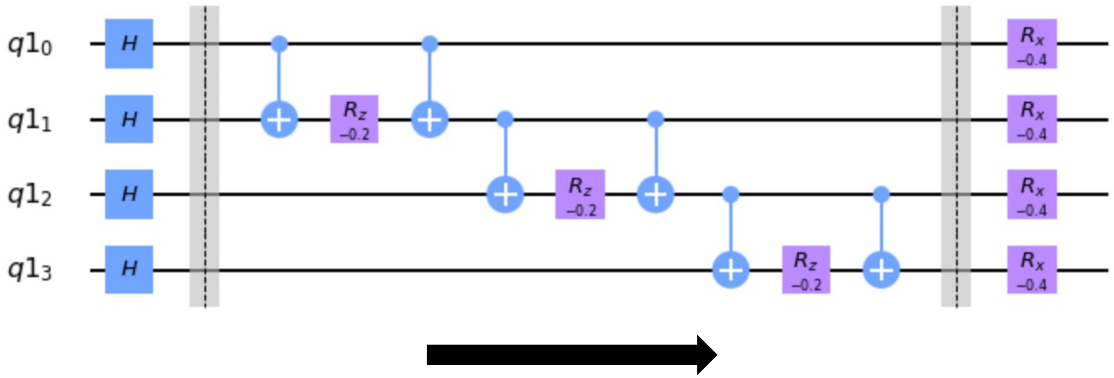
Digital Quantum Simulation

** gauge-invariant excited energy eigenstate*

Application to spectroscopy : measuring the excited state* spectra through dynamical process

Q1. which process depending on the energy gap?

degree of freedom



unitary time evolution ~ action of unitary gates

Q2. how to encode quantum fields into spin qubits?

Q3. how to encode interactions into discrete sets of gates?

Fermi Golden Rule

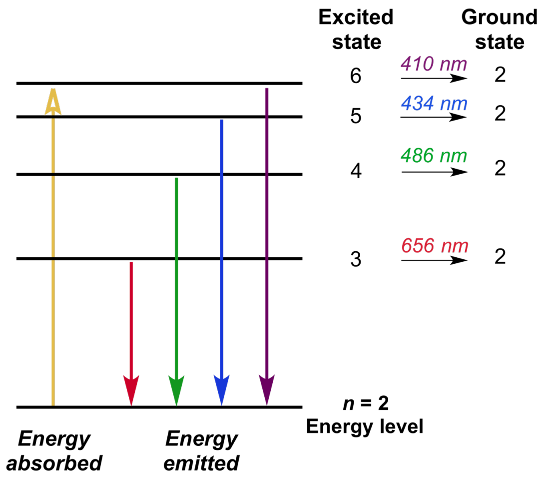
Transition rate from an initial state to a final state under the time-dependent (sinusoidal) perturbation $\Delta H = \Delta V\ \sin(\omega t)$

$$\dot{P}_{i\to f} = \frac{2\pi}{\hbar^2}\left|\langle f\,|\Delta V|\,i\rangle\right|^2 \delta(E_f - E_i - \omega)$$

selection rule

e.g. parity, momentum (approximate)

frequency matching with spectral gap



[Dirac 1927], [Fermi 1950]
e.g. [Gottfried, Yan 03]

Quantum Simulation for Spectroscopy

** gauge-invariant excited energy eigenstate*

Application to spectroscopy : meausuring the excited state* spectra (gap)

[Senko, Smith, Richerme, Lee, Campbell, Monroe 14]

excited state vacuum

$$\dot{P}_{i\to f} = \frac{2\pi}{\hbar^2} |\langle f\,|\Delta V|\, i\rangle|^2\, \delta(E_f - E_i - \omega)$$

selection rule frequency matching

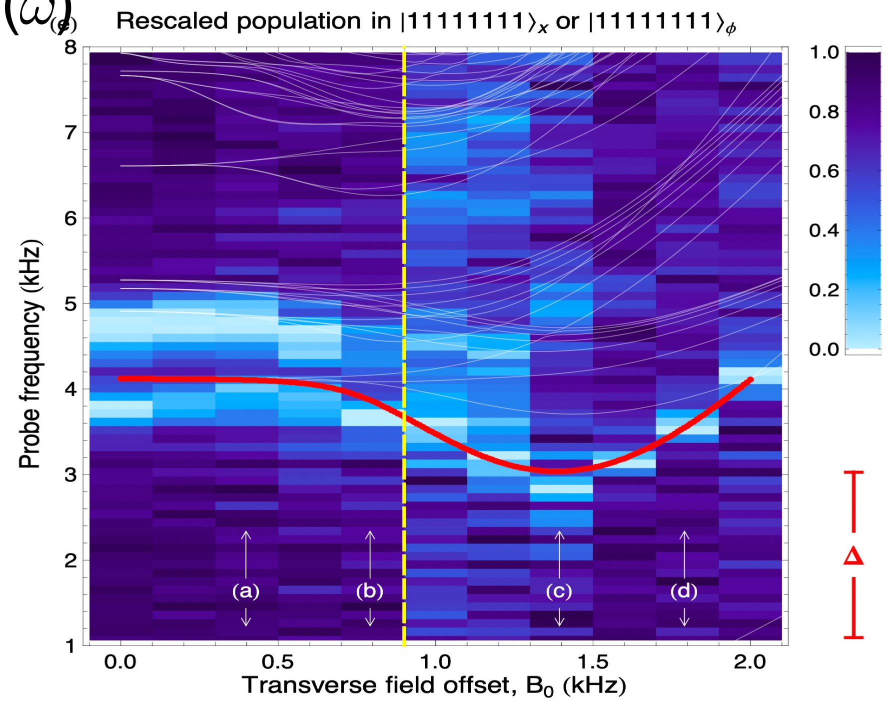
approximated ground state $|\widetilde{\text{vac}}\rangle = |11\cdots 1\rangle$

NB **Practically, they measured**

$$P_{\text{vac-vac}}(\omega) = |\langle \underset{\langle\widetilde{\text{vac}}|}{\cancel{\text{vac}}}|\,\mathcal{T} e^{-i\int dt H + \Delta V \sin(\omega t)}\,|\underset{|\widetilde{\text{vac}}\rangle}{\cancel{\text{vac}}}\rangle|^2$$

- $\omega \simeq E_f - E_i$: transition occurs.
- $\omega \neq E_f - E_i$: vacuum remains.

energy gap (ω)



$B(t) = B_0 + B_p \sin(\omega t)$

a parameter in Hamiltonian

$$H = \sum_{i<j} J_{ij} X_i X_j + B(t) \sum_i Y_i$$

Quantum Simulation for Spectroscopy

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excited state vacuum

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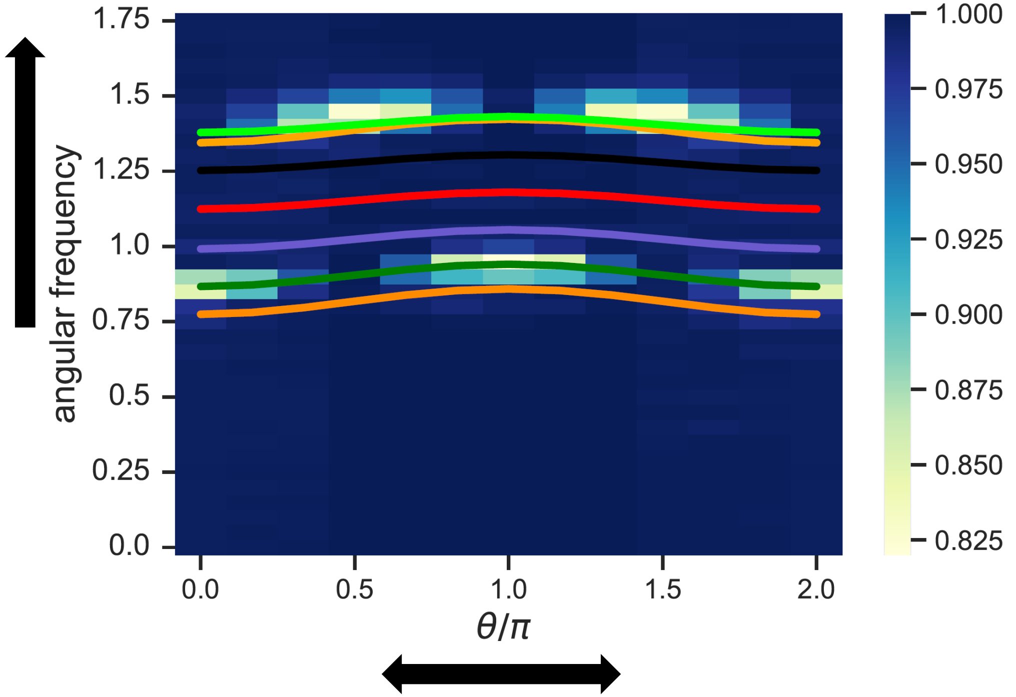

a parameter in Hamiltonian

NB **For general model, we measure**

$$P_{\text{vac-vac}}(\omega) = |\langle \text{vac}|\mathcal{T}e^{-i\int dt H + \Delta V \sin(\omega t)}|\text{vac}\rangle|^2$$

- $\omega \simeq E_f - E_i$: transition occurs.
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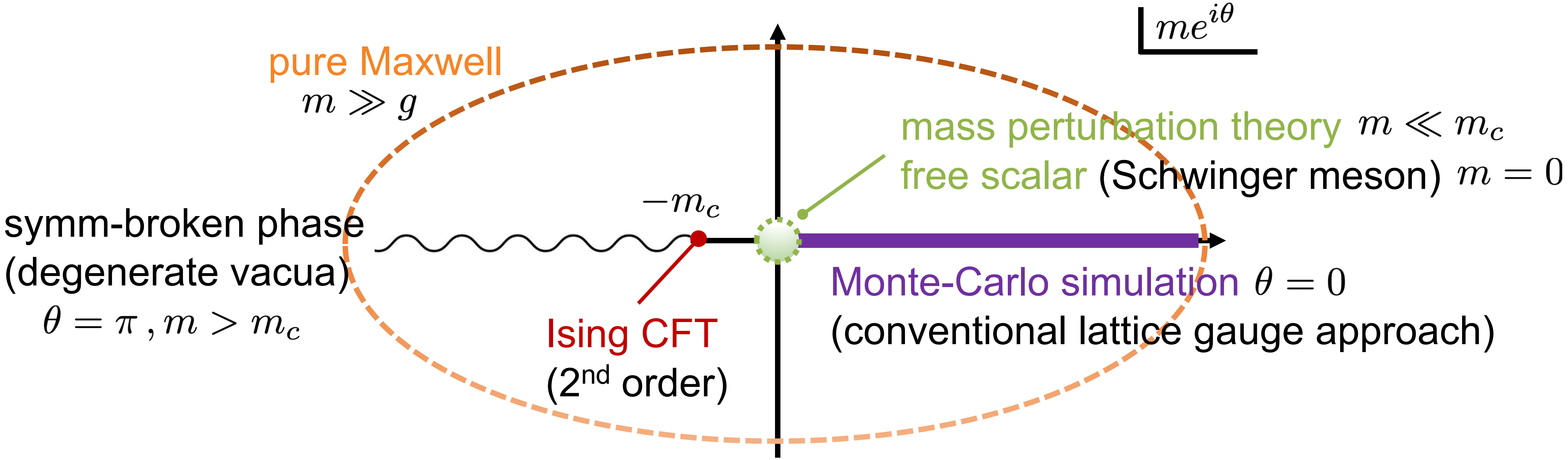
[4] **Discussion** : Advantage / Drawback, Complexity Analysis

Schwinger Model, i.e. 2d QED

[1] Lagrangian density [Schwinger 1951, 1962]

$$\mathcal{L} = \frac{1}{2g^2}F_{01}^2 + \frac{\theta}{2\pi}F_{01} + \overline{\psi}i\gamma^\mu(\partial_\mu + iA_\mu)\psi - m\overline{\psi}\psi$$

[2] Phase diagram and accessibility by (exact / numerical) techniques



NB duality with massive sine-Gordon model (bosonization) [Coleman 1976]

Qubit Description of Schwinger Model

Qubit-mapped Hamiltonian (after solving Gauss constraint with OBC)

$$H_{\text{chiral}} = \frac{J}{2}\sum_{n=1}^{N-2}\sum_{k<n}(N-n-1)Z_k Z_n + \frac{1}{2}\sum_{n=0}^{N-2}\left\{w-(-1)^n\frac{m_{\text{lat}}}{2}\sin\theta\right\}(X_n X_{n+1}+Y_n Y_{n+1})$$

$$+\frac{m_{\text{lat}}\cos\theta}{2}\sum_{n=0}^{N-1}(-1)^n Z_n + \frac{J}{2}\sum_{n=0}^{N-2}(n+1 \bmod 2)\sum_{l=0}^{n} Z_l$$

e.g. [Chakraborty, Honda, Izubuchi, Kikuchi, Tomiya 20]

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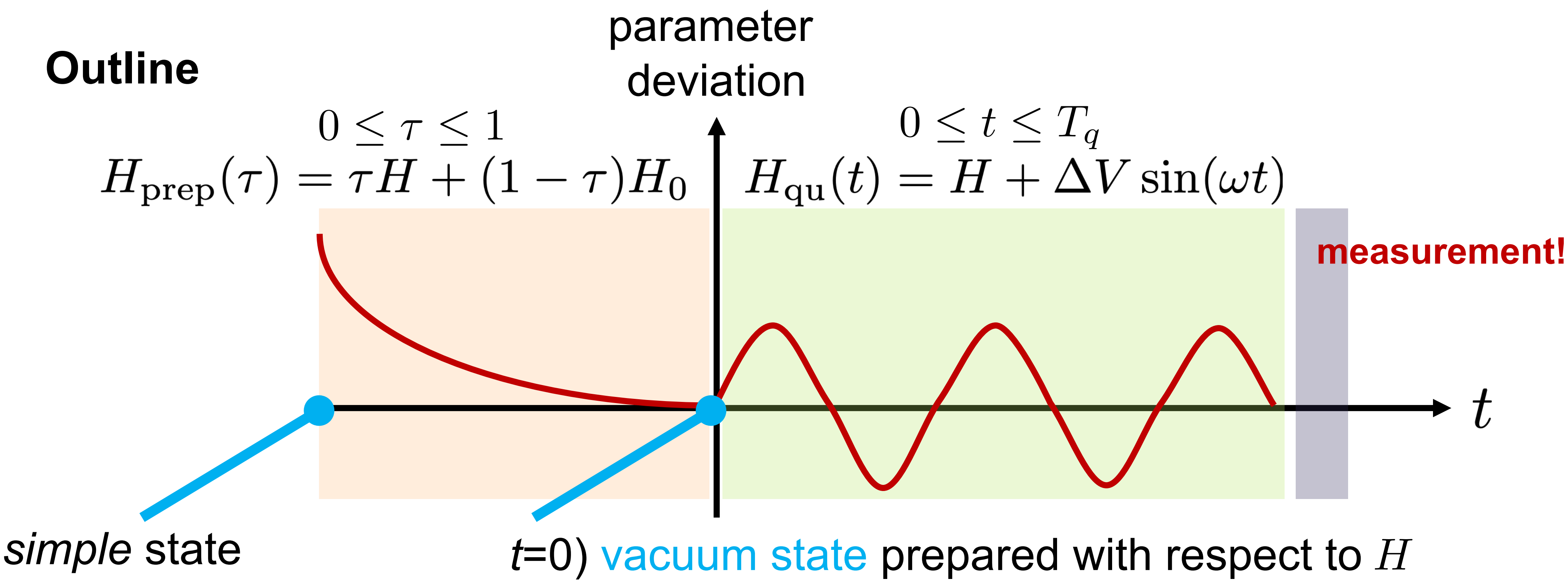
Coherent Imaging Spectroscopy

Outline



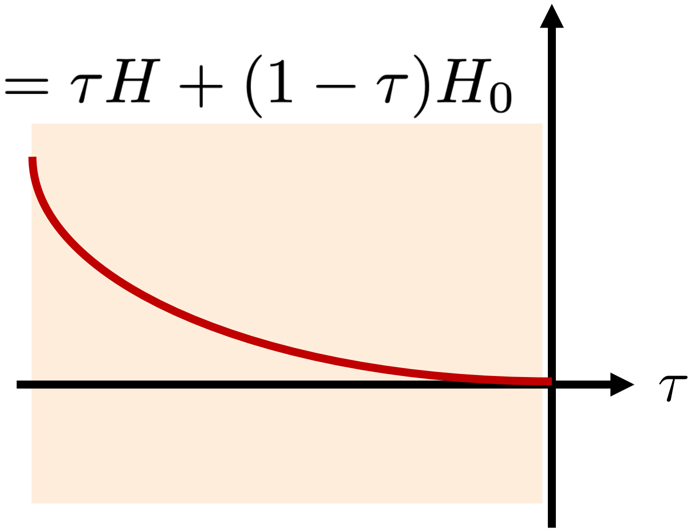
1. Ground state preparation by adiabatic procedure
2. Time-evolution with quenched Hamiltonian $H_{\text{qu}} = H + \Delta H(t;x)$
3. Measurement of the vacuum persistency probability (or vac-to-vac probability)

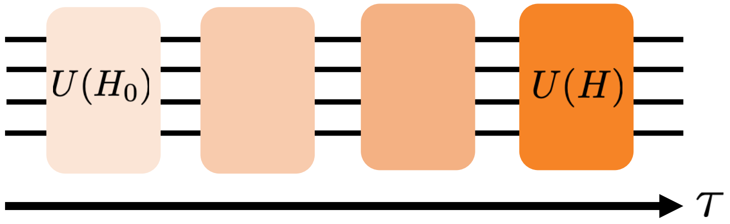
$$P_{\text{vac-vac}}(\omega) = |\langle \text{vac}| \mathcal{T} e^{-i\int dt H + \Delta V \sin(\omega t)} |\text{vac}\rangle|^2$$

Adiabatic State Preparation

One of the powerful heuristics in finding out the **ground state** of Hamiltonian

$$H_{\text{prep}}(\tau) = \tau H + (1-\tau) H_0$$





[1] Prepare the vacuum of **well-known Hamiltonian**, e.g. Ising spin chain

[2] Tune its parameter slowly so that they becomes those of Hamiltonian of interest

$$H_0 = \frac{M_0}{2} \sum_{n=0}^{N-1} (-1)^n Z_n$$

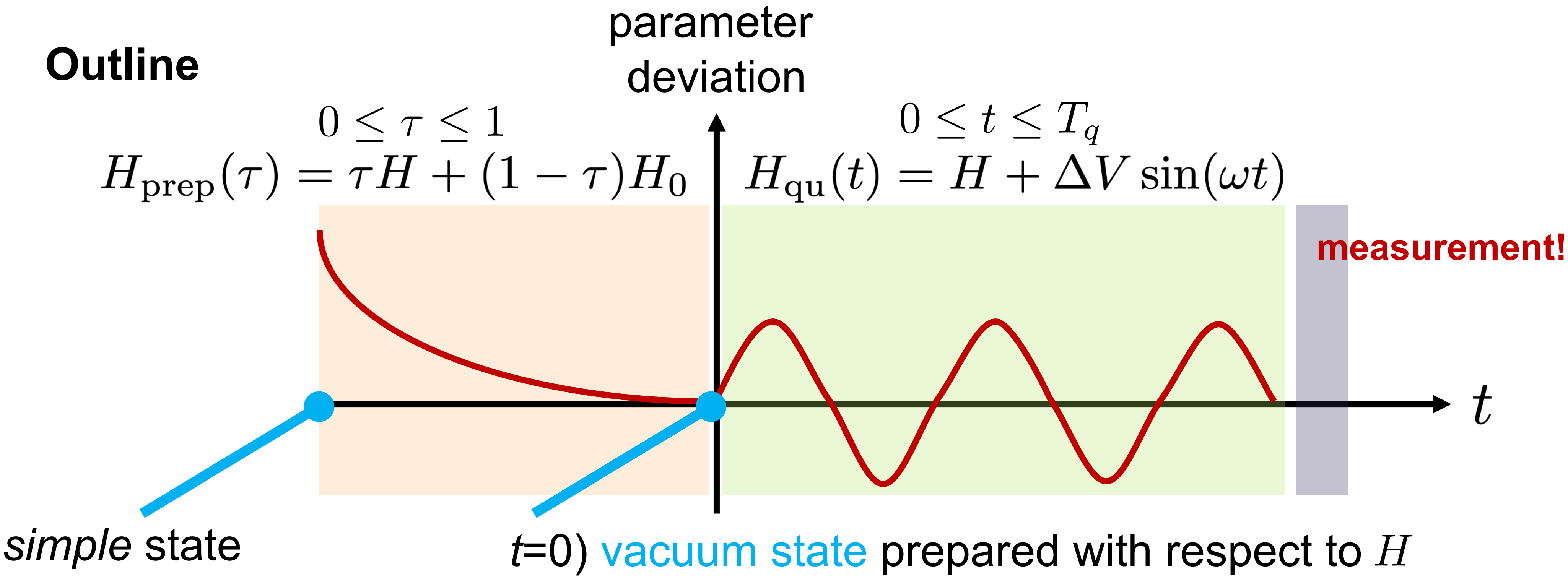
$$|\text{vac}_0\rangle = |1010 \cdots 101\rangle$$

more complicated Hamiltonian

$$H_{\text{chiral}} = \frac{J}{2} \sum_{n=1}^{N-2} \sum_{k<n} (N-n-1) Z_k Z_n + \frac{1}{2} \sum_{n=0}^{N-2} \left\{ w - (-1)^n \frac{m_{\text{lat}}}{2} \sin\theta \right\} (X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m_{\text{lat}} \cos\theta}{2} \sum_{n=0}^{N-1} (-1)^n Z_n + \frac{J}{2} \sum_{n=0}^{N-2} (n+1 \bmod 2) \sum_{l=0}^{n} Z_l$$

Coherent Imaging Spectroscopy

Outline



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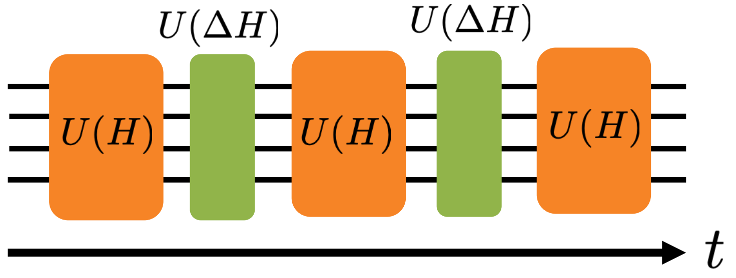
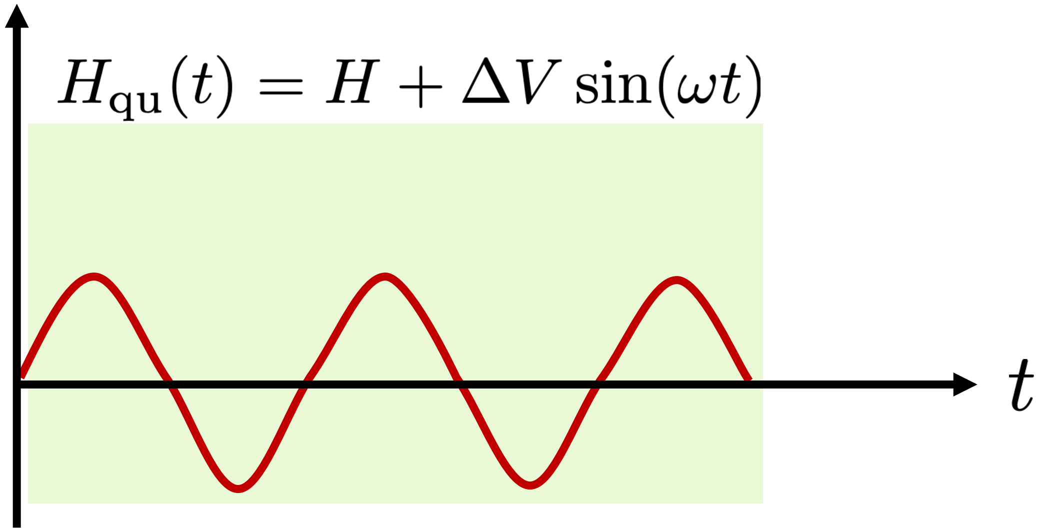
Suzuki-Trotter Decomposition

Discrete time evolution due to gate-based construction of quantum circuit

$$U(t;t_0) = e^{-i\int_{t_0}^{t} d\tau H_{\mathrm{qu}}(\tau)}$$





NB In a gate-based quantum circuit, we use a finite set of gates in generating (infinitely many) unitary gates. The followings are examples of 1-qubit gates;

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

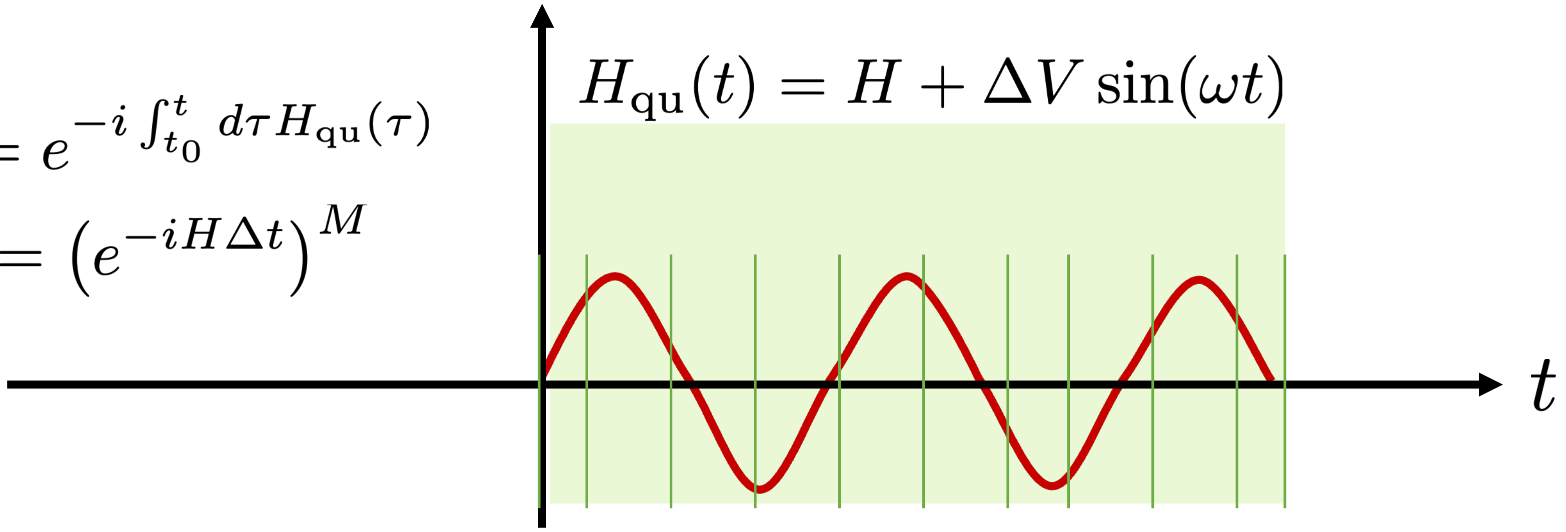
$$R_X(\theta) = e^{-i\theta\frac{X}{2}}, \quad R_Y(\theta) = e^{-i\theta\frac{Y}{2}}, \quad R_Z(\theta) = e^{-i\theta\frac{Z}{2}}$$

Suzuki-Trotter Decomposition

Discrete time evolution due to gate-based construction of quantum circuit

$$U(t;t_0) = e^{-i\int_{t_0}^{t} d\tau H_{\mathrm{qu}}(\tau)}$$

$$e^{-iHT} = \left(e^{-iH\Delta t}\right)^M$$



e.g. Ising spin chain $\quad H_{ZZ} = -J\sum_i Z_i Z_{i+1}\,, \quad H_X = -h\sum_i X_i$

$$e^{-i(H_{ZZ}+H_X)\Delta t} \simeq e^{-iH_{ZZ}\Delta t}e^{-iH_X\Delta t} + \mathcal{O}\left(\frac{1}{M}\right)$$

$$e^{-i\theta Z_i Z_j} = CX^{(i,j)}\, R_Z^{(j)}(2\theta)\; CX^{(i,j)}$$

simple R_X gate

External Quench with(out) spatial modulation

chirality matrix

eg. pseudo-scalar condensate operator $\Delta V = \int dx\, f(x)\overline{\psi}(x)\gamma_5\psi(x)$

charged Dirac fermion

Jordan-Wigner transformation

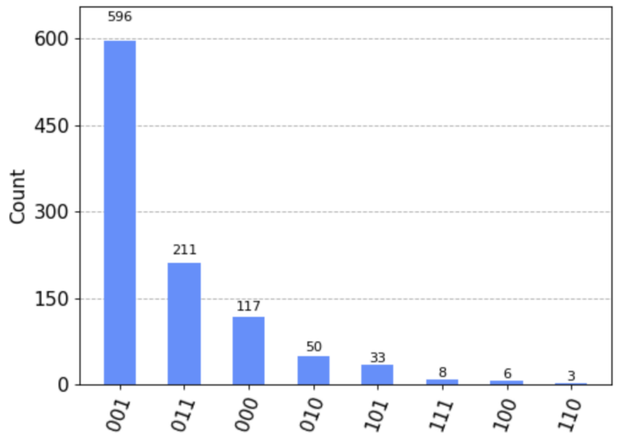
$$\mathcal{L}_{\text{eff}} = \frac{1}{2g^2}F_{01}^2 + \overline{\psi}i\gamma^\mu(\partial_\mu + iA_\mu)\psi - m\overline{\psi}e^{i\theta\gamma^5}\psi$$

$$\Delta H = \frac{B_p}{2}\sum_{n=0}^{N-2}(-1)^{n+1} f_n \sin(\omega t)\left(X_n X_{n+1} + Y_n Y_{n+1}\right)$$

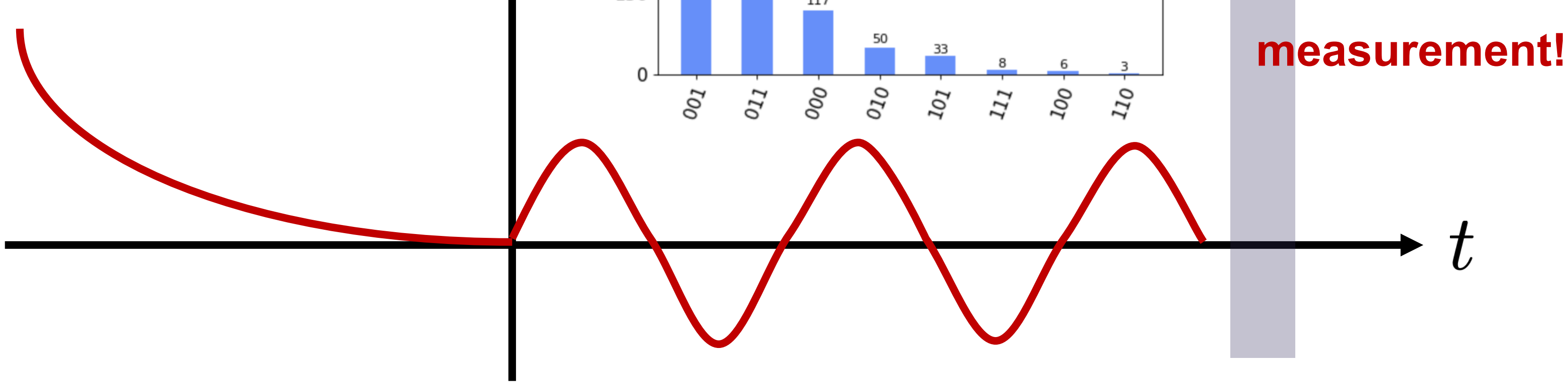
$$f_n = 1\,, \sin\left(\frac{\pi p n}{N-1}\right), \cos\left(\frac{\pi p n}{N-1}\right)$$

Probabilistic Outcome of Quantum Computation

Outcome : Probability Histogram!



[IBM Qiskit Documentation]



- We can measure the state population following the quench.
- Measurement basis is Z-eigenstate basis ≠ Eigenvectors of Hamiltonian

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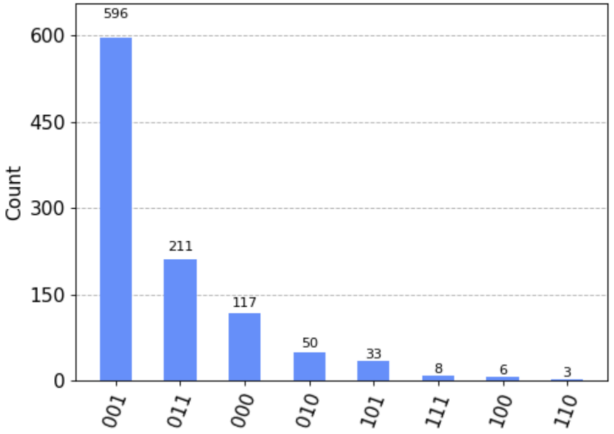

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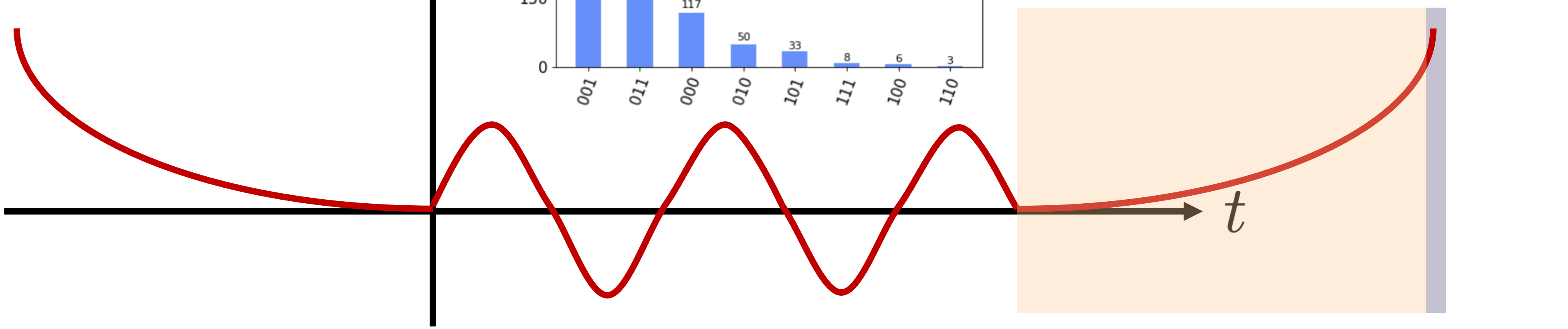

- We can measure the state population following the quench.
- Measurement basis is Z-eigenstate basis ≠ Eigenvectors of Hamiltonian
- Take the conjugate process of adiabatic vacuum preparation, then by reading |*101010..1*> population, vac-to-vac probability can be read off!

$$H_0 = \frac{M_0}{2}\sum_{n=0}^{N-1}(-1)^n Z_n$$

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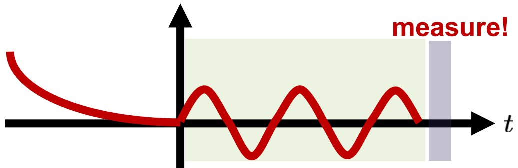
Preliminary

Pseudo-scalar condensate quench



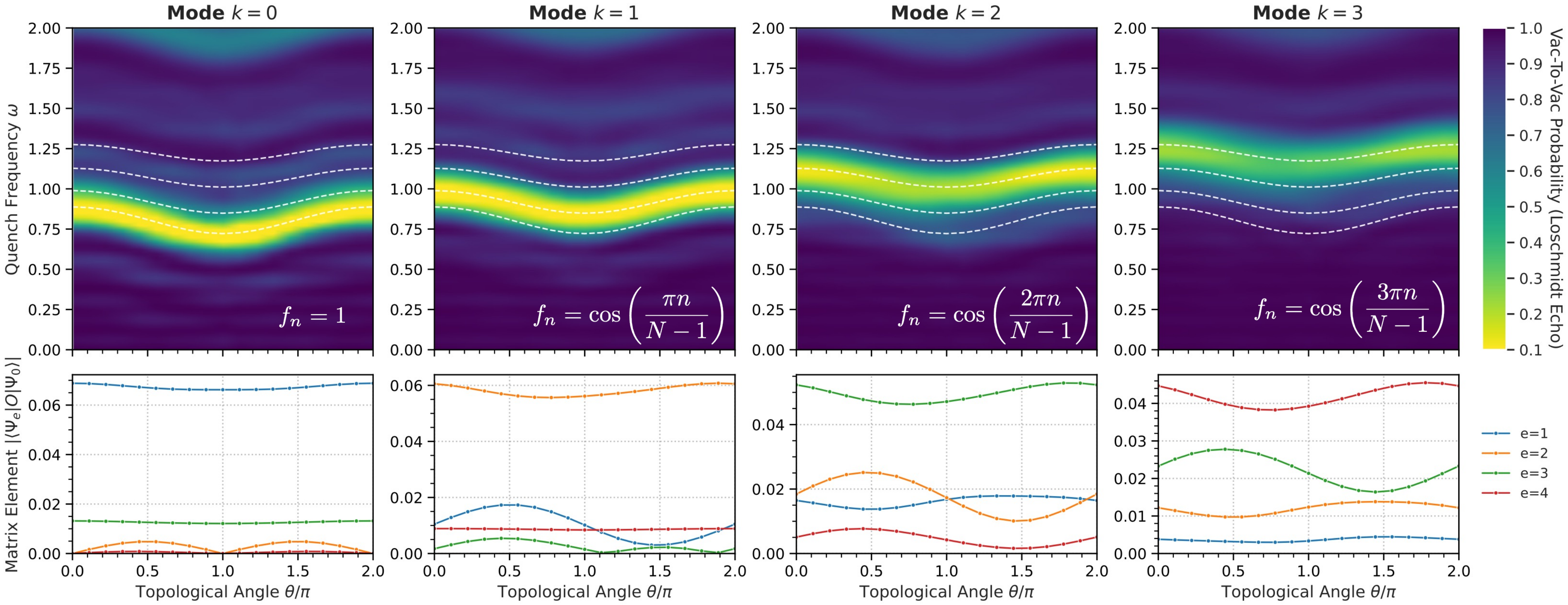
$$\Delta V = \int dx\, f(x)\overline{\psi}(x)\gamma_5\psi(x)$$

$$\dot{P}_{i\to f} = \frac{2\pi}{\hbar^2}\left|\langle f\,|\Delta V|\,i\rangle\right|^2 \delta(E_f - E_i - \omega)$$

$$\Delta H = \frac{B_p}{2}\sum_{n=0}^{N-2}(-1)^{n+1} f_n \sin(\omega t)\left(X_n X_{n+1} + Y_n Y_{n+1}\right)$$

Vacuum-to-vacuum probability with 4 low excited states' energy spectra

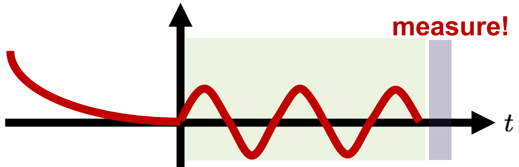
$$P_{\text{vac-vac}}(\omega) = \left|\langle \text{vac}|\mathcal{T}e^{-i\int dt H + \Delta V\,\sin(\omega t)}|\text{vac}\rangle\right|^2$$



$N = 11\,, L = 9\,, B_p = 0.035\,, T = 25\,, M = 70\,, m = 0.05$

`QISKIT statevector , QuSpin`

Pseudo-scalar condensate quench



$$\Delta V = \int dx\, f(x)\overline{\psi}(x)\gamma_5\psi(x)$$

$$\dot{P}_{i\to f} = \frac{2\pi}{\hbar^2}\left|\langle f\,|\Delta V|\,i\rangle\right|^2 \delta(E_f - E_i - \omega) \qquad \Delta H = \frac{B_p}{2}\sum_{n=0}^{N-2}(-1)^{n+1} f_n \sin(\omega t)\left(X_n X_{n+1} + Y_n Y_{n+1}\right)$$

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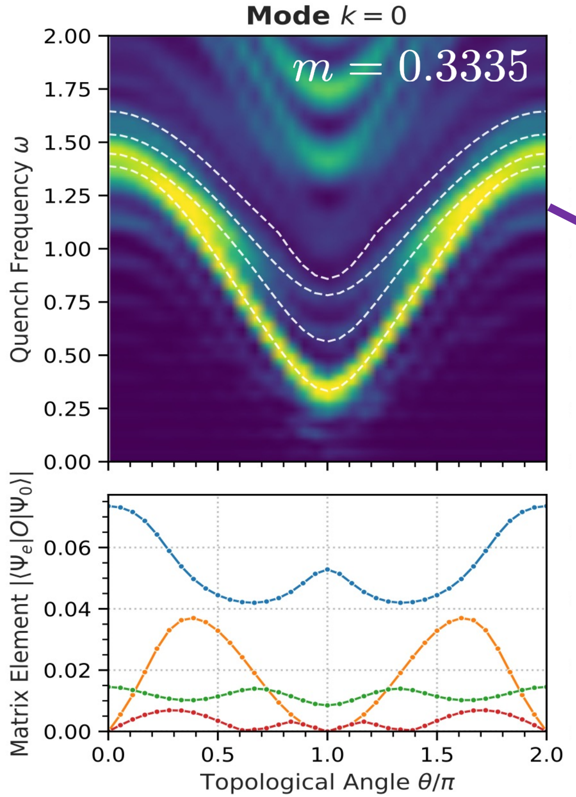

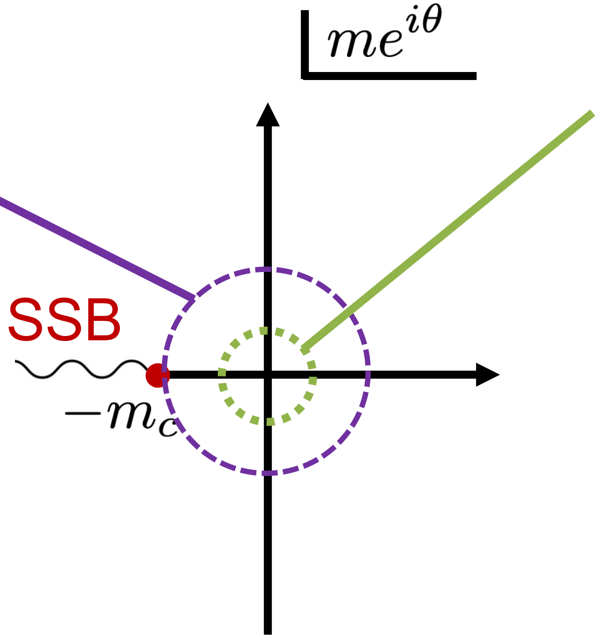

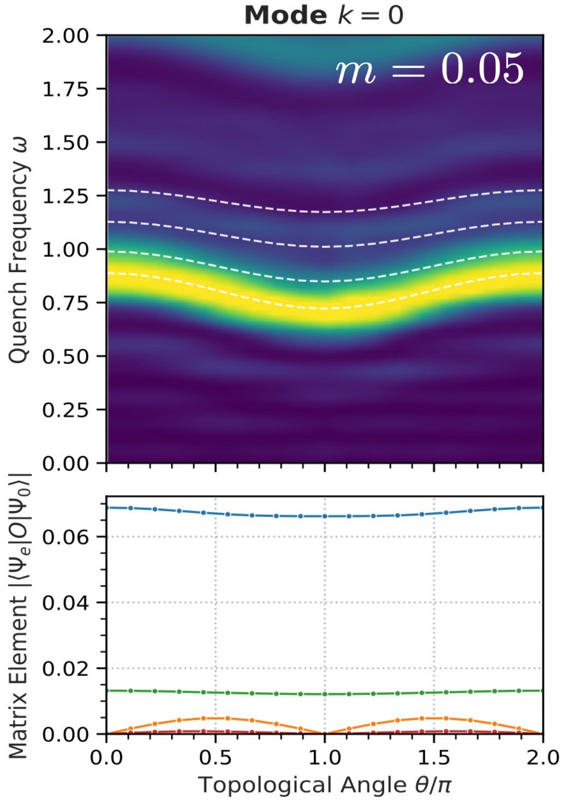

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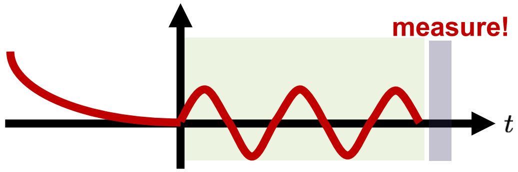


PROs and CONs

$$H_{\rm qu}(t) = H + \Delta V \sin(\omega t)$$

Advantage :

[1] can directly explore high-energy regime

[2] can filter quantum number by choosing operator and weights

Technically, we have other advantage like…

[1] result is robust under the infidelity during the initial state preparation

[2] (empirically) small shot number can result in a nice-looking plot

Complexity Analysis

Five time scales associated with the simulation

[1] Trotterization frequency $\omega_{ST} = \frac{2\pi}{\Delta t_{ST}}$

[2] quench frequency $\omega \sim \Delta E_{\rm gap}$

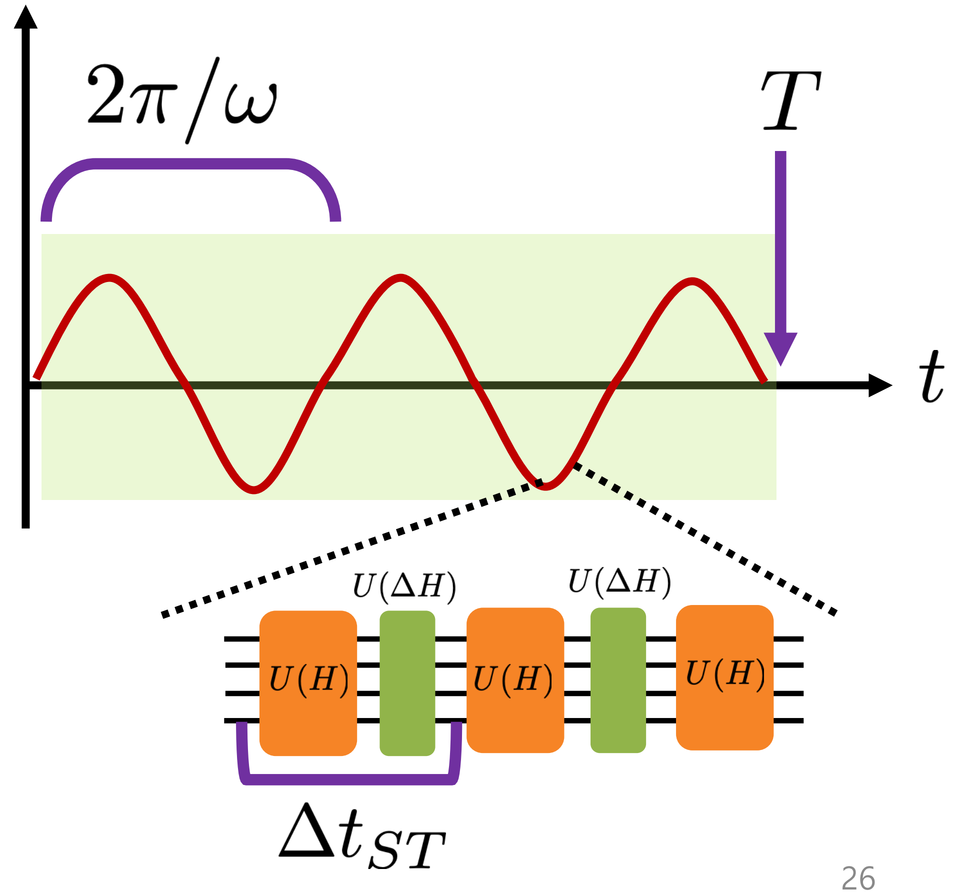
[3] simulation time scale $\Omega = \frac{2\pi}{T}$

[4] Resolution in quench frequency $\Delta\omega$

[5] characteristic scale from perturbation theory

$$\gamma^2 B_p^2 = |\langle \text{exc}\,|\Delta V|\,\text{vac}\rangle|^2$$

$$H_{\rm qu}(t) = H + \Delta V \sin(\omega t)$$



Complexity Analysis

Lessons

[1] Hierarchy between temporal scales

$$\Delta\omega < \Omega < \omega < \omega_{ST}$$

parameters	symbol	value	remark	parameters	symbol	value	remark
the number of qubits	N	11	odd integer	overall strength of the quench	B_p	0.03500	
lattice spacing	a	0.90000		the inverse lattice constant	w	0.55556	$w = 1/(2a)$
the length of interval	L	9.0000	$L = (N-1)a$	quench frequency gap	$\Delta\omega$	0.02500	
the number of steps	M	70		steps for adiabatic preparation	M_{adia}	490	$M_{\text{adia}}/M = 7$
quench time	T	25.00000		IR frequency cutoff	Ω	0.25133	$\Omega = 2\pi/T$
Trotterization time	Δt_{ST}	0.35714	$\Delta t_{ST} = T/M$	Trotteriztation frequency	ω_{ST}	17.59292	$\omega_{ST} = 2\pi/\Delta t_{ST}$

[2] Gate complexity (# of CNOT or CZ gates acting on two-qubit)

$$\mathcal{N}_{CZ} \sim \mathcal{O}\left(MN^2\right) > \mathcal{O}\left(\left(\omega\sqrt{P_{\text{th}}}/\gamma B_p\right)^{\frac{3}{2}} N^2\right)$$

Summary

[1] Hamiltonian formulation on a digital quantum processor simulates a state transition induced by external sinusoidal quench.

[2] Vacuum persistency probability signals the existence of excited state whose energy is identical to the frequency of quench.

- Selection rule on the quantum number matters.
- No-signal ≠ Nothing; new lesson from the various quench operators

[3] Perturbation theory-based complexity analysis gives *N*-polynomial complexity.

[4] More detailed physics analysis on the excited spectra on Schwinger / Ising model should be carried.

Thank you