Neutron spectrum deconvolution by Bayesian Methods

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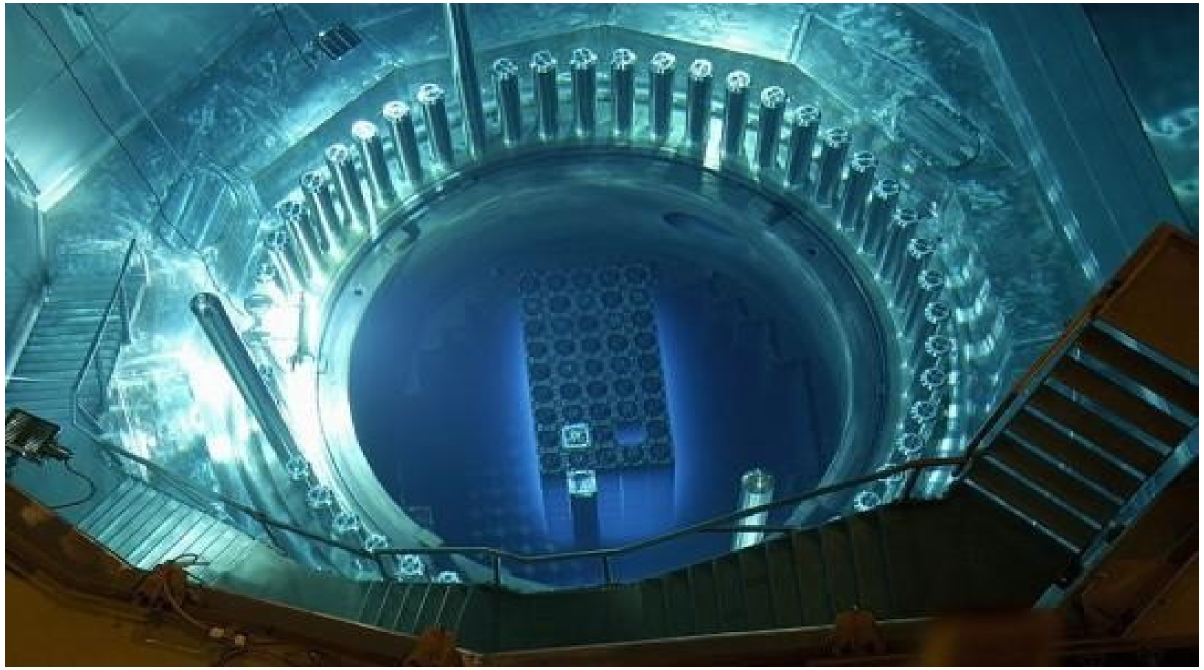


Motivation

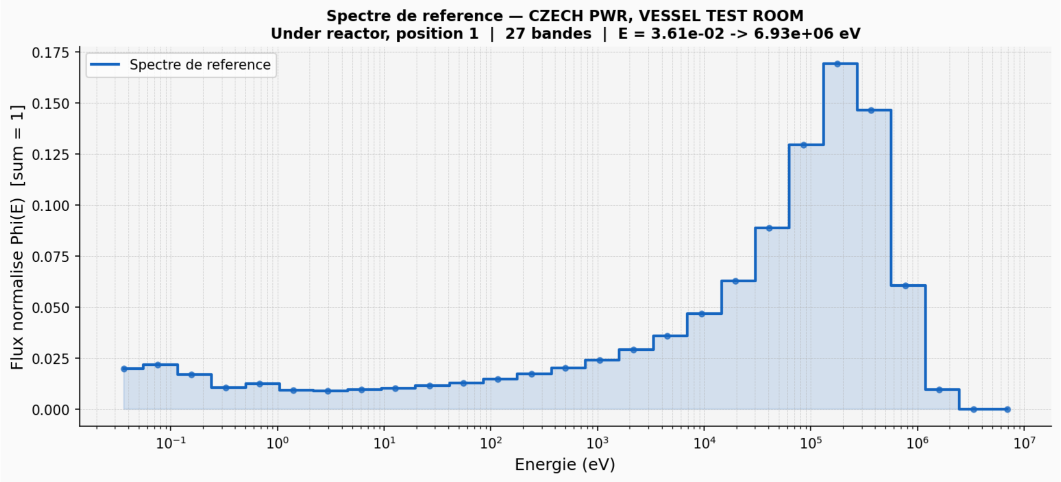
Nuclear reactor diagnostics

- Neutron spectrum is important for reactor dosimetry and diagnostics (e.g. how fuel evolves over burnup, absorption in U and structural materials etc)
- Measurement by activation foils/fission chambers

EPR, Flamanville, France (Gen III+, 1300MWe)

Challenges

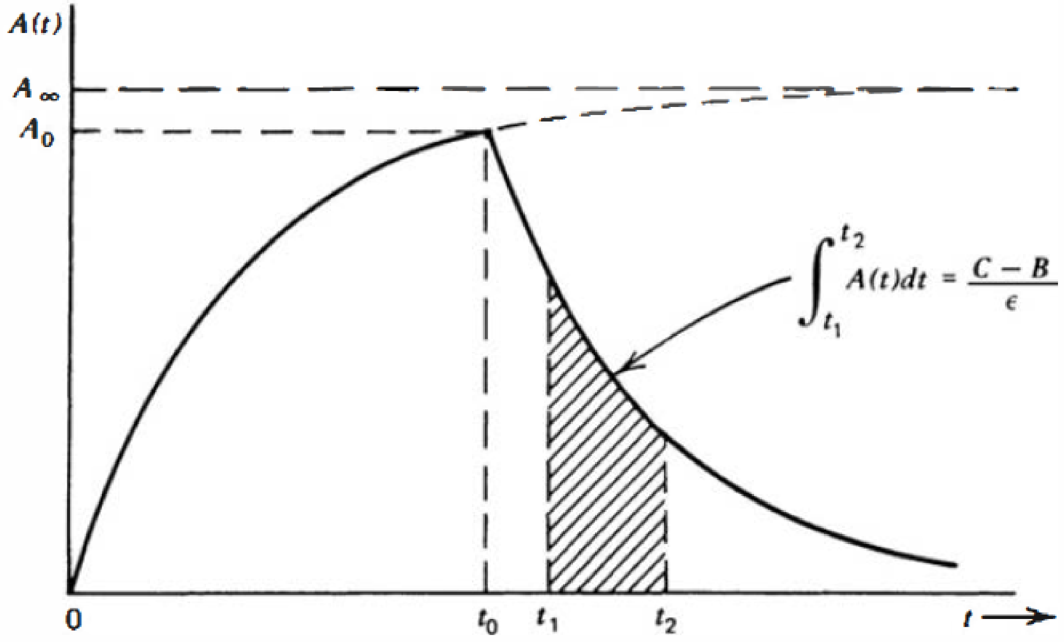
- Deconvolution problem is ill-posed
  - multiple solutions which can be non physical
- Methods rely on a-priori knowledge of the spectrum (MAXED, GRAVEL)



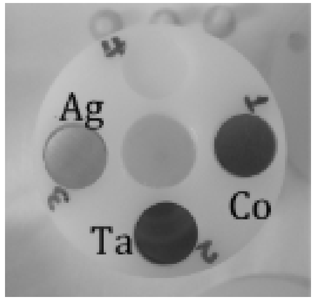
Methodology

The activation method

- The saturated activity is directly related to the neutron flux incident on the foil
- The number of decays can be experimentally measured by gamma spectroscopy
- Used in intense fields, mostly blind in gamma rays
- Indirect method (off line)



✓ Using different dosimeters we can construct a linear system. The solution of the inverse problem will yield the neutron spectrum



$$A_\infty = R = \varphi \Sigma_{act} V$$

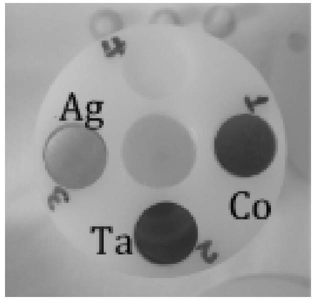
$$A_\infty = \frac{\lambda(C-B)}{\epsilon(1-e^{-\lambda t_0})e^{\lambda t_0}(e^{-\lambda t_1} - e^{-\lambda t_2})}$$

G. Knoll, Radiation detection and measurement, 2010

Methodology

Neutron spectrum deconvolution

- Requires solving an ill-posed inverse problem
- Most probable solution not always physical (needs a level of smoothness and is constrained to only-positive values)



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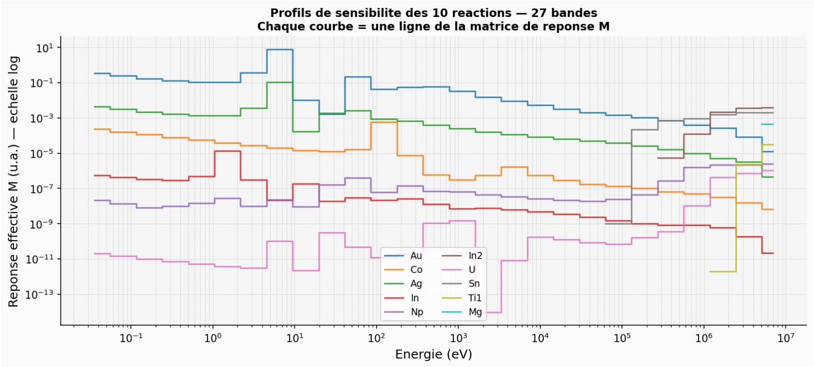
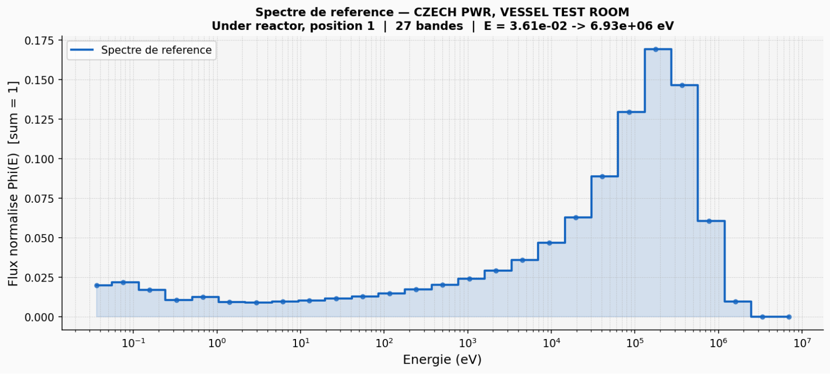


x



Activities

Response matrix
(cross sections)

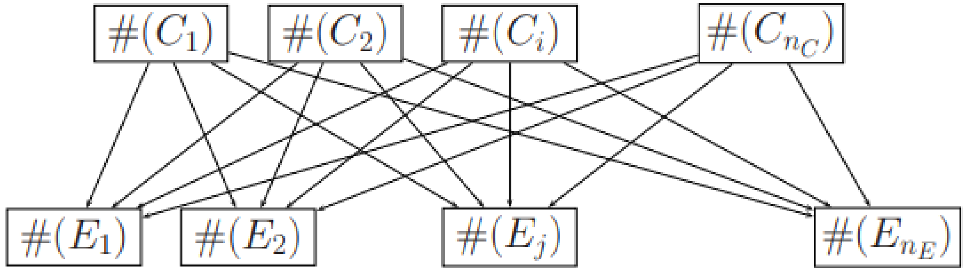
Neutron flux

- Ill-posed problem : $\dim(\Phi) = 27 > \dim(A) = 10 \rightarrow$ not a unique solution
- Use of particular minimisation algorithms requiring a reference spectrum (MAXED, GRAVEL etc.)
- **Can we bypass the requirement of a reference spectrum ? And what would be the reconstruction performance ?**

Methodology

D’Agostini Iterative Bayesian Unfolding

- Iterative method
- Uses Bayes theorem and applied a correction to the initial spectrum
- The problem of initial prior can be overcome by the iterative procedure
- Widely used in HEP



How it functions ?

- Views the problem as causes (neutron spectrum bins) and effects (activities)
- What is the probability that a cause comes from any effect ?

$$P^{(k)}(C_j \mid E_i) = \frac{P(E_i \mid C_j)\, P^{(k)}(C_j)}{\sum_\ell P(E_i \mid C_\ell)\, P^{(k)}(C_\ell)}$$

→

$$x_j^{(k+1)} = \frac{1}{\varepsilon_j} \sum_i \left[\frac{P(E_i \mid C_j)\, P^{(k)}(C_j)}{\sum_\ell P(E_i \mid C_\ell)\, P^{(k)}(C_\ell)} \right] y_i$$

Element ij of the response matrix

Prior

Efficiency – What fraction of events do we observe ?

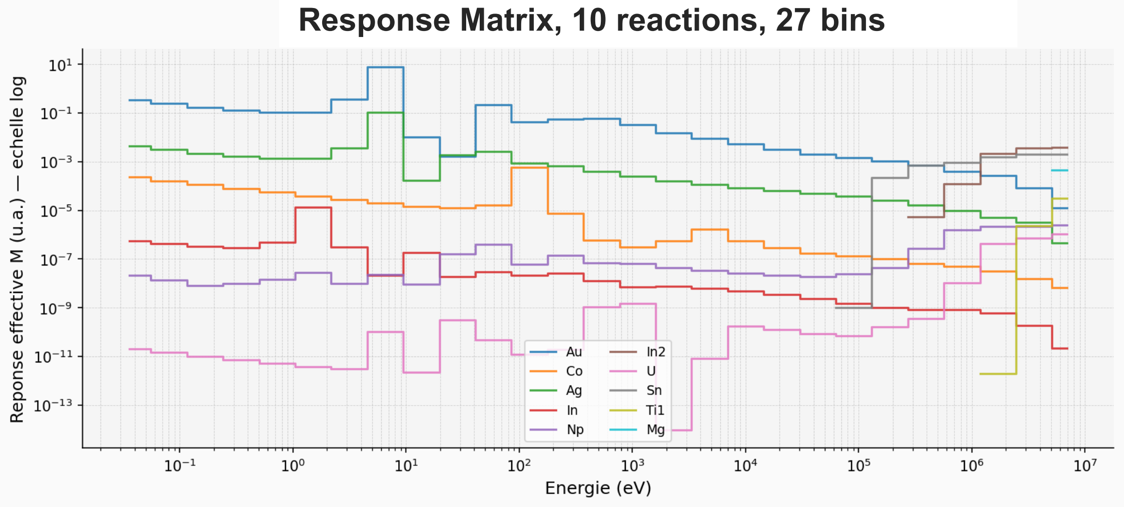
$$\sum_i P(E_i \mid C_j) = \varepsilon_j$$

G. D’Agostini, NIMA 362 487 (1995)
G. D'Agostini, arXiv:1010.0632 (2018)

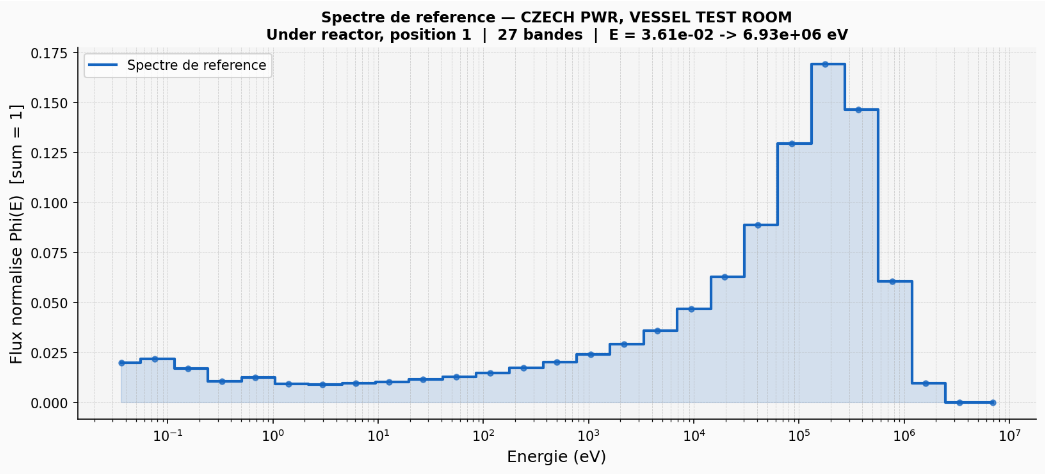
Response Matrix construction

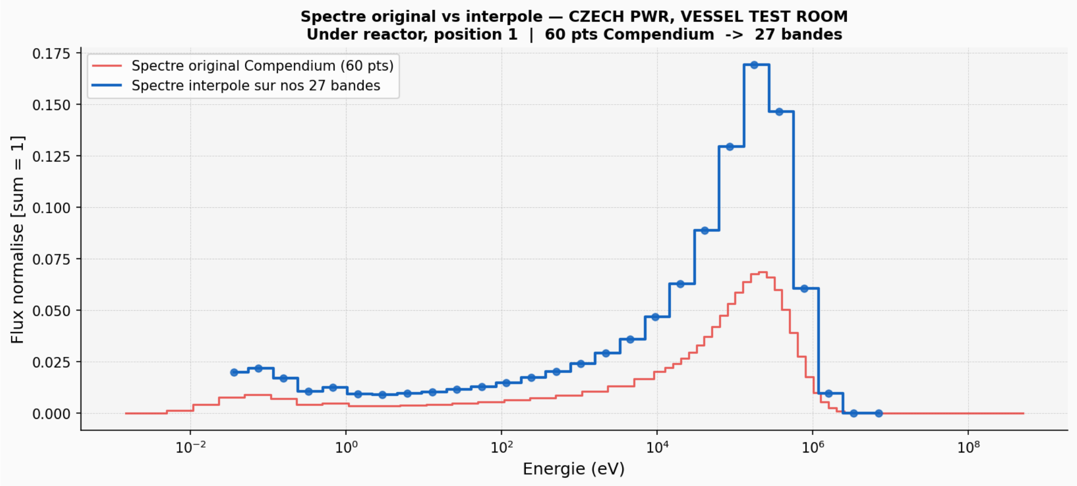
Reactions

Material	Reaction	Energy range	Half life
Au	$^{197}Au(n,\gamma)^{198}Au$	thermal + epithermal	2,7 d
Co	$^{59}Co(n,\gamma)^{60}Co$	thermal + epithermal	5,27 y
Ag	$^{109}Ag(n,\gamma)^{110m}Ag$	thermal + epithermal	249,8 d
In	$^{115}In(n,\gamma)^{116m}In$	thermal + epithermal	54,3 min
Np	$^{237}Np(n,f)^{137}Cs$	≥ 0,5 MeV	30 y
In	$^{115}In(n,n')^{115m}In$	≥ 1,1 MeV	4,5 h
U	$^{238}U(n,f)^{137}Cs$	≥ 1,4 MeV	30 y
Zn	$^{64}Zn(n,p)^{64}Cu$	≥ 2,8 MeV	12,7 h
Ti	$^{46}Ti(n,p)^{46}Sc$	≥ 3,7 MeV	83,8 d
Mg	$^{24}Mg(n,p)^{24}Na$	≥ 6,5 MeV	15 h



Test spectra



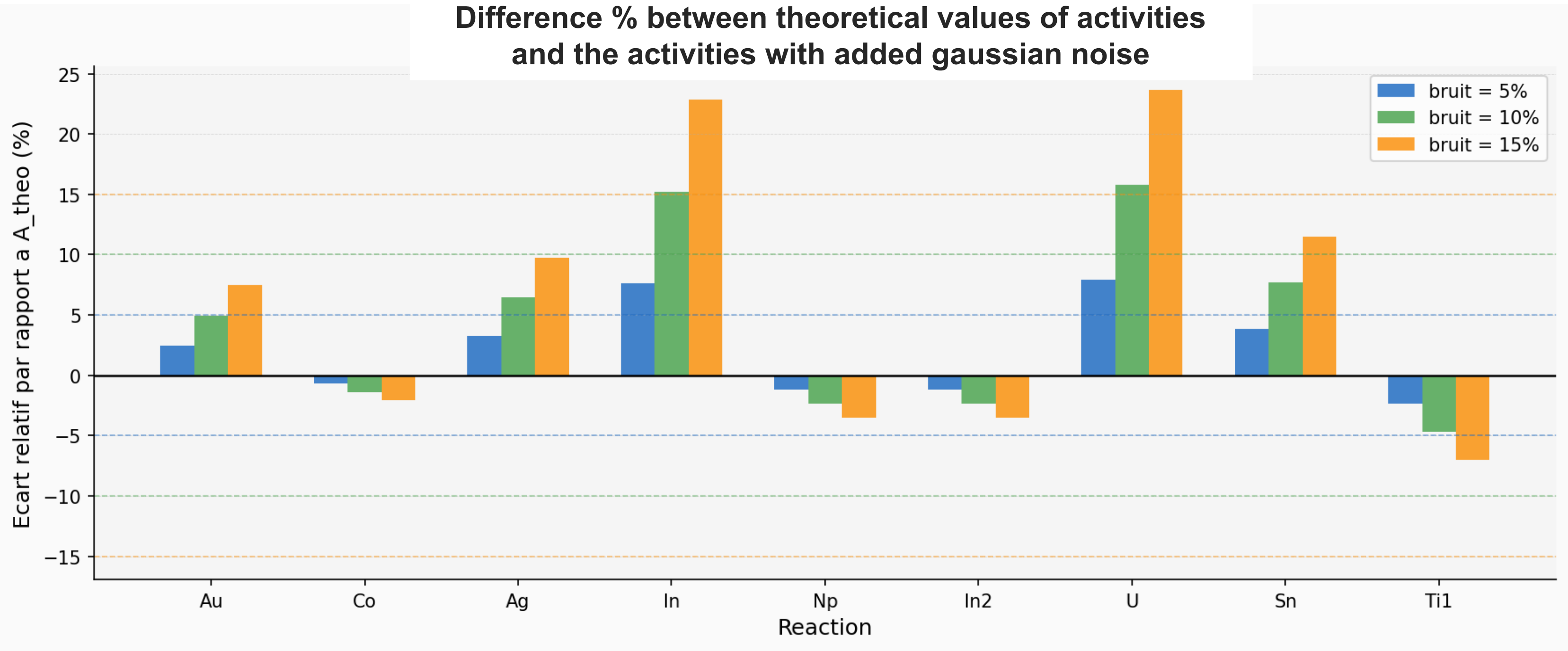


IAEA Compendium database TRS-403 - International reference for measured neutron spectra

- **Example :** Czech PWR, Vessel Test Room → *Under reactor, position 1*
- **Initial spectrum binning at 60 bins -** Interpolation for 27 bandes in logarithmic scale (25 meV → 10 MeV)
- **Used as the true spectrum to evaluate the reconstruction**

Activities

- In order to create a dataset for activities, we will solve the direct problem (true spectrum multiplied by the response matrix) and then we will introduce a gaussian noise
- Noise is added to approximate experimental conditions
- Total flux will be taken as 10^6 neutrons/cm²/s (zero power reactor)



Results

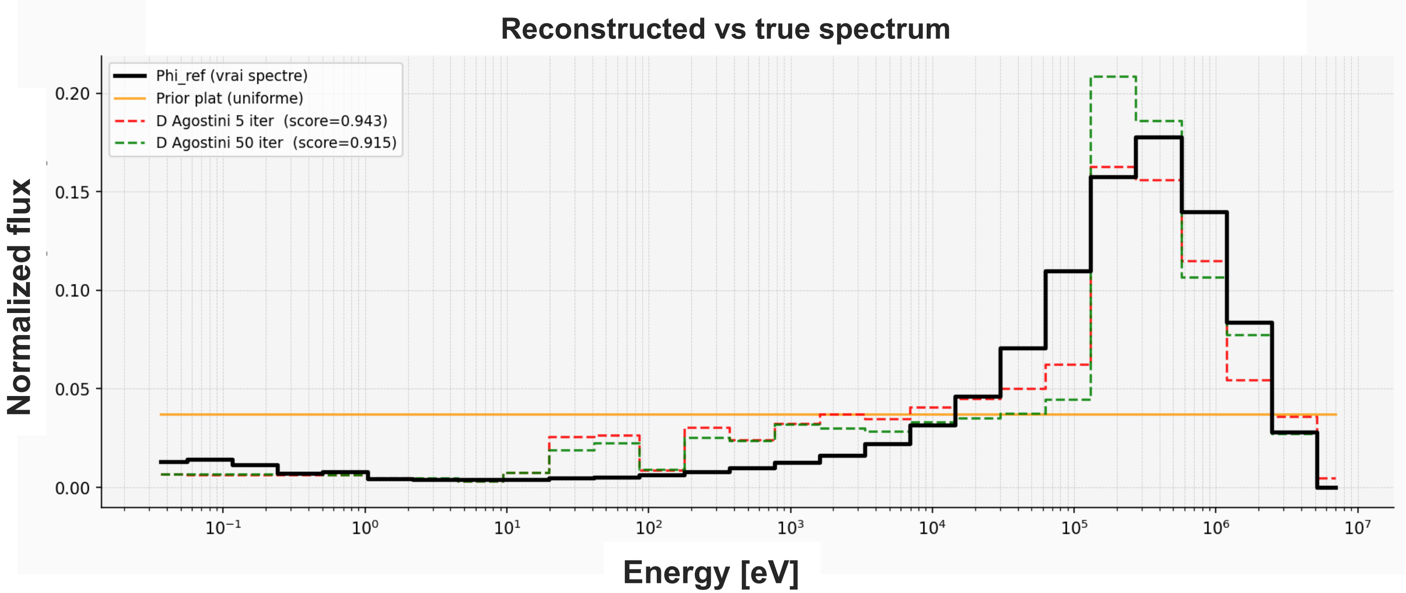
Spectrum : Czech PWR Reactor Hall, Platform of reactor cap, 27 bins

Metris	valeur
Cosine similarity	0.966
R^2	0.889
Combined score = 0.7*Cos. Sim. + 0.3*R^2	0.943

Cosine similarity 0.966 → Shape well reproduced

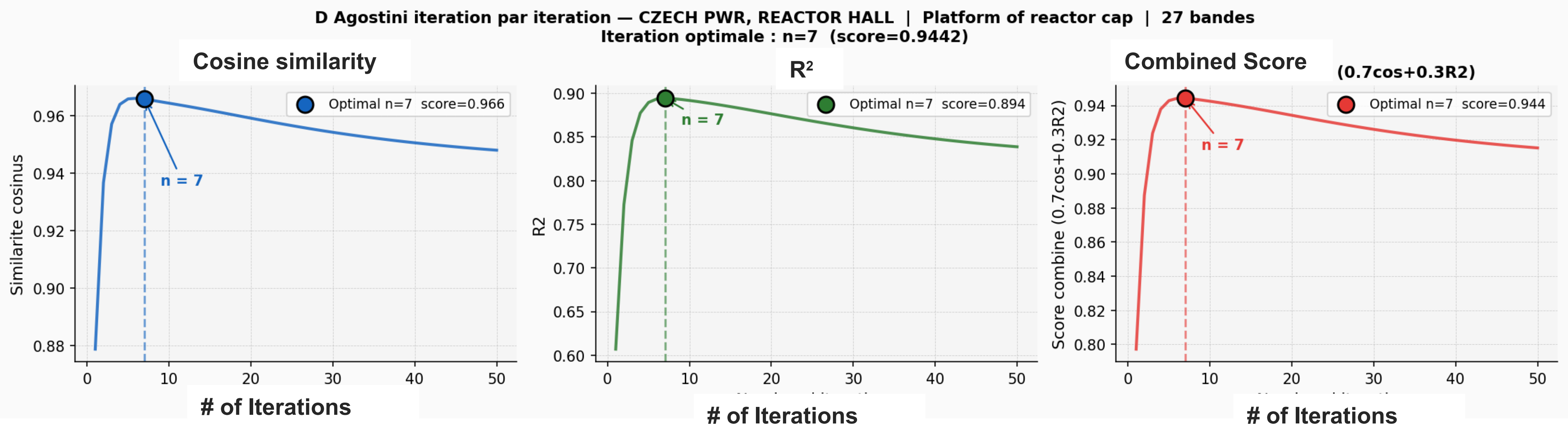
R^2 0.889 → Deviation satisfactory

Score 0.943 → Good overall reproduction



Results

Effect of Iterations



- Best performance in the first 10 iterations
- Noisy results after a large number of iterations (>50)

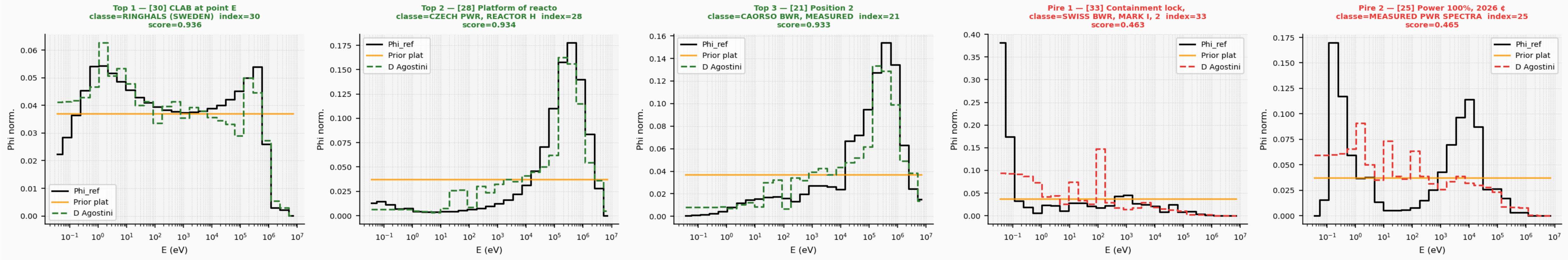
Results

Global Test for reactor spectra : 63 spectra from the IAEA Compendium database (classes 21-35 : PWR, BWR, gas cooled reactors)

Metric	Value
Mean Combined score	0.756
Mean Cosine Similarity	0.868
Mean R^2	0.496

- **41% of the spectra are well reproduced, having a combined score ≥ 0.8**
- **90% of the spectra have an acceptable value for the combined score (≥ 0.6)**

D'Agostini, 5 iterations – 27 bins



Conclusions

- Overall good reproduction of spectra even by starting without information (flat spectrum)
- Performance acceptable for the majority of reactor spectra in the compendium database
- Difficulty in reproducing spectra with high variation/slopes

Future Directions

- Testing of other methods (Fully Bayesian Unfolding, MCMC)
- Testing of different priors + comparison with classical unfolding methods
- Effect of noise in activities – How much uncertainty is acceptable for a good reproduction ?
- Optimisation/Effect of binning

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Internship at Nuclear Measurement Laboratory (Hanine Rhimi)