Modeling the Transition from Hadron Gas to Deconfined Quark Matter via the Albright–Kapusta–Young Switch Function

G. A. Tsalis and Ch. C. Moustakidis

Department of Theoretical Physics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

June 5, 2026

Hybrid stars

- Hybrid stars may have a hadron matter part.
- Hybrid stars could contain quark matter.
- Hybrid stars EOS combines a Hadron matter part (Divaris et al., 2024) and quark matter part, CFL model (Blaschke et al., 2022) using a switch function.

Hadron matter

Hadron matter is governed by parameter $\eta = \left(k_0 L^2\right)^{1/3}$

- k_0 is incompressibility parameter.
- L is slope parameter
- η defines the stiffness of hadronic EOS

The Hadronic part of the Hybrid EOS:

$$\varepsilon_b(n,a) = E_0 n + \frac{k_0}{18 n_0^2} n \left(n - n_0\right)^2 + \left(J + \frac{L}{3n_0}\left(n - n_0\right)\right) n a^2$$

$$P_b = \frac{K_0}{9n_0^2} n^2 \left(n - n_0\right) + \alpha^2 \frac{L}{3n_0} n^2$$

$$S(n_b) = J + \frac{L}{3n_0}(n_b - n_0)$$

$J = 32 MeV$, $n_0 = 0.16 fm^{-3}$, $E_0 = -16 MeV$

Hadron matter

Electron contribution - total energy density and pressure for hadron matter in chemical equilibrium:

$$\mathcal{E}_e = \frac{(m_e c^2)^4}{8\pi^2(\hbar c)^3}\left[z(2z^2+1)\sqrt{1+z^2} - \ln\left(z+\sqrt{z^2+1}\right)\right]$$

$$P_e = \frac{(m_e c^2)^4}{24\pi^2(\hbar c)^3}\left[z(2z^2-3)\sqrt{1+z^2} + 3\ln\left(z+\sqrt{z^2+1}\right)\right]$$

$$\mathcal{E}_{\rm tot} = \mathcal{E}_b + \mathcal{E}_e, \qquad P_{\rm tot} = P_b + P_e$$

$$z = \frac{(\hbar c)(3\pi^2 n_e)^{1/3}}{m_e c^2}, \quad n_e = x_p n$$

$$x_p(n) = \frac{1}{2} - \frac{1}{4}\left([2\beta(\gamma+1)]^{1/3} - [2\beta(\gamma-1)]^{1/3}\right)$$

$$\beta = 3\pi^2 n\left(\frac{\hbar c}{4S(n)}\right)^3, \qquad \gamma = \left(1+\frac{2\beta}{27}\right)^{1/2}$$

Quark matter

Quark matter EOS is dictated by 3 parameters (Blaschke et al., 2022):

- a_4 coefficient.
- Δ diquark pairing gap
- B_{eff} effective bag pressure

Constituents of quark matter EOS by (Alford et. al.2005):

$$P_q(\mu) = \frac{\xi_4 a_4}{4\pi^2}\left(\frac{\mu}{3}\right)^4 + \frac{\xi_{2a}\Delta^2 - \xi_{2b} m_s}{\pi^2}\left(\frac{\mu}{3}\right)^2 + \mu_e^4 - B_{eff}$$

CFL model:

- $\xi_4 = 3$, $\xi_{2a} = 3$, and $\xi_{2b} = 34$
- $m_u = m_d = m_s = 0$
- $\mu_e = 0$

Quark matter-CFL model

CFL model:

$$P_q(\mu) = \frac{3}{4\pi^2} a_4 \left(\frac{\mu}{3}\right)^4 + \frac{3}{\pi^2} \Delta^2 \left(\frac{\mu}{3}\right)^2 - B_{eff}$$

$$\varepsilon_q(\mu) = -P_q(\mu) + \left(\frac{\mu}{3}\right) n_q(\mu) = \frac{9}{4\pi^2} a_4 \left(\frac{\mu}{3}\right)^4 + \frac{3}{\pi^2} \Delta^2 \left(\frac{\mu}{3}\right)^3 + Beff$$

Expressing the two parts of EOS as function of the same variable:

$$\mu = (\varepsilon + P)/n$$

Mixing Hadron and Quark part - Switching Function

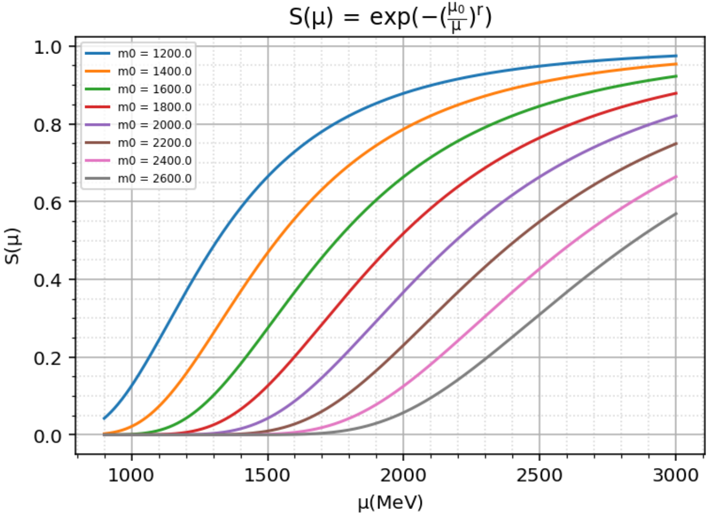
Combining the two distinct parts (Hadron - Quark) $\rightarrow$ Hybrid EOS.
The switch function:

$$S(\mu) = \exp\left[-\left(\frac{\mu_0}{\mu}\right)^r\right]$$

Why this?

- Abandons the need of pressure equality between the phases (interpolation) of Maxwell and Gibbs construction.
- Mixed phase where pressure is composed of contributions from Hadron and Quark matter simultaneously, but in different proportion for various baryon densities.
- Switch function accounts for the partial pressure of each component (Hadron-Quark matter). One thermodynamic variable μ.
- Infinitely differentiable. It does not introduce any artificial phase transition of any order. (Maxwell like).
- Unlike first order transition (Maxwell), where 2 phases are in contact.
- One single phase whose ground state is governed by parameter μ.

Switch function Diagram



The smaller the μ_0, the faster the switch function approaches 1.

Crossover EOS

Crossover EOS using the aforementioned switch function:

- Combines the Hadron EOS with Quark EOS using a switch function
- $S(\mu) = \exp\left[-\left(\frac{\mu_0}{\mu}\right)^r\right]$
- It does not introduce thermodynamical phases transition of any order

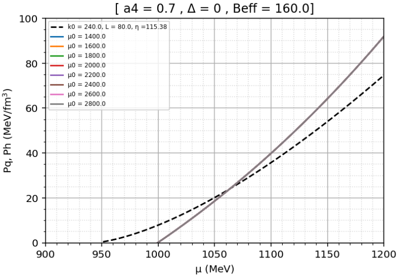
$$P(\mu) = S(\mu)P_q(\mu) + [1 - S(\mu)]P_h(\mu)$$

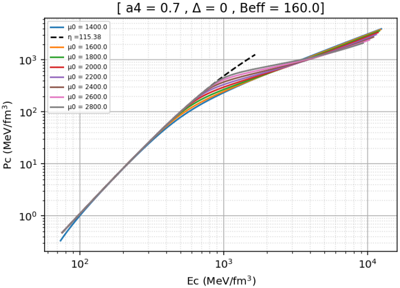
$$n(\mu) = S(\mu)n_q(\mu) + [1 - S(\mu)]n_h(\mu) + S'(\mu)[P_q(\mu) - P_h(\mu)]$$

$$\varepsilon(\mu) = S(\mu)\varepsilon_q(\mu) + [1 - S(\mu)]\varepsilon_h(\mu) + r\left(\frac{\mu_0}{\mu}\right)^{r+1} S(\mu)[P_q(\mu) + P_h(\mu)]$$

As the value of switch function ranges monotonically, between [0,1], the pressure as function of chemical potential, derives from hadron matter in the beginning, but in the end comes from quark matter.
In reality, in our approach, the two phases coexist, but in a changing proportion.

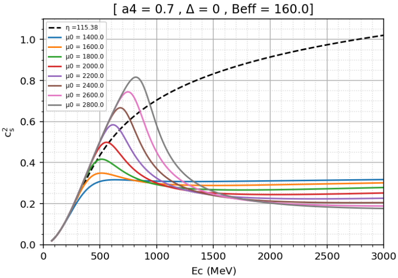
Results 1 Hadronic - 1 Quark EOS different μ_0

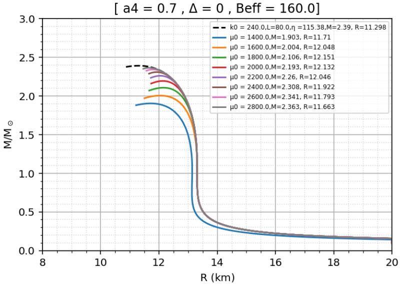




- the value of μ_0 does not affect the crossing point.
- the effect of quark matter becomes significant at lower values of energy density.

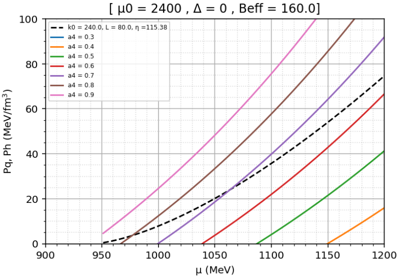
Results 1 Hadronic - 1 Quark EOS different μ_0

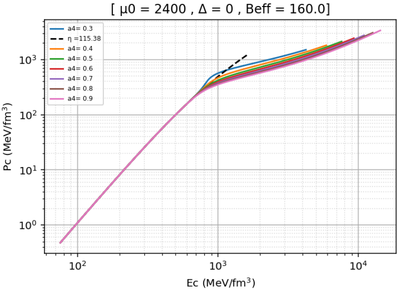




- As the value of μ_0 gets smaller the M_{max} gets smaller.
- The lowest value of μ_0 produces a very different result than the pure neutron matter.

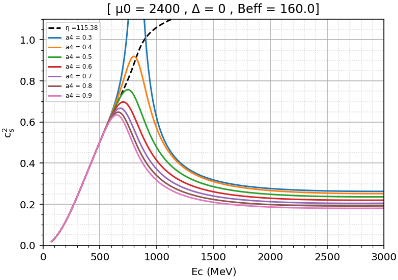
Results 1 Hadronic - 7 Quark EOS different a_4

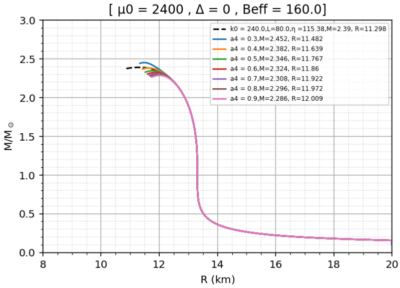




- low values of a_4 may describe no crossing point or unphysical situation where for increasing chemical potential the system switches from quark to hadronic matter.

Results 1 Hadronic - 7 Quark EOS different a_4

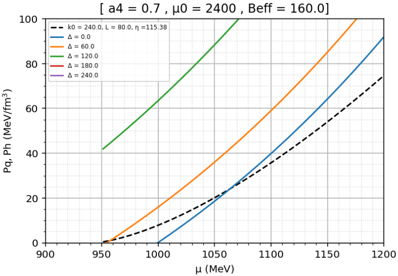


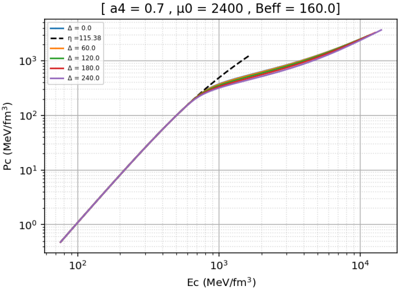


For $a_3 = 0.3$ the speed of sound exceeds the limit of 1 set from hadronic EOS. It is unphysical.

That is depicted in the kink in P_c with respect to E_c for $a_{3=0.3}$ which makes the transition impossible.

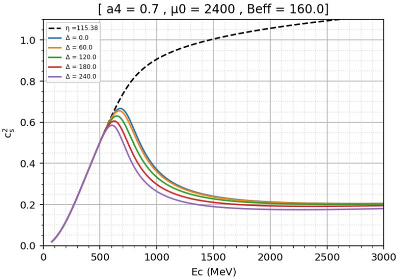
Results 1 Hadronic - 5 Quark EOS different Δ

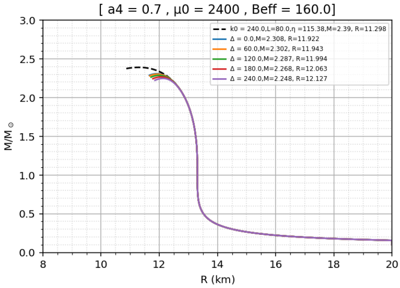




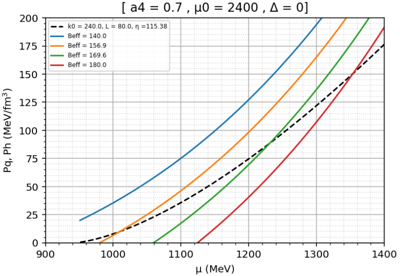
- Only for $\Delta = 0$ there is a clear crossing point. The other cases are impossible.

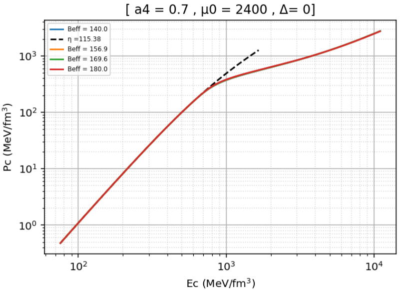
Results 1 Hadronic - 5 Quark EOS different Δ





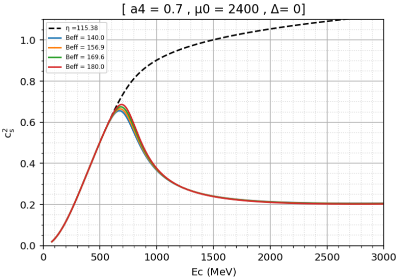
Results 1 Hadronic - 4 Quark EOS different B_{eff}

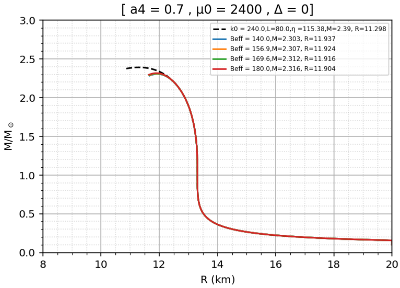




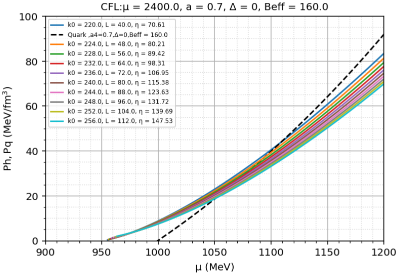
- there as transitions for the most of the different value of B_{eff}.
- However the changes in the value of B_{eff} have minor impact.

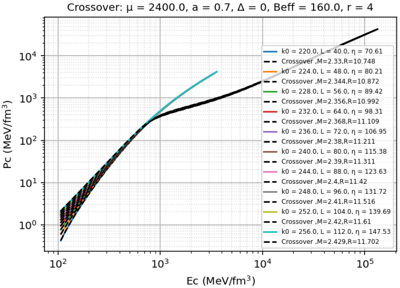
Results 1 Hadronic - 4 Quark EOS different B_{eff}

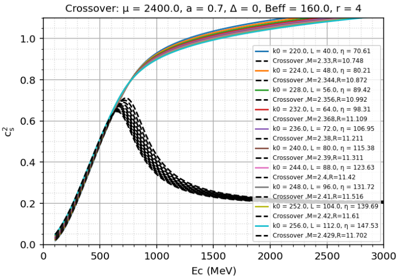


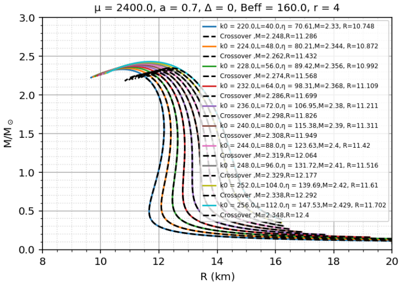


Results 10 Hadronic - 1 Quark EOS different η

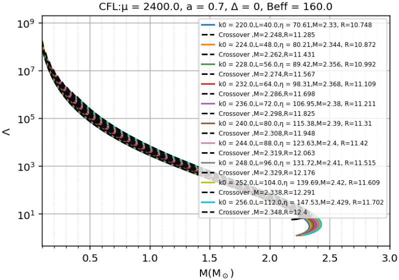


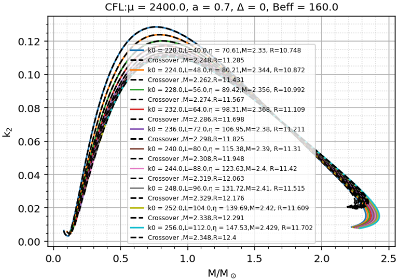






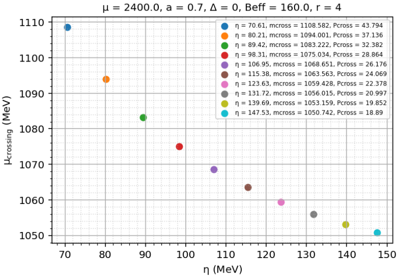
- increasing η, slightly higher M_{max}
- As it has been seen in the previous cases, the phase transition softens The EOS so for Hybrid stars M_{max} gets lower.

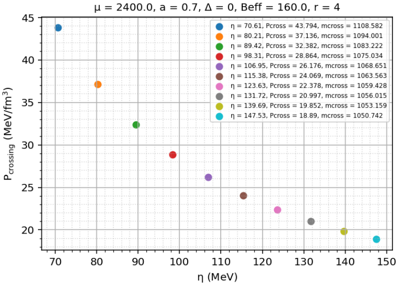




The presence of quark matter and its effect on tidal deformability Λ and love number k_2

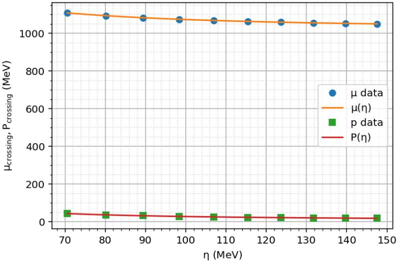
Results 10 Hadronic - 1 Quark EOS different η





- As η value gets greater, the values of μ and P where the crossing occurs get lower.

Results 10 Hadronic - 1 Quark EOS different η



$$\mu_{cross}(\eta) = -4.356 * 10^{-9}\eta^5 + 3.07 * 10^{-6}\eta^4 - 0.0008957\eta^3$$
$$+0.1378\eta^2 - 11.58\eta + 1486$$

$$P_{cross}(\eta) = -3.311 * 10^{-9}\eta^5 + 2.217 * 10^{-6}\eta^4 - 0.0006098\eta^3$$
$$+0.08749\eta^2 - 6.751\eta + 249.6$$

Conclusions and what is next!

To summarize:

- Switch function allows the coexistence of hadronic and quark phase, but in different ratio with respect to increasing chemical potential.
- Switch function does not introduce any artificial phase transitions.
- With increasing μ system have to switch from hadronic to quark matter, otherwise it is something unphysical.
- The contribution of quark matter softens the EOS, lowers the mass.

Future prospects:

- Compare the results with experimental data
- Adding the variable of temperature in switch function

Thank you