Small Radius Inclusive Jet Production through NNLO+NNLL QCD

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Structure of inclusive jet cross sections

- Single-inclusive jet cross section as a function of anti-k_T R
- At small R, $\ln^n R$ shows up to all orders:

$$\sigma = c_{0,0} + \alpha_s(c_{1,1}\ln R + c_{1,0}) + \alpha_s^2(c_{2,2}\ln^2 R + c_{2,1}\ln R + c_{2,0}) + \mathcal{O}(\alpha_s^3)$$

- These logs can slow down or even spoil series convergence
- Resummation: include logs to all orders at certain accuracy
- LL: $\alpha_s^n \ln^n R$, NLL: $\alpha_s^n \ln^{n-1} R$, NNLL: $\alpha_s^n \ln^{n-2} R$, ...
- Also large logs from soft emissions
- We will resum logs of R, but not soft/threshold logs

Previous work: a rough timeline of dijet/inclusive jet

- Threshold resummation (NLO+NLL)
  Kidonakis, Sterman (1997); Kidonakis, Oderda, Sterman (1998);
  Laenen, Oderda, Sterman (1998); Kidonakis, Owens (2000)

- $\ln R$ resummation (NLO+LL, NLO+NLL):
  Dasgupta, Dreyer, Salam, Soyez (2014, 2016);
  Kang, Ringer, Vitev (2016); Dai, Kim, Leibovich (2016)

- Fixed-order (NNLO):
  Currie, Glover, Pires (2016); Currie et al. (2017, 2018);
  Bellm et al. (2019); Czakon, van Hameren, Mitov, Poncelet (2019);
  Chen, Gehrmann, Glover, Huss, Mo (2022)

- Threshold+$\ln R$ resummation (NLO+$\text{NLL}_{\text{thresh.}}$+$\text{NLL}_{\ln R}$):
  Liu, Moch, Ringer (2017, 2018);
  Moch, Eren, Lipka, Liu, Ringer (2018)

Why do we care?

Example of single-inclusive jet production at NNLO

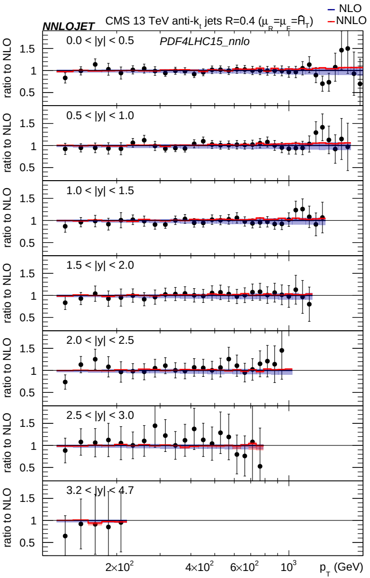
For this example:

- Small NNLO corrections ✓
- Reduced error band ✓
- Th. error $<$ exp. error ✓
- NNLO within NLO band ✓

FO NNLO seems sufficient for $R = 0.4$ (similar for $R = 0.7$)

Theory precision already exceeds experimental precision

So why bother with resummation?



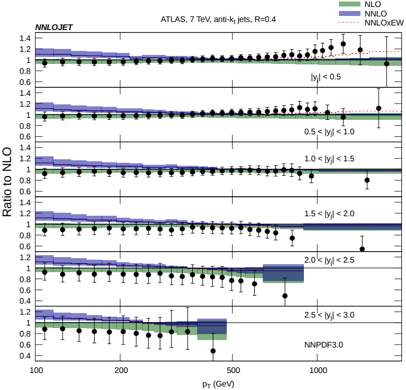
Plot taken from arXiv:1807.03692

Another example at NNLO

For this example:

- Small NNLO corrections ?
- Reduced error band ✗
- Th. error < exp. error ✗
- NNLO within NLO band ✗

So what changed?

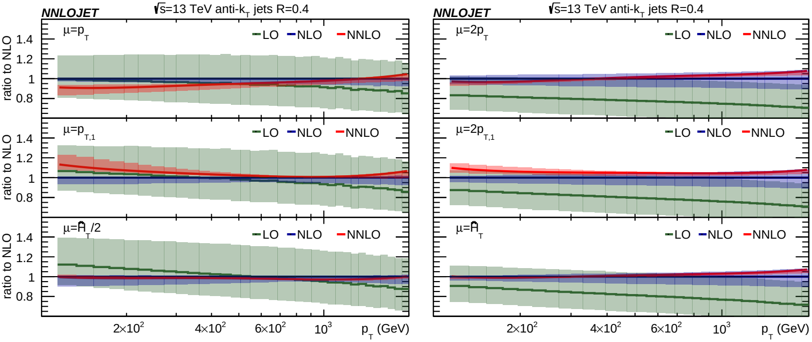


Plot taken from arXiv:1611.01460

Scale choice matters

Quality of apparent convergence depends strongly on scale choice!

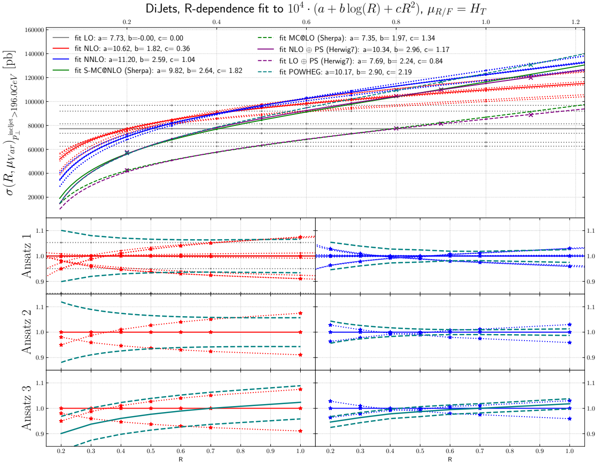
So does a ‘good’ choice solve the problem? Or is this an illusion?



Plot taken from arXiv:1807.03692

Scale uncertainties as a function of R

Accidental cancellations lead to unrealistic uncertainties



Plot taken from arXiv:1903.12563

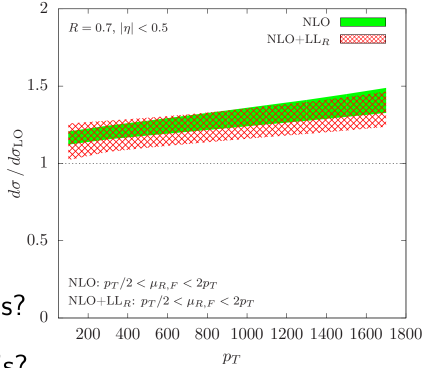
Resummation helps

NLO+LL yields larger
(i.e. more realistic)
uncertainties than FO NLO

⇒ Resummation is potential solution

- Does this hold for higher orders?
- Does this hold for all ‘reasonable’ R’s?
- What about ‘unreasonable’, small R’s?
  Surely, resummation should reduce scale uncertainties there.

Plot taken from arXiv:1606.06732



Now (hopefully) everyone cares.
How do we do NNLO+NNLL?

Factorisation

- As $R \to 0$, single-inclusive cross section develops large logs $\ln^m R$
- In this limit, the cross section factorises: Lee, Moult, Zhang (2024)

$$\frac{d\sigma_{\text{jet}}}{dp_T} = \sum_{i=g,u,\bar{u},\ldots} \int_0^1 \frac{dz}{z} \frac{d\sigma_i}{dp_T}(p_T/z, \mu_J) J_i\left(z, \ln\frac{p_T R}{z\,\mu_J}\right) + \mathcal{O}\left(R^2 \ln^m R\right)$$

- This is (almost) identical to fragmentation:

$$\frac{d\sigma_h}{dp_T}(p_T) = \sum_{i=g,u,\bar{u},\ldots} \int_0^1 \frac{dz}{z} \frac{d\sigma_i}{dp_T}(p_T/z, \mu_J) D_{i\to h}(z, \mu_{Fr}) + \mathcal{O}\left(\frac{\Lambda^n_{\text{QCD}}}{p_T^n}\right)$$

- J_i is the 'FF' for producing a jet with radius R from parton i
- $\Rightarrow$ Swap out e.g. pion FF for 'jet FF' to get jet p_T spectra
- All R-dependence contained in the jet functions

Resummation

- All R-dependence contained in the jet functions
- $\Rightarrow$ Only need to resum logs in the jet functions
- As for FFs, the jet functions satisfy DGLAP equations:

$$\frac{dJ_i}{d\ln\mu^2}\left(z, \ln\frac{p_T^2 R^2}{z^2\mu^2}, \mu\right) = \int_z^1 \frac{dy}{y} P_{ij}^{\mathrm{T}}(y) J_j\left(\frac{z}{y}, \ln\frac{y^2 p_T^2 R^2}{z^2\mu^2}, \mu\right)$$

- Follow standard procedure:
  1. Evaluate the J_i at $\mu \sim R\, p_T \Rightarrow$ No large logs
  2. Evolve to $\mu \sim p_T$ using DGLAP $\Rightarrow$ Resums logs of R
  3. Combine evolved J_i with partonic cross sections at $\mu \sim p_T$
- Match to FO to include power corrections in R through NNLO

NNLO computations with STRIPPER

- STRIPPER framework: Monte Carlo code for the numerical computation of fully differential NNLO cross sections

  Czakon (2010, 2011); Czakon, Heymes (2014); Czakon, van Hameren, Mitov, Poncelet (2019)

- Fully general: only process-specific part: two-loop amplitudes

- In particular already used to compute jet cross sections at NNLO

  Czakon, van Hameren, Mitov, Poncelet (2019)

- Extended to support fragmentation a few years ago

  Czakon, TG, Mitov, Poncelet (2021)

- Any process with any number of identified hadrons supported!

- Today: replace FFs$\rightarrow$jet functions $\Rightarrow$ small-R resummation!

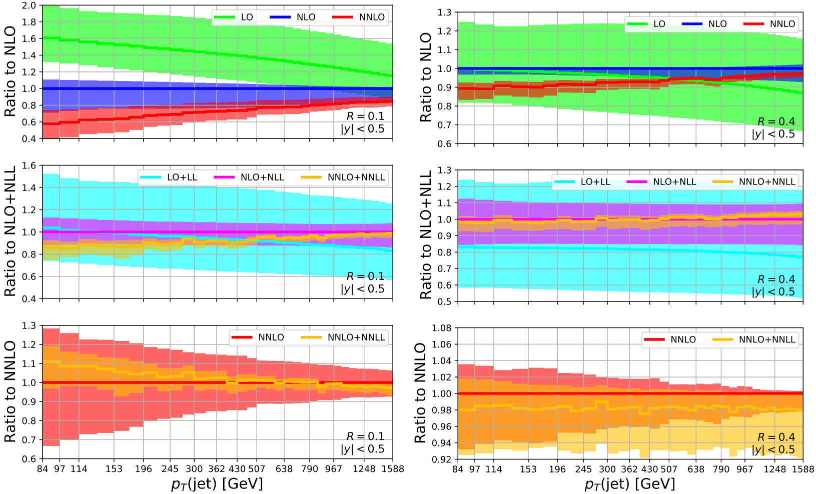
- Can now convolve hard functions with arbitrary 1D distributions

The measurement

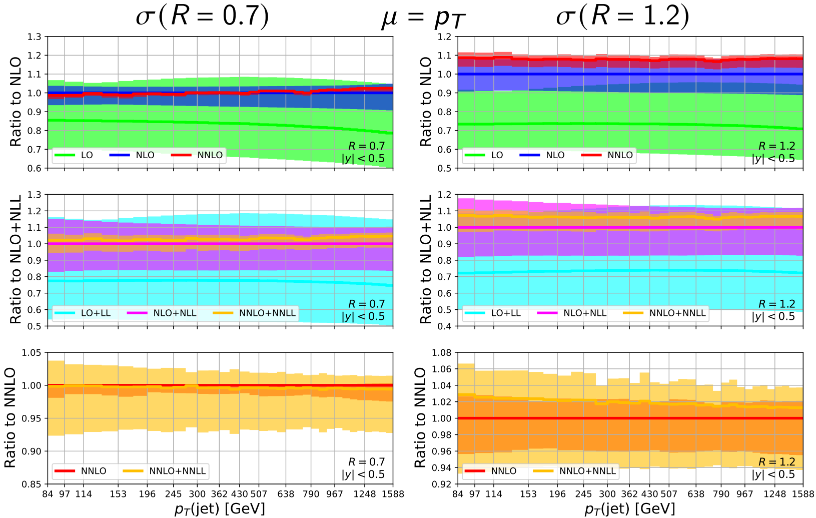
- Ultimately want to compare to data
- arXiv:2005.05159: '3D' measurement of inclusive jets by CMS
- Double-differential in p_T and y for $R = 0.1, 0.2, ..., 1.2$
- Absolute spectra not provided; only ratio's w.r.t. $R = 0.4$
- Will use same binning and cuts to facilitate comparison
- Non-perturbative effects very large - must be included!
- NP corrections provided by CMS (obtained using parton showers)
- Important additional source of uncertainty

Results: cross sections at 13 TeV LHC

$\sigma(R = 0.1)$ $\mu = p_T$ $\sigma(R = 0.4)$

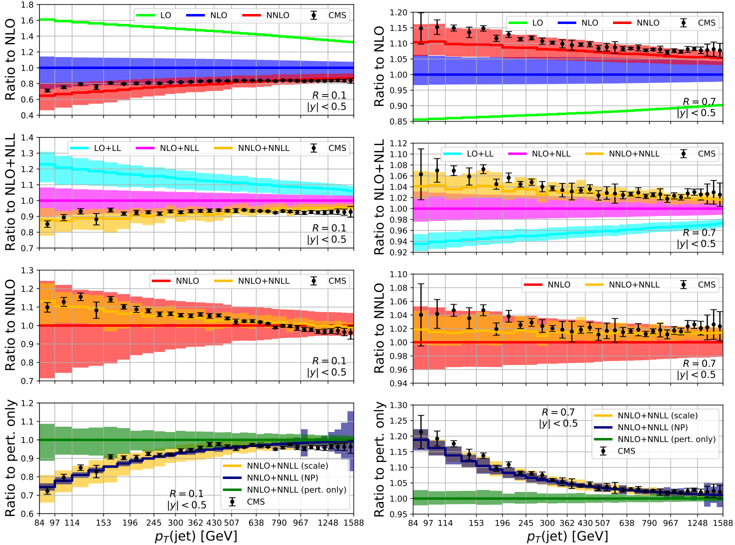


Results: cross sections at 13 TeV LHC



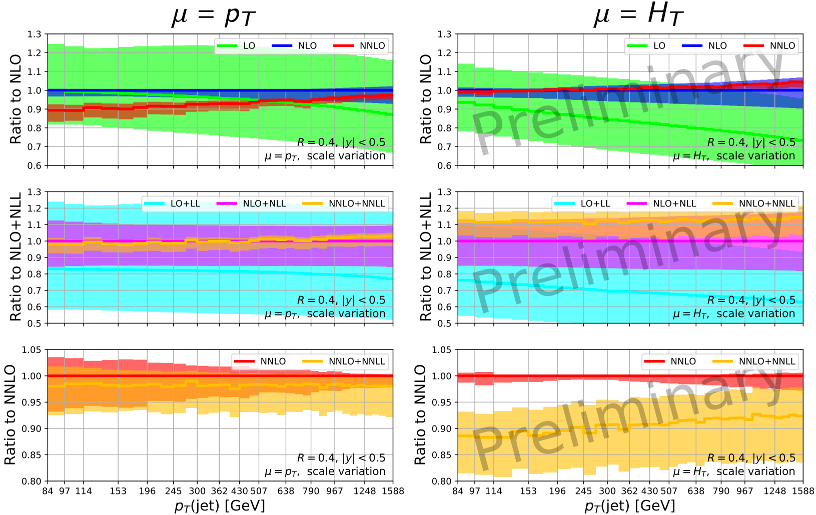
Results: cross section ratios at 13 TeV LHC

$\sigma(R=0.1)/\sigma(R=0.4)$ $\mu = p_T$ $\sigma(R=0.7)/\sigma(R=0.4)$

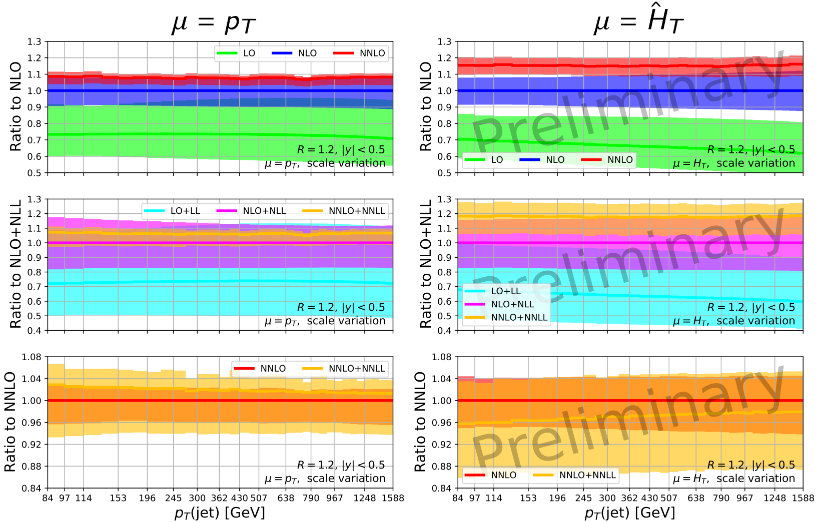


What does this look like for $\mu = \hat{H}_T$?

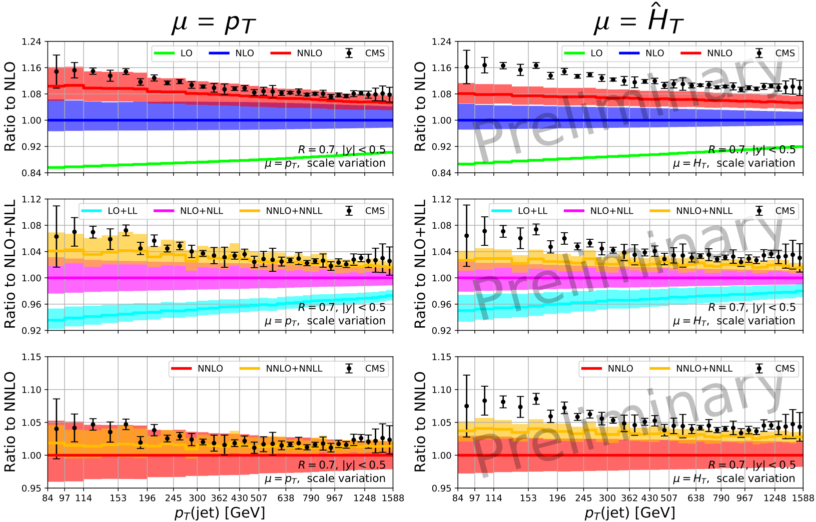
Scale choice p_T vs. $\hat{H}_T$: absolute spectra for $R = 0.4$



Scale choice p_T vs. $\hat{H}_T$: absolute spectra for $R = 1.2$



Scale choice p_T vs. $\hat{H}_T$: $\sigma(R=0.7)/\sigma(R=0.4)$



So resummation gives more reliable uncertainties, but this might still not be good enough.

And what if I cannot (or do not want to) do resummation?

An alternative to scale uncertainties

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Beyond Scale Variations: Perturbative Theory Uncertainties from Nuisance Parameters

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arXiv:2411.18606

Theory nuisance parameters

- Suppose we have an anomalous dimension γ:

$$\gamma = \sum_{i=0}^{\infty} \gamma_i \left(\frac{\alpha_s}{\pi}\right)^{i+1}$$

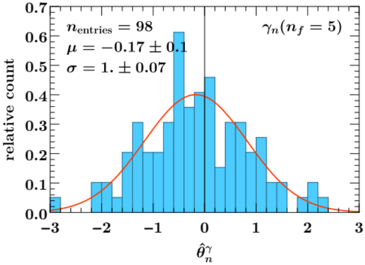
- First term proportional to C_F (quarks) or C_A (gluons)
- At leading colour: each power of α_s comes with another C_A
- $\Rightarrow$ Define new coefficients $\hat{\gamma}_n$:

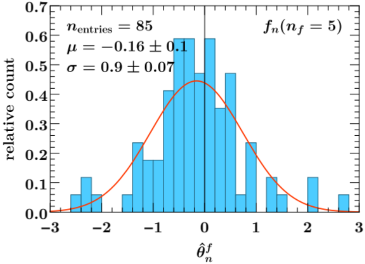
$$\gamma_n \equiv C_i C_A^n \hat{\gamma}_n$$

- Expect $\hat{\gamma}_n$ to be $\mathcal{O}(1)$
- How are these $\hat{\gamma}_n$ distributed across all known series?

Theory nuisance parameters

- $\hat{\gamma}_n$ normally distributed!
- Similar observation for matrix-element constants f_n (extra factor n!)
- Also holds for each order separately
- Frank's idea: estimate uncertainty on scalar series by treating next order as 0 ± 1 (after proper normalisation)





Histograms taken from arXiv:2411.18606

Theory nuisance parameters

- Cross sections are not scalar series
- Dependence on kinematics, involves squared quantities, summing over spin, colour, partonic channels, ...
- But: can often decompose problem into scalar series in some approximation
- Here: threshold expansion: Kidonakis, Sterman (1997); Laenen, Oderda, Sterman (1998); ...

$$\frac{d\sigma}{dp_T} = f_a \otimes f_b \otimes J_c(R) \otimes \frac{d\hat{\sigma}_{ab\to c}}{dp_T}\,, \qquad g(N) \equiv \int_0^1 x^{N-1} g(x)\, dx$$

$$\frac{d\hat{\sigma}_{ab\to c}}{dp_T} = \Delta_a \otimes \Delta_b \otimes \Delta_c \otimes j_d \otimes \mathrm{Tr}[\boldsymbol{H}_{ab\to cd}\boldsymbol{S}] + \mathcal{O}\left(\frac{1}{N}\right)$$

- Here: Mellin moment $N \gtrsim 5 \Rightarrow$ Good enough approximation!

Theory nuisance parameters

- Threshold expansion:

$$\frac{d\hat{\sigma}_{ab\to c}}{dp_T} = \Delta_a \otimes \Delta_b \otimes \Delta_c \otimes j_d \otimes \mathrm{Tr}[\boldsymbol{H}_{ab\to cd}\boldsymbol{S}] + \mathcal{O}\left(\frac{1}{N}\right)$$

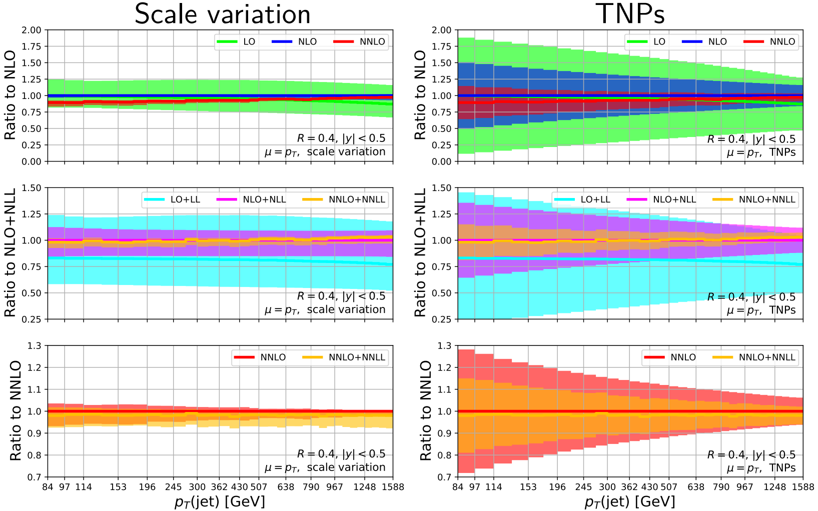
- All ingredients can be written in terms of scalar series, e.g.:

$$\Delta_a(N_{0a}) = R_a\big(\alpha_s(\mu)\big) \times \exp\left[\int_{\sqrt{\hat{s}}/N_{0a}}^{\mu} \frac{d\mu'}{\mu'}\left(A_a(\alpha_s(\mu'))\ln\left(\frac{\mu'^2 N_{0a}^2}{\hat{s}}\right) - \frac{1}{2}D_a\big(\alpha_s(\mu')\big)\right)\right]$$

- Threshold expansion of jet function similar
- $\Rightarrow$ Can apply idea of theory nuisance parameters!

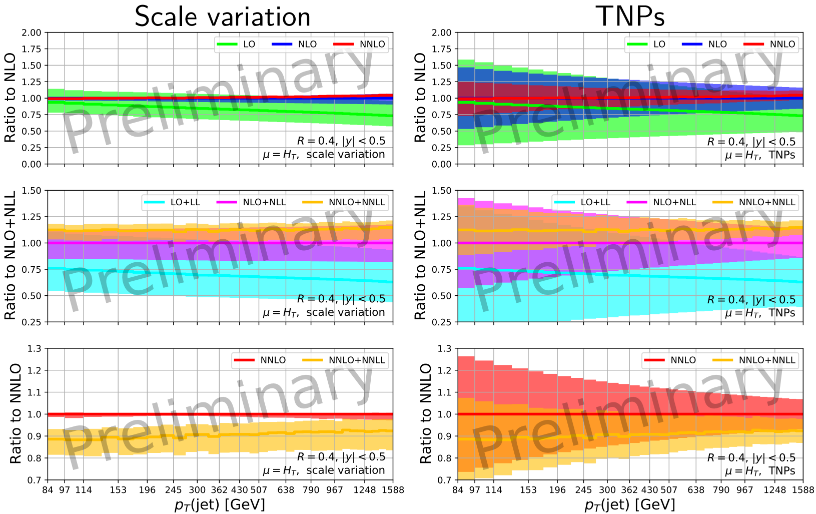
Theory nuisance parameters vs. scale variation

$\mu = p_T$

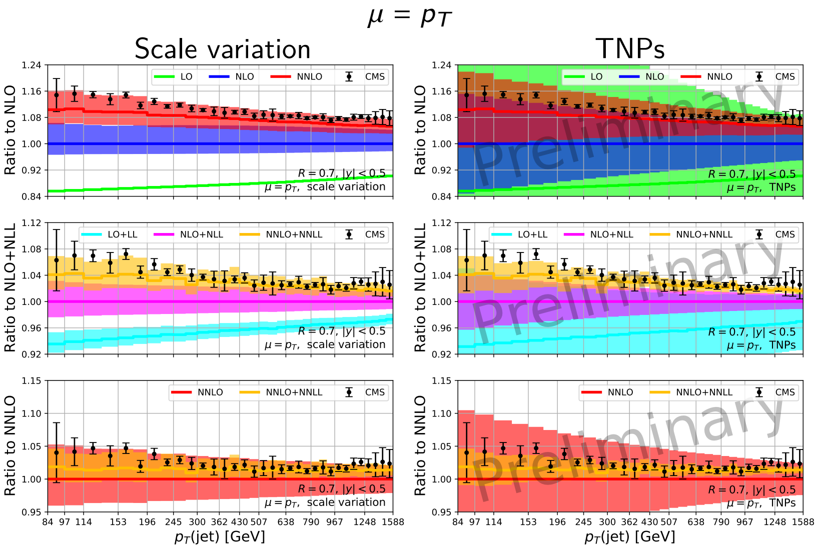


Theory nuisance parameters vs. scale variation

$$\mu = \hat{H}_T$$

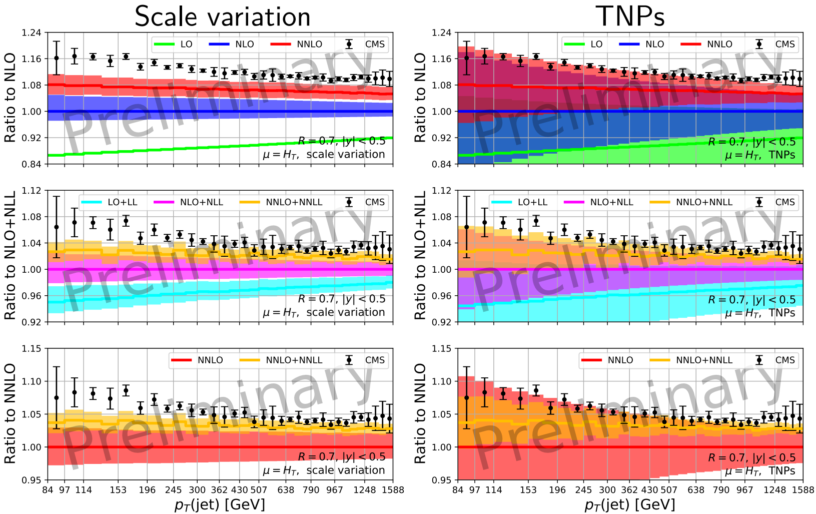


Theory nuisance parameters vs. scale variation



Theory nuisance parameters vs. scale variation

$$\mu = \hat{H}_T$$

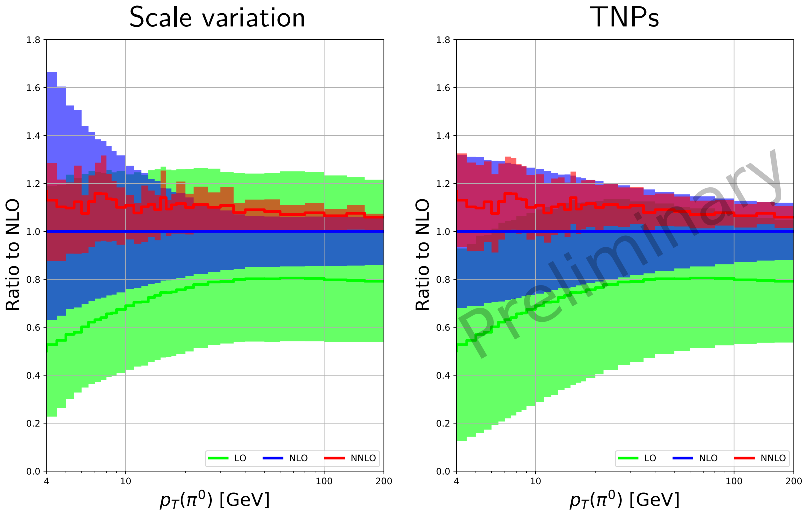


Boost confidence in TNP errors

$\Longrightarrow$ Apply approach to other processes

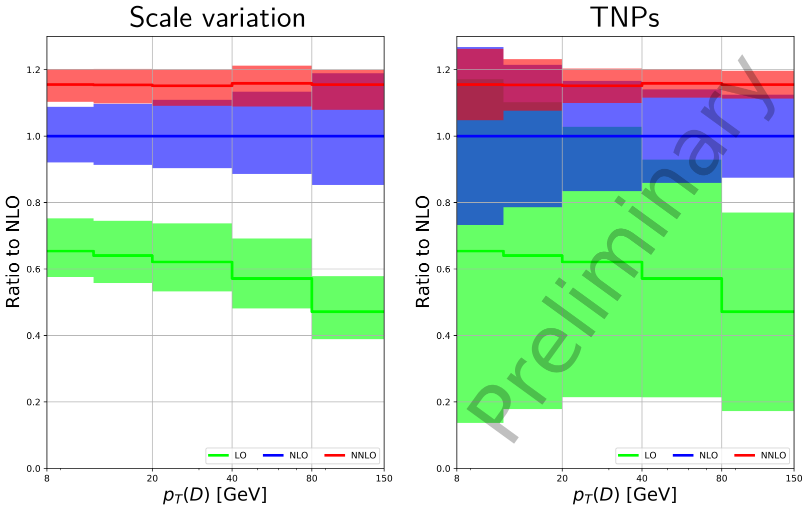
Check 1: π^0 p_T spectrum at the 8 TeV LHC

Based on the results of arXiv:2503.11489 Czakon, TG, Mitov, Poncelet



Check 2: $D^{\pm}$ p_T spectrum for $p\,p \longrightarrow W^{\pm} D^{\mp}$ at 13 TeV

Based on the results of arXiv:2510.24525 TG, Poncelet, Muškinja



Conclusion

- First NNLO+NNLL calculation of small radius jets at the LHC
- Reduced or more reliable uncertainties w.r.t. FO NNLO
- Better agreement with data w.r.t. both FO NNLO and NLO+NLL
- TNPs provide even more robust uncertainties
- TNP uncertainties similar to scale uncertainties for resummed results
- Checked reliability of TNP approach in other processes
- TNPs allow proper statistical interpretation, correlations, etc.
- $\Rightarrow$ Can be taken into account in e.g. PDF fits

Backup

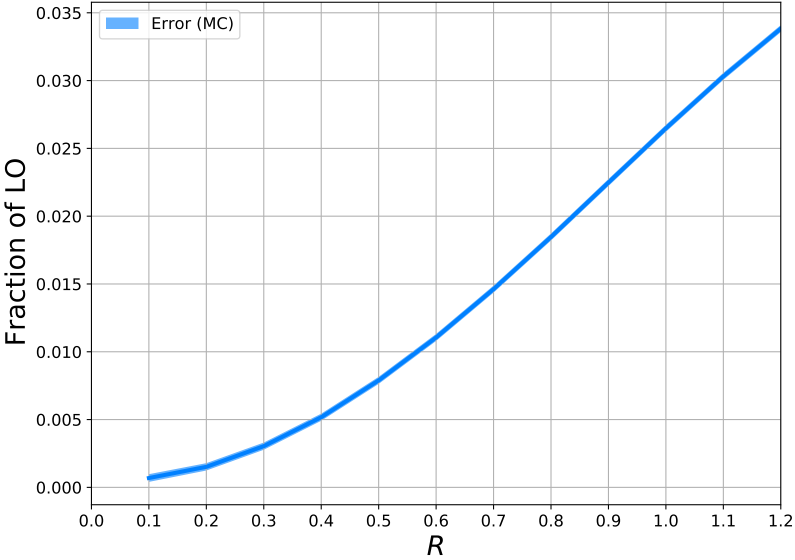
Perturbative power corrections

- Factorisation valid up to power corrections:

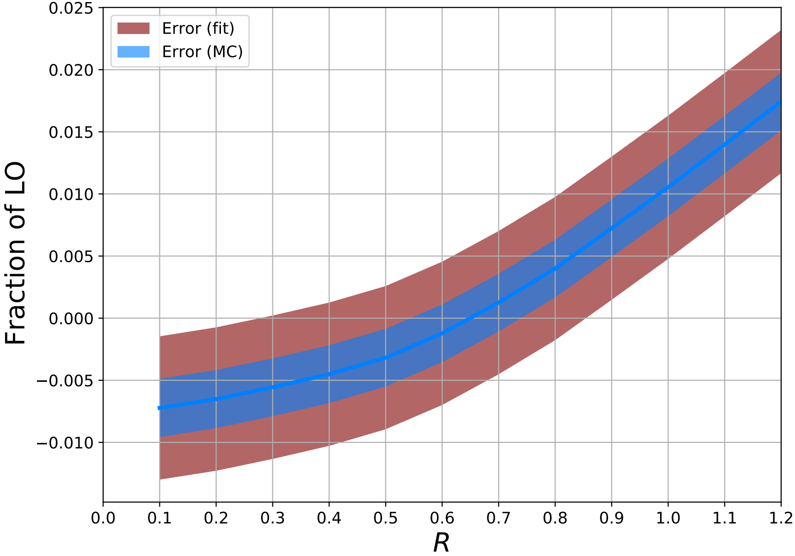
$$\frac{d\sigma_{\text{jet}}}{dp_T} = \sum_{i=g,u,\bar{u},\ldots} \int_0^1 \frac{dz}{z} \frac{d\sigma_i}{dp_T}(p_T/z, \mu_J) J_i\left(z, \ln \frac{p_T R}{z\,\mu_J}\right) + \mathcal{O}(R^2 \ln^m R)$$

- How big are they?
- Can they safely be neglected beyond FO?
- When can they even be neglected at FO?
- Is $R = 0.4$ 'small'? What about $R = 0.7$?

Perturbative power corrections at NLO



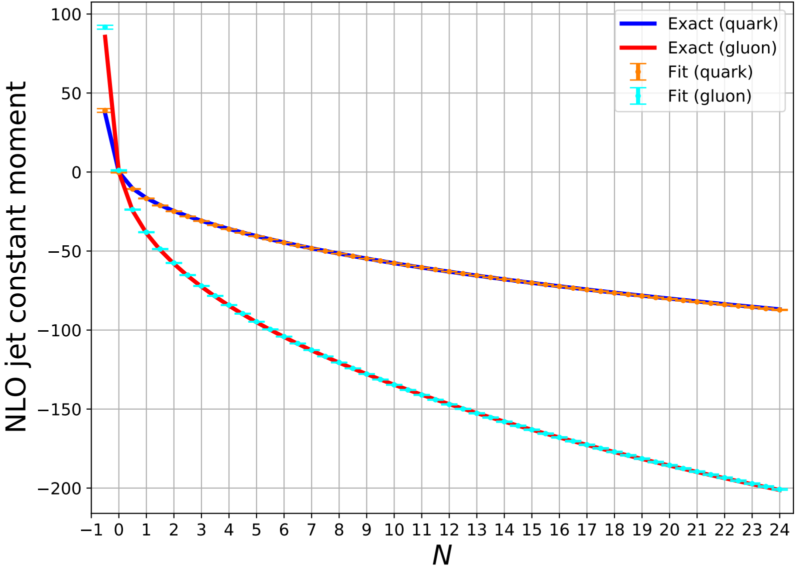
Perturbative power corrections at NNLO



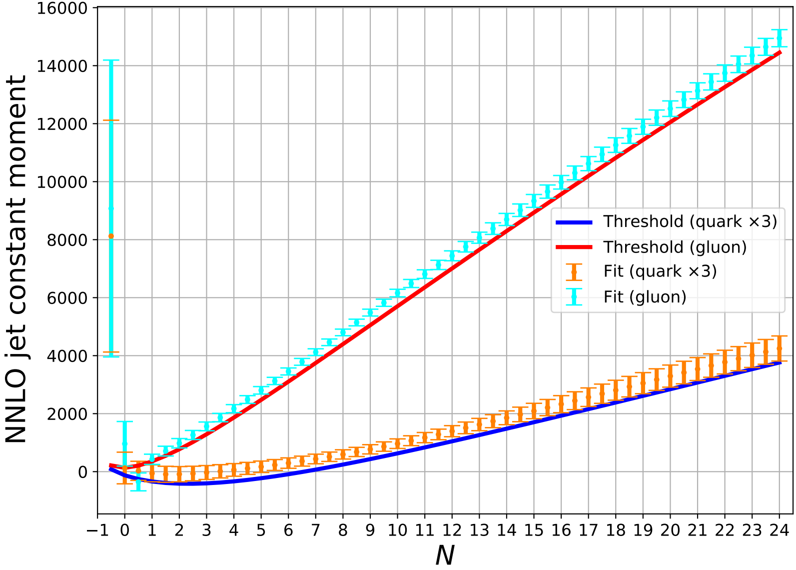
NNLO jet constant

- NNLL part of NNLO jet functions, i.e. $\mathcal{O}(\ln^0 R)$ not known
- But: can compute exact, fixed-order NNLO cross section
- Extract unknown terms by comparing exact and factorised result!
- In practice: cross section moments double-differential in y and $\hat{H}_T$
- Also split up cross section according to initial-state partons
- Allows to disentangle quark and gluon-initiated jet very well
- Computed at $R = 0.1$, power corrections found to be negligible
- Obtained the first 50 half-integer moments of both J_q and J_g

Cross-check: NLO jet constant



Result: NNLO jet constant



Approach

- DGLAP evolution performed by truncating at high order
- Converges well and gives precise control over included terms
- Matching trivial: σ = (exact NNLO) + (LP beyond NNLO)
- Requires convolutions with many different distributions
- In practice: α_s^5 for LL and NLL and α_s^4 for NNLL terms
- $\Rightarrow$ Need convolutions with $\left(\frac{\ln^5(1-x)}{1-x}\right)_+$
  $\Rightarrow$ Need very robust and stable code
- STRIPPER generalised to support arbitrary distributions