Semileptonic Heavy Decays

Aurelia Zuchanke

University of Liverpool

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Intro

Background

- Motivation
- CKM matrix
- Semileptonic Decays

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- Semileptonic Decays

Our work

- Functional forms
- Regulated Laplace Filter
- Fitting and averaging

Motivation

- Want to find Standard Model breakdown
- Energy frontier a while away
- Precision indirect tests
- Want processes with precise theory and experiment
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$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ c \end{bmatrix} \tag{1}$$

Features of CKM

- CKM Matrix strictly unitary in SM
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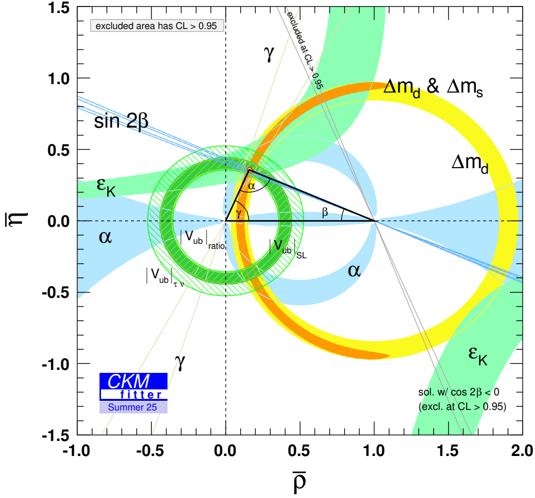
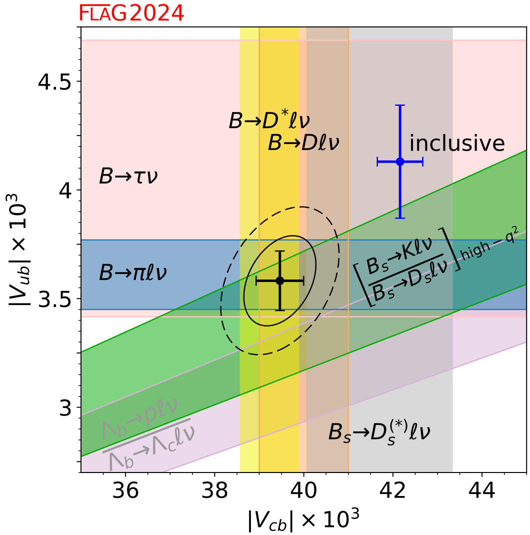
- Tension between inclusive and exclusive decay
- Potentially BSM contribution
- Sister project focused on inclusive decay

Unitarity Triangle



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

Inclusive VS Exclusive

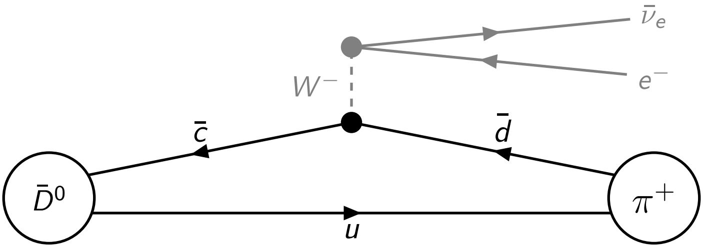


Taken from FLAG 2024

Determining CKM

- Requires input from experiment and theory
- Semileptonic decays
- Sufficiently simple to be effectively modelled on lattice
- Frequently appear in experimental data set
- Allow determination of non perturbative form factors
- Combine with experimental measure to get CKM elements

Semileptonic Decays



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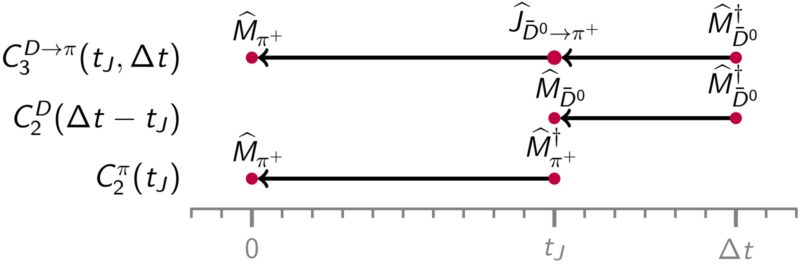

Functional Forms

$$C_2(t) = \sum_n \frac{A_n A_n^*}{2E_n} e^{-E_n t} \tag{3}$$

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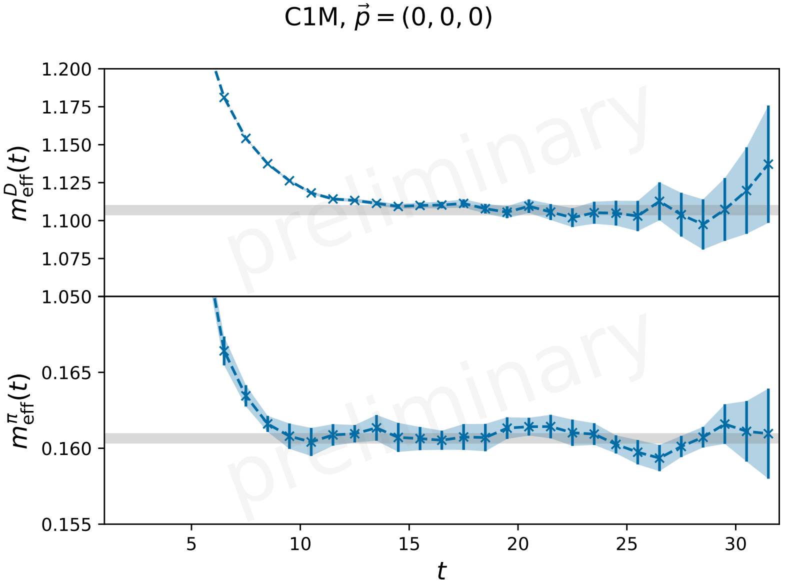
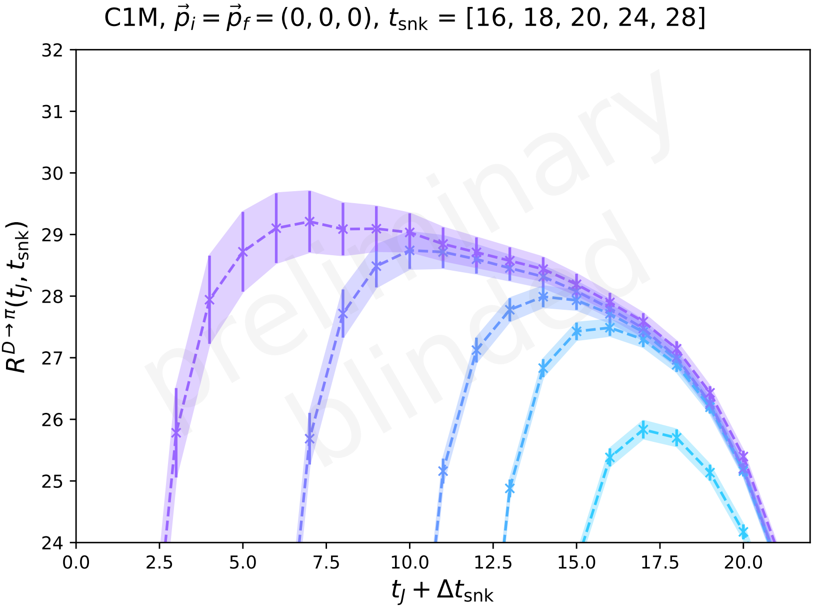
$$\frac{C_3(t_J, \Delta t)}{C^I_2(\Delta t - t_J) C^F_2(t_J)} = \frac{A^J_{00}}{A^I_0 A^F_0} + \dots \tag{6}$$

Two-Point Function



Three-Point Function Ratio



Laplace Filter Recap

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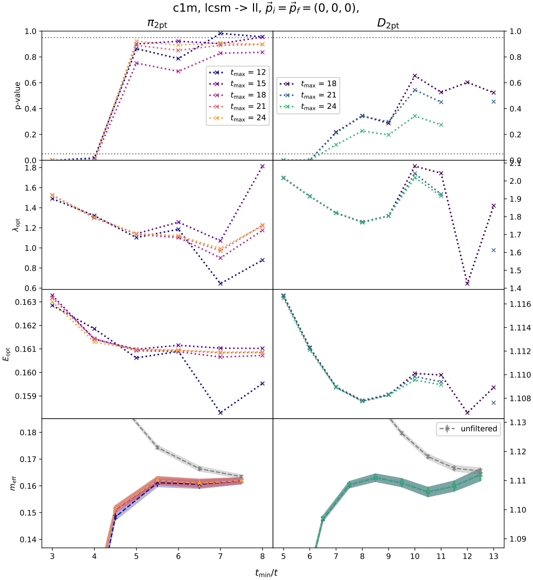
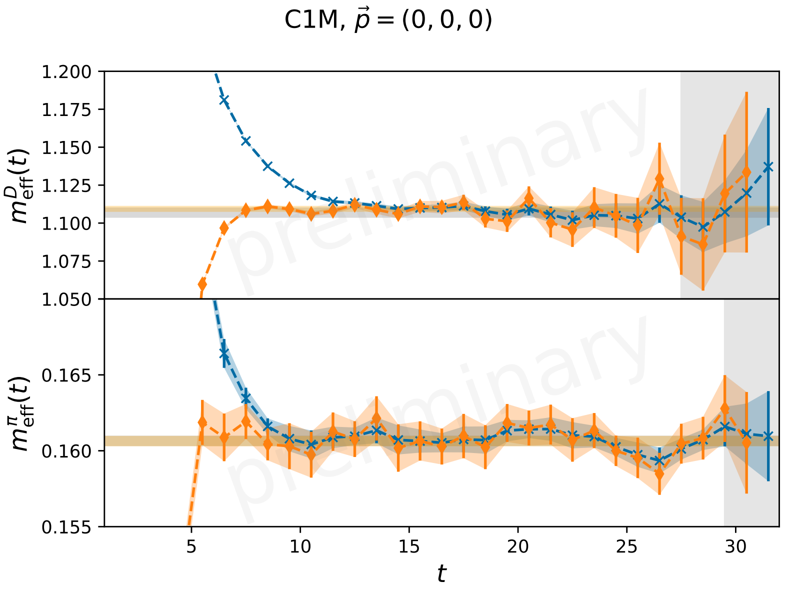
$$D_\lambda \sum_n a_n e^{-E_n t} = \sum_n a_n(\lambda^2 - \tilde{E}_n{}^2)e^{-E_n t} \tag{10}$$

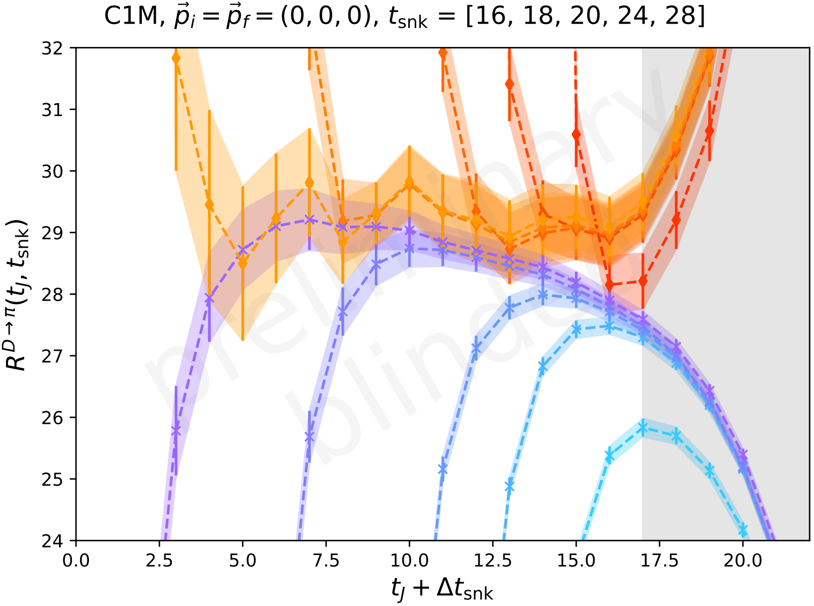
Filter Parameter Optimisation



Two-Point Filtered



Three-Point Function Ratio Filtered

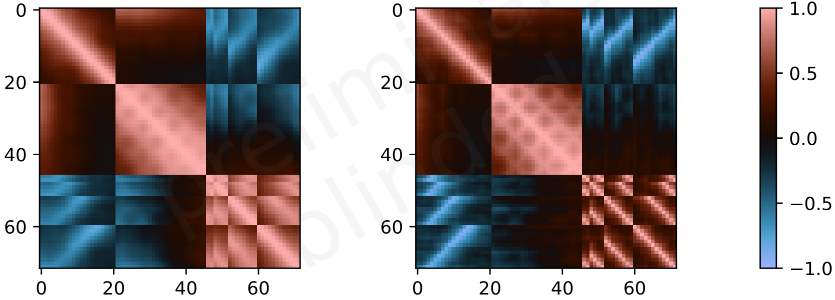


Fitting

- Need to actually fit data
- Goal is Laplace ground state vs two state
- Data sets not independent
- Need to do joint fit
- The more fit simultaneously the better
- Two-point and three-point, but also different momenta etcetera
- Potential poor conditioning
- Laplace filter might help

Correlation Matrix

C1M, $\vec{p}_i = \vec{p}_f = (0, 0, 0)$, t_{snk} = [16, 18, 20, 24, 28]



Combining Fits

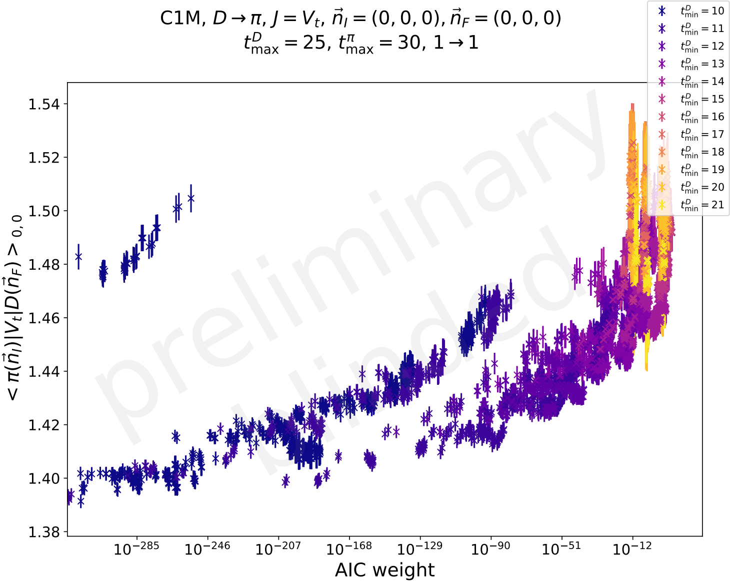
- Produce a large range of p-value "acceptable" fits
- Want do produce a single value
- Also want to quantify systematic error from choice
- Akaike information criterion
- Use relative weighting for "acceptable" fits

$$\overline{E_0} = \frac{\sum wE_0}{\sum w} \tag{11}$$

$$w \propto e^{-\frac{1}{2}\mathrm{AIC}} \tag{12}$$

$$\propto e^{-\frac{1}{2}(\chi^2 - 2N_{\mathrm{DoF}} + \{1|2\}N_{\mathrm{fit\ params}})} \tag{13}$$

Preliminary fitting



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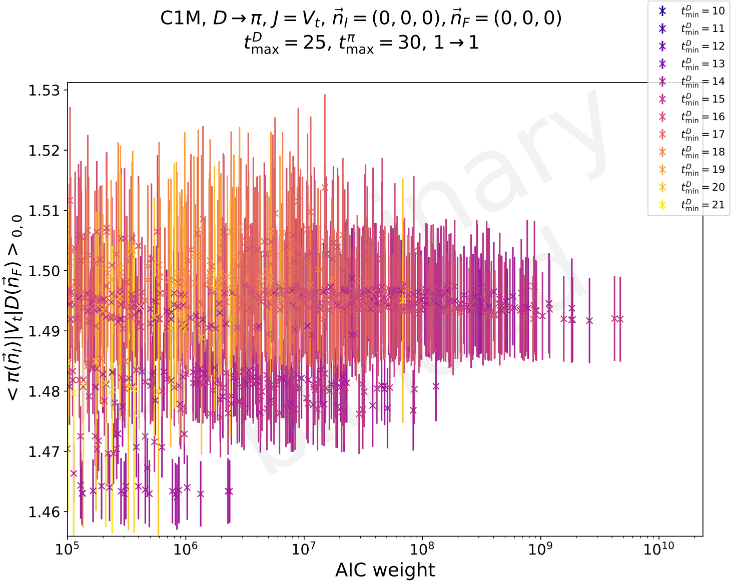

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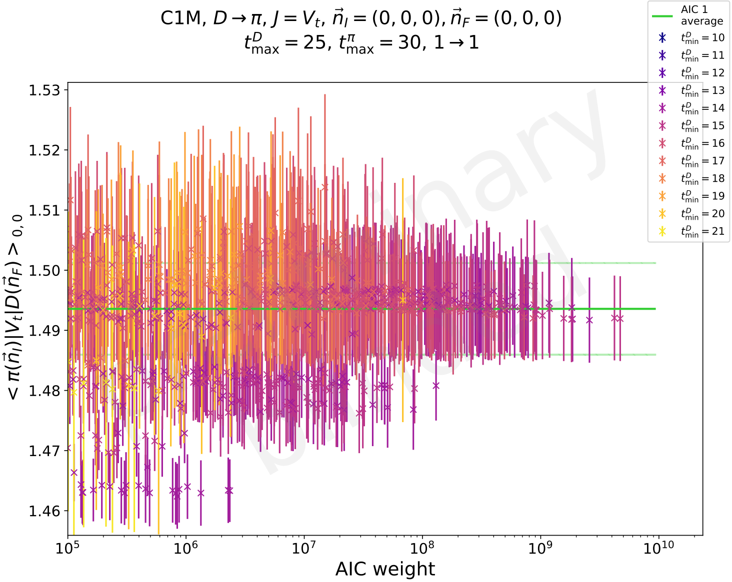

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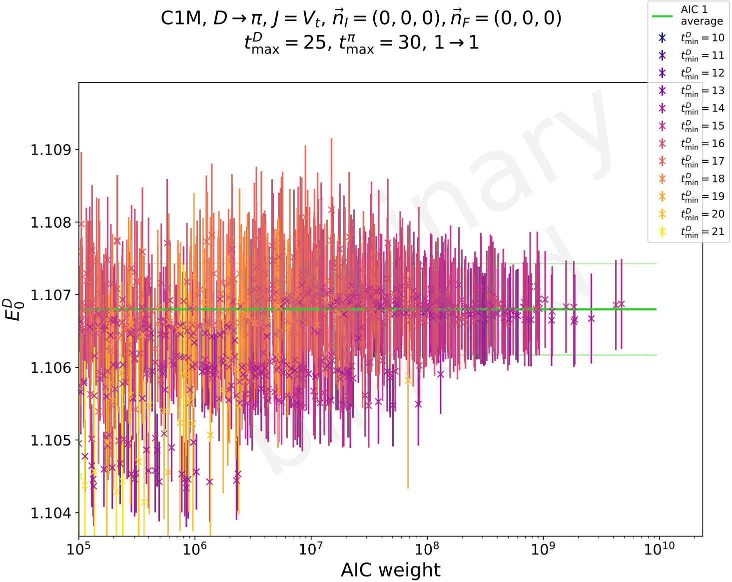

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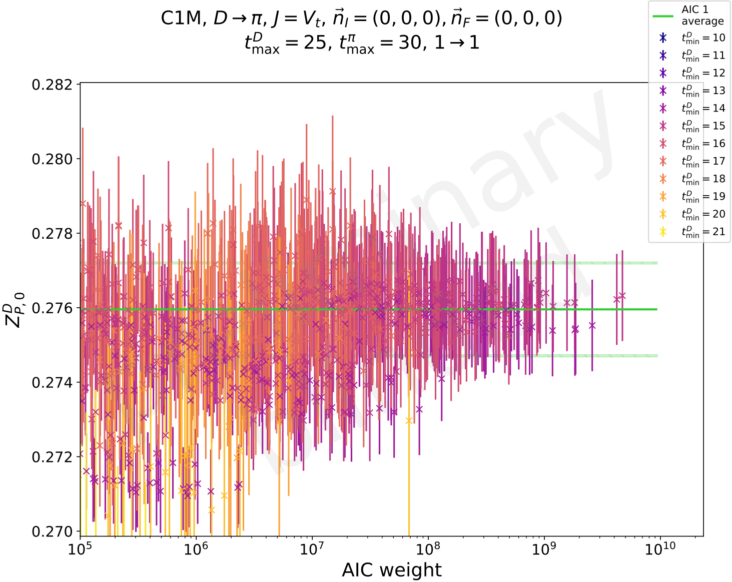

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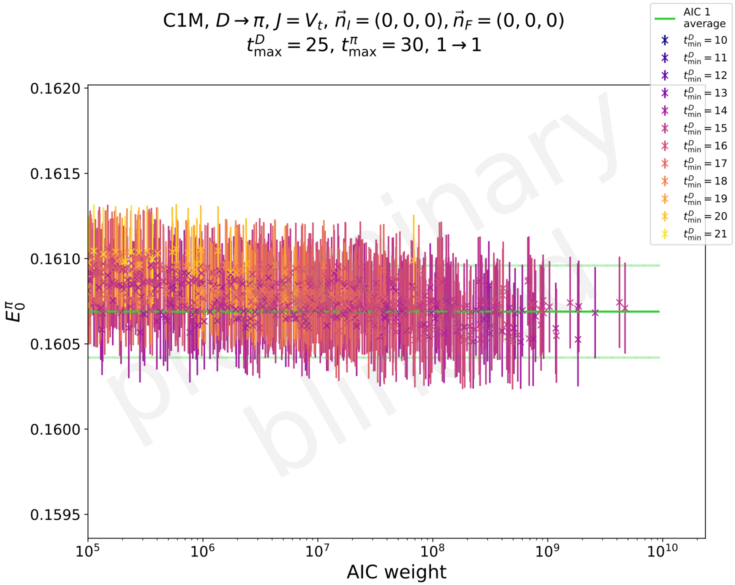

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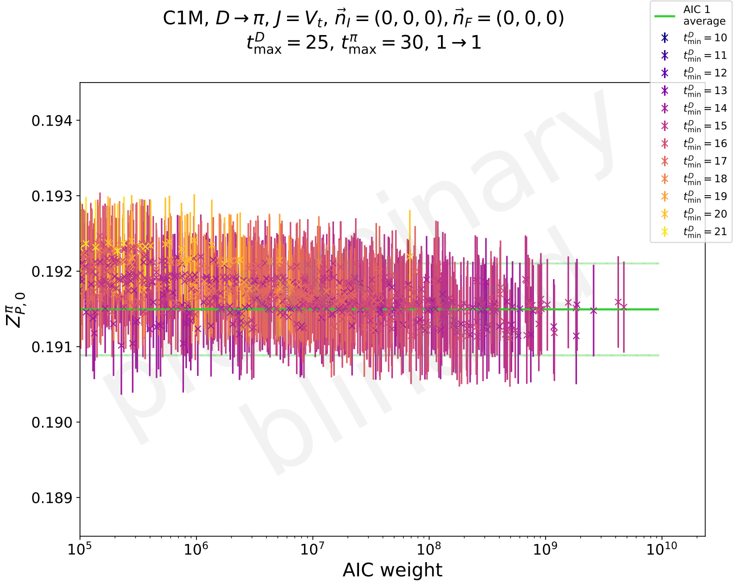

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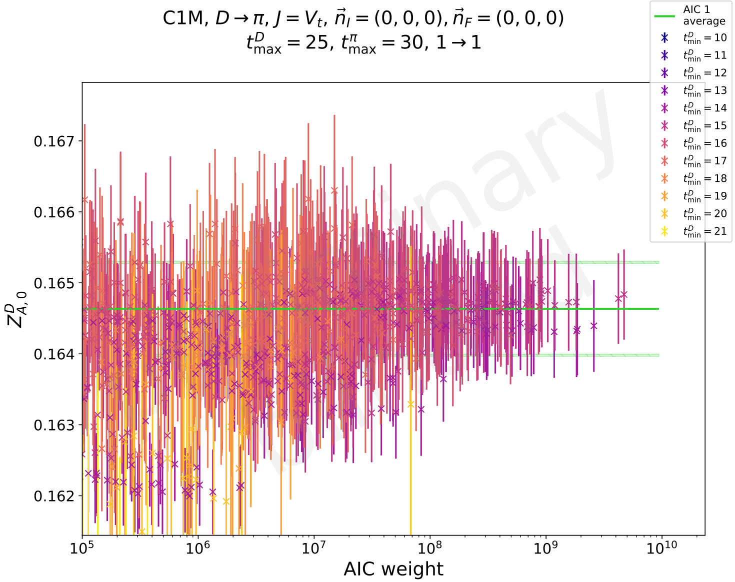

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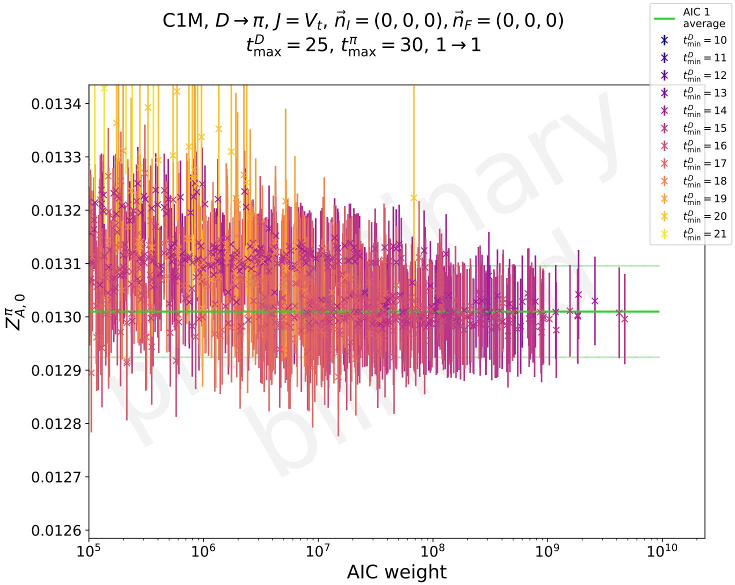

Future Work

Next steps

- Finalise optimal filtering choice
- Develop initial guess mechanism
- Actually perform fits
- Fit other sample data sets
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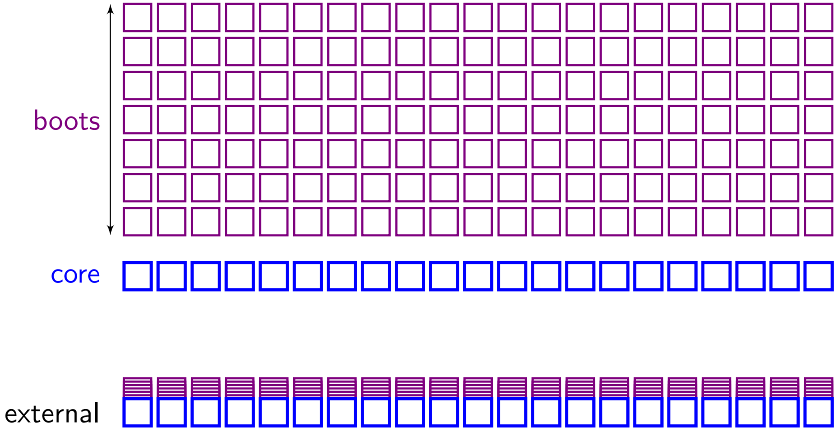
Further expansions

- Expand to different momenta
- Continuum limit
- Other decay process
- Vector final states
- Clean/expand fitting framework

Summary

- Precision tests most fruitful avenue for BSM search in near future
- CKM matrix a promising avenue
- Precision from lattice semileptonic decays
- Laplace filter potentially powerful tool
- Imminent work comparing two state fit to novel Laplace method
- $D \to \pi$ to follow
- Many channels/processes to explore

Bonus: Bootstrap Code Framework



Bonus: Fitting Structure

- Hierarchical HDF5 file
- Arbitrary multi level tagging

```
GLOBAL_ARGUMENTS___FIT1___FIT2...

ARGUMENT1=VALUE1__ARGUMENT2=VALUE2__...
```

Bonus: Fitting Structure

```
boot_num=100__max_boot_num=10000__seed=12345__ensemble=
    C1M___fit=2pt__process=ll_to_ll__gamm=
    Gamma5_Gamma5__momentum=0_0_0 __data_path=
    C1M_lcsm_to_ll_P.h5/mes/Fnsq0/
    F_Gamma5_Gamma5__lap_filters=0 __t_i=10__t_f=30
    __num_states=1__sign=1
```

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```
boot_num=100__max_boot_num=10000__seed=12345__ensemble=C1M
    ___fit=2pt__process=ll_to_ll__gamma=
    Gamma5_Gamma5__momentum=0_0_0 __data_path=
    C1M_lcsm_to_ll_P.h5/mes/Fnsq0/
    F_Gamma5_Gamma5__lap_filters=1 __t_i=10__t_f=30
    __num_states=1__sign=1 ___fit=2pt__process=
    lcsm_to_lcsm__gamma=Gamma5_Gamma5__momentum=0_0_0
    __data_path=C1M_lcsm_to_ll.h5/mes/Fnsq0/
    I_Gamma5_Gamma5__lap_filters=1 __t_i=5__t_f=30
    __num_states=1__sign=1 ___fit=3pt_ratio__process=
    lcsm_to_ll__current=V_T__ini_gamma=Gamma5_Gamma5
    __fin_gamma=Gamma5_Gamma5__ini_momentum=0
    _0_0__fin_momentum=0_0_0 __thr_data_path=
    C1M_lcsm_to_ll_P.h5/semi/nIsq=0_nFsq=0_nFdotnI=0/Src-
    Gamma5_Cur-V_T/data/ __ini_data_path=C1M_lcsm_to_ll.h5/
    mes/Fnsq0/I_Gamma5_Gamma5 __fin_data_path=
    C1M_lcsm_to_ll.h5/mes/Fnsq0/
    F_Gamma5_Gamma5__ini_filters=1 __fin_filters=1__t_fin=9
    __t_fin=7__dTs=16_18_20_24_28__num_states=1_to_1
```