

Third Harmonic Cavity Design for the Electron-Ion Collider



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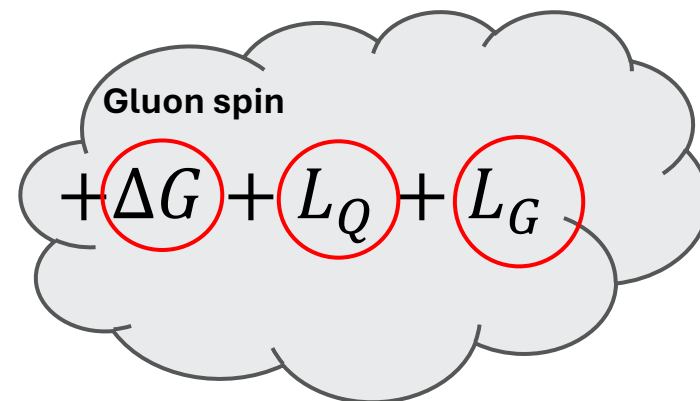
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Engineering



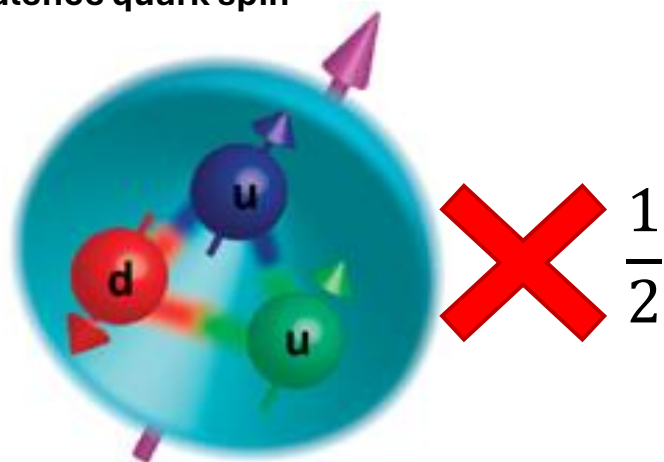
Quark and gluon
orbital angular
momentum

Protons, were thought to derive their $\frac{1}{2}$ spin primarily from their constituent valence quarks, uud

$$\frac{1}{2} \approx \frac{1}{2} \Delta \sum (\Delta u + \Delta d + \Delta s)$$



Valence quark spin



In 1988, deep inelastic scattering (DIS) experiment revealed valence quarks contribute **less than 30% of proton spin**¹

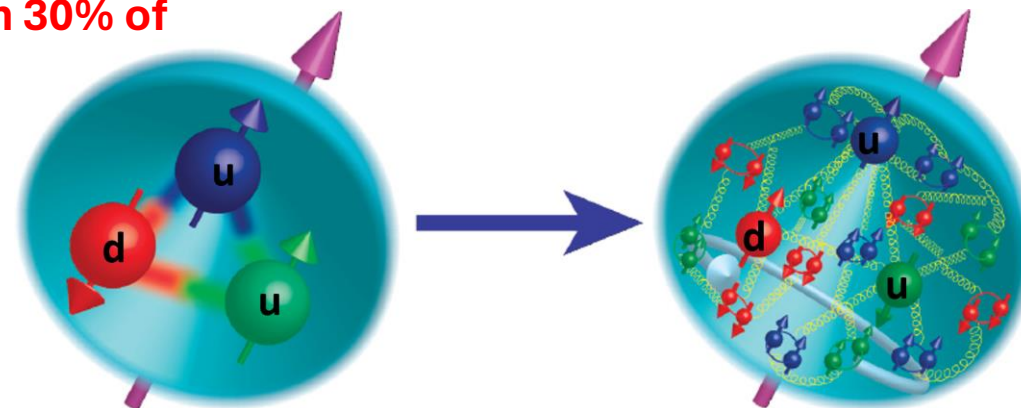
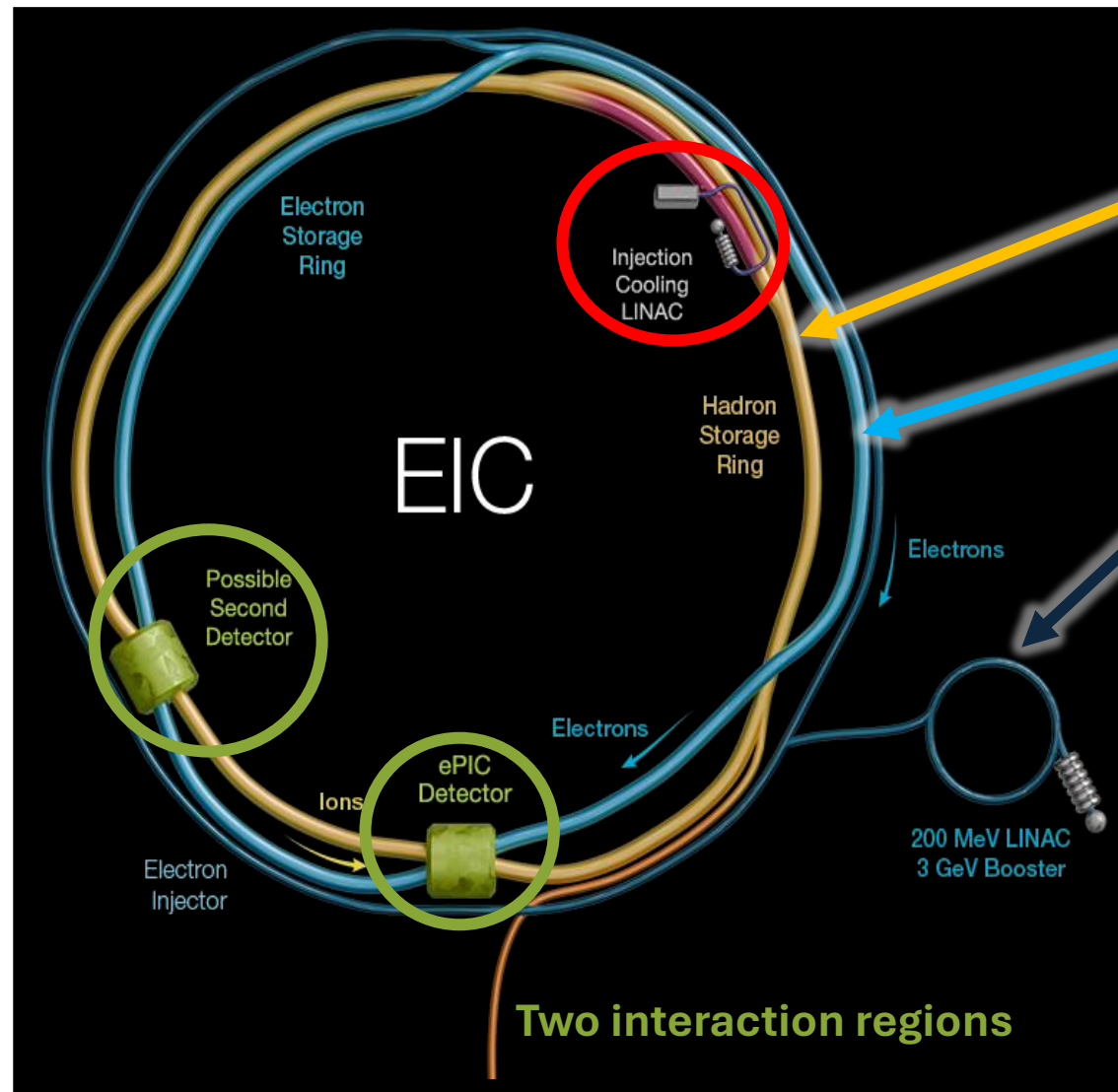


Figure 1: Evolving understanding of proton spin origin. **(Left)** proton was composed of three valence quarks, with total spin $\frac{1}{2}$. **(Right)** proton contains quarks, as well as a gluonic sea of quark-antiquark pairs; the total spin is composed of that of the elementary spins².

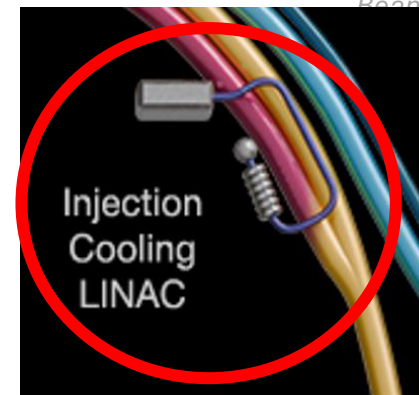
¹Lipkin, Harry J. 1988. "Flavor Symmetry, EMC Results and the Spin Content of the Proton." Physics Letters. B 214 (3): 429–33; ²White Paper on the Electron-Ion Collider in Preparation for the NSAC Long Range Plan." 2023

The Electron Ion Collider



The existing 3.4km rings of the **Relativistic Heavy Ion Collider** are used to build:

- **Hadron Storage Ring (HSR):** storing polarised protons at energies 41-275 GeV.
- **Electron Storage Ring (ESR):** storing polarised electrons at energies 5 GeV -18 GeV.
- **Rapid-Cycling Synchrotron (RCS):** a pre-injector to accelerate polarised electrons from 200MeV to 5 GeV-18 GeV⁴.



Beam parameters operating at maximum luminosity⁴.

	e ⁻	p ⁺
Energy [GeV]	10	275
Current [A]	2.5	1
Polarisation [%]		≥70
Luminosity [10 ³⁴ cm ⁻² s ⁻¹]		1

Heavy ions do not have an intrinsic damping mechanism to counteract emittance growth when subjected to long stores in collider rings

Emittance growth occurs due to: **Intra Beam Scattering** → **Degrades luminosity**

Vertical emittance grows from **0.3 μm to 2.5 μm in 2 hours⁵**

External **fast cooling mechanism** required to counteract this⁶

18 - 275 GeV

p^+

1.

Modulator

Proton time-of-flight

$E < E_0$

$E_0 > E$

3.

Kicker

p^+ to experiment

e^-

Electron amplification

2.

e^- to dump

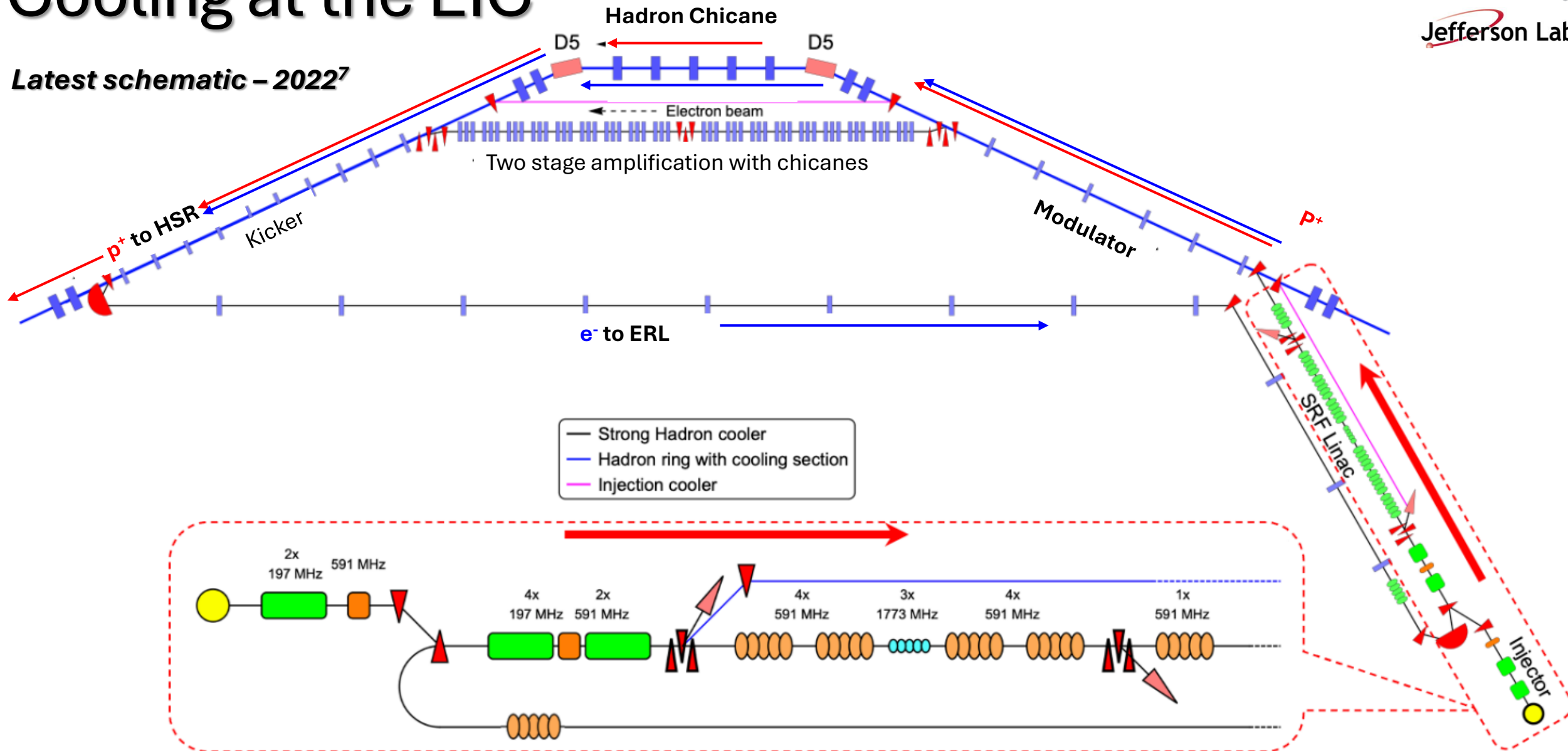
50 - 150 MeV

⁵COHERENT ELECTRON COOLING PHYSICS FOR THE EIC. doi:https://doi.org/10.18429/JACoW-IPAC2024-THYD1

⁶Ratner, D. (2013). Microbunched Electron Cooling for High-Energy Hadron Beams. Physical Review Letters, 111(8). Bergan, W.F., Mayes, C., Wang, N. and Stupakov, G. (2024).

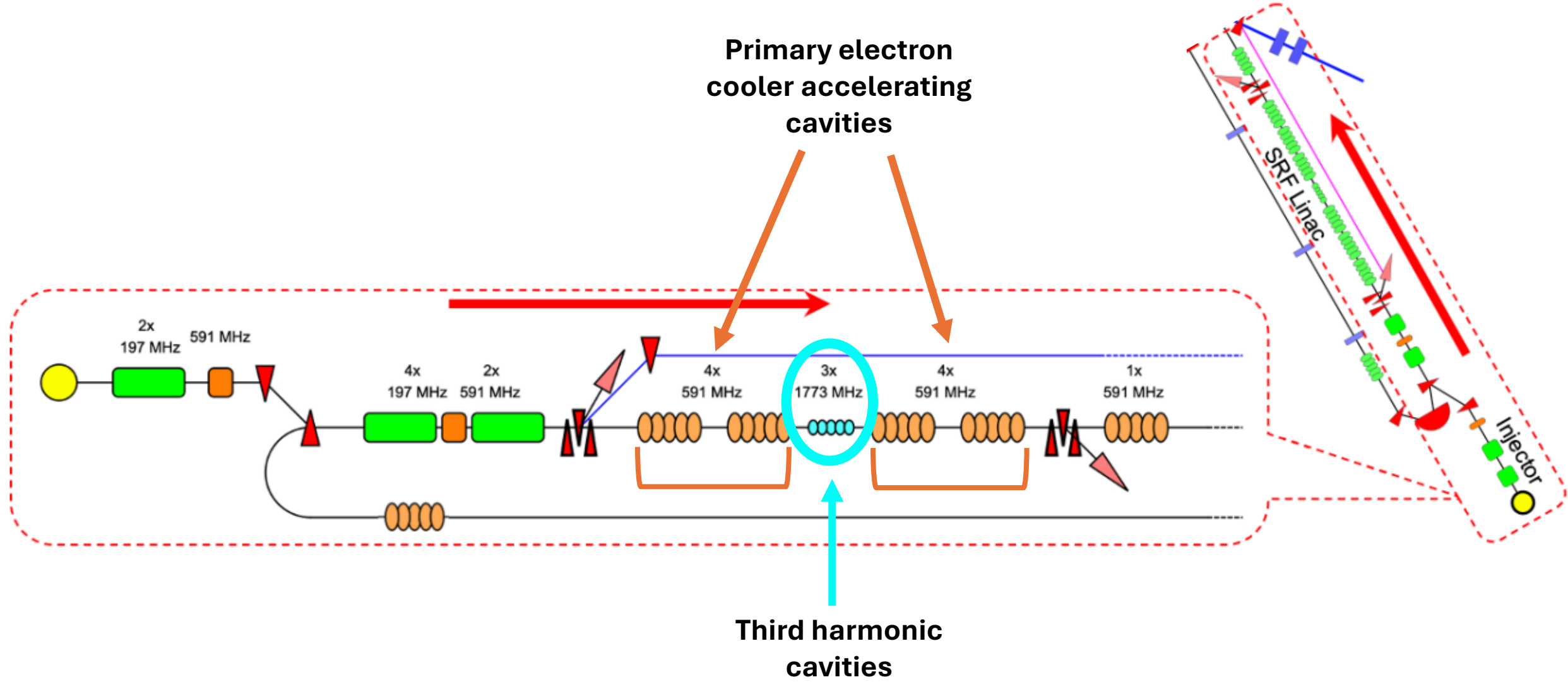
Cooling at the EIC

Latest schematic – 2022⁷



⁷Wang, E, W Bergan, F Willeke, S Benson, K Deitrick, Thomas Jefferson, C Mayes, D Douglas, C Gulliford, and N Taylor. n.d. "ELECTRON ION COLLIDER STRONG HADRON COOLING INJECTOR and ERL.

Cooling at the EIC



Energy Recovery Linacs & BBU

ERL operation

1. Beam accelerated by SRF cavities
2. After use, beam is recirculated back to cavities with π phase difference
3. Beam is decelerated, returning its energy back to cavities

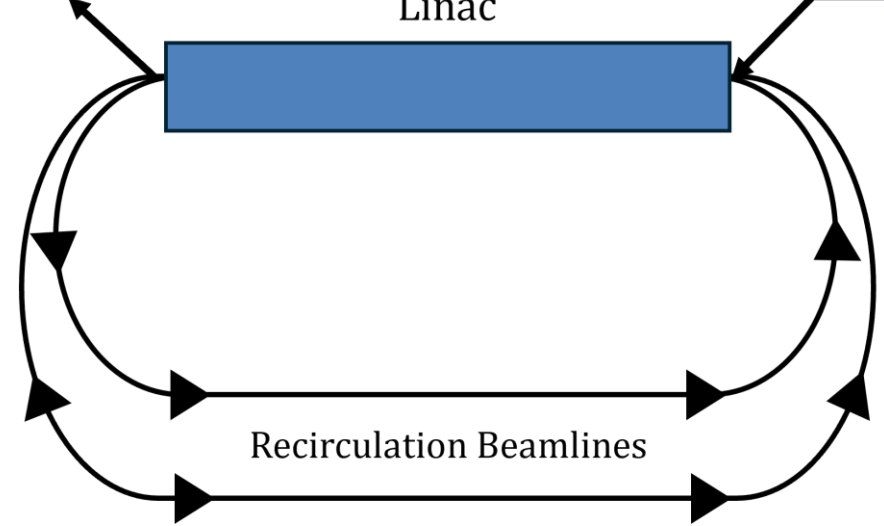
Beam Dump



Linac



Injector



Beam Break Up Instability

Bunch enters cavity off axis



Deflecting HOM is excited



Bunch experiences transverse kick



Bunch returns to cavity with larger transverse offset



Bunch receives another transverse kick



Bunch decelerated, HOM energy grows



Positive feedback loop

Limits maximum operable beam current

Compromises complete energy recovery

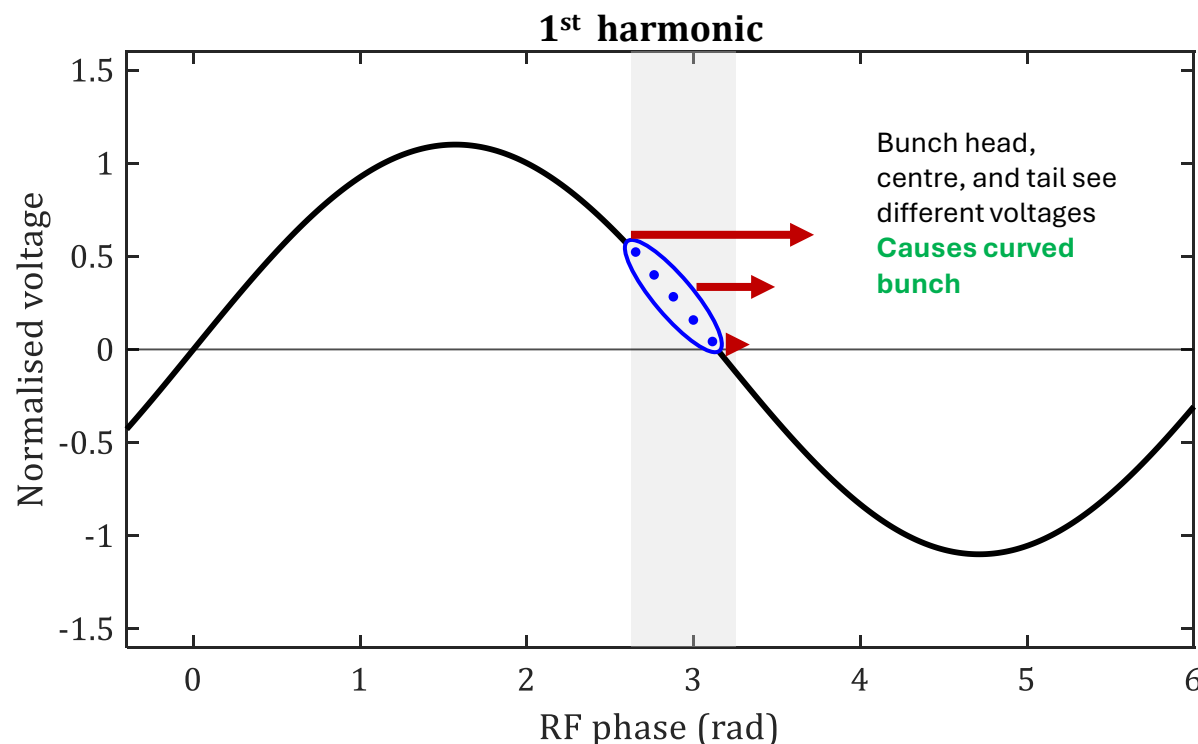
Deflecting modes must be sufficiently damped

Motivation for Third Harmonic Cavities

Cooler **electron bunch length is 4-5 cm**,
occupying 36 degrees of 591 MHz RF voltage



- **Curved electron bunch**
- **Poor cooling efficiency**



1st harmonic

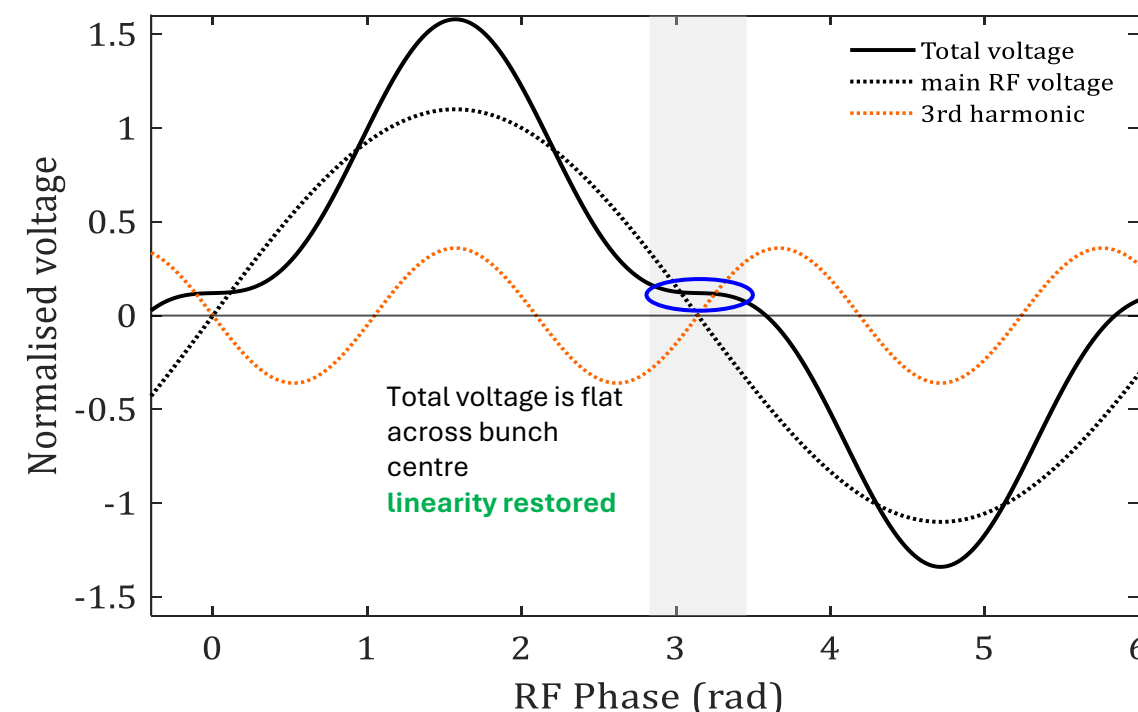
$$V(\phi) = V_{RF} [\sin(\phi + \phi_s)]$$

$$\phi = \frac{\omega_{RF}}{c} z + \phi_s$$

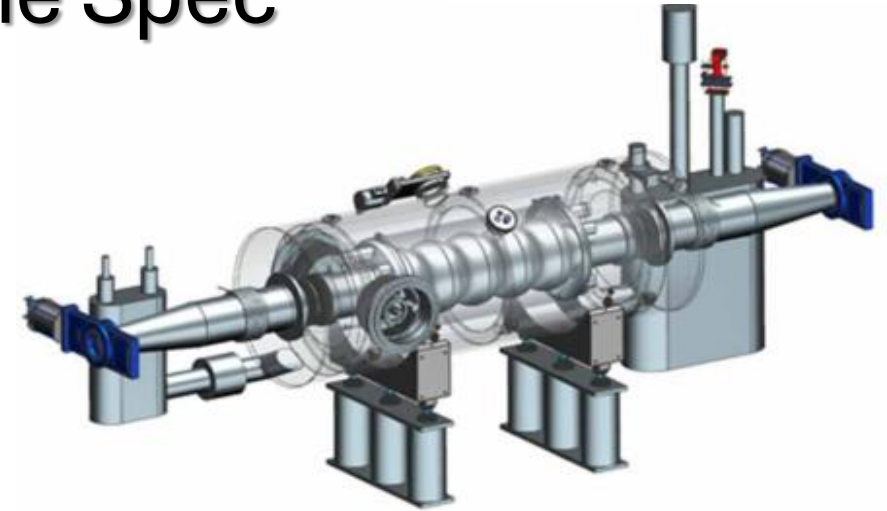
1st + 3rd harmonic

$$V(\phi) = V_{RF} \left[\sin(\phi + \phi_s) + \frac{V_3}{V_{RF}} \sin(n\phi + n\phi_h) \right]$$

1st harmonic + 3rd harmonic



1773MHz Cavities: The Spec



591MHz SRF 5-cell cavity module for the ERL cooler¹⁰

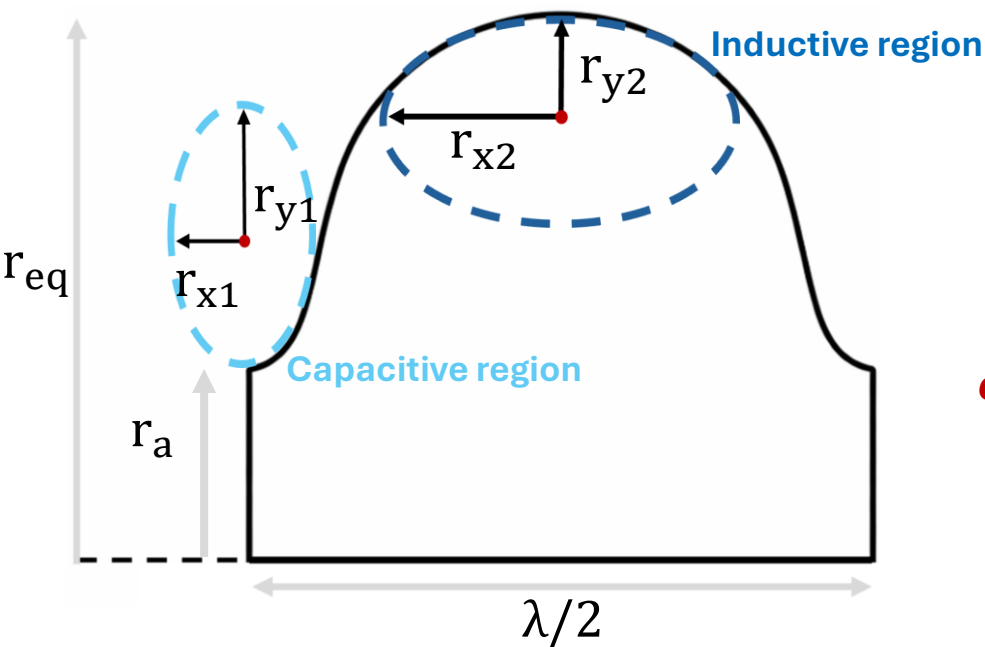
Parameter	Value
Frequency [MHz]	1773
$V_{5\text{-cell}}$ [MV]	6.33
E_{pk}/E_{acc}	2.77
B_{pk}/E_{acc} [mT/MV/m]	5.54
E_{acc} [MV/m]	15.4

⁹ Rimmer, R. (2021). An Overview of RF Systems for the EIC. *JaCoW*. doi:<https://doi.org/10.18429/JACoW-IPAC2021-MOPAB385>

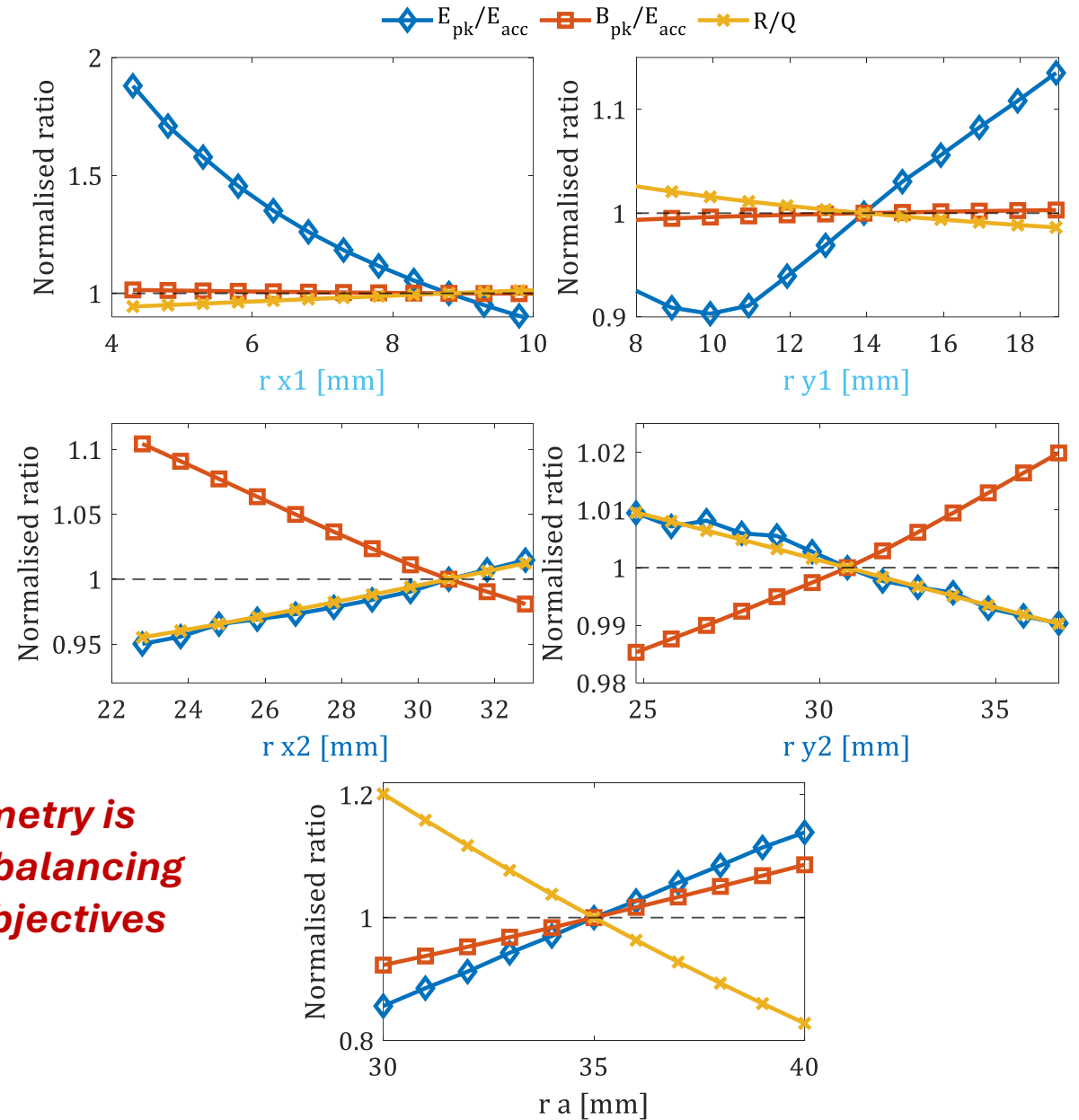
¹⁰ Rimmer, R. and Daly, E. (2022). CAVITY AND CRYOMODULE DEVELOPMENTS FOR EIC. *JaCoW*. doi:<https://doi.org/10.18429/JACoW-eeFACT2022-WEXAS0101>

CAVITY OPTIMISATION GOALS

- 1. E_{pk}/E_{acc}** – surface electric field required to produce $E_{acc} \downarrow$
- 2. B_{pk}/E_{acc}** – surface magnetic field required to produce $E_{acc} \downarrow$
- 3. R/Q** – how efficiently RF energy is transferred to beam $\uparrow$



Cavity geometry is optimised by balancing competing objectives





Cavity design requires **simultaneous optimisation of conflicting objectives**

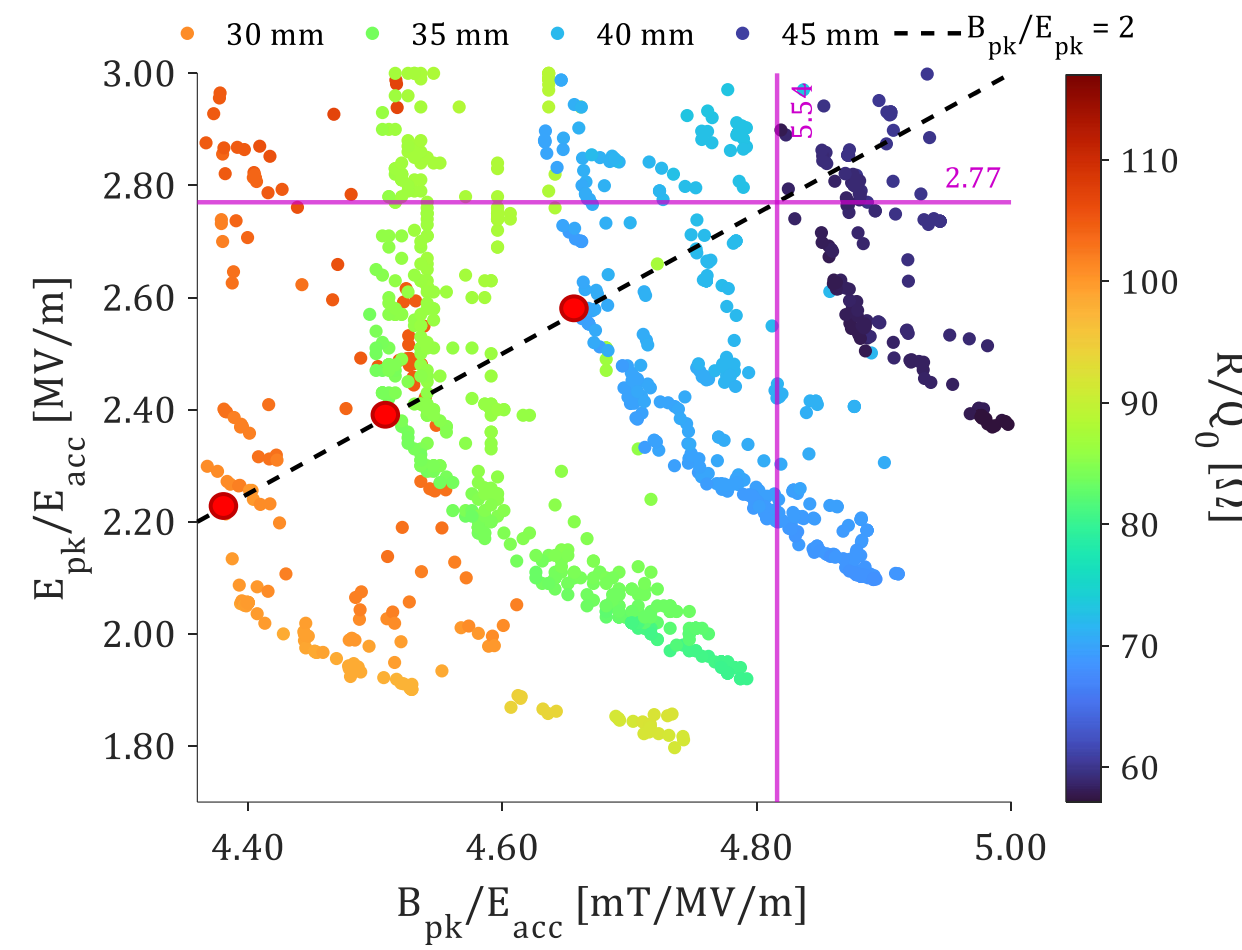
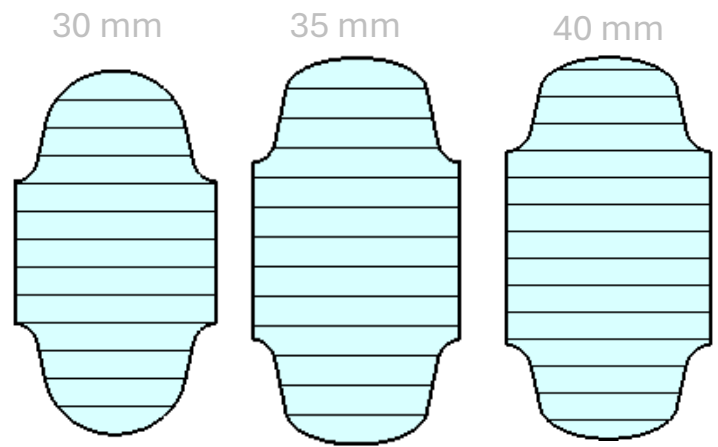
A design is **Pareto-optimal** if one objective cannot be improved without degrading another.

Design objectives:

$$\begin{aligned} \min. \quad & \begin{cases} E_{pk}/E_{acc} \\ B_{pk}/E_{acc} \end{cases} \\ \max. \quad & \{R/Q\} \end{aligned}$$

- Multi-objective genetic algorithm is used to identify a set of **Pareto-optimal geometries** satisfying objectives
- Each point is an individual design candidate from the optimisation process, where cell elliptical parameters were varied with increasing aperture radius.

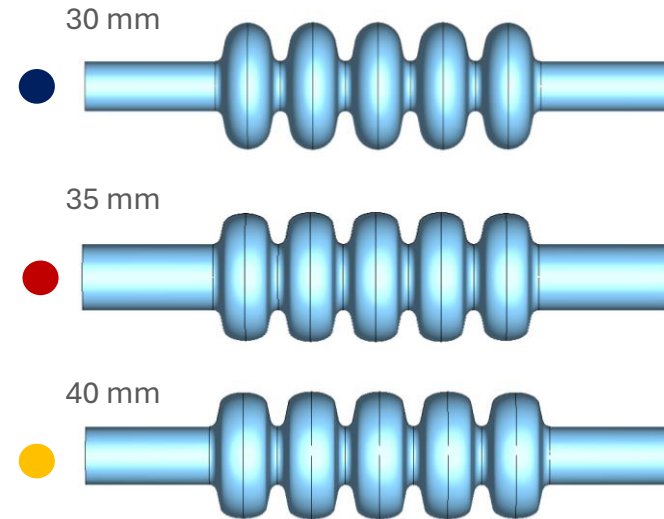
Candidate geometries



Trapped Mode Analysis

Initial studies of dipole behaviour begins with a standard 5-cell structure

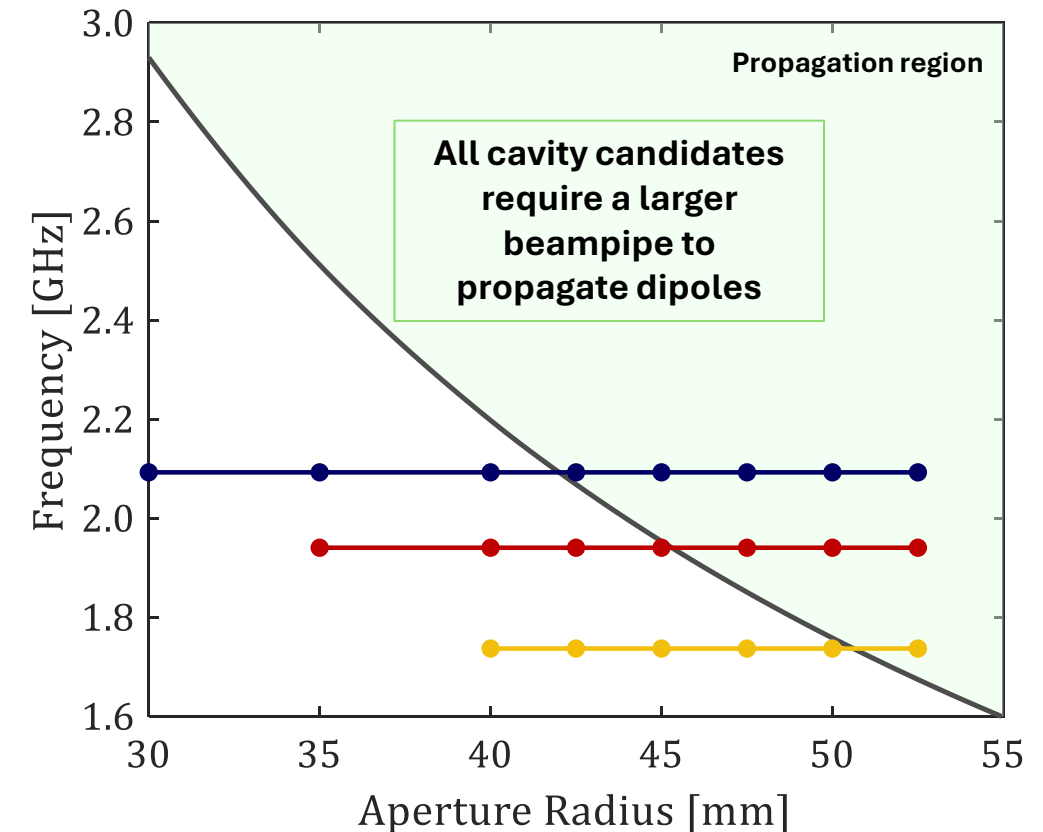
cavity aperture radius = beampipe radius



Lowest HOM frequency is trapped within all candidates

Beampipe radius sets TE11 cut-off frequency

TE11 cut-off frequency in circular waveguide

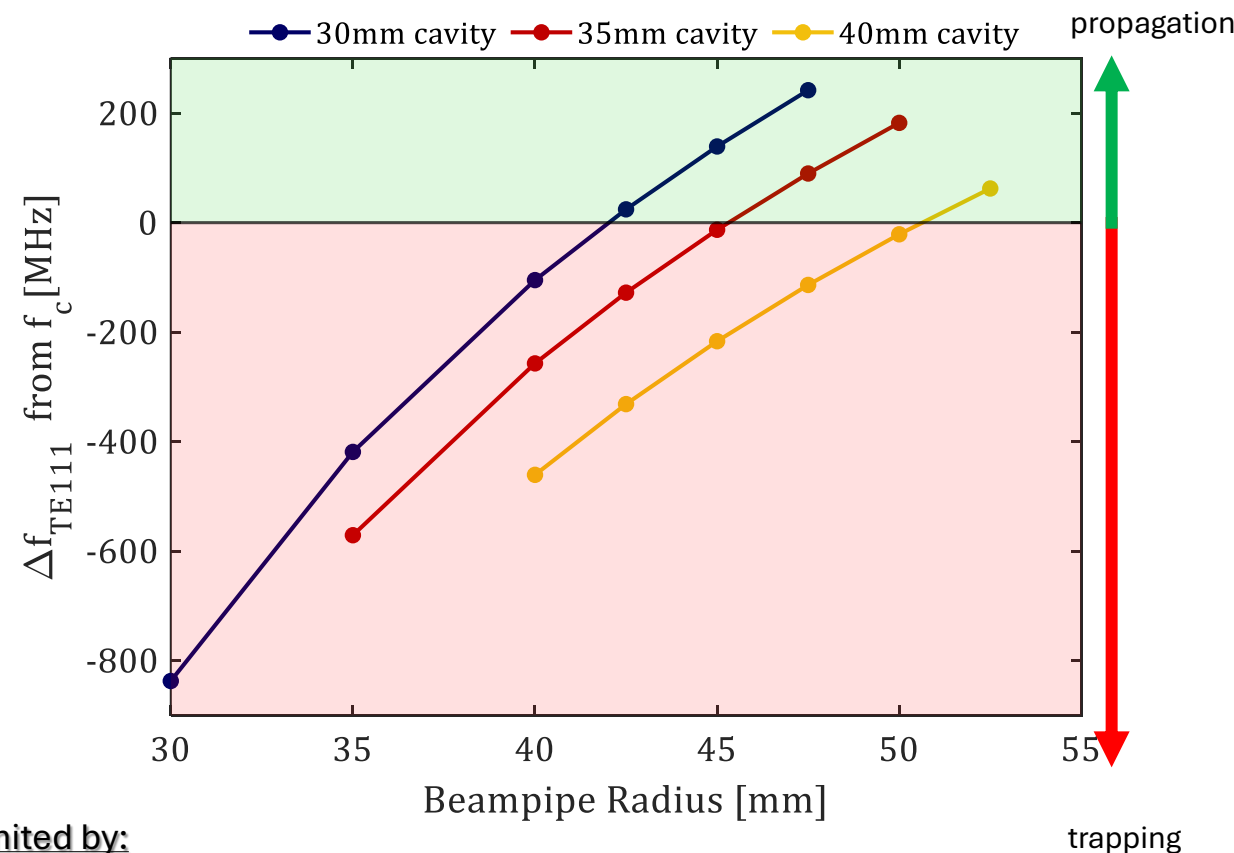
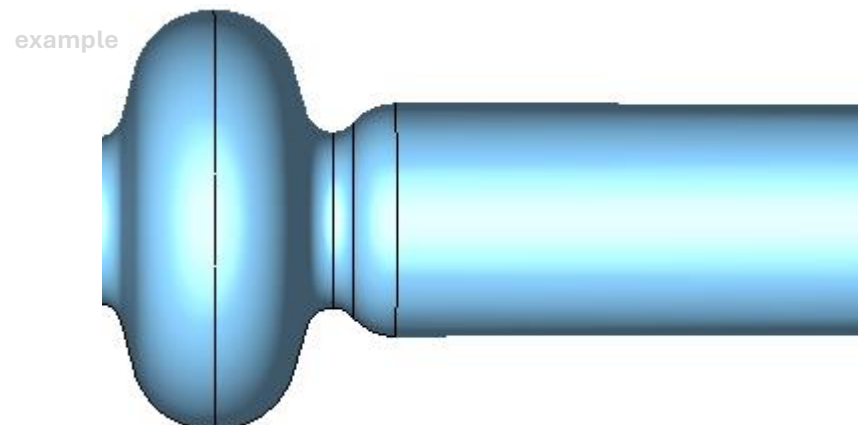
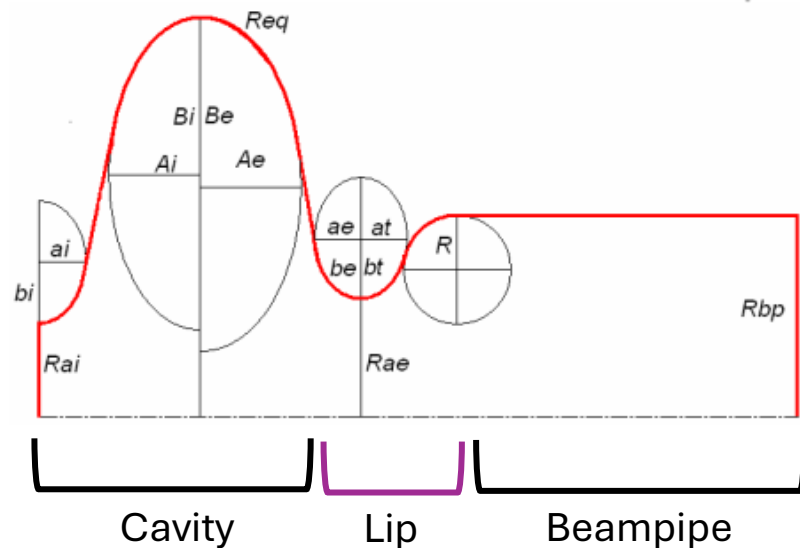


Each 5-cell configuration satisfies

- ✓ Field Flatness $\geq 98\%$
- ✓ Transit Time Factor ≥ 0.7
- ✓ Evenly distributed surface fields across all cells (close to 1:1)

Trapped Mode Analysis

Using a larger beampipe requires an optimised 'lip' transition between cavity and beampipe

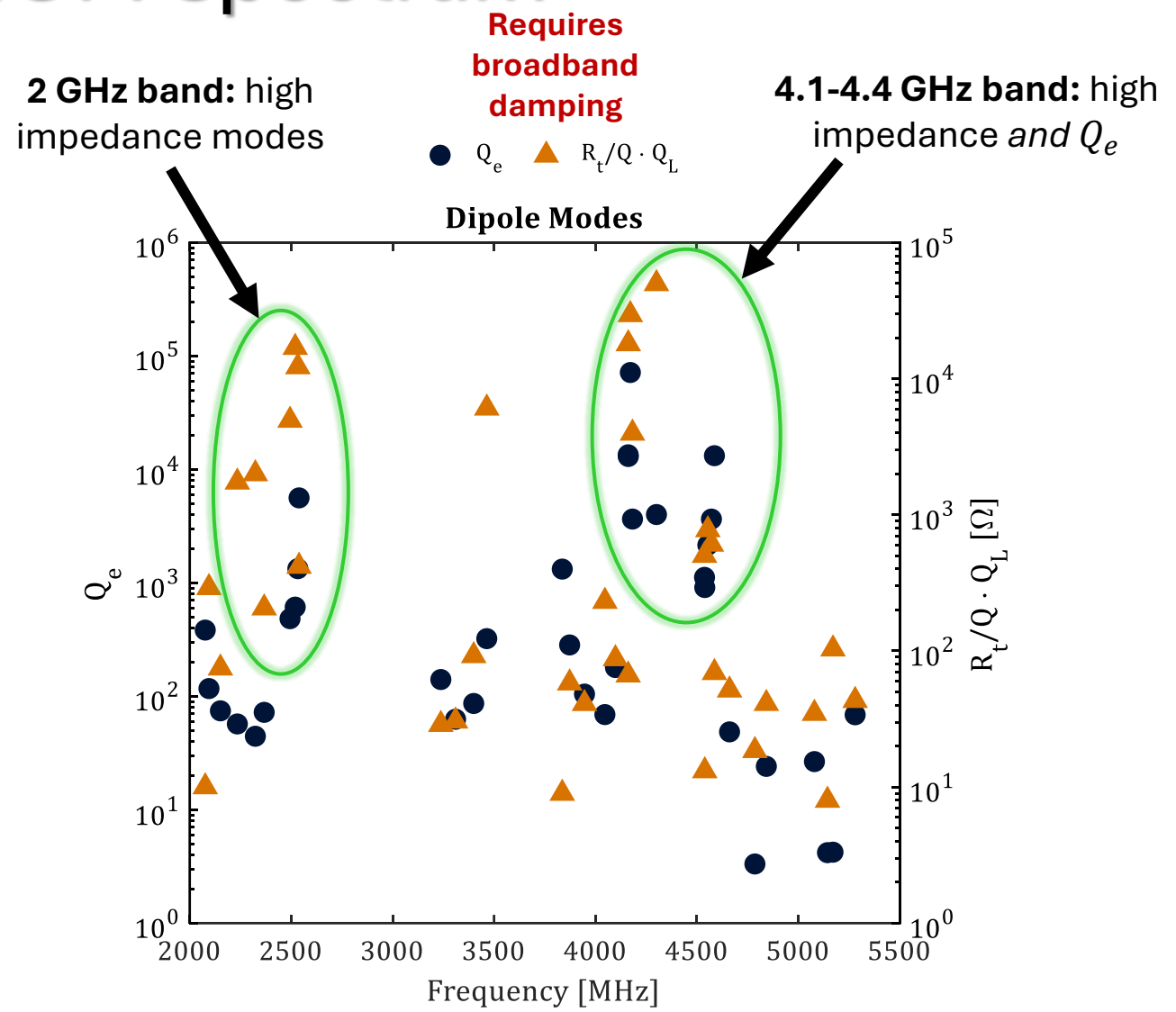
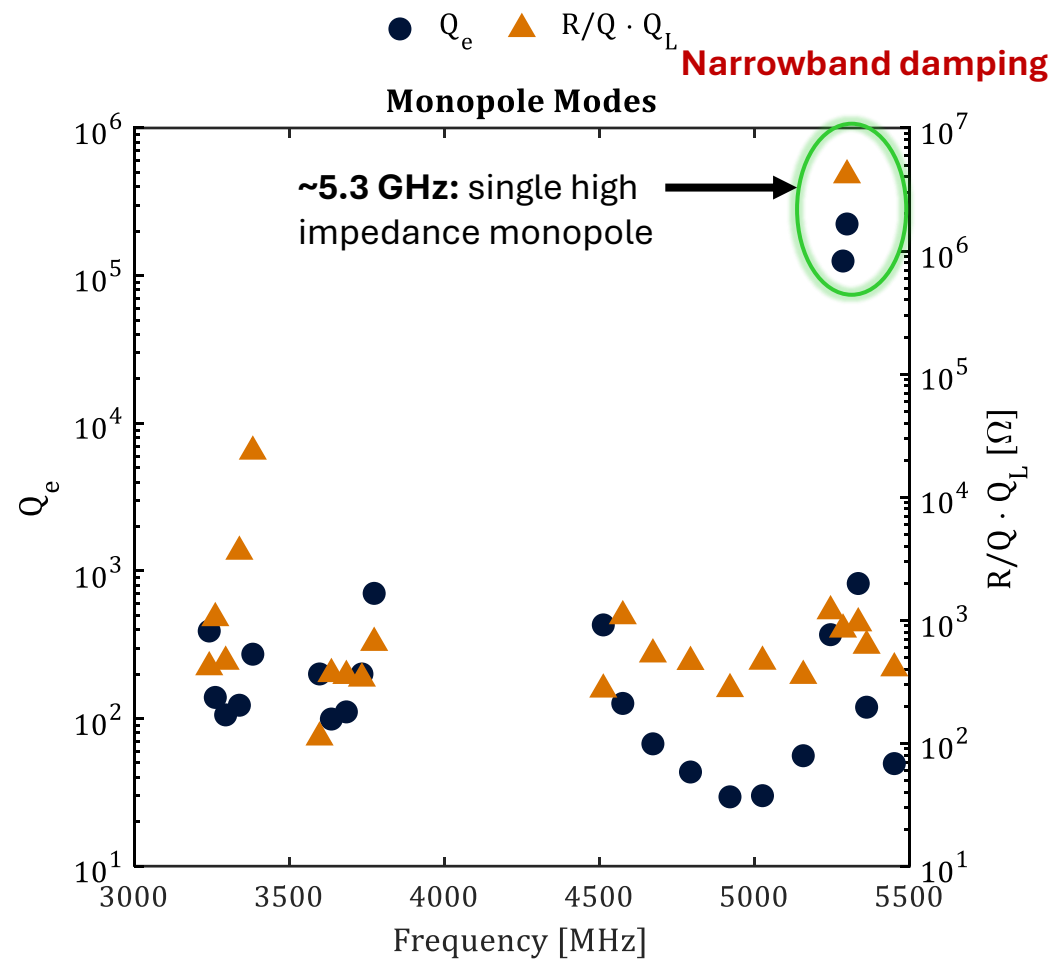


Maximum aperture limited by:

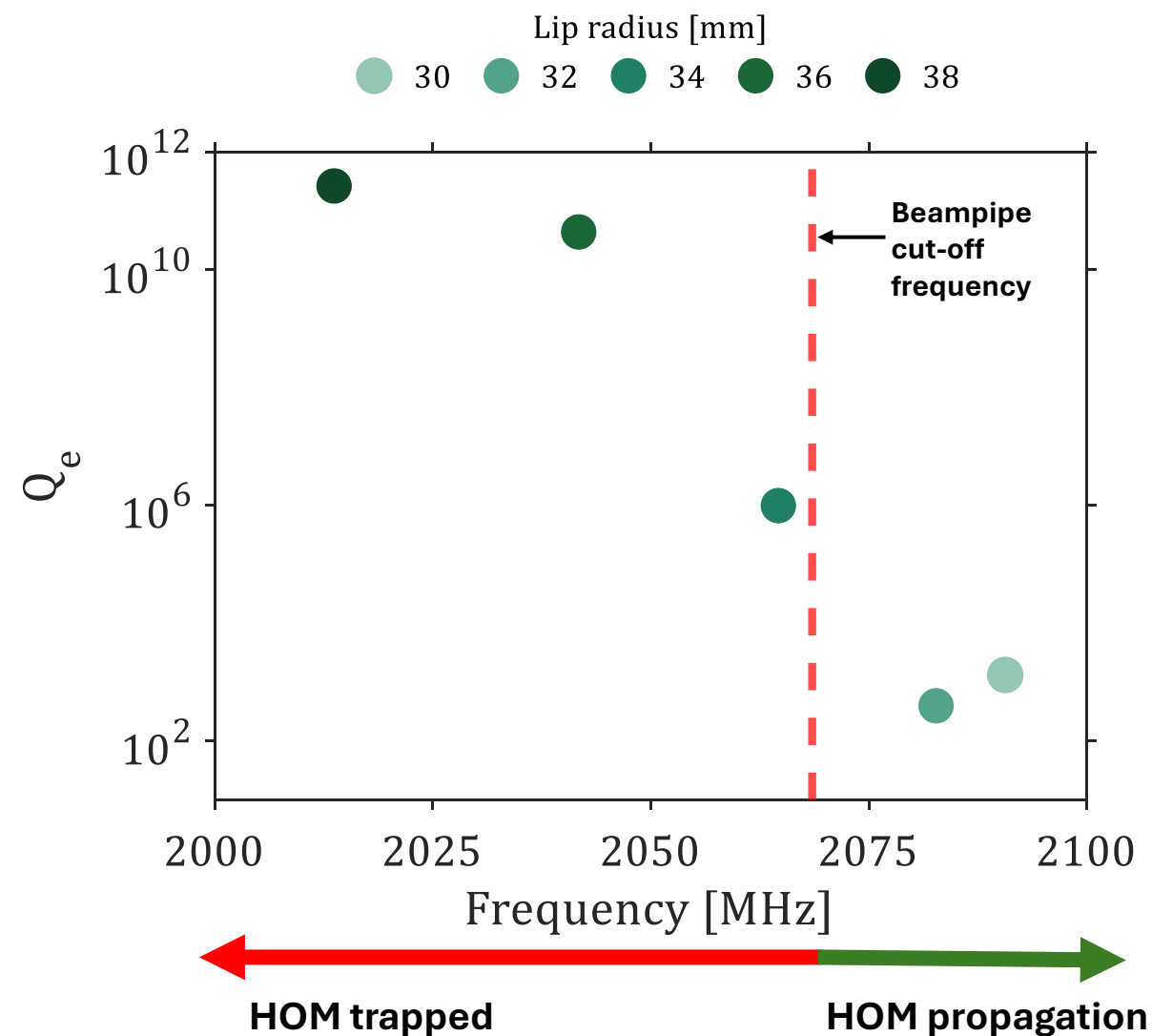
- Evanescent decay of TM010 mode into beampipe
- TM010 cut-off
- Cavity sandwiched between 591 MHz cavities –beampipe will have to taper down

BLA configuration based on beampipe
optimised for TM010 trapping ($Q_e > 10^{10}$)

Baseline HOM Spectrum



Lowest Dipole Frequency Propagation



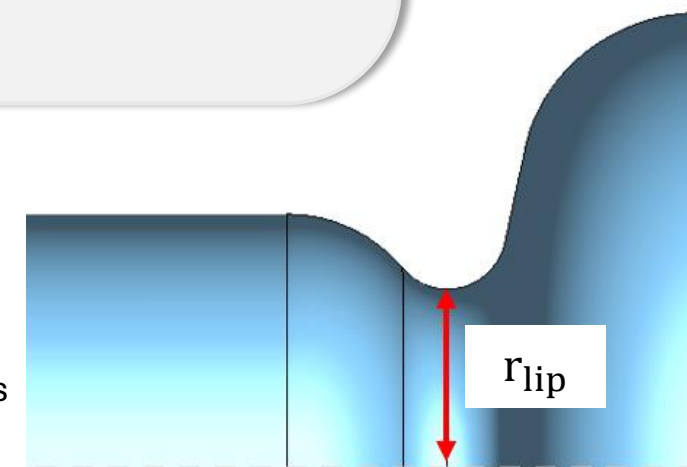
Key Takeaway

Lip radius determines whether the first dipole is trapped or propagates

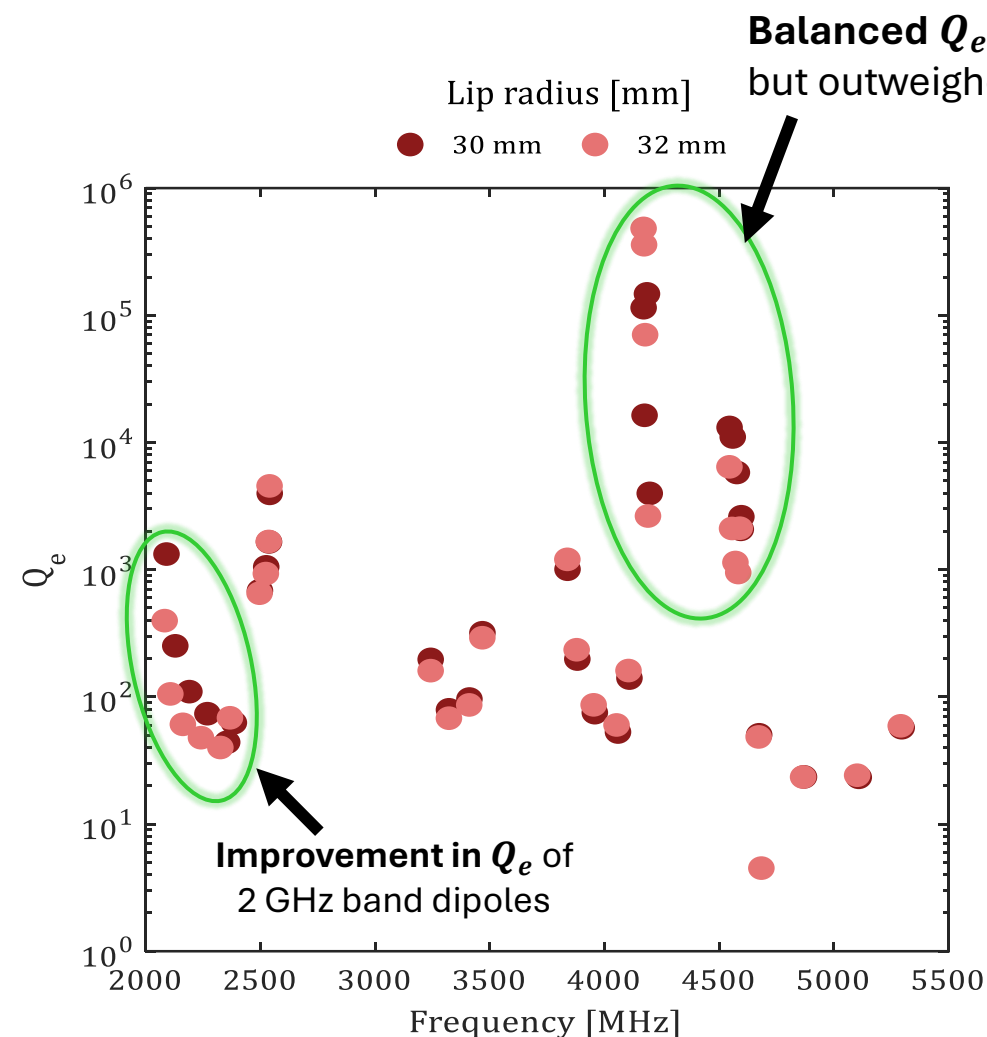
➤ Using a smaller lip increases dipole frequency to push above cut-off

- 30 mm and 32 mm lip shows promise for damped Q_e however,
- Q_e and impedance dipole response to lip radius 2.0GHz – 5.4 GHz must be evaluated

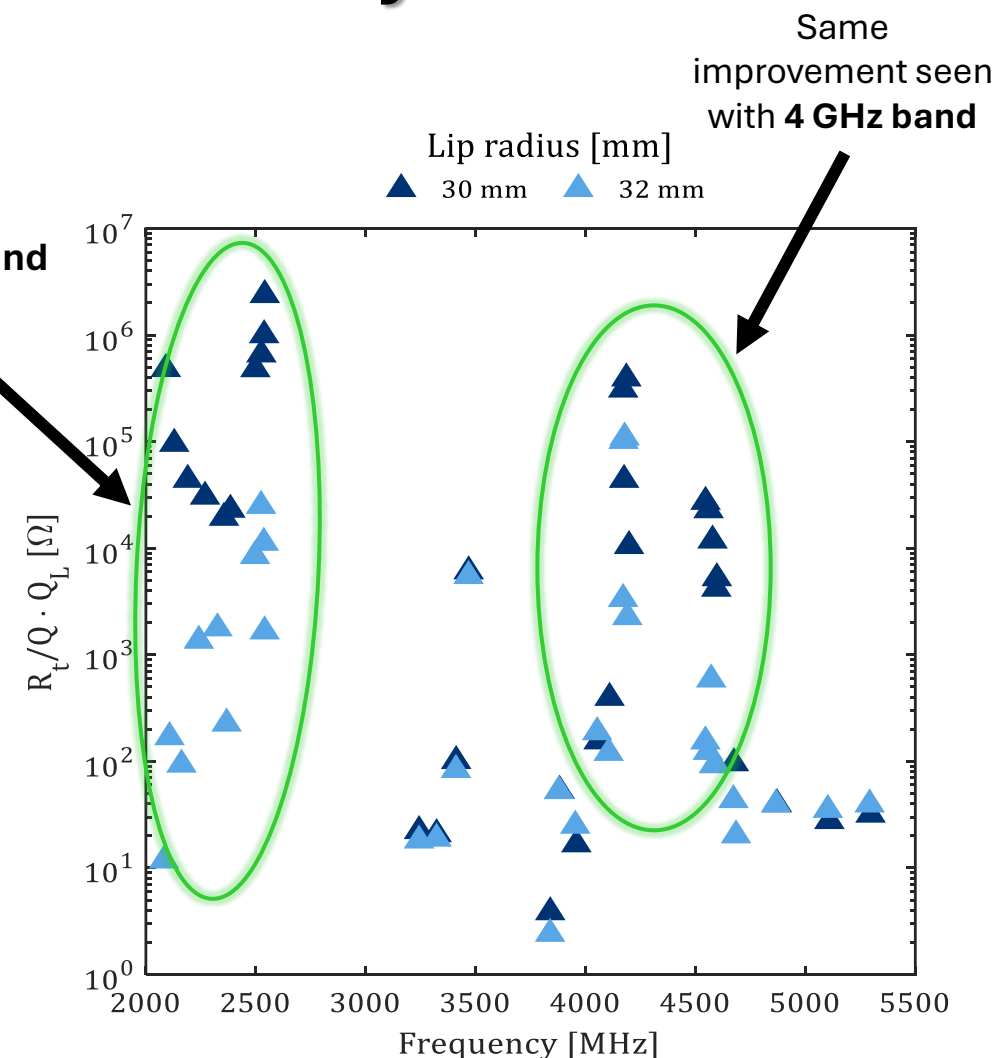
Lip radius defined from beamline axis

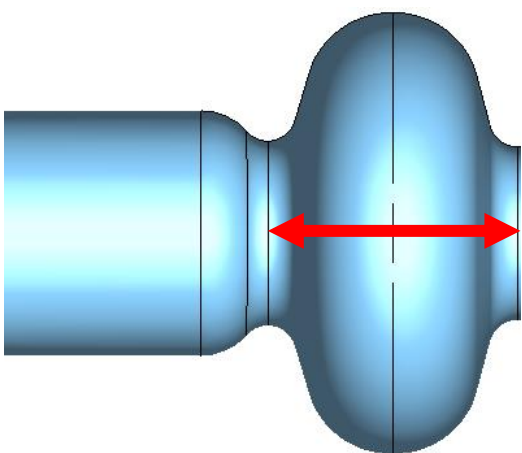


Mode Trapping in Bottleneck Geometry



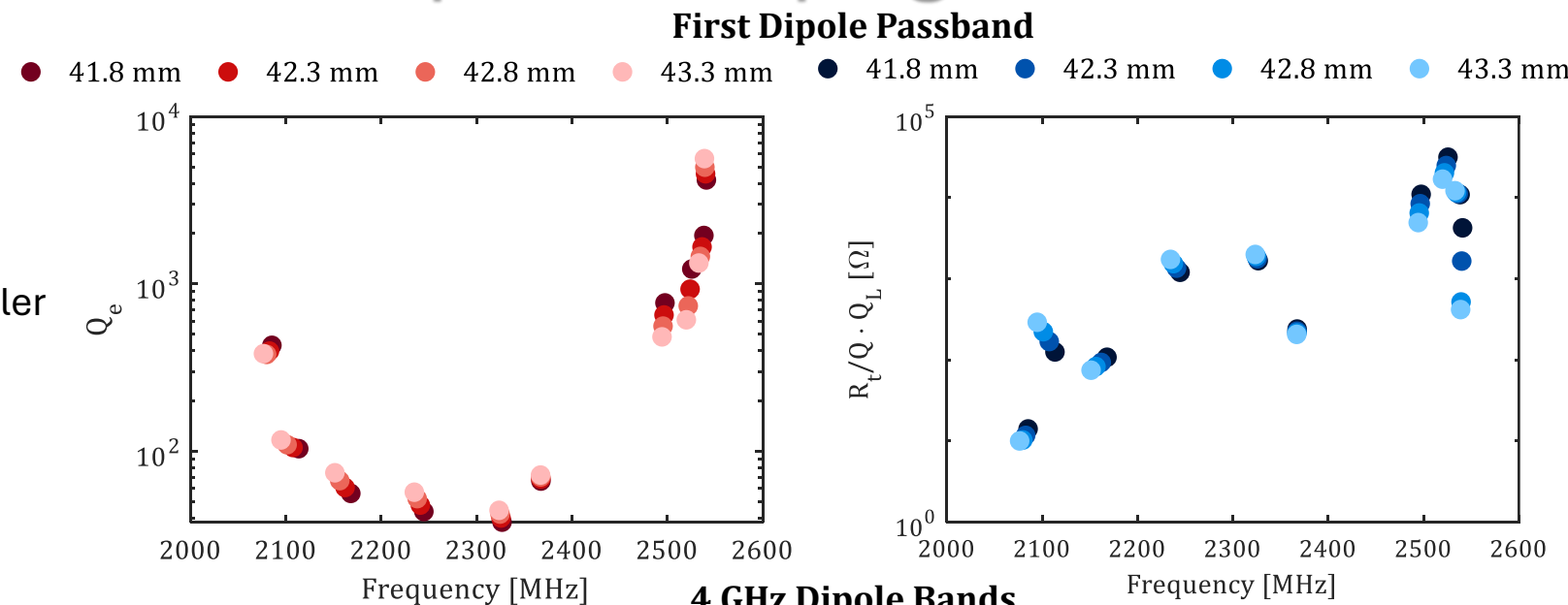
A huge impedance improvement GHz band with larger lip





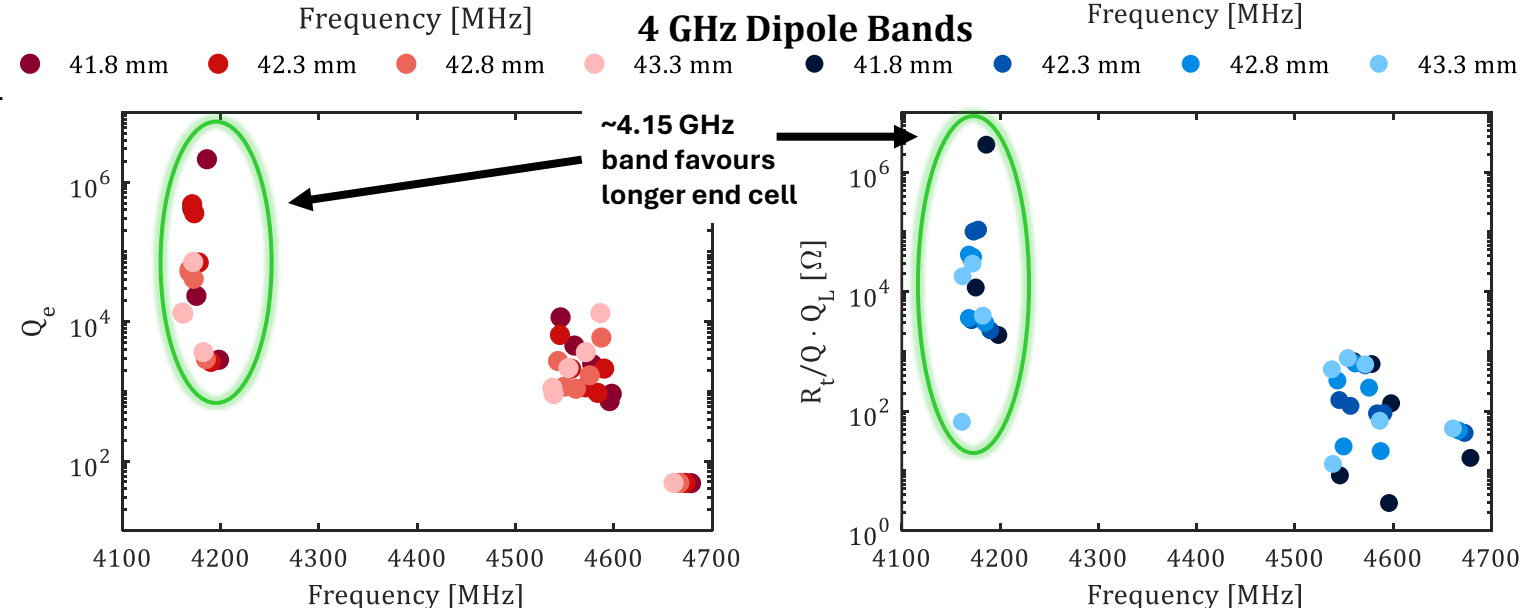
End cell length

2 GHz bands: smaller end cell length favours Q_e



4 GHz bands: longer end cell length improves both Q_e and impedance

We require broadband damping of Q_e and impedance, with stronger impedance weighting for BBU
longer end cell preferred

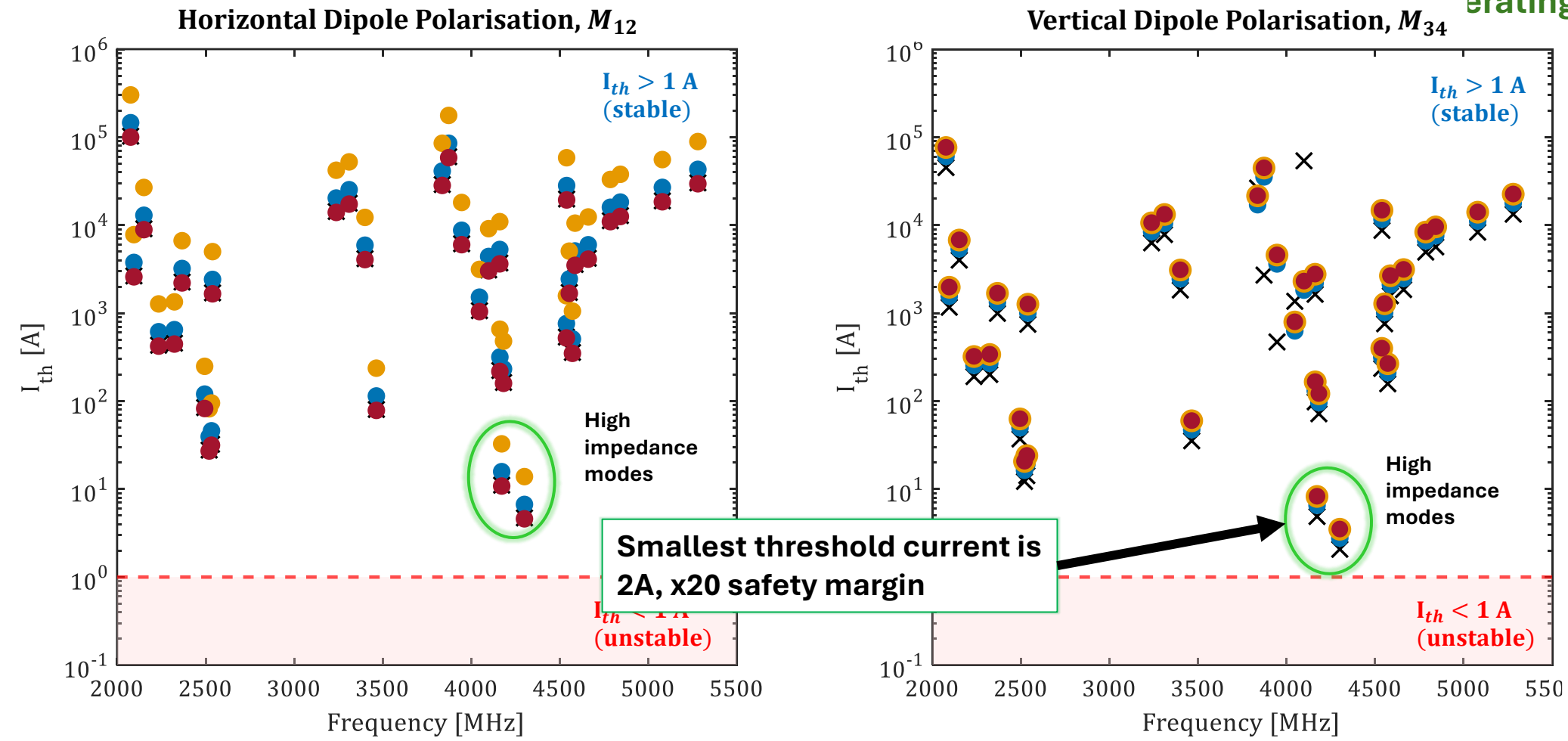


Beam Break-Up Instability

Minimum beam current to drive regenerative beam break up for each cavity¹¹ ($I_b = 100$ mA)

All modes stable at
erating current

× 1st cav ● 2nd cav ● 3rd cav ● 4th cav



Threshold currents calculated using cavity R_t/Q , Q_L , HOM frequency, and lattice recirculation optics (M_{12} , M_{34})

¹¹ S. Sentiya (2025) BBU Parameters for EIC-ERL, Private communication,

Waveguide Consideration

Beampipe absorber shows
**good promise for damping
dipole modes**

however,

In a recirculatory machine,
**available beamline real
estate is limited**



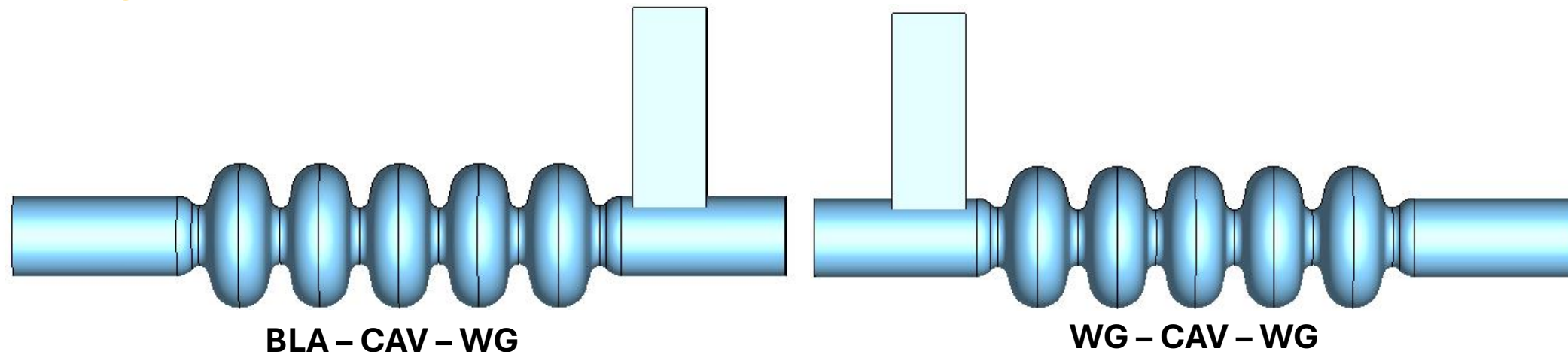
Beamline absorber
requires warm section



**Requires
more space**

Can adding a waveguide provide equivalent or
improved HOM damping than a BLA?

Possible configuration



- **Break rotational symmetry** → introduce hybrid modes → complex WG system
- **Local field enhancement** around beampipe-WG interface
- **Multipacting**

Summary

- Developed 5-cell 3rd harmonic cavity for EIC cooler
- Optimised end-cell and cavity-beampipe geometry for built-in HOM damping
- All BBU threshold currents at least 1 order of magnitude above I_{th}

Future Work

- Evaluate if a waveguide provides improved damping for optimisation of space utility
- Perform thermal simulations of BLA with ferrite HOM loads
- Change of direction – coaxial coupler for EIC crab cavity

Thank you for listening

Back-up Slides

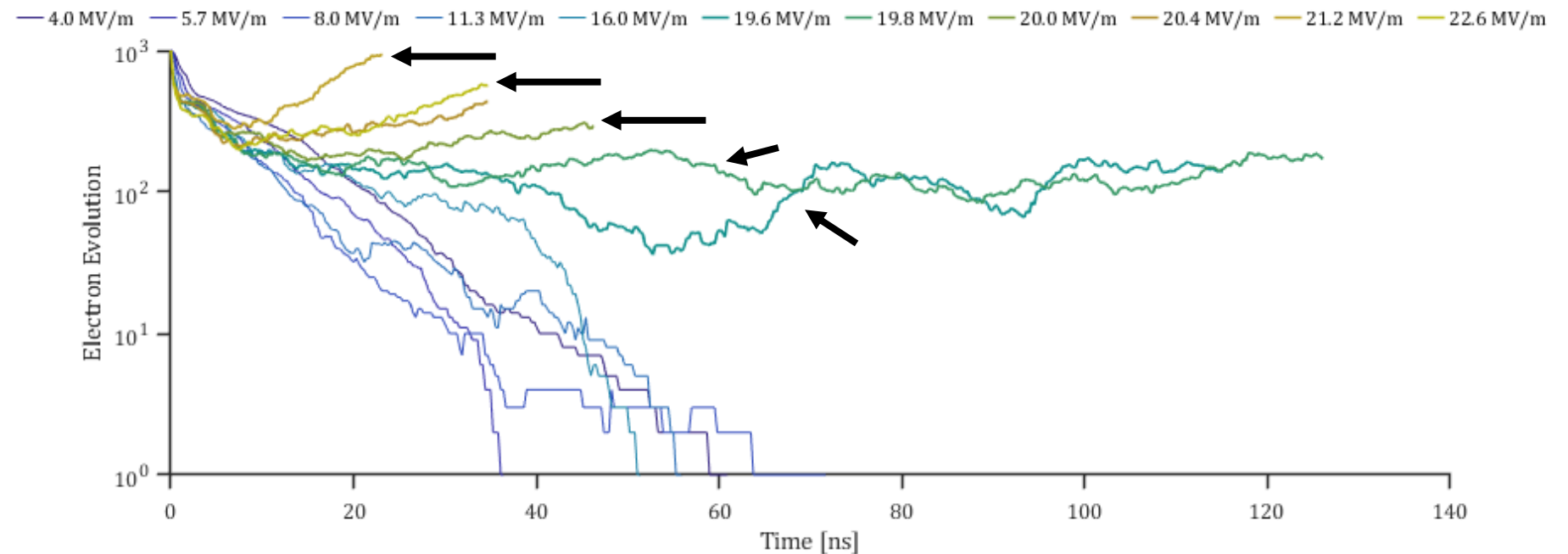
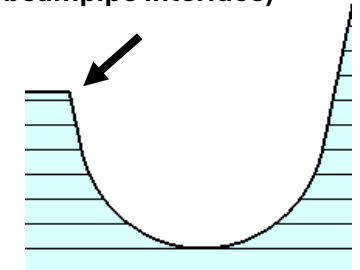
Multipacting in the Beampipe

Table X: SEY parameters of high purity Nb
following 300° C bakeout.

Parameter	Value
SEY _{max}	1.5
SEY ₀	0.8
Impact energy at first crossover [eV]	75
Maximum impact energy [eV]	300
Initial number of electrons	1000

Multipacting
field levels

Unoptimised lip (cavity-
beampipe interface)



Operating $E_{pk} = 35 \text{ MV/m}$

Figure 10: Electron evolution starting from a homogenous distribution of 1000 electrons as a function of rising E_p surface fields on 5-cell cavity with no lip optimisation. Nb (2K) SEY curve: 300° C bakeout.

It is energetically favourable for electrons to exist in a local electric minima due to the associated lower potential (Φ), which attracts electrons

$$\Phi_{well} = \left(\frac{\eta}{2\omega_0} \right)^2 \cdot |\mathbf{E}_{minima}|^2$$

$$\eta = \frac{e_e}{m_e} r_0(t)$$

Miller force

$$\ddot{r}_0(t) = -\nabla\Phi$$

‘Calm corners’ are eliminated by encouraging electron drift in the direction of the decaying field.

$$\Delta(R_{bp} - R_{ap}) = 12.5mm$$

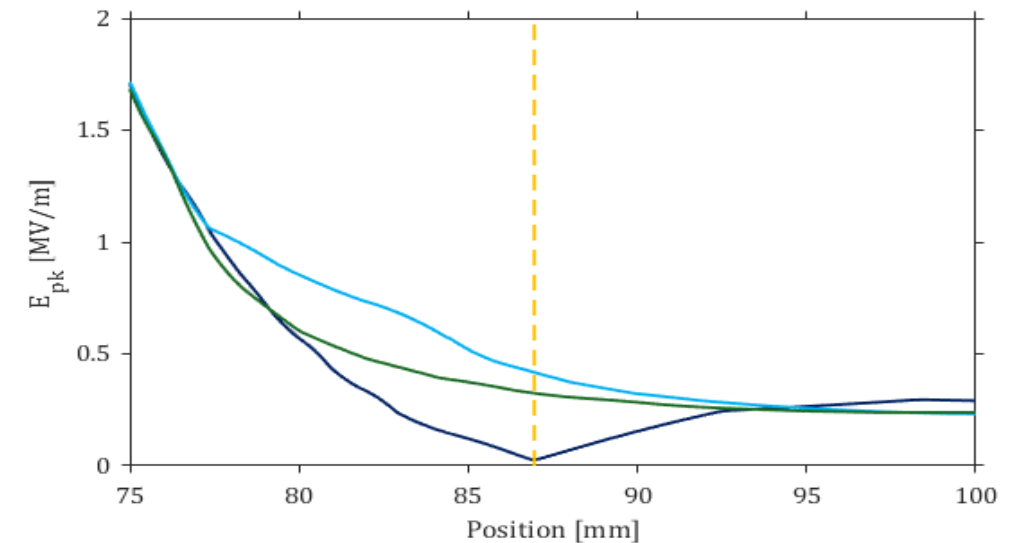
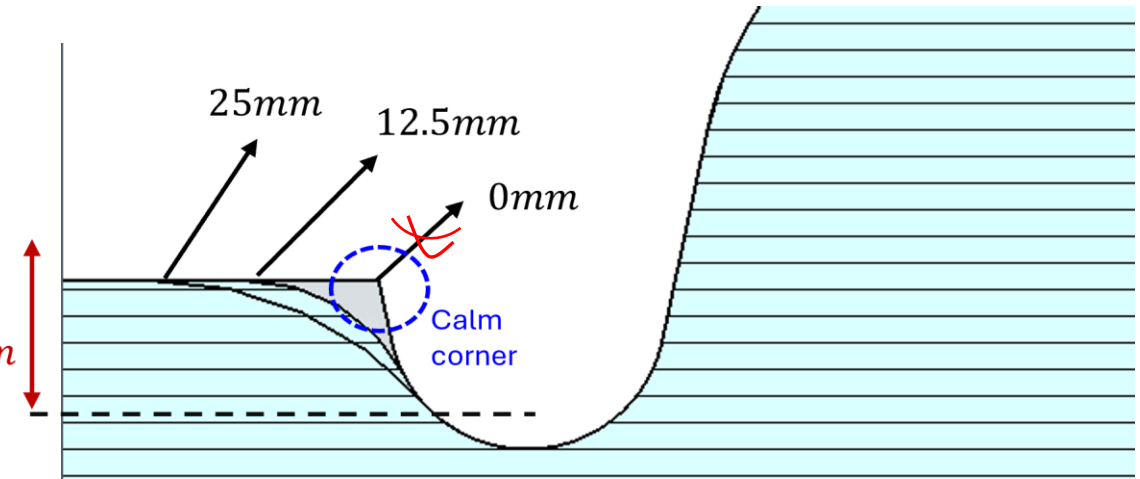


Figure 12: (top) Blend at beampipe-lip transition region of 0mm, 12.5mm, and 25mm (bottom) Elimination of electric field minima at MP location.

BBU ERL Parameters

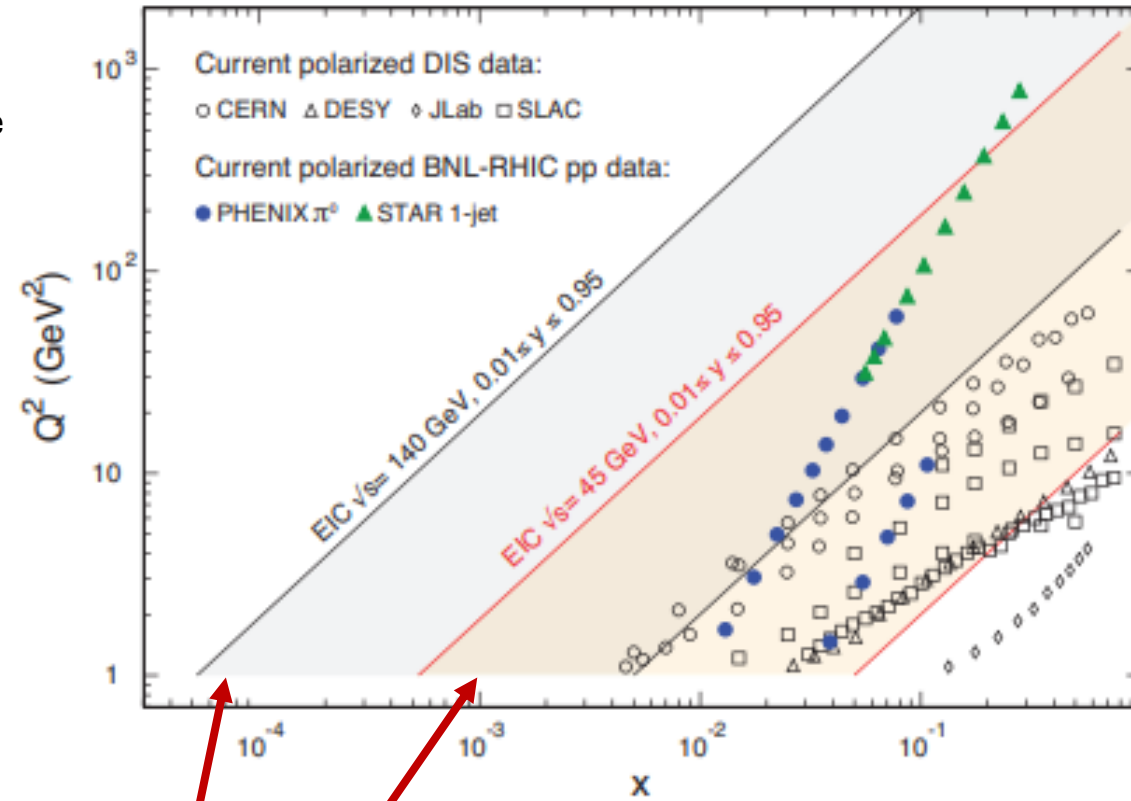
$$I_{th} = \frac{-2E}{\left(\frac{\omega}{c}\right) \left(\frac{R_{\perp}}{Q_0}\right) Q_l M_{12} \sin(\omega T_r)}$$

Cavity	Energy [MeV]	M_{12}	Energy [MeV]	M_{34}
1 st	25.052	-2.40998727	25.025	5.47015190
2 nd	26.462	-1.79034846	26.462	4.32234179
3 rd	27.899	0.9118264	27.889	3.61400521
4 th	29.336	2.89340006	29.336	3.77933567

Revolution time = 2.526298722343194e-06 s

EIC DIS data

Q^2 - negative squared four momentum transfer carried by the exchanged virtual photon during scattering event



Achieving this region requires **maximising total available energy at collision point**

x -fraction of proton momentum carried by a struck quark or gluon (Bjorken scaling variable)