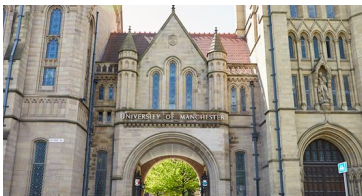


Precision PDFs: Opportunities and Challenges



Manchester Bohr Seminar

Thomas Cridge

6th March 2026

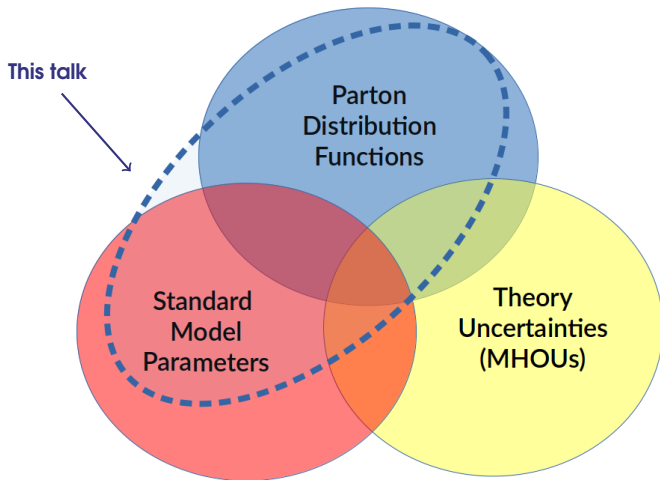


The University of Manchester

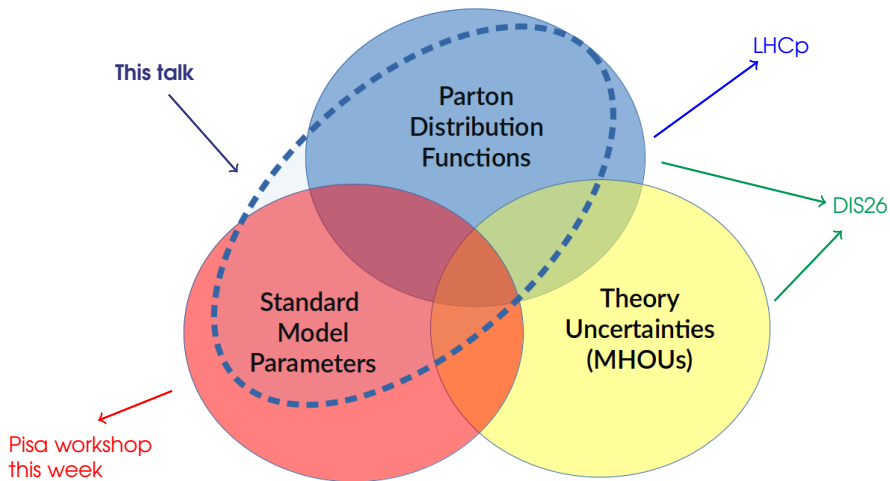


In collaboration with MSHT colleagues - T.C., L.A. Harland-Lang and R.S. Thorne.

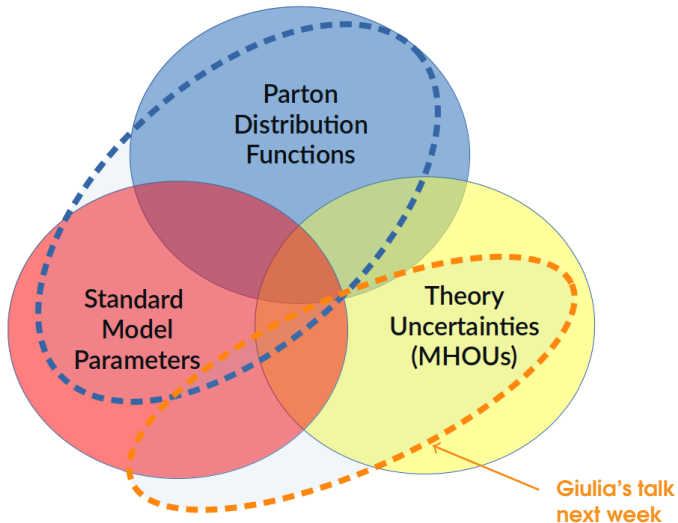
My Research



My Research



My Research



Outline - Precision PDFs

- 1 Overview - Where do we stand?
- 2 Experimental Data Progress
- 3 Methodological Developments
- 4 Theoretical Advances
- 5 Simultaneous Parameter Determination
- 6 Conclusions

More information in: 2012.04684, 2207.04739, 2306.14885, 2309.11269, 2312.07665, 2312.12505, 2404.02964, 2407.07944, 2411.05373, 2508.06603, 2510.03753, 2510.09321, 2512.06092, 2602.06908, etc, by TC, L.A. Harland-Lang and R.S. Thorne (MSHT) and others.

Overview

- Why are PDFs important?
- Where do we stand?

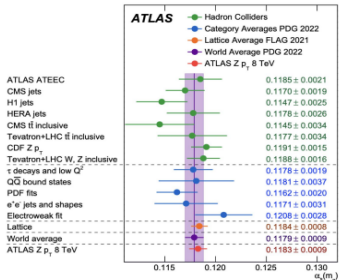
Precision PDFs - Introduction:

- PDFs are a **key input and output** of many calculations/measurements.
- Moreover, they are often a **dominant contribution to the uncertainty**.
- In era of *precision LHC physics*, we need **precise and accurate PDFs**.

1. Precision EW:

ATLAS α_s determination

$$\Delta_{TOT} = 0.0009, \Delta_{PDF} = 0.0005$$



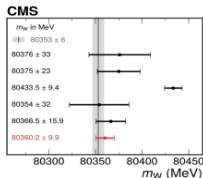
ATLAS collaboration, arXiv: 2309.12986

CMS M_W determination

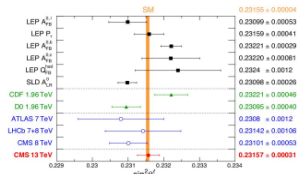
$$\Delta_{TOT} = 9.9 \text{ MeV}$$

$$\Delta_{PDF} = 4.4 \text{ MeV}$$

Electroweak fit
 PRD 110 (2024) 035001
 LEP combination
 Phys. Rep. 532 (2013) 119
 D0
 PRL 108 (2012) 151804
 CDF
 Science 376 (2022) 6589
 LHCB
 JHEP 01 (2022) 036
 ATLAS
 arXiv:2403.15085
 CMS
 This work



CMS collaboration, arXiv: 2412.13872



CMS collaboration, arXiv: 2408.07622

CMS $\sin^2\theta_W$ determination

$$\Delta_{TOT} = 31 \times 10^{-4}$$

$$\Delta_{PDF} = 27 \times 10^{-4}$$

Precision PDFs - Introduction:

- PDFs are a **key input and output** of many calculations/measurements.
- Moreover, they are often a **dominant contribution to the uncertainty**.
- In era of *precision LHC physics*, we need **precise and accurate PDFs**.

2. Higgs:

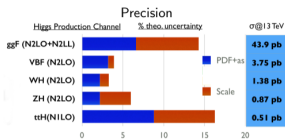
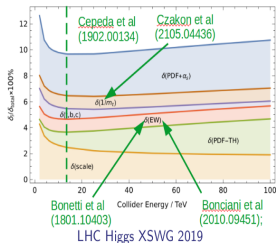
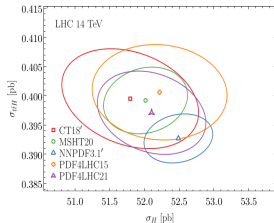


Image: E. Nocera

$$\text{Often } \delta_{\text{PDF}} \sim \frac{1}{2} \delta_{\text{tot}}$$

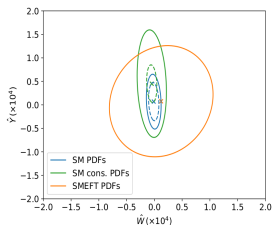
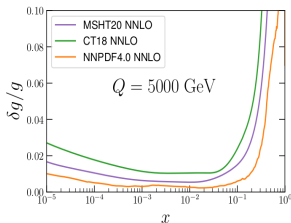
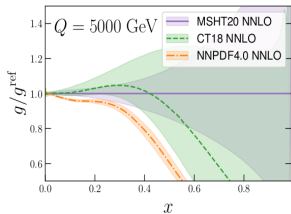


PDF4LHC21: 2203.05506

Precision PDFs - Introduction:

- PDFs are a **key input and output** of many calculations/measurements.
- Moreover, they are often a **dominant contribution to the uncertainty**.
- In era of *precision LHC physics*, we need **precise and accurate PDFs**.

3. BSM:



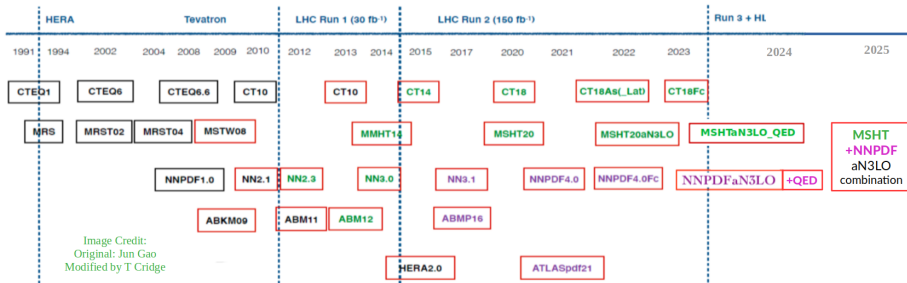
If looking in high energy tails for new BSM resonances, e.g. dijet searches, then gluon PDF uncertainty blows up
 \Rightarrow limiting sensitivity to new physics.

SimuNet: 2104.02723

Or if looking for small deviations from SM, need precise PDFs to separate out new physics.

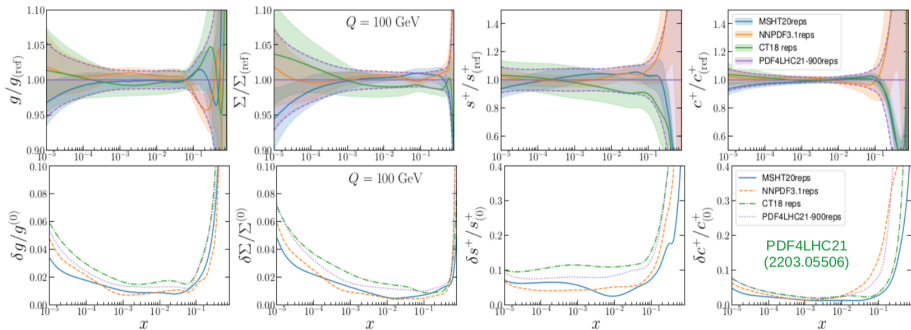
Precision PDFs - Status:

- Several different PDF analysis groups – ABM, ATLASPDF, CJ, **CT**, HERAPDF, JAM, **MSHT**, **NNPDF**, PDF4LHC and others.

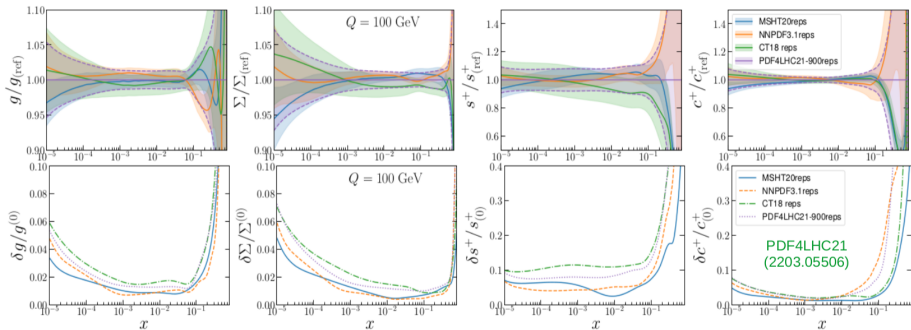


- Continuous Improvements made, e.g.:
 - ▶ Experimentally - More data.
 - ▶ Methodologically - Improved Fit and uncertainties. Closure Testing.
 - ▶ Theoretically - Higher Orders (aN3LO), theory uncertainties, QED.
- Here I will focus on **MSHT**.

PDF Comparison:

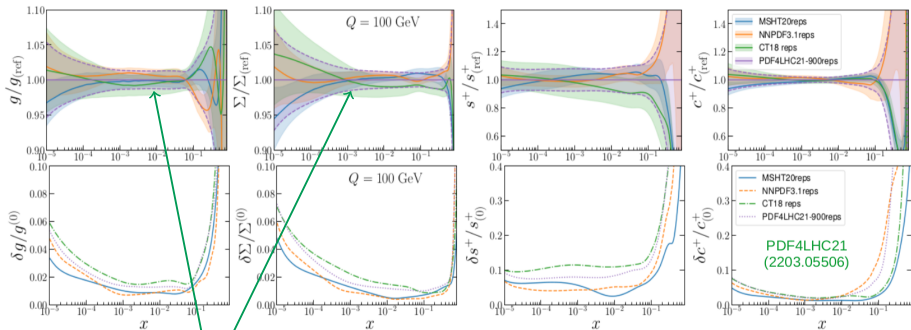


PDF Comparison:



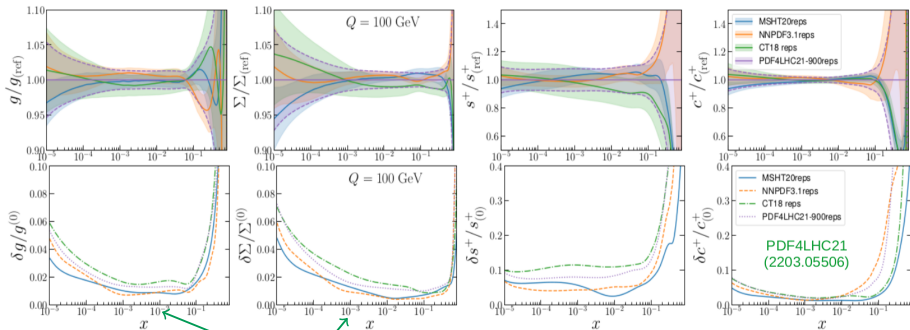
- Good agreement + PDF uncertainties $\mathcal{O}(\%)$ in data region.

PDF Comparison:



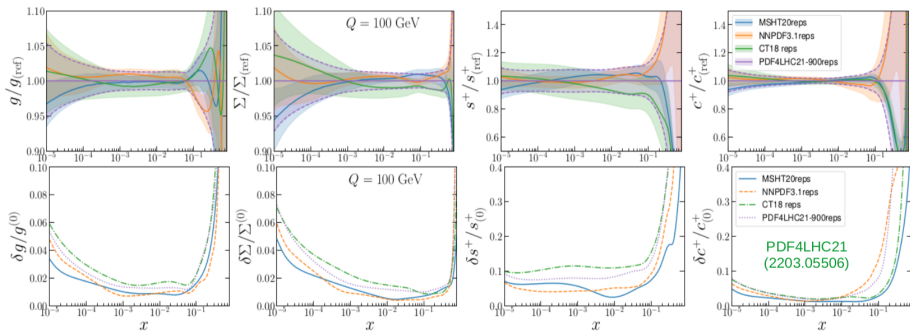
- Good agreement + PDF uncertainties $\mathcal{O}(\%)$ in data region.

PDF Comparison:



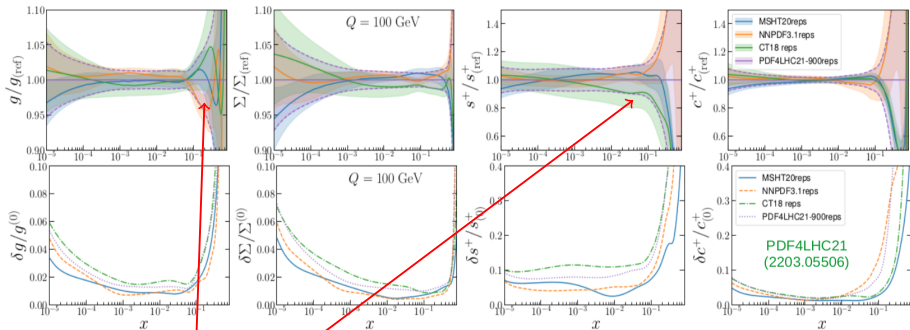
- Good agreement + PDF uncertainties $\mathcal{O}(\%)$ in data region.

PDF Comparison:



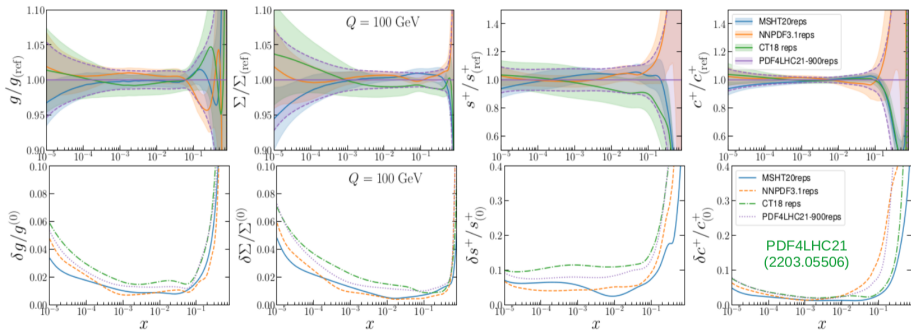
- Good agreement + PDF uncertainties $\mathcal{O}(\%)$ in data region.
- Differences exist elsewhere, e.g. high x gluon, strangeness.

PDF Comparison:



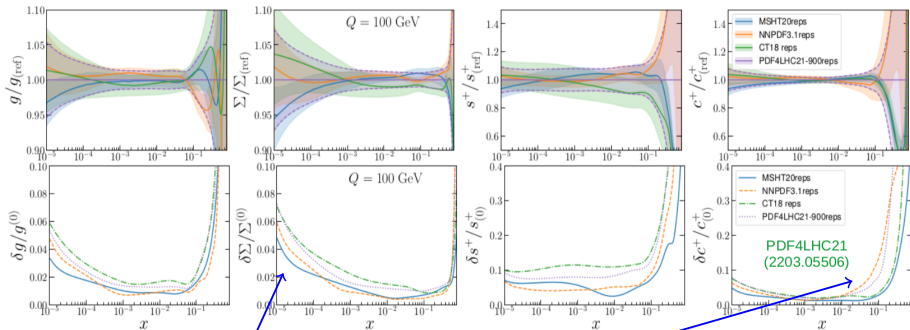
- Good agreement + PDF uncertainties $\mathcal{O}(\%)$ in data region.
- Differences exist elsewhere, e.g. high x gluon, strangeness.

PDF Comparison:



- Good agreement + PDF uncertainties $\mathcal{O}(\%)$ in data region.
- Differences exist elsewhere, e.g. high x gluon, strangeness.
- Uncertainties vary in size, often based on methodology/choices made.

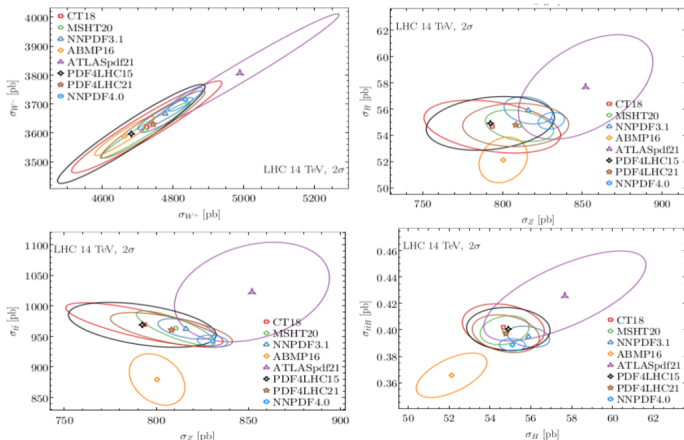
PDF Comparison:



- Good agreement + PDF uncertainties $\mathcal{O}(\%)$ in data region.
- Differences exist elsewhere, e.g. high x gluon, strangeness.
- Uncertainties vary in size, often based on methodology/choices made.

Cross-section Comparison:

- Similar level of agreement holds at level of total cross-sections:
- Particularly amongst global PDF sets:



Snowmass PDF Article: 2203.13923

Experimental Progress

- New LHC Data

- Non LHC

 - (e.g. Fermilab Seaquest - see backup)

- Future Colliders (see backup)

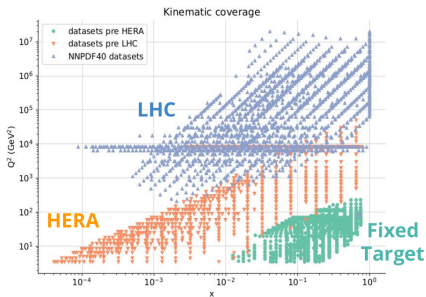
More information in articles: [arXiv:hep-ph/2012.04684](https://arxiv.org/abs/hep-ph/2012.04684), [2312.12505](https://arxiv.org/abs/2312.12505), [2309.11269](https://arxiv.org/abs/2309.11269), and [2512.06092](https://arxiv.org/abs/2512.06092).

Kinematic Coverage of Data:

- Global PDF fits - broad coverage of (x, Q^2) plane from complementary experiments \Rightarrow **robust** (central values and uncertainties!).
- Huge range of LHC Run 1 + 2 data already included.

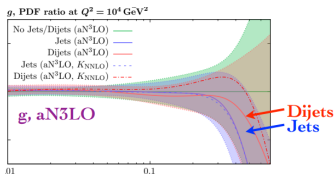
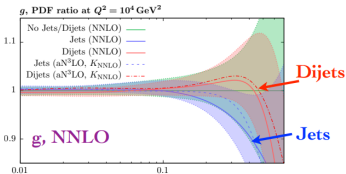
Process	Subprocess	Partons	x range
$\ell^+ n/p \rightarrow \ell^+ + X$	$\gamma^* q \rightarrow q$	q, \bar{q}, g	$x \gtrsim 0.01$
$\ell^+ n/p \rightarrow \ell^+ + X$	$\gamma^* d/u \rightarrow d/u$	d/u	$x \gtrsim 0.01$
$pp \rightarrow \mu^+ \mu^- + X$	$u\bar{u}, d\bar{d} \rightarrow \gamma^*$	\bar{q}	$0.015 \lesssim x \lesssim 0.35$
Fixed Target $pn/pp \rightarrow \mu^+ \mu^- + X$	$(u\bar{u})/(u\bar{u}) \rightarrow \gamma^*$	d/\bar{u}	$0.015 \lesssim x \lesssim 0.35$
$v(\bar{v})N \rightarrow \mu^-(\mu^+) + X$	$W^+q \rightarrow q'$	q, \bar{q}	$0.01 \lesssim x \lesssim 0.5$
$\nu N \rightarrow \mu^+ + X$	$W^+s \rightarrow c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu}N \rightarrow \mu^+ + X$	$W^+ \bar{s} \rightarrow \bar{c}$	\bar{s}	$0.01 \lesssim x \lesssim 0.2$
$e^+ p \rightarrow e^+ + X$	$\gamma^* q \rightarrow q$	g, q, \bar{q}	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \rightarrow \nu + X$	$W^+(d, s) \rightarrow [u, c]$	d, s	$x \gtrsim 0.01$
Collider DIS $e^+ p \rightarrow e^+ c\bar{c} + X$	$\gamma^* c \rightarrow c, \gamma^* g \rightarrow c\bar{c}$	c, g	$10^{-4} \lesssim x \lesssim 0.01$
$e^+ p \rightarrow e^+ b\bar{b} + X$	$\gamma^* b \rightarrow b, \gamma^* g \rightarrow b\bar{b}$	b, g	$10^{-4} \lesssim x \lesssim 0.01$
$e^+ p \rightarrow \text{jet} + X$	$\gamma^* g \rightarrow q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$pp \rightarrow \text{jet} + X$	$gg, q\bar{q}, q\bar{q} \rightarrow 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
Tevatron $pp \rightarrow (W^+ \rightarrow \ell^+ \nu) + X$	$u\bar{d} \rightarrow W^+, d\bar{u} \rightarrow W^-$	u, d, \bar{u}, \bar{d}	$x \gtrsim 0.05$
$pp \rightarrow (Z \rightarrow \ell^+ \ell^-) + X$	$u, d, d\bar{d} \rightarrow Z$	u, d	$x \gtrsim 0.05$
$pp \rightarrow \bar{t} + X$	$q\bar{q} \rightarrow \bar{t}$	q	$x \gtrsim 0.1$
$pp \rightarrow \text{jet} + X$	$gg, q\bar{q}, q\bar{q} \rightarrow 2j$	g, q	$0.001 \lesssim x \lesssim 0.5$
$pp \rightarrow (W^+ \rightarrow \ell^+ \nu) + X$	$u\bar{d} \rightarrow W^+, d\bar{u} \rightarrow W^-$	$u, d, \bar{u}, \bar{d}, g$	$x \gtrsim 10^{-3}$
$pp \rightarrow (Z \rightarrow \ell^+ \ell^-) + X$	$q\bar{q} \rightarrow Z$	q, \bar{q}, g	$x \gtrsim 10^{-3}$
$pp \rightarrow (Z \rightarrow \ell^+ \ell^-) + X, p_{\perp}$	$g\bar{g}(\bar{q}) \rightarrow Zq(\bar{q})$	g, q, \bar{q}	$x \gtrsim 0.01$
LHC $pp \rightarrow (\gamma^* \rightarrow \ell^+ \ell^-) + X, \text{Low mass}$	$q\bar{q} \rightarrow \gamma^*$	q, \bar{q}, g	$x \gtrsim 10^{-4}$
$pp \rightarrow (\gamma^* \rightarrow \ell^+ \ell^-) + X, \text{High mass}$	$q\bar{q} \rightarrow \gamma^*$	\bar{q}	$x \gtrsim 0.1$
$pp \rightarrow W^+ \bar{c}, W^- c$	$sg \rightarrow W^+ c, \bar{g}g \rightarrow W^- \bar{c}$	s, \bar{s}	$x \sim 0.01$
$pp \rightarrow \bar{t} + X$	$g\bar{g} \rightarrow \bar{t}$	g	$x \gtrsim 0.01$
$pp \rightarrow D, B + X$	$g\bar{g} \rightarrow c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-3}, 10^{-5}$
$pp \rightarrow J/\psi, \Upsilon + pp$	$\gamma^*(gg) \rightarrow c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-3}, 10^{-5}$
$pp \rightarrow \gamma + X$	$g\bar{g}(\bar{q}) \rightarrow \gamma q(\bar{q})$	g	$x \gtrsim 0.005$

Typically $N_{\text{dataset}} \gtrsim 50 - 60$ and $N_{\text{pts}} \gtrsim 5000$.



New LHC Data:

- Impact of new data being assessed, both 13TeV and alternative 7-8TeV.
- Jets: Inclusive jet vs dijet production at up to aN3LO (more later).



χ^2/N_{pts}	Total jets	Total dijets
N_{pts}	643	266
NNLO	1.67	1.13
aN3LO	1.63	1.04

MSHT, 2312.12505.

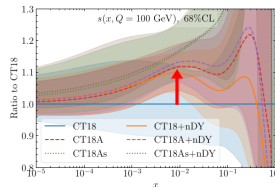
CT, 2307.11153.

- Top: 13TeV data studied, some impact + complementary to jet data.

- Drell-Yan data:

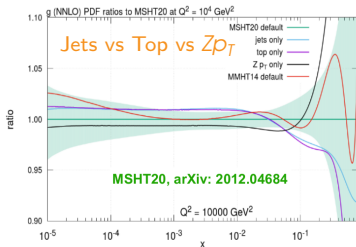
- ▶ All groups observe enhanced strangeness from precision 7-8TeV data.
- ▶ CT observe same from 13TeV data.

CT, 2305.10733.

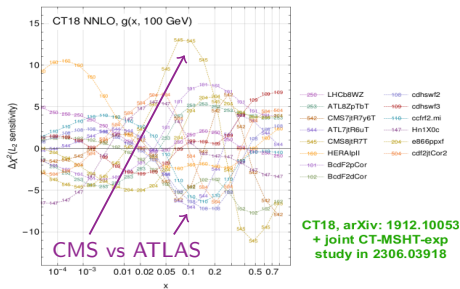


Data Challenges: Data Tensions

- Wide range of data in global PDFs \Rightarrow robust + reduces uncertainties.
- But global fits are **adversarial** - datasets pull in different directions \Rightarrow **data tensions** both between different data types (e.g. **jet vs top vs Zp_T**) and different datasets of same type (e.g. **ATLAS vs CMS**).



MSHT global	LO	NLO	NNLO	aN3LO
χ^2/N_{pts} (4363)	2.57	1.33	1.17	1.14



- **Limits improvements in precision** + causes $\chi^2/N_{pts} > 1$.
- Many reasons, sometimes connected with treatment of systematics.

Methodological Developments

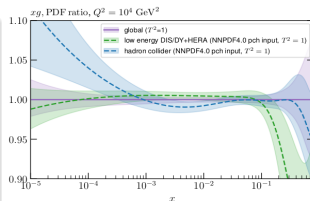
- Uncertainty treatment
 - Errors on errors
 - Closure Testing

More information in articles: [arXiv:hep-ph/2407.07944](https://arxiv.org/abs/hep-ph/2407.07944), [2408.11423](https://arxiv.org/abs/2408.11423), [1909.10541](https://arxiv.org/abs/1909.10541), [2012.04684](https://arxiv.org/abs/2012.04684).

Challenge: Uncertainty Quantification

- How to quantify PDF uncertainties in presence of tensions?
- Size of uncertainties not affected at all by tensions using a “textbook” $\Delta\chi^2 = 1$ (“ $T = 1$ ”).

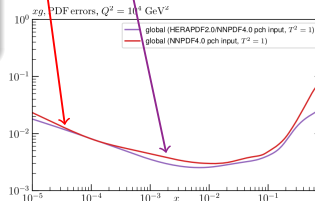
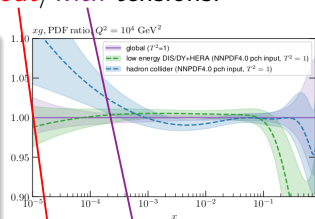
- $\Delta\chi^2 = 1$ would apply if:
 - ▶ Complete Statistical compatibility between all datasets.
 - ▶ Completely faithful evaluation of exp. uncertainties of each dataset.
 - ▶ Theoretical calculations to match these data exactly.



Challenge: Uncertainty Quantification

- How to quantify PDF uncertainties in presence of tensions?
- Size of uncertainties not affected at all by tensions using a “textbook” $\Delta\chi^2 = 1$ (“ $T = 1$ ”). E.g. generate data **without**/**with** tensions:

- $\Delta\chi^2 = 1$ would apply if:
 - ▶ Complete Statistical compatibility between all datasets.
 - ▶ Completely faithful evaluation of exp. uncertainties of each dataset.
 - ▶ Theoretical calculations to match these data exactly.



Harland-Lang, TC, R.S. Thorne 2407.07944

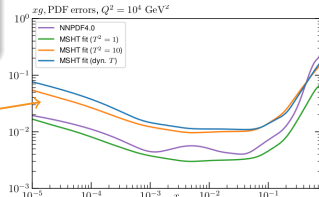
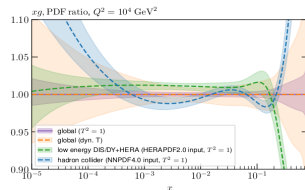
Challenge: Uncertainty Quantification

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- $\Delta\chi^2 = 1$ would apply if:
 - ▶ Complete Statistical compatibility between all datasets.
 - ▶ Completely faithful evaluation of exp. uncertainties of each dataset.
 - ▶ Theoretical calculations to match these data exactly.

- MSHT (and CT) use a **tolerance to account for dataset tensions + other effects** (similar idea to PDG rescaling):

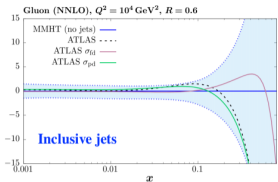
$$\Delta\chi^2 = T^2 \quad (T > 1).$$



Harland-Lang, TC, R.S. Thorne 2407.07944

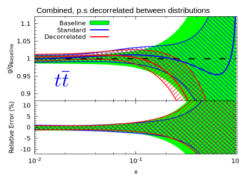
Treatment of Experimental Systematics:

- As LHC datasets become increasingly **systematics dominated**, we become sensitive to precise **treatment of systematics + their correlations**.
- “Two point systematics” where two MCs/models used and difference taken as fully correlated are particularly problematic.



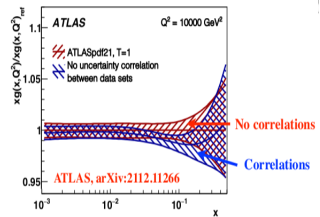
S. Bailey et al., arXiv:2012.04684

n_{dat}	default	part. decorr.	full decorr.
140	χ^2/N_{pts}	1.89	0.83



S. Bailey and LHL, arXiv:1909.10541

n_{dat}	default	stat. uncorr.	p.s. uncorr.
25	χ^2/N_{pts}	7.00	1.80



- Currently investigating better correlation models...
- But how precisely do we know these systematic errors anyway?
 \Rightarrow “Errors on errors”.

PDF Uncertainties - Errors on errors:

- “Error on error” framework - how uncertain are estimates of error.
- Model variances of uncertainty as **Gamma-distributed**:

$$L(\mu, \theta) = P(y|\mu, \theta) \prod_{i=1}^N \frac{1}{\sqrt{2\pi}\sigma_{u,i}} \exp(-(u_i - \theta_i)^2 / 2\sigma_{u,i}^2) \frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)} \nu_i^{\alpha_i - 1} \exp(-\beta_i \nu_i)$$

where $\alpha_i = 1/(4r_i^2)$, $\beta_i = 1/(4r_i^2\sigma_{u,i}^2)$,

r_i is relative uncertainty on systematic error \Rightarrow “error on error”.

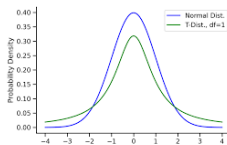
- Model equivalent to **Student's t-distribution**:

$$L(\mu, \theta) = P(y|\mu, \theta) \prod_{i=1}^N \frac{\Gamma((\nu_i + 1)/2)}{\sqrt{\nu_i\pi}\Gamma(\nu_i/2)} \left(1 + \frac{t_i^2}{\nu_i}\right)^{-(\nu_i+1)/2}$$

where $t_i = (u_i - \theta_i)/\sqrt{\nu_i}$, $\nu_i = 1/(2r_i^2)$,

- So we can treat the **nuisance parameters for correlated systematics as t-distributed!**

“Errors on errors” - G. Cowan 1809.05778.

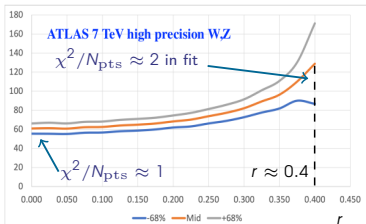
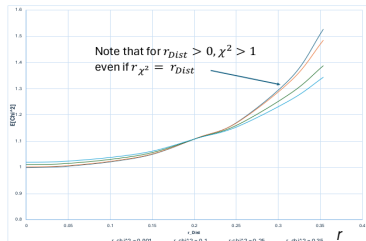


See M. Reader DIS24 for more details.

Errors on errors:

Preliminary!

- Taking Student's t-distribution compute expectation value of χ^2/N_{pts} as r ("error on error") increases.
- $E[\chi^2/N_{\text{pts}}]$ increases to > 1 as r increases, particularly if assuming $r = 0$ in fit, but even if taking "correct" $r \neq 0$.
- Would explain why (at least partly) PDF fits observe $\chi^2/N_{\text{pts}} > 1$.
- Perhaps $T^2 \sim 2 - 5$ could result from such uncertainties?
- How does this impact χ^2/N ? - Try on precision ATLAS 7 TeV W, Z data:
- Observed $\chi^2/N_{\text{pts}} \approx 2$, requires $r \approx 0.4$.

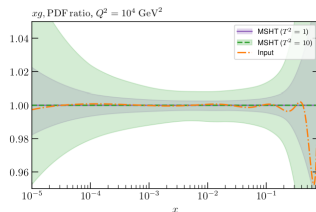
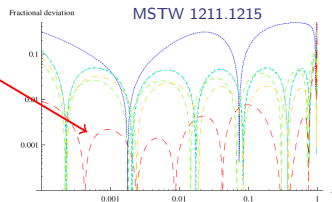


M. Reader 2408.12922 and MSHT, Reader et al, in preparation.

Closure Testing:

- Other methodological PDF uncertainties \Rightarrow test by “closure testing”.
- Idea: Generate pseudodata (fluctuated/unfluctuated) + attempt to fit it.
- MSHT parameterisation in Chebyshevs** chosen to give few per mille precision.
- Test 1: Analyse MSHT parameterisation uncertainty \rightarrow generate pseudodata with NNPDF4.0 and fit.
- Fit quality very close to 0, in fact better than NNPDF achieve:

	χ^2	χ^2/N_{pts}
Fit quality:	2.4	0.0005
- Excellent agreement, particularly in data regions \Rightarrow **parameterisation uncertainty very small** for MSHT.



Harland-Lang, TC, R.S. Thorne 2407.07944

Closure Testing:

- What if we used fewer parameters?
- Fails closure \Rightarrow **care required fitting with fewer parameters: precision vs accuracy.**
- Test 2: Fit MSHT parameterisation using NNPDF data/theory (using public code).
- **Fit quality with MSHT parameterisation slightly improved** cf NNPDF central replica.

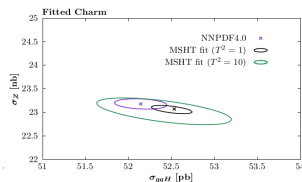
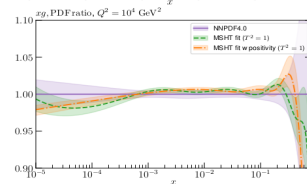
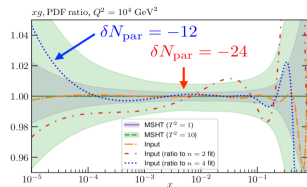
	NNPDF4.0	MSHT fit	MSHT fit (w positivity)
χ_{t0}^2	5692.1 (1.233)	5645.2 (1.222)	5651.0 (1.224)

$$\Delta\chi_{t0}^2: \quad \underline{-46.9(0.011)} \quad \underline{-41.1(0.009)}$$

- PDFs in some cases similar, but not always consistent within a $T^2 = 1$ uncertainty.

★ Quark flavour decomposition: $\sigma(\text{NNPDF}) \sim \sigma(\text{MSHT}, T^2 = 1)$

★ Gluon (singlet at intermediate x): $\sigma(\text{MSHT}, T^2 = 1) \lesssim \sigma(\text{NNPDF}) \lesssim \sigma(\text{MSHT}, T^2 = 10)$



Harland-Lang, TC, R.S. Thorne 2407.07944

Theoretical Advances

- Inclusion of theory uncertainties
 - Approximate N3LO
 - QED effects

More information in articles: [arXiv:hep-ph/2207.04739](https://arxiv.org/abs/hep-ph/2207.04739), [2312.07665](https://arxiv.org/abs/2312.07665), [2406.16188](https://arxiv.org/abs/2406.16188), [2408.10008](https://arxiv.org/abs/2408.10008), [2411.05373](https://arxiv.org/abs/2411.05373), [2508.06603](https://arxiv.org/abs/2508.06603), [2510.03753](https://arxiv.org/abs/2510.03753), [2510.09321](https://arxiv.org/abs/2510.09321), [2602.06908](https://arxiv.org/abs/2602.06908).

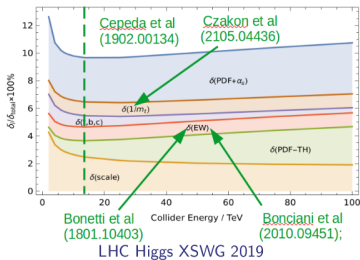
Motivation

- Experiments more precise \Rightarrow need more precise (and accurate!) theory.

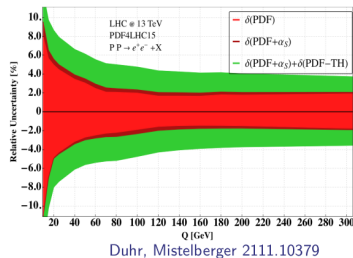
Motivation

- Experiments more precise \Rightarrow need **more precise (and accurate!) theory.**
- Progress for N3LO cross-sections: Higgs (ggF, VBF, VH), DY(NC, CC).

Higgs - ggF:



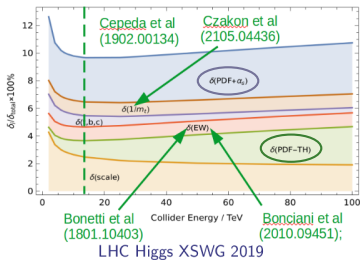
Drell-Yan - NC:



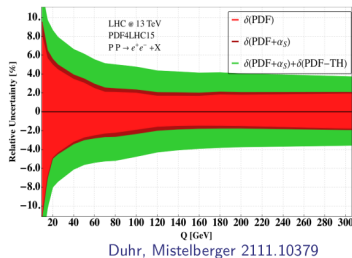
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Higgs - ggF:



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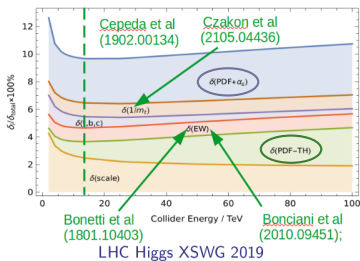


- PDFs at N3LO with theory uncertainties were becoming a bottleneck.

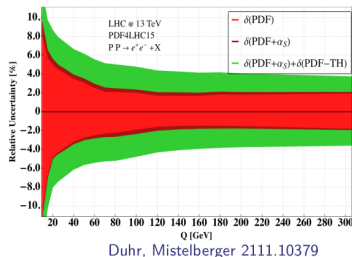
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Higgs - ggF:



Drell-Yan - NC:

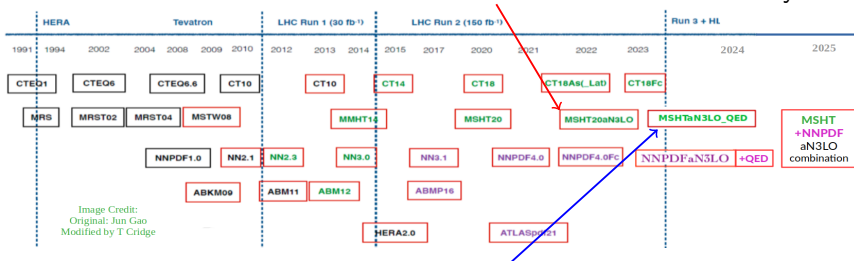


- PDFs at N3LO with theory uncertainties were becoming a bottleneck.
- Two steps required for *more accurate and precise PDFs*:

- Higher order PDFs \Rightarrow aN3LO (“approximate N3LO”).
- Theoretical uncertainties from missing higher orders \Rightarrow MHO.

Approximate N3LO PDFs Available

- World first aN3LO PDF set \Rightarrow MSHT20aN3LO available for ~ 3 years.



- Also available for ~ 1.5 years: MSHT20an3lo+QED (more later).
- First time at approximate N3LO (“aN3LO”) and first inclusion of theory uncertainties in PDFs at highest order.

Basic Idea:

- ▶ Include all known N3LO terms into the PDFs (a lot!)
 - ▶ Include theoretical uncertainty for the missing ingredients
- \Rightarrow aN3LO PDFs + theory uncertainty.

What do we need to know for N3LO PDFs?

- Need to know:

- ▶ **Splitting functions** - at 4-loop to evolve PDFs in (x, Q^2) :

$$P(x, \alpha_s) = \alpha_s P^{(0)}(x) + \alpha_s^2 P^{(1)}(x) + \alpha_s^3 P^{(2)}(x) + \alpha_s^4 P^{(3)}(x) + \dots$$

- ▶ **Transition Matrix Elements (TMEs)** - at 3-loop to change number of PDF flavours at heavy quark mass (m_h) thresholds.

$$f_\alpha^{n_f+1}(x, Q^2) = [A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2)](x)$$

- ▶ **Coefficient Functions for DIS** - at 3-loop to determine structure functions.

$$F_2(x, Q^2) = \sum_{\alpha \in H, q, g; \beta \in q, H} (C_{\beta, \alpha}^{VF, n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2))$$

- ▶ **Hadronic cross-section k-factors** - at N3LO.

$$\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \dots \equiv \sigma_{N3LO} + \dots$$

- Much already known, **only a few remaining missing pieces.**

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Mellin moments, small x ,
high x limits

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Light flavour known, heavy
flavour high Q^2 known,
approx for low Q^2 .

- ▶ **Hadronic cross-section k-factors** - at N3LO.

$$\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \dots \equiv \sigma_{N3LO} + \dots$$

Very little known, PDFs
need differential with cuts.

- Much already known, **only a few remaining missing pieces.**

Approach in MSHT20aN3LO:

- Include known info (already a lot, even more now) for each ingredient.
- Use Theoretical Nuisance Parameters (TNPs) to include effect of unknown pieces, added to fit a la experimental nuisance parameters.
- Consider PDF fit probability - add N3LO theory and theory uncertainty:

$$P(T|D) \propto \exp\left(-\frac{1}{2} \sum_{k=1}^{N_{pt}} \frac{1}{s_k^2} (D_k - T_k - \sum_{\alpha=1}^{N_{corr}} \beta_{k,\alpha} \lambda_\alpha)^2 + \sum_{\alpha=1}^{N_{corr}} \lambda_\alpha^2\right)$$

Experimental Nuisance parameters

Approach in MSHT20aN3LO:

- Include known info (already a lot, even more now) for each ingredient.
- Use Theoretical Nuisance Parameters (TNPs) to include effect of unknown pieces, added to fit a la experimental nuisance parameters.
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$$\begin{aligned}
 P(T|D) &\propto \exp\left(-\frac{1}{2} \sum_{k=1}^{N_{pt}} \frac{1}{s_k^2} (D_k - T_k - \sum_{\alpha=1}^{N_{corr}} \beta_{k,\alpha} \lambda_\alpha)^2 + \sum_{\alpha=1}^{N_{corr}} \lambda_\alpha^2\right) \\
 &\propto \exp\left(-\frac{1}{2} \sum_{k=1}^{N_{pt}} \frac{1}{s_k^2} (D'_k - \tilde{T}_k - \sum_{t=1}^{N_{TNPs}} u_{k,t} \theta'_t)^2 + \sum_{\alpha=1}^{N_{corr}} \lambda_\alpha^2 + \sum_{t=1}^{N_{TNPs}} \theta'_t{}^2\right)
 \end{aligned}$$

Experimental Nuisance parameters
Theory Nuisance Parameters

- Upgrade theory, $T \rightarrow \tilde{T}$ to now contain known N3LO info (aN3LO) and allow to vary by theory nuisance parameters, TNPs - θ' .
- **Probes precisely the missing higher order terms.**

(Applications more widely - e.g. theory uncertainty for $Z p_T$ spectrum and $\alpha_S - T.C.$, G. Marinelli, F. Tackmann 2506.13874.)

Splitting Functions:

- Splitting functions now almost completely known at N3LO (for pheno).
- MSHT20aN3LO: 4 mellin moments for $P_{qq, qg, gq, gg}^{PS}$, 8 for P_{qq}^{NS} .
- Now 10 moments for all singlet P_{ij} .
- Consider MSHT:

- ▶ Add varying parameter for missing piece of each N3LO ingredient.

E.g. for P_{qg}^3 :

$$P_{ab}^{(3)}(x) = \sum_{i=1}^k A_i f_i(x) + f_e(x, \rho_{ab})$$

Construct from known Mellin moments.

Variational parameter as unknown coefficient.

Contains exact small (and high) x info e.g. from resummation.

Known structure

where

$$f_e(x, \rho_{qg}) = \frac{C_A^3}{3\pi^4} \left(\frac{82}{81} + 2\zeta_3 \right) \frac{1}{2} \frac{\ln^2(1/x)}{x} + \rho_{qg} \frac{\ln 1/x}{x}$$

- ▶ Uncertainty on aN3LO comes through **varying functional basis** $f_i(x)$ and **varying unknown coefficient** (“theory nuisance parameter” - TNP).

⇒ aN3LO PDF + theory uncertainty.

Theory Uncertainty via TNP

Advantages of TNP method:

- Probes precisely the missing higher order terms.
- Allows inclusion of known N3LO information (a lot) without needing to wait for remaining few pieces.
- Can be included in PDF fit in same way experimental data are.
- No requirement for scale variations - can underestimate MHOU, issue of correlation between PDF fit and use [11].
- Exactly same data can be included at all orders - no need to raise Q^2 cut on data to enable downwards scale variations.
- Output eigenvectors include theory uncertainty from missing higher orders out-of-the-box \Rightarrow using MSHT20aN3LO PDF set exactly as previous sets includes theory uncertainty for no extra user effort.

- Applications also more widely - e.g. theory uncertainty for $Z p_T$ spectrum and α_S . F. Tackmann SCET 2019; and T.C., G. Marinelli, F. Tackmann 2506.13874.

4.1 - Impact of aN3LO on PDFs

More information in articles: [arXiv:hep-ph/2207.04739](https://arxiv.org/abs/hep-ph/2207.04739), [2312.07665](https://arxiv.org/abs/2312.07665), [2406.16188](https://arxiv.org/abs/2406.16188), [2408.10008](https://arxiv.org/abs/2408.10008), [2411.05373](https://arxiv.org/abs/2411.05373), [2510.09321](https://arxiv.org/abs/2510.09321).

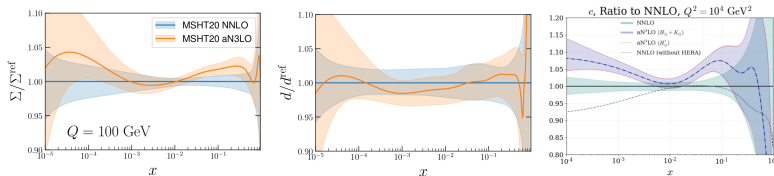
Impacts of aN3LO on MSHT PDFs:

- MSHT (and NNPDF) - **Improvement order by order of fit quality:**

MSHT χ^2/N_{pts} (4363)	LO	NLO	NNLO	aN3LO
		2.57	1.33	1.17

$\Delta\chi^2$ improves by
 ~ -150 in original aN3LO,
 $(\sim -85$ after new updates
 - more later).

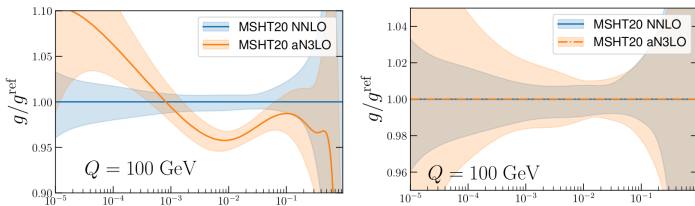
- Mild effects on light quark PDFs:



- Generally **percent level impacts of aN3LO** in data region, only mild effects on quarks - same seen in NNPDF.
- Inclusion for first time of **theoretical uncertainty from MHOUs enlarges PDF uncertainty at small x**.
- Heavy quarks raised** at low and high x , similarly singlet raised at high x .

Impacts of aN3LO on MSHT PDFs:

- Larger effect on gluon PDF:



- Observed **impact on gluon of aN3LO is larger** than for quarks, with $\sim 5\%$ dip around 10^{-2} . Cf NNPDF also sees a dip here but of $\sim 2\%$.
- **Inclusion of theoretical uncertainty enlarges gluon uncertainty**, including in data (Higgs) region.
- Whilst **PDF uncertainty is larger**, it's **more complete** (inclusion of theoretical error), **accurate and reliable**.
- May expect larger effects on gluon, particularly at small x due to NNLO \rightarrow N3LO at small x : $P_{gg}^{(2)} \sim \log x/x$, cf $P_{gg}^{(3)} \sim \log^3 x/x$.

What do we need to know for N3LO PDFs?

- Need to know:

- **Splitting functions** - at 4-loop to evolve PDFs in (x, Q^2) :

$$P(x, \alpha_s) = \alpha_s P^{(0)}(x) + \alpha_s^2 P^{(1)}(x) + \alpha_s^3 P^{(2)}(x) + \alpha_s^4 P^{(3)}(x) + \dots$$

Mellin moments, small x ,
high x limits

- **Transition Matrix Elements (TMEs)** - at 3-loop to change number of PDF flavours at heavy quark mass (m_h) thresholds.

$$f_\alpha^{n_f+1}(x, Q^2) = [A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2)](x)$$

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- **Coefficient Functions for DIS** - at 3-loop to determine structure functions.

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Light flavour known, heavy
flavour high Q^2 known,
approx for low Q^2 .

- **Hadronic cross-section k-factors** - at N3LO.

$$\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \dots \equiv \sigma_{N3LO} + \dots$$

Very little known, PDFs
need differential with cuts.

- Much already known, **only a few remaining missing pieces.**

What do we need to know for N3LO PDFs?

- Need to know:

- **Splitting functions** - at 4-loop to evolve PDFs in (x, Q^2) : Mellin moments, small x ,
high x limits

New moments for
 $P_{qg, gq, gg}, P_{qq}^{PS}$
(FHRMUVV)

$$\leftarrow P(x, \alpha_s) = \alpha_s P^{(0)}(x) + \alpha_s^2 P^{(1)}(x) + \alpha_s^3 P^{(2)}(x) + \alpha_s^4 P^{(3)}(x) + \dots$$

- **Transition Matrix Elements (TMEs)** - at 3-loop to change number of PDF flavours at heavy quark mass (m_h) thresholds. Mellin moments, small x ,
high x limits

$A_{Hq}^{(3)}, A_{qg, H}, A_{gg, H}$
since determined.
(Blümlein et al)

$$\leftarrow f_\alpha^{n_f+1}(x, Q^2) = [A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2)](x)$$

- **Coefficient Functions for DIS** - at 3-loop to determine structure functions. Light flavour known, heavy
flavour high Q^2 known,
approx for low Q^2 .

No further info.

$$\leftarrow F_2(x, Q^2) = \sum_{\alpha \in H, q, g; \beta \in q, H} (C_{\beta, \alpha}^{VF, n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2))$$

- **Hadronic cross-section k-factors** - at N3LO. Very little known, PDFs
need differential with cuts.

N3LO DY γ distribution,
not yet usable in PDFs
(Chen et al)

$$\leftarrow \sigma = \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \dots \equiv \sigma_{N3LO} + \dots$$

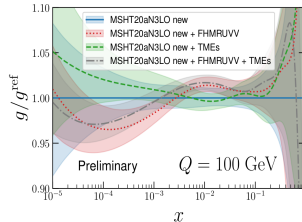
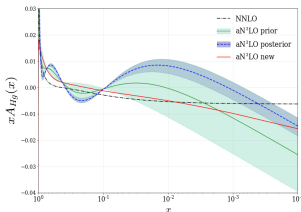
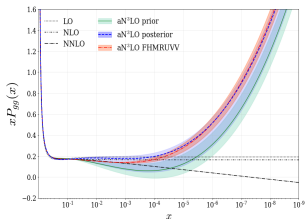
References in backup

- Much already known, **only a few remaining missing pieces.**

Impact of Updated $P_{ij} + A_{ij}$ on aN3LO PDFs:

- How do updated FHMURVV splitting functions and new results for TMEs $A_{Hg}^{(3)}$, $A_{qq,H}^3$, $A_{gg,H}^3$ since MSHT20aN3LO affect the aN3LO PDFs?:

N.B. Preliminary as no uncertainty included for MHOV in $P_{ij}^{(3)}$ here or $A_{ij}^{(4)}$.



- P_{ij} and TMEs largely consistent within uncertainties with MSHT approximations. Thanks to G. Magni for providing TME grids. FHMURVV + TME references in backup.
- Impact on PDFs limited**, in MSHT see rise of $\sim 1.5\%$ at $x \approx 10^{-2}$ in gluon, (cf NNPDF sees reduction of $\sim 1\%$ at $x \approx 10^{-3}$.)
- aN3LO/NNLO ratio moves closer between MSHT and NNPDF.

TC, Harland-Lang, Thorne 2510.09321.

4.2 - QED Effects in PDF fits

More information in articles: [arXiv:hep-ph/2207.04739](https://arxiv.org/abs/hep-ph/2207.04739), [2312.07665](https://arxiv.org/abs/2312.07665), [2411.05373](https://arxiv.org/abs/2411.05373), [2508.06603](https://arxiv.org/abs/2508.06603), [2602.06908](https://arxiv.org/abs/2602.06908).

What about QED effects?:

- Naively $\alpha_{\text{QED}}(M_Z) \sim \alpha_S^2(M_Z)$, therefore need to include QED evolution \Rightarrow a **photon PDF**. Also photon-initiated and EW corrections.
- Global PDFs do this via **LUXQED**:

$$x\gamma(x, Q_0^2) = \frac{1}{2\pi\alpha(Q_0^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{Q_0^2} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[\left(zP_{\gamma,q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) - z^2 F_L(x/z, Q^2) \right] - \alpha^2(Q_0^2) \left(z^2 + \ln(1-z) z P_{\gamma,q}(z) - \frac{2x^2 m_p^2 z}{Q_0^2} \right) F_2(x/z, Q_0^2) \right\},$$

Manohar et al 1607.04266

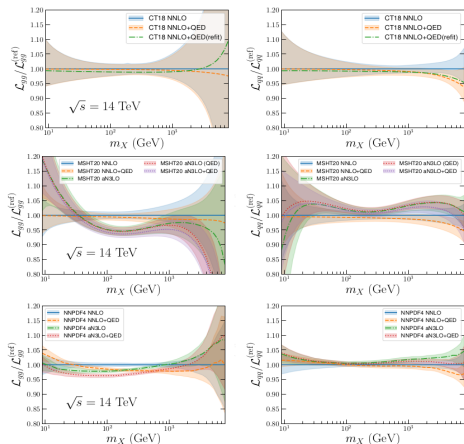
- Adding a γ PDF requires momentum:

$$\int_0^1 dx (\Sigma(x, Q^2) + g(x, Q^2)) = 1 \Rightarrow \int_0^1 dx (\Sigma(x, Q^2) + g(x, Q^2) + \gamma(x, Q^2))$$

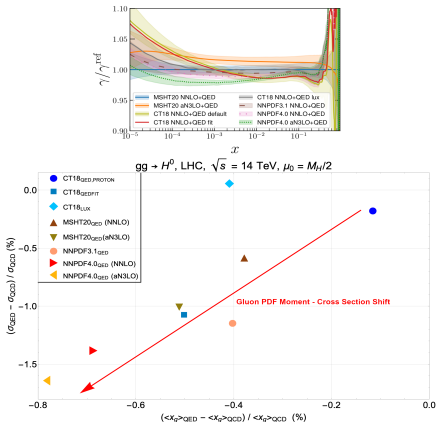
- Various subtleties in application:
 - ▶ Scale at which LUXQED is applied.
 - ▶ EW corrections included.
 - ▶ How momentum is obtained from the other partons.
 - ▶ Baseline QCD PDFs.

PDF Impacts of QED corrections:

- However some differences in where the momentum comes from:



TC et al 2508.06603, 2602.06908



- All PDFs depleted at high x in quarks (m_X in L_{qq}) due to $q \rightarrow q\gamma$.
- **Gluon reduced by photon momentum**, reduces gluon-initiated xsecs.

aN3LO QCD + QED:

- All groups now provide NNLO + QED PDF sets. MSHT produced the first aN3LO + QED PDFs - highest possible theoretical precision.
- Need to combine aN3LO QCD evolution and $\mathcal{O}(\alpha, \alpha\alpha_S, \alpha^2)$:

$$\begin{aligned}
 \text{QED} \quad P_{ij} &= \frac{\alpha}{2\pi} P_{ij}^{(0,1)} + \frac{\alpha\alpha_S}{(2\pi)^2} P_{ij}^{(1,1)} + \left(\frac{\alpha}{2\pi}\right)^2 P_{ij}^{(0,2)} \\
 \text{NNLO QCD} \quad &+ \frac{\alpha_S}{2\pi} P_{ij}^{(1,0)} + \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ij}^{(2,0)} + \left(\frac{\alpha_S}{2\pi}\right)^3 P_{ij}^{(3,0)} \\
 \text{aN3LO QCD} \quad &+ \left(\frac{\alpha_S}{2\pi}\right)^4 P_{ij}^{(4,0)}.
 \end{aligned}$$

- Impact on fit at NNLO and aN3LO, **substantial fit quality improvement remains true after adding QED**:

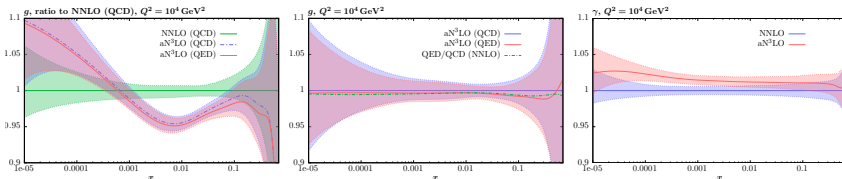
	χ^2/N_{pt} aN ³ LO (QED)	$\Delta\chi^2_{\text{aN}^3\text{LO}}$ QED-QCD	$\Delta\chi^2_{\text{NNLO}}$ QED-QCD	$\Delta\chi^2_{\text{QCD,QED}}$ aN ³ LO-NNLO
Total	5323.6/4534	(+3.6)	(+17.3)	(-209.3, -223.1)

T.C., L.A. Harland Lang, R.S. Thorne 2312.07665
Now also NNPDF 2406.01779.

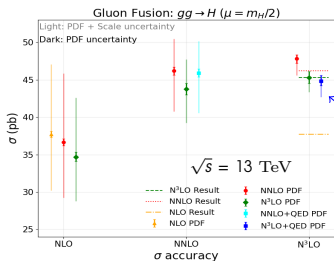
aN3LO QCD + QED:

T.C., L.A. Harland Lang, R.S. Thorne 2312.07665.

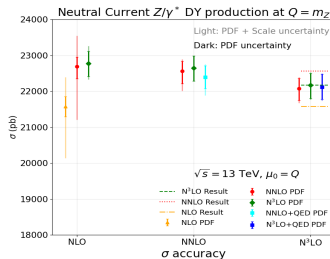
- Impact small relative to aN3LO QCD corrections in most regions.
- Effect of adding QED similar when applied to NNLO and aN3LO.



- Knock-on impact on cross-sections, ggF Higgs (left), Z (right):

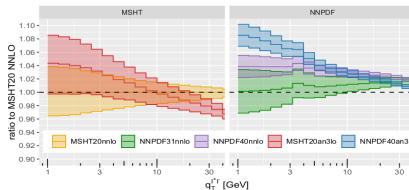
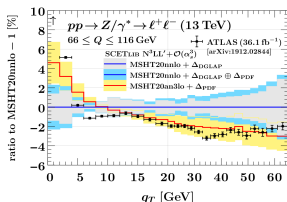


ggH xsec reduced by $\approx 1\%$ by QED effects.



Drell-Yan production - Transverse Momentum:

- $Z \rho_T$ spectrum - wish to use aN3LO PDFs to match resummation accuracy in predictions for $Z\rho_T$ spectrum at low q_T :
- MSHT20aN3LO and NNPDFaN3LO PDFs have **same impact on shape of q_T spectrum**:



- **Substantial aN3LO PDF effect on N3LL'/N4LL q_T spectrum.**

Left: SCETlib - Johannes Michel LHC EW WG meeting Sep 2022.

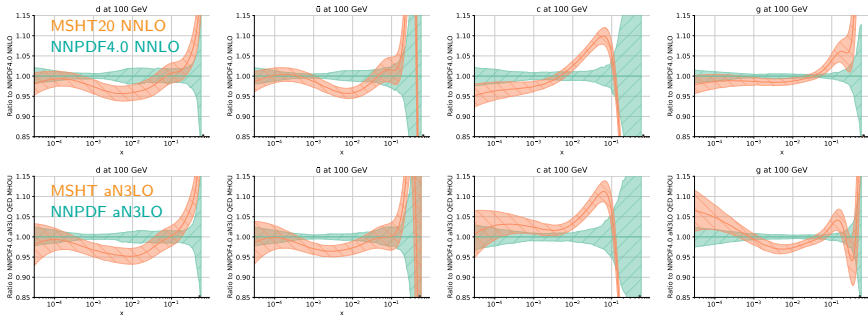
Centre: CuTe-MCFM - Tobias Neumann Loops and Legs March 2024

4.3 - α N³LO PDF Combination + effects on Cross-sections

More information in articles: [arXiv:hep-ph/2207.04739](https://arxiv.org/abs/hep-ph/2207.04739), [2312.07665](https://arxiv.org/abs/2312.07665), [2406.16188](https://arxiv.org/abs/2406.16188), [2408.10008](https://arxiv.org/abs/2408.10008), [2411.05373](https://arxiv.org/abs/2411.05373), [2510.09321](https://arxiv.org/abs/2510.09321).

How do MSHT and NNPDF aN3LO compare?

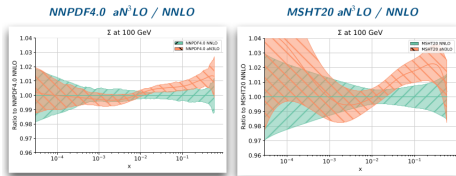
- Compare NNLO PDFs (top) and aN3LO PDFs (bottom):



- Most differences already present at NNLO.

- Largest aN3LO effect is on the gluon, both see suppression but $\sim 5, 2\%$ in MSHT, NNPDF.

- aN3LO/NNLO ratios generally qualitatively similar between groups.



Combination of the aN3LO PDFs:

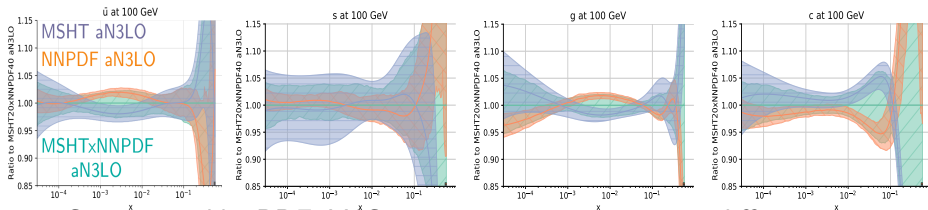
- Given the different aN3LO PDFs, a **conservative approach to estimate the total aN3LO PDF uncertainty** is to combine them.

Central value: unweighted average of two sets

Uncertainty: accounts for both individual PDF sets' uncertainties and any differences in their central values.

- À la PDF4LHC21, constructed by combining 100 replicas of MSHT and NNPDF aN3LO sets.

(See TC et al - MSHT and NNPDF - 2411.05373)



- Caveat** - unlike PDF4LHC21 no attempt to minimise differences, e.g. different heavy quark masses \Rightarrow use only at sufficiently large Q^2 .

aN³LO PDF effects on Cross-sections: Higgs

- Largest aN³LO effect is on Higgs production in gluon-gluon fusion.

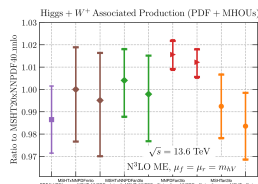
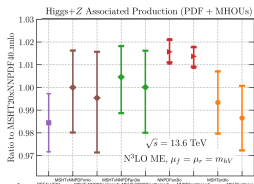
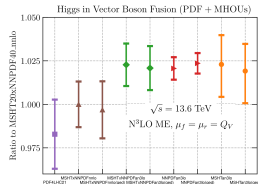
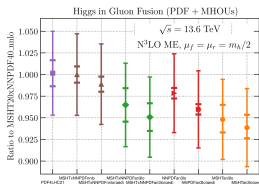
PDF	MSHT	NNPDF	MSHTxNNPDF
ggF xsec shift relative to NNLO	-5%	-2%	-4%
aN ³ LO	-5%	-2%	-4%
aN ³ LO+QED	-6%	-4%	-5%

- Smaller aN³LO effects for quarks and hence for VBF, ZH, W[±]H.

- VBF: aN³LO+QED result in +2.5%.

- ZH and W[±]H: all observe ≈ +0.5% from aN³LO+QED.

- Consistent trends in all cases between MSHT and NNPDF aN³LO PDFs.



(See TC et al - MSHT and NNPDF - 2411.05373)

Simultaneous determination of SM parameters with PDFs

- Strong Coupling
- Top mass
- PDF Profiling (see backup)

More information in articles: [arXiv:hep-ph/2106.10289](https://arxiv.org/abs/hep-ph/2106.10289), [2306.14885](https://arxiv.org/abs/2306.14885), [2404.02964](https://arxiv.org/abs/2404.02964), [2512.06092](https://arxiv.org/abs/2512.06092)
+ Ongoing Work...

Determining $\alpha_S(M_Z^2)$ jointly with PDFs:

- Default PDFs provided at fixed $\alpha_S(M_Z^2) = 0.118$, but can also **allow α_S to be a free parameter** in the fit.
- **Global fit** nature of PDFs \Rightarrow individual datasets have different dependences on α_S , but **robust** determination utilising all datasets.
- $\alpha_S(M_Z^2)$ sensitivity in PDF fit comes from:

- ▶ Direct $\alpha_S(M_Z^2)$ dependence in coefficient functions.

$$F(x, Q^2) = \sum_{i=q,\bar{q},g} [C_i(\alpha_S(Q^2)) \otimes f_i(Q^2)](x) \quad \mathbf{C}(\alpha_S) = \alpha_S^1 [\mathbf{C}_0 + \alpha_S \mathbf{C}_1 + \alpha_S^2 \mathbf{C}_2 + \alpha_S^3 \mathbf{C}_3 + \dots]$$

- ▶ Indirect $\alpha_S(M_Z^2)$ dependence through PDF evolution.

$$\frac{d\mathbf{f}}{d \ln \mu_f^2} \equiv \frac{d}{d \ln \mu_f^2} \begin{pmatrix} \Sigma \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix} \equiv \mathbf{P} \otimes \mathbf{f}$$

TC, L.A. Harland-Lang,
R.S. Thorne: 2404.02964.

$$\mathbf{P}(x, \alpha_s) = \alpha_s \mathbf{P}^{(0)}(x) + \alpha_s^2 \mathbf{P}^{(1)}(x) + \alpha_s^3 \mathbf{P}^{(2)}(x) + \alpha_s^4 \mathbf{P}^{(3)}(x) + \dots$$

First PDF $\alpha_S(M_Z^2)$ determination at aN3LO.
May lower with updates/TMCs, work ongoing...

- The **best fit values** are found to be:

$$\alpha_{S, \text{NNLO}}^{\text{new}}(M_Z^2) = 0.1171$$

$$\alpha_{S, \text{aN3LO}}^{\text{new}}(M_Z^2) = 0.1170$$

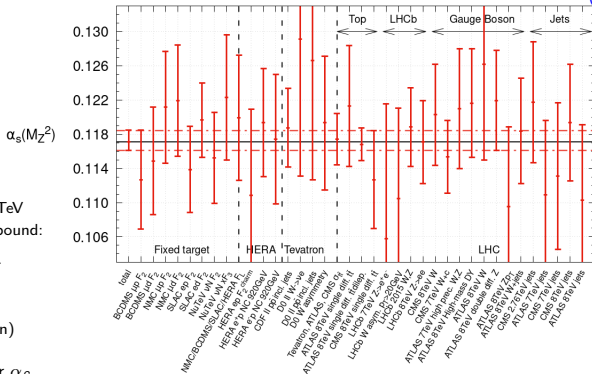
MSHT20 α_S bounds - NNLO

Consistent with α_S bounds seen in previous studies, and between orders (NNLO and aN3LO).

BCDMSp data strongest constraint upwards: $\Delta\alpha_S(M_Z^2) = +0.0014$.

SLACp and ATLAS 8TeV Zp_T both give upper bound: $\Delta\alpha_S(M_Z^2) = +0.0018$.

CMS/ATLAS (dilepton) $f\bar{f}$ single diff. would give lower/same upper α_S bound, but not used.



ATLAS 8 TeV Z data gives lower bound: $\Delta\alpha_S(M_Z^2) = -0.0010$.

NMC deuteron, ATLAS 8 TeV High Mass DY give lower bounds of $\Delta\alpha_S(M_Z^2) = -0.0017, -0.0018$.

● Therefore upper/lower bounds are $+0.0014/-0.0010$ at NNLO.

TC, L.A. Harland-Lang,
R.S. Thorne: 2404.02964.

$$\alpha_{S,NNLO}(M_Z^2) = 0.1171 \pm 0.0014$$

Consistent with World Average of 0.1180 ± 0.0009 .

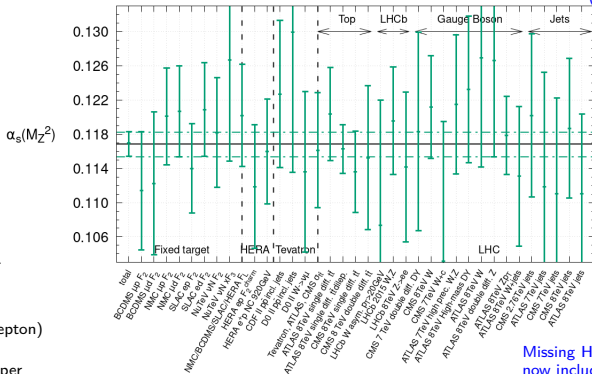
MSHT20 α_S bounds - aN3LO

Consistent with α_S bounds seen in previous studies, and between orders (NNLO and aN3LO).

BCDMSp data strongest constraint upwards: $\Delta\alpha_S(M_Z^2) = +0.0013$.

F_2^C provides upwards bound of: $\Delta\alpha_S(M_Z^2) = +0.0020$.

CMS and ATLAS (dilepton) $t\bar{t}$ single diff. would give slightly higher upper α_S bounds, but not used.



SLAC deuteron data gives lower bound: $\Delta\alpha_S(M_Z^2) = -0.0016$.

NMC deuteron, ATLAS 8 TeV Z both give lower bounds of $\Delta\alpha_S(M_Z^2) = -0.0017$.

Missing Higher Order Uncertainties now included, in particular causes some LHC bounds to weaken as unknown N3LO K-factors.

● Therefore upper/lower bounds are $+0.0013/-0.0016$ at aN3LO.

TC, L.A. Harland-Lang,
R.S. Thorne: 2404.02964.

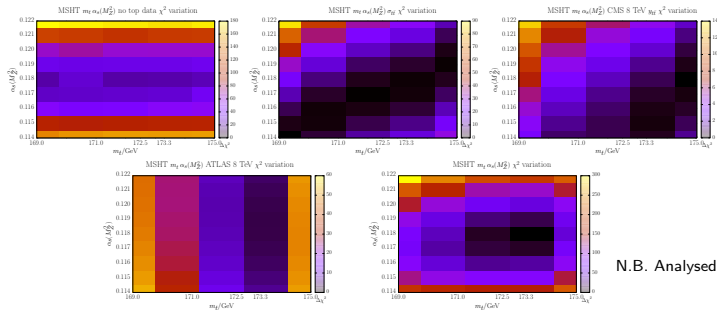
$$\alpha_{S,aN3LO}(M_Z^2) = 0.1170 \pm 0.0016$$

Consistent with World Average of 0.1180 ± 0.0009 .

MSHT20 Top mass α_S Correlation

TC and M.A. Lim: arXiv:2306.14885.

- How much correlation between m_t and α_S is there in a PDF fit?



N.B. Analysed at NNLO here.

- Without top data (upper left), no m_t sensitivity.
- Total xsec data and rapidity-differential data (upper centre, upper right) show m_t - α_S correlation.
- Data differential in p_T or $m_{H\tau}$ (lower left) much less correlated.
- Overall for total fit (lower right) very limited correlation seen \Rightarrow can extract m_t at fixed α_S . Obtain $m_t^{\text{pole}} = 173.0 \pm 0.6 \text{ GeV}$.

Conclusions

Conclusions

- **Precise and accurate PDFs are vital** for physics at the LHC and beyond.
- Challenges faced on experimental, methodological and theoretical fronts.
- **Experimentally** - understanding of experimental uncertainties, tensions.
- **Methodological** - stress testing of methodology, parameterisation, uncertainties.
- **Theoretically** - higher orders in QCD, QED corrections, inclusion of theory uncertainties.
- Much **work ongoing in PDF community**, welcome collaboration with exp/theory colleagues.

More information: arXiv:hep-ph/2012.04684, 2207.04739, 2306.14885, 2309.11269, 2312.07665, 2312.12505, 2404.02964, 2407.07944, 2411.05373, 2508.06603, 2510.03753, 2510.09321, 2512.06092, 2602.06908, etc by TC, L.A. Harland-Lang and R.S. Thorne (MSHT) and others.

Conclusions

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Thankyou!

Any Questions?

More information: arXiv:hep-ph/2012.04684, 2207.04739, 2306.14885, 2309.11269, 2312.07665, 2312.12505, 2404.02964, 2407.07944, 2411.05373, 2508.06603, 2510.03753, 2510.09321, 2512.06092, 2602.06908, etc by TC, L.A. Harland-Lang and R.S. Thorne (MSHT) and others.

Backup Slides



The University of Manchester



Acknowledgements: T. Cridge wishes to thank the Royal Society for support through his Royal Society University Research Fellowship: Grant URF\R1\251540..

References:

MSHT PDFs:

- L.A. Harland-Lang, T.C., R.S. Thorne - 2510.03753.
 - T.C., L.A. Harland-Lang, R.S. Thorne - 2510.09321.
 - L.A. Harland-Lang, T.C., P. Newman, R.S. Thorne, K. Wichmann - 2512.06092.
 - J. McGowan, T.C., L.A. Harland-Lang, R.S. Thorne - 2207.04739.
 - T.C., L.A. Harland-Lang, R.S. Thorne - 2312.07665
 - T.C., L.A. Harland-Lang, R.S. Thorne - 2312.12505
 - R. D. Ball et al - NNPDF - 2402.18635.
 - T.C., L.A. Harland-Lang, R.S. Thorne - 2404.02964
 - A. Barontini, N. Laurenti, J. Rojo - 2406.01779.
 - A. Cooper-Sarkar, T. Cridge et al - 2406.16188.
 - T. Cridge et al - MSHT and NNPDF - 2411.05373.
 - A. Cooper-Sarkar, T. Cridge et al - 2508.06603.
 - T. Cridge, L.A. Harland-Lang, R.S. Thorne - 2510.09321.
 - L.A. Harland-Lang, T. Cridge, R.S. Thorne - 2510.03753.
 - L.A. Harland-Lang, T. Cridge et al - 2512.06092.
 - T. Cridge, J.C. Martinez, J. Huston - 2602.06908.
- ### aN3LO Splitting Functions:
- S.O. Moch et al - 1707.08315.
 - J. Davies et al - 1610.07477.

- T. Gehrmann et al - 2308.07958.
- T. Gehrmann et al - 2310.12240.
- G. Falcioni et al - 2310.01245.
- S.O. Moch et al - 1805.09638.
- S.O. Moch et al - 2111.15561.
- G. Falcioni et al - 2302.07593.
- G. Falcioni et al - 2307.04158.
- S.O. Moch et al - 2310.05744.
- G. Falcioni et al - 2404.09701.
- G. Falcioni et al - 2409.02870.
- G. Falcioni et al - 2410.08089.

N3LO Transition Matrix Elements:

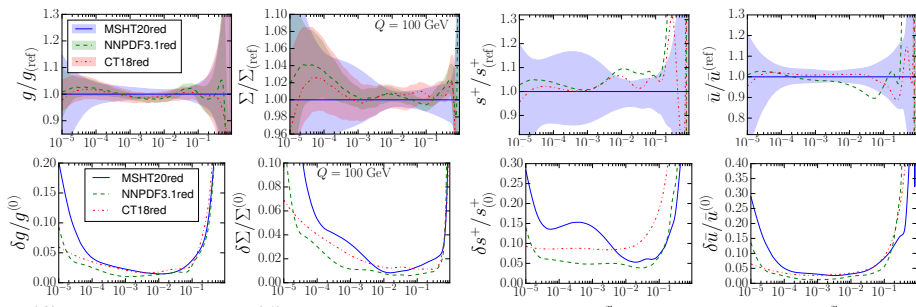
- H. Kawamura et al - 1205.5727.
- I. Bierenbaum et al - 0904.3563.
- J. Ablinger et al - 1406.4654.
- J. Ablinger et al - 1409.1135.
- J. Blümlein et al - 2107.06267.
- J. Ablinger et al - 1405.4259.
- J. Ablinger et al - 1409.1435].
- J. Ablinger et al - 1402.0359.
- J. Ablinger et al - 2211.05462.
- J. Ablinger et al - 2311.00644.
- J. Ablinger et al - 2403.00513.

N3LO DIS Coefficient Functions:

- J.A.M. Vermaseren, A. Vogt and S.O. Moch - 0504242.

PDF Uncertainties:

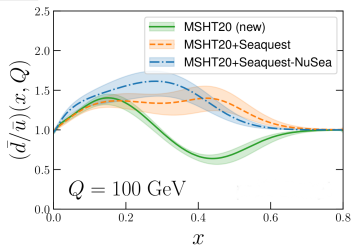
- Given precision demands placed on PDFs there is an increasing focus on PDF uncertainties.
- Different groups have different methods - neural network, parameterisation, tolerance, etc. Also include different data and theory.
- PDF4LHC21 benchmarking revealed with the same input data, groups report similar central values but different PDF uncertainties:



Fermilab Seaquest at NNLO and aN3LO: Preliminary!

- Seaquest (E906) fixed target DY data at high x : $\sigma_D/\sigma_H \sim 1 + \bar{d}/\bar{u}$.
- **Raises high x \bar{d}/\bar{u} . Tension with NuSea(E866) which pulls it down.**

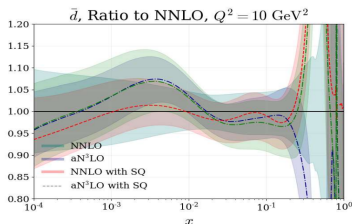
NNLO:



- NuSea χ^2/N_{pts} : $0.65 \rightarrow 1.27$, when Seaquest added. Rest of data worsens by $\Delta\chi^2 = +9$.
- **General Q: How to deal with datasets that are in tension?**

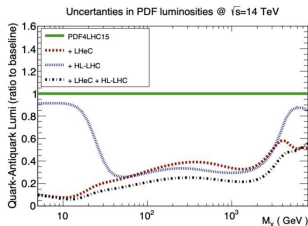
aN3LO:

- **Similar effect to at NNLO.**
- **Improves NNLO-aN3LO consistency once Seaquest data is added.**

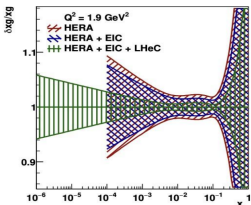


Complementarity of Future Colliders:

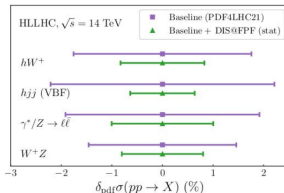
- **HL-LHC**: Huge stats can reduce PDF uncertainties by factor of 2-3.
- **EIC**: Precision can reduce high x PDF, gluon and even α_S uncertainties.
- **LHeC**: Precision DIS could boost this further, particularly at low x/m_X .
- **Forward Physics Facility at HL-LHC**: use neutrinos - production probes charm/gluon at high and low x , ν DIS probes strangeness.



LHeC/FCC-eh Study Group 2007.14491
(see also Abdul Khalek et al 1906.10127)



LHeC Input to EPPSU arXiv:2503.17727



FPF Input to EPPSU arXiv:2503.19010

But:

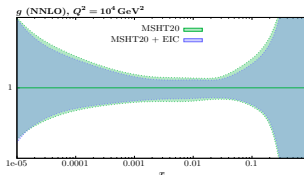
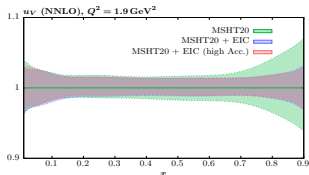
- Assumes consistency of data (limits PDF precision).

Harland-Lang, TC, Thorne
EIC: 2309.11269, 2512.06092

Future Colliders: EIC

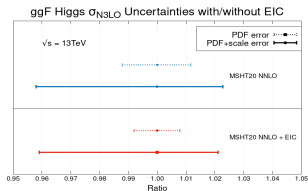
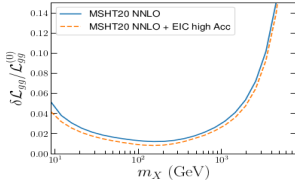
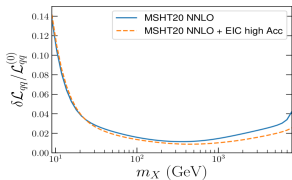
TC et al, 2309.11269

- Impact of EIC pseudodata on MSHT20 - *high x lower Q^2 sensitivity.*



Here considered only inclusive DIS at NNLO, more opportunities from SIDIS, F_L , $F_2^{C,D}$.

- Up valence uncertainty reduced at high $x \Rightarrow \delta\mathcal{L}_{qq}$ reduced at high m_X .
- Gluon uncertainty reduced across range of x .
- $\delta\mathcal{L}_{gg}$ reduced across $x \Rightarrow$ smaller PDF uncertainty for $gg \rightarrow H$.

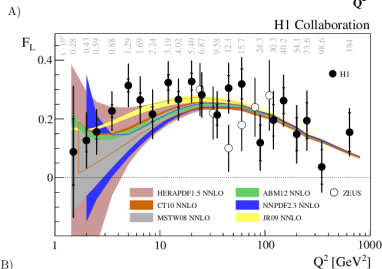
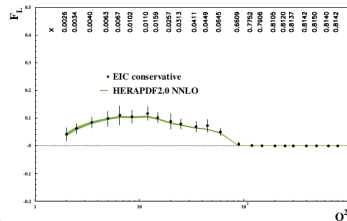
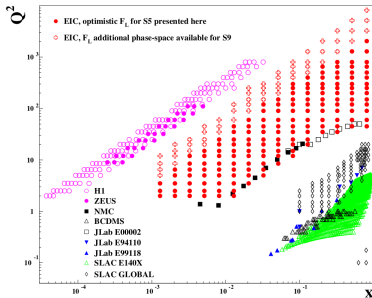


- Studies also show constraints possible on α_S . Harland-Lang, TC, R.S. Thorne 2512.06092

EIC Further PDF Constraints - F_L in HERAPDF:

- Additional direct sensitivity to the gluon from F_L measurements. $F_L \sim \alpha_S g$
- Possible over larger range than HERA.
- Separate by Rosenbluth method using:

$$\sigma_{red}^{NC} \sim F_2 - \frac{Y^2}{Y_+} F_L$$

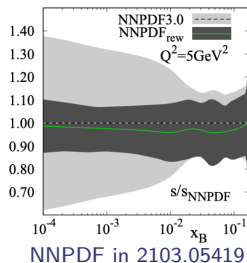
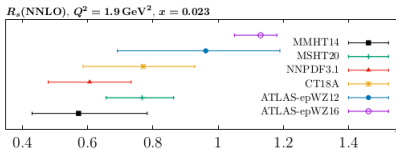
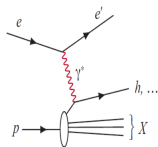


Jiménez-López, Newman, Wichmann 2412.16123

EIC Further PDF Constraints - Strangeness:

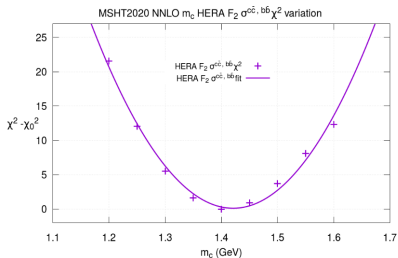
- Limited strangeness sensitivity from inclusive DIS EIC measurements.
- Use **SIDIS** - parton content of outgoing hadron is connected to fragmenting parton and via CC/NC vertex to the **parton in the proton**.
- Pickup uncertainties from fragmentation functions.
- Similar to ν DIS already used from NuTeV, which provides main constraint on $s - \bar{s}$ asymmetry, and from future FPF at CERN.
- **Proton strangeness observed to be larger at LHC.**
- **Further s constraints come from charm jets.**

(see e.g. Arratia, Hobbs et al in 2006.12520)

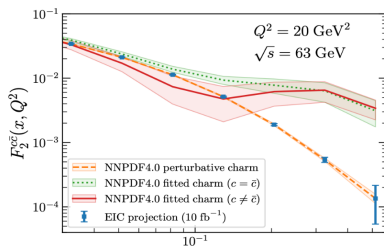


EIC Further PDF Constraints - Heavy Quarks:

- Measurements of **charm and bottom structure functions** will be extended to **higher x** .
- Gives **sensitivity to high x heavy quark PDFs**, and to **heavy quark masses**. E.g. used HERA data in MSHT (lower left).
- Recent suggestions of a **fitted charm component of proton** at high x by NNPDF, using EMC F_2^C and LHCb ($Z+c$) data.
- Several questions in community about this \Rightarrow can be resolved by EIC.



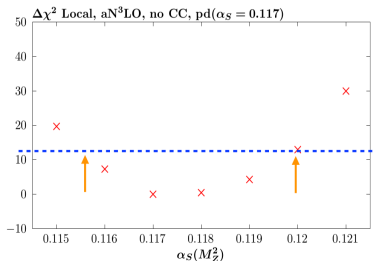
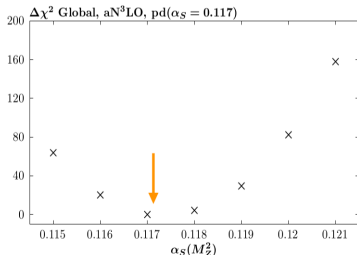
MSHT, TC et al 2106.10289



NNPDF, R.D. Ball et al 2311.00743

Determination of α_S at EIC:

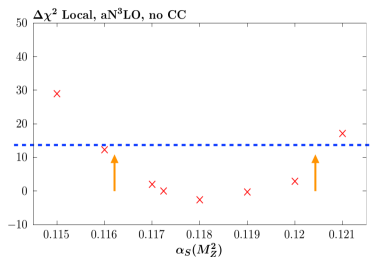
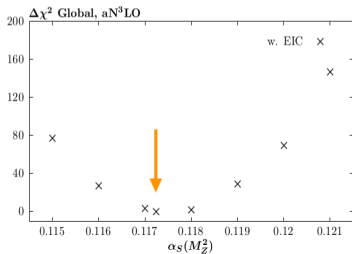
- Utilise same pseudodata now generated at aN³LO and with $\alpha_S(M_Z^2) = 0.117$ - consistent with PDFs. **Fit simultaneously PDF+ α_S .**
- Examine χ^2 profile of EIC pseudodata to determine its bounds on α_S .



- Bounds set via dynamical tolerance $\Delta\chi^2 < (1 - \frac{\xi_{68}}{\xi_{50}} \chi_0^2) \sim 13$ for EIC NC data. Upper bound on α_S found not competitive.
- Lower bound ~ -0.0015 competitive \Rightarrow would be best lower bound, better than SLAC/NMC $d \sim -0.0016/17$ which currently set limits.
Harland-Lang, TC, Thorne et al (work in progress) - Preliminary!

Determination of α_S at EIC:

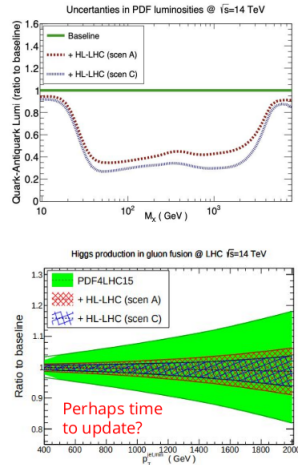
- What happens if preferred α_S value different to global PDF fit?
- Instead generate pseudodata with $\alpha_S(M_Z^2) = 0.118$ and repeat.
- Now **global $\alpha_S(M_Z^2)$ best fit is shifted up**, from 0.1170 to 0.1172.



- Now **sets tighter lower bound than before ~ -0.0012** (as it prefers larger $\alpha_S(M_Z^2)$) **and weaker upper bound.**
- **Interplay of preferred α_S and uncertainty on bounds is often neglected.**
- Improved precision on α_S directly reduces uncertainties on many key SM processes. Harland-Lang, TC, Thorne et al (work in progress) - Preliminary!

Future Colliders - HL-LHC

- Huge statistics of **HL-LHC will reduce PDF uncertainties** across wide range by factor of 2-3.
- Quark-antiquark improvement -> **precision SM $m_W, \sin^2\theta_w$** .
- Gluon uncertainty improvement -> **Higgs**.
- Large x uncertainty reductions -> **BSM** searches.
- Caveats:
 - Only subset of HL-LHC processes!
 - **Depends on understanding of systematics + correlations** within/between datasets
 - Assumes consistency of data



Abdul Khalek et al arXiv:1810.03639

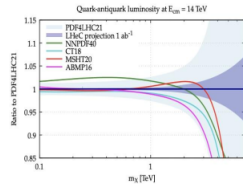
From TC talk at ESPPU Update 14th May 2025 - <https://indico.cern.ch/event/1527574/>

Future Colliders - LHeC

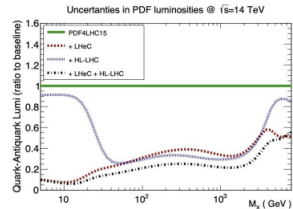
- **LHeC clean DIS environment** - reduce PDF uncertainties, particularly at low x (mX). Informs FCChh (related FCCeh).
- Much of impact from initial 50 fb^{-1} (1 year of data).
- **Combination of HL-LHC+LHeC complementary**, reduction in PDF uncertainties from LHeC would improve SM precision at HL-LHC (+ also Higgs/BSM):
 - $\Delta M_W = \pm 3 \text{ MeV}$
 - $\Delta \sin^2 \Theta_W = \pm 0.00008$
- Potential for DIS only PDFs - simpler systematics.
- Caveats:
 - Assumes consistency of data, upper plot tolerance of 1 and limited parameterisation.

N.B. Can also be constrained independently at LHeC as can α_s

More info on Precision QCD at LHeC in Daniel Britzger's talk in 2 weeks time



LHeC Input to EPPSU arXiv:2503.17727

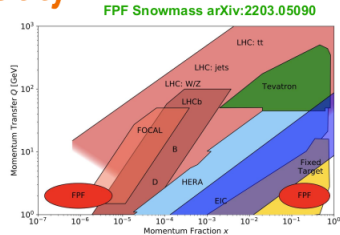
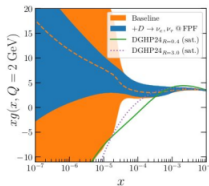
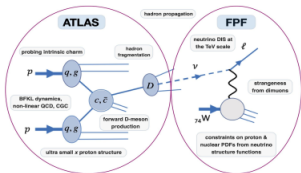


LHeC/FCC-eh Study Group 2007.14491
(see also Abdul Khalek et al 1906.10127)

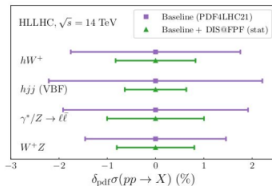
From TC talk at ESPPU Update 14th May 2025 - <https://indico.cern.ch/event/1527574/>

Future Colliders - Forward Physics Facility

- Exploit **HL-LHC produced neutrinos**.
- Production -> **charm, gluon** at high and low x .
- Interaction in ν DIS -> **strangeness**.
- Significant uncertainty reductions and knock-on impacts on key LHC processes.
- **Complementary with HL-LHC**. Informs also FCC.



FPF Input to EPPSU arXiv:2503.19010



From TC talk at ESPPU Update 14th May 2025 - <https://indico.cern.ch/event/1527574/>

MSHT work on aN3LO PDFs:

- 1 First Approximate N3LO PDF set plus with inclusion of theoretical uncertainties for missing higher orders (MHOUs) - [2207.04739](#).
- 2 Analysis of experimental pulls on PDFs including MSHT20aN3LO PDFs, with CT and others, [2306.03918](#).
- 3 MSHT20aN3LO QCD + QED - combining aN3LO QCD with QED PDF evolution for highest accuracy PDFs, [2312.07665](#).
- 4 Analysis of high x gluon data in aN3LO PDFs - Jets, dijets, Z p_T , clear preference for aN3LO, [2312.12505](#).
- 5 First Determination of α_s at aN3LO in PDF fit, [2404.02964](#).
- 6 Les Houches SM WG Report - Impact of aN3LO PDFs on ggF Higgs Uncertainties, [2406.00708](#), with theory colleagues and NNPDF.
- 7 Benchmarking of QCD evolution at Approximate N3LO with theory (FHMRUVV), NNPDF and other colleagues (also in Les Houches SM WG Report), [2406.16188](#).
- 8 Approximate N3LO Combination + Higgs Pheno Study [2411.05373](#).

aN3LO effects on the PDF fit:

- aN3LO (and theory uncertainties) have impact on PDF fit.
 - ▶ MSHT (and NNPDF) - **Improvement order by order of fit quality:**

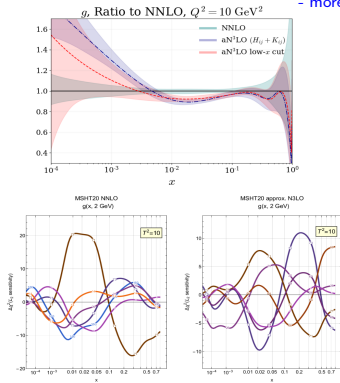
MSHT χ^2/N_{pts} (4363)	LO	NLO	NNLO	aN3LO
		2.57	1.33	1.17

$\Delta\chi^2$ improves by
 ~ -150 in original aN3LO,
 $(\sim -85$ after new updates
 - more later).

- MSHT - DY data (in particular $Z p_T$ data) notably better fit.
- **Reduced tensions between data** also seen.
- MSHT - Dijet data also better fit at aN3LO than NNLO.

T.C. et al, 2312.12505

- **High precision data requires high precision theory.**



L2 study 2306.03918

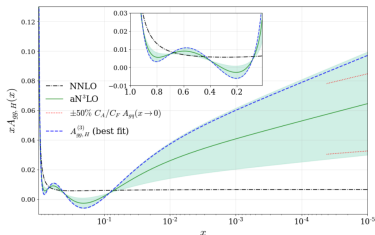
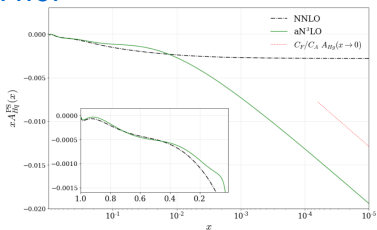
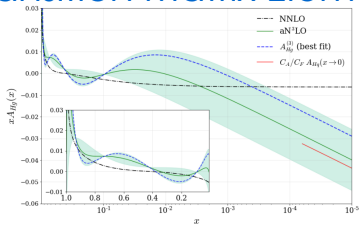
Transition Matrix Elements

- General perturbative expansion for heavy quark TME:

$$A_{ij}(\mu^2/m_h^2) = \delta_{ij} + \sum_{l=1}^{\infty} \alpha_S^l \sum_{k=0}^l \log^k(\mu^2/m_h^2) a_{ij}^{(l,k)} \quad (1)$$

- Apply at $\mu = m_h$, then only $a_{ij}^{(l,0)}$ relevant, other powers determined by lower orders in any case.
- $a_{ij}^{(3,0)} = a_{ij}^{(3,0),0} + n_f a_{ij}^{(3,0),1}$, focus on n_f -independent pieces.
- At time of MSHTaN3LO: $A_{Hq}^{PS,(3)}$, $A_{gq,H}^{(3)}$ known completely.
- For others we knew:
 - ▶ Even low-integer N Mellin Moments (4-8)
 - constrain intermediate and high x via $\int_0^1 dx x^{N-1} P(x)$.
 - ▶ Form at low x , in some case low and high x limits.
- Deal with as for Splitting functions \Rightarrow 1 TNP for $A_{Hg}^{(3)}$, $A_{qq,H}^{NS,(3)}$, $A_{gg,H}^{(3)}$.
- $A_{Hg}^{(3)}$, $A_{qq,H}^{NS,(3)}$, $A_{gg,H}^{(3)}$ since determined, former not in NNPDF.

Transition Matrix Elements:

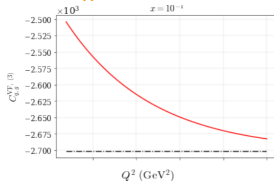
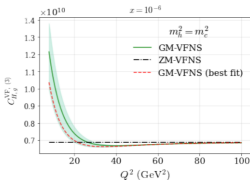


- $A_{Hq}^{PS,(3)}$, $A_{qq,H}^{(3)}$ known completely, need to be approximated (without uncertainty) due to complex form. $A_{Hg}^{(3)}$, $A_{qq,H}^{NS,(3)}$, $A_{gg,H}^{(3)}$ have one theory nuisance parameter each at low x .

DIS Coefficient Functions

- Needed to produce N3LO Structure Functions, we know:
 - ▶ Light flavour contributions to coefficient functions (CFs) known.
 - ▶ Heavy flavour contributions to heavy flavour coefficient function:
 - 1 Zero Mass ($Q^2 \rightarrow \infty$) case (ZM-VFNS) known exactly.
 - 2 Massive case $Q^2 \leq m_H^2$ (FFNS) approximations known.
- Low- Q^2 FFNS CFs $C_{H,\{q,g\}}$ - known LL small x and mass threshold info, but unknown NLL small $x \Rightarrow$ introduce TNP's c_q^{NLL} and c_g^{NLL} :

$$C_{H,i}^{(3),NLL}(Q^2 \rightarrow 0) \propto c_i^{NLL} \left[-4 \frac{1}{x} + c_i^{LL} \frac{\ln 1/x}{x} \right], \text{ for } i = q, g.$$



- Heavy flavour contribution to light quark CFs unknown at low Q^2 , use known high Q^2 info + lower order behaviour. Very small generally.

DIS Coefficient Functions

- Need to **interpolate low and high Q^2** to generate full **General-Mass Variable Flavour Number Scheme** (GM-VFNS) for all Q^2 .
- Use regression model and guidance from lower orders for interpolation.
- **Include Transition Matrix Elements at aN3LO** so full **cancellation of PDF discontinuities** in the structure functions.
- Therefore some DIS coefficient functions **inherit some uncertainty** bands from these, e.g. $C_{H,g}^{VF,(3)}$ from $A_{Hg}^{(3)}$:

$$C_{H,g}^{VF,(3)} = C_{H,g}^{FF,(3)} - C_{H,g}^{VF,(2)} \otimes A_{gg,H}^{(1)} - C_{H,H}^{VF,NS+PS,(2)} \otimes A_{Hg}^{(1)} \\ - C_{H,g}^{VF,(1)} \otimes A_{gg,H}^{(2)} - C_{H,H}^{VF,(1)} \otimes A_{Hg}^{(2)} - C_{H,H}^{VF,(0)} \otimes A_{Hg}^{(3)}$$

- $C_{Hq}^{VF,(3)}$ and $C_{Hg}^{VF,(3)}$ have uncertainties from C_q^{NLL} and C_g^{NLL} parameters,
 $C_{Hq}^{VF,(3)}$ and $C_{qq,NS}^{VF,(3)}$ inherit uncertainty from $A_{Hg}^{(3)}$ and $A_{qq,NS}^{(3)}$.

Hadronic K-factors

Ingredient 4

- **N3LO calculations** becoming available but not yet for PDF fits:
 - ▶ **Drell-Yan** - Inclusive + some differential - not yet usable for PDFs.
 - ▶ **Higgs** - ggF, VBF and VH - doesn't go in PDFs.
 - ▶ **Top** (aN3LO) - soft gluon resummation approximation.
- Overall, **much less known** than for other N3LO PDF fit ingredients.
- Challenge: include a theory uncertainty for unknown N3LO into PDFs.
- Parameterise N3LO k-factor as combination of **NLO and NNLO k-factors**, α_1, α_2 coeffs incorporating MHOUs into PDF uncertainties:

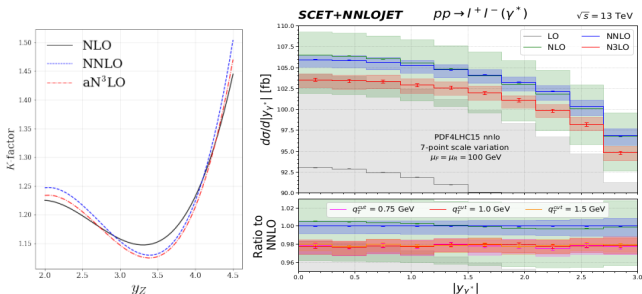
$$K^{N3LO/LO} = K^{NNLO/LO} (1 + \alpha_1 \mathcal{N}^2 \alpha_S^2 (K^{NLO/LO} - 1) + \alpha_2 \mathcal{N} \alpha_S (K^{NNLO/LO} - 1))$$

- **Default** prior is $\alpha_1, \alpha_2 = 0$, i.e. **no N3LO correction**.
- Categorise all hadronic processes into **5 types - jets (or dijets), Drell-Yan, top, vector boson p_T /jets, and dimuon** \Rightarrow **10 TNP's added**.
- Very limited impact on aN3LO PDFs, more complex parameterisations possible (M.A. Lim, R. Poncelet: 2412.14910) .

Hadronic K-factors - Drell-Yan

1 Drell-Yan (DY)

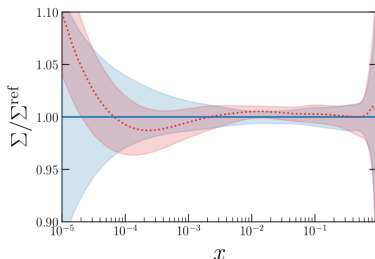
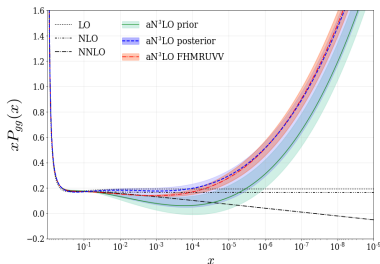
- Fit prefers a $\approx 1\%$ decrease in the N3LO k-factors relative to NNLO.
- Improved perturbative convergence with aN3LO PDFs.
- In qualitative agreement with recent N3LO results for NC DY⁵³.



- **Key point:** Method allows N3LO info. on any piece to be incorporated as it becomes available, rather than needing to wait for all info. - e.g. can include N3LO k-factors as they become available for PDFs.

Impact of New Moments on the aN3LO PDFs:

- How much do the new mellin moments calculated since MSHT20aN3LO affect the aN3LO PDFs?: N.B. Preliminary as no uncertainty included for remaining MHOU in $P_{ij}^{(3)}$.

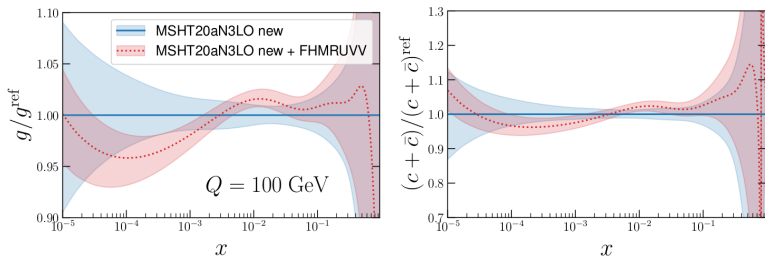


- Splitting functions largely consistent, e.g. P_{gg} very good agreement.
- Only P_{gq} (not shown) significantly changed, has highest power unknown small x log. Partly anticorrelated with P_{gg} , limited PDF impact.
- PDFs move by up to $\sim 1\sigma$, very little impact on quark singlet.

See TC et al 2510.09321.

Impact of New Moments on the aN3LO PDFs:

- How much do the new mellin moments calculated since MSHT20aN3LO affect the aN3LO PDFs?: N.B. Preliminary as no uncertainty included for remaining MHO in $P_{ij}^{(3)}$.

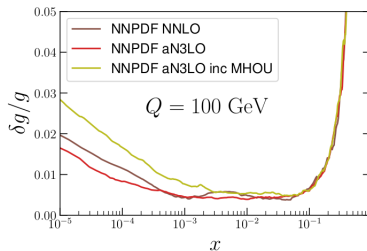
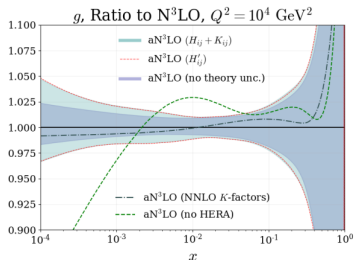


- MSHT sees rise of $\sim 1.5\%$ at $x \approx 10^{-2}$ in gluon, (cf NNPDF sees reduction of $\sim 1\%$ at $x \approx 10^{-3}$.)
- aN3LO/NNLO ratio moves closer between MSHT and NNPDF (e.g. L_{gg} rises by $\sim 1\%$ for MSHT and lowers by $\sim 0.5\%$ for NNPDF.)
- Charm (and bottom) PDF slightly altered, reflecting gluon changes.

See TC et al 2510.09321.

Impact of aN3LO + MHO on PDF uncertainties:

- aN3LO and theory uncertainties from Missing Higher Orders (MHOUs) impact PDF errors.
- MSHT (left) and NNPDF (right) both see **added theory uncertainty increasing PDF uncertainties at low x** , e.g. gluon:
- Whilst **PDF uncertainty is larger**, it's **more accurate and reliable**.



Theory Nuisance Parameter Summary

- So in total, we add **20 added theory nuisance parameters**, on top of 51 central PDF parameters (which give 32 PDF uncertainty parameters).
- Now have **52 eigenvectors** (32 as before + 20 new theory).

Origin	Parameters	Number of Added Parameters
Splitting Functions - $P_{qq}^{(3)}, P_{qq}^{NS,(3)}, P_{qq}^{PS,(3)}, P_{gg}^{(3)}, P_{gg}^{(3)}$	$\rho_{qg}, \rho_{qq}^{NS}, \rho_{qq}^{PS}, \rho_{gq}, \rho_{gg}$	5
Transition Matrix Elements - $A_{Hg}^{(3)}, A_{qq,H}^{NS,(3)}, A_{gg,H}^{(3)}$	$a_{Hg}, a_{qq,H}^{NS}, a_{gg,H}$	3
DIS Coefficient Functions - $C_{H,q}^{(3),NLL}, C_{H,g}^{(3),NLL}$	C_q^{NLL}, C_g^{NLL}	2
Hadronic K-factors - Drell-Yan Top Jets p_T Jets Dimuon	DY_{NLO}, DY_{NNLO} Top_{NLO}, Top_{NNLO} Jet_{NLO}, Jet_{NNLO} $p_T Jet_{NLO}, p_T Jet_{NNLO}$ $Dimuon_{NLO}, Dimuon_{NNLO}$	$5 \times 2 = 10$

- Using **MSHT20an3lo_as118** eigenvectors as usual naturally incorporates MHOUs at aN3LO into the PDF uncertainties.

N.B. We find the penalties on these parameters are almost all $< 1 \Rightarrow$ conservative priors set.

Perform aN3LO fit - fit quality:

- Perform aN3LO fit with identical dataset to MSHT20 NNLO PDF fit.
- Overall fit quality (4363 points)

χ^2/N_{pts}	LO	NLO	NNLO	aN3LO
	2.57	1.33	1.17	1.14

Smooth fit improvement with order and amount of improvement reducing with order - as we might hope.

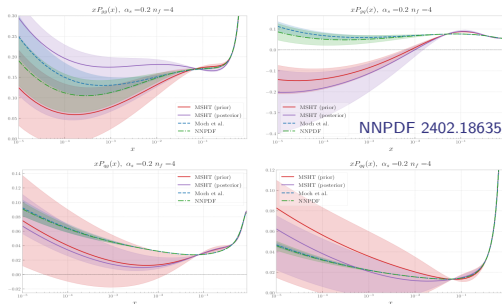
- Improvement in fit quality from NNLO to aN3LO is $\Delta\chi^2 = -154.4$.
- Much larger than number of parameters (20) introduced.

Dataset type	Total χ^2/N_{pts}	$\Delta\chi^2$ from NNLO	$\Delta\chi^2$ from NNLO (but no N3LO k-factors)
DIS datasets	2580.9/2375	-90.8	-86.2
Drell-Yan datasets	1065.4/864	-12.8	+10.4
Dimuon datasets	125.0/170	-1.2	+0.5
Top datasets	75.1/71	-4.2	-2.5
$V p_T / V + \text{jets}$ datasets	138.0/144	-77.2	-54.7
Inclusive Jets datasets	963.6/739	+21.5	+42.2
Total	4957.2/4363	-154.4	-83.6

- Over half of fit improvement occurs **without N3LO k-factors freedom**.
- Average TNP penalty $0.460 < 1$. **Fit able to describe data well with known info and only small departures around prior.**

Comparison of aN3LO Splitting Functions:

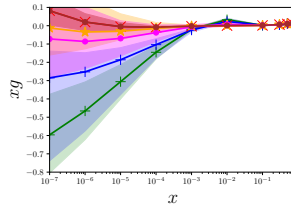
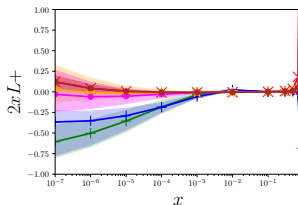
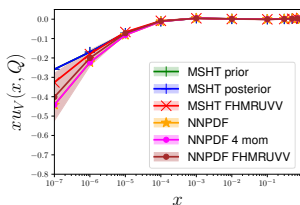
- Overall consistent trends observed across PDFs, largest impact in both MSHT and NNPDF is suppression of gluon at intermediate x .
- But size of the effect observed is different, can we understand this?
- One significant ingredient is the splitting functions, how do these compare?:



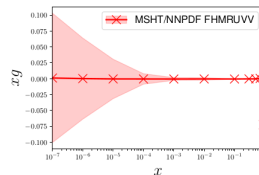
- Newer results are within MSHT uncertainties, exception $P_{gq} \Rightarrow$ highest power unknown log, impact partly anticorrelated with P_{gg} , latter more important. MSHT don't impose Casimir scaling here, NNPDF do.

Impact of aN3LO evolution on PDFs:

- N3LO evolution benchmarking - use toy PDFs, no fit or other issues:



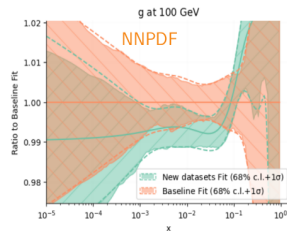
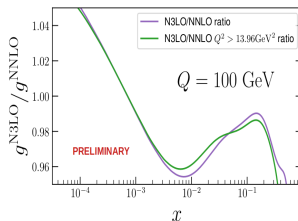
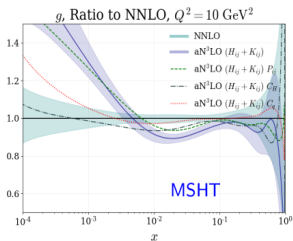
- Agreement down to 10^{-3} with \lesssim (few) % impacts over data region.
- Differences with larger uncertainties at (very) low x .
- New information (FHMRUVV) provides some additional constraints but still consistent with previous determinations.
- Per mille agreement when using the same splitting functions (right).



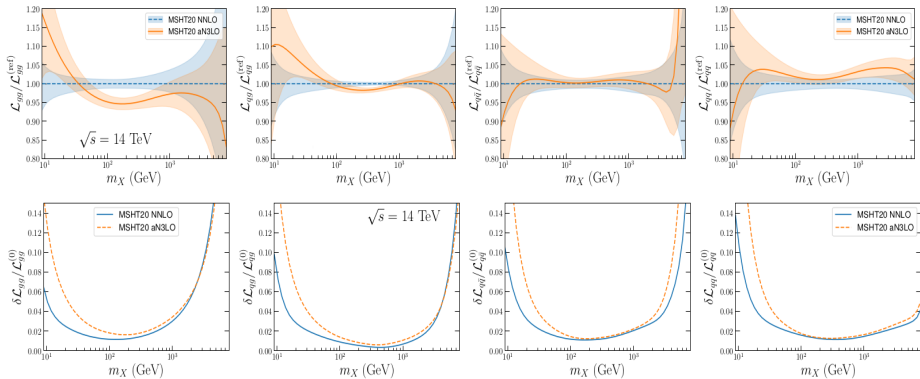
Les Houches Proceedings (2406.00708) and article - 2406.16188.

Examination of aN3LO effects on the gluon:

- Not only evolution, **other N3LO ingredients also important** (lower left) - **heavy flavour transition matrix elements and CFs also impact gluon.**
- Different implementation of theory uncertainties - theory nuisance parameters or scale variations. Different Q^2 cut in DIS? (lower centre).
- Plus we have the **usual sources of PDF differences** (as at NNLO):
 - ▶ Methodology, e.g. perturbative vs fitted charm, positivity can cause $\sim 1\%$ differences in the gluon.
 - ▶ Data, e.g. new 13TeV jet data lower NNPDF gluon (lower right).



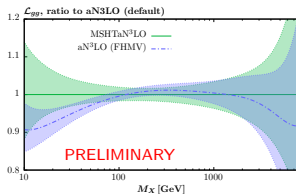
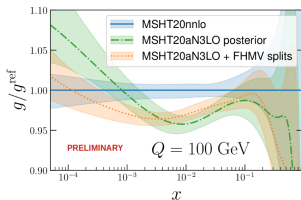
aN3LO PDF luminosities:



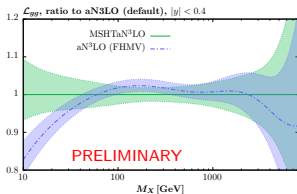
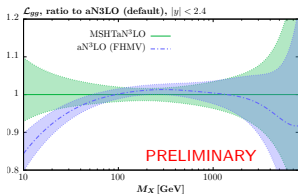
- PDF changes have implications for PDF luminosities for phenomenology.
- gg luminosity reduced around 100GeV and increased at 10GeV.
- Luminosity uncertainties enlarged (and more so at lower invariant masses) due to inclusion of aN3LO and PDF theory uncertainties.

gg Luminosity with FHMV and rapidity cuts:

- Despite rise in gluon PDF with FHMV splitting functions[26-30], very little change in gg luminosity as integrating over rapidity (x).

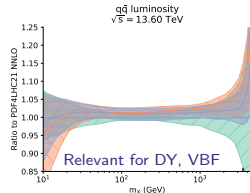
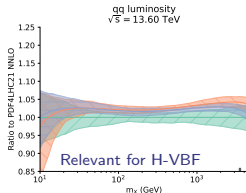
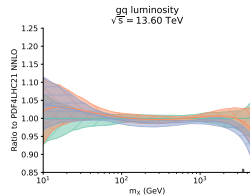
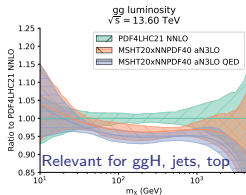


- Adding rapidity cuts - no change with $|y| < 2.4$ (lower left), only with stricter cuts (e.g. $|y| < 0.4$, lower right) do we see the lumi rise.



PDF Luminosities of the aN3LO combined sets:

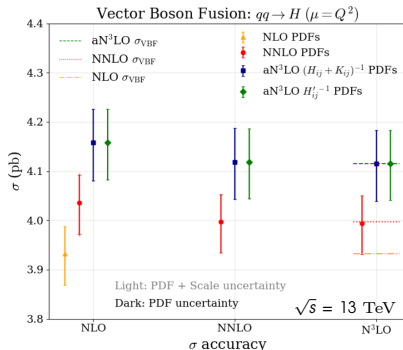
- Compare combined PDF set luminosities for aN³LO and aN³LO+QED with PDF4LHC21 NNLO benchmark:
- Most significant aN³LO effect is **gluon-gluon lumi reduction of $\sim 3.5\%$** at m_H .
- **Further suppressed by $\sim 1.5\%$ by QED effects**
 \Rightarrow due to reduction in gluon momentum, seen also at NNLO.
- Also rise in aN³LO quark-quark lumi at large m_X .
- **QED corrections similar in magnitude to aN³LO effects for quark sector.**



(See TC et al - MSHT and NNPDF - 2411.05373)

Impact on Higgs cross-sections - VBF:

- Consider impact of our aN3LO PDFs on known N3LO Higgs production in vector boson fusion²⁷:



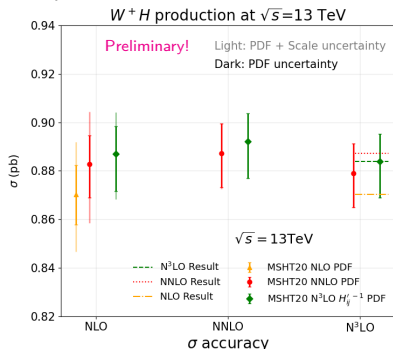
N.B. For scale variations - do μ_R and μ_F at NNLO but only μ_R at aN3LO as PDF uncertainty from MHOs already in PDF eigenvectors.

Results obtained using proVBFH code^{48,57}.

- Increase in σ using aN3LO PDFs, occurs due to enhanced charm and light quarks at high x .
- VBF more reliant on quark sector - changes less ($\sim 2.5\%$, cf $\sim 5\%$ for ggF) with PDF order as more data constraints on quarks.

Impact on VH cross-sections:

- Consider impact of our aN3LO PDFs on VH associated production (Higgsstrahlung) at LHC, e.g. W^+H at 13 TeV:



N.B. For scale variations - do μ_R and μ_F at NNLO but only μ_R at aN3LO as PDF uncertainty from MHOs already in PDF eigenvectors.

Results obtained using the `n3lox` code⁴⁹.

- Result with aN3LO PDFs raised slightly**, reflects increased quarks at high X , antiquarks at low X and strange and charm.
- N3LO σ + aN3LO PDF result very close to NNLO σ + NNLO PDF result, increased **stability in predictions**.

aN3LO PDF effects on Xsecs:

- aN3LO PDFs can be used:
 - 1 With N3LO matrix elements to **compute cross-sections more precisely**.
 - 2 To **evaluate the PDF uncertainty from using NNLO PDFs** with N3LO cross-sections (previous “highest order” available).
- Before aN3LO PDFs, often N3LO cross-section with NNLO PDFs taken as “highest order” result + $\Delta_{\text{NNLO}}^{\text{approx}}$ for PDF MHO uncertainty.

$$\Delta_{\text{NNLO}}^{\text{approx}} = \frac{1}{2} \left| \frac{\sigma_{\text{NNLO-PDF}}^{\text{N3LO-xsec}} - \sigma_{\text{NLO-PDF}}^{\text{NNLO-xsec}}}{\sigma_{\text{NNLO-PDF}}^{\text{NNLO-xsec}}} \right|$$

- 2 Compare with $\Delta_{\text{NNLO}}^{\text{exact}}$:

$$\Delta_{\text{NNLO}}^{\text{exact}} = \left| \frac{\sigma_{\text{N3LO-PDF}}^{\text{N3LO-xsec}} - \sigma_{\text{NNLO-PDF}}^{\text{N3LO-xsec}}}{\sigma_{\text{N3LO-PDF}}^{\text{N3LO-xsec}}} \right|$$

PDF set	Δ	$\sigma(\text{gg} \rightarrow \text{H})$	$\sigma(\text{H VBF})$
MSHT20	$\Delta_{\text{NNLO}}^{\text{exact}}$	5.3%	2.3%
	$\Delta_{\text{NNLO}}^{\text{approx}}$	1.4%	1.3%
NNPDF4.0	$\Delta_{\text{NNLO}}^{\text{exact}}$	2.2%	1.3%
	$\Delta_{\text{NNLO}}^{\text{approx}}$	0.2%	0.2%
MSHT20xNNPDF4.0	$\Delta_{\text{NNLO}}^{\text{exact}}$	3.3%	2.3%
	$\Delta_{\text{NNLO}}^{\text{approx}}$	1.6%	0.5%

- $\Delta_{\text{NNLO}}^{\text{approx}}$ is very unreliable, underestimating $\Delta_{\text{NNLO}}^{\text{exact}}$ for ggF and VBF.

(See TC et al - MSHT and NNPDF - 2411.05373)

Impact of TMCs and Higher Twists

- **Target mass corrections** - $\mathcal{O}(m^2/Q^2)$ corrections, known results:

$$F_2^{\text{TMC}}(x, Q^2) \approx \frac{x^2}{\xi^2 r^3} F_2^{(0)}(\xi, Q^2) \left[1 + \frac{6x\mu\xi}{r} (1 - \xi)^2 \right] \quad \xi = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2/Q^2}}$$

- **Higher twist** - more general class of $\mathcal{O}(1/Q^2)$ corrections. Models exist, but we choose to parameterise freely:

$$F_2^{\text{HT}}(x, Q^2) = F_2^{\text{LT}}(x, Q^2) \left(1 + \frac{D_i(x)}{Q^2} \right)$$

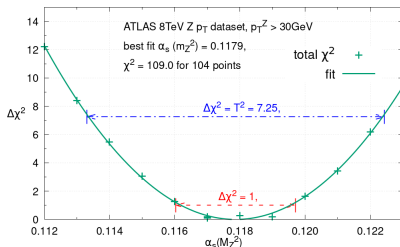
- In standard (“baseline”) MSHT we cut $Q^2 > 2 \text{ GeV}^2$, $W^2 > 15 \text{ GeV}^2$, here investigate lower cuts $W^2 > 5 \text{ GeV}^2 \Rightarrow$ also means more DIS data.

aN3LO	Baseline cuts, no TMC, no HT	lower cuts, no TMC no HT	lower cuts, TMC, no HT	lower cuts, TMC, HT	Baseline cuts, TMC, HT
Fixed Target	2201.1 (1.11)	2590.7 (1.15)	2511.1 (1.12)	2464.3 (1.10)	2116.0 (1.06)
Global	6214.7 (1.23)	6646.4 (1.26)	6526.4 (1.23)	6469.8 (1.22)	6121.7 (1.21)

- **Negligible TMC impact for standard cuts but needed for lower cuts.**
- **Higher twists** improve χ^2/N_{pts} even for standard cuts, mops up some MHO. **Limited PDF impact.** Harland-Lang, TC, Reader, Thorne, 2510.03753

MSHT20 ATLAS 8 TeV $Z p_T$ α_S dependence

- ATLAS 8 TeV $Z p_T$ data with $p_T^Z > 30$ GeV is in the MSHT PDF fit.
- What bounds does it offer within the global PDF fit on $\alpha_S(M_Z^2)$?

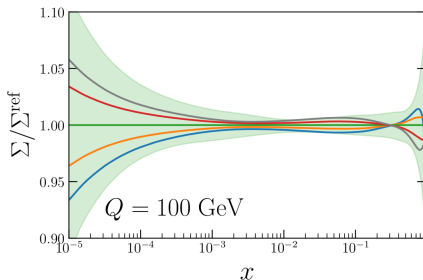
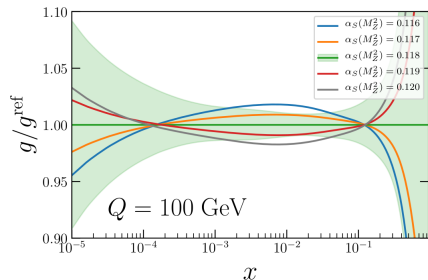


- If you do individual dataset extraction you use $\Delta\chi^2 = 1$ for bounds.
- If you do do in a global fit, factoring in tensions with other data you use $\Delta\chi^2 = T^2 = 7.25$ for bounds.
- $p_T^Z > 30$ GeV not very constraining on $\alpha_S(M_Z^2)$ in global PDF fit.
- ATLAS $Z p_T \alpha_S$ result used $p_T^Z < 29$ GeV part of spectrum. Used MSHT20 aN3LO PDFs to correspond to accuracy used in resummation.

MSHT20 PDF α_S dependence

Forte, Kassabov: 2001.04986

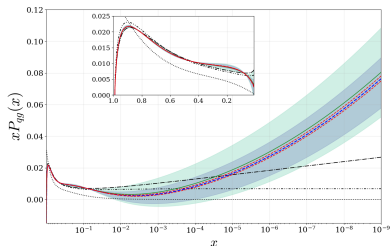
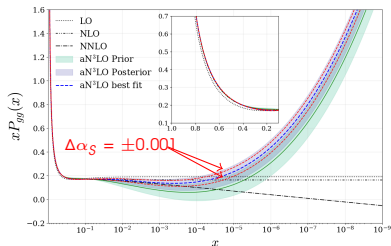
- Correlations between PDFs and $\alpha_S \Rightarrow$ necessity of global fit.



- Changes generally within PDF uncertainties for $\Delta\alpha_S(M_Z) \approx \pm 0.001$.
- Gluon anti-correlated with $\alpha_S(M_Z^2)$ for $x \lesssim 0.1$ as maintains $dF_2/dQ^2 \sim \alpha_S g$. Implies correlated at high $x \gtrsim 0.1$ by momentum sum rule.
- Larger effect at low Q^2 as less evolution distance.
- Smaller effects on quarks, reduced/increased at high/low x by splitting. s less impacted, at high x may absorb some of change.

MSHT20 aN3LO pieces α_S dependence

- Technical aside - we have theory “nuisance” parameters for the few unknown (at time of publication) theoretical ingredients at aN3LO.
 - How do the posteriors change with $\alpha_S(M_Z^2)$?
- Most change very little. Splitting functions show some change:

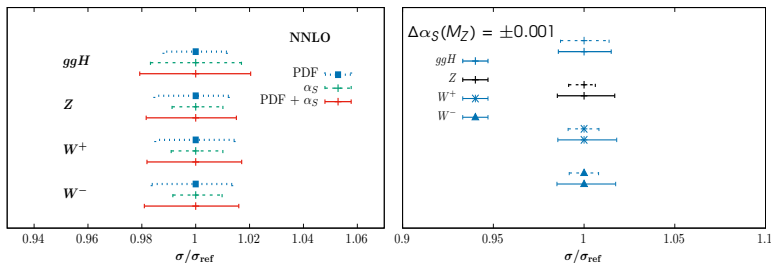


- Changes within uncertainties, well within for everything except P_{gg} - shows most change, as expected it's the most correlated with α_S .
- As seen in dataset χ^2 profiles, unknown aN3LO K-factors weaken α_S sensitivity. May obtain stronger bounds as these become known.

T.C., L.A. Harland-Lang, R.S. Thorne 2404.02964.

MSHT20 Cross-section PDF + α_S dependence

- Cross-section uncertainties at NNLO/aN3LO (left/right) at the LHC.

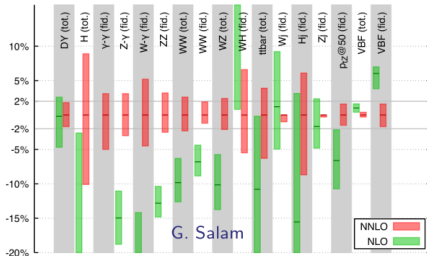


- Direct α_S uncertainty through xsec is small for DY. Total α_S sensitivity larger due to change of PDFs with α_S .
- Direct α_S uncertainty of ggF Higgs is larger ($\sim 2 - 3\%$), reduced by anti-correlation of gluon with α_S .
- Higher energies sample lower X quarks \Rightarrow larger α_S uncertainties.
- Interplay of direct and indirect (through PDFs) effects \Rightarrow importance of treating PDFs+ α_S together.

T.C., L.A. Harland-Lang, R.S. Thorne 2404.02964.

NLO and NNLO Cross-section Scale Variations

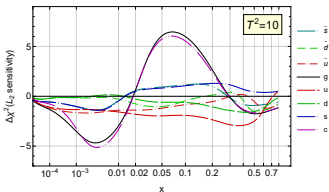
- For many processes NLO scale variations were not sufficient to incorporate NNLO result.



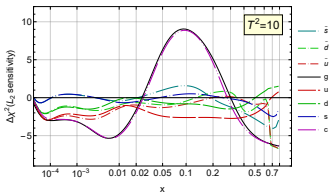
- Is there a better way to do this?

NNLO and aN3LO Data "Pulls" - L_2 Sensitivities - g

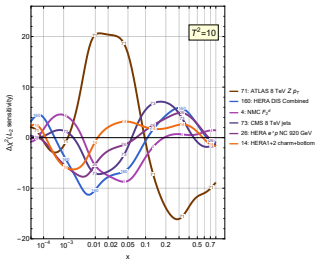
MSHT20 NNLO
CMS 8 TeV jets (73), Q=100 GeV



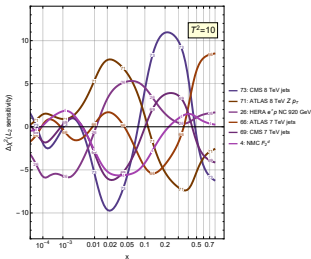
MSHT20 approx. N3LO
CMS 8 TeV jets (73), Q=100 GeV



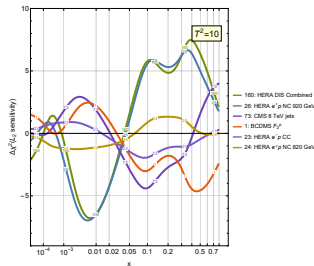
MSHT20 NNLO
g(x, 2 GeV)



MSHT20 approx. N3LO
g(x, 2 GeV)

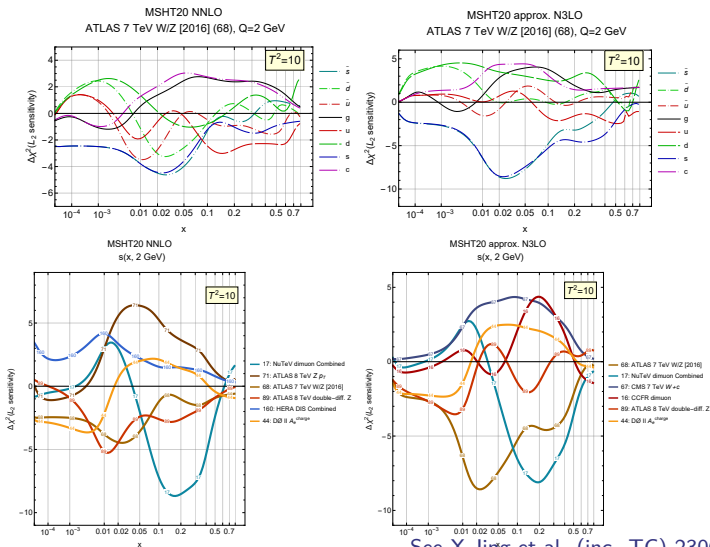


MSHT20 NNLO reduced
g(x, 2 GeV)



See X. Jing et al. (inc. TC) 2306.03918.

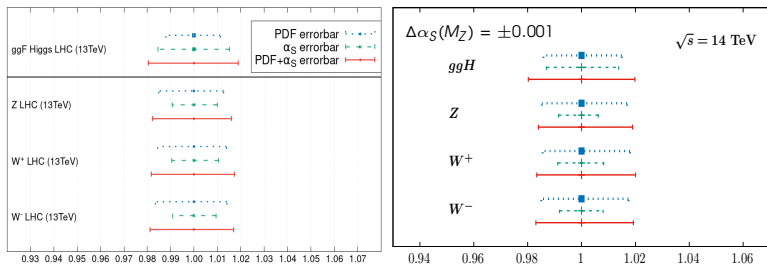
NNLO and aN3LO Data “Pulls” - L_2 Sensitivities -

 s^+


See X.Jing et al. (inc. TC) 2306.03918.

MSHT20 Cross-section PDF + α_S dependence

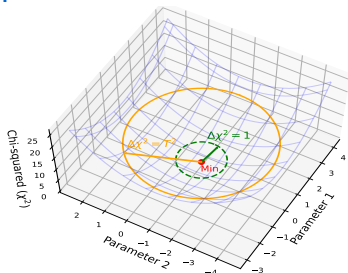
- Cross-section uncertainties at NNLO/aN3LO (left/right) at the LHC.



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Profiling PDFs with Tolerance:

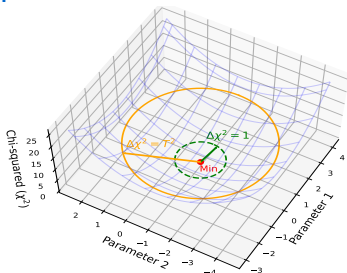
- Increasingly experimental collaborations profile PDFs:
 - See impact of new data on PDFs.
 - Extract SM parameters, e.g. $\alpha_S(M_Z)$.
- What this means is construct a joint χ^2 of new data + PDF:



$$\chi_{\text{total}}^2 = \overbrace{\sum_{i,j=1}^{N_{\text{new data}}} (D_i - E_i + \sum_k \Gamma_{ik}^{\text{PDF}} \theta_k^{\text{PDF}}) C_{ij}^{-1} (D_j - E_j + \sum_l \Gamma_{jl}^{\text{PDF}} \theta_l^{\text{PDF}})}^{\text{new data}} + \overbrace{\sum_k (\theta_k^{\text{PDF}})^2}_{\text{PDF constraints}}.$$

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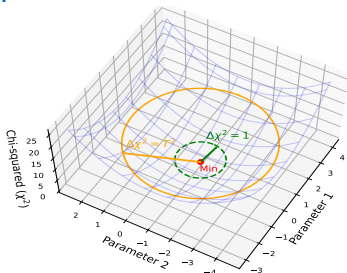


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- Issue - Last term $\sum_k (\theta_k^{\text{PDF}})^2$ represents χ^2 penalty to move PDF from its best fit, given by data constraints already in PDFs.
- But many PDFs (MSHT, CT) use a “tolerance” so $\theta_k^{\text{PDF}} = \pm 1$ means $\Delta\chi^2 \neq 1$

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- But many PDFs (MSHT, CT) use a “tolerance” so $\theta_k^{\text{PDF}} = \pm 1$ means $\Delta\chi^2 \neq 1$, but $\Delta\chi^2 = T^2 \Rightarrow$ need T_k^2 .

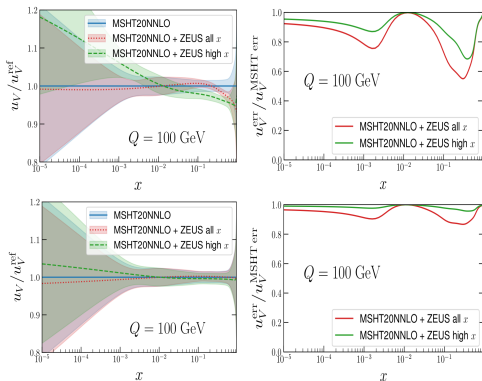
PDF Uncertainty - Tolerance in Profiling

- Consider PDF impact of high- x ZEUS data, $T = 1 \rightarrow$ clearly overestimates pull (left column) and uncertainty reduction (right column) relative to $T = \text{dyn}$.

L.A. Harland-Lang, T.C., M.
Reader, R.S. Thorne: 2510.03753

- Profiling also relies on two assumptions:

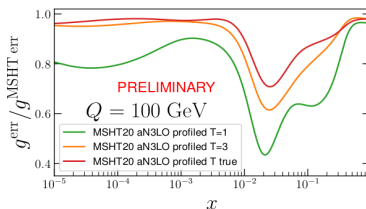
- PDF uncertainties, representing constraints of previous data in PDFs, are consistently included alongside the new data.
- The PDFs are not altered significantly (i.e. central values moved $\lesssim 1\sigma$ and uncertainties not substantially changed).



Profiling PDFs and extracting $\alpha_S(M_Z)$

- Same occurs in extracting $\alpha_S(M_Z)$ from Zp_T .

(Use Asimov setup as in TC, G. Marinelli, F.J. Tackmann 2506.13874)



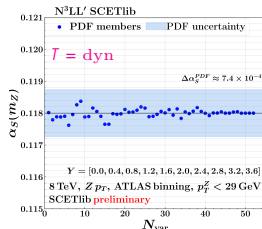
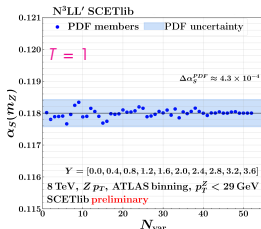
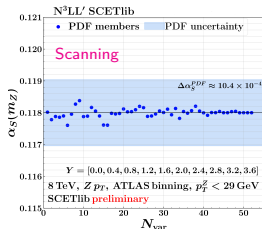
- Gluon constrained by Zp_T data, though overestimated with $T = 1$.
- Underestimated profiled PDF uncertainty propagates into α_S PDF uncertainty \Rightarrow **Must include tolerance when profiling PDFs.**
- Challenge: How to include this effect without including a tolerance also in α_S uncertainty for new data (as is done in global PDF α_S extractions where aim is instead to extract combined $\alpha_S(M_Z)$).

$$\chi_{\text{total}}^2 = \sum_{i,j=1}^{N_{\text{new data}}} (D_i - E_j(\alpha_S) + \sum_k \Gamma_{ik}^{\text{PDF}} \theta_k^{\text{PDF}}) C_{jj}^{-1} (D_j - E_j(\alpha_S) + \sum_l \Gamma_{jl}^{\text{PDF}} \theta_l^{\text{PDF}}) + \sum_k^{N_{\text{ev}}} T_k^2 (\theta_k^{\text{PDF}})^2.$$

PDF Uncertainty - Tolerance in Profiling

Preliminary!

- How can I propagate this to α_s uncertainty? Suggestion (WIP):
- Two-step profiling:**
 - 1 First profile PDFs with factor of T^2 and $\Delta\chi^2 = T^2$ to correctly account for existing PDF uncertainties \Rightarrow obtain **consistently profiled PDFs**.
 - 2 Then perform fit with $\Delta\chi^2 = 1$ for other uncertainties and obtain PDF uncertainty by scanning profiled PDFs as new input set.



- In terms of PDF uncertainty:
 $T = 1 \leq$ Two-step profiling $T = dyn \leq$ Offset/scanned input PDFs.