Neutron star dissipative tidal deformation originated by the bulk viscosity of npe matter

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34th Annual Symposium of the Hellenic Nuclear Physics Society
5 -6 June 2026, Lamia, University of Thessaly, Physics Department

Outline

- Theoretical Framework - Equation of State (EoS)
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- Tidal deformation of Neutron Stars
- Dissipative Tidal Deformability
- Preliminary Results
- Remarks

Equation of State (EoS)

- The EoSs describing neutron-star matter are constructed within the momentum-dependent interaction (MDI) model.
- In the present work, we choose two parametrizations for symmetric nuclear matter (named SNM-1 and SNM-2); MDI-1 consists of SNM-1 and MDI-2,3,4 with SNM-2.
- We reproduce the empirical saturation properties of nuclear matter, namely the binding energy per nucleon $E_0 = -16\ \mathrm{MeV}$, the incompressibility $K_0 = 250\ \mathrm{MeV}$, the effective nucleon mass $m_N^*/m_N = 0.8$, and the optical potential at infinite momentum $U_0^\infty = 55\ \mathrm{MeV}$.
- The isovector sector is constrained by imposing

$$E_{\mathrm{sym}}(2n_0) = 51 \pm 13\ \mathrm{MeV}, \tag{1}$$

which is consistent with current χEFT calculations. Here, n_0 denotes the saturation density.

Equation of State (EoS)

- For neutron-star matter, we consider cold, catalyzed matter composed of neutrons, protons, and electrons. The composition of matter at each baryon density is determined by imposing charge neutrality and chemical equilibrium. Since only electrons are included as leptonic degrees of freedom

$$n_p = n_e, \quad \text{and} \quad \mu_n - \mu_p = \mu_e, \tag{2}$$

- The energy density and pressure of the EoS are determined as

$$\begin{aligned} \mathcal{E}(n, x_p) &= \mathcal{E}_{\rm MDI}(n, x_p) + \mathcal{E}_e(n_e), \\ P(n, x_p) &= n^2 \frac{\partial E_{\rm MDI}(n, x_p)}{\partial n} + P_e(n_e), \end{aligned} \tag{3}$$

  where $\mathcal{E}_{\rm MDI}(n, x_p)$ is the nucleonic contribution obtained from the MDI interaction, $\mathcal{E}_e(n_e)$ and $P_e(n_e)$ are the contributions of a degenerate relativistic electron gas, and x_p is the proton fraction.
- All EoSs considered here remain causal, $c_s < c$.

Direct Urca process - Restore chemical equilibrium

Our study considers npe matter (superconductivity and superfluidity are neglected).
We also assume that neutrinos are absent, i.e. the matter is neutrino transparent (our investigation is employed to temperatures of a few MeV).
In npe matter, we consider only the direct Urca (dUrca) mechanism for the restoration of chemical equilibrium, described by two simultaneously evolving processes

$$n \to p + e^- + \bar{\nu}_e, \tag{4}$$

$$p + e^- \to n + \nu_e, \tag{5}$$

which is kinematically allowed only if the Fermi momenta of the participating particles satisfy the inequality $k_{Fn} < k_{Fp} + k_{Fe}$.
The direct Urca operates only if the proton fraction exceeds the threshold value $x_p \simeq 0.11$.

Direct Urca - MDI EoSs

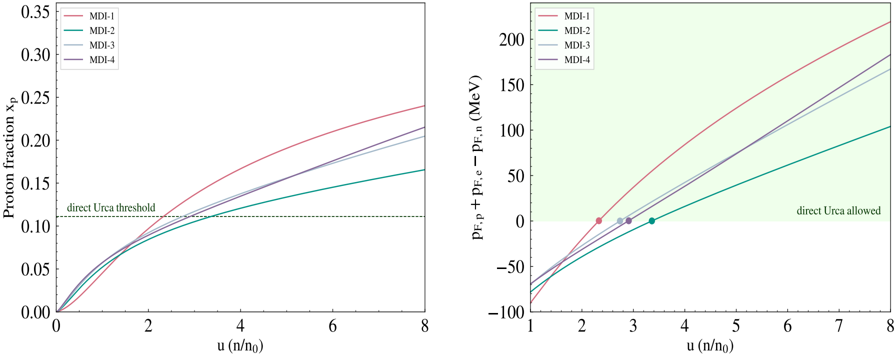


Figure: 4 MDI EoS with different proton fraction behavior lead to different baryon density threshold for dUrca to switch on inside neutron star core.

Transport coefficients

Our formalism follows that of Sawyer (PRD 39, 3804 (1989)), Haensel, Levenfish, and Yakovlev, Astron. and Astroph., 357, 1157 (2000).
We name $\delta\mu \equiv \mu_n - \mu_p - \mu_e$ the measure of the chemical equilibrium (for $\delta\mu = 0$ we are in β-equilibrium, while for $\delta\mu \neq 0$ the npe matter is subject to compressions/decompressions; instantaneous values of μ_i).

$$\delta\mu = \left(\frac{\partial\delta\mu}{\partial n_b}\right)\delta n_b + \left(\frac{\partial\delta\mu}{\partial x}\right)\delta x_p = \frac{C_e}{n_b}\delta n_b + B_{ee}\delta x_p, \tag{6}$$

Our study restricts on the linear (subthermal) regime, i.e. the amplitude $|\delta\mu|$ of oscillations is smaller than $k_B T$.
We define the transport coefficients $C = C_e$ and $B = (\partial\delta\mu/\partial\alpha)_{n_b} = -\frac{B_{ee}}{2}$ ($\alpha = 1 - 2x_p = I$ is the isospin asymmetric nuclear matter parameter),

$$C_e = n_{b,0}\left(\frac{\partial\delta\mu}{\partial n_b}\right)_{x_p} \quad \text{and} \quad B_{ee} = \left(\frac{\partial\delta\mu}{\partial x_p}\right)_{n_b}, \tag{7}$$

C_e and B_{ee} have to be evaluated for x_p and n_b in beta equilibrium.

Bulk Viscosity of npe matter

The bulk viscosity for the subthermal case is expressed as

$$\zeta = -\frac{\lambda_{npe} C^2}{\omega^2 + 4\lambda_{npe}^2 B^2/n_b^2}, \tag{8}$$

The rate of the direct Urca process is given by

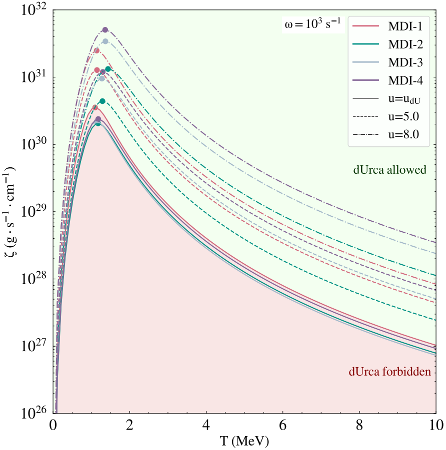
$$\lambda_{npe} = -3.5 \times 10^{40} \frac{m_n^*}{m_n} \frac{m_p^*}{m_p} \left(x_p \frac{n_b}{n_0} \right)^{1/3} T_9^4 \Theta_{npe} \, (\mathrm{cm}^{-5}\ \mathrm{g}^{-1}\ \mathrm{s}), \tag{9}$$

where $T_9 = T/10^9\ K$ is the temperature in units of 10^9 K.
With m_τ^*/m_τ ($\tau = n, p$) we denote the effective masses of protons and neutrons, and ω to be the oscillation frequency of npe matter (we assume harmonic oscillations of fixed ω around the β-equilibrium $n_{b,0} \equiv n_b$)

$$n_b(t) = n_{b,0} + \Re(\delta n_b e^{i\omega t}) = n_{b,0} + \delta n_b \cos \omega t, \tag{10}$$

Bulk Viscosity ζ of npe matter vs temperature

- Resonant behavior; Max bulk viscous reaction at a specific T.
- The resonant peak is altered by ω.



Tidal effects during inspiral of a binary neutron star system

- Gravitational-waves (GW) from the late phase of a binary neutron star system are among the most significant sources for a GW detector
- The quadrupole moment is given by

$$Q_{ij} = -\lambda E_{ij}, \text{ with } \lambda = \frac{2}{3} k_2 \frac{R^5}{G}, \text{ where } \Lambda = \frac{2}{3} k_2 C^{-5}, \tag{11}$$

- The induced quadrupole moment depends on neutron star structure.
- Tidal deformability λ depends on radius R (the smaller the star, the harder to deform) and k_2; the latter encodes the reaction of the star to the tidal field which depends on the structure of the neutron star.

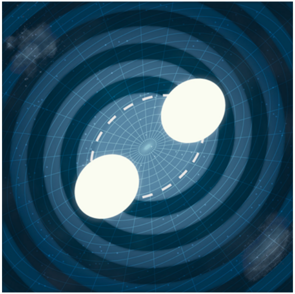


Figure: Artist's visualization of binary neutron star inspiral. Credit: Megan Janeski

Non-equilibrium dissipative effects of neutron stars

- The tidal response of a Neutron Star can be visualized as 'tidal bulge', not aligned to the gravitational field of its companion (see Figure).
- The extent of this misalignment is described (at first approximation) by the **tidal lag time** $\tau_{\rm d}$.

- Universal feature of self-gravitating astrophysical objects.
- The tidal misalignment torques the two neutron stars, *heating them up* through tidal-viscous heating (Nature Astronomy 8, 1277 (2024)).
- Investigate Λ for finite temperature EoSs in our previous work: Phys. Lett. B, 832, 137267 (2022).

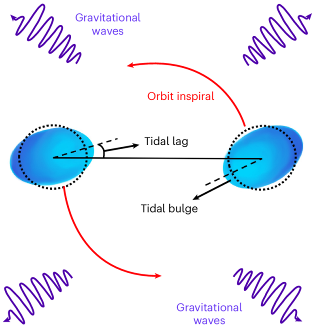


Figure: J.L. Ripley, A. Hegade, R.S. Chandramouli, and N. Yunes, *Nature Astronomy, 8, 1277, 2024*

Tidal lag time

- The viscous effects cause a time delay between the induced quadrupole moment of a body and the imposed gravitational field (PRD 108, 103037 (2023)).
- We consider $\tau_{\rm d}$ to be constant, and from linear tidal response we obtain for the *bulk viscosity* of npe matter

$$\tau_{d,A} = \frac{p_{2,A} v_A R_A}{G m_A} = \frac{p_{2,A}}{C_A} \cdot \frac{\langle\zeta\rangle}{\langle\varepsilon\rangle} \quad (s), \tag{12}$$

where $p_{2,A}$ is a dimensionless number within the range 0.02-0.2 for bulk viscous matter, $C_A = Gm_A/R_A c^2$ is the (dimensionless) compactness of the neutron star, and v_A is an averaged quantity which has the dimensions of the kinematic viscosity, given by

$$v_A \equiv \frac{\langle\zeta\rangle}{\langle\rho\rangle} = c^2 \frac{\langle\zeta\rangle}{\langle\varepsilon\rangle} \tag{13}$$

Tidal lag time - Considerations

- The $p_{2,A}$ is currently unknown for realistic EoS
- In our study we set $p_{2,A}$ to be constant.
- The $\langle\zeta\rangle$ is the volume averaged bulk viscosity. In our study we consider only matter with neutrons, protons, and electrons (no muons), as well we investigate the bulk viscosity driven by the direct Urca process.
- no modified Urca currently included in the investigation, since for inner core $\lambda_{npe}^{dUrca} \sim 10^8 T_9^{-2} \lambda_{npe}^{mUrca}$; e.g. $u = 4$ becomes $\lambda_{npe}^{dUrca} \sim 10^9 T_9^{-2} \lambda_{npe}^{mUrca}$.
- i.e. we investigate the bulk viscosity originated by the restoring action of direct Urca as a *potential source of dissipation* that could tidally heating up neutron stars in the inspiral

Average Bulk Viscosity - Considerations

- We define the volume average quantities as

$$\langle A \rangle = \frac{3}{R^3} \int_0^R A(r) r^2 dr, \tag{14}$$

for any function $A(r)$, i.e. for the bulk viscosity we obtain

$$\langle \zeta_{dUrca} \rangle \equiv \langle \zeta \rangle = \frac{3}{R_{dUrca}^3} \int_0^{R_{dUrca}} \zeta(r) r^2 dr, \tag{15}$$

- Because the dUrca operates only for kinematically allowed values of Fermi momenta (that corresponds to $x_p \geq 1/9$ for npe matter in beta-equilibrium), we numerically integrate up to a specific region inside the neutron star core, defined by the limit value of R_{dUrca}. **This cut-off depends on the specific EoS.**
- The $\langle \varepsilon \rangle$ is the average energy density of the star, dependent on the specific EoS.

Tidal lag time - Measurability from GW detectors

- The tidal lag time imprints a signature on the emitted gravitational waves signal of binary neutron star mergers.
- **Current measurability from advanced LIGO** $\tau_\mathrm{d} \geq 20\ \mu\mathrm{s}$ (Ripley, Hegade and Yunes PRD 108, 103037 (2023))
- There is **no correction** to the (conservative) Λ, which is measured by GW events.
- The contribution of τ_d to the GW phase is captured through a new quantity; the dimensionless **dissipative tidal deformability** Ξ.
- Ξ can be mapped through the viscosities of NS (bulk or/and shear viscosity).
- Potential measurable by the ground-based GW detectors (upper constraint of $\Xi \leq 1200$, see Ripley et al. Nature Astronomy 8, 1277 (2024)).
- New tool for examining neutron star matter.

Dissipative tidal deformability

- Ξ introduces viscous correction to the GW phase
- Phenomenologically expressed as

$$\Xi_A = \frac{2}{3}\frac{k_{2,A}}{C_A^6}\frac{c\tau_{d,A}}{R_A} = \Lambda_A \frac{c^3}{G}\frac{\tau_{d,A}}{m_A}, \tag{16}$$

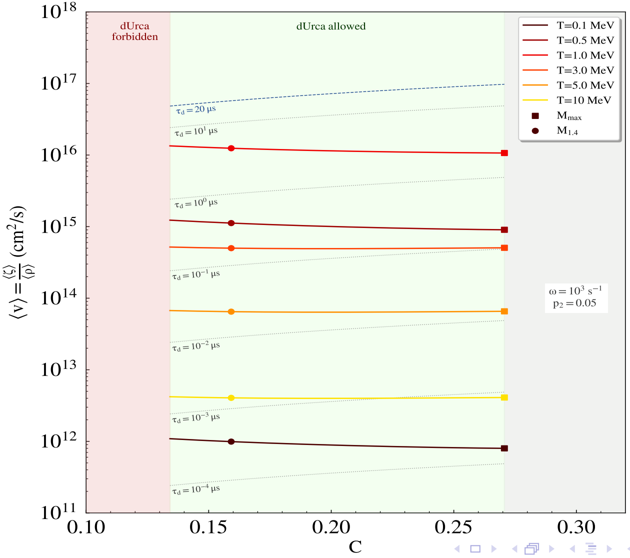
- Ξ depends on the 6th power of compactness and in general its dependence can be summarized to: Λ_A, $\tau_{d,A}$, and m_A.
- Effective dissipative tidal deformability of binary neutron star systems

$$\bar{\Xi} = f_1(\eta)\frac{\Xi_A + \Xi_B}{2} + g_1(\eta)\frac{\Xi_A - \Xi_B}{2}, \tag{17}$$

where $f_1(\eta) = 8(2\eta^2 - 4\eta + 1)$ and $g_1(\eta) = -8\sqrt{1-4\eta}(1-2\eta)$, with $\eta = m_A m_B/(m_A + m_B)^2$

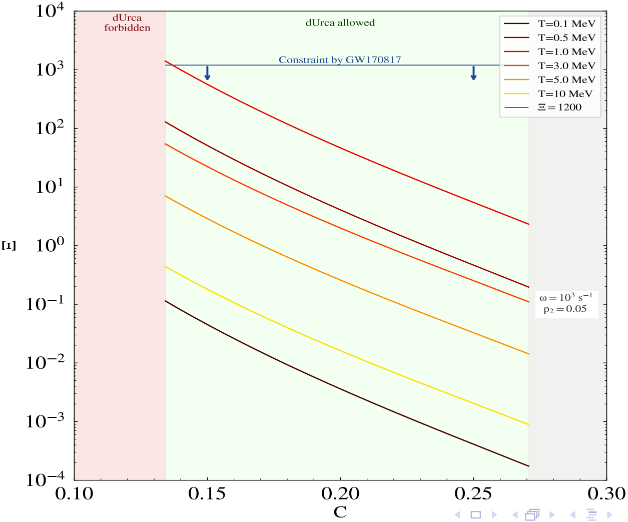
- $\tilde{\Lambda}$ and $\bar{\Xi}$ are both present in the gravitational waveform, i.e. measurable quantities.

Iso-contours of τ_d set paths for $\langle v \rangle$, EoS dependent (MDI-1).



Preliminary Results for Dissipative Tidal Deformation

Mapping the Ξ for various T across the allowed dUrca in the core
Orders of magnitude effect from Temperature



Remarks

- The volume-average bulk viscosity presents a resonant-like behavior related to the temperature T (originated by the corresponding behavior of npe matter bulk viscosity).
- Consequently, τ_d, $\langle v \rangle$, and Ξ share the same behavior on T.
- An observation of specific compactness (i.e. measured mass and radius of neutron stars), together with a potential measurement of τ_d can indicate where the neutron star "sits", providing an estimate for $\langle v \rangle$.
- Caution: degeneracy on Temperature; i.e. for each value of $\langle v \rangle$ corresponds two values of T.
- For the same value of τ_d the heavier neutron stars correspond to higher values of $\langle v \rangle$.
- Even with $p_{2,A} = 0.05$ there is a temperature $\mathrm{T} \approx 1 \ \mathrm{MeV}$ that can lead to values of τ_d close to the current detection threshold of 20 μs.
- Need for further upgrade on the terrestrial GW detectors;

Remarks

- Sensitivity from the EoS; Need to further study the potential impact of different EoSs.
- Effect of ω should be taken into account.
- Ξ is affected by orders of magnitude from T.
- For small T the values of Ξ are very small, i.e. the cold EoS produces almost negligible contribution of dissipative process to GW phase.
- The lighter neutron stars (small compactness) lead to higher contribution of tidal dissipative deformation, for fixed T.
- A measurement of a high enough Ξ could be used as a tool for finite temperature presence. Need to study more EoSs.
- Extra tool for investigate dense nuclear matter in neutron star environments.

Thank you for your attention!

Collaborators: Assist. Prof. dr. sc. P.S. Koliogiannis (University of Zagreb), and Prof. Dr. Ch.C. Moustakidis (AUTh)

This work is supported by the Croatian Science Foundation under the project Relativistic Nuclear Many-Body Theory in the Multimessenger Observation Era (HRZZ-IP-2022-10-7773) and by the European Union – NextGenerationEU through the National Recovery and Resilience Plan 2021-2026 – Institutional grant of University of Zagreb Faculty of Science (Nuclear Astrophysics).