Towards a precise determination of α_s from the Z boson q_T spectrum

Bohr Seminar 13.03.26
University of Manchester, UK

Giulia Marinelli
DESY, Hamburg

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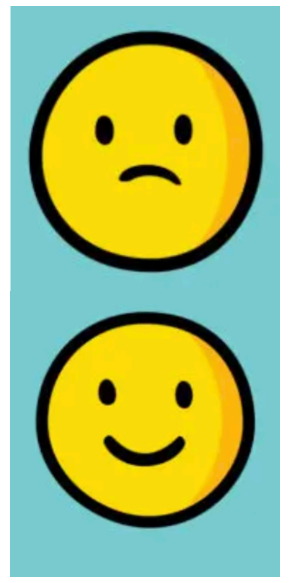
Outline

1. **Introduction and Motivation: the Drell-Yan process**

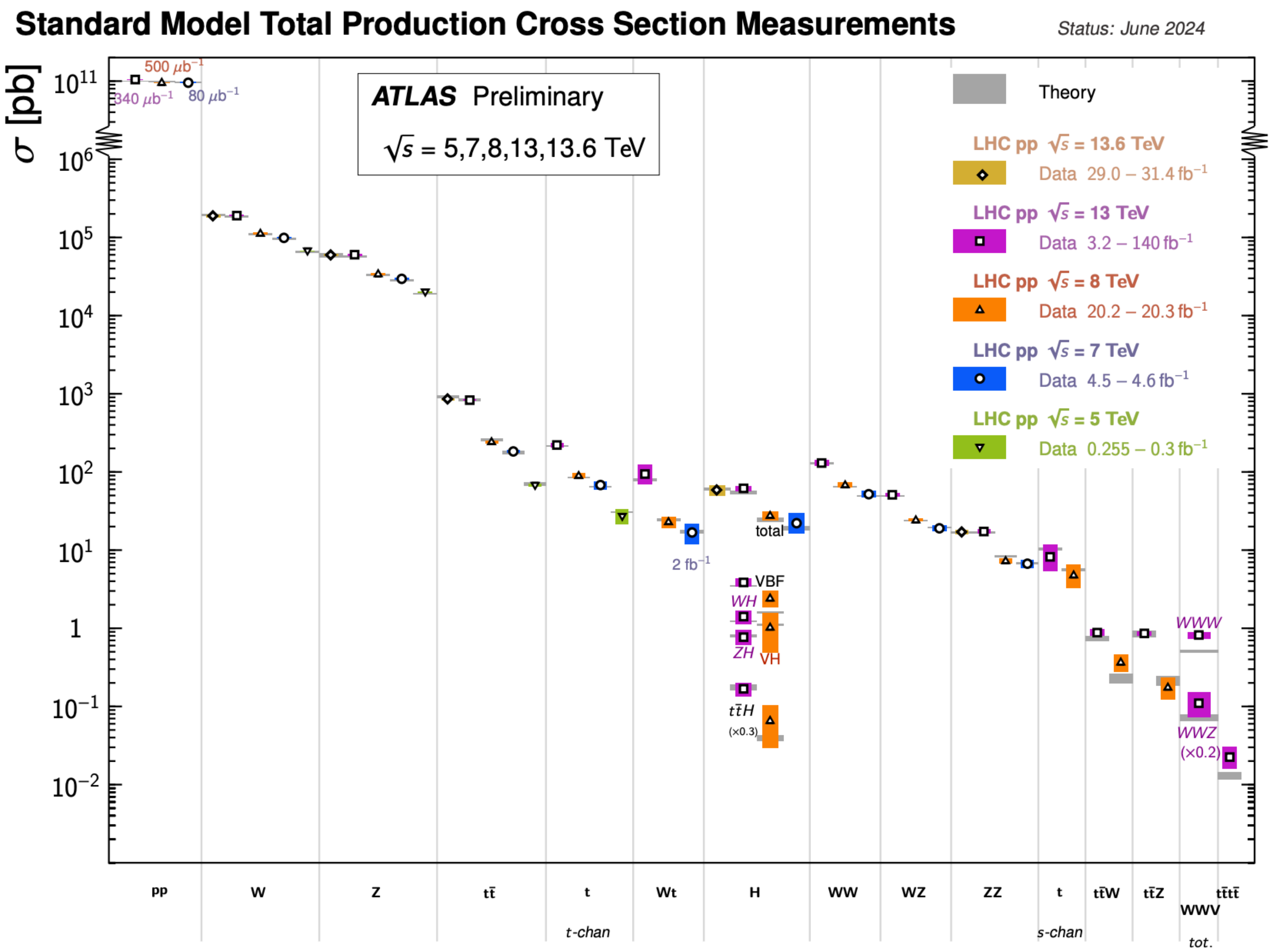
2. **Theory Uncertainty for precision measurements**
   - usual scale variations approach
   - novel Theory Nuisance Parameters (TNPs) approach

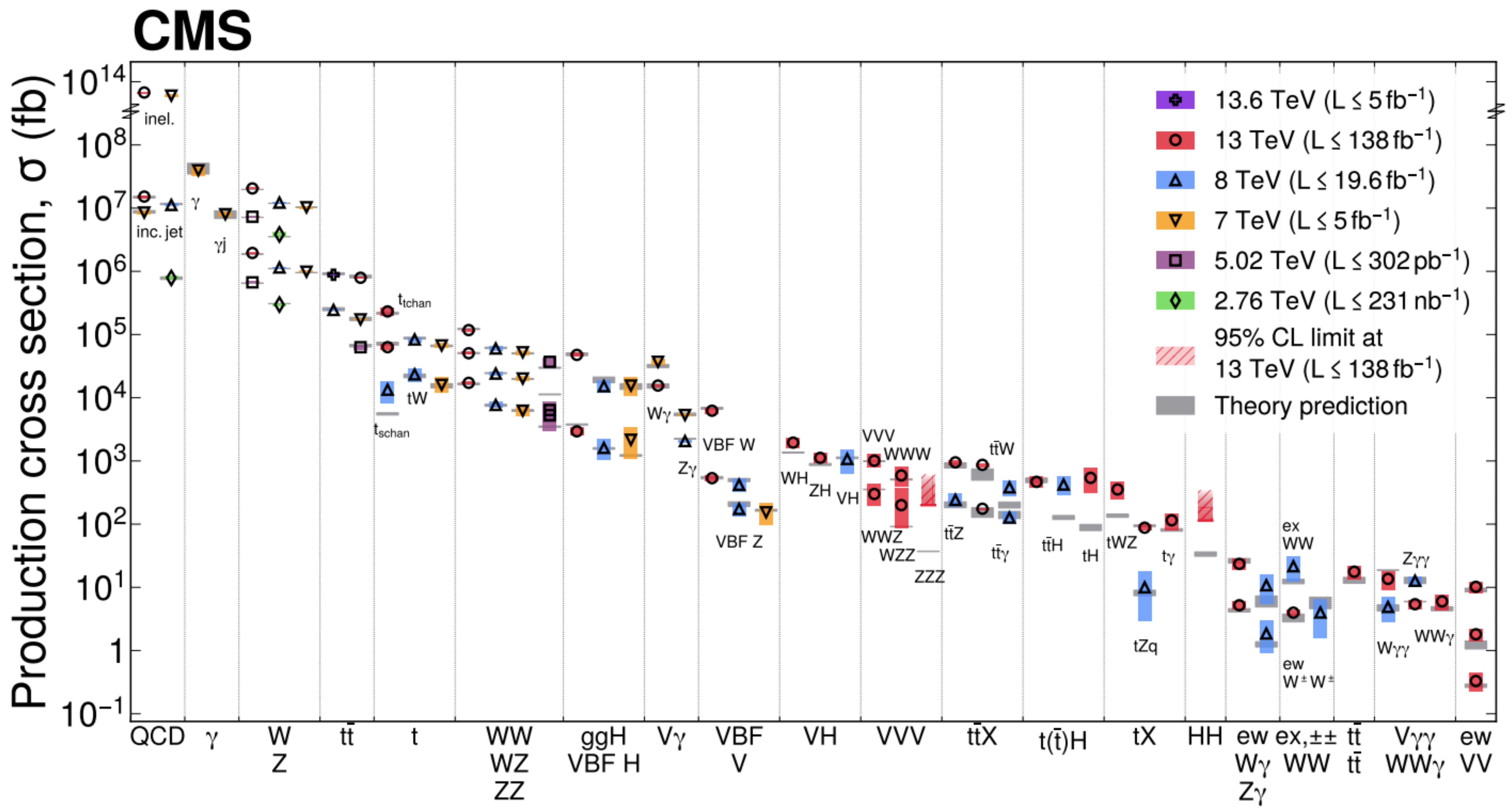


3. **Extraction of α_s from the $Z\, q_T$ spectrum**

Introduction & Motivation

Wonderful measurements of Standard Model processes:





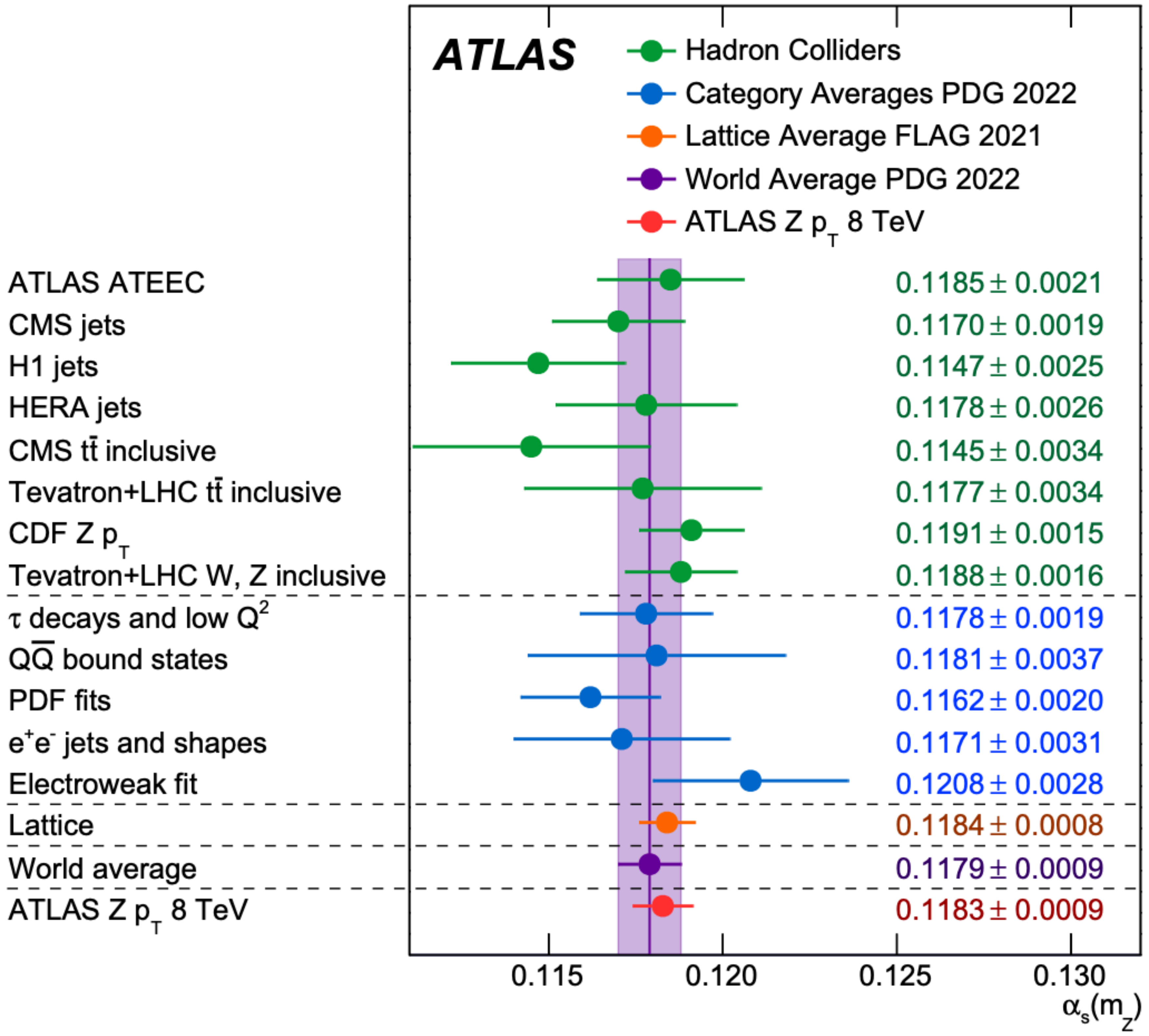
» Good agreement with theory predictions

» Rich program for the future both SM testing and new physics searches

» LHC not only discovery machine, but also a **precision machine**!

Introduction & Motivation

Drell-Yan process has a special role:

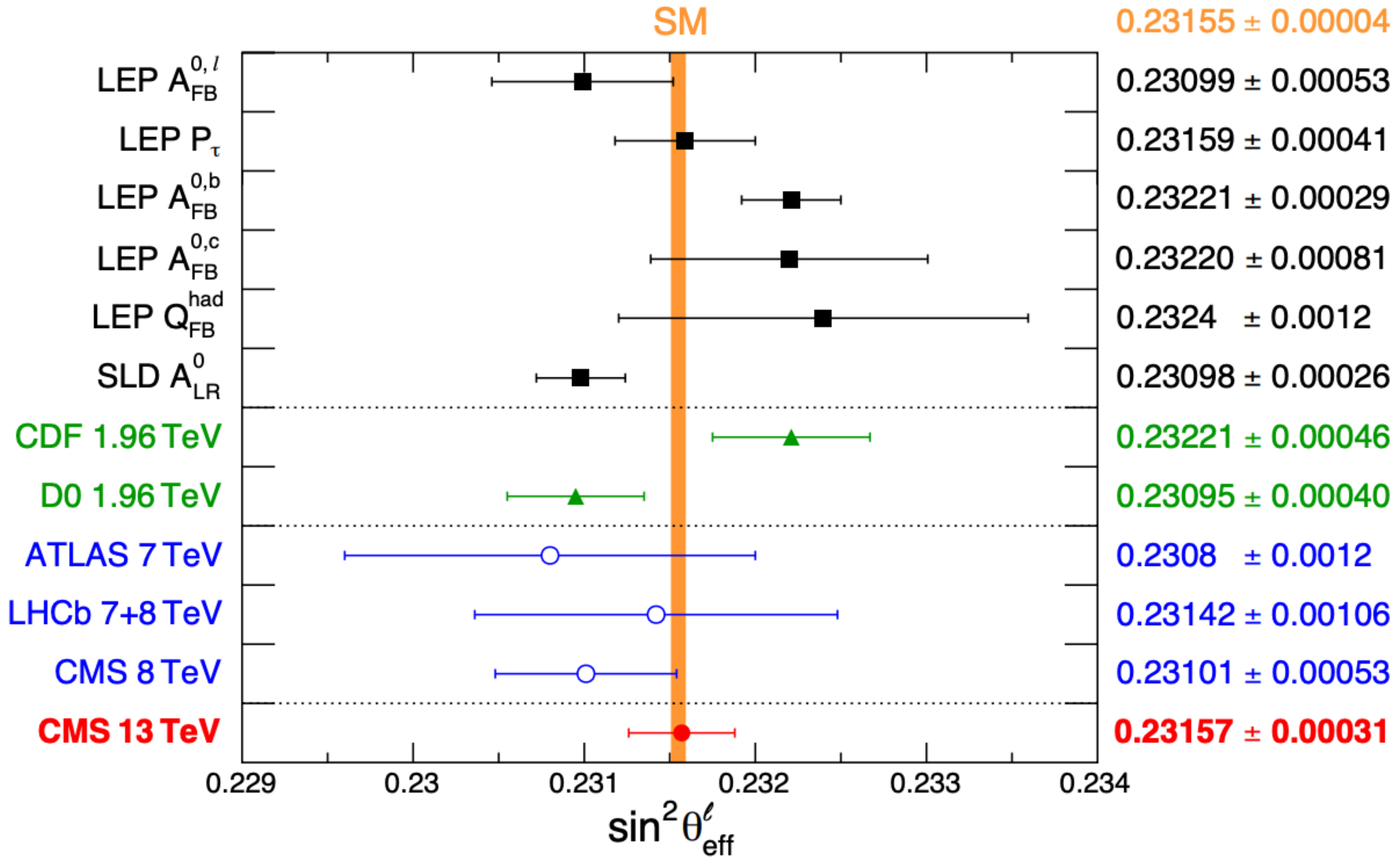


$\alpha_s(m_Z)$

[ATLAS '23, '24]

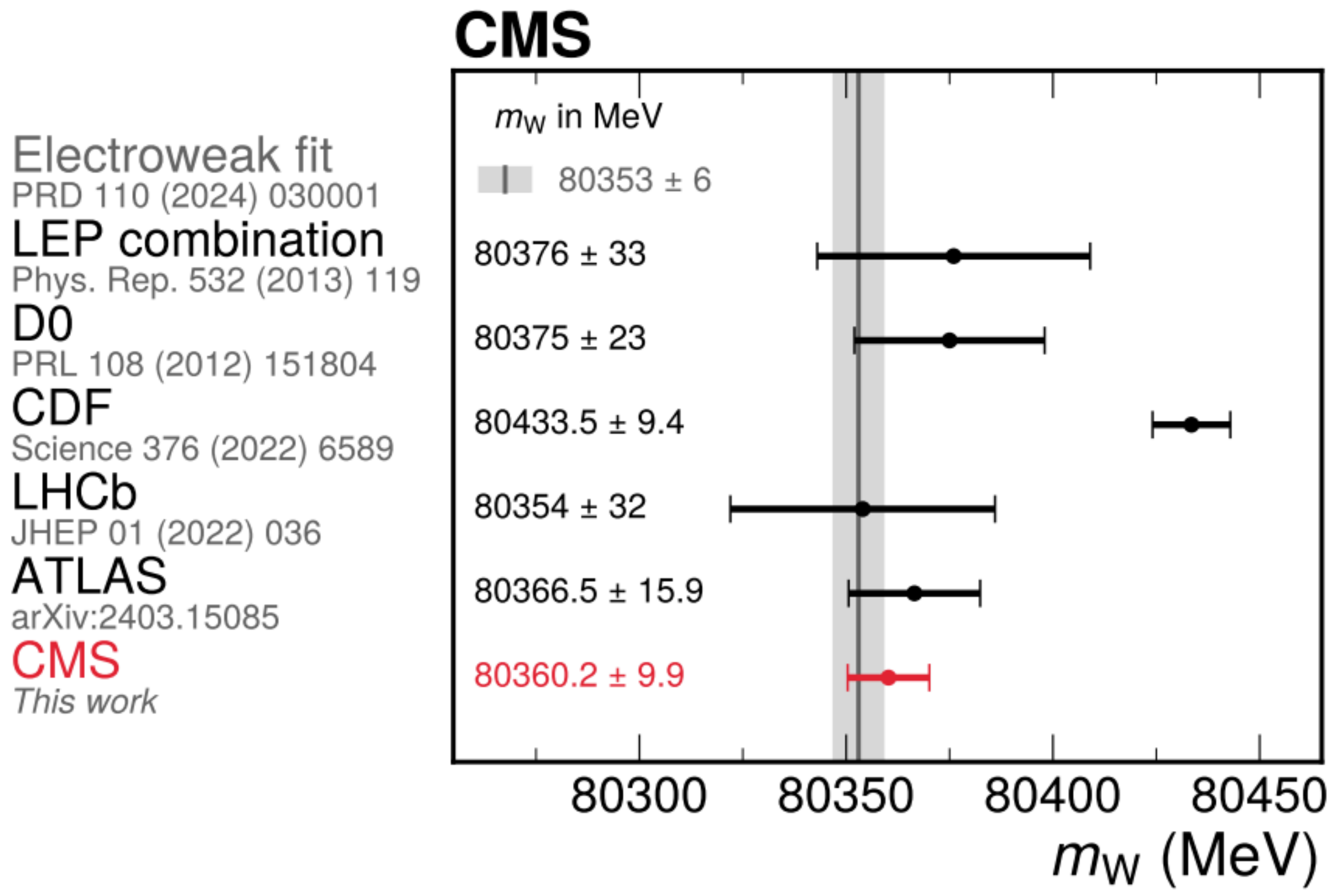
$\sin^2\theta^{\ell}_{\text{eff}}$

[CMS '24]



m_W

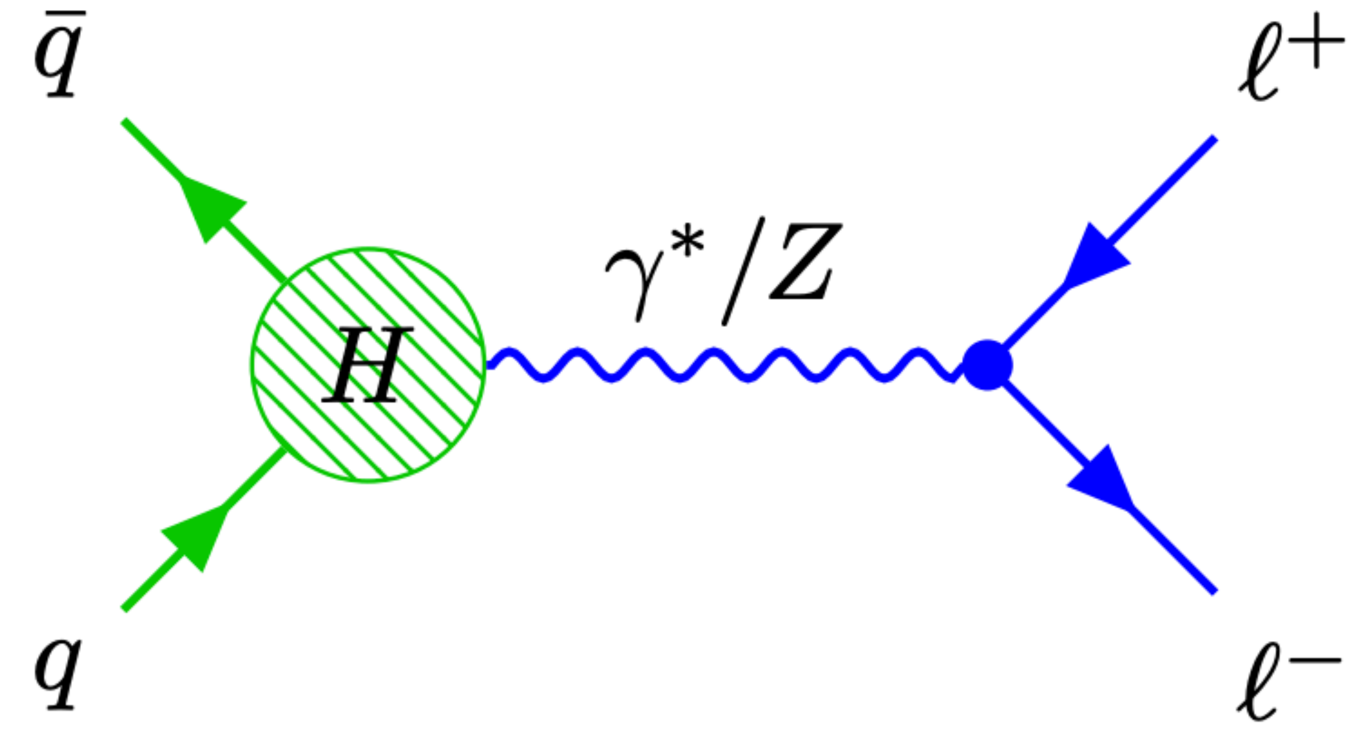
[CMS '24]



Three of the SM fundamental parameters measurements have been performed at LHC!

Color singlet q_T spectrum crucial observable

Introduction: Drell-Yan q_T spectrum



- Wide-ranging applications, many precise measurements:

  ATLAS '20, ATLAS '24, CMS '17, CMS '19, LHCb '16, ...

- Many theory requirements to reach $\mathcal{O}(1\%)$ level precision:

$$d\sigma = d\sigma^{\text{resum}} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{q_T^2}\right) + \mathcal{O}\left(\frac{m_q^2}{q_T^2}\right) + \mathcal{O}(\alpha^{\text{ISR/EW}}) + \mathcal{O}(\alpha_{\text{em}}^{\text{FSR}}) + \text{Parton Distribution Functions (PDFs)}$$

resummation

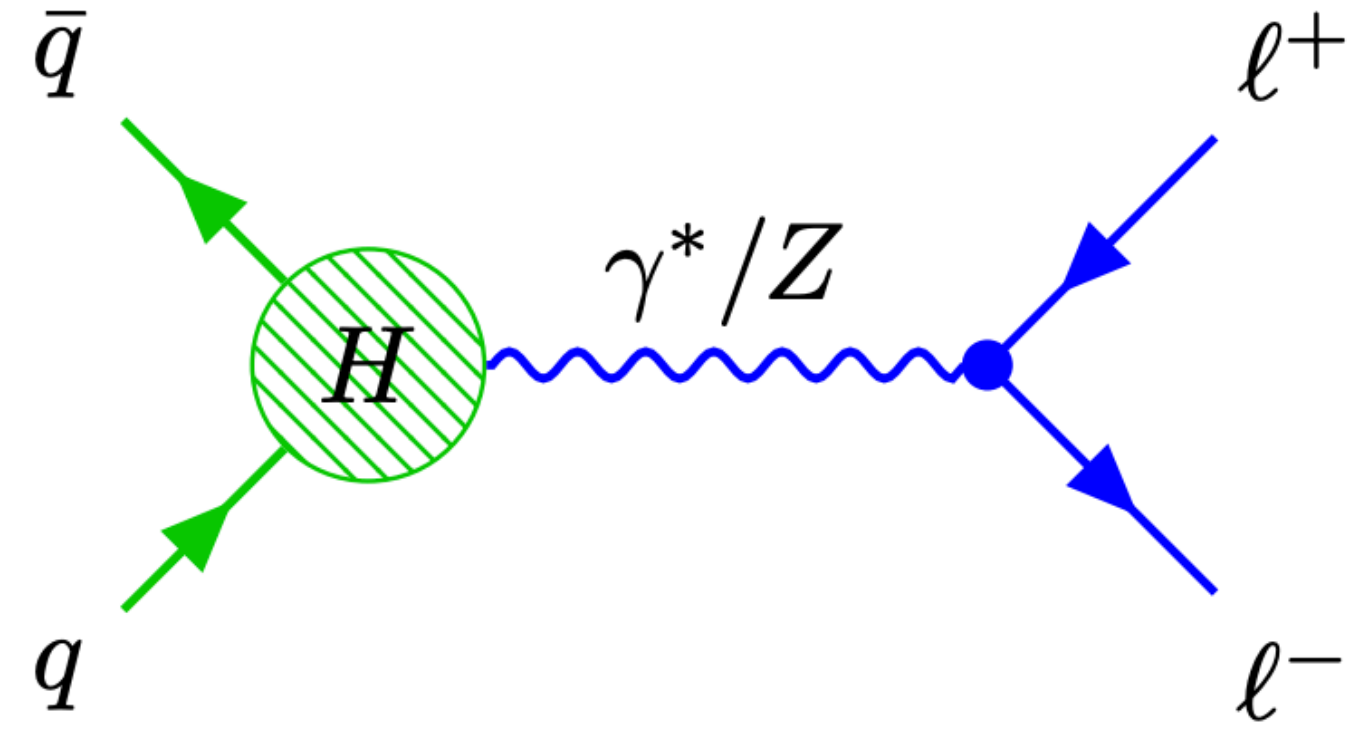
pert. corrections

nonpert. effects

quark mass

EW corrections

Introduction: Drell-Yan q_T spectrum



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$$+\mathcal{O}\left(\frac{m_q^2}{q_T^2}\right)$$

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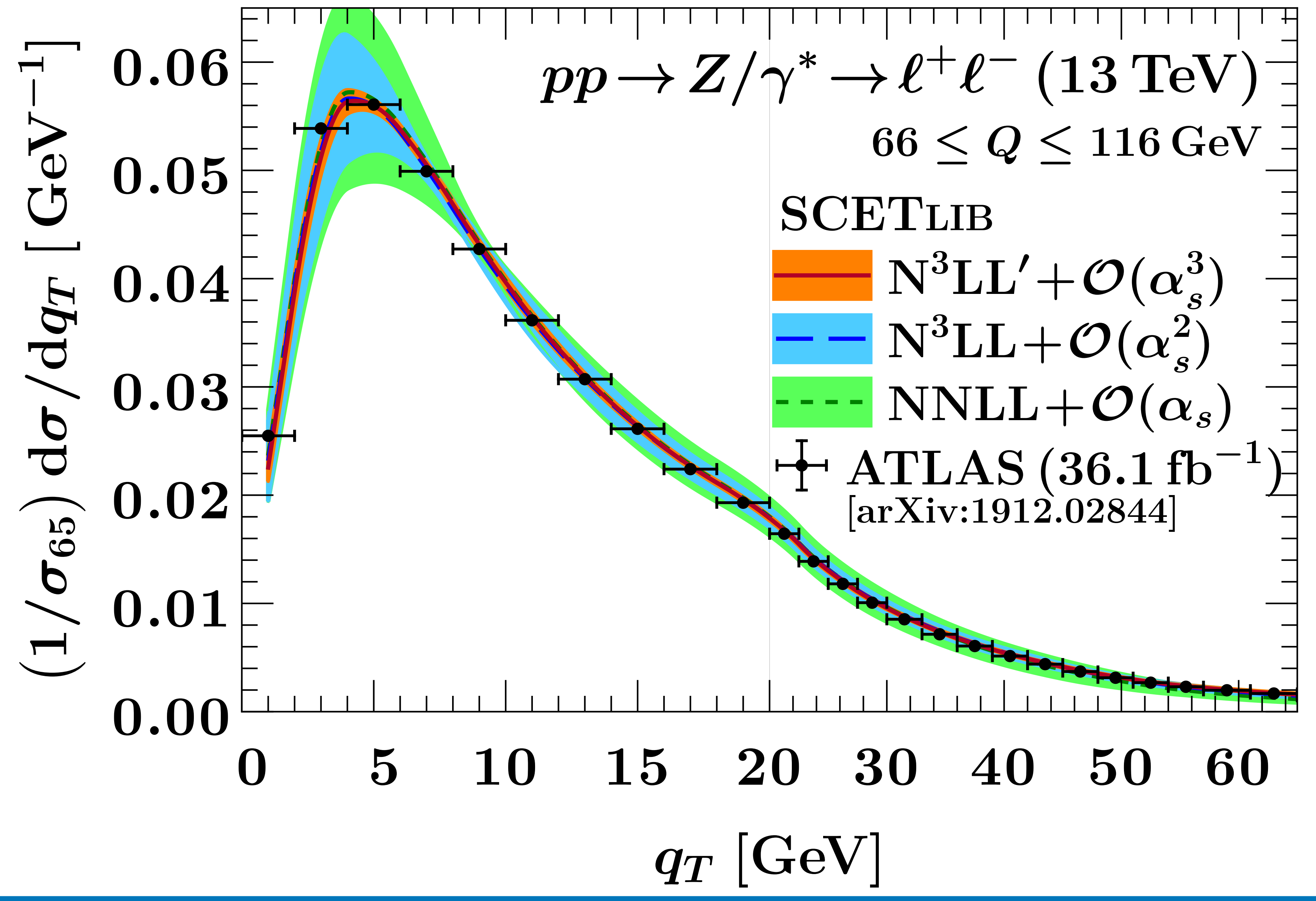
$$+\mathcal{O}(\alpha^{\text{ISR/EW}}) + \mathcal{O}(\alpha_{\text{em}}^{\text{FSR}})$$

EW corrections

$+$ Parton Distribution Functions (PDFs)



$\longrightarrow$ **N^{3}LL$'$/ approx N^{4}LL**

[Billis, Michel, Tackmann '25 - Moos, Scimeni, Vladimirov, Zurita '24 - Camarda, Cieri, Ferrera '23 - ...]

Theory Uncertainty

Theory uncertainty

- Every theory prediction needs its theory uncertainty:

$$\Delta_{\text{theo}} \gg \Delta_{\text{exp}}$$

"quite embarrassing"

Major source of uncertainty: missing higher orders (MHO)

Consider a series expansion in a small parameter α: $f(\alpha) = f_0 + \alpha f_1 + \alpha^2 f_2 + \mathcal{O}(\alpha^3)$

LO : $f(\alpha) = \hat{f}_0 \pm \Delta f$

NLO : $f(\alpha) = \hat{f}_0 + \alpha \hat{f}_1 \pm \Delta f$

$\longrightarrow$ Δf uncertainty due to the series of the unknown true values $\hat{f}_n$

[nice overview of exp-theo status for DY SM@LHC25 Buonocore's talk]

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- **Meaningful** Theory Uncertainties:

1. must reflect our degree of knowledge (or ignorance)
2. provide correct **correlations** for different predictions
3. have a statistical meaning needed for the interpretation of experimental measurements

[nice overview of exp-theo status for DY SM@LHC25 Buonocore's talk]

Scale variations approach

MHOs uncertainty always detemined through:

[Cacciari, Houdeau '11 - Bonvini '20 - Duhr, Huss, Mazeliauskas, Szafron '21]

- scale variations
- scale variation with bayesian approach
- series acceleration

→ based on the same approach, easy to implement and use but with many known limitations

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Method: given $f(\alpha)$, make a variable transformation

$$\tilde{\alpha}(\alpha) = \alpha[1 + b_0\alpha + b_1\alpha^2 + \mathcal{O}(\alpha^3)]$$

defining a different prediction

$$\tilde{f}(\tilde{\alpha}) = \tilde{f}_0 + \tilde{f}_1\tilde{\alpha} + \tilde{f}_2\tilde{\alpha}^2 + \mathcal{O}(\tilde{\alpha}^3)$$

LO : $\tilde{f}(\tilde{\alpha}) = \tilde{f}_0 = \hat{f}_0$

NLO : $\tilde{f}(\tilde{\alpha}) = \hat{f}_0 + \tilde{\alpha}\hat{f}_1 = \hat{f}_0 + \hat{f}_1\alpha^2 + b_0\hat{f}_1\alpha^2 + b_1\hat{f}_1\alpha^3 + \ldots$

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LO : $f(\alpha) = \hat{f}_0 \pm \Delta f$

NLO : $f(\alpha) = \hat{f}_0 + \alpha\hat{f}_1 \pm \Delta f$

Take the difference between the two "schemes":

LO : $\Delta f(\alpha) = 0$

NLO : $\Delta f(\alpha) = b_0\hat{f}_1\alpha^2 + b_1\hat{f}_1\alpha^3 + \mathcal{O}(\alpha^4)$

Scale variations approach

Estimating MHOs uncertainty by approximating them by some linear combination of known lower-order terms $[f_2 \approx b_0 \hat{f}_1]$

- ✅ $\Delta f(\alpha)$ is genuinely of higher order

Scale variations approach

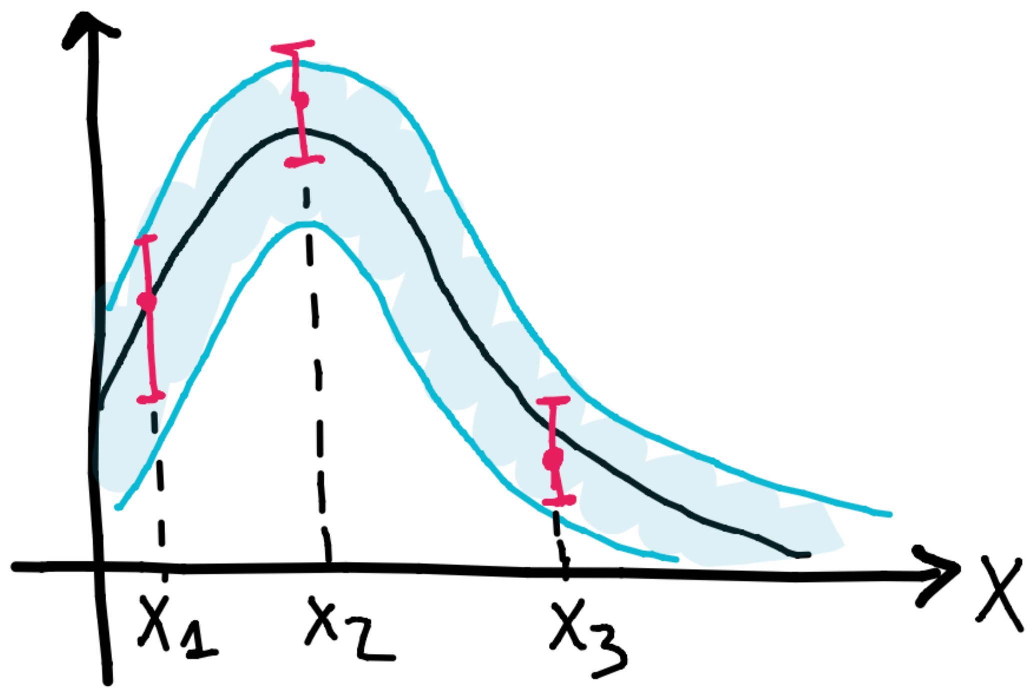
Estimating MHOs uncertainty by approximating them by some linear combination of known lower-order terms $[f_2 \approx b_0 \hat{f}_1]$

- ✅ $\Delta f(\alpha)$ is genuinely of higher order
- ❌ nothing guarantees this is any good
- ❌ f_{n+1} generally more complex internal structure than $f_{\leq n}$
- ❌ b_0 (b_n) are just arbitrary constants and usually the same for any f
- ❌ b_n are not actual physical parameter with a true value
- ❌ correlation and shape uncertainties?

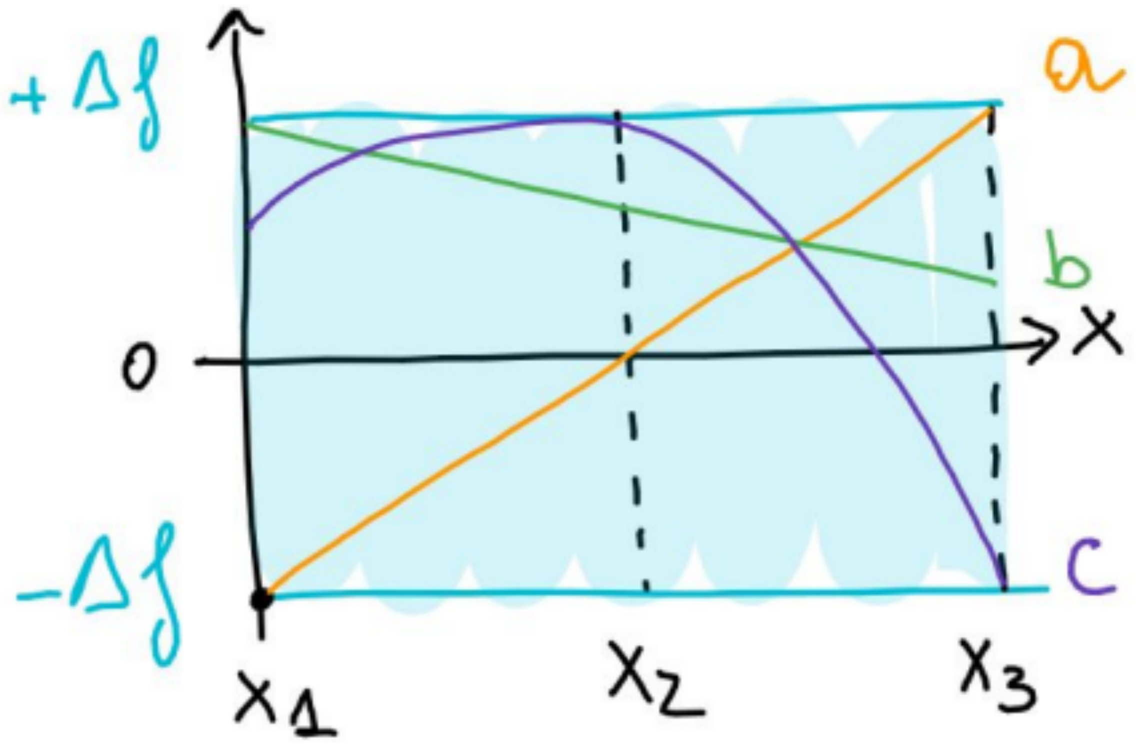
Now imagine $\alpha \equiv \alpha_s(\mu_0)$ and $\tilde{\alpha} \equiv \alpha_s(\mu)$: $b_0 = \frac{\beta_0}{2\pi} \ln \frac{\mu}{\mu_0}$, and why vary μ by 2?

Correlation with Scale Variations

For a differential spectrum, each bin is a separate prediction as it is a separate measurement!
With scale variations:



scanning over
scales



? ~~correlation~~ ?
? shape ?

	ρ_{12}	ρ_{13}	ρ_{23}
a	0	−1	0
b	1	1	1
c	1	−1	−1

» Scanning over scale variations that fill the band is like scanning over several *ad hoc* correlation models

⟶ scale variations cannot give the correct shape (and therefore correlation):
that's why we take envelopes!

⟶ to get correct correlation: breakdown into *independent uncertainty components* required

Theory Nuisance Parameters (TNPs)

[Tackmann '24]

Theory Nuisance Parameters (TNPs) in a nutshell

1 Parametrize the uncertainty by the missing highest piece

$$\mathrm{N^{2+1}LO}: \; f^{\mathrm{pred}}(\alpha,\theta_3) = \hat{f}_0 + \alpha\,\hat{f}_1 + \alpha^2\,\hat{f}_2 + \alpha^3 f_3\left(\theta_3\right)$$

using theory nuisance parameters θ_n ;

- θ_n have physical true value $\hat{\theta}_n$, such that $\hat{f}_n = f_n(\hat{\theta}_n)$ ... and therefore encode correct theory correlations
- TNPs well-defined parameters with true but unknown value

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2 To define θ_n , account for the internal structure of f_3 : find a suitable parameterization and break down internal structure until remaining unknown $f_{n,i}$ are numbers

- Drell-Yan resummed q_T spectrum case: [considering SCET factorization]

$$q_T \frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \left[H \times B_a \otimes B_b \otimes S \right] (\alpha_S, L \equiv \ln q_T/m_Z) + \mathcal{O}\left(\frac{q_T^2}{m_Z^2}\right)$$

leading power q_T dependence known to all orders

Theory Nuisance Parameters (TNPs) in a nutshell

2

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$F = \{H, B, S\}$ solution to RGE equations

$$F\left(\alpha_S, L\right) = F(\alpha_S) \exp \int_0^L \mathrm{d}L' \left\{ \Gamma[\alpha_S(L')] L' + \gamma_F [\alpha_S(L')] \right\}$$

boundary conditions

anomalous dimensions

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boundary conditions — anomalous dimensions

Identify a parameterization that captures correlations and tightens the theory constraint

$$F(\alpha_S) = 1 + \sum_{n=1} \left(\frac{\alpha_S}{4\pi}\right)^n F_n \qquad \gamma(\alpha_S) = \sum_{n=0} \left(\frac{\alpha_S}{4\pi}\right)^{n+1} \gamma_n$$

$$F_n(\theta_n^f) = 4C_r(4C_A)^{n-1}(n-1)!\, \theta_n^f \qquad \gamma_n(\theta_n^\gamma) = 4C_r(4C_A)^n\, \theta_n^\gamma$$

where C_r leading color factor, C_A^{n-1} leading $n-$loop color factor

» Total of seven TNPs: Γ_{cusp}, γ_μ, γ_ν, $H(\alpha_s)$, $S(\alpha_s)$, B_{qq}, B_{qg}

Theory Nuisance Parameters (TNPs) in a nutshell

2 Beam function non-trivial case: up to 5 one-dim functional series for beam functions* (+ DGLAP splitting function)

$$\tilde{b}_i(x,\alpha_s) = \sum_j \int \frac{\mathrm{d}z}{z} \left[\hat{I}_{ij,0}(z) + \hat{I}_{ij,1}(z) + I_{ij,2}\left(z, \theta_2^{B_{ij}}\right) \right] f_j\left(\frac{x}{z}\right)$$

to be changed in the future!

* at the moment, using functional known form $I_{ij,n}(z,\theta_n^{B_{ij}}) = \frac{3}{2}\theta_n^{B_{ij}}\hat{I}_{ij,n}(z)$

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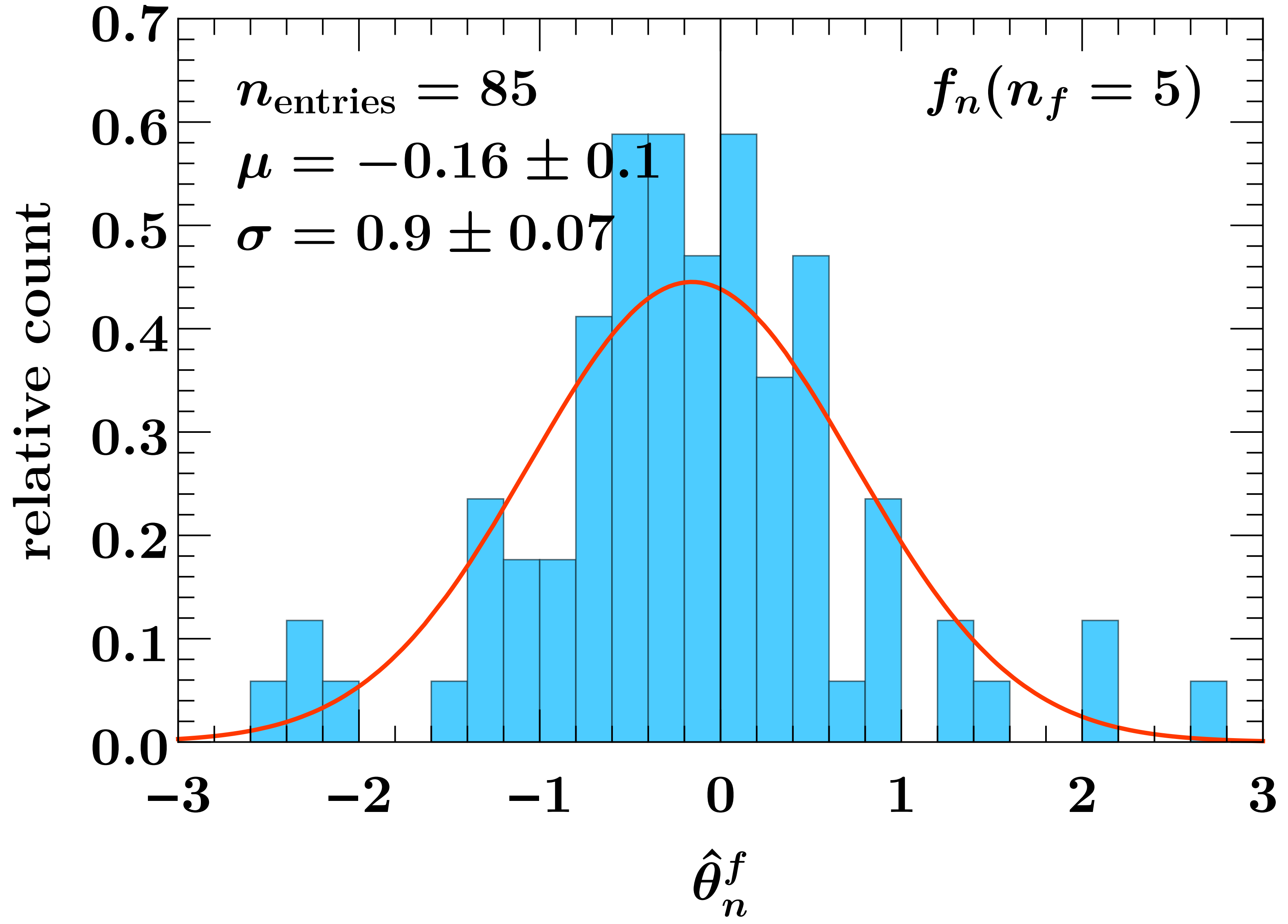
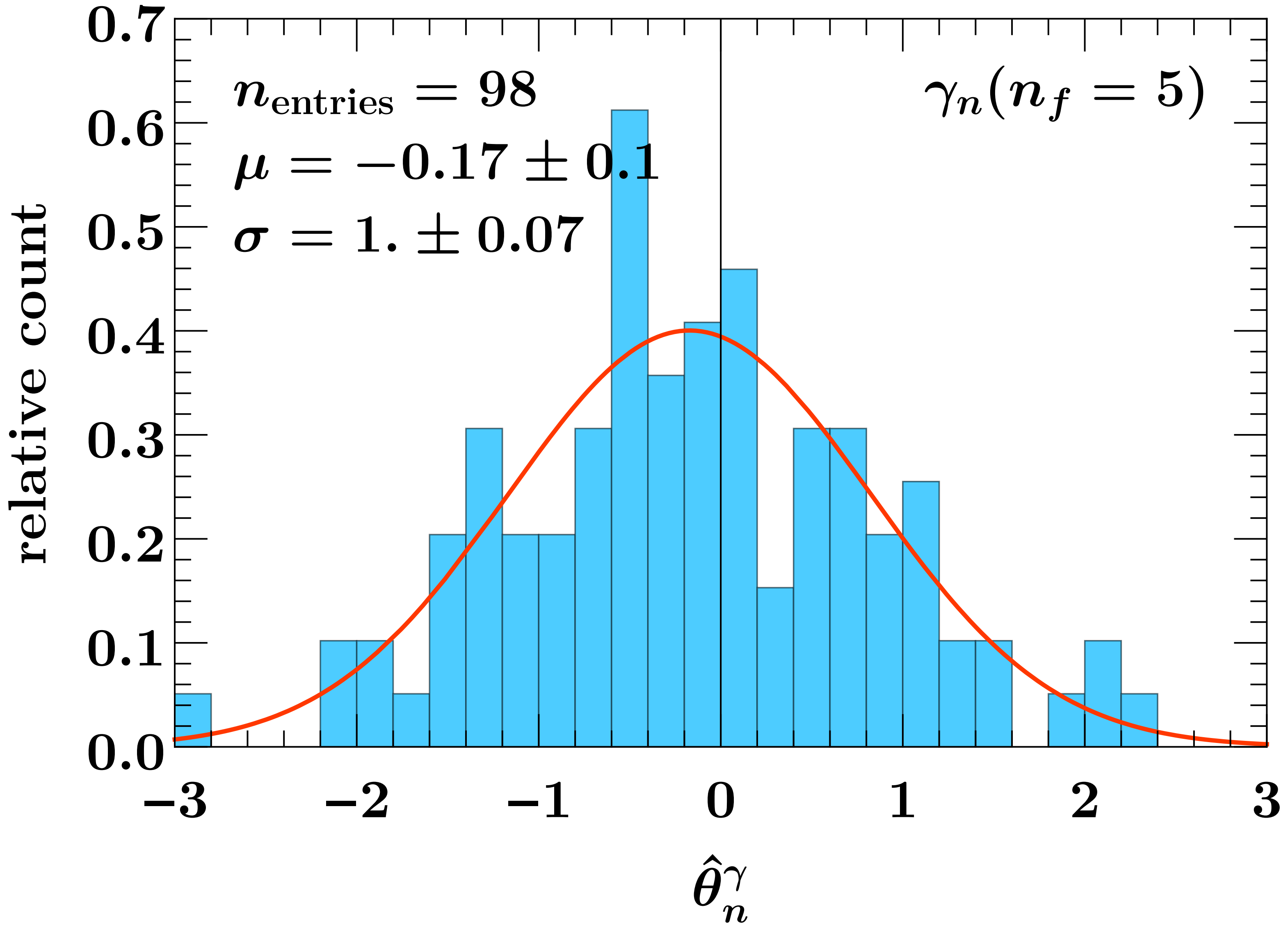
3 With these normalizations, expected natural size $|\hat{\theta}_n| \lesssim 1$ ⟶ $\theta_n = 0 \pm 1$

$\Delta\theta_n = 1$ theory constraint

68% theory CL



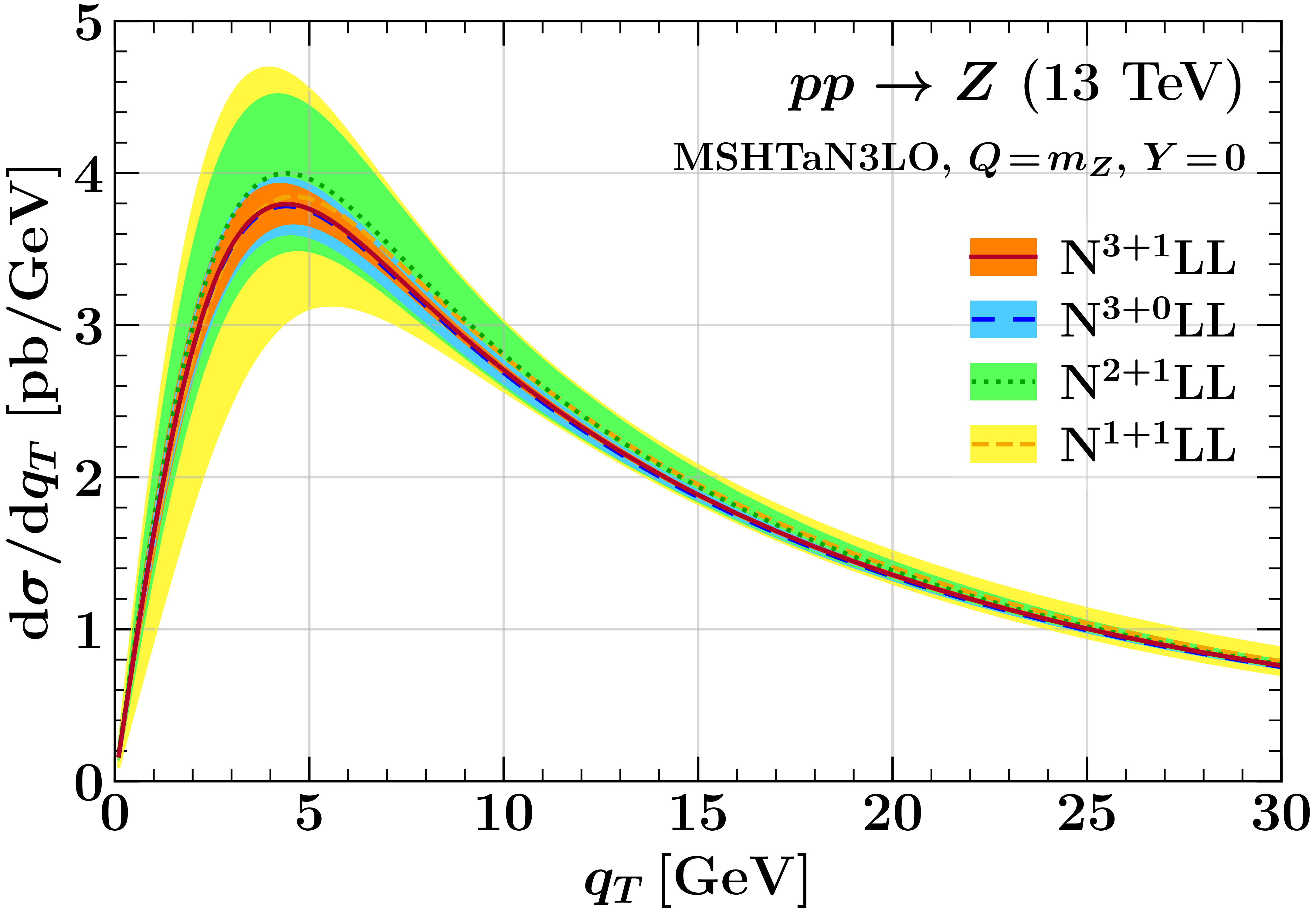


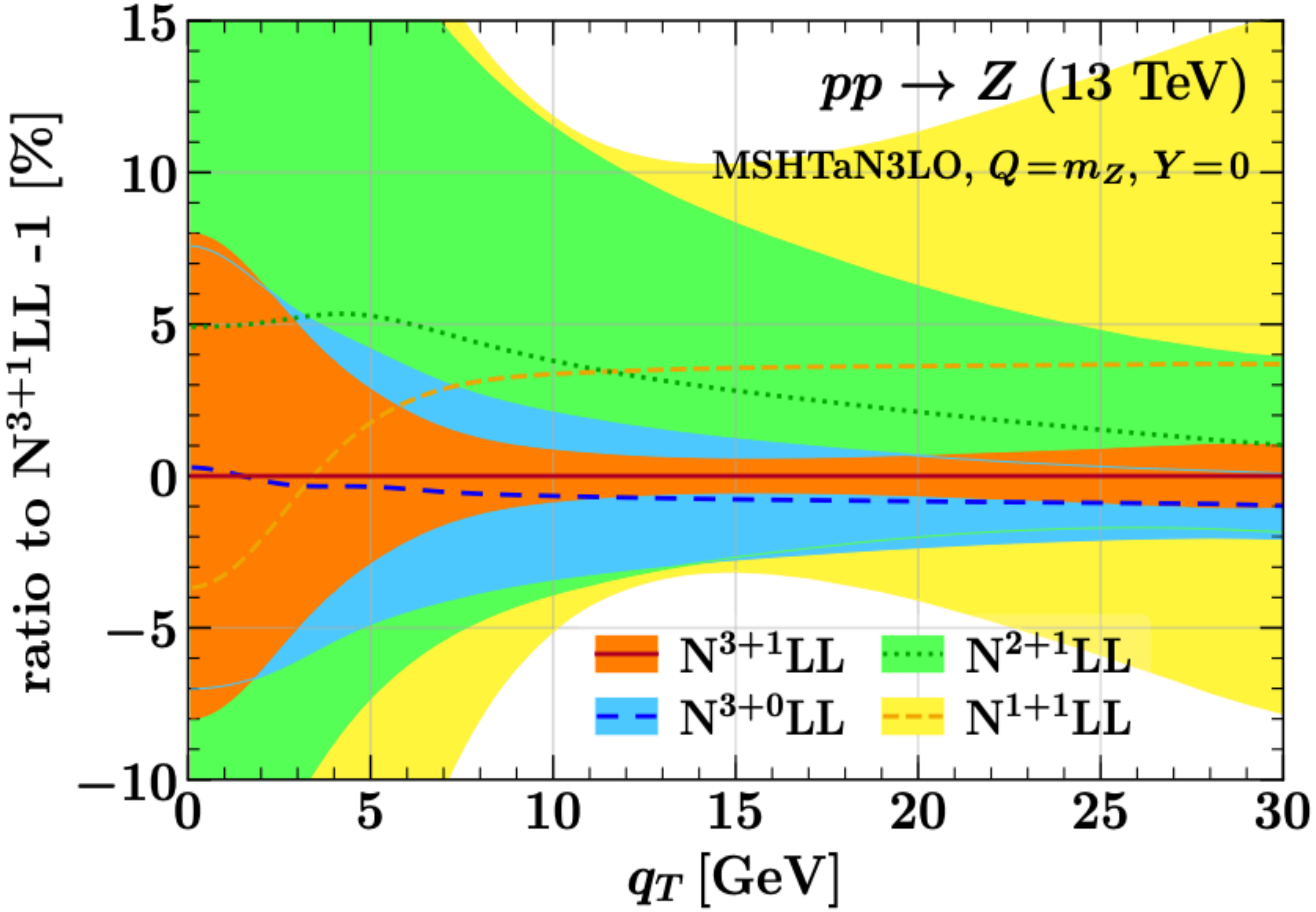
[look at other known $n-$loop coefficients from population sample here]

TNPs for Drell-Yan q_T spectrum

$N^{k+1}LL$: $N^{k+1}LL$ resummed structure, highest-order boundary conditions/anomalous dim. as TNPs

Comparing different orders at **95% theory CL** ($\Delta\theta_n = \pm 2$)



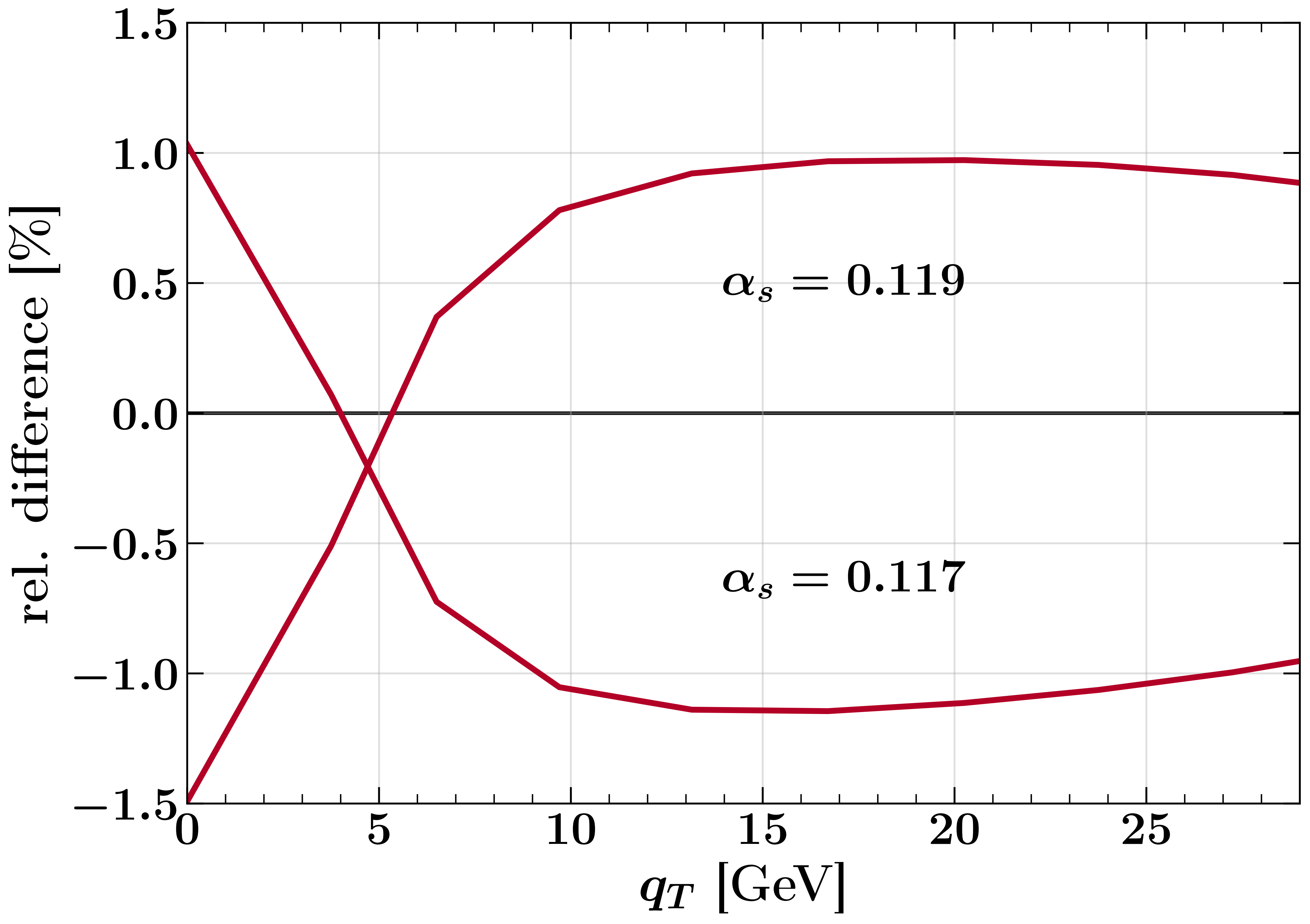


Theory uncertainties in the extraction of α_s from Drell-Yan at small q_T

[Cridge, GM, Tackmann '25]

Measuring $\alpha_s(m_Z)$ from low q_T Drell-Yan spectrum

- LHC as precision machine: Drell-Yan has a special role
- Description of the q_T spectrum at $\mathcal{O}(1\%)$ level precision requires
  1. many theory inputs: resummation, nonperturbative effects, PDFs, perturbative corrections, quark mass & EW corrections



α_s **is** a %-level **shape effect**!

whether $\Delta\alpha_s$ is 0.5% or 2% depends on correlation

2. Extraction of α_s is a theoretical game: need of accounting for **bin-by-bin correlations**!

Asimov Fits

Asimov fits: standard procedure to estimate expected uncertainties in a fully controlled setting, rationale similar to the *closure tests*

- using pseudodata (or Asimov data, or toy-data)
- results of the fits *not affected* by statistical fluctuations and possible subleading/higher-order effects present in the real data
  - → theory model correctly describes pseudodata with a minimum $\chi^2 = 0$ (or very close)
- study the *dominant* sources of uncertainty and their impact on the extracted α_s
  can neglect subleading effects:
  - → affecting the small q_T spectrum at few-% level, their associated uncertainty is subdominant with respect to the dominant ones
  - → still necessary for fitting **real data**

In practice:
nonsingular, quark mass and EW corrections neglected in our pseudodata and theory model

Asimov Fits for $\alpha_s(m_Z)$ from Z q_T spectrum

Our theory inputs:

→ SCETlib N^{3+1}LL and N^{4}LL *only* resummed contribution

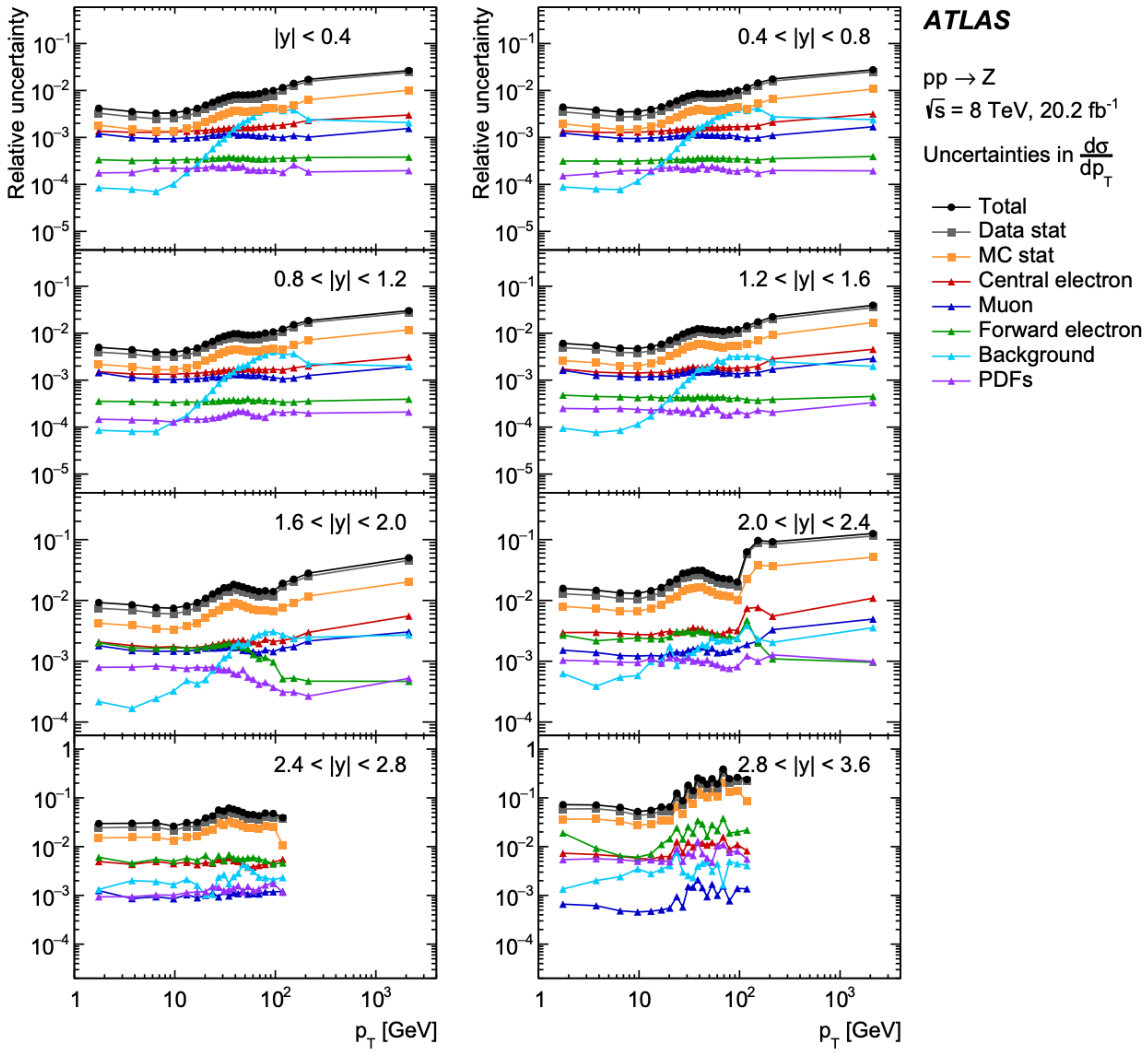
Our toy data:

→ Data defined as central theory prediction [$\alpha_s = 0.118$]
[fixed nonp. params discussed later, MSHT20aN3LO PDF set]

→ Using ATLAS exp. uncert. and complete correlations [arXiv:2309.09318]

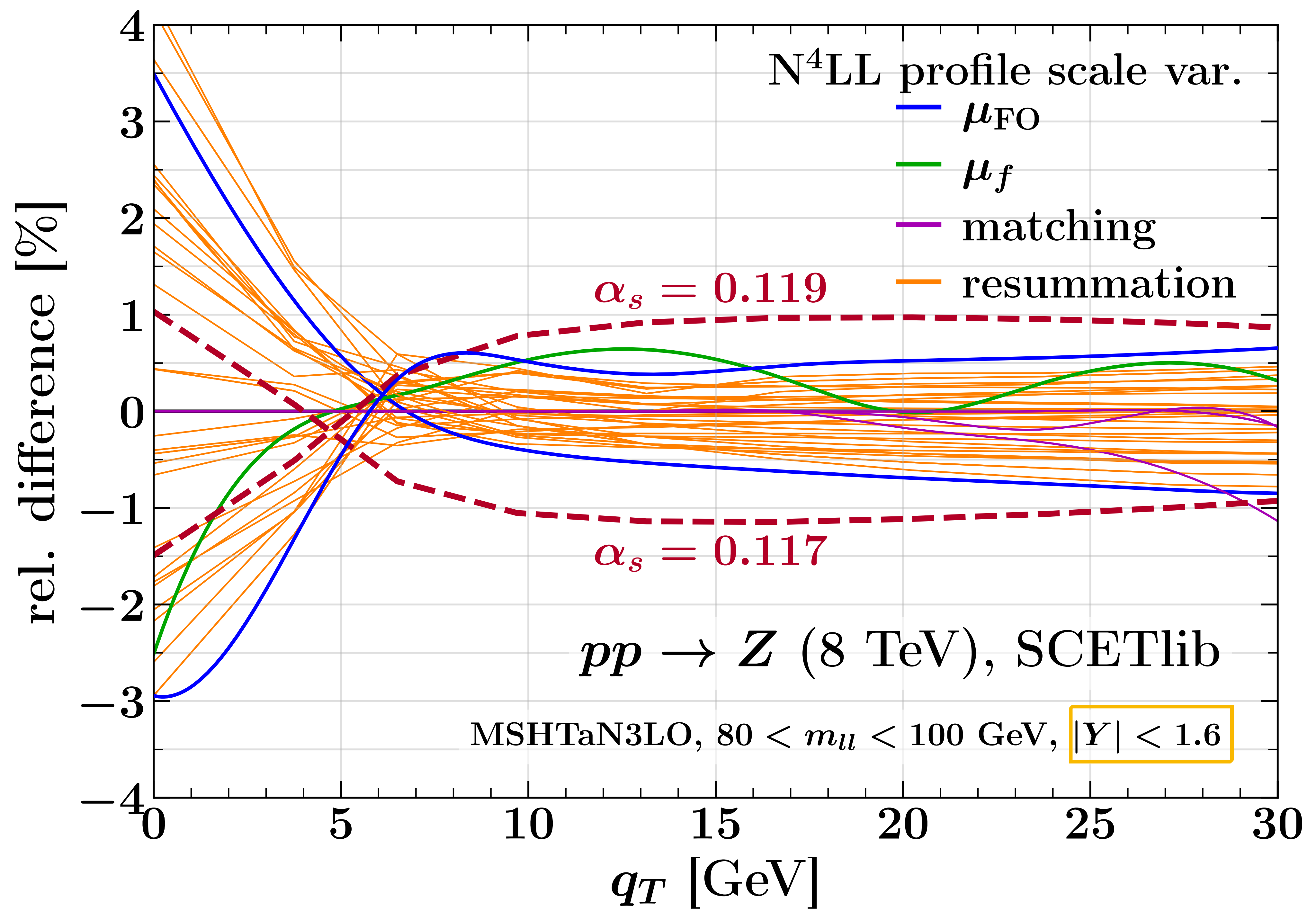
→ 72 data points in ATLAS binning:
9 q_T bins in [0,29] GeV for each 8 Y bin in [0.0,3.6]
[integrated in q_T, Y and Q]

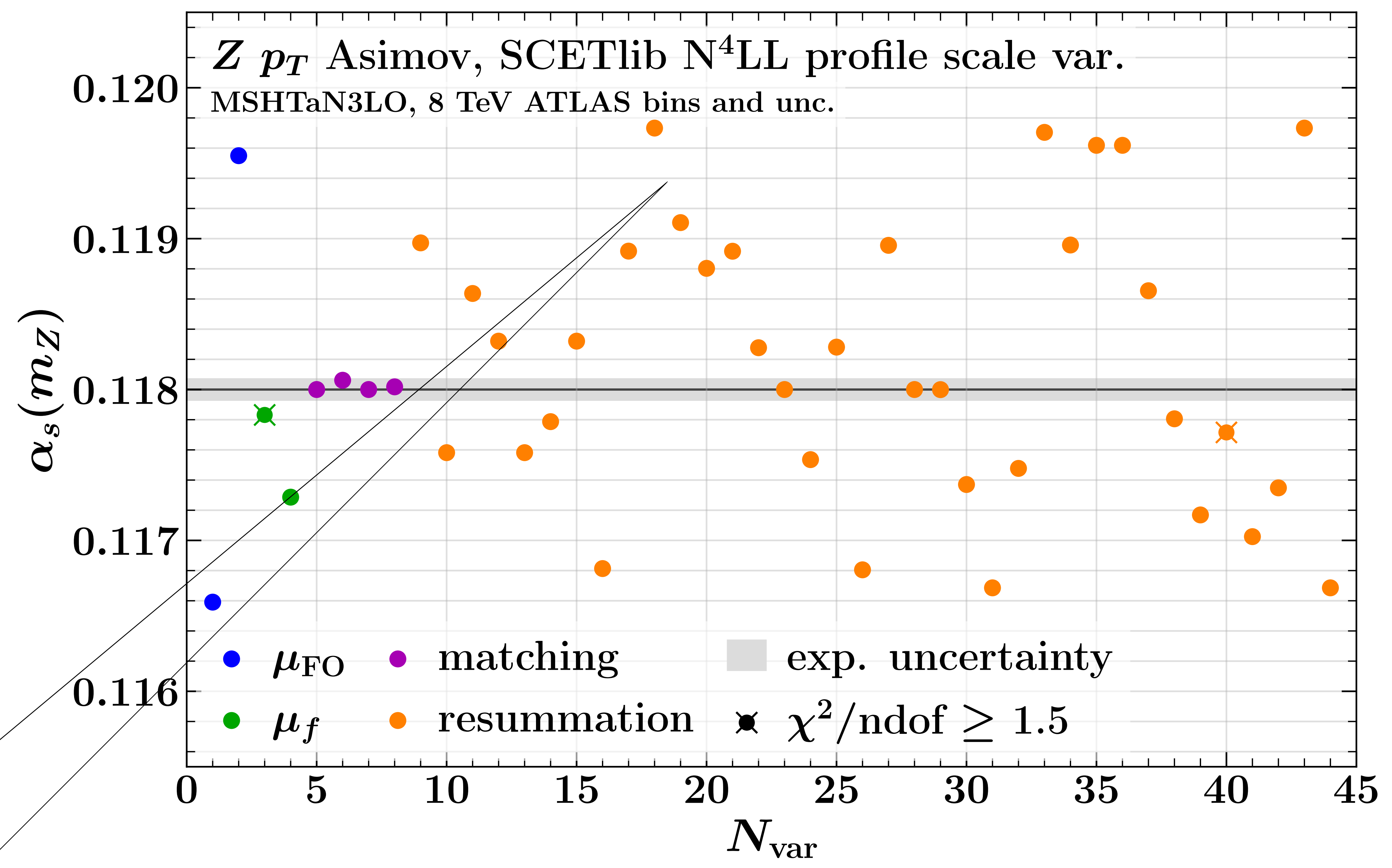
→ Using Minuit (and Minos) as minimizer for the fit



Perturbative uncertainty: scale variations

Fitting only $\alpha_s(m_Z)$ as a concrete example:





Perturbative uncertainty	Absolute uncertainty on $\alpha_s(m_Z)$ in units of 10^{-3}	
	ATLAS '23	Our estimate of expected size
Scale variations	±0.42	±2.43

Sum envelopes of different types: $\Delta_{\text{scale}} = 2.43$

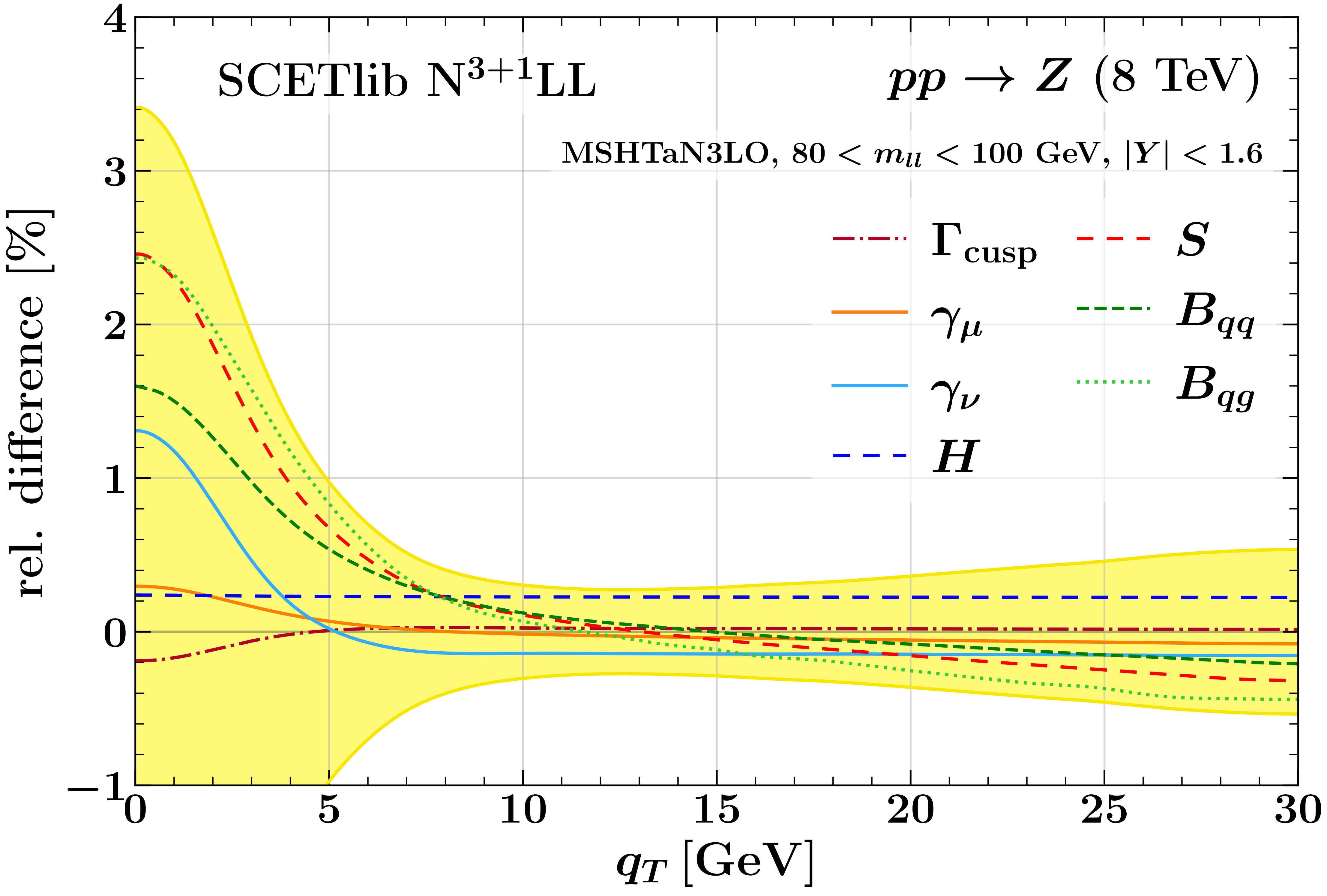
Naive envelope: $\Delta_{\text{scale}} = 1.73$

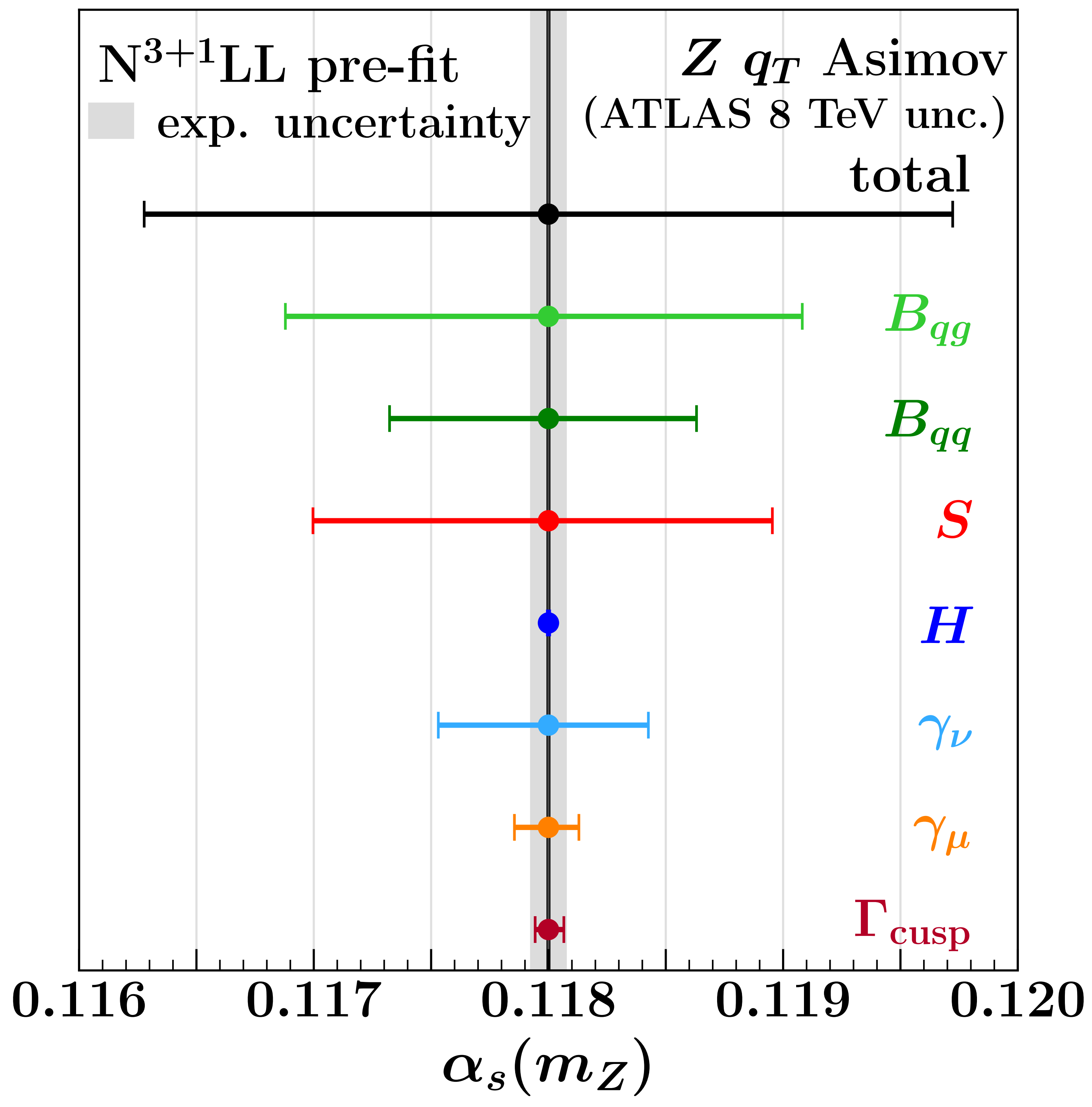
This is not our final expected perturbative uncertainty!

❌ **Scale variations are just insufficient for this purpose!**

*uncertainties in units of 10^{-3}

Perturbative uncertainty: TNP scanning





Only fitting α_S

Data as N^{3+1}LL

at $\alpha_s = 0.118$

- Breakdown into independent sources of uncertainty, varying each TNP by $\Delta\theta_n = \pm 1$(68% CL)

  encoding bin-by-bin correlation

- Repeat the α_s fit for every TNP variation [scanning or off-set]

Sum in quadrature: $\Delta^{\rm expect}_{\rm pert} = 1.75$

Indication that SCETlib scale variations are realistic [$\Delta_{\rm scale} = 1.73$]

*uncertainties in units of 10^{-3}

Perturbative uncertainty: TNP profiling

$$\chi^2_{\text{total}} = \sum_{ij} (y_i - \lambda_i)^T C^{-1}_{ij} (y_j - \lambda_j) + \sum_i \frac{(\theta_i - 0)^2}{\Delta\theta_i^2}$$

- Profiling: fitting α_s *together* with all TNPs
  - → TNPs are proper parameters, included in the fit with Gaussian constraint $\theta_n = 0 \pm 1$
  - → allows data to constrain TNPs and thereby reduce theory uncertainty

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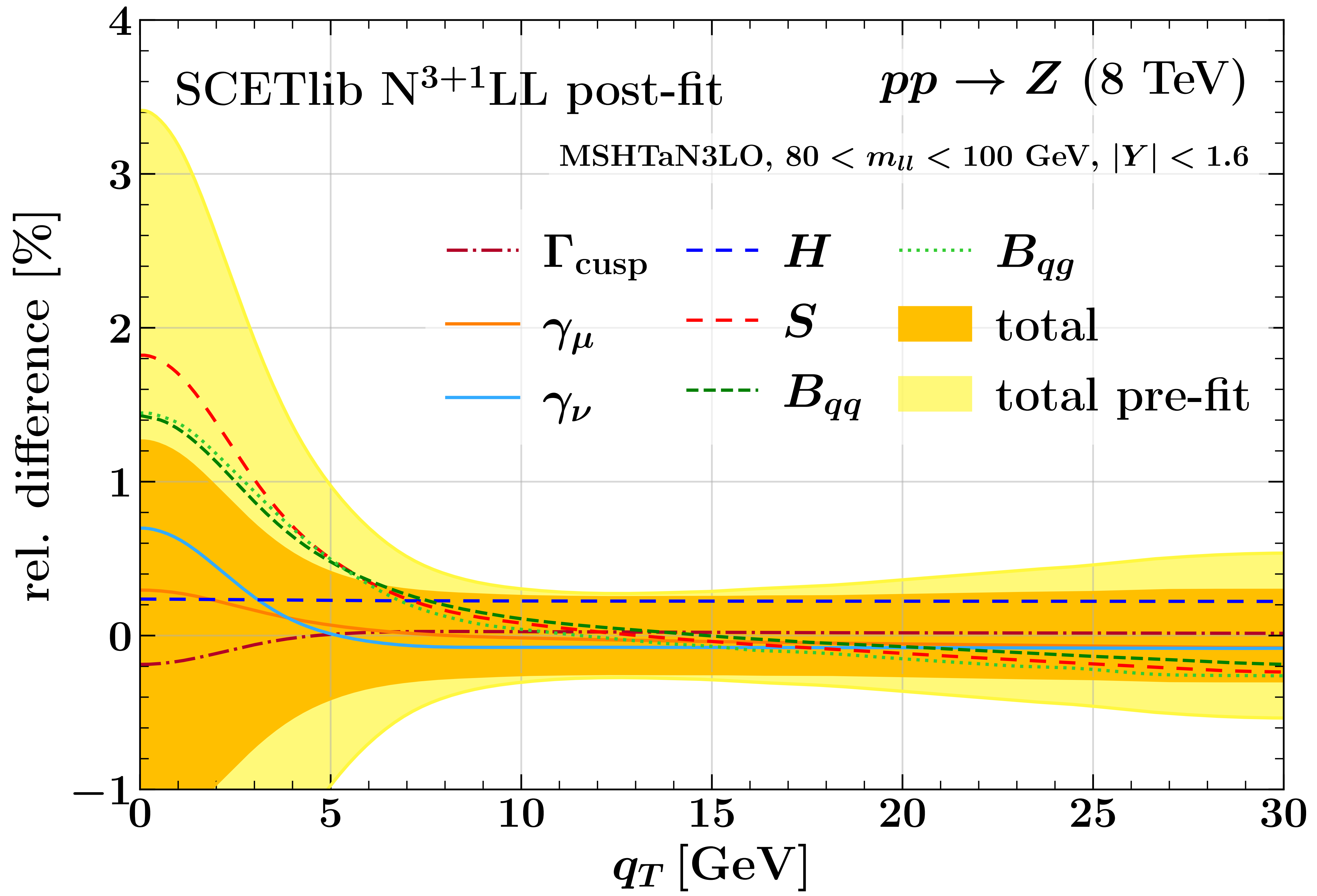
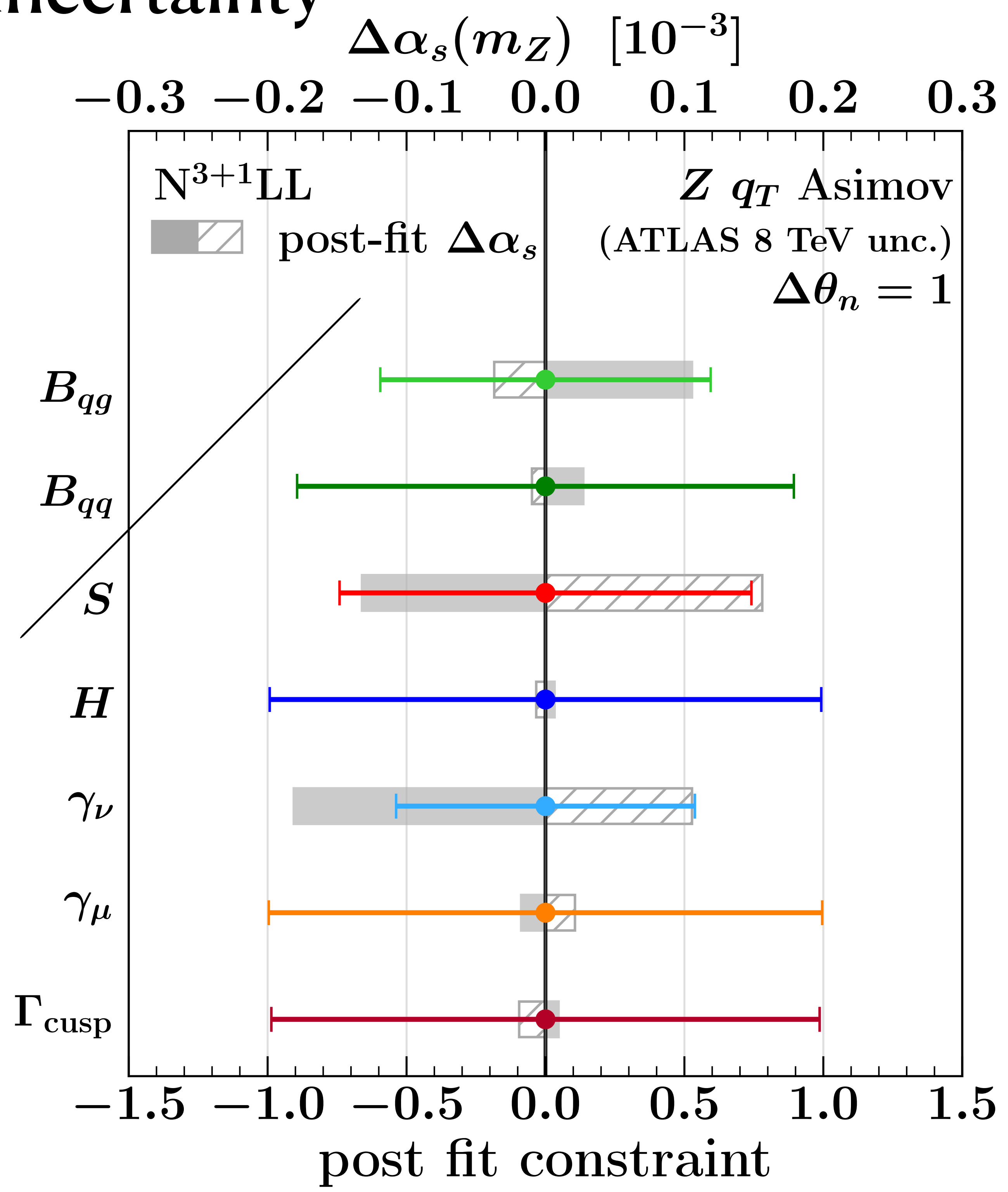




Only fitting α_S

Data as N^{3+1}LL

at $\alpha_s = 0.118$

grey:
TNP down variation

dashed grey:
TNP up variation

- Reduction in the uncertainty as expected: induced correlation between α_s and TNPs!

$\Delta^{\text{expect}}_{\text{pert}} = 0.45$

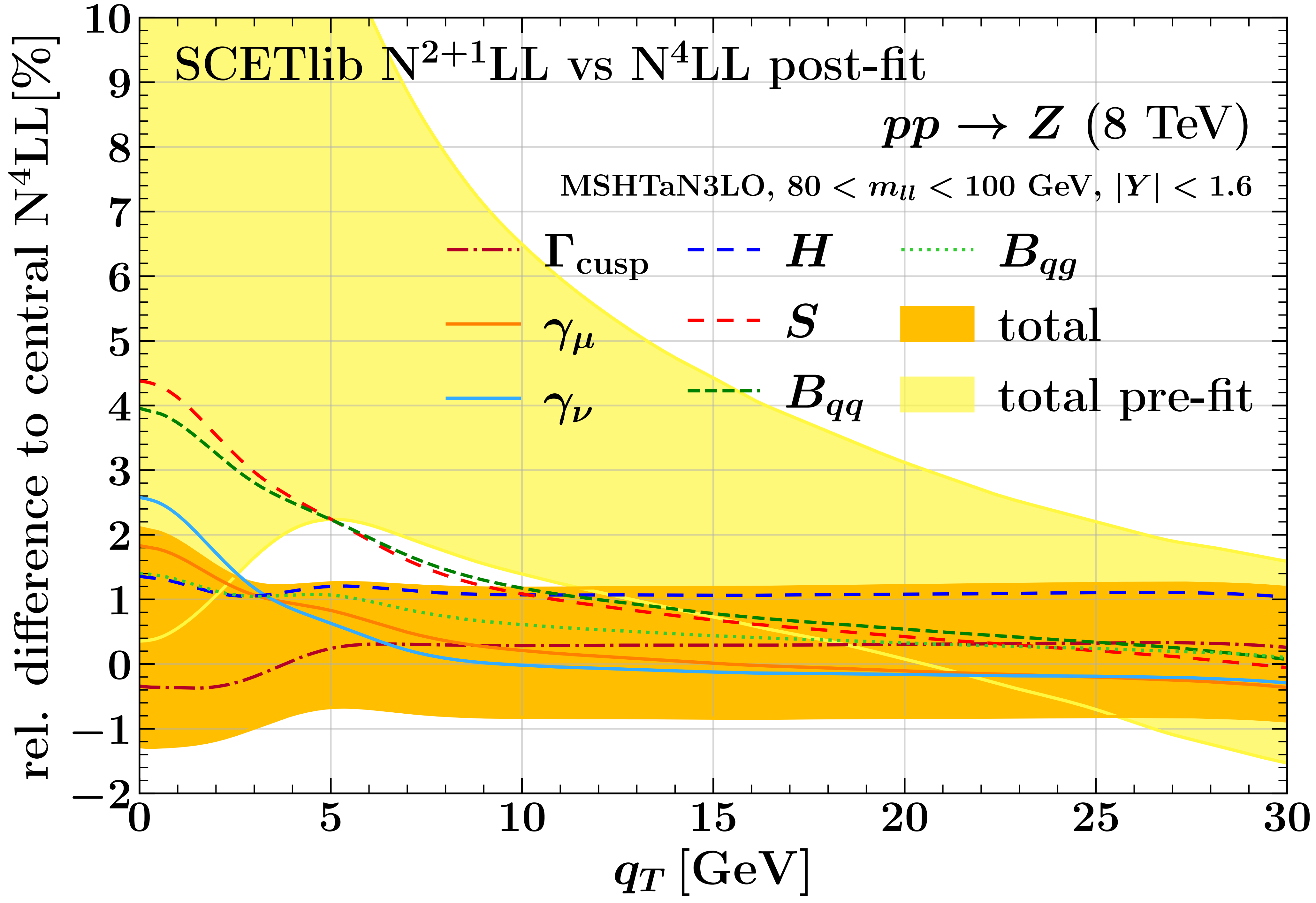
*uncertainties in units of 10^{-3}

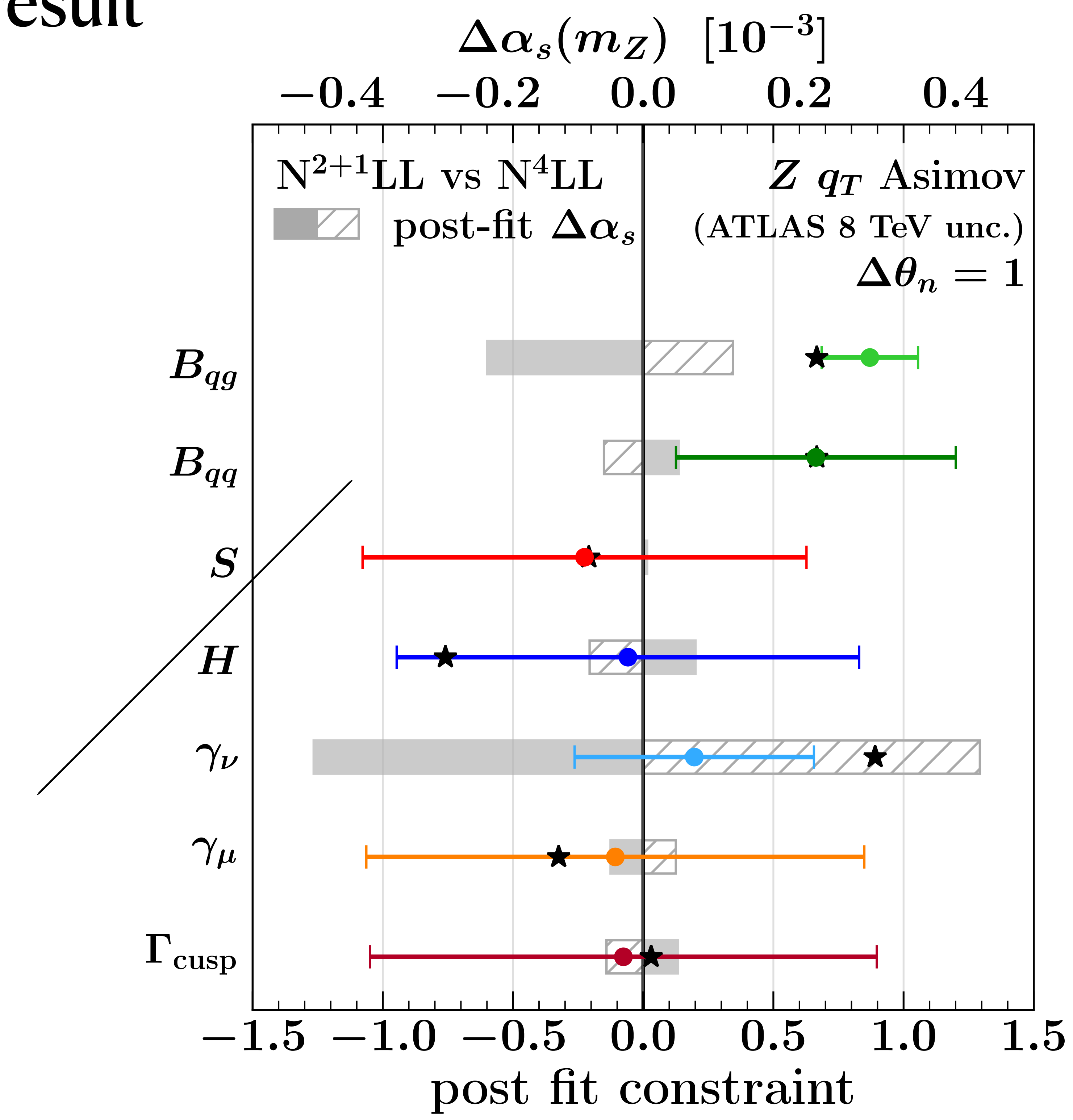
Perturbative uncertainty: TNP profiling

- Profiling lower order against higher order: $\mathrm{N}^{2+1}\mathrm{LL}$

Data: central $\mathrm{N}^4\mathrm{LL}$ prediction at $\alpha_s = 0.118$

→ simulates the fit to real data, which contains the all-order result





- $\mathrm{N}^{2+1}\mathrm{LL}$ **strongly** pulled toward correct true values [★] — indication that the order is not enough for the data

- post-fit prediction for q_T spectrum driven by constraints from data

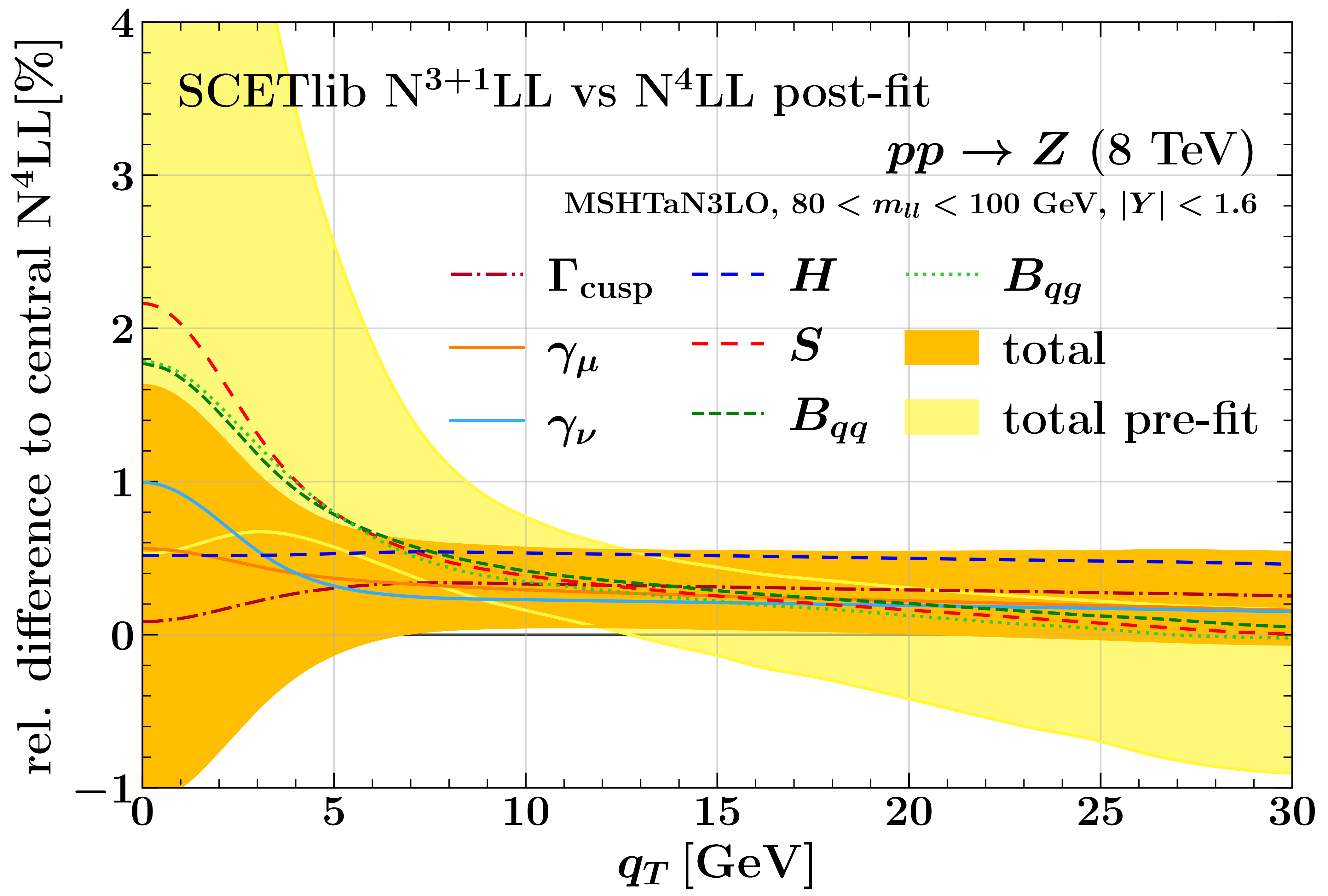
$$\Delta^{\rm expect}_{\rm pert} = {}^{+0.66}_{-0.62}$$

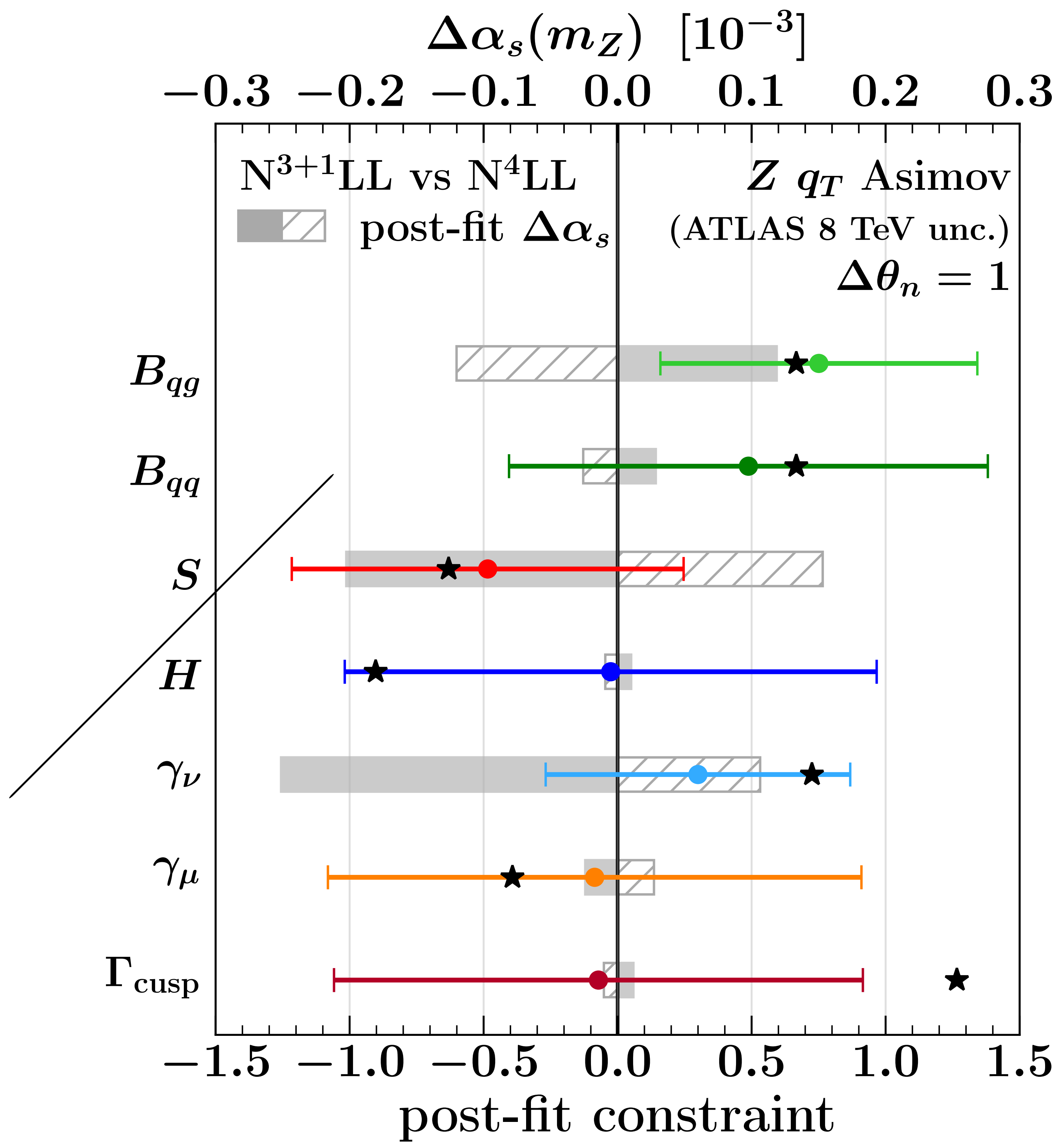
*uncertainties in units of 10^{-3}

Perturbative uncertainty: TNP profiling

- Profiling lower order against higher order: N^{3+1}LL

Data: central N^{4}LL prediction at $\alpha_s = 0.118$





- N^{3+1}LL pulled toward correct true values [★]
- post-fit prediction for q_T spectrum driven by constraints from data

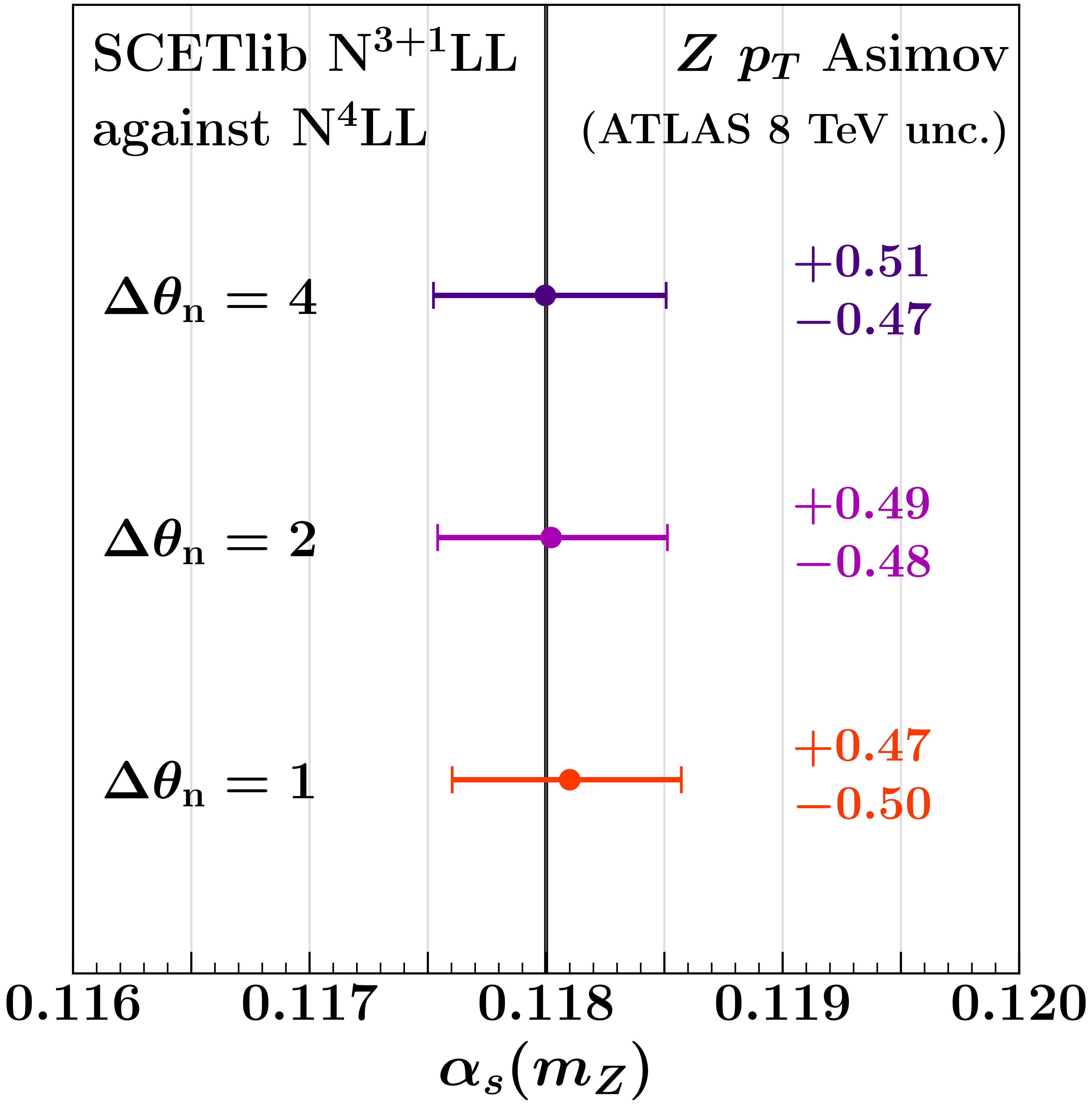
$$\Delta^{\text{expect}}_{\text{pert}} = {}^{+0.47}_{-0.50}$$

*uncertainties in units of 10^{-3}

Perturbative uncertainty: TNP profiling with different $\Delta\theta_n$

Data: central N^{4}LL prediction at $\alpha_s = 0.118$

- Change the prior theory constraint: using now $\theta_n = 0 \pm \Delta\theta_n$ with $\Delta\theta_n = 1, 2, 4$ and fit again



→ the effect relative to the theory constraint strongly depends on the power of the experimental constraint

→ data reduces dependence on theory constraint and associated potential bias

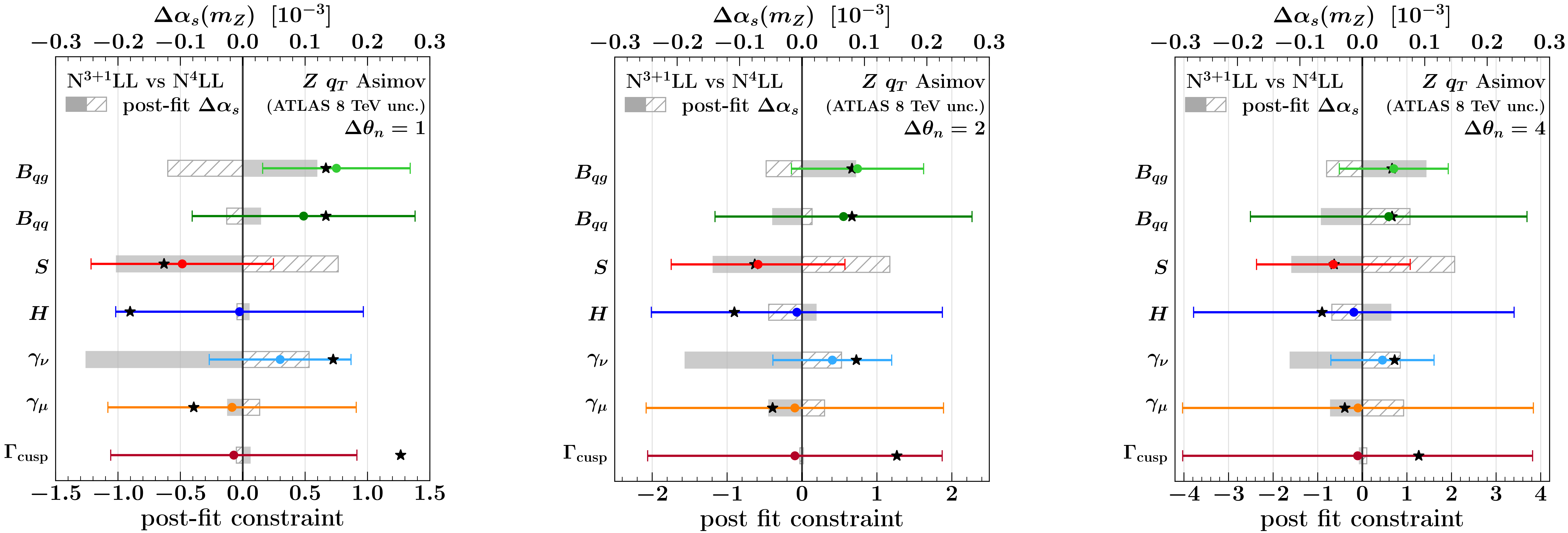
→ only slight difference in the uncertainties when relaxing the TNP constraint

Precise theory constraint does not matter here

*uncertainties in units of 10^{-3}

Perturbative uncertainty: TNP profiling with different $\Delta\theta_n$

Data: central N^{4}LL prediction at $\alpha_s = 0.118$



Post-fit constraints on TNPs become even more consistent with true values!

Nonperturbative uncertainty

For $\Lambda_{\rm QCD} \ll 1/b_T \sim q_T \ll Q$, can systematically expand nonperturbative effects in an OPE

$$\tilde f_i(x, b_T, \mu, Q) = \boxed{\tilde f_i^{(0)}(x, b_T, \mu, Q)} \left\{ 1 + \boxed{b_T^2 \left[\Lambda_{2,i}(x) + \lambda_2^\zeta \ln \frac{b_T Q}{b_0}\right] + \mathcal{O}(\Lambda_{\rm QCD}^4 b_T^4)} \right\}$$

resummed perturbative part

nonperturbative contributions to the CS kernel and TMD PDF

At large b_T, no constraint from OPE, following Collins and Rogers '14:

$$\ln \tilde f_i^{\rm np}(x, b_T \to \infty) = -\,{\rm const} \times b_T$$
$$\tilde\gamma_\zeta^{\rm np}(b_T \to \infty) = -\,{\rm const}$$

At small b_T:

$$\tilde f_i^{\rm np}(x, b_T) = 1 + \Lambda_{2,i}(x)\, b_T^2 + \mathcal{O}(\Lambda_{\rm QCD}^4\, b_T^4)$$
$$\tilde\gamma_\zeta^{\rm np}(b_T) = \lambda_2^\zeta\, b_T^2 + \mathcal{O}(\Lambda_{\rm QCD}^4\, b_T^4)$$

Nonperturbative uncertainty

For $\Lambda_{\rm QCD} \ll 1/b_T \sim q_T \ll Q$, can systematically expand nonperturbative effects in an OPE

$$\tilde{f}_i(x, b_T, \mu, Q) = \tilde{f}_i^{(0)}(x, b_T, \mu, Q) \left\{ 1 + b_T^2 \left[\Lambda_{2,i}(x) + \lambda_2^\zeta \ln \frac{b_T Q}{b_0} \right] + \mathcal{O}(\Lambda_{\rm QCD}^4 b_T^4) \right\}$$

resummed perturbative part

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$$\tilde{\gamma}_\zeta^{\rm np}(b_T \to \infty) = -\,{\rm const}$$

At small b_T:

$$\tilde{f}_i^{\rm np}(x, b_T) = 1 + \Lambda_{2,i}(x)\, b_T^2 + \mathcal{O}(\Lambda_{\rm QCD}^4\, b_T^4)$$

$$\tilde{\gamma}_\zeta^{\rm np}(b_T) = \lambda_2^\zeta\, b_T^2 + \mathcal{O}(\Lambda_{\rm QCD}^4\, b_T^4)$$

Every nonperturbative model has to satisfy the OPE expansion!

$$\tilde{f}^{\rm np}(b_T) = \exp\left[-\,\Lambda_\infty\, b_T\, \tanh\left(\frac{\Lambda_2}{\Lambda_\infty} b_T + \frac{\Lambda_4}{\Lambda_\infty} b_T^3 \right) \right]$$

No prior constraint

$$2\tilde{\gamma}_\zeta^{\rm np}(b_T) = -\,\lambda_\infty\, \tanh\left(\frac{\lambda_2}{\lambda_\infty} b_T^2 + \frac{\lambda_4}{\lambda_\infty} b_T^4 \right)$$

+ *Lattice QCD** constraints [details in Peter's talk]

λ_∞, Λ_∞ determine $b_T \to \infty$ behaviour, $\lambda_{2,4}$, $\Lambda_{2,4}$ quadratic/quartic small b_T coefficients

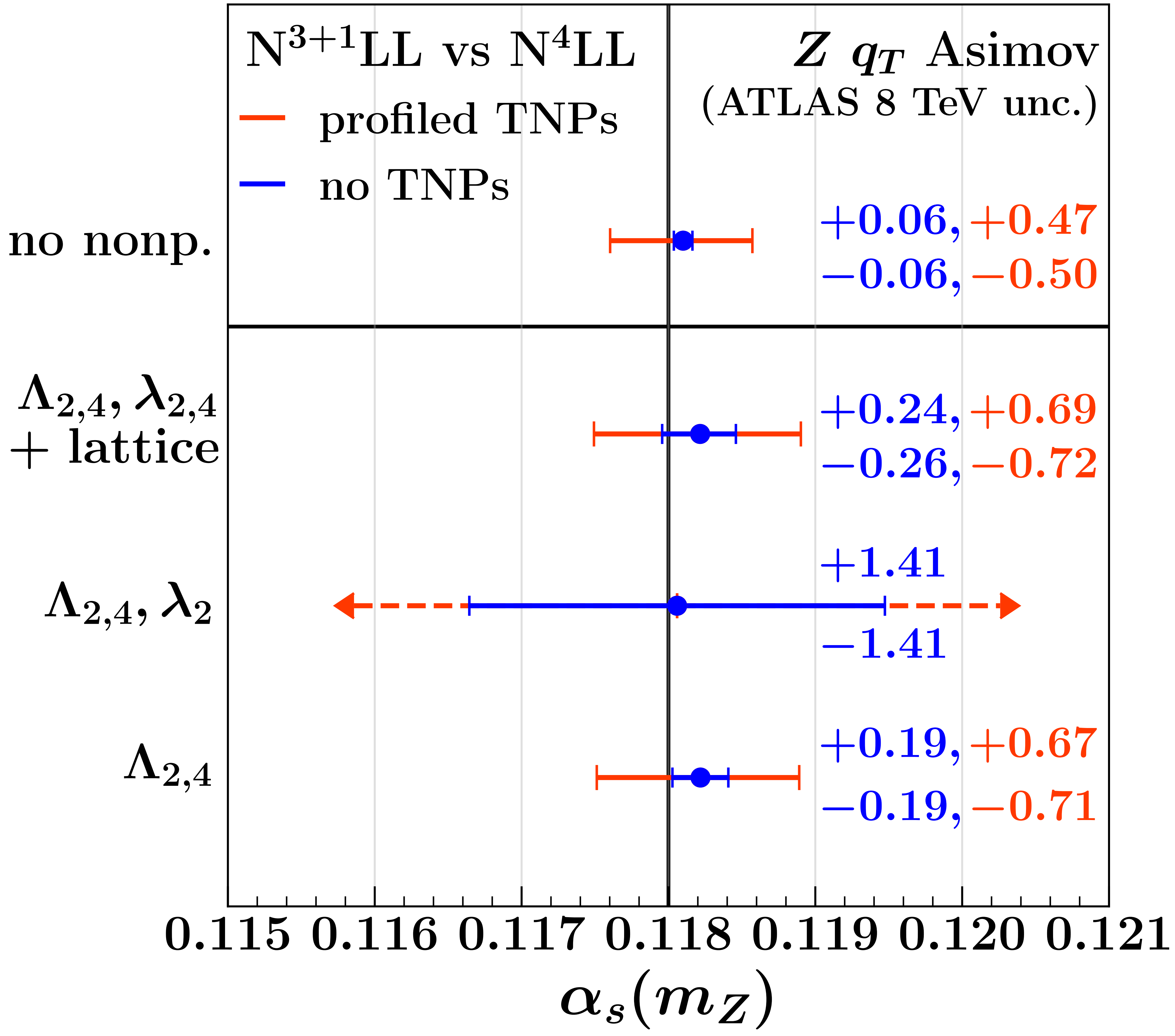
*MIT, RQCD23, LPC

Nonperturbative uncertainty

fit unc. only: fitting *only* α_s and nonp.

profiled TNPs: α_s + nonp. + TNPs

Data: central N^{4}LL prediction at $\alpha_s = 0.118$



→ TMD PDF + λ_2 and no further constraint leads to an unstable fit

→ CS kernel constrained by lattice QCD data allows to a joint fit between CS and TMD nonperturbative!

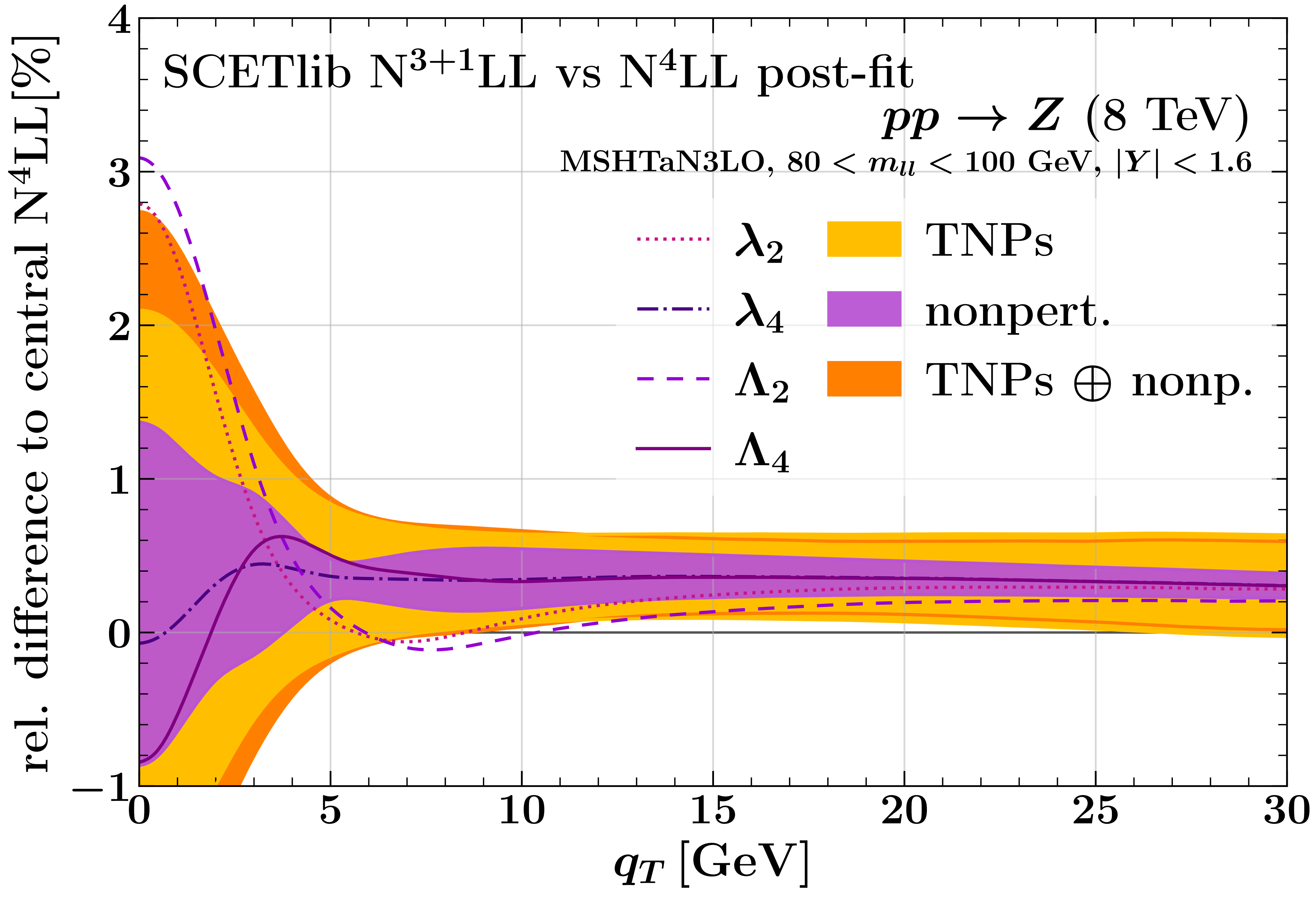
→ this is possible *especially* thanks to the constraining power of the data!!!

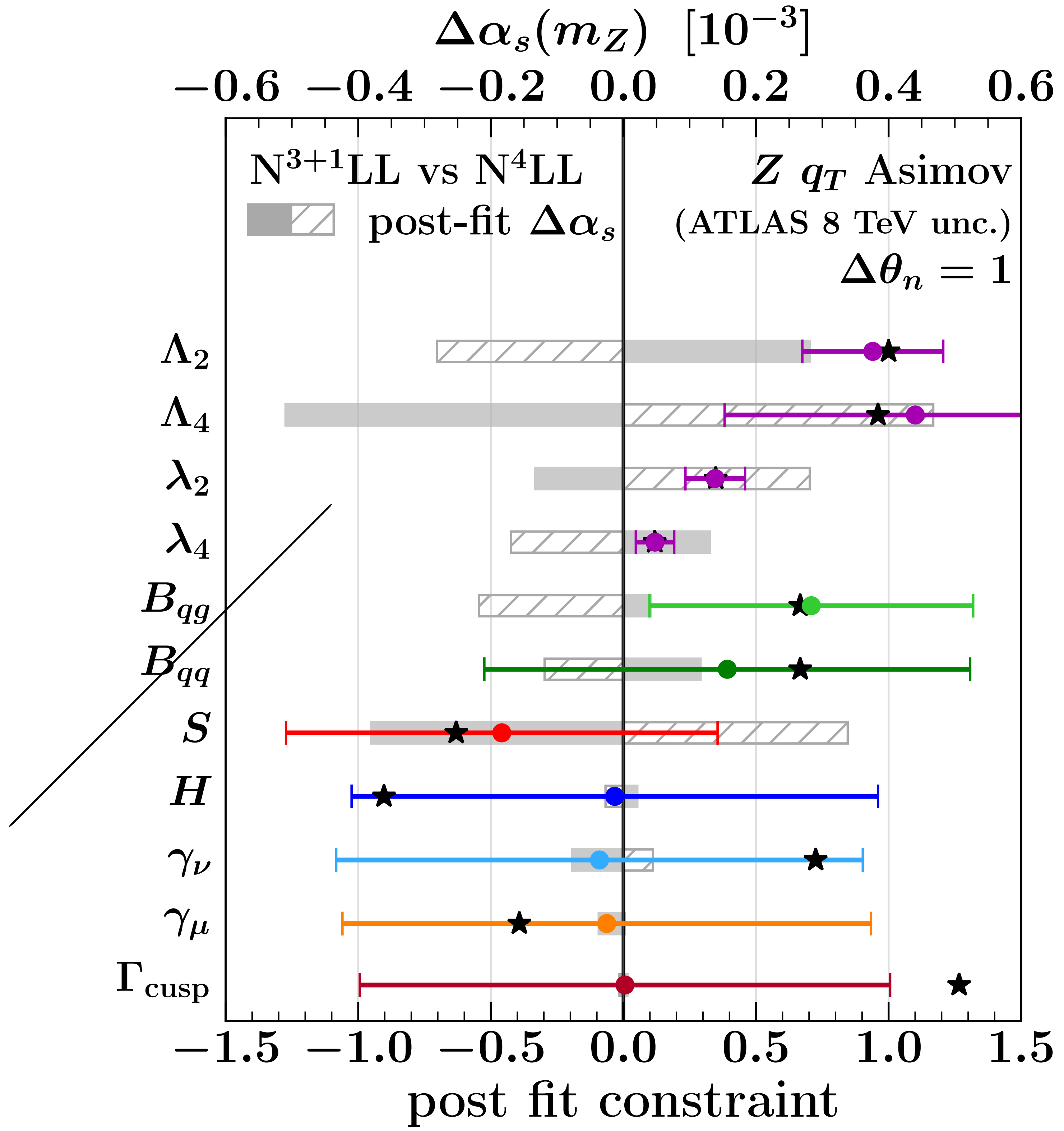
*uncertainties in units of 10^{-3}

Nonperturbative uncertainty

- Fitting nonperturbative and including lattice QCD constraints

Data: central N^{4}LL prediction at $\alpha_s = 0.118$





- N^{3+1}LL pulled toward correct true values [★]
- Data now also constrain nonp. params., therefore less constraint on TNPs

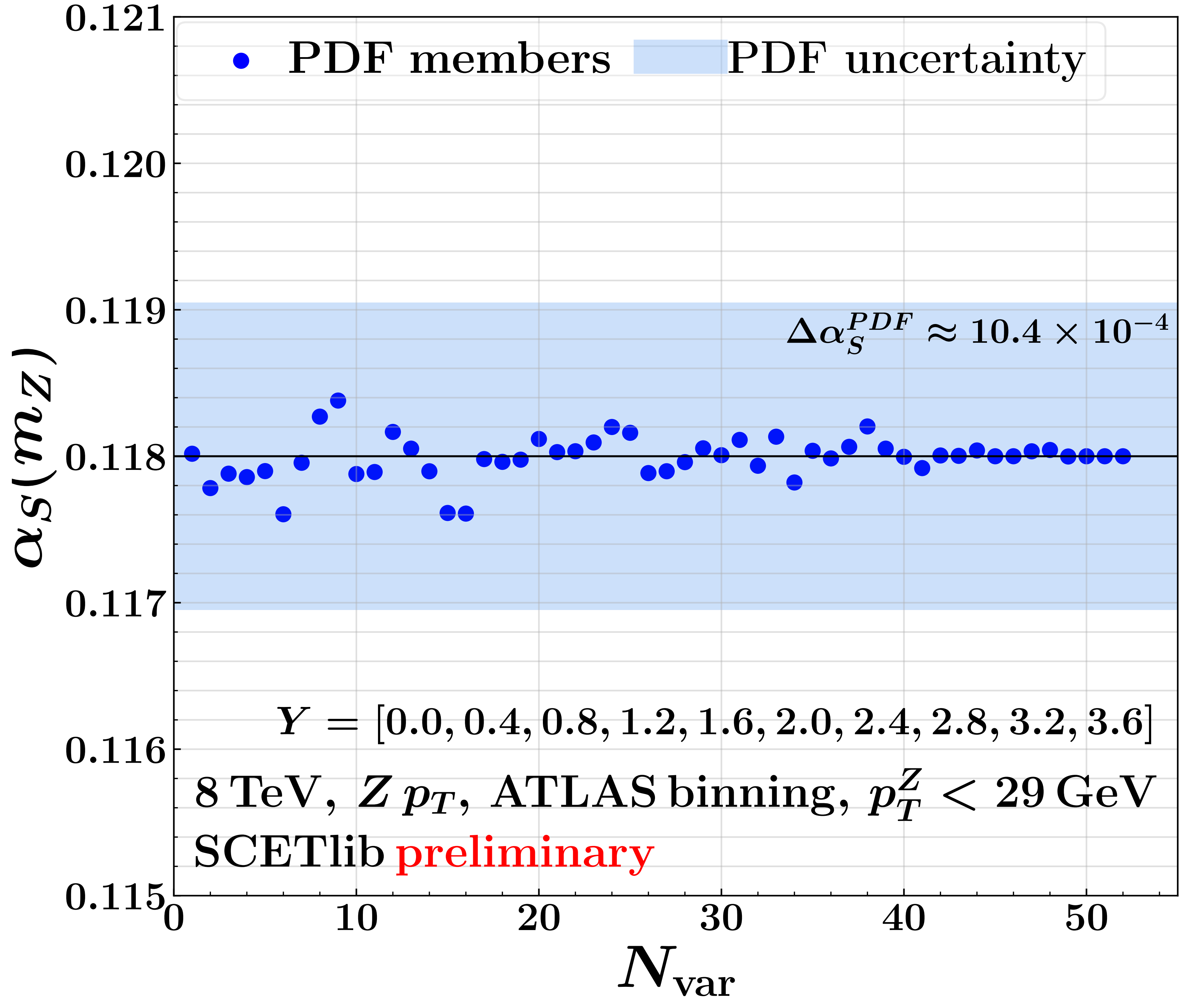
$$\Delta_{\rm np}^{\rm expect} = ^{+0.24}_{-0.26}$$

PDF uncertainty

Fitting only $\alpha_s(m_Z)$:

Data: central $N^{3+1}LL$ prediction at $\alpha_s = 0.118$

PRELIMINARY



→ $\Delta^{\mathrm{expect}}_{\mathrm{PDF}} = 1.04$ through scanning

[scanning/off-set over PDF eigenvector variations]

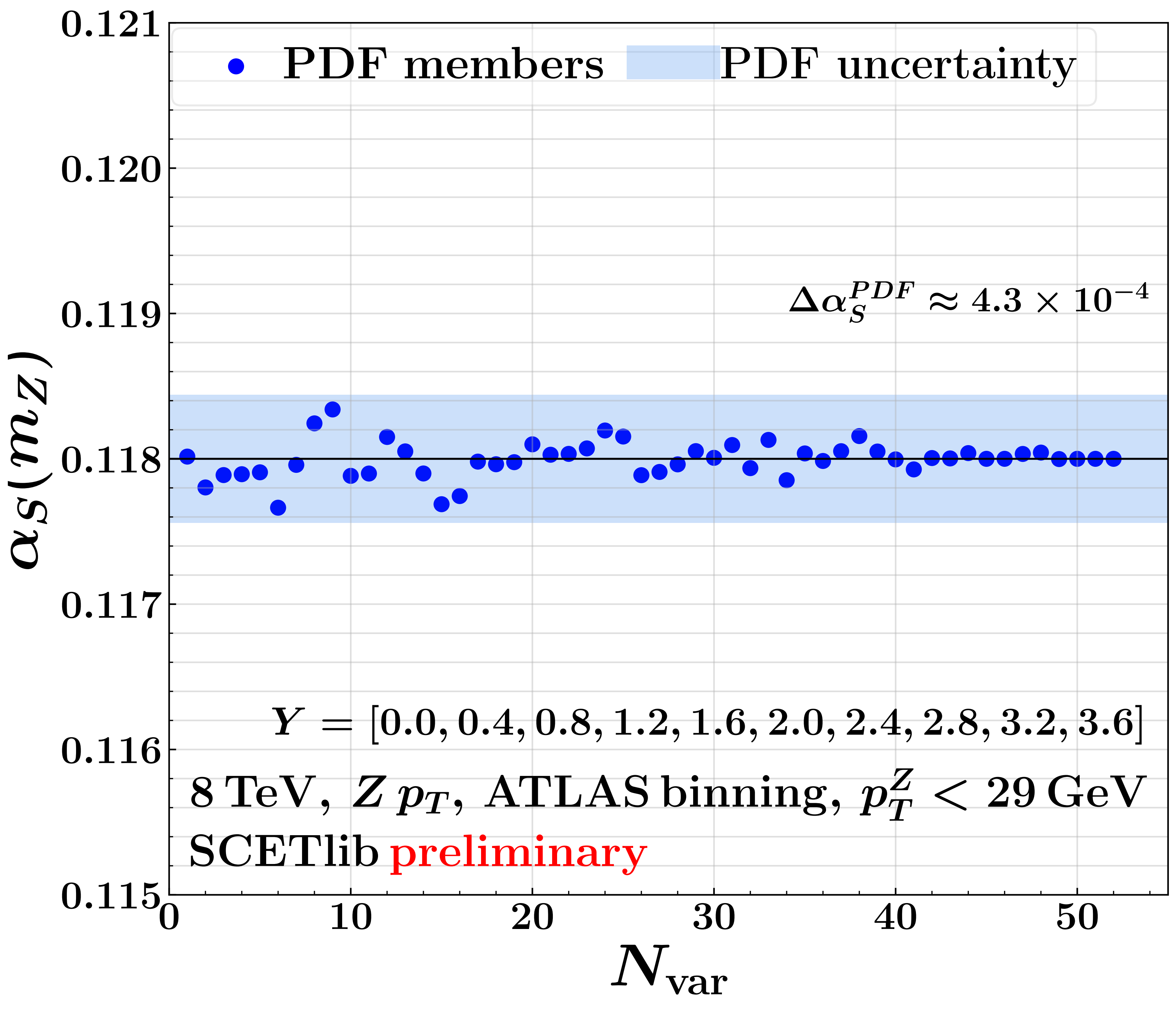
*uncertainties in units of 10^{-3}

PDF uncertainty

Fitting only $\alpha_s(m_Z)$:

Data: central N^{3+1}LL prediction at $\alpha_s = 0.118$

PRELIMINARY



→ $\Delta_{\text{PDF}}^{\text{expect}} = 1.04$ through scanning
[scanning/off-set over PDF eigenvector variations]

→ $\Delta_{\text{PDF}}^{\text{expect}} = 0.43$ through **incorrect profiling** (fitting α_s+PDFs)
[exploiting the precision data to constrain also PDFs]

$T = 1$

$$\chi^2_{\text{total}} = \sum_{ij} (y_i - \lambda_i)^T C_{ij}^{-1} (y_j - \lambda_j) + \sum_i \frac{(\theta_i - 0)^2}{\Delta\theta_i^2} + \sum_k T_k^2 (\theta_k^{\text{PDF}})^2$$

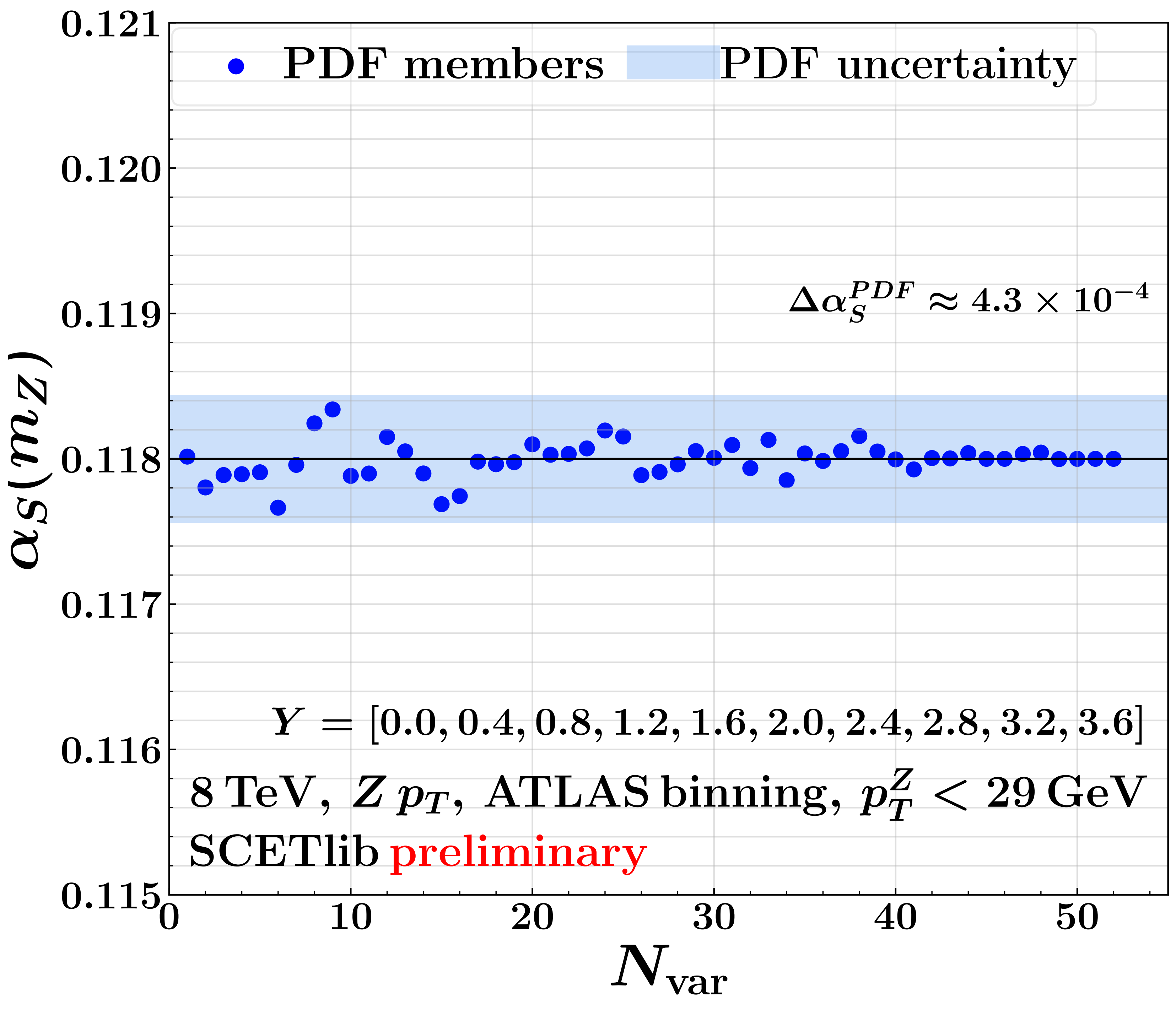
*uncertainties in units of 10^{-3}

PDF uncertainty

Fitting only $\alpha_s(m_Z)$:

Data: central $N^{3+1}LL$ prediction at $\alpha_s = 0.118$

PRELIMINARY



→ $\Delta^{\text{expect}}_{\text{PDF}} = 1.04$ through scanning
[scanning/off-set over PDF eigenvector variations]

→ $\Delta^{\text{expect}}_{\text{PDF}} = 0.43$ through **incorrect profiling** (fitting α_s+PDFs)
[exploiting the precision data to constrain also PDFs]

$T = 1$

Profiling requires:

1. PDFs central value and uncertainties not substantially changed
2. PDFs uncertainties included consistently alongside the new data ($Z\, q_T$)

» Large reduction in the uncertainty, just because we are including the wrong tolerance (T) factor

PDFs uncertainty given by $\Delta\chi^2 = T^2$, and not by $\Delta\chi^2 = 1$

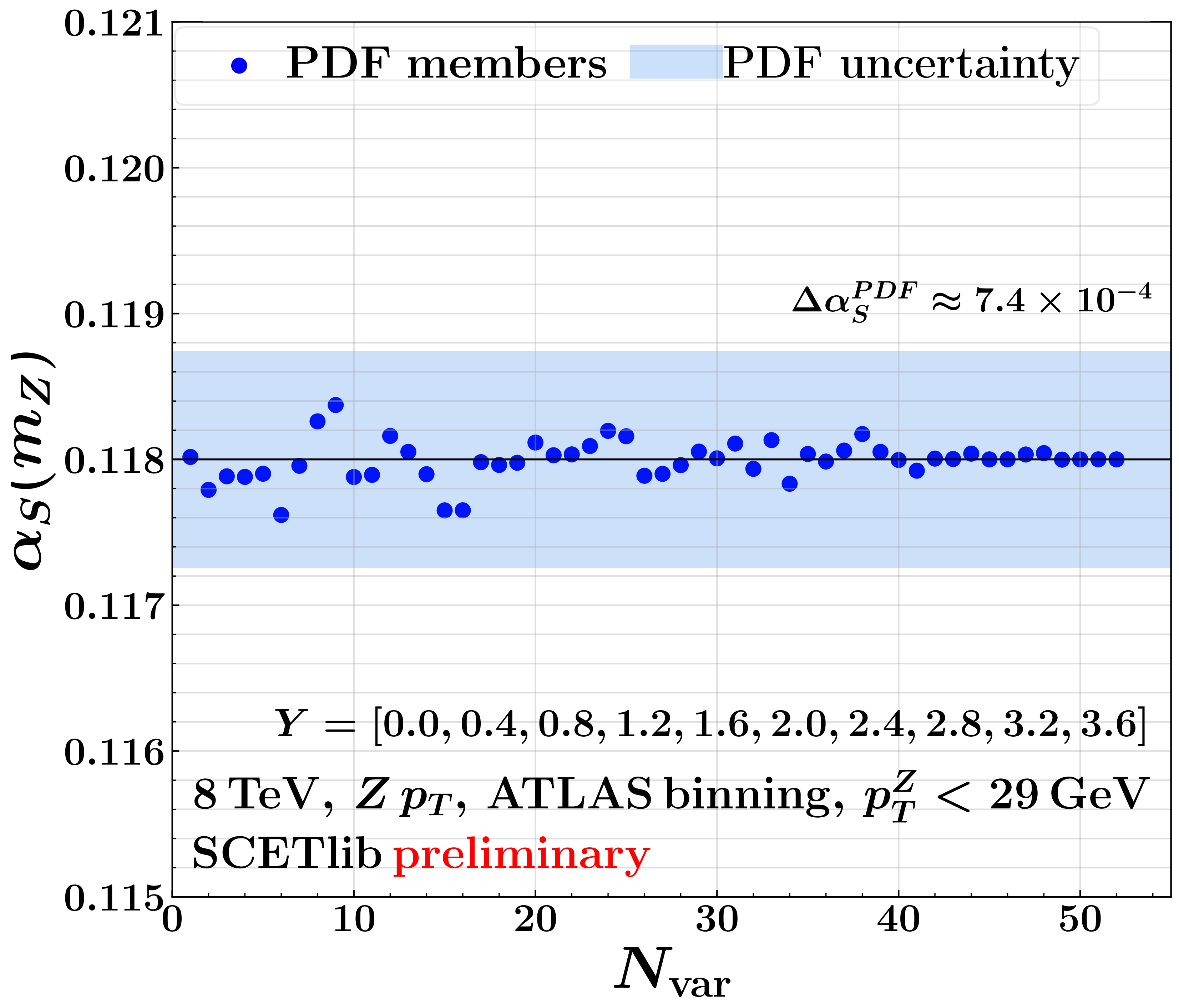
*uncertainties in units of 10^{-3}

PDF uncertainty

Fitting only $\alpha_s(m_Z)$:

Data: central $N^{3+1}LL$ prediction at $\alpha_s = 0.118$

PRELIMINARY



→ $\Delta^{\mathrm{expect}}_{\mathrm{PDF}} = 1.04$ through scanning
[scanning/off-set over PDF eigenvector variations]

→ $\Delta^{\mathrm{expect}}_{\mathrm{PDF}} = 0.43$ through **incorrect profiling** (fitting α_s+PDFs)
[exploiting the precision data to constrain also PDFs] $T = 1$

→ $\Delta^{\mathrm{expect}}_{\mathrm{PDF}} = 0.74$ through **profiling** (fitting α_s+PDFs)
and considering the **correct tolerance** $T \sim 3$

» Adding the correct tolerance reduces both pull on central value and uncertainty reduction!

how to propagate it to α_s uncertainty? Two-step profiling

*uncertainties in units of 10^{-3}

Summary and next steps

Correlations are fundamental for the interpretation of precision measurements:
having meaningful theory uncertainty is as important as having meaningful experimental one!

1. Theory Nuisance Parameters perfect candidate to describe theory uncertainty and *correlations*

2. Highly relevant and needed for the extraction of α_s from the $Z\, q_T$ spectrum:
   first applications work as advertised, very promising for the actual α_s fit!

3. *Moving to real data*: many things we need to add
   - ⟶ nonsingular contribution [work in progress]
   - ⟶ quark-mass effects: can generate a bias on α_s [work in progress]
   - ⟶ QED/EW corrections

$$\Delta\alpha_s^{\text{expect}} \simeq \underbrace{\pm\, 0.5_{\text{Sing.TNPs}} \pm ???_{\text{nons}} \pm ???_{m_q} \pm ???_{\text{QED/EW}}}_{\text{perturbative uncertainty}} \underbrace{\pm\, 0.3_{\text{nonp}} \pm 0.7_{\text{PDFs}}}_{\text{nonperturbative uncertainty+ PDFs}}$$

*uncertainties in units of 10^{-3}

Backup slides

Resummation details

Leading power cross section: $VV' = \{\gamma\gamma, \gamma Z, Z\gamma, ZZ, W^+W^+, W^-W^-\}$ $\quad x_{a,b} = \frac{Q}{E_{\rm cm}} e^{\pm Y}$

$$\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}^4 q} = \frac{1}{2E_{\rm cm}^2} L_{VV'}(q^2) \sum_{a,b} H_{VV'\,ab}(q^2, \mu) \times \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{\mathrm{i}\vec{b}_T\cdot\vec{q}_T} \tilde{B}_a(x_a, b_T, \mu, \nu/Q)\, \tilde{B}_b(x_b, b_T, \mu, \nu/Q)\, \tilde{S}(b_T, \mu, \nu)$$

leptonic tensor: $L_{VV'}(q^2)$

hard function: $H_{VV'\,ab}(q^2, \mu)$

beam functions: $\tilde{B}_a(x_a, b_T, \mu, \nu/Q)\, \tilde{B}_b(x_b, b_T, \mu, \nu/Q)$

soft function: $\tilde{S}(b_T, \mu, \nu)$

Unprimed vs primed counting:

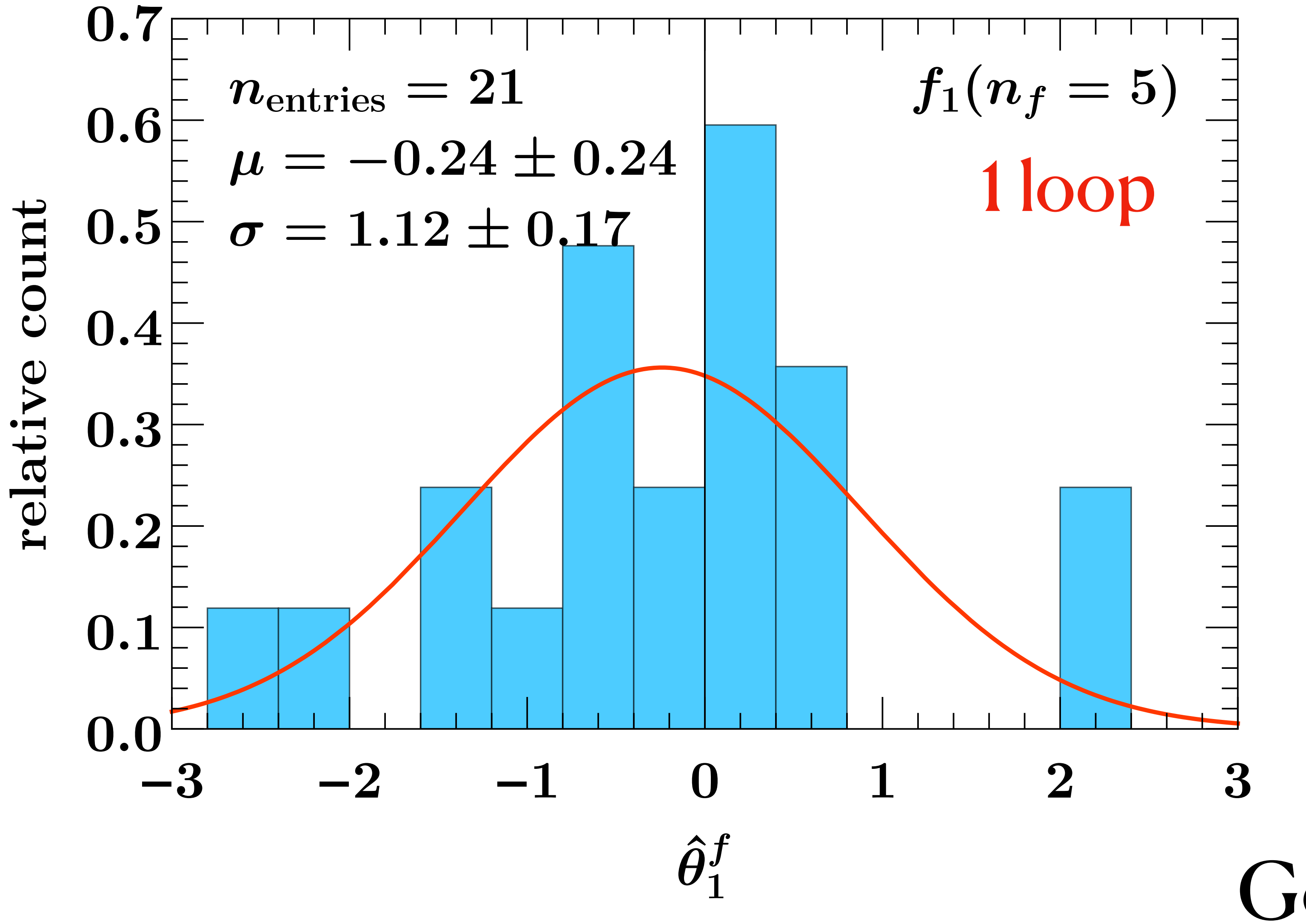
- primed orders boundary condition added to α_s^n higher

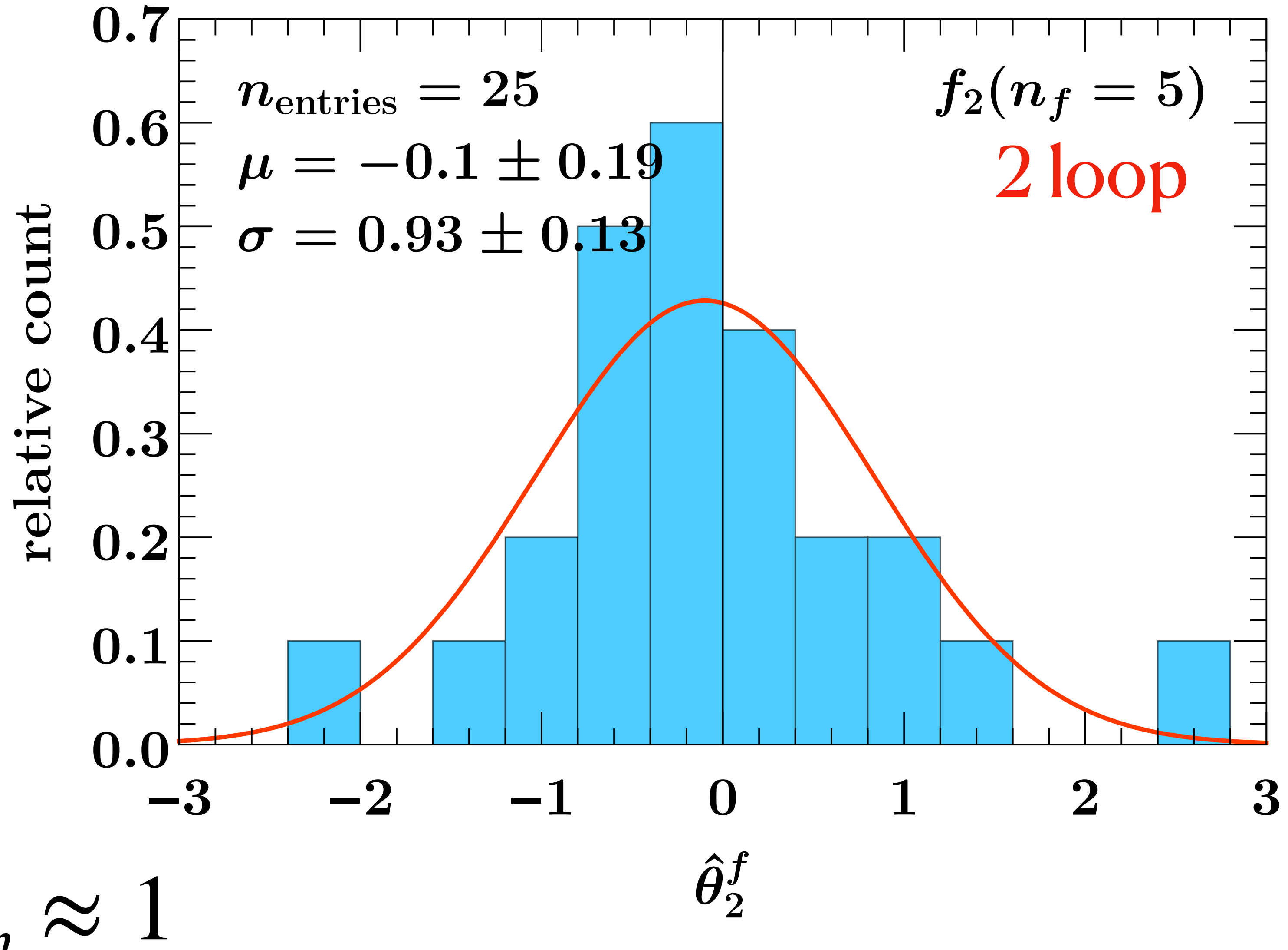
Order	Boundary cond. (FO singular)	Anomalous dimensions γ_i (noncusp)	$\Gamma_{\rm cusp}$, β	FO matching (nonsingular)
LL	1	-	1-loop	-
NLL	1	1-loop	2-loop	-
NLL′ (+NLO$_0$)	α_s	1-loop	2-loop	α_s
NNLL (+NLO$_0$)	α_s	2-loop	3-loop	α_s
NNLL′ (+NNLO$_0$)	α_s^2	2-loop	3-loop	α_s^2
N^{3}LL (+NNLO$_0$)	α_s^2	3-loop	4-loop	α_s^2
N^{3}LL′ (+N^{3}LO$_0$)	α_s^3	3-loop	4-loop	α_s^3
N^{4}LL (+N^{3}LO$_0$)	α_s^3	4-loop	5-loop	α_s^3

[from Michel's talk]

TNPs for Boundary Conditions

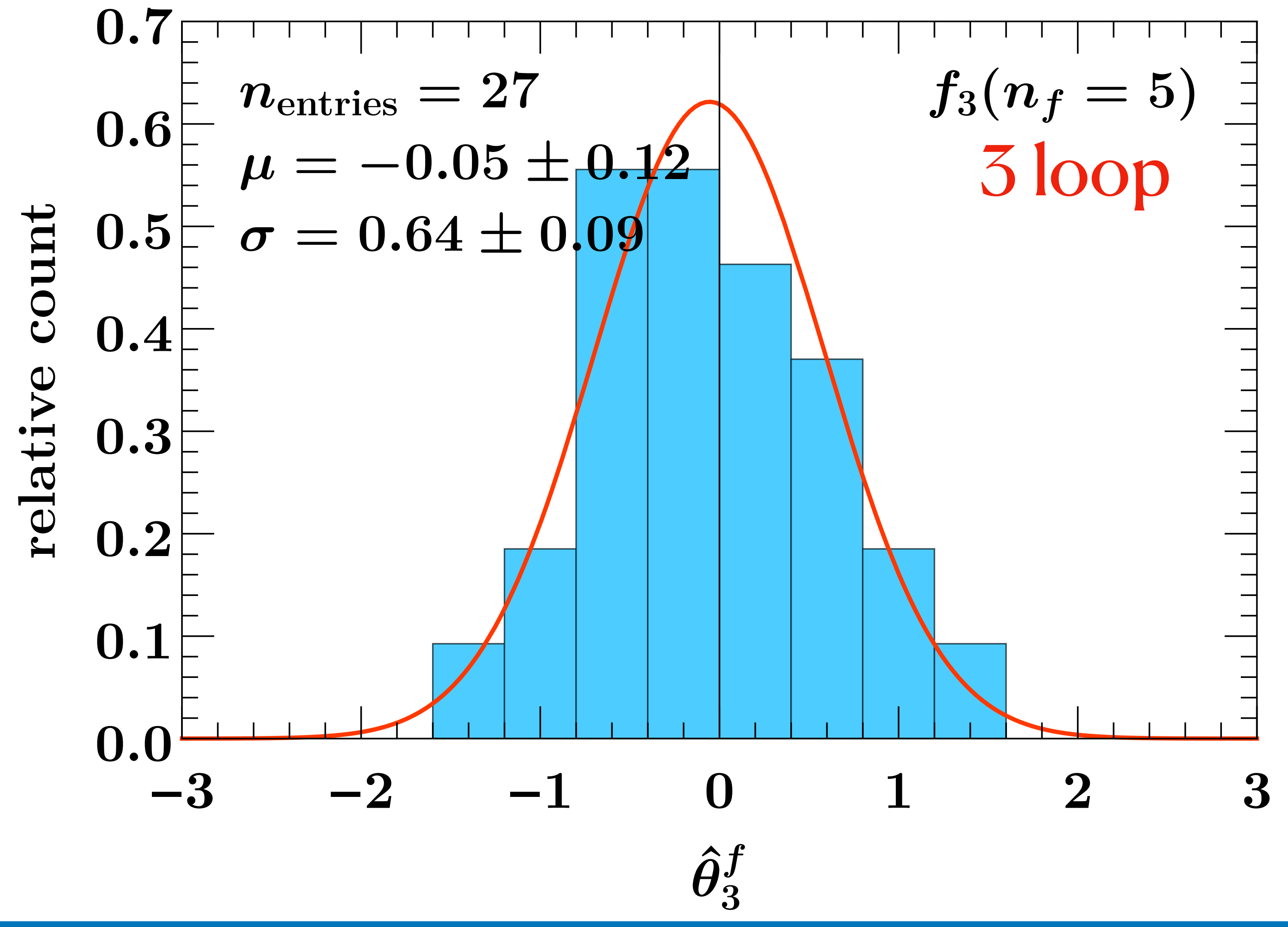
$$F_n(\theta_n) = 4C_r(4C_A)^{n-1}(n-1)!\,\theta_n^f$$

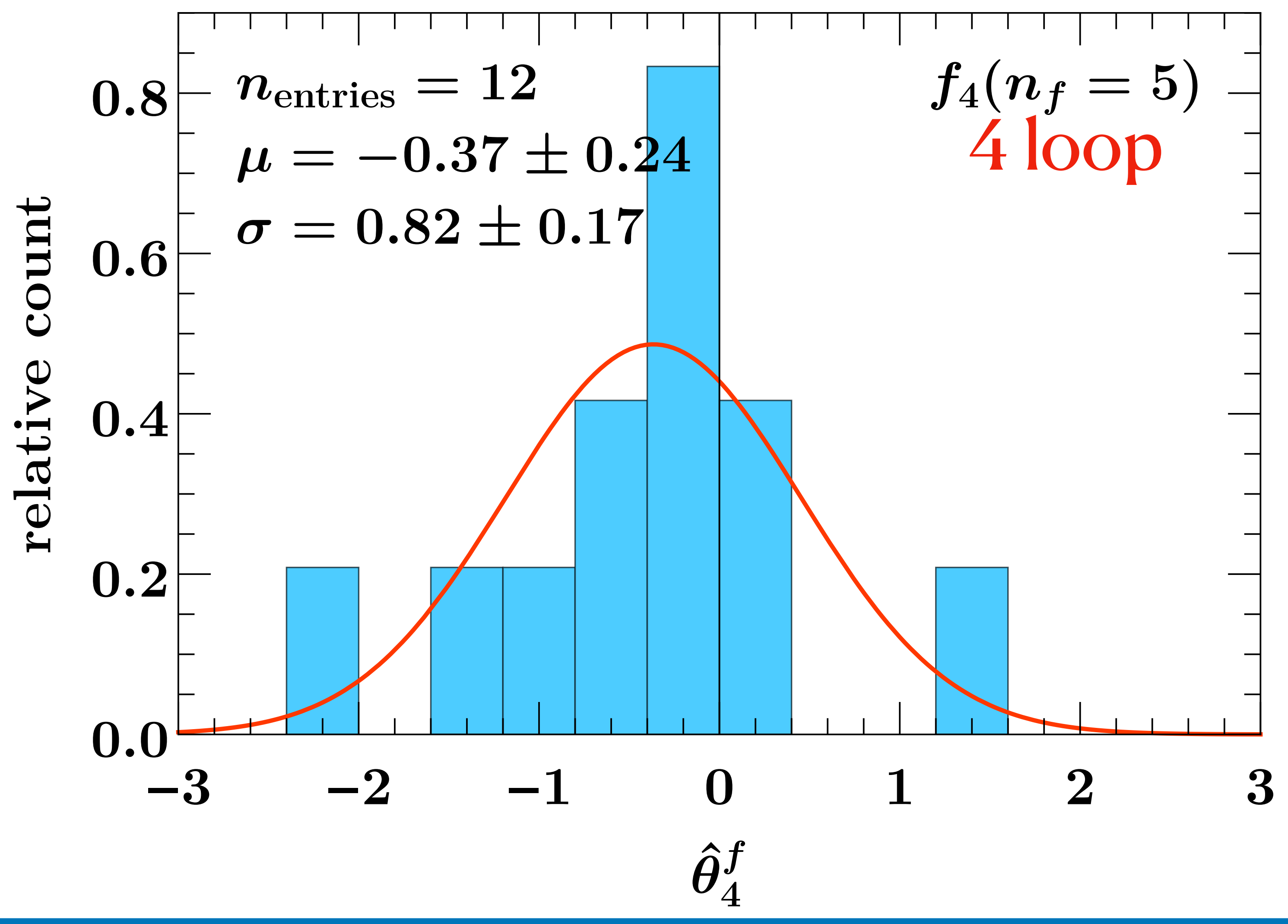




Good fit to a Gaussian with $\theta_n \approx 0$ and $\Delta\theta_n \approx 1$

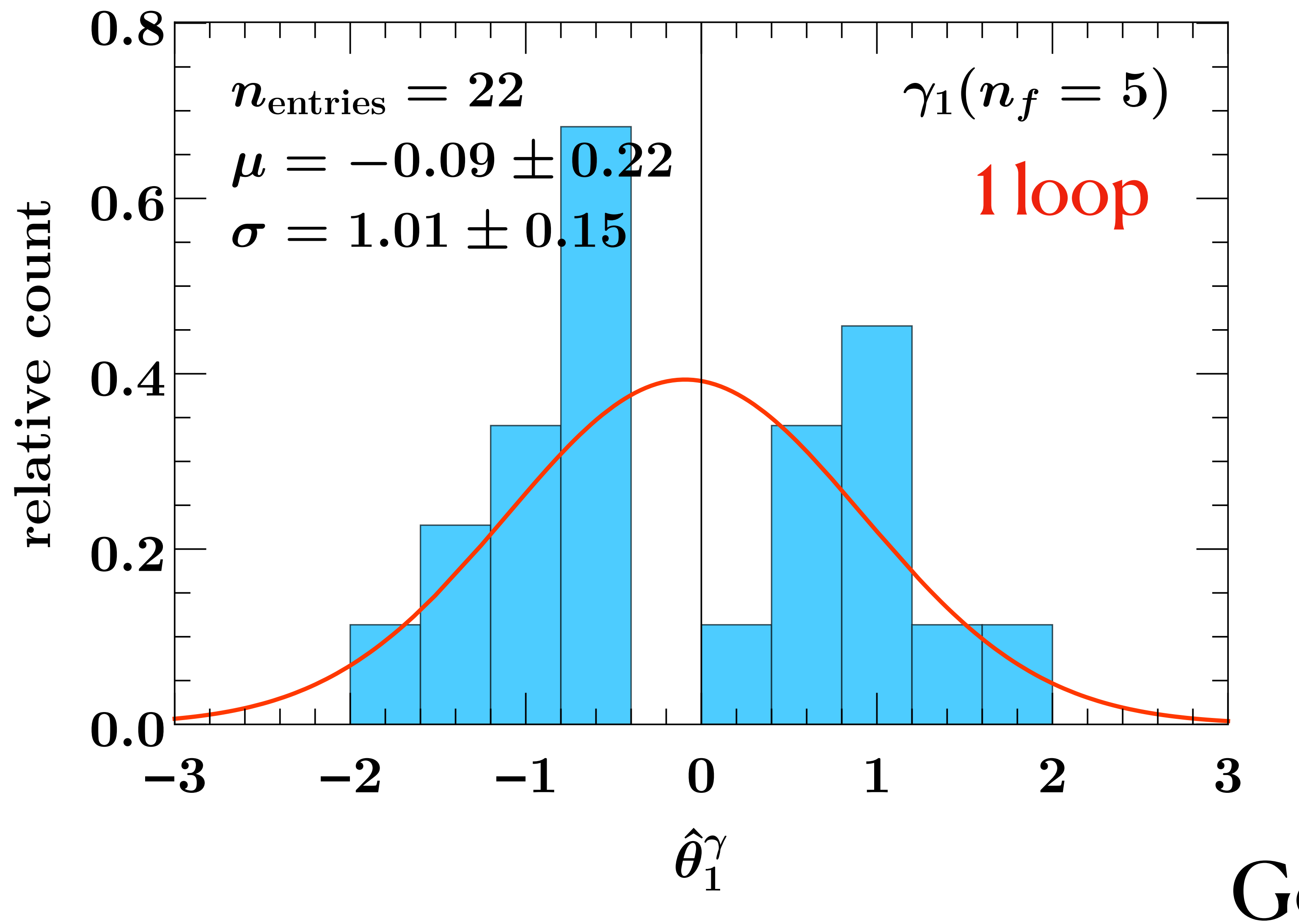
✓

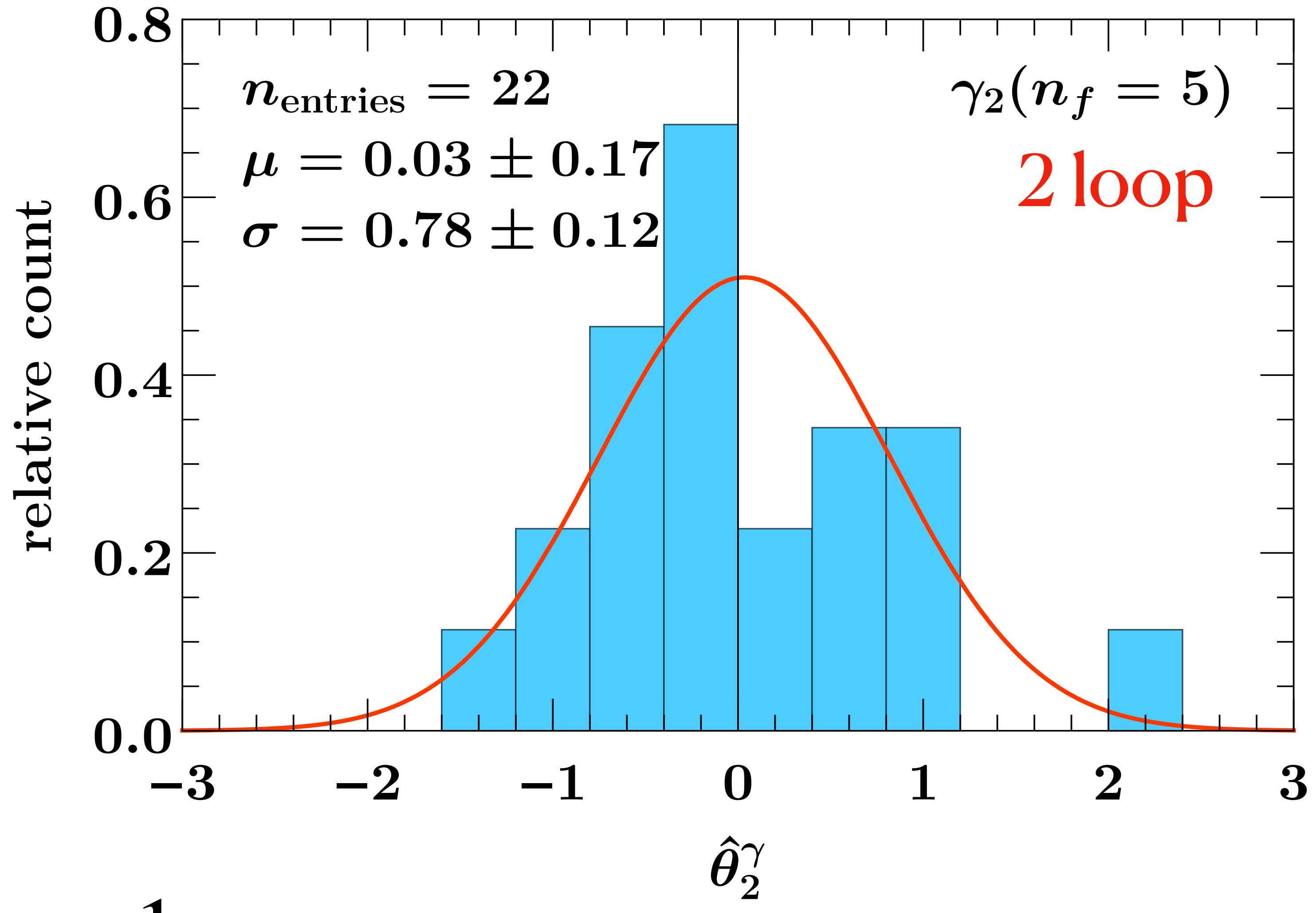




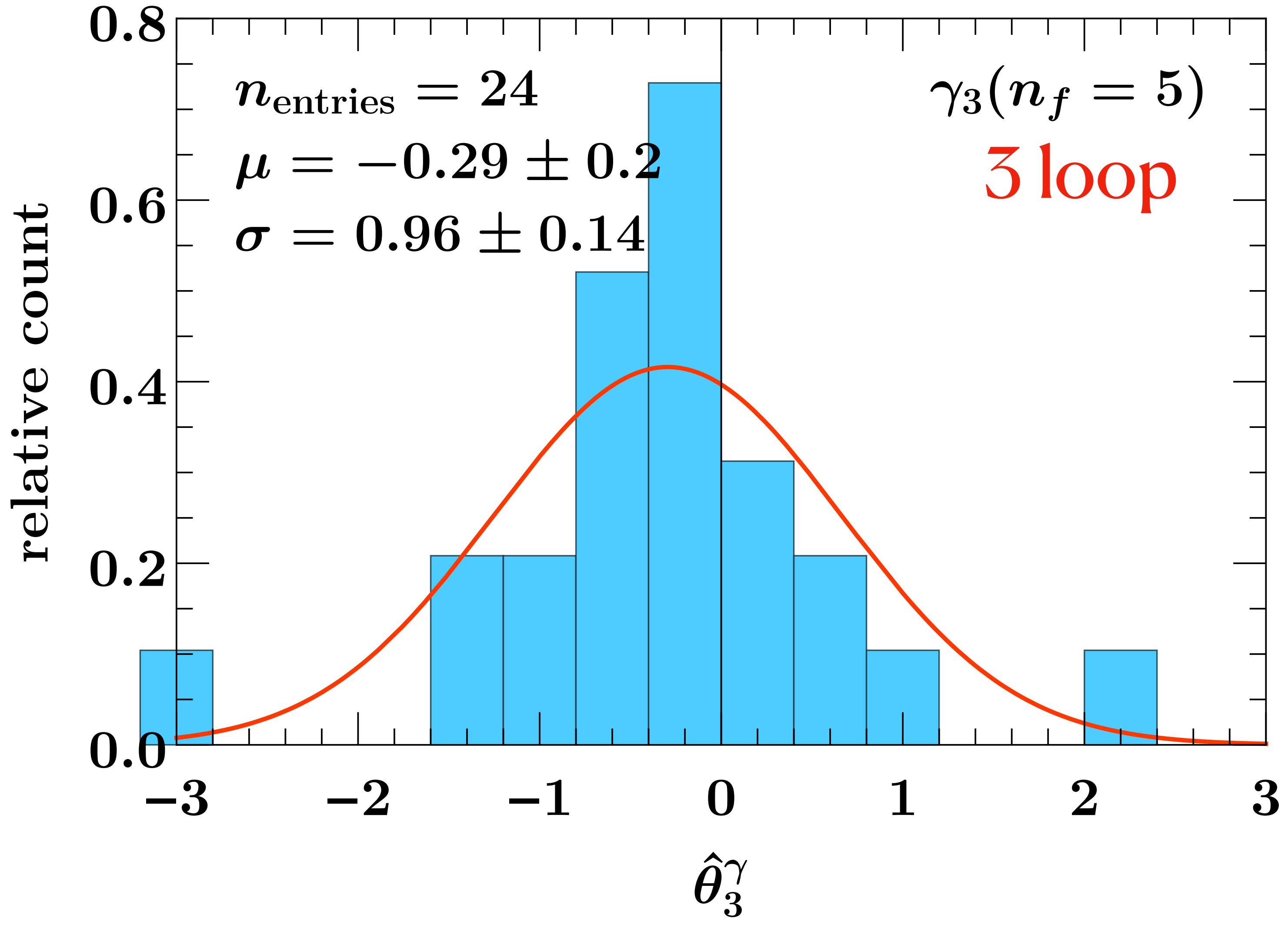
TNPs for Anomalous Dimensions

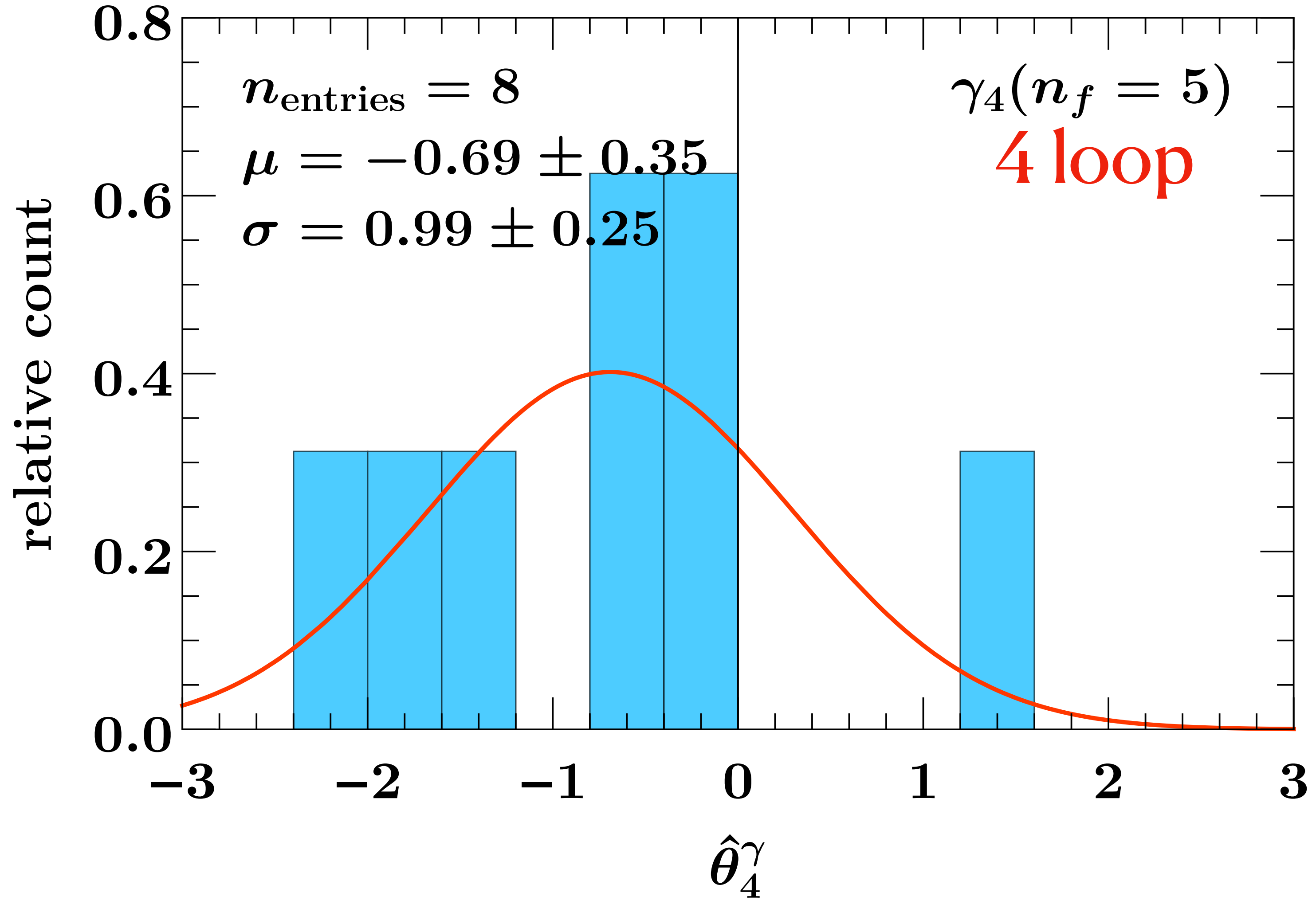
$$\gamma_n(\theta_n) = 4C_r(4C_A)^n\,\theta_n^\gamma$$





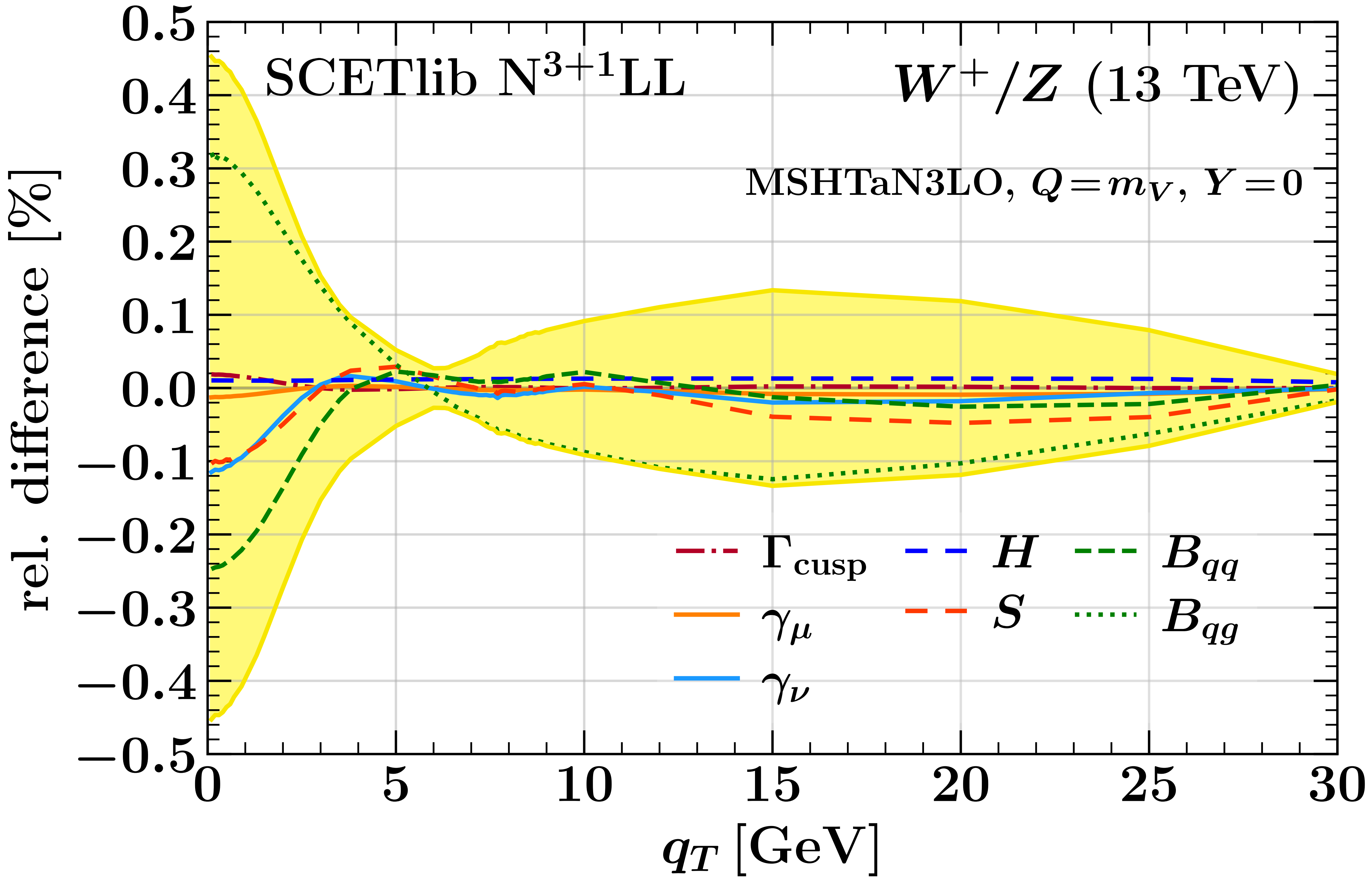
Good fit to a Gaussian with $\theta_n \approx 0$ and $\Delta\theta_n \approx 1$





TNPs correlation for Drell-Yan q_T spectrum

Relative impacts on W/Z*:



» uncertainties very similar for Z and W processes: same TNPs for both

⟶ each TNP impacts are 100% correlated between the processes: nice cancellation in the ratio!

*just for illustration: only leading **massless** contribution

PDF uncertainty: two-step profiling

Very generically:

$$\chi^2_{\text{total}} = \sum_{i,j} (y_i - \lambda_i)^T C^{-1}_{ij} (y_j - \lambda_j) + \sum_i \frac{(\theta_i - 0)^2}{\Delta\theta_i^2} + \sum_k T_k^2 (\theta_k^{\text{PDF}})^2$$

data

theory model

(containing also PDFs!)

To propagate the PDF uncertainty on the PoI:

1 First profile PDFs including T factor to account for existing PDF uncertainties

→ obtain consistently profiled PDFs

2 Perform a fit with $\Delta\chi^2 = 1$ and obtain PDF uncertainty by scanning profiled PDFs from step 1 as a new input set

→ PDF uncertainty on α_s enlarged relative to incorrectly profiled $T = 1$ case, but still reduced substantially relative to the simple scanned case

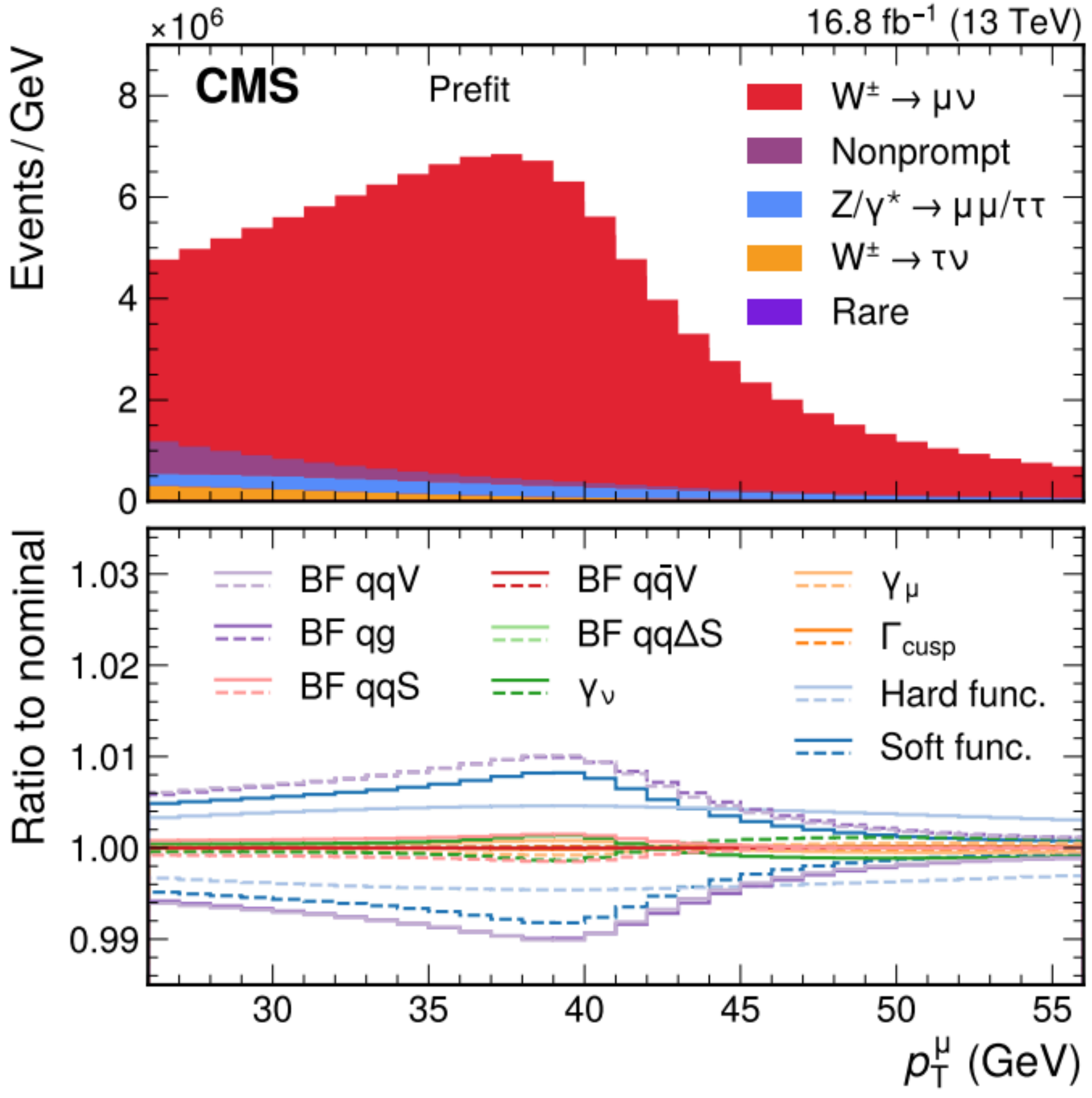
[WIP]

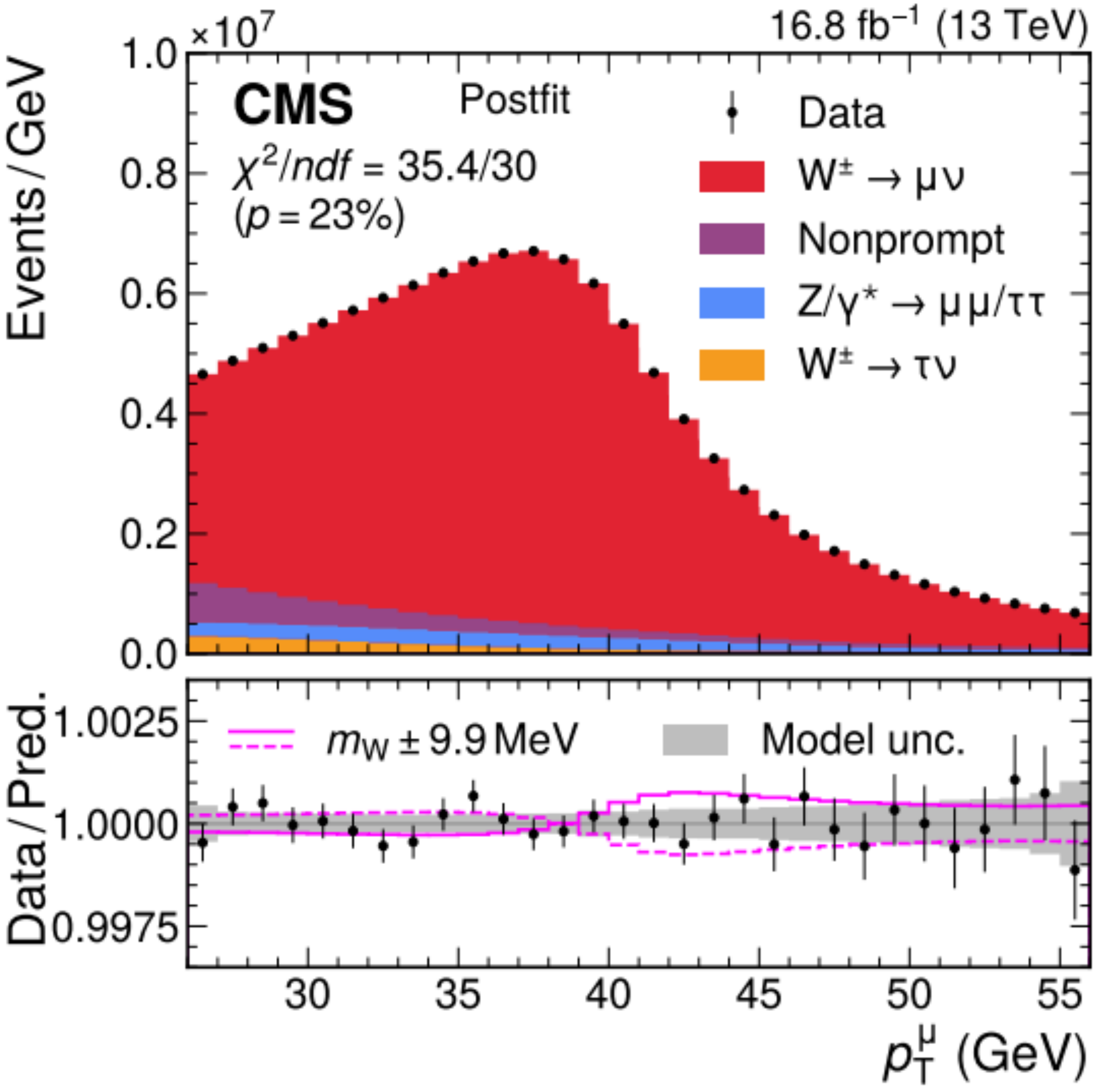
CMS W mass measurement

[CMS-SMP-23-002]

Perturbative uncertainties in the resummed prediction: N^{3+0}LL* SCETlib

contribution of all theoretical and experimental uncert. before and after profiling





* used in the CMS m_W determination

CMS *W* mass measurement

[CMS-SMP-23-002]

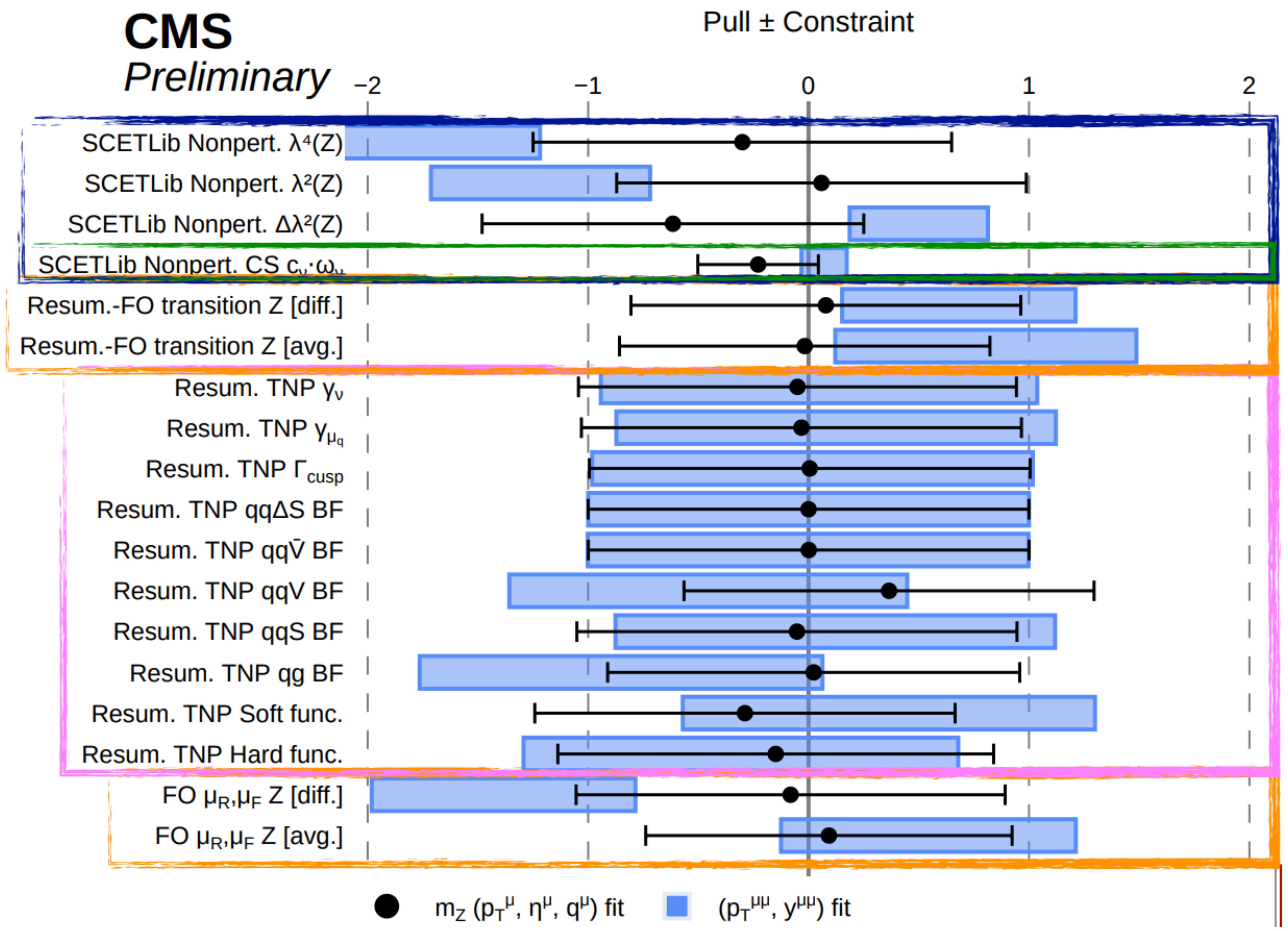
$m_W = 80360.2 \pm 9.9\text{MeV}$

$N^{3+0}LL$ is an approximation of $N^{3+1}LL$:

$$f(\alpha,\theta_4) = \hat{f}_0 + \hat{f}_1\alpha + \hat{f}_2\alpha^2 + [\hat{f}_3 + \alpha_0 f_4(\theta_4)]\alpha^3$$

consider the N^3LL structure but absorb the $N^{3+1}LL$ TNPs uncert. term into the N^3LL structure

» limited effect on the overall size of theory uncert. but correlation approximated by lower order structure



→ if possible prefer the $N^{m+1}LL$ prescription!

Source of uncertainty	Impact (MeV) Nominal	Global
Muon momentum scale	4.8	4.4
Muon reco. efficiency	3.0	2.3
W and Z angular coeffs.	3.3	3.0
Higher-order EW	2.0	1.9
p_T^V modeling	2.0	0.8
PDF	4.4	2.8
Nonprompt background	3.2	1.7
Integrated luminosity	0.1	0.1
MC sample size	1.5	3.8
Data sample size	2.4	6.0
Total uncertainty	9.9	9.9

from Long's talk

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erc

European Research Council

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