Can local hidden-variable theories be excluded at colliders?

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20th March 2026

OUTLINE

What are Local Hidden Variable Theories?

Can LHVTs describe collider data?

Testing Locality vs Non-locality and
Entanglement vs Non-Entanglement at Colliders

OUTLINE

What are Local Hidden Variable Theories?

Historical Development

Beginning…

Paper by EPR
May 1935

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

… the description of reality as given by a wave function is not complete.

Reply by Bohr
October 1935

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

… quantum-mechanical description of physical phenomena would seem to fulfil, within its scope, all rational demands of completeness.

Historical Development

Gedankenexperiment

Proposed by
Bohm-Aharonov
November 1957

PHYSICAL REVIEW VOLUME 108, NUMBER 4 NOVEMBER 15, 1957

Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky

D. Bohm and Y. Aharonov
Technion, Haifa, Israel
(Received May 10, 1957)

A brief review of the physical significance of the paradox of Einstein, Rosen, and Podolsky is given, and it is shown that it involves a kind of correlation of the properties of distant noninteracting systems, which is quite different from previously known kinds of correlation. An illustrative hypothesis is considered, which would avoid the paradox, and which would still be consistent with all experimental results that have been analyzed to date. It is shown, however, that there already is an experiment whose significance with regard to this problem has not yet been explicitly brought out, but which is able to prove that this suggested resolution of the paradox (as well as a very wide class of such resolutions) is not tenable. Thus, this experiment may be regarded as the first clear empirical proof that the aspects of the quantum theory discussed by Einstein, Rosen, and Podolsky represent real properties of matter.

… (QM) involves a kind of correlation of the properties of distant noninteracting systems, which is quite different from previously known kinds of correlation.

An illustrative hypothesis is considered, which would avoid the (EPR) paradox, and which would still be consistent with all experimental results …

Bohm-Aharonov Thought Experiment

Spin-singlet state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Entangled system of two spin 1/2 particles

Measure S_z

$S_z = \uparrow$ 50%

$S_z = \downarrow$ 50%

Measure S_z

$S_z = \downarrow$

$S_z = \uparrow$

Bohm-Aharonov Thought Experiment

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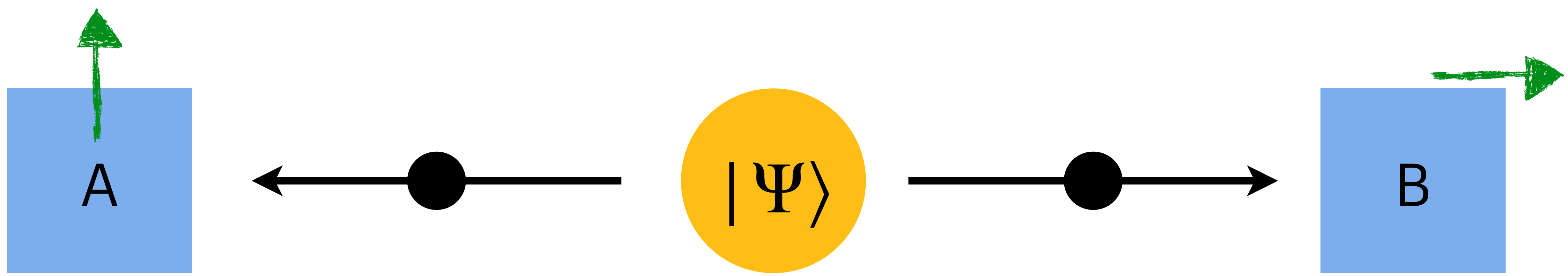
Entangled system of two spin 1/2 particles



Measure S_z

$S_z = \uparrow$ 50%

$S_z = \downarrow$ 50%

$$[S_x, S_z] \neq 0$$

$$|\hat{z}\pm\rangle = \frac{1}{\sqrt{2}}\left(|\hat{x}+\rangle \pm |\hat{x}-\rangle\right)$$

Measure S_x

$S_x = \leftarrow$ 50%

$S_x = \rightarrow$ 50%

Bohm-Aharonov Thought Experiment

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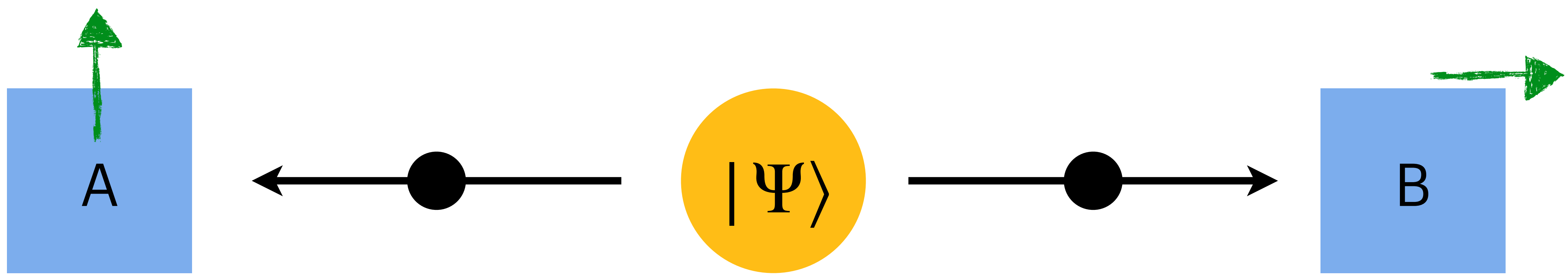

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Measure S_x

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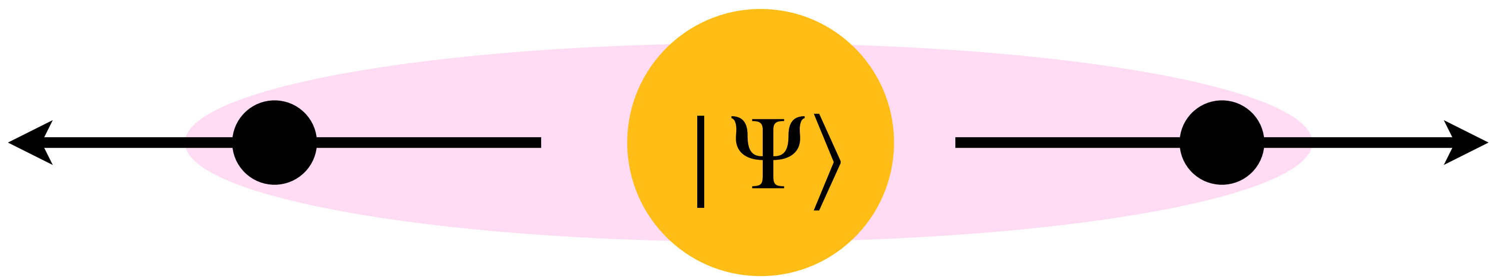
A "Non-Local" Correlation

between the two measurements

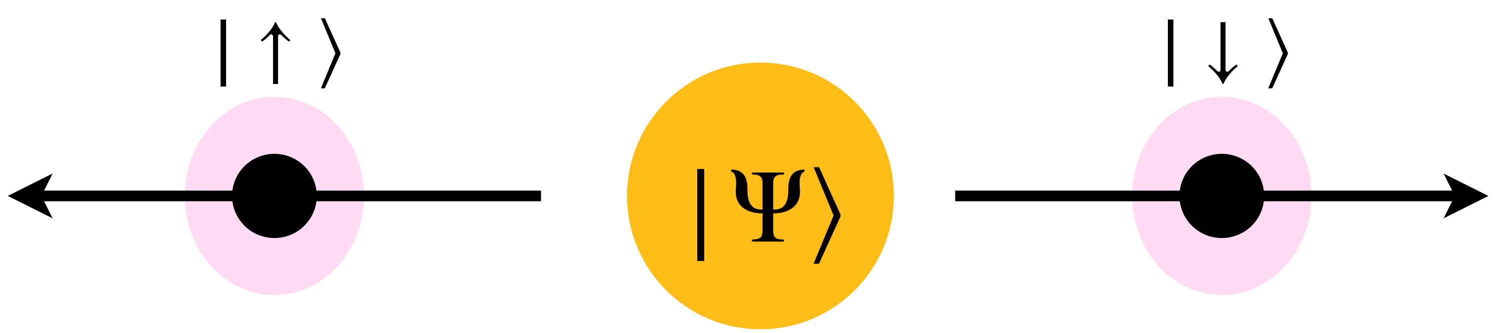
Local Hidden Variable Theories

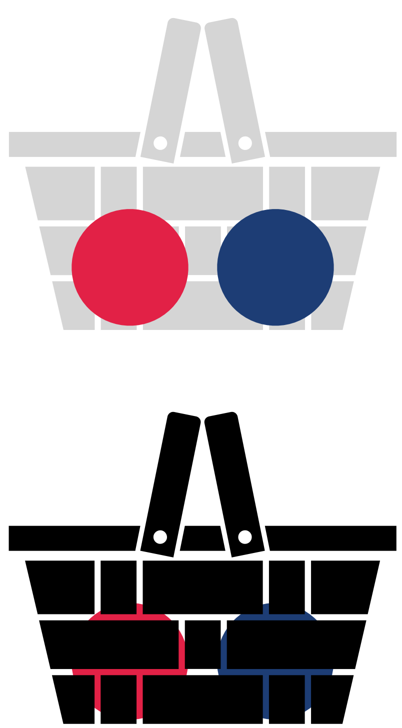


"Entangled" — information connected despite separation
State of the particle "not defined" unless measured.

Example:

Alternative...



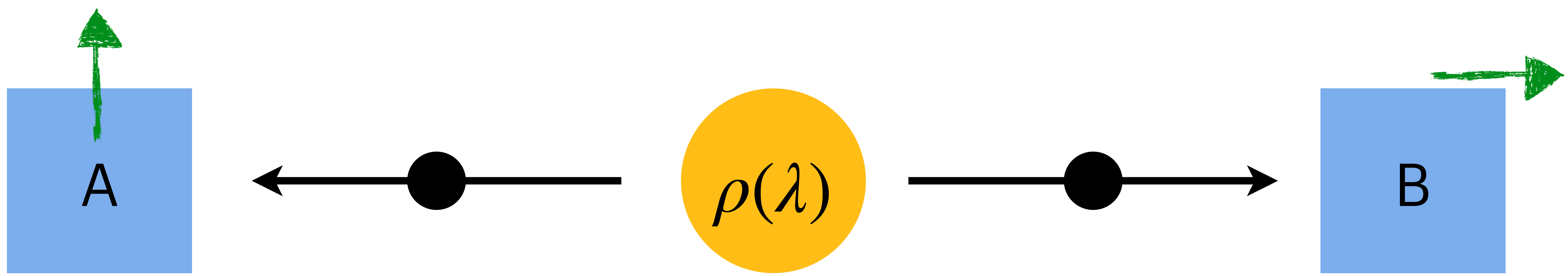


Particle produced with a definite state.
States are just anti-correlated.
Measurements independent of each other.

Local Hidden Variable Theories

Particles emitted with definite spins, *eg.*, ($S_z = \uparrow$, $S_x = \leftarrow$)

Definite results obtained due to additional hidden variables, λ (λ_i)



Measure S_z

$A(\hat{z}, \lambda) = \pm 1$

Measure S_x

$B(\hat{x}, \lambda) = \pm 1$

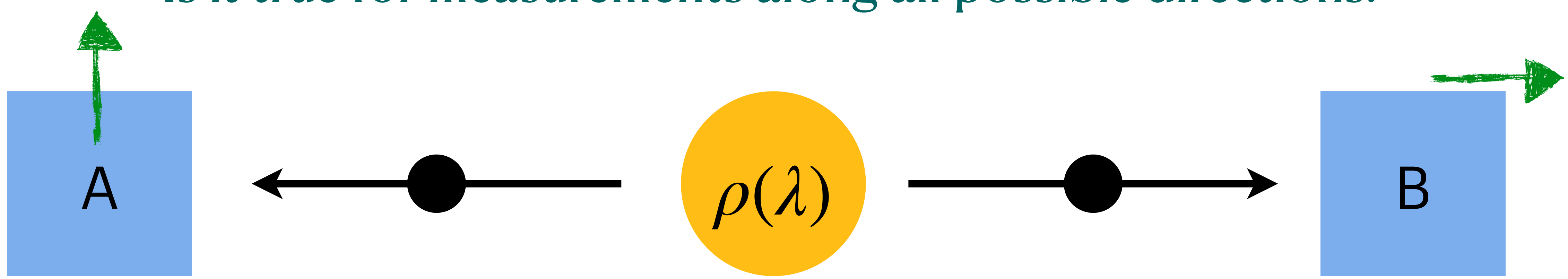
Can we reproduce the QM predictions with a local theory with hidden variables?

Local Hidden Variable Theories

Can we reproduce the QM predictions with a local theory with hidden variables?

In this particular case, YES!

Is it true for measurements along all possible directions?



Measure S_z

$A(\hat{z}, \lambda) = \pm 1$

$$(S_z = \uparrow\,,\ S_x = \leftarrow) \Leftrightarrow (S_z = \downarrow\,,\ S_x = \rightarrow)$$

$$(S_z = \uparrow\,,\ S_x = \rightarrow) \Leftrightarrow (S_z = \downarrow\,,\ S_x = \leftarrow)$$

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$$(S_z = \downarrow\,,\ S_x = \rightarrow) \Leftrightarrow (S_z = \uparrow\,,\ S_x = \leftarrow)$$

Measure S_x

$B(\hat{x}, \lambda) = \pm 1$

Bell's Inequality

Incompatibility of LHVT and QM

Proved by Bell
November 1964

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

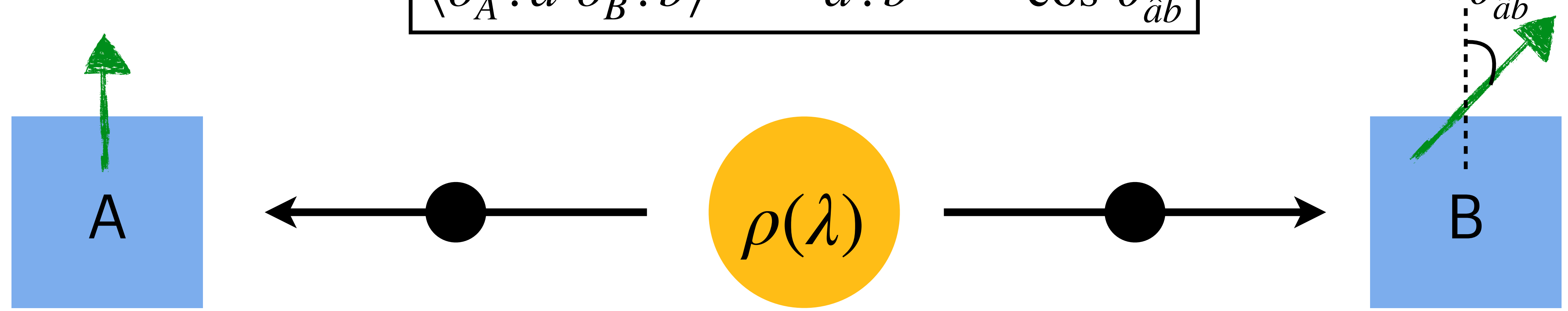
I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

Bell's Inequality

QM correlation between
the spin measurements

$$\langle \vec{\sigma}_A \,.\, \hat{a} \;\; \vec{\sigma}_B \,.\, \hat{b} \rangle = -\, \hat{a} \,.\, \hat{b} = -\cos\, \theta_{\hat{a}\hat{b}}$$



Measure spin
along $\hat{a}$

$$A(\hat{a}, \lambda) = \pm\, 1$$

Correlation in LHVT

$$P(\hat{a}, \hat{b}) = \int d\lambda \rho(\lambda) A(\hat{a}, \lambda) B(\hat{b}, \lambda)$$

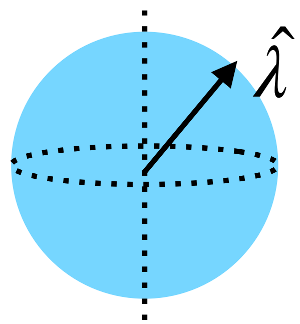
$$\int d\lambda \rho(\lambda) = 1$$

$$P(\hat{a}, \hat{b}) = -\, 1 + \frac{2}{\pi} \theta_{\hat{a}\hat{b}}$$

Measure spin
along $\hat{b}$

$$B(\hat{b}, \lambda) = \pm\, 1$$

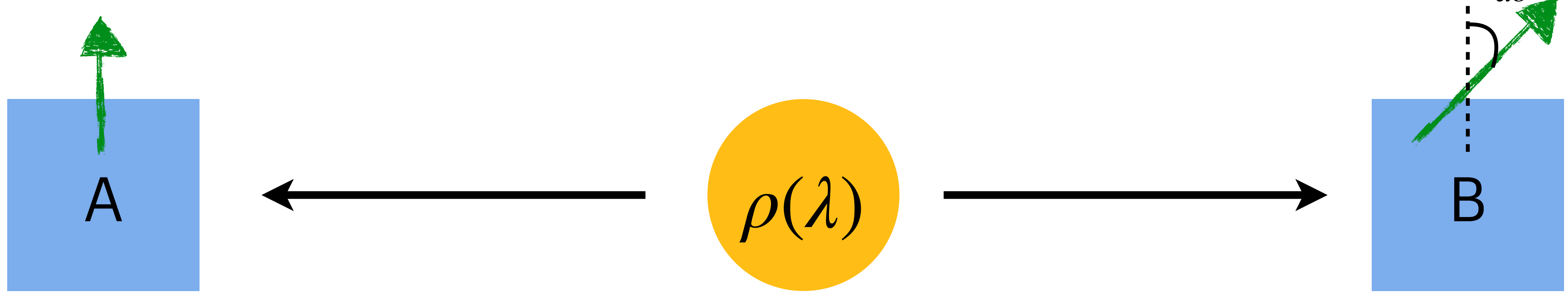
Bell's Inequality



$$\rho(\lambda) = \frac{1}{4\pi}$$

$$A(\hat{a}, \hat{\lambda}) = \text{sign } \hat{a} \,.\, \hat{\lambda}$$

$$B(\hat{b}, \hat{\lambda}) = -\text{ sign } \hat{b} \,.\, \hat{\lambda}$$



Measure spin along $\hat{a}$

$$A(\hat{a}, \lambda) = \pm 1$$

Correlation in LHVT

$$P(\hat{a}, \hat{b}) = -1 + \frac{2}{\pi}\theta_{\hat{a}\hat{b}}$$

Measure spin along $\hat{b}$

$$B(\hat{b}, \lambda) = \pm 1$$

QM correlation

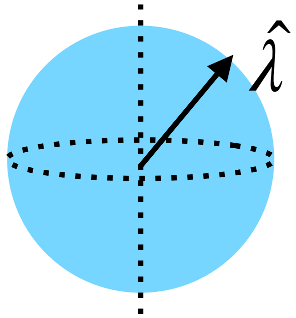
$$\langle \vec{\sigma}_A \,.\, \hat{a} \; \vec{\sigma}_B \,.\, \hat{b} \rangle = -\cos \theta_{\hat{a}\hat{b}}$$

matches when

$$P(\hat{a}, \hat{a}) = -P(\hat{a}, -\hat{a}) = -1$$

$$P(\hat{a}, \hat{b}) = 0 \text{ , if } \hat{a} \,.\, \hat{b} = 0$$

Bell's Inequality



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$$A(\hat{a}, \hat{\lambda}) = \text{sign } \hat{a}\,.\,\hat{\lambda}$$

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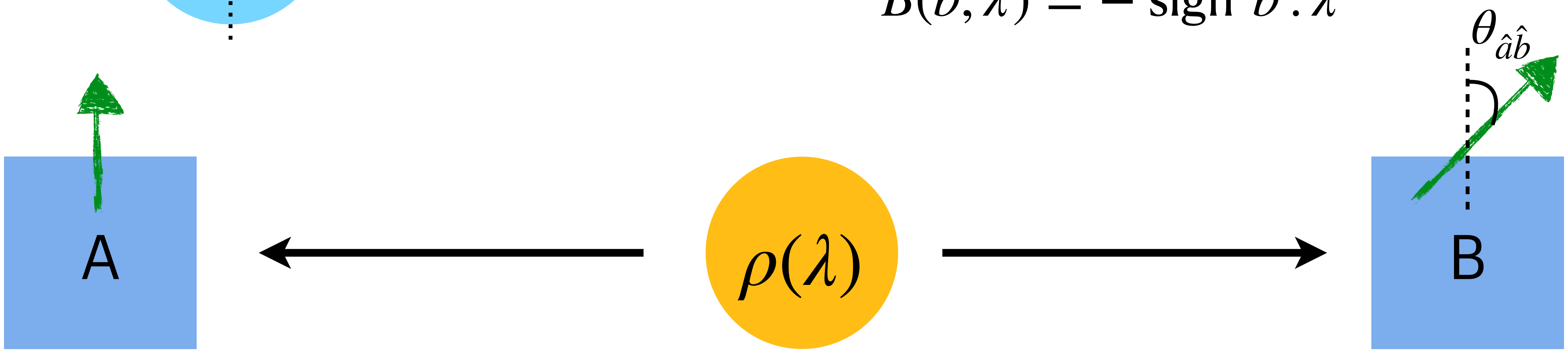


Measure spin along $\hat{a}$

$$A(\hat{a}, \lambda) = \pm 1$$

Correlation in LHVT

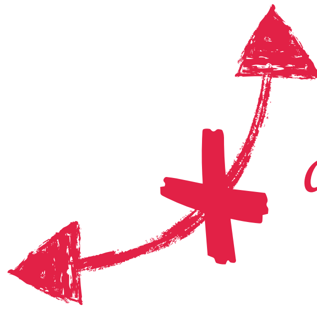
$$P(\hat{a}, \hat{b}) = -1 + \frac{2}{\pi}\theta_{\hat{a}\hat{b}}$$

Measure spin along $\hat{b}$

$$B(\hat{b}, \lambda) = \pm 1$$

QM correlation

$$\langle \vec{\sigma}_A\,.\,\hat{a}\ \vec{\sigma}_B\,.\,\hat{b} \rangle = -\cos\,\theta_{\hat{a}\hat{b}}$$

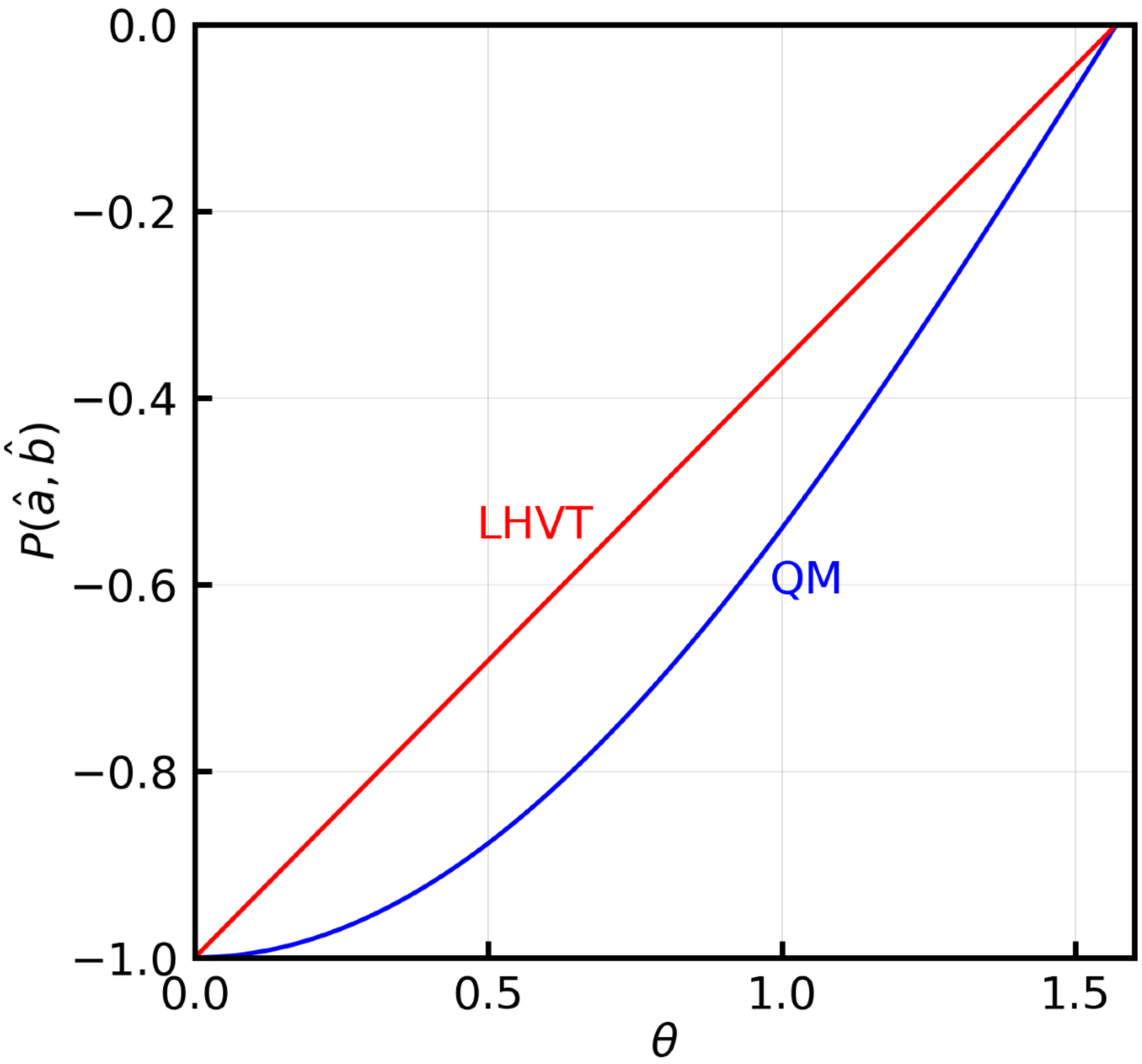


does not match

$$P_{\text{QM}}(\hat{a}, \hat{b}) = -\frac{1}{\sqrt{2}}$$

$$P_{\text{LHVT}}(\hat{a}, \hat{b}) = -\frac{1}{2}\,,\ \text{if } \theta_{\hat{a}\hat{b}} = \frac{\pi}{4}$$

Bell's Inequality



Bell's Inequality

$$1 + P(\hat{b}, \hat{c}) \geq |P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{c})|$$

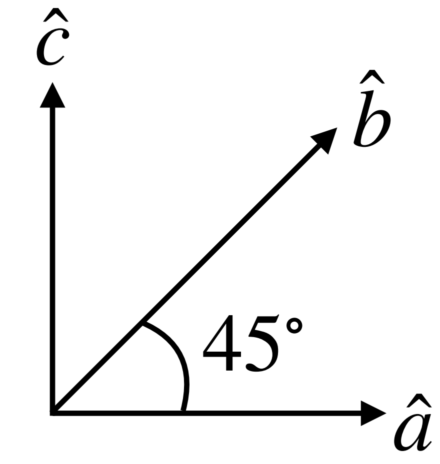
- Bell's inequality is satisfied by all Local Hidden Variable Theories (LHVT)*.

* Role of momentum cuts discussed later.

- Quantum mechanics can satisfy or violate the above inequality.

$$1 - \cos\,\theta_{\hat{b}\hat{c}} \geq |\cos\,\theta_{\hat{a}\hat{b}} - \cos\,\theta_{\hat{a}\hat{c}}|$$

$$1 - \frac{1}{\sqrt{2}} \not\geq \frac{1}{\sqrt{2}}$$



- Therefore, QM $\neq$ LHVT

The Photon Experiments

Clauser, Horne, Shimony, Holt

August 1969

PRL 23, 880 (1969)

The CHSH inequality:

$$|P(\hat{a},\hat{b}) - P(\hat{a}',\hat{b}) + P(\hat{a},\hat{b}') + P(\hat{a}',\hat{b}')| \leq 2$$

Freedman and Clauser

February 1972

PRL 28, 938 (1972)

Observation of violation of Bell's inequality using photons

Aspect, Dalibard, Roger

September 1982

PRL 49, 1804 (1982)

Locality loophole closed in tests of Bell's inequality using photons

Pan, Bouwmeester, Weinfurter, and Zeilinger

May 1998

PRL 80, 3891 (1998)

The Photon Experiments

Clauser, Horne, Shimony, Holt
August 1969
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May 1998
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Nobel Prize in 2022

OUTLINE

What are Local Hidden Variable Theories?

- *Local theories with additional hidden variables* that produce *correlated measurements*, yet are *inherently different from QM*, as demonstrated by Bell.

Can LHVTs describe collider data?

Higher Energies? And Fermions?

The only tests of Bell's inequality violation were done with photons in the eV energy range.

- *Does anything change at higher energies or correspondingly shorter length scales?*

- *Could fermions be different from bosons in this context?*

** Stern-Gerlach won't work for charged spin-1/2 particles!
The Lorentz force acting on charged particles masks the spin-dependent splitting.

Colliders

Fermion Pairs at Colliders

$e^+e^- \to Z^0 \to \tau^+\tau^-$ Correlated τ-spins

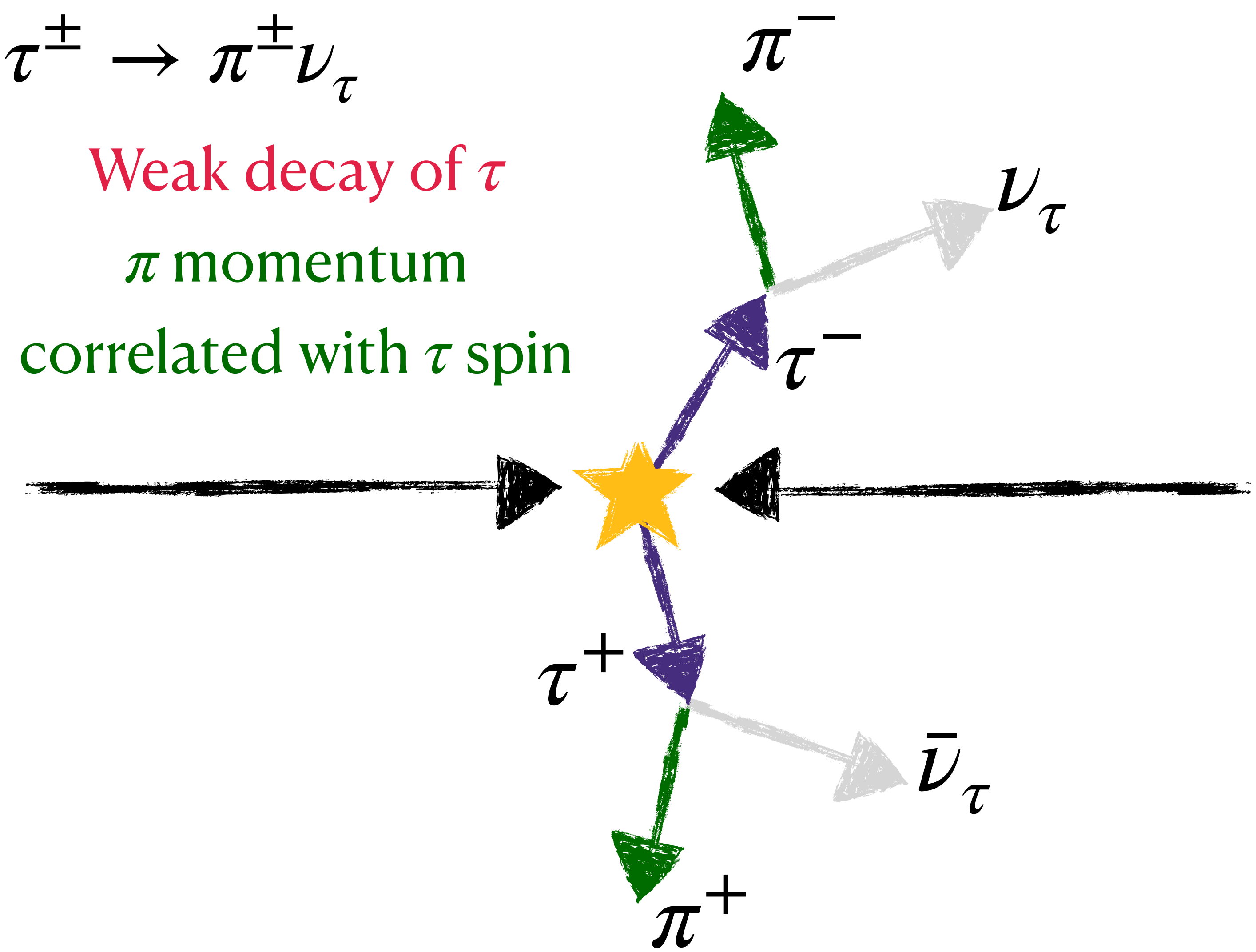
Testing locality at colliders via Bell's inequality?
Abel, Dittmar and Dreiner, PLB 280 (1992) 304-312

$\tau^\pm \to \pi^\pm \nu_\tau$

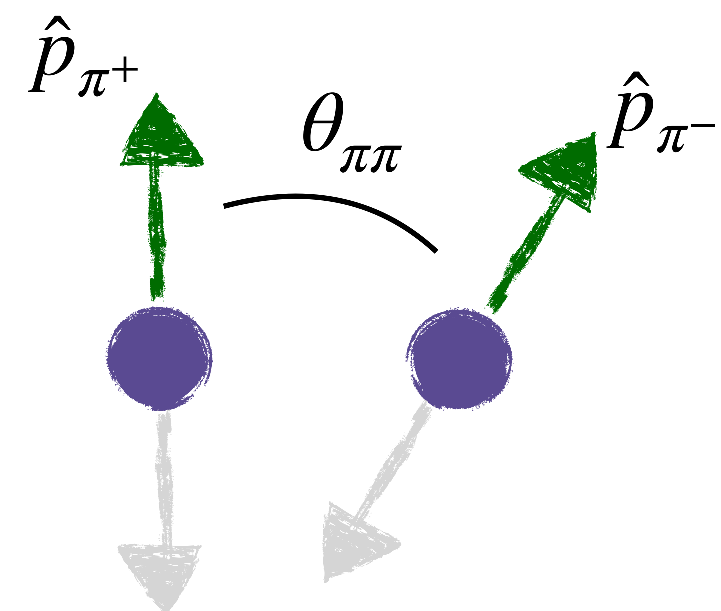
Weak decay of τ

π momentum

correlated with τ spin



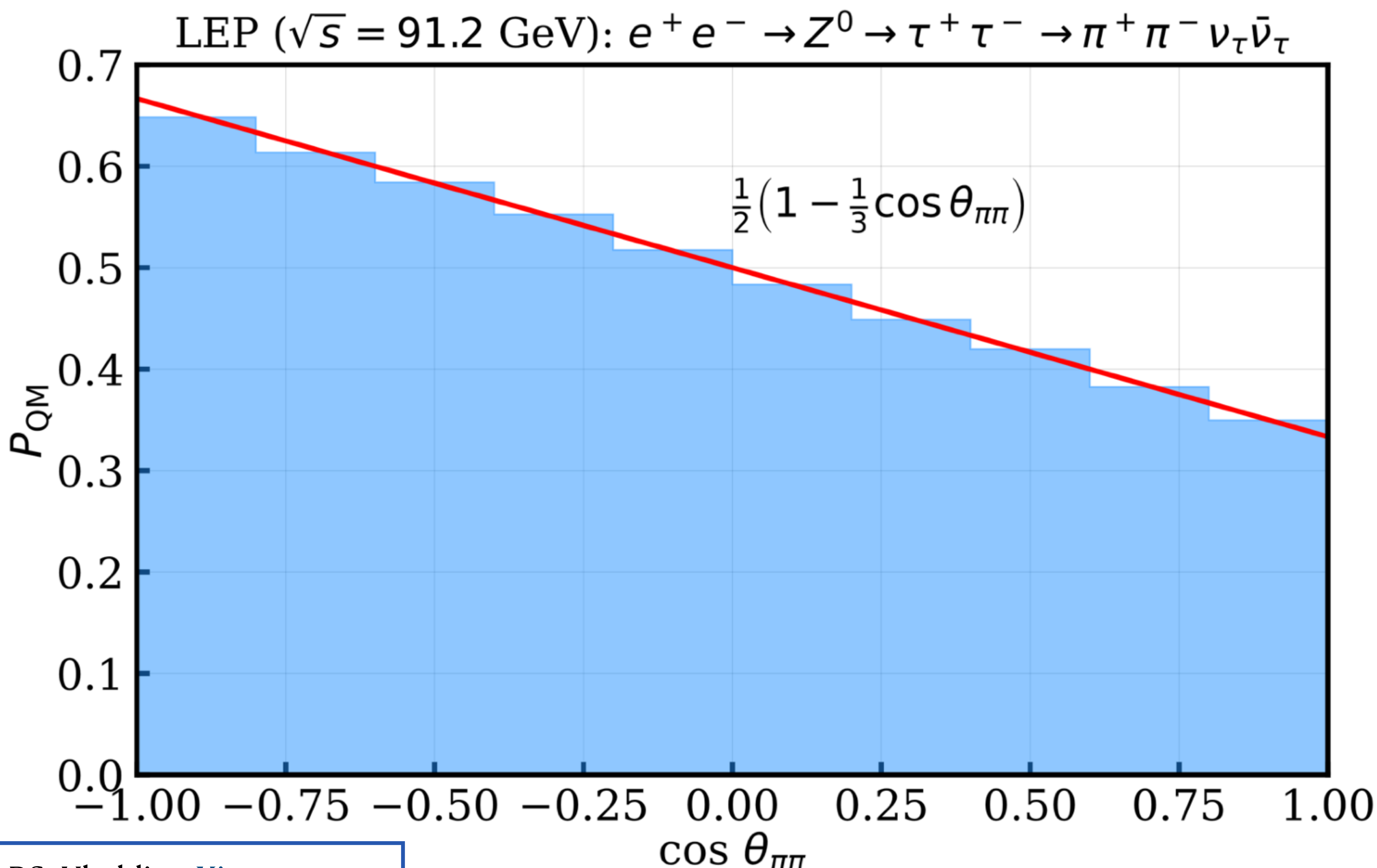
Boosting each pion to the respective tau lepton rest frame



From Standard Model, $$\frac{d\sigma}{d\cos\theta_{\pi\pi}}(e^+e^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau) = A(1 - \frac{1}{3}\cos\theta_{\pi\pi})$$

$e^+e^- \to Z^0 \to \tau^+\tau^-$

$$P_{\rm QM}(Z \to \tau\tau) \equiv \frac{d\sigma/d\cos\theta_{\pi\pi}(e^+e^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau)}{\sigma(e^+e^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau)} = \frac{1}{2}\left(1 - \frac{1}{3}\cos\theta_{\pi\pi}\right)$$



Abel, Dreiner, RS, Ubaldi, arXiv:2507.15949

$e^+e^- \to Z^0 \to \tau^+\tau^-$

Bell's Inequality?

$$P_{\text{QM}}(Z \to \tau\tau) = \frac{1}{2}\left(1 - \frac{1}{3}\cos\theta_{\pi\pi}\right)$$

$$1 + P(\hat{b}, \hat{c}) \geq |P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{c})|$$

$$1 + \frac{1}{2} - \frac{1}{6}\cos\,\theta_{\hat{b}\hat{c}} \geq |-\frac{1}{6}\cos\,\theta_{\hat{a}\hat{b}} + \frac{1}{6}\cos\,\theta_{\hat{a}\hat{c}}|$$

$$9 - \cos\,\theta_{\hat{b}\hat{c}} \geq |\cos\,\theta_{\hat{a}\hat{b}} - \cos\,\theta_{\hat{a}\hat{c}}|$$

Inequality satisfied for all values of θ

* Also satisfies the CHSH inequality: $|P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{b}') + P(\hat{a}', \hat{b}) + P(\hat{a}', \hat{b}')| \leq 2$

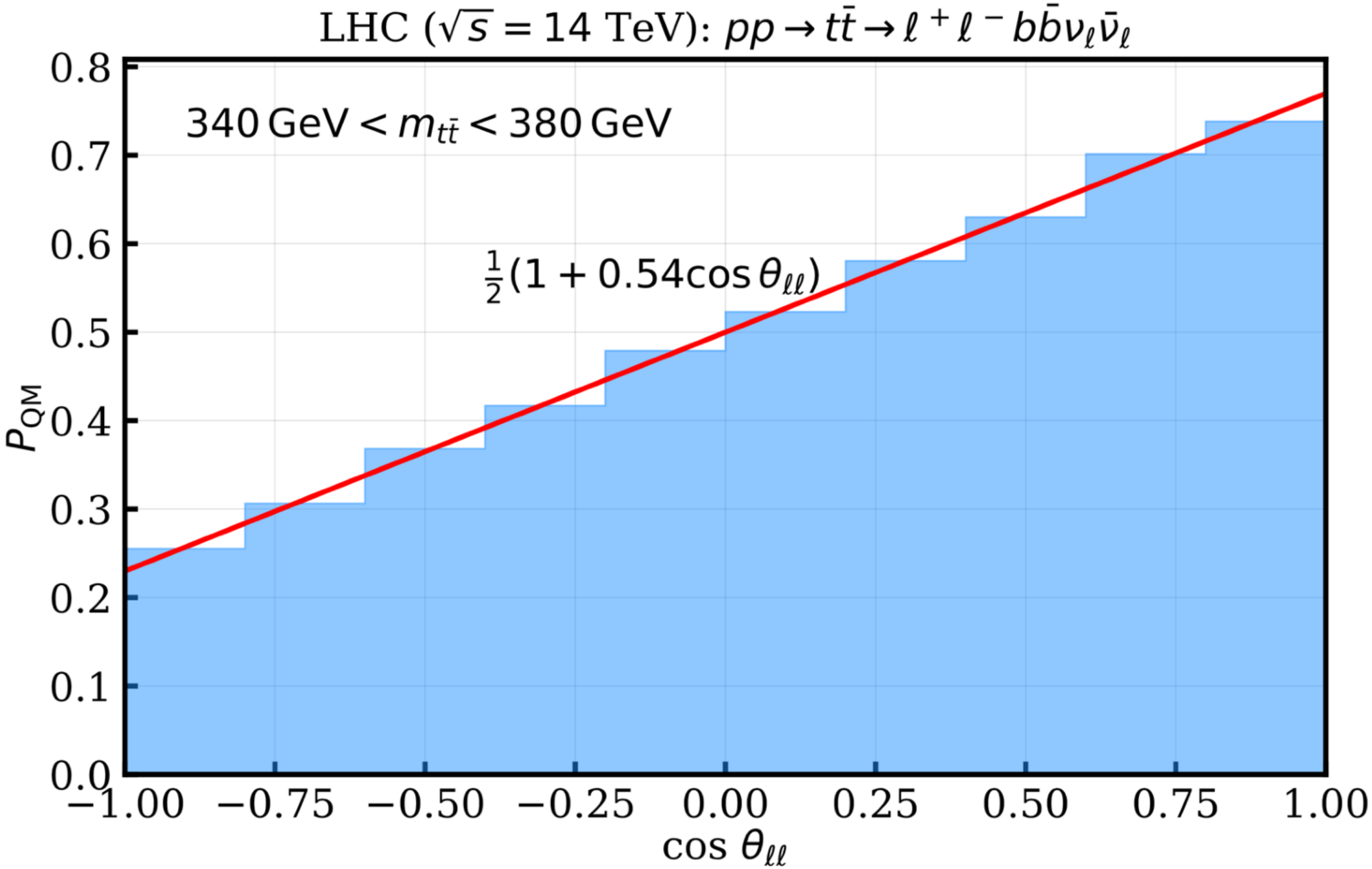
$pp \to t\bar{t}$

$t \to b\ell\nu_\ell$

Bell's inequality satisfied for all values of θ

CHSH inequality also satisfied

Abel, Dreiner, RS, Ubaldi, arXiv:2507.15949



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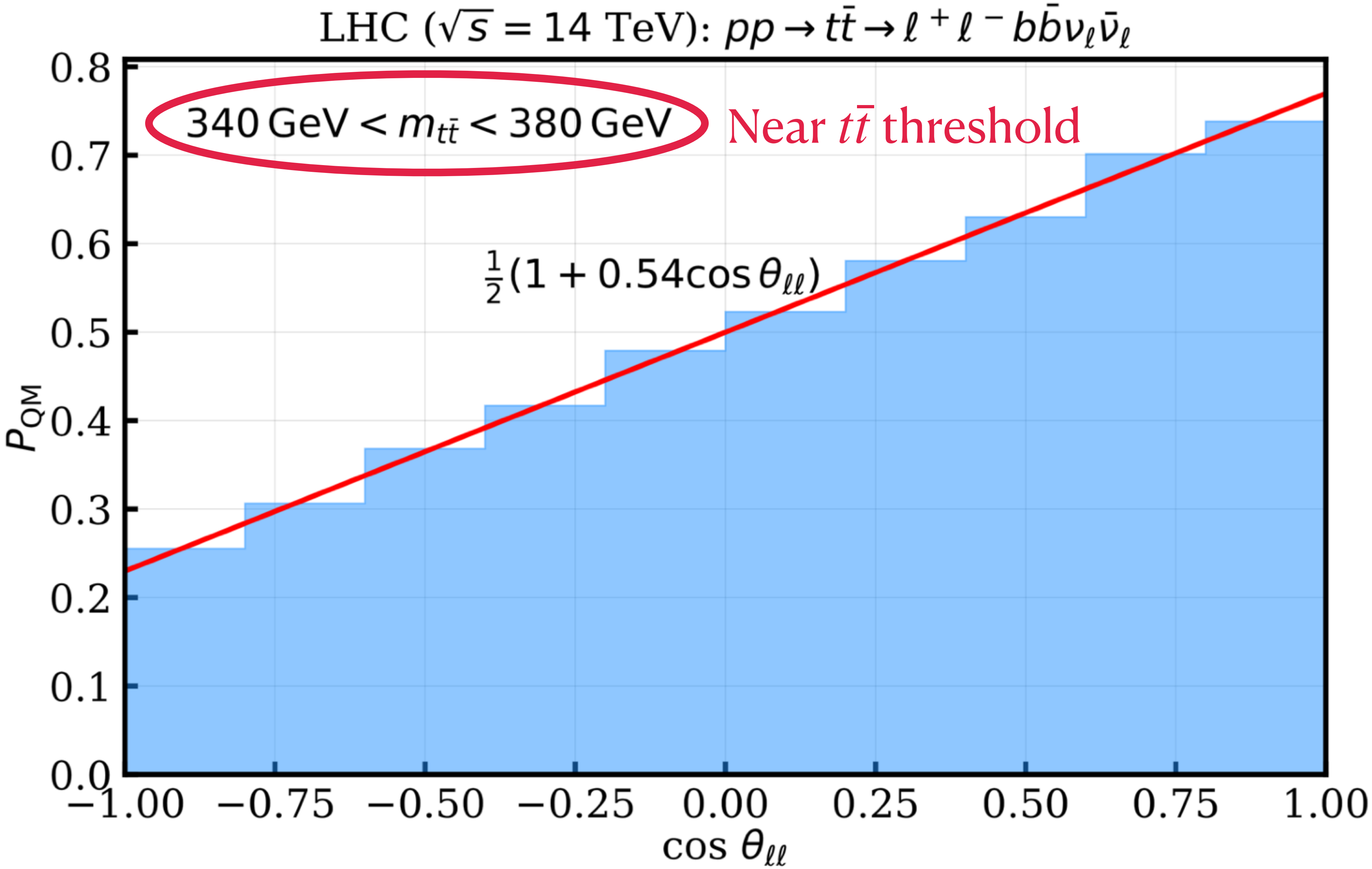

OUTLINE

What are Local Hidden Variable Theories?

- *Local theories with additional hidden variables* that produce *correlated measurements*, yet are *inherently different from QM*, as demonstrated by Bell.

Can LHVTs describe collider data?

- *Yes.*
- *But how?* We know the collider data matches with our QM expectation!

Kasday's Construction

L. Kasday, Experimental test of quantum predictions for widely separated photons, (1971) 195

Compton scattering of photons

Detectors A and B — if they detect the scattered photon, assign +1
else, assign -1

QM provides the probability distribution of scattered photons: $F(\hat{p}_a, \hat{p}_b)$

Can we write an LHVT to explain the data?

Let us identify the hidden variables as the directions of the scattered photons

$$\hat{\lambda}_a, \hat{\lambda}_b \equiv \hat{p}_a, \hat{p}_b$$

$$A(\hat{p}_a, \hat{\lambda}_a) = \delta(\hat{p}_a - \hat{\lambda}_a) \qquad B(\hat{p}_b, \hat{\lambda}_b) = \delta(\hat{p}_b - \hat{\lambda}_b)$$

Assign the hidden variables the same probability distribution as that of the momenta of the scattered photons: $\rho(\hat{\lambda}_a, \hat{\lambda}_b) \equiv F(\hat{\lambda}_a, \hat{\lambda}_b)$

This construction of an LHVT can explain the experimental data!

Why and when can we do this?

REVISIT

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Assign the hidden variables the same probability distribution as that of the momenta of the scattered photons: $\rho(\hat{\lambda}_a, \hat{\lambda}_b) \equiv F(\hat{\lambda}_a, \hat{\lambda}_b)$

This construction of an LHVT can explain the experimental data!

Why and when can we do this?

QM provides the probability distribution of scattered photons: $F(\hat{p}_a, \hat{p}_b)$

This is only true when the measured quantities correspond to commuting observables, like momenta.

Otherwise, QM does not provide a joint probability distribution function for non-commuting observables, like S_x, S_y, S_z.

The photon experiments measured the spin of the photons using polarisers. Therefore, such a construction of LHVT is not possible to explain the data, and the data is consistent with QM.

$e^+e^- \to Z^0 \to \tau^+\tau^-$

LHVT: Applying Kasday's construction

Testing locality at colliders via Bell's inequality?
Abel, Dittmar and Dreiner, PLB 280 (1992) 304-312

$$P_{\rm QM}(Z \to \tau\tau) \equiv \frac{d\sigma/d\cos\theta_{\pi\pi}(e^+e^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau)}{\sigma(e^+e^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau)} = \frac{1}{2}\left(1 - \frac{1}{3}\cos\theta_{\pi\pi}\right)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-}(e^+e^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau) \equiv f(\hat{p}_{\pi^+}, \hat{p}_{\pi^-})$$

We know this joint probability distribution since

$$[(\hat{p}_{\pi^\pm})_i, (\hat{p}_{\pi^\pm})_j] = 0\,,\ \forall\, i, j$$
$$[(\hat{p}_{\pi^+})_i, (\hat{p}_{\pi^-})_j] = 0\,,\ \forall\, i, j$$

Each τ carries a set of hidden variables, $\{\hat{\lambda}_e, \hat{\lambda}_\mu, \hat{\lambda}_\pi, \hat{\lambda}_\rho, \ldots\}$, depending on the decay mode.

For the decay $\tau^\pm \to \pi^\pm\nu_\tau$, the hidden variable $\hat{\lambda}_{\pi^\pm}$ tells it to decay such that $\hat{p}_{\pi^\pm} \equiv \hat{\lambda}_{\pi^\pm}$

We can make the following assignments:

$$\hat{p}_{\pi^\pm} \equiv \hat{\lambda}_{\pi^\pm} \qquad \rho(\hat{\lambda}_{\pi^+}, \hat{\lambda}_{\pi^-}) \equiv f(\hat{p}_{\pi^+}, \hat{p}_{\pi^-})$$

The resulting LHVT can describe the collider data.

$e^+e^- \to Z^0 \to \tau^+\tau^-$

How to get the spin correlation of the τ pair?

Testing locality at colliders via Bell's inequality?
Abel, Dittmar and Dreiner, PLB 280 (1992) 304-312

The QM expectation includes *s* and *p*-wave scattering for a system of two spin-½ particles.

$$P_{\mathrm{QM}} = c_1 + c_2 \underbrace{\langle (\vec{\sigma}_{\tau^+} . \hat{p}_{\pi^+})(\vec{\sigma}_{\tau^-} . \hat{p}_{\pi^-}) \rangle}_{P_{\sigma^\tau \sigma^\tau}}$$

$$P_{\sigma^\tau \sigma^\tau} \equiv 6 \left[P_{\mathrm{QM}}(Z \to \tau\tau) - \frac{1}{2} \right] = -\cos\theta_{\pi\pi}$$

$P_{\sigma^\tau \sigma^\tau}$ violates Bell's Inequality.

But in deriving the form of $P_{\sigma^\tau \sigma^\tau}$, we have already used QM.

Therefore, this cannot be considered a test of QM!

OUTLINE

What are Local Hidden Variable Theories?

- *Local theories with additional hidden variables* that produce *correlated measurements*, yet are *inherently different from QM*, as demonstrated by Bell.

Can LHVTs describe collider data?

- *Yes.*
- One can *always construct an LHVT to describe the collider data* when measuring *commuting observables* — follows from Kasday's construction.

Testing Locality vs Non-locality and Entanglement vs Non-Entanglement at Colliders

$pp \to t\bar{t}$

Spin density matrix:

$$\rho_{t\bar{t}} = \frac{1}{4}\left(\mathbf{I}_4 + B_i^+\sigma_i \otimes \mathbf{I}_2 + B_i^-\mathbf{I}_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j\right)$$

$\mathbf{I}_n$: $n \times n$ identity matrix

σ_i : Pauli matrices for the top spins

$\mathbf{B}^+$ and $\mathbf{B}^-$: spin polarization vectors of the top and anti-top quarks

$\mathbf{C}$: matrix encoding the spin correlation between the two particles

Entanglement marker: $D \equiv \mathbf{Tr}[\mathbf{C}]/3$

Peres-Horodecki Criterion for Entanglement: $D < -\frac{1}{3}$

Derived from the non-separability criterion for density matrices

A. Peres, PRL 77, 1413–1415 (1996)

P. Horodecki, PLA 232, 333 (1997)

Only applicable to spin density matrix

$pp \to t\bar{t}$

Colliders:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+\Omega_-}(pp \to t\bar{t} \to b\ell^+\nu_\ell \bar{b}\ell^-\bar{\nu}_\ell) = \frac{\left(1 + \overline{\mathbf{B}}^+ \cdot \hat{\mathbf{q}}_+ - \overline{\mathbf{B}}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot \overline{\mathbf{C}} \cdot \hat{\mathbf{q}}_-\right)}{4\pi^2}$$

$\hat{\mathbf{q}}_\pm$: final state anti-lepton and lepton momentum directions in the rest frame of their parent top quark

$\Omega_\pm$: the solid angle associated with the anti-lepton and the lepton momenta

$\overline{\mathbf{B}}^\pm, \overline{\mathbf{C}}$: extracted from a fit to the measurements at colliders

The analysis reconstructs the top quarks from the detector-level charged leptons, jets and MET

ATLAS, Nature 633, pages 542–547 (2024)

Ellipse method, a geometric approach to analytically calculate the neutrino momenta

Betchart, Demina, Harel, Nucl. Instrum. Meth. A 736, 169–178 (2014)

$\overline{D} \equiv \mathbf{Tr}[\overline{\mathbf{C}}]/3$

$\overline{D} = -0.537 \pm 0.002(\text{stat.}) \pm 0.019(\text{syst.})$

$pp \to t\bar{t}$

Identify $\mathbf{B}^{\pm} = \overline{\mathbf{B}}^{\pm}$ and $\mathbf{C} = \overline{\mathbf{C}}$

Then $D = \overline{D}$

$$D = -0.537 \pm 0.002(\text{stat.}) \pm 0.019(\text{syst.})$$

ATLAS, Nature 633, pages 542–547 (2024)

$D < -\frac{1}{3}$

"Entangled"

The *maximal parity-violating* nature of the electroweak charged current interactions in the SM ⇒

the final-state leptons carry the full spin information of their parent top quarks, i.e. the analysing power is 1 *within the SM*.

Can we do this assignment without assuming QM?

P. Bechtle, C. Breuning, H. K. Dreiner, C. Duhr, arXiv:2507.15947

The Case of Massive Vector Bosons

Massive vector bosons have 3 possible outcomes!

The Collins-Gisin-Linden-Massar-Popescu (CGLMP) Inequality

D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, PRL 88 (2002), no. 4 040404

$$I_3 \equiv +[P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)]$$
$$-[P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)] \leq 2$$

However the CGLMP inequality refers directly to spin measurements (in our case with results $\pm 1, 0$)

Not angles of polarimeter settings, $\hat{a}, \hat{b}, \hat{c}$, etc., as for Bell.

We applied the Bell's inequality to only the transverse spin components of W/Z bosons
$\Rightarrow$ satisfies the inequality

Kasday's LHVT construction should be applicable here as well!

Can LHVT's Violate Bell's Inequality?

Let us start with a possible LHVT:

$$P(\hat{\mathbf{p}}_a \mid \hat{\boldsymbol{\lambda}}) = \frac{1}{\sqrt{2}}\left(1 + i\sqrt{3c}(\hat{\mathbf{p}}_a \cdot \hat{\boldsymbol{\lambda}})\right) \qquad \rho(\hat{\boldsymbol{\lambda}}) = \frac{1}{4\pi}$$

$$\begin{aligned} P(\hat{\mathbf{p}}_a, \hat{\mathbf{p}}_b) &= \int d^2\hat{\lambda}\, \frac{1}{8\pi}\left(1 - i\sqrt{3c}(\hat{\mathbf{p}}_a \cdot \hat{\boldsymbol{\lambda}}) - i\sqrt{3c}(\hat{\mathbf{p}}_b \cdot \hat{\boldsymbol{\lambda}}) - 3c(\hat{\mathbf{p}}_a \cdot \hat{\boldsymbol{\lambda}})(\hat{\mathbf{p}}_b \cdot \hat{\boldsymbol{\lambda}})\right) \\ &= \frac{1}{2}\left(1 - c\ \hat{\mathbf{p}}_a \cdot \hat{\mathbf{p}}_b\right) \end{aligned}$$

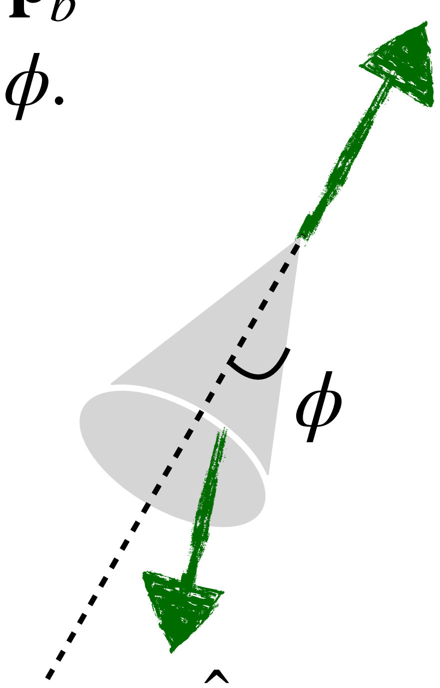
This reproduces similar form of normalised differential distribution that we obtained for $t\bar{t}$ and $\tau^+\tau^-$

But in collider measurements, we often put momentum cuts.

How does the scenario change?

Can LHVT's Violate Bell's Inequality?

We measure particles along $\hat{\mathbf{p}}_a$ and only accept particles along $\hat{\mathbf{p}}_b$ that lie in a backwards cone from $\hat{\mathbf{p}}_a$ with some opening angle ϕ.



$$P(\hat{\mathbf{p}}_b \mid \hat{\lambda}) = \frac{1}{\sqrt{2}}\left(1 + i\sqrt{3c}\,(\hat{\mathbf{p}}_b \cdot \hat{\lambda})\right)\Theta(\cos\phi - \cos(\pi - \theta_{ab}))$$

$$= \frac{1}{\sqrt{2}}\left(1 + i\sqrt{3c}\,(\hat{\mathbf{p}}_b \cdot \hat{\lambda})\right)\Theta(\cos\phi + \cos\theta_{ab})$$

$$\rho(\hat{\lambda}) = \Theta(\cos\phi + \cos\theta_{ab})\,\frac{1}{2\pi(1-\cos\phi)}$$

$$P'(\hat{\mathbf{p}}_a, \hat{\mathbf{p}}_b) = \frac{\Theta(\cos\phi + \cos\theta_{ab})}{1-\cos\phi}\left(1 - c\;\hat{\mathbf{p}}_a \cdot \hat{\mathbf{p}}_b\right)$$

Non-local cut — one measurement depends on the setting of another

$$1 \geq \frac{|1 - c\cos\theta_{ac}|}{1-\cos\phi}$$

Does not satisfy Bell's inequality when $\phi \to 0$

Conclusion

Testing Locality vs Non-locality and Entanglement vs Non-Entanglement at Colliders

"The observed result is more than five standard deviations from a scenario without entanglement…"

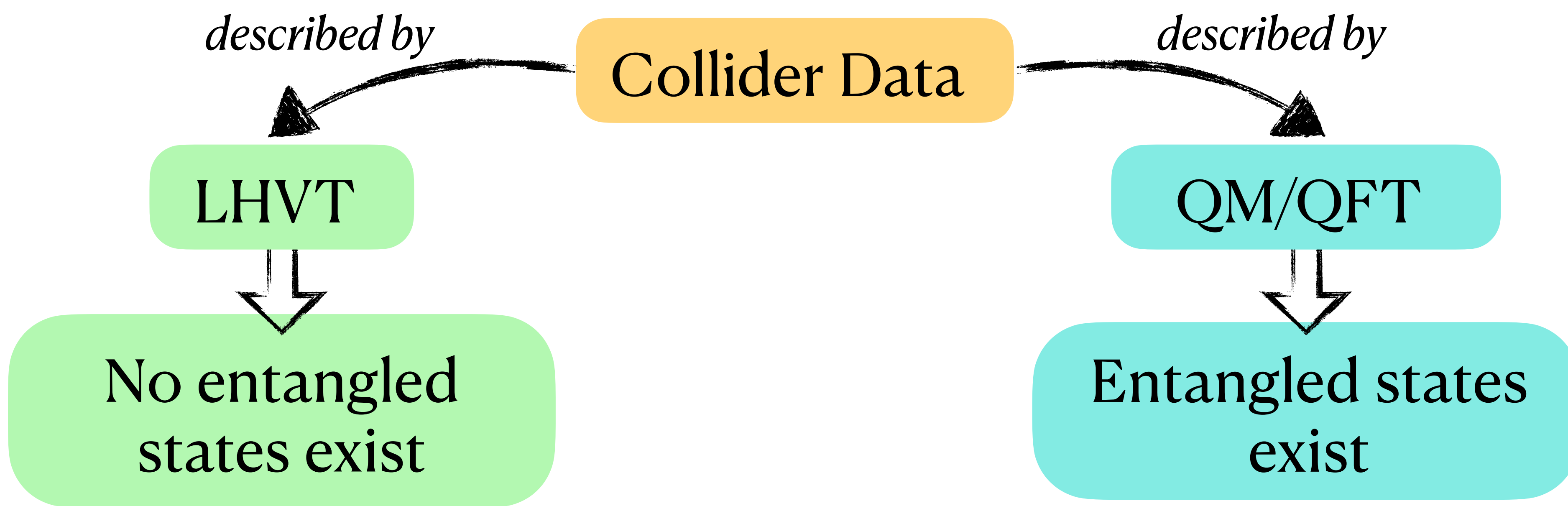
ATLAS, Nature 633, pages 542–547 (2024)

"The exclusion of LHVT is found to be statistically significant at a level exceeding 5.2σ in the testing of three Bell-like inequalities."

BESIII, Nature Commun. 16 (2025) 4948

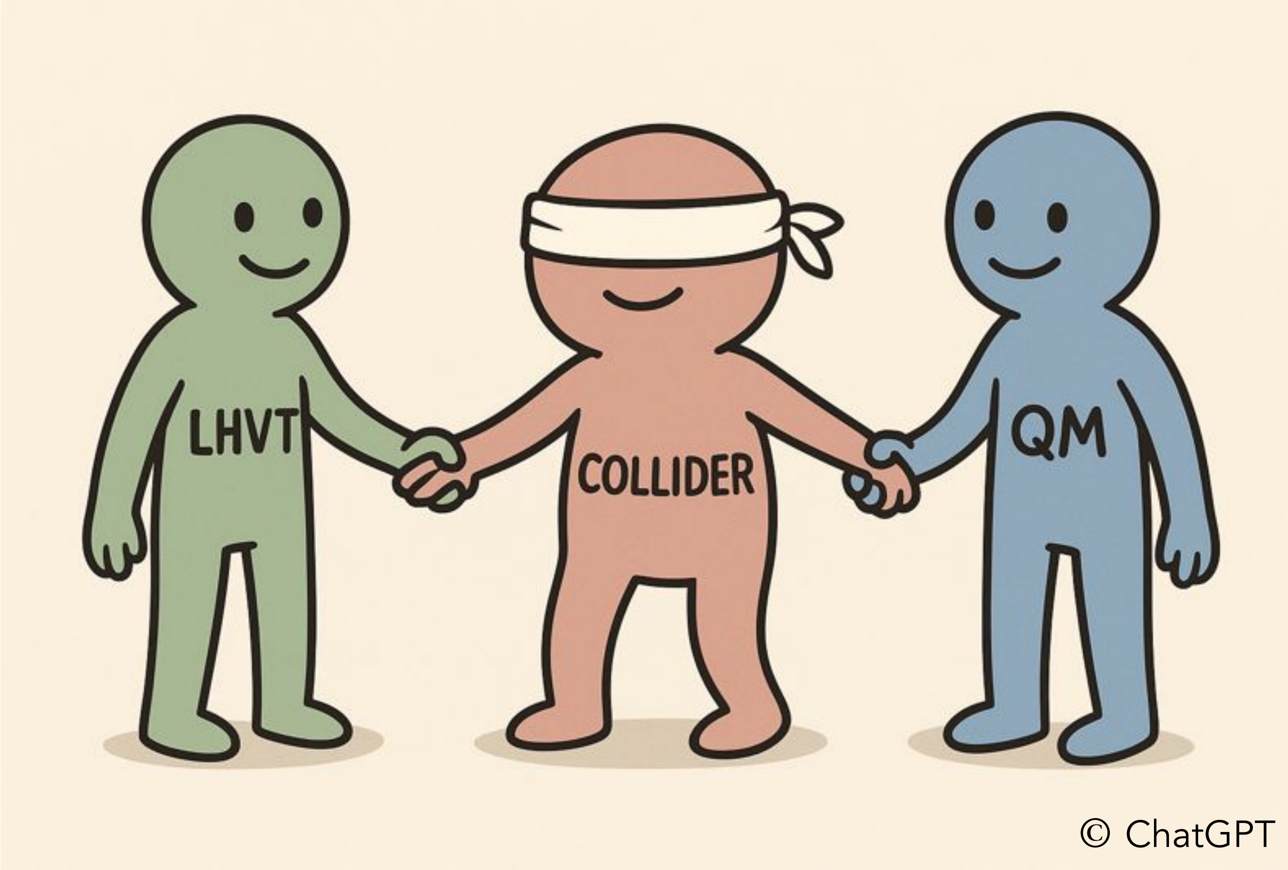
Conclusion

Testing Locality vs Non-locality and Entanglement vs Non-Entanglement at Colliders



How to exclude a local non-entangled theory based on collider data?

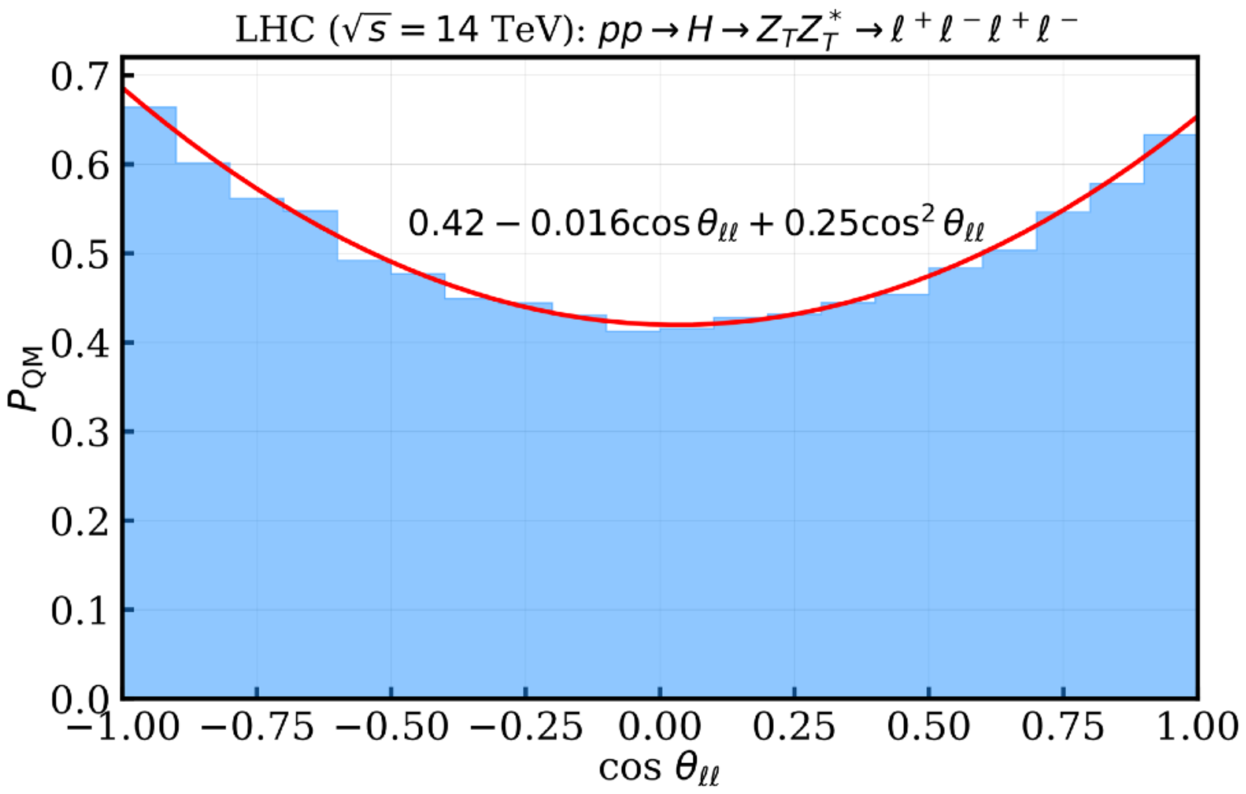
It is not possible to test for locality via Bell's inequality or for entanglement versus non-entanglement at colliders, in cases where only final state momenta are measured.

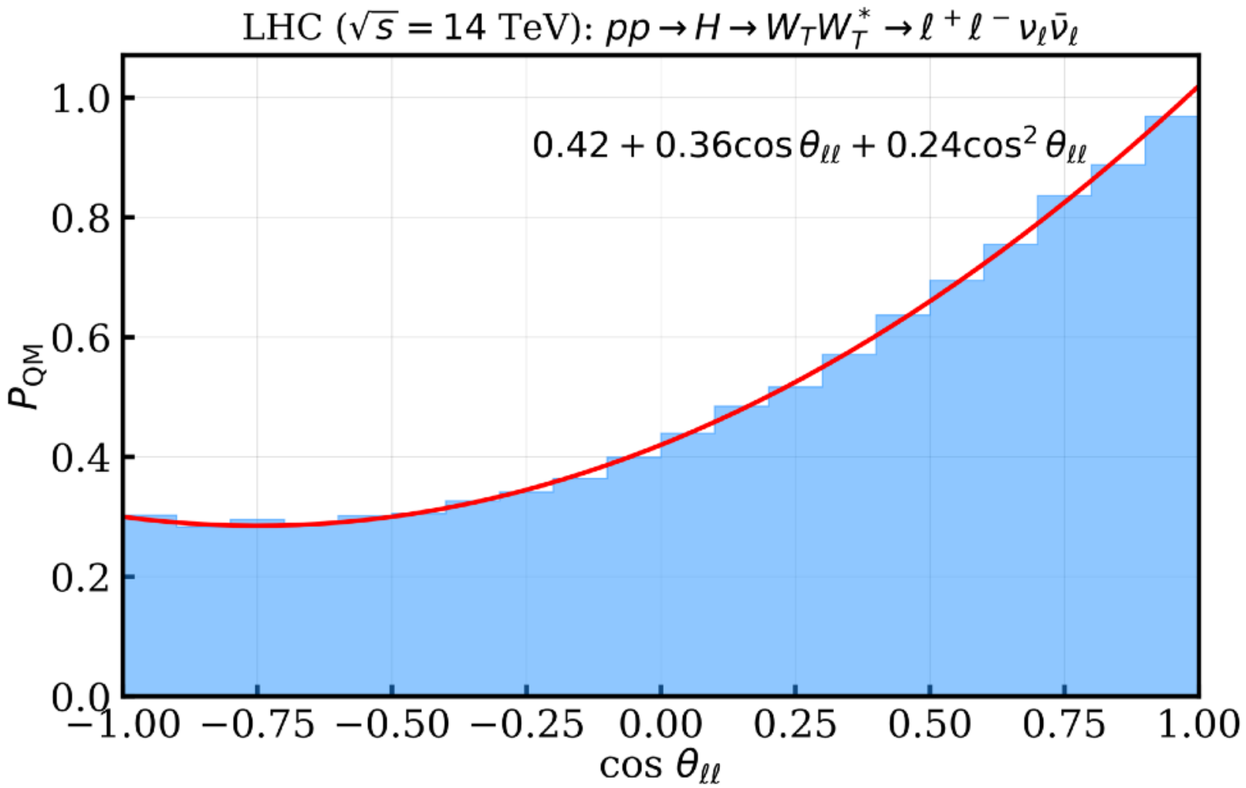


Thank you for your attention!

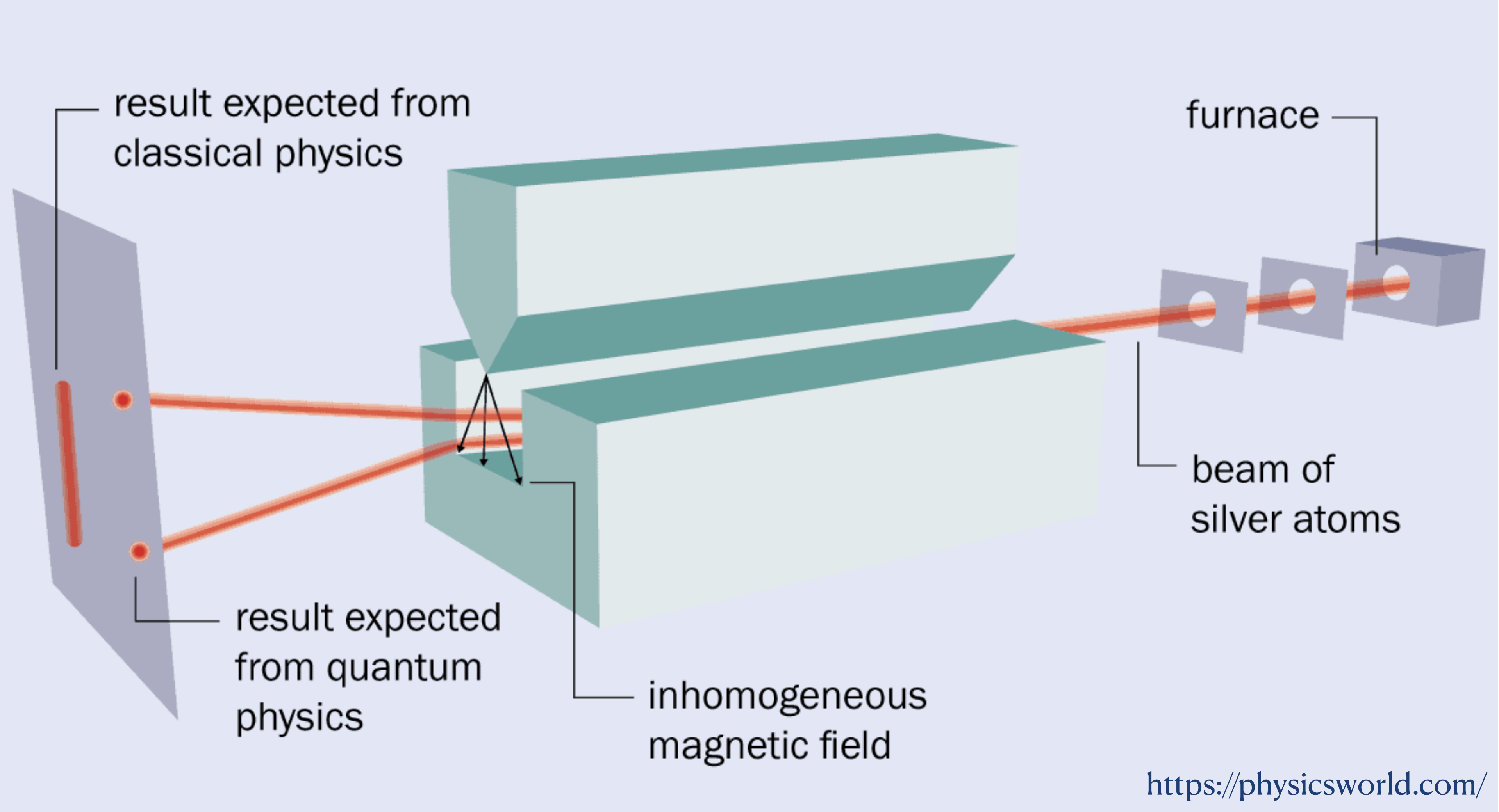
Backup

The Subtle Case of Massive Vector Bosons





Stern-Gerlach Experiment



Impossible to measure spin-½ with Stern-Gerlach setup for charged particles, i.e. electrons (or taus)

The Theory of Atomic Collisions, H. Massey and N. Mott, pages 61-64

Bell's inequality violation in meson systems

$$\Upsilon(4S) \to B^0\bar{B}^0$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\,|B^0\rangle_1\,|\bar{B}^0\rangle_2 - |\bar{B}^0\rangle_1\,|B^0\rangle_2\right]$$

Observation of Bell Inequality violation in B mesons
J.Mod.Opt. 51 (2004) 991

Belle

Measurement of EPR-type flavour entanglement in
$\Upsilon(4S) \to B^0\bar{B}^0$ *decays*
Phys.Rev.Lett. 99 (2007) 131802

refuted via a Kasday-type construction

Bramon, Escribano and Garbarino

Bell's inequality tests: from photons to B–mesons
J.Mod.Opt. 52 (2005) 1681-1684

Bell's inequality tests with meson-antimeson pairs
Found.Phys. 36 (2006) 563-584