Quantum Error Correction for Noise Mitigation in Wave-like Dark Matter Search

Hajime Fukuda (University of Tokyo)

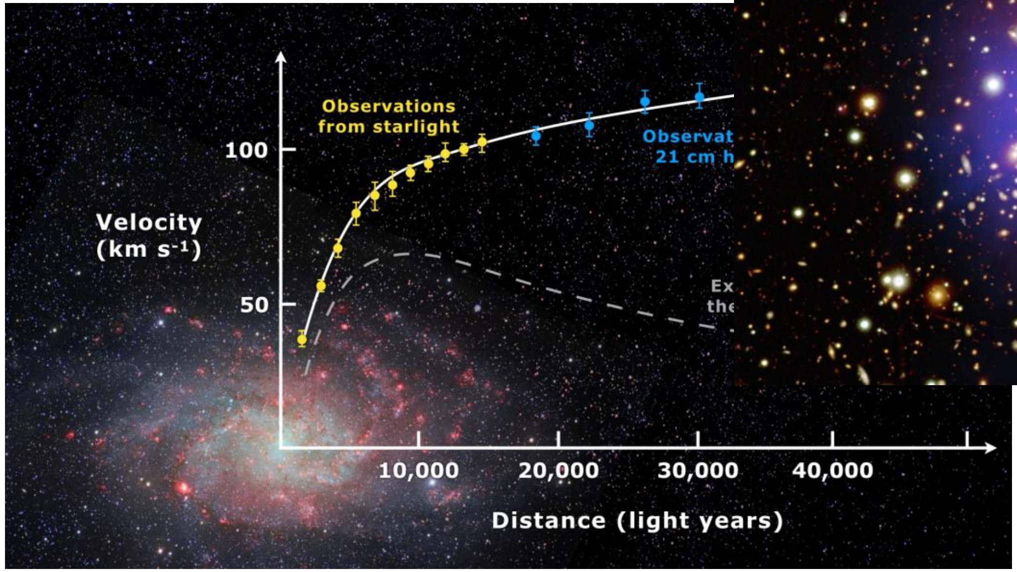
Based on

HF+, PRD 113 (2026) 5, 055014, arXiv:2510.01816

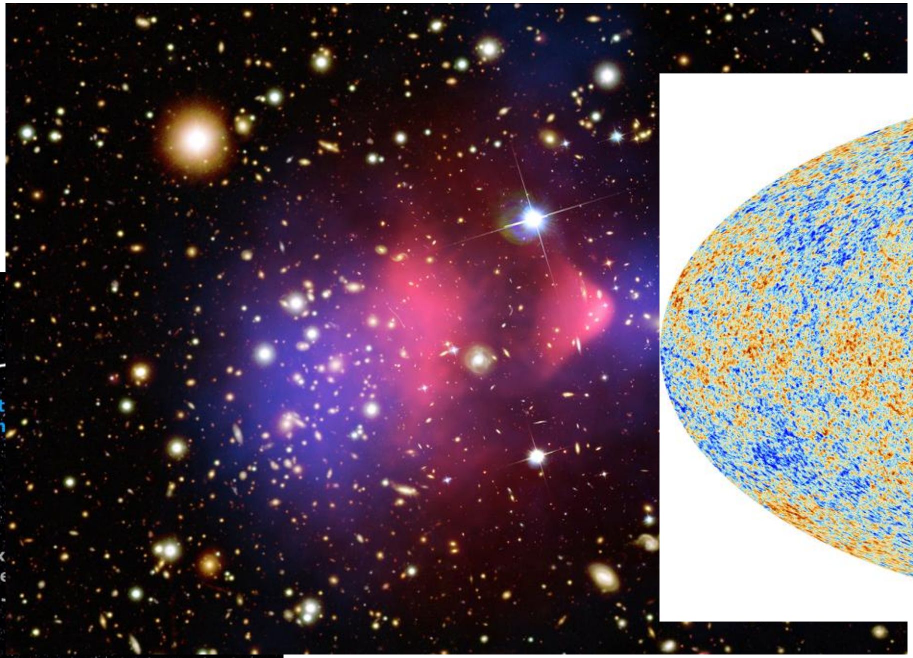
HF, Moroi, Sichanugrist PRL 137 (2026) 2, 021001, arXiv:2511.03253

Introduction

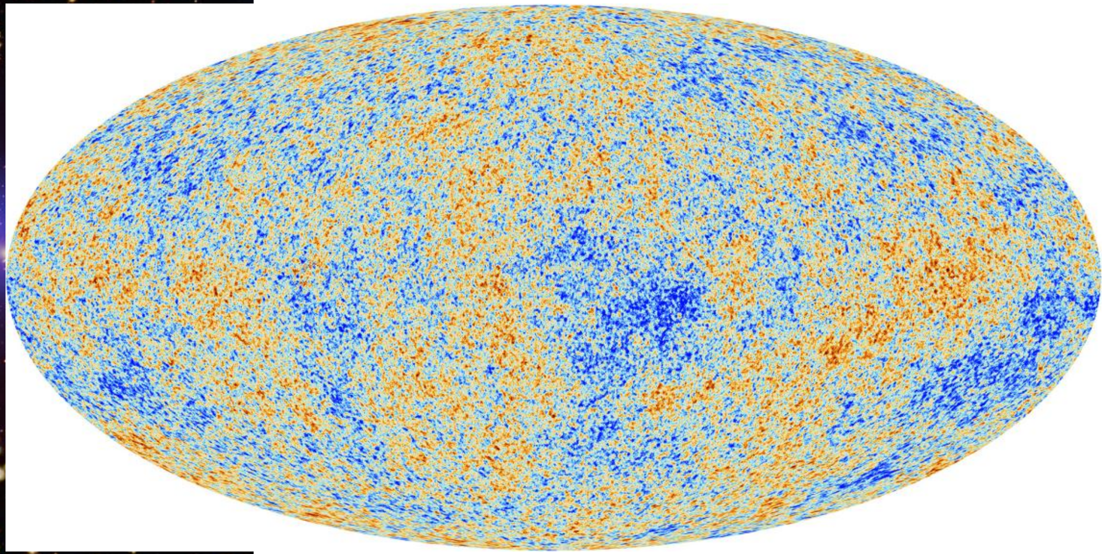
Dark Matter in the Universe



Rotation velocity of galaxy
From Wikimedia Commons



Bullet cluster, From ESA
Pink: X-ray observation
Blue: Gravitational lensing

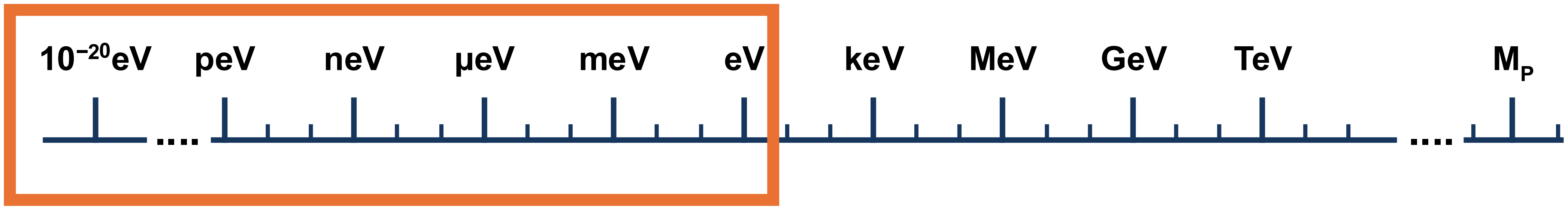


CMB fluctuation, From ESA

Wave-like DM

What is the mass of DM?

A Huge Range of Possible Masses!



Focusing on the Ultralight Regime
High occupation number ➔ **DM behaves like a wave** (just like classical light).

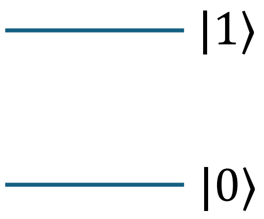
Use of Quantum Sensors

Detecting weak waves ➔ **Quantum Sensors**

- e.g. resonant cavity, superconducting qubit, NV center, ion trap, ⋯

Why do we use quantum sensors?

- Very small energy gaps ($\mathrm{GHz} \sim \mu\mathrm{eV}$ for superconducting qubit)
- Can be insensitive to the unknown DM phase
- We may manipulate the sensor states: **use of entanglement**



I’m going to talk about how we may exploit the sensor entanglement

Before that, let me start from the interaction b/w DM and sensors

DM-Sensor Interaction

- We want to know DM-SM interaction strength, e.g.,
  - $\frac{a}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu}$ for axion-like particles, $-\frac{1}{2}\varepsilon F_{\mu\nu}F'^{\mu\nu}$ for dark photons
- The interaction is reduced to an effective potential
  - Since qubits are two-level systems, it is written using Pauli matrices
- The details vary, but let us assume the potential is proportional to σ_X
- The qubit Hamiltonian is

$$H = H_0 + \Delta H, \qquad H_0 = -\frac{1}{2}\omega\sigma_z, \qquad \Delta H = 2\eta\sigma_X \sin mt$$

All the information we need is in η!
→We want to estimate η using qubits

Time Evolution of Qubits in DM Wave

- We first move to the interaction picture

$$H_I = e^{iH_0 t} \Delta H e^{-iH_0 t}$$
$$= \eta \sigma_X \cos(m - \omega)t + \cancel{(\text{high freq mode})} \simeq \eta \sigma_X$$

Here, we assume DM is resonant to the qubit frequency

e.g., terms proportional to $\cos(m + \omega)t$
such terms are averaged out to zero

- Starting from $|0\rangle$, $|\psi(t)\rangle \simeq (1 - iH_I t)|0\rangle = |0\rangle - i\eta t|1\rangle$
- We measure $|1\rangle$ against $|\psi(t)\rangle$ to extract η

If we include the phase φ,
$|\psi(t)\rangle \simeq |0\rangle - i\eta t e^{i\varphi}|1\rangle$

- In real experiments, however, **noise** is unavoidable

Use of Entanglement

As long as I know, there could be two possible advantages:

1. **Quantum Version of Signal Processing**

   HF et al., 2506.19614
   Shu et al., 24; HF et al., 2511.03253, 2510.01816; Freiman et al., 25

2. **Enhancing Signals**

   Ito et al. 23; Sichanugrist, HF, et al., 23; Zheng et al., 25; He et al., 25

As an example of 1, I'm going to talk about noise suppression protocols

Note: Its is strictly theoretical example. (Not ready for experiment now)

Quantum Noise on Qubits

What are Quantum Noises?

- What is “quantum” noise in the first place?
  - An operation that changes the state of a sensor with a certain classical probability.

$$|\psi(t_1)\rangle \to \begin{cases} |\psi(t_1)\rangle & \text{probability } 1-p \\ E|\psi(t_1)\rangle & \text{probability } p \end{cases}, E\text{: Noise operator}$$

“**classical probability**”
is just like coin tossing.
No interference.

- We may use density matrices for a simpler description

$$\rho(t_1) \to (1-p)\rho(t_1) + p\, E\rho(t_1)E^\dagger$$

- E need not be Hermitian

State	**Density matrix**
$\|\psi\rangle$	$\rho = \sum_i c_i \|\psi_i\rangle\langle\psi_i\|, 0 \le c_i \le 1, \mathrm{Tr}\rho = 1$
$\langle A\rangle = \langle\psi\|A\|\psi\rangle$	$\langle A\rangle = \mathrm{Tr}\, A\rho$
$p = \|\langle\phi\|\psi\rangle\|^2$	$p = \langle\phi\|\rho\|\phi\rangle$

Examples: Excitation Noise

- Consider a sensor excited by its interaction with the environment
- Clearly, the state should be classical, not a quantum superposition
- Hence, this process is not unitary evolution but quantum noise
  - $E = a^\dagger = |1\rangle\langle 0|$, not Hermitian

$$|0\rangle \to \begin{cases} |0\rangle & \text{probability } 1-p \\ |1\rangle & \text{probability } p \end{cases}$$

or

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1-p & 0 \\ 0 & p \end{pmatrix}$$

(100% $|0\rangle$) (100(1-p)% $|0\rangle$, 100p% $|1\rangle$)

Example: Thermal Relaxation Noise

- We can consider a more generic noise:
$$E = \left\{1, \sigma_Z, a = |0\rangle\langle 1|, a^\dagger = |1\rangle\langle 0|\right\}$$
  - 1: No noise
  - σ_Z: "Decoherence" noise (erasing the phase information)
  - a: De-excitation noise
  - $a^\dagger$: Excitation noise

- This is a model of thermal relaxation.

Excitation Noise and Signals

- Among various noises, excitation noise is most important:

<u>signal</u>

$$|0\rangle \to |\psi\rangle = |0\rangle - i\eta e^{i\varphi} t|1\rangle$$

$$p_1 = |\langle 1|\psi\rangle|^2 = \eta^2 t^2$$

<u>noise</u>

$$|0\rangle \to \begin{cases} |0\rangle & \text{probability } 1-p \\ |1\rangle & \text{probability } p \end{cases}$$

$$p_1 = p$$

φ: **Unknown** DM phase offset – expected to changes time to time.

- Can we use off-diagonal terms? – No, since φ is random.

We can't distinguish them with a single sensor!

Excitation Noise w/ Many Sensors

- With N sensors, the situation changes

<u>signal</u>

$$|0\rangle \to |0\rangle - i\eta e^{i\varphi} t|1\rangle$$
$$|0\rangle \to |0\rangle - i\eta e^{i\varphi} t|1\rangle$$
$$\vdots$$
$$|0\rangle \to |0\rangle - i\eta e^{i\varphi} t|1\rangle$$

All qubits are excited

<u>noise</u>

$$|00\ldots0\rangle \to \begin{cases} |00\ldots0\rangle \\ |10\ldots0\rangle \\ |01\ldots0\rangle \\ \vdots \\ |00\ldots1\rangle \end{cases}$$

A **particular** qubit is excited

Q: Can we distinguish them by this structure?

Correcting Quantum Noise

Measurements with W-State

Shu, Xu, Xu 24, HF+. 25, Freiman et al, 25

- Let's take a detailed look at the signal state
$$|\psi\rangle \simeq |00\ldots0\rangle + i\eta t\boxed{(|10\ldots0\rangle + |01\ldots0\rangle + \cdots + |00\ldots1\rangle)} + \mathcal{O}(\eta^2 t^2)$$
- In classical setup, we repeat measurement with $|10\ldots0\rangle$, $|01\ldots0\rangle$, $\cdots$.
- However, a single measurement actually suffices; we can measure by
$$|W\rangle \equiv \boxed{\frac{1}{\sqrt{N}}(|10\ldots0\rangle + |01\ldots0\rangle + \cdots + |00\ldots1\rangle)}$$
W-state
- The rate does not change up to $\mathcal{O}(\eta^2 t^2)$
- How about the noise state?

W-State and Noise State

- With noise, the state is one of the following:

$$|00\ldots0\rangle \rightarrow \begin{cases} |00\ldots0\rangle \\ |10\ldots0\rangle \\ |01\ldots0\rangle \\ \\ |00\ldots1\rangle \end{cases}$$

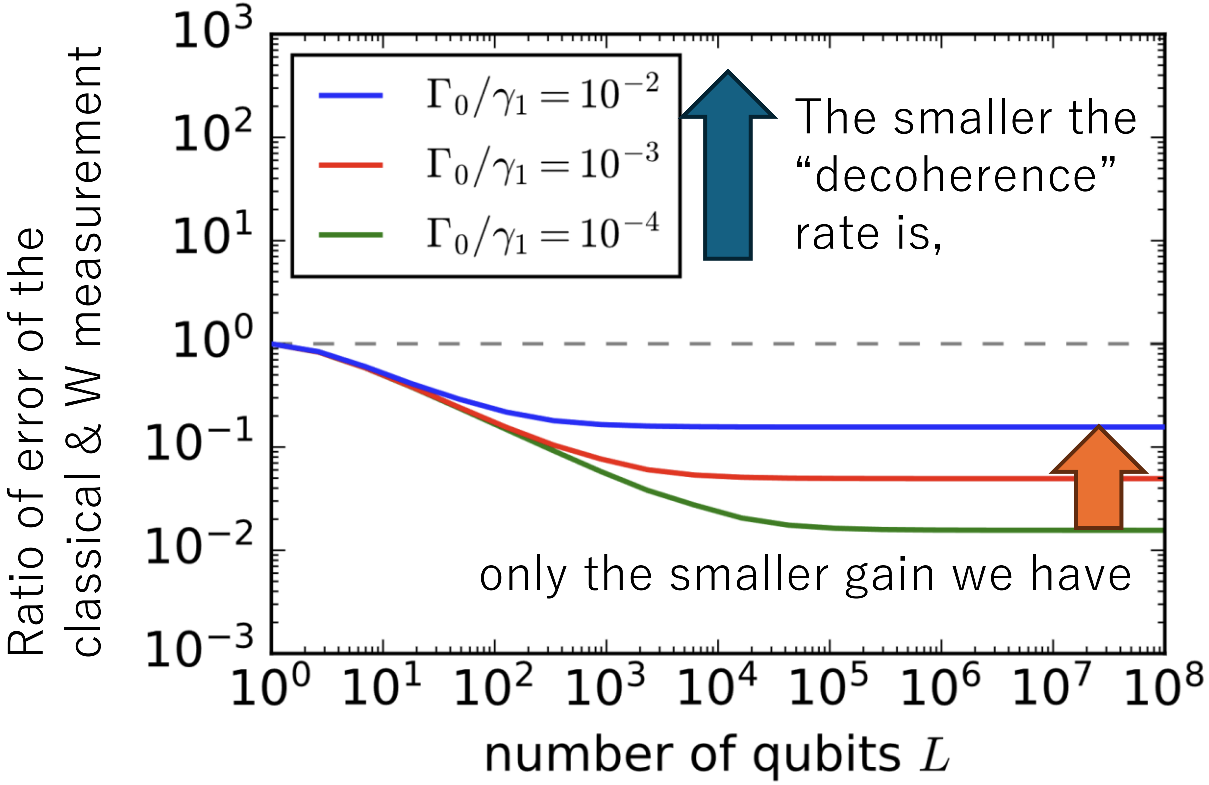
$$\langle 10\ldots0|10\ldots0\rangle = 1, \quad \langle W|10\ldots0\rangle = \frac{1}{\sqrt{N}}$$

W-state "takes correlations" quantum mechanically.

- In classical setup, we repeat measurement with $|10\ldots0\rangle$, $|01\ldots0\rangle$, $\cdots$.
- With W-state, the inner product is suppressed!: $\langle W|10\ldots0\rangle = 1/\sqrt{N}$
- **The W state suppresses noise while preserving the signal!**

Result for W

- With N qubits, under the thermalization error
- Take the ratio of the standard deviation of estimator of η
  - It is roughly "the ratio of the measurement error of η"



Γ_0: Excitation noise rate
γ_1: Other noise rate

When the Excitation Noise is Too Large…

HF+ 25

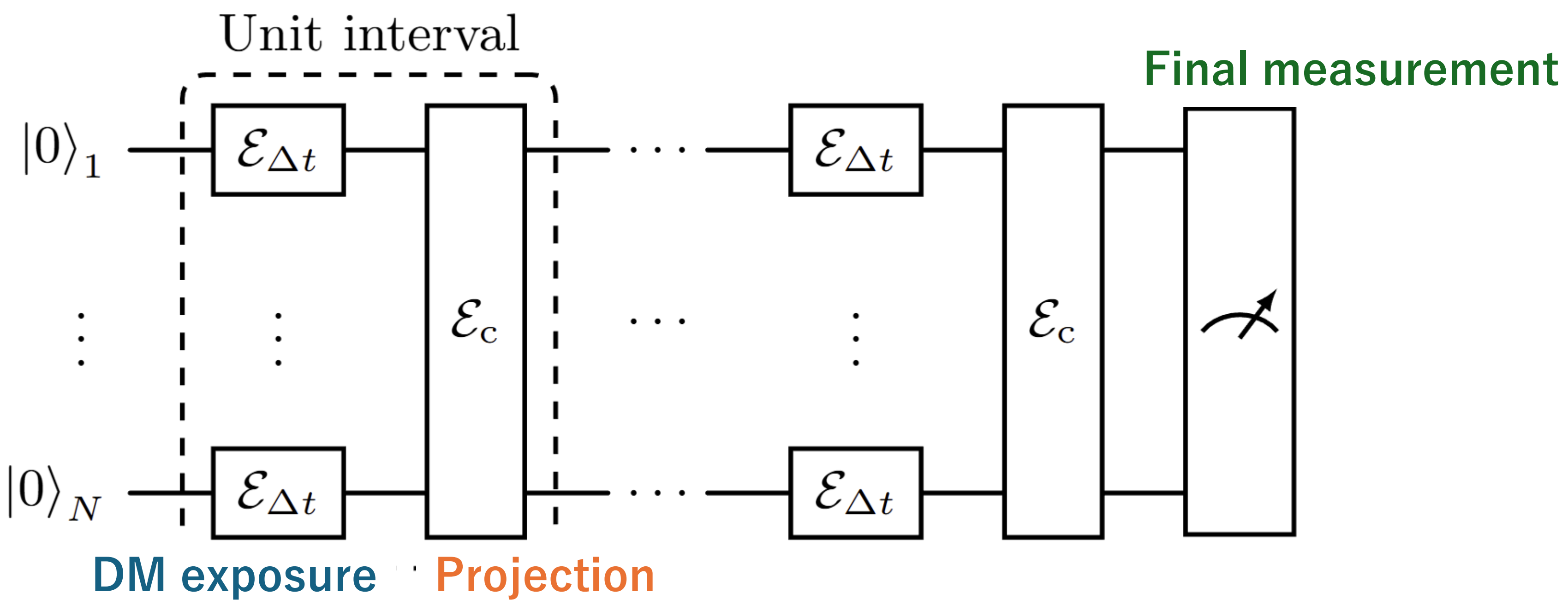
- **W-state is no more useful when the noise is too large**
- With a W-state, part of the signal is lost under noise:

$$(1 - iH_I t_1)|0\rangle \to a_1^\dagger (1 - iH_I t_1)|0\rangle$$

- W-state cannot measure η from these states
  - W-state is one-qubit-excited state, but multiple qubits are excited
- However, this error is not fundamental; it appears only because the measurement is performed in the W-state basis.
- Can we avoid this limitation by **continuously projecting onto the subspace spanned by $|\mathbf{0}\rangle$ and $|\boldsymbol{W}\rangle$**?

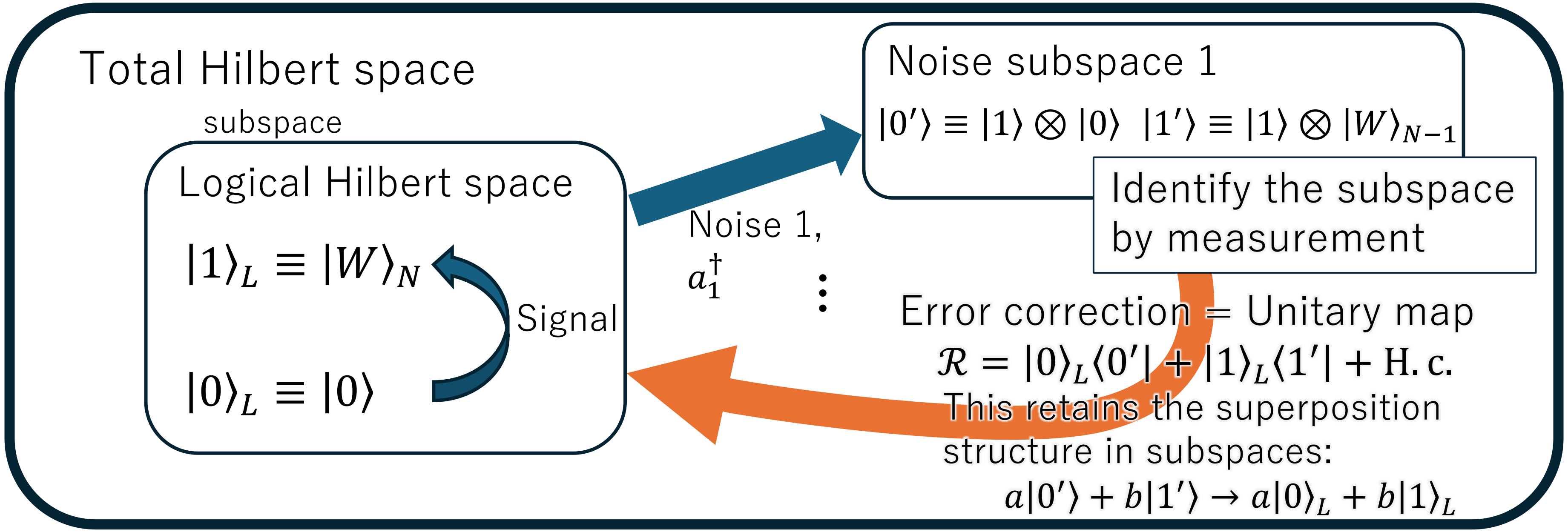
Measurement Protocol

- We want to measure DM with continuous projection
- A schematic illustration of the measurement is shown below:



Quantum Error Correction

Projection onto some subspace = QEC



What is the noise subspace and how can we identify it?

Signal/Noise Subspace

- Let us focus on the Hilbert space with/without noise

$$(1 - iH_I t_1)|0\rangle, \qquad a_1^\dagger(1 - iH_I t_1)|0\rangle$$

superposition of $|0\rangle$ and $|W\rangle_N$ superposition of $|1\rangle \otimes |0\rangle$ and $|1\rangle \otimes |W\rangle_{N-1}$

  - $|1\rangle \otimes |0\rangle$ and $|W\rangle_N$ are almost orthogonal：
    - $\langle W|(|1\rangle \otimes |0\rangle) = \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$
  - $|1\rangle \otimes |W\rangle_{N-1}$ is 2-qubit-excited states
- States with/without noise lives almost orthogonal subspaces

$$\{|0\rangle, |W\rangle_N\}, \qquad \{|1\rangle \otimes |0\rangle, |1\rangle \otimes |W\rangle_{N-1}\}$$

- For orthogonal subspaces, projection operators can distinguish them.
- Here, the subspaces are “almost” orthogonal, so a POVM measurement can instead distinguish them “almost” perfectly.

Error Detection for DM Search

- The error detection processes are a bit complicated…

Projective meas. : $|\psi\rangle \rightarrow P_i|\psi\rangle$, P_i is a projection op. $P_i^2 = P_i$. Successful with $p = \langle\psi|P_i|\psi\rangle$

POVM meas. : $|\psi\rangle \rightarrow M_i|\psi\rangle$, M_i is (normalized) "positive op". Successful with $p = \left\langle\psi\left|M_i^\dagger M_i\right|\psi\right\rangle$

$$M_0 \equiv |0\rangle\langle 0| + |W\rangle\langle W|\,, \quad \boxed{\text{No noise}}$$

$$M_i \equiv \sqrt{\frac{N}{N-1}}\,|i\rangle_\perp\langle i|_\perp + \frac{\sqrt{N(N-1)}}{N-2}\,|2,i\rangle_\perp\langle 2,i|_\perp \quad \boxed{\text{Error on } i\text{-th qubit}}$$

$$M_S \equiv |S\rangle\langle S|\,, \quad \boxed{\text{Rest}}$$

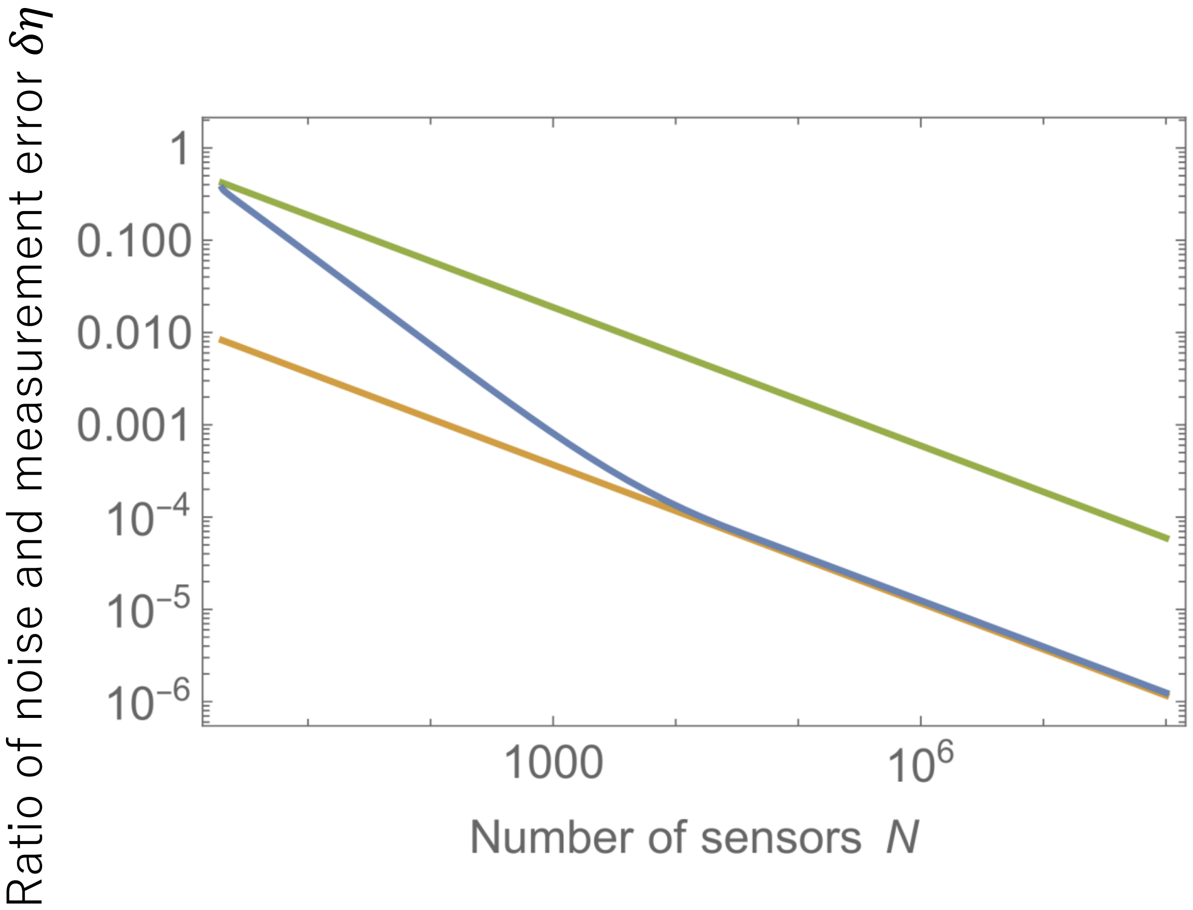
$$|i\rangle_\perp \equiv |i\rangle - \frac{1}{\sqrt{N}}|W\rangle\,,$$

$$|2,i\rangle_\perp \equiv |2,i\rangle - \sqrt{\frac{2}{N}}|S\rangle\,.$$

$|S\rangle$: 2 qubit excitation state
$|i\rangle_\perp$: almost i-th excited state, but orthogonal to W
$|2,i\rangle_\perp$: almost $|i\rangle \otimes |W\rangle_{N-1}$, but orthogonal to $|S\rangle$

Numerical Result

HF, Moroi, Sichanugrist 25, PRL



For simplicity, we consider only excitation noises
(No improvement expected for W-measurement)

Green: No QEC
Orange: σ_Y measurement with known φ (impossible)
Blue: With QEC

⇒**With QEC, we achieved the best sensitivity!**

Measuring σ_Y

$$|\psi\rangle = |0\rangle - i\eta e^{i\varphi} t|1\rangle$$

$\langle\sigma_Y\rangle \sim \eta t \sin\varphi$ but φ is random!

Summary

- Excitation noise are crucial for light DM search with quantum sensors
- We may reduce the noise by using entangled sensors
- By using the QEC-like protocol, we achieve the best sensitivity

Backup

No-go Theorem

- In quantum measurement theory, a no-go theorem was known stating that QEC cannot improve sensitivity.

[Sekatski et al., 2017, Demkowicz-Dobrzański et al., 2017, Zhou et al., 2017]

- This is because correcting the noise also corrects away the signal.

- However, this assumes infinitely coherent signals
- Due to the finite coherence, some measurement is impossible for DM

e.g., Measuring $\boldsymbol{\sigma_Y}$

$|\psi\rangle = |0\rangle - i\eta e^{i\varphi} t|1\rangle$ $\quad\langle\sigma_Y\rangle \sim \eta t \sin\varphi$ but φ is random!

Signal-Noise Distinction

Shu, Xu, Xu 24, **HF**, Moroi, Nitta et al. 25, Freiman et al, 25

- Q. Can we distinguish noises and signals?

$$-iH_I t|0\rangle \sim \sum_i \sigma_X^i\,|0\rangle \qquad\qquad a_i^\dagger|0\rangle \qquad \left(\rho = \sum a_i^\dagger|0\rangle\langle 0|a_i\right)$$

Signal: **All** qubits are excited

Noise: A **particular** qubit is excited

- Off-diagonal terms are absent → entangled states can distinguish!

	Normal measurements：Do projection measurements with $\lvert 10\ldots 0\rangle, \lvert 01\ldots 0\rangle, \ldots \lvert 00\ldots 1\rangle$ and sum the probability	Projection measurement by $\lvert W\rangle \equiv \frac{1}{\sqrt{N}}(\lvert 10\ldots 0\rangle + \lvert 01\ldots 0\rangle + \cdots \lvert 00\ldots 1\rangle)$
$\eta\sum_i \sigma_X^i\,\lvert 0\rangle$	$p = \eta^2 + \eta^2 + \ \ldots \eta^2 = N\eta^2$	$p = \left(\eta\cdot\frac{N}{\sqrt{N}}\right)^2 = N\eta^2$
$\sqrt{\gamma}a_1^\dagger\lvert 0\rangle$	$p = \gamma$	$p = \frac{\gamma}{N}$