# Reconstructing Compact Objects Equation of State (EoS) via Machine and Deep Learning

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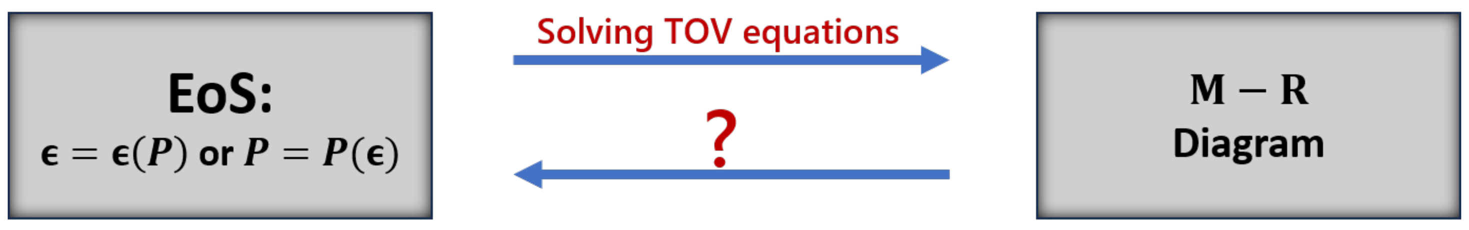
## Introduction

The TOV equations can be considered as a process, which correlates an equation of state (EoS) of a Compact Star to a $M-R$ diagram. The **forward process** involves establishing the relation between energy density and pressure, through an EoS model, and then proceed to solve TOV equations and generate a $M-R$ diagram. A **backward (inverse) process**, that allows one to obtain the EoS from a $M-R$ curve, has proved extremely difficult to find. In our study, we aim to develop such a machine or deep learning model, that returns data of the equation of state, when data from the respective $M-R$ graph are given, successfully posing as this inverse process.




## Data Generation I - Hadronic Stars

In order to address the problem thoroughly, we need a systematic and dense coverage, both of the $P-\epsilon$ and $M-R$ spaces. To do so, for Hadronic Stars, we incorporate the methodology of mock polytropic EoSs, with some adjustments to ensure zero causality violation. More specifically, we construct EoSs with the following form:

$$\epsilon_{hadronic}(P) = \begin{cases} \epsilon_{crust}(P), & P < P_{crust-core} \\ \epsilon_{main}(P), & P_{crust-core} \leq P < P_0 \\ \epsilon_{poly}(P), & P_0 \leq P \leq P_{tr} \\ \epsilon_{poly}(P_{tr}) + P - P_{tr}, & P_{tr} < P \leq P_n \end{cases} \quad (1)$$

where:

- $\epsilon_{crust}$: EoS of the crust (4 layers),
- $\epsilon_{main}$: either the **HLPS-2** or **HLPS-3**, as an intermediate EoS,
- $\epsilon_{poly}$: the interval $[P_0, P_n]$ is divided into $n$ segments, each of which is governed by a distinct polytropic EoS: $P = K\rho^{\Gamma}$. The values of $\Gamma$ are arbitrarily chosen to be: $\{1, 2, 3, 4\}$,
- $\epsilon_{lin}$: if causality is violated at some pressure $P_{tr} < P_n$, the EoS shifts from polytropic to linear behavior via *Maxwell* transition, with fixed slope at $c_s/c = 1$ or $(c_s/c)^{-2} = d\epsilon/dP = 1$.

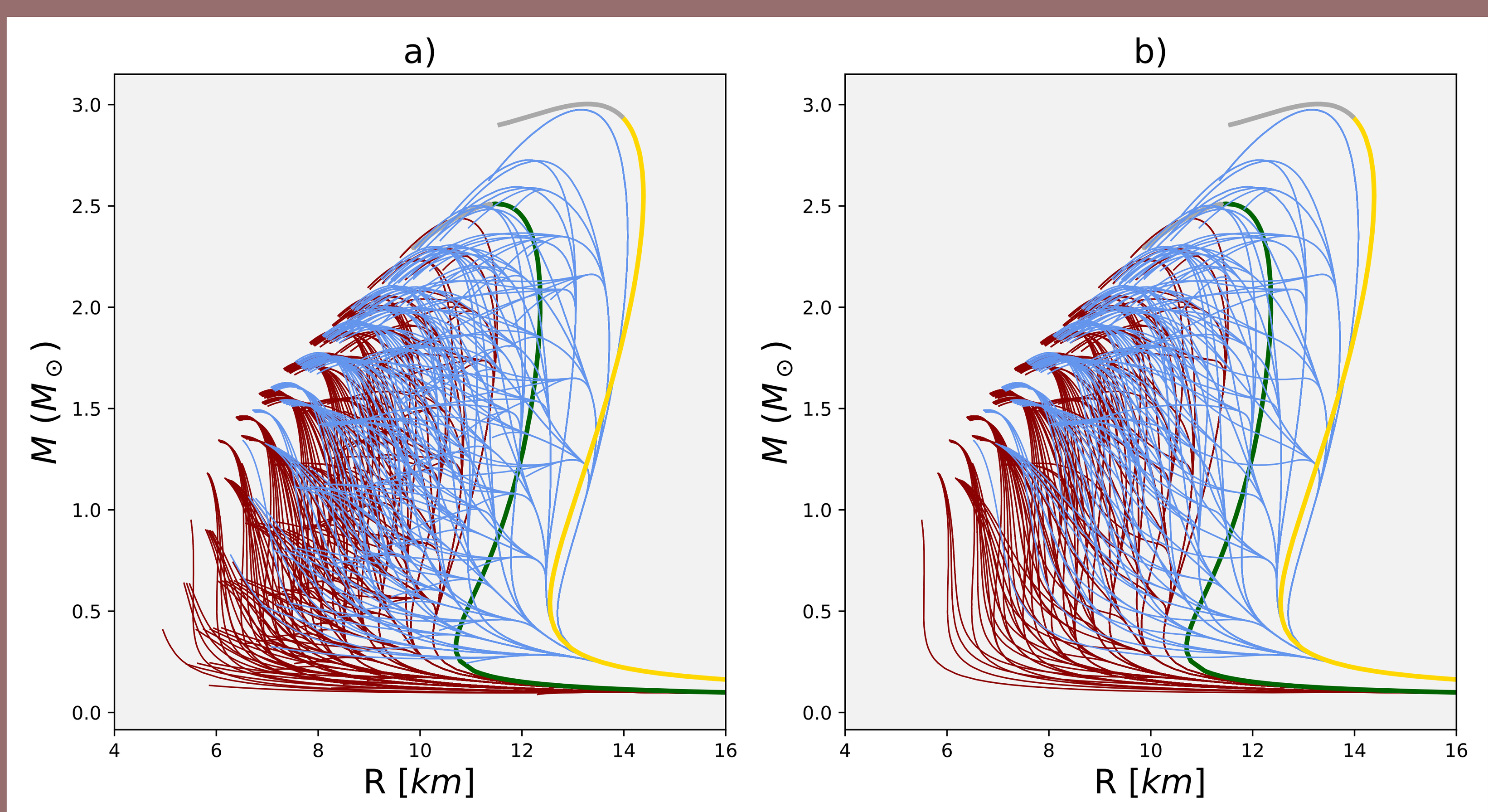



**Figure 1:** ]Plots of the $M-R$ curves of mock Neutron Stars EoSs. a) The $M-R$ curves of 512 mock EoSs, derived from HLPS-2 and HLPS-3 'main' EoSs, for all $\Gamma$ combinations in 4 mass density segments. b) The $M-R$ curves of the 304 out of the 512 mock EoSs, exceeding the pressure of 850 $MeV \cdot fm^{-3}$. The $M-R$ curves of HLPS-2 (green) and HLPS-3 (yellow), are also included, with gray endings marking the violation of causality.

## Data Generation II - Quark Stars

Regarding Quark Stars, we employed two different models of confined strange quark matter:

- **MIT-bag model**: $\epsilon = 3P + 4B$, where $B$ is the so-called "bag constant", having pressure dimensions ($MeV \cdot fm^{-3}$).
- **CFL model**: $\epsilon = 3P + 4B - \frac{9\alpha\mu^2}{(\hbar c)^3\pi^2}$. In addition to the bag constant $B$, the gap parameter $\Delta$ is introduced (with energy dimensions $MeV$) via the $\alpha$ constant, in order to describe the superconductive **Color-Flavor-Locked** (**CFL**) phase of strange quark matter at high densities. In this phase, quarks of all three flavors and colors form *Cooper* pairs and have the same *Fermi* momentum.

We scan the $M-R$ space, by allowing the parameters $B$ and $\Delta$ taking values across a fairly large interval $[60, 250]$ and by using a step of 0.5 $MeV \cdot fm^{-3}$ (MIT-bag) or the steps 5 $MeV \cdot fm^{-3}$ and 10 $MeV$ (CFL), respectively. The results are shown in Figure 2.

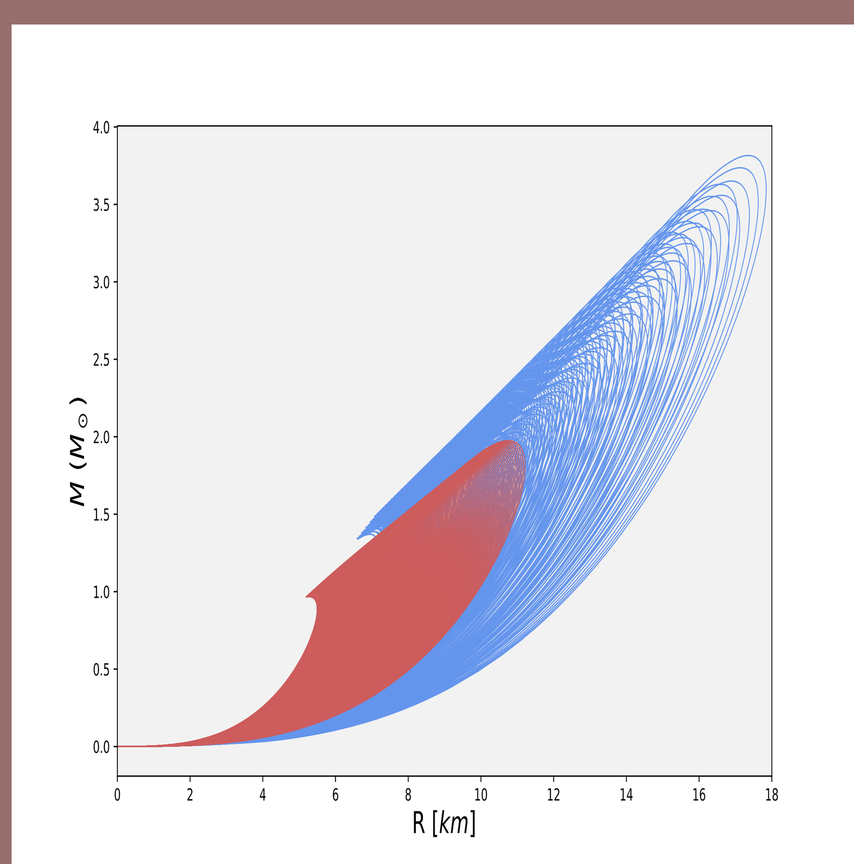
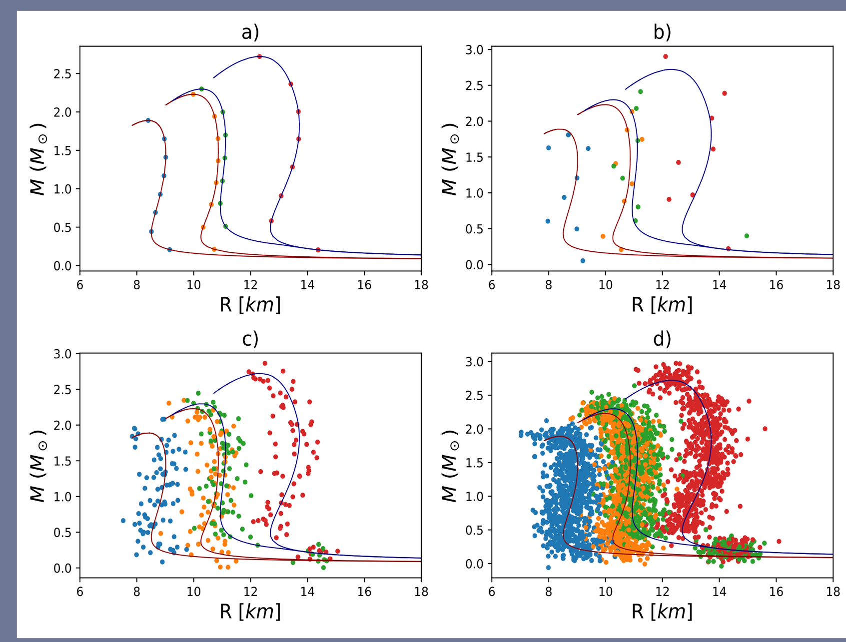



**Figure 2:** Plots of the $M-R$ curves of Quark Stars EoSs. The 381 $M-R$ curves of MIT bag model EoSs are depicted with red color, while the 510 $M-R$ curves of CFL model EoSs are depicted with blue color. In total we get 891 curves scanning the $M-R$ space.

## Data Preparation

- **Sampling X data:**

**Figure 3:** Sampling example of mass and radius data, using 8 points from the M-R curves of four mock polytropic EoSs. The respective M-R curves are plotted too. The graphs depict: a) the noise-free basic observation of M-R points for each EoS, b) 1 random M-R observation per EoS, c) 10 random M-R observations per EoS and d) 100 random M-R observations per EoS. Each random observation is produced by adding normal distributed observational noise: $\Delta M \sim 0.1 M_{\odot}$ and $\Delta R \sim 0.5 km$, to the basic observation.




- **Sampling Y data:** We collected energy density $\epsilon$ values at 12 pressure $P$ points.
- **Row-wise shuffling** of the $M-R$ points to eliminate correlations in X data.
- **Shuffling** of EoSs' observations to prevent data leakage.

## Machine and Deep Learning Regression Algorithms

- We investigated the performance of four "tree-based" machine learning algorithms, for our regression purposes: **Decision Tree**, **Random Forest**, **Gradient Boosting XGBoost**.
- We also built **Deep Neural Networks** models, with three hidden layers, and compared their performance to the machine learning ones.
- We combined the modern techniques of **Cross-Validation (5-fold)** and **Grid Search** over the hyperparameters, in order to fine-tune our models and boost their accuracy.
- We used the **MSLE** and **MSE** metrics as assessment criteria.

## General Results

- The results from the training of our regression models are presented below:

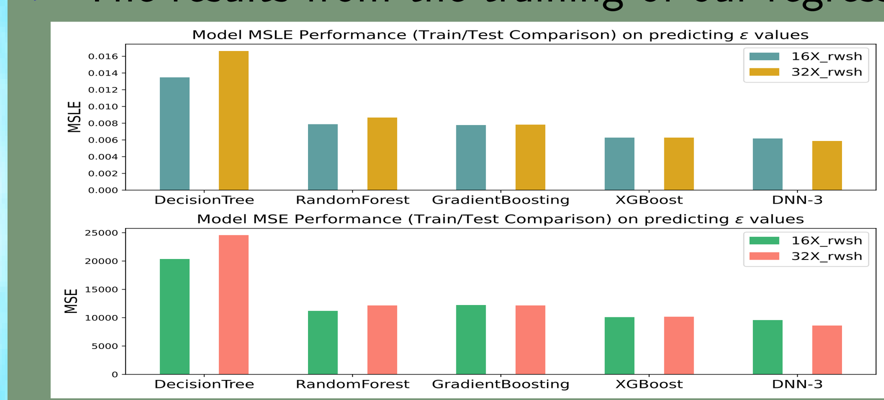



**Figure 4:** MSLE and MSE results from training on Hadronic Stars Data

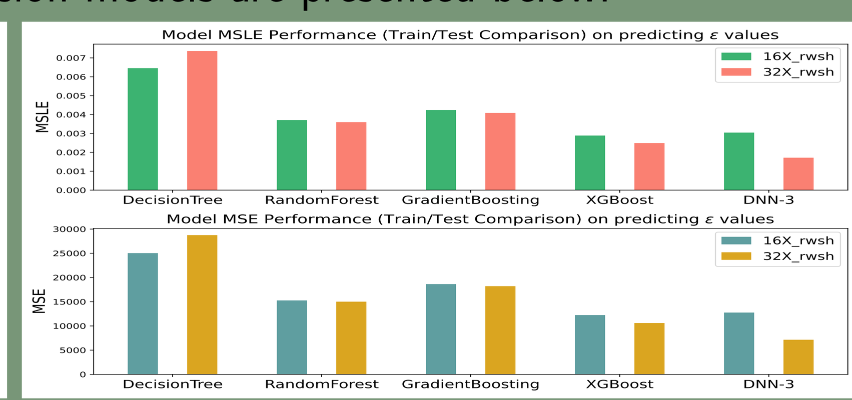



**Figure 5:** MSLE and MSE results from training on Quark Stars Data

## Reconstruction I - Hadronic EoSs

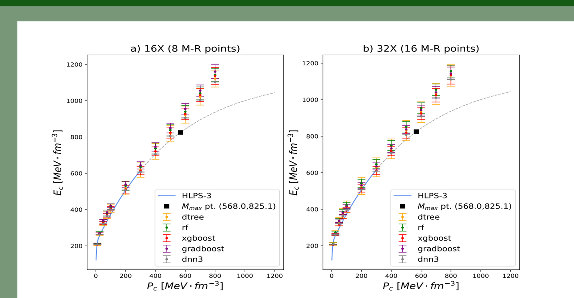


**Figure 6:** HLPS-3 EoS

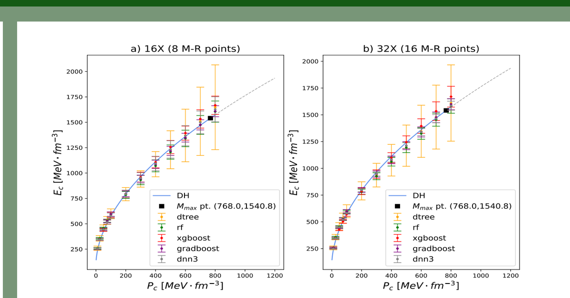


**Figure 7:** DH EoS

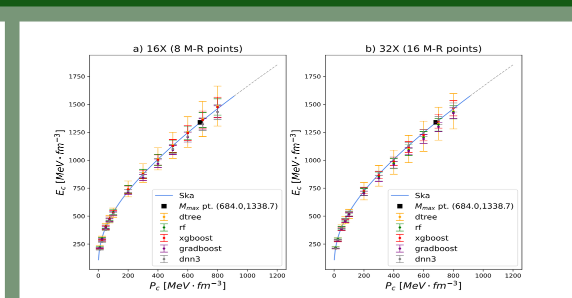


**Figure 8:** Ska EoS

## Reconstruction II - Quark Matter EoSs

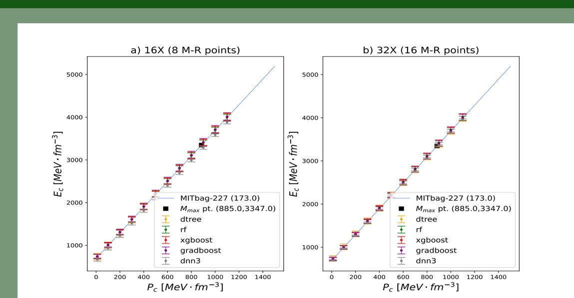


**Figure 9:** MITbag-227 EoS

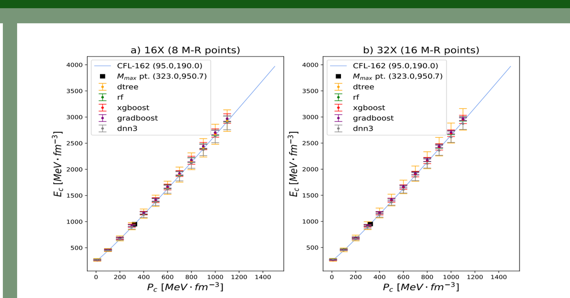


**Figure 10:** CFL-162 EoS

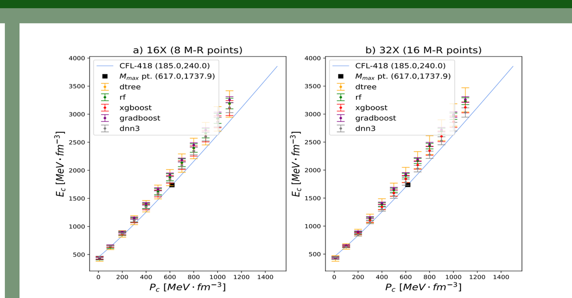


**Figure 11:** CFL-418 EoS

## Conclusions/Future Work

- **Algorithms:** *Decision Trees* exhibit the highest error and variance. *XGBoost* and Deep Neural Network models consistently achieve the highest accuracy, while the ensemble tree-methods *Random Forest* and *Gradient Boosting* provide stable intermediate performance.
- **8 vs 16 points:** Using 8 points is generally sufficient, as 16 points offer negligible improvement and can occasionally degrade algorithmic performance. This indicates that Equations of State can be effectively reconstructed using less data from the M-R curve.
- **Variance and Reliability:** Reconstructions are highly accurate at low pressures and throughout the stable branch, though variance increases at high densities and causality-violating regions. This trend underscores the inherent physical uncertainty and algorithmic sensitivity at extreme mass densities.
- **Improvements:** Future work would focus on denser sampling around the maximum mass point, investigating fewer M-R points, and identifying new features to enhance EoS capture. Efficiency could be improved by better tuning of hyperparameters, deploying advanced ML/DL architectures, and predicting the maximum mass point to distinguish between stable and unstable star configurations.

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