Thermal effects on hadron-quark phase transition and the structure of proto-compact stars

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Overview

- Finite temperature in neutron stars
- Open problems
- Modeling the equation of state
- Results
- Conclusions

The role of temperature in neutron star (NS) calculations

- Old Neutron Stars are 'cold'. Thermal energy is negligible compared to the Fermi energy. Temperature effects can be neglected.
- Thermal effects are signifficant in the cases of supernova matter, protoneutron stars and neutron star merger remnants.
- Temperature range during the inspiral phase of a merger $\rightarrow$ Still an open problem.

Evolution of proto-neutron stars (PNS)

- **Stage 1**: $S_b = 1\ k_B$ and $Y_l = 0.4$.
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- **Stage 3**: $T = 0$ K.
  The compact star temperature falls. Thermal effects become negligible.

Phase transitions in PNS cores - open issues

Phase transitions in dense and hot matter (isentropic configurations and Maxwell construction) are often modeled in the same way as in the cold case:

1. Evaluation of $P - \mu_B$ curves for each phase.
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!!! The condition for thermal equilibrium is not taken into account.

Scope

In the present work we aim to reconcile the aforementioned issue (thermal disequilibrium) resolving this thermodynamic inconsistency.

Hadronic matter

For the hadronic phase we employ a Skyrme effective interaction. The relevant energy density

$$\mathcal{E} = \mathcal{E}(\tau_n, \tau_p, n_n, n_p) \tag{1}$$

depends on

$$n_i = 2\int \frac{d^3k}{(2\pi)^3} f_i(n_i, k, T), \qquad \tau_i = 2\int \frac{d^3k}{(2\pi)^3} k^2 f_i(n_i, k, T) \tag{2}$$

and

$$f_i(n_i, k, T) = \left\{1 + \exp\left[\frac{e_i(n_i, k, T) - \mu_i(n_i, T)}{T}\right]\right\}^{-1}, \tag{3}$$

Hadronic matter

The entropy density s_i has the same functional form as that for a non-interacting gas system, given by the equation

$$s_i = -2 \int \frac{d^3k}{(2\pi)^3} [f_i \ln(f_i) + (1 - f_i) \ln(1 - f_i)]. \tag{4}$$

and then the pressure can be evaluated as

$$P = -\mathcal{E} + \sum_{i=n,p} (Ts_i + \mu_i n_i) \tag{5}$$

To evaluate the total EOS one has to include **leptonic contributions** and impose the conditions for **chemical equilibrium** and **charge neutrality**.

Quark matter

For the quark phase we employed the vector MIT bag model described the following Lagrangian density

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{vec}}, \tag{6}$$

where

$$\mathcal{L}_0 = \left\{ \sum_q \bar{\psi}_q (i\gamma_\mu \partial^\mu - m_q)\psi_q - B \right\} \Theta, \tag{7}$$

$$\mathcal{L}_{\text{vec}} = \left\{ -g_v \sum_q \bar{\psi}_q \gamma_\mu V^\mu \psi_q + \frac{1}{2} m_V^2 V_\mu V^\mu \right\} \Theta, \tag{8}$$

Quark matter

Applying RMF one can derive the single-particle energy of q quarks

$$e_q = \sqrt{k^2 + m_q^2} + g_v V^0, \tag{9}$$

$$m_v V_0 = \sum_{q=u,d,s} g_v n_q. \tag{10}$$

Then,

$$n_q = 2\int \frac{d^3k}{(2\pi)^3} f_q, \quad \mathcal{E}_Q = \sum_{q=u,d,s} \mathcal{E}_q - \langle \mathcal{L} \rangle, \quad P_Q = \sum_{q=u,d,s} P_q + \langle \mathcal{L} \rangle,$$

and

$$\mathcal{E}_q = 2\int \frac{d^3k}{(2\pi)^3} e_q f_q, \quad P_q = \frac{2}{3}\int \frac{d^3k}{(2\pi)^3} k \frac{\partial e_q}{\partial k} f_q, \quad \langle \mathcal{L} \rangle = -B + \frac{1}{2} m_V^2 V_0^2.$$

Phase transition modeling

We follow a Maxwell-like construction assuming large surface tension between the two phases. Then, the conditions that need to be satisfied are.

1. Without neutrinos

$$P^h = P^q, \quad \mu_B^h = \mu_B^q, \quad T^h = T^q$$

2. With neutrinos

$$P^h = P^q, \quad \bar{\mu}^h = \bar{\mu}^q, \quad T^h = T^q,$$

with

$$\bar{\mu} = \mu_B + Y_l \mu_\nu \tag{11}$$

Results I - Phase diagram

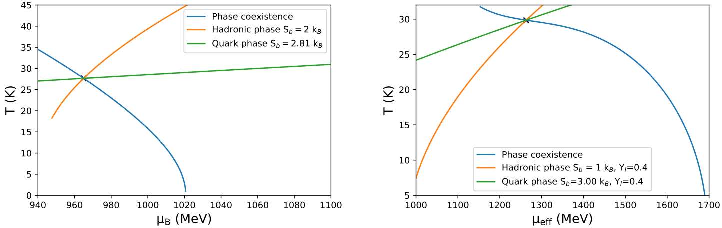

Figure: Phase diagrams related to the thermodynamic conditions of Stages 1 and 2.

Results II - Clausius-Clapeyron (CC) equation

Another way to test the thermodynamical consistency of the constructed EOSs is via the CC equation

$$\frac{dP_{\rm co}}{dT_{\rm co}} = \frac{S_b^h - S_b^q}{\frac{1}{n_b^h} - \frac{1}{n_b^q}} \tag{12}$$

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Stage	LHS	RHS	Relative difference
1	-53.44(...)	-53.11(...)	0.6%
2	-0.197(...)	-0.197(...)	0.3%

Table: CC verification

Results III - Hybrid equations of state

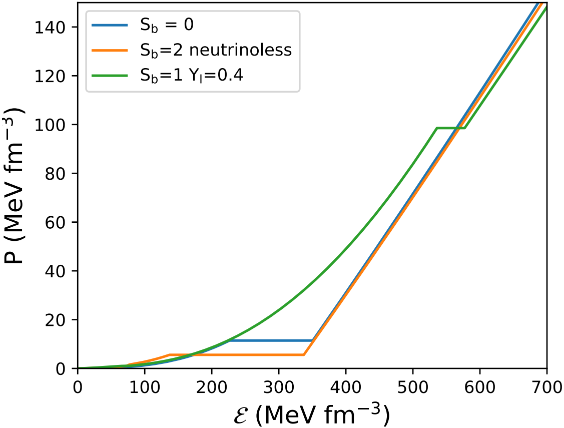

Figure: EOSs constructed in this study

Results IV - Proto-hybrid star structure

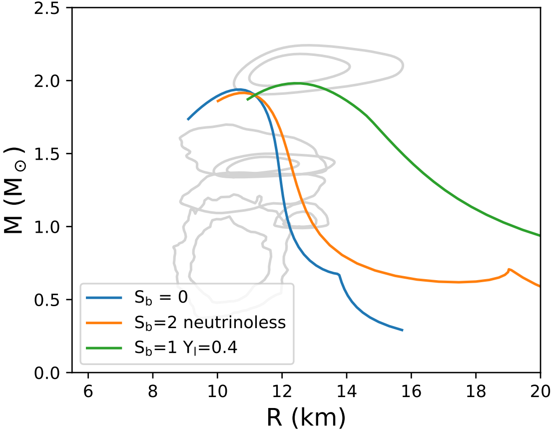

Figure: Mass-radius diagrams.

Summary and future research

- We have seen that the consistent thermodynamic description of proto-hybrid stars requires the consideration of different entropy per nucleon.
- Finite temperature shifts the phase transition onset to lower baryon density.
- The inclusion of neutrino trapping shifts the onset of the phase transition to larger baryon density.
- Thermal twin stars may exist (for the neutrinoless case). What about their evolution?
- Future Gibbs construction of stage 1, assuming Global lepton charge conservation.

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Thank you for your attention !!!