

Precision theory predictions for top and heavy flavour at colliders

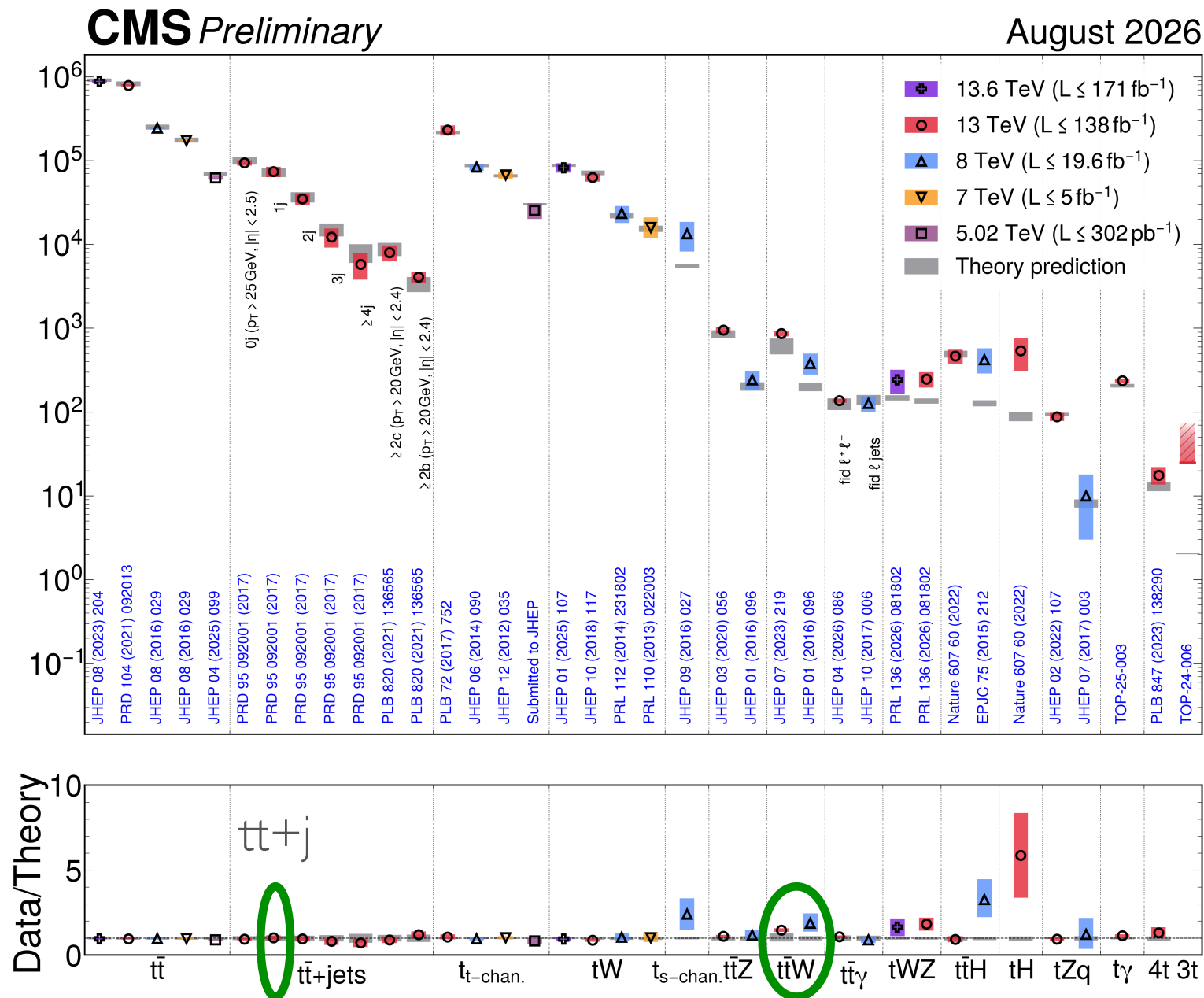
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QCD@LHC
Oxford, 30th September 2026

FARE
RICERCA IN ITALIA
FRAMEWORK PER L'ATTRAZIONE E IL RAFFORZAMENTO
DELLE ECCELLENZE PER LA RICERCA IN ITALIA

Precision physics with massive quarks



key SM parameters

$$m_t, \alpha_s, y_t, y_b$$

initial state quark PDFs

top interacting with W,Z,H, γ

Higgs vacuum stability

not just the top...

bb, cc, Wbb, Zbb, Hbb

Reliable theoretical predictions

We need to ensure theory predictions have **reliable uncertainties** and can be evaluated **efficiently across the full phase space**

[see also Signorile-Signorile's talk]

will need at least NNLO QCD - **fully differential**

may even need N3LO in some cases (e.g. tt)

recent examples for precision
top/heavy flavour physics

NNLO: ttj and ttW

NNLO+PS : bbH, ttH

approximations are always necessary

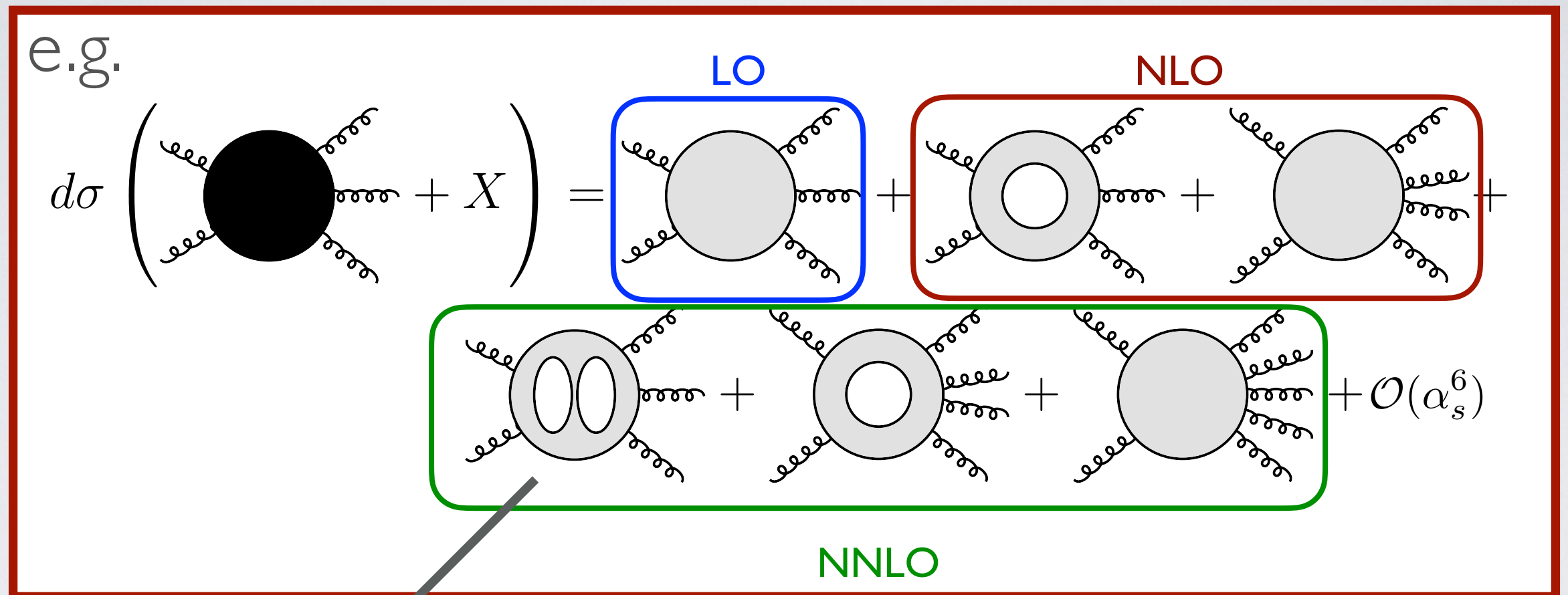
full mass effects?

full colour effects?

full spin effects (decays)?

Aim for today: see how these computations have been done to understand where the complexity is coming from and what we can do to improve

Theory challenges at NNLO



often missing

$$\mathcal{A} = C.I$$

**maturing IR
subtraction methods**

STRIPPER, Antenna, nested-soft, ...

State of the art for multiscale two-loop corrections

remarkable progress in recent years for many multiscale processes

$$\mathcal{A} = C.I$$



$$\mathcal{F} = c.f$$

finite remainder

$$pp \rightarrow 3j, pp \rightarrow 3\gamma, pp \rightarrow \gamma\gamma j, pp \rightarrow \gamma jj, pp \rightarrow W+2j, \\ pp \rightarrow Wbb, pp \rightarrow Hbb, pp \rightarrow W\gamma\gamma$$

Abreu, Agarwal, Badger, Buccioni, Chawdhry, Chicherin, Czakon, de Laurentis, Febres-Cordero, Gambuti, Gehrmann, Henn, Lo Presti, Klinkert, Kuschke, Manteuffel, Ma, Mitov, Page, Peraro, Poncelet, Schabinger, Sotnikov, Tancredi, Zhang, ...

tensor reduction to master integrals

integration-by-parts identities [Chetykin, Tkachov] [Laporta]
solve over **finite fields** e.g. FiniteFlow, FireFly, FIRE, Kira
optimized systems (Blade, NeatIBP etc.)

analytic reconstruction

extract rational coefficients through multiple **finite field**
evaluations (in parallel) [Thiele interpolation]
optimize with linear relations, partial fractioning etc.

master integral expansions

identity function basis via **differential equations** for master integrals
canonical 'ε'-factorized form (Henn)

optimized numerical evaluation

fast one-form integral representation (e.g. **pentagon functions**)

Integration by parts: The algorithm to calculate β -functions in 4-loops, Chetykin, Tkachov Nucl.Phys.B 192 (1981) 159-204

technology also relevant $3L \ 2 \rightarrow 2, 4L \ 2 \rightarrow 1, \dots$

State of the art for multiscale two-loop corrections

finite field methods for amplitudes

$$\mathcal{F} = c.f$$

↑

IBPs: Schabinger, Manteuffel (2014)

Amplitudes: Peraro (2016, 2019)

- floating point arithmetic $\rightarrow$ large cancellations = loss of precision
- modular arithmetic $\rightarrow$ exact, no loss of precision

$$-3(\bmod 5) = 2 \qquad \frac{1}{2}(\bmod 97) = 49$$

- rational function reconstruction: through repeated evaluations modulo a set of (large) prime numbers the polynomial coefficients in numerator and denominator can be determined exactly (using chinese remainder theorem)
- number of evaluations grows fast with the number of scales / complexity of the amplitude

public tools: FiniteFlow, FireFly,...

FiniteFlow uses modular graph structure

State of the art for multiscale two-loop corrections

differential equations

Kotikov (1996) Gehrmann, Remiddi (2000)

$$\mathcal{F} = c.f \uparrow$$

- integrals are closed under IBPs:

$$dI = M(\epsilon, x).I$$

Henn (2013)

- one may find a particular basis such that

$$dI = \epsilon M(x).I \quad (\text{Canonical form})$$

- if in addition $M(x) = \sum_w \text{dlog}(w)$ it is possible to expand to multiple polylogarithms
 $\Rightarrow$ path to indep. basis with good numerical evaluation
- in the general non-dlog case we may encounter Elliptic curves (genus 1), hyperelliptic curves (genus > 1), K3 surfaces, higher-dimensional Calabi–Yau varieties, ...

this is the case for most amplitudes with internal masses

$\Rightarrow$ no clear path to good numerical evaluation

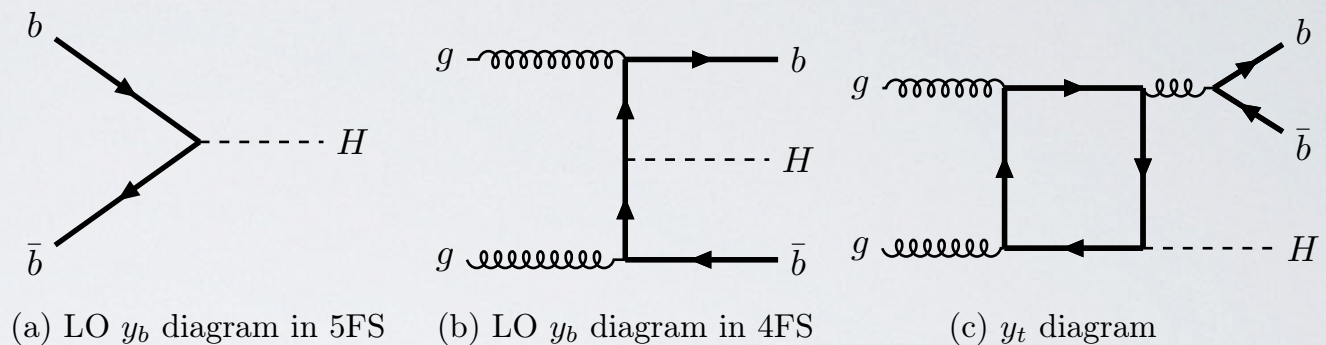
Two-loop corrections to bbH

Biello, Mazzitelli, Sankar, Wiesemann, Zanderighi [JHEP 04 \(2025\) 088](#)

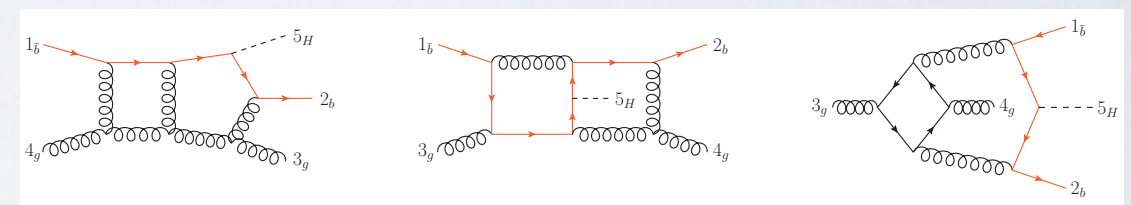
y_b, y_t , background to HH

NNLO+PS
via MINNLO_{PS}

[see talks by Biello, Savoini ($t\bar{t}H$)]

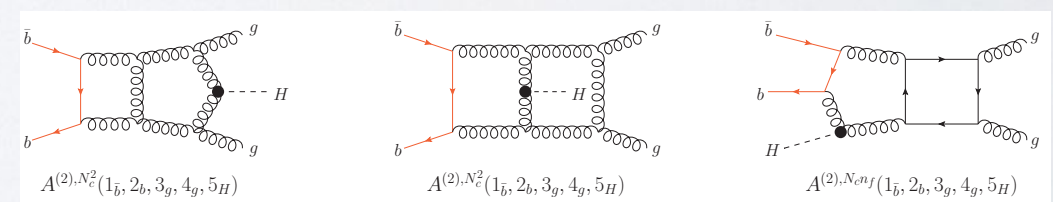


full colour corrections computed in
massless b limit (non-zero y_b)



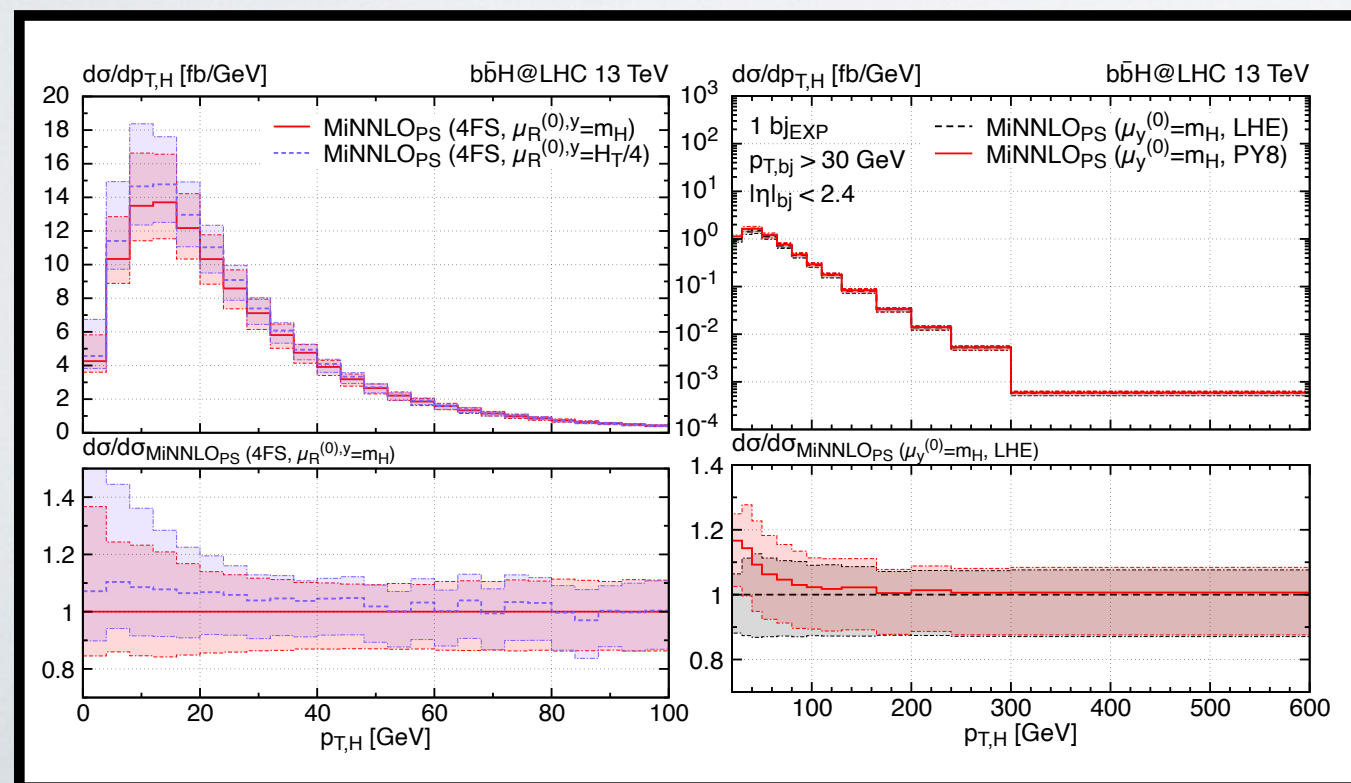
SB, Hartanto, Poncelet, Wu, Zhang, Zoia [JHEP 03 \(2025\) 066](#)

potential improvements for y_t dependence
using H -gluon coupling at $m_t \rightarrow \infty$

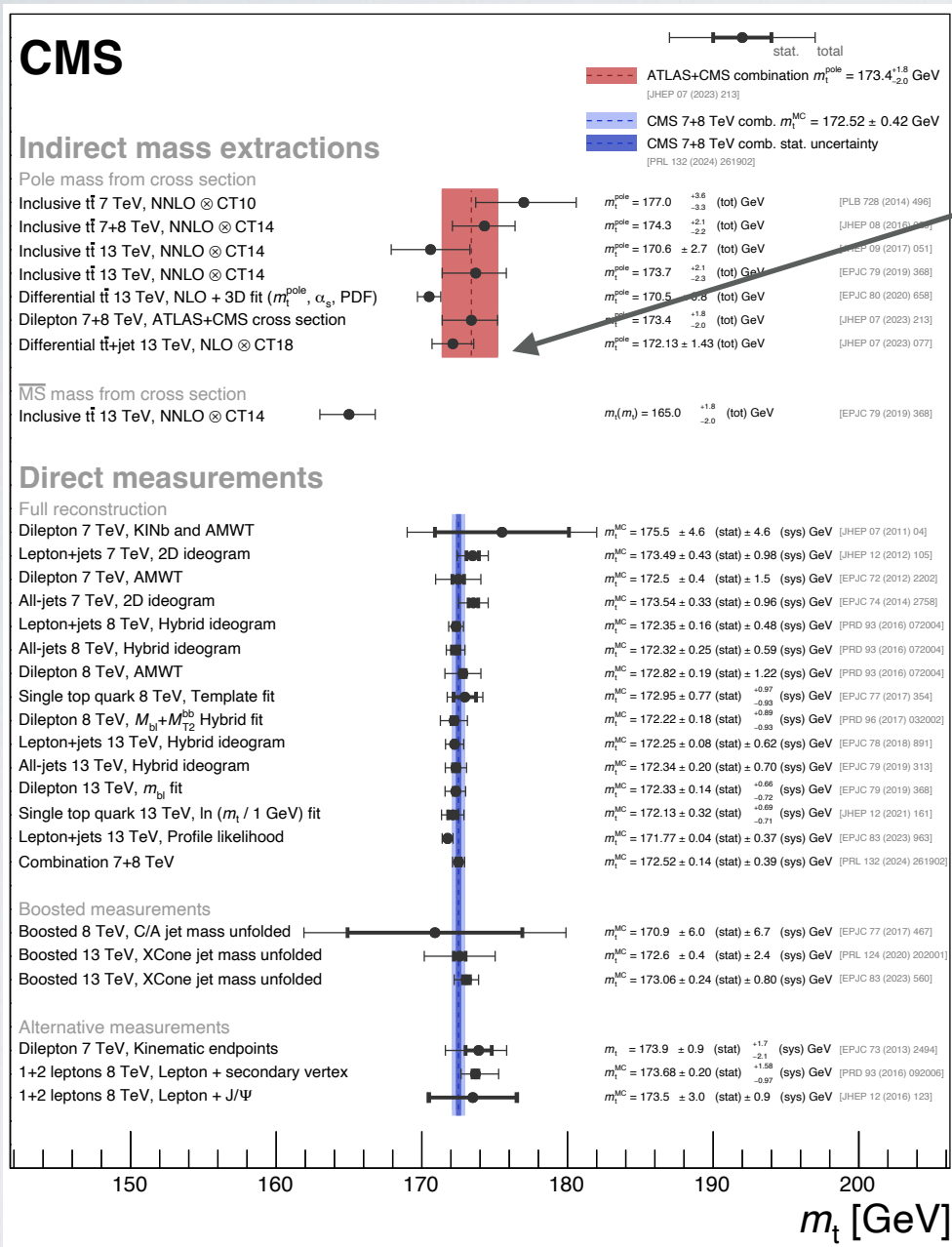


Hartanto, Poncelet [SciPost Phys. 21 \(2026\) 022](#)

c.f. H2j De Laurentis et al. [JHEP 07 \(2026\) 251](#)



Two-loop corrections to tt+jet



m_t from
normalised $t\bar{t}$
distributions

$$\mathcal{R}(m_t^{\text{pole}}, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{-jet}}} \frac{d\sigma_{t\bar{t}+1\text{-jet}}(m_t^{\text{pole}}, \rho_s)}{d\rho_s} \quad \rho_s = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}}$$

$$S(\rho_s) = \sum_{\Delta=\pm 5-10 \text{ GeV}} \frac{|\mathcal{R}(170 \text{ GeV}, \rho_s) - \mathcal{R}(170 \text{ GeV} + \Delta, \rho_s)|}{2|\Delta|\mathcal{R}(170 \text{ GeV}, \rho_s)}$$

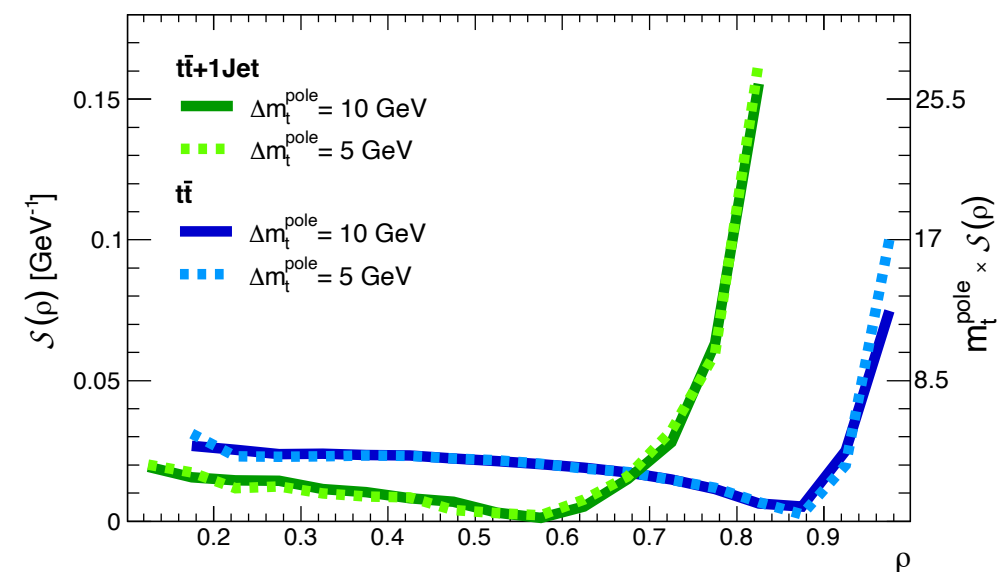


Fig. 5. The sensitivity $S(\rho_s)$ of $\mathcal{R}$ with respect to the top-quark mass as defined in Eq. (5).

Alioli et al. [1303.6415](#)

mass extraction
summary [2403.01313](#)

$pp \rightarrow t\bar{t}j$ NNLO
step towards $pp \rightarrow t\bar{t}$
@ N3LO

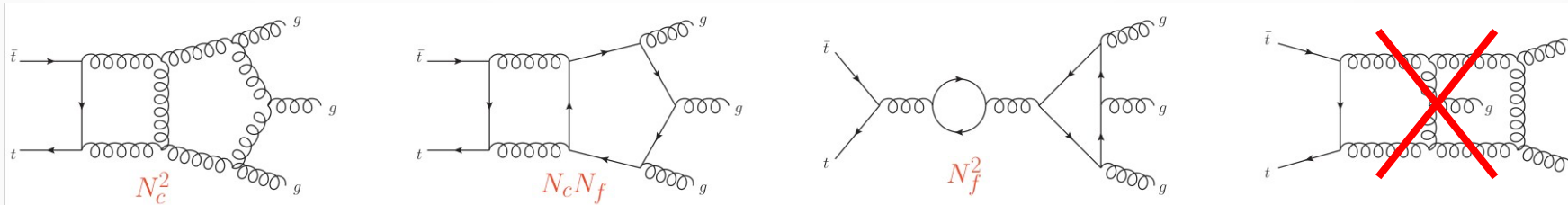
Two-loop corrections to tt+jet

Leading colour amplitude

SB, Becchetti, Brancaccio,, Hartanto, Zoia [JHEP 03 \(2025\) 070](#)

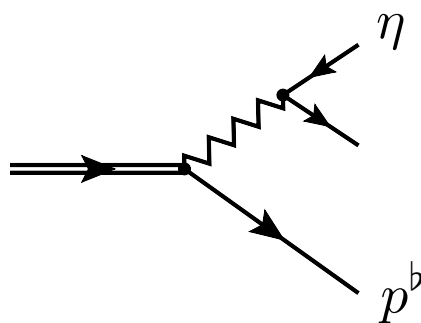
SB, Becchetti, Brancaccio, Czakon, Hartanto, Poncelet, Zoia [JHEP 05 \(2026\) 044](#)

$$\mathcal{A}_{\text{LC}}^{(L)}(1_{\bar{t}}, 2_t, 3_g, 4_g, 5_g) = \sqrt{2} \bar{g}_s^3 n^L \sum_{\sigma \in Z_3} (t^{a_{\sigma(3)}} t^{a_{\sigma(4)}} t^{a_{\sigma(5)}})^{\bar{i}_1}_{i_2} A^{(L)}(1_{\bar{t}}, 2_t, \sigma(3)_g, \sigma(4)_g, \sigma(5)_g)$$



diagrams	tot	LC
tree-level	16	6
1-loop	384	77
2-loop	11370	1357

helicity amplitudes encode
spin correlations in the
narrow width approximation



$$\overline{u}_{\pm}(p, m; n) = \frac{\langle \eta \mp | (\not{p} + m) | \eta \mp | p^b \pm \rangle}{\langle \eta \mp | p^b \pm \rangle}$$

e.g. Melnikov, Schulze '09

4D projectors [Tancredi, Peraro '19,'20]

see also 1L ttH [Buccioni, Kreer, Liu, Tancredi '23]

$$\mathcal{A}^{(L)h_3 h_4 h_5} = \sum_{i=1}^4 \Gamma_i \mathcal{G}_i^{(L)h_3 h_4 h_5}$$

$$\mathcal{G}_i^{(L)h_3 h_4 h_5} = \sum_{i=1}^4 (\Gamma^\dagger \Gamma)_{ij}^{-1} [\Gamma_j^\dagger \mathcal{A}^{(L)}]^{h_3 h_4 h_5}$$

“contracted helicity amplitude”

$\Gamma_1 = m_t^2 \bar{u}_2 v_1$

$\Gamma_2 = m_t \bar{u}_2 \not{p}_3 v_1$

$\Gamma_3 = m_t \bar{u}_2 \not{p}_4 v_1$

$\Gamma_4 = \bar{u}_2 \not{p}_3 \not{p}_4 v_1$

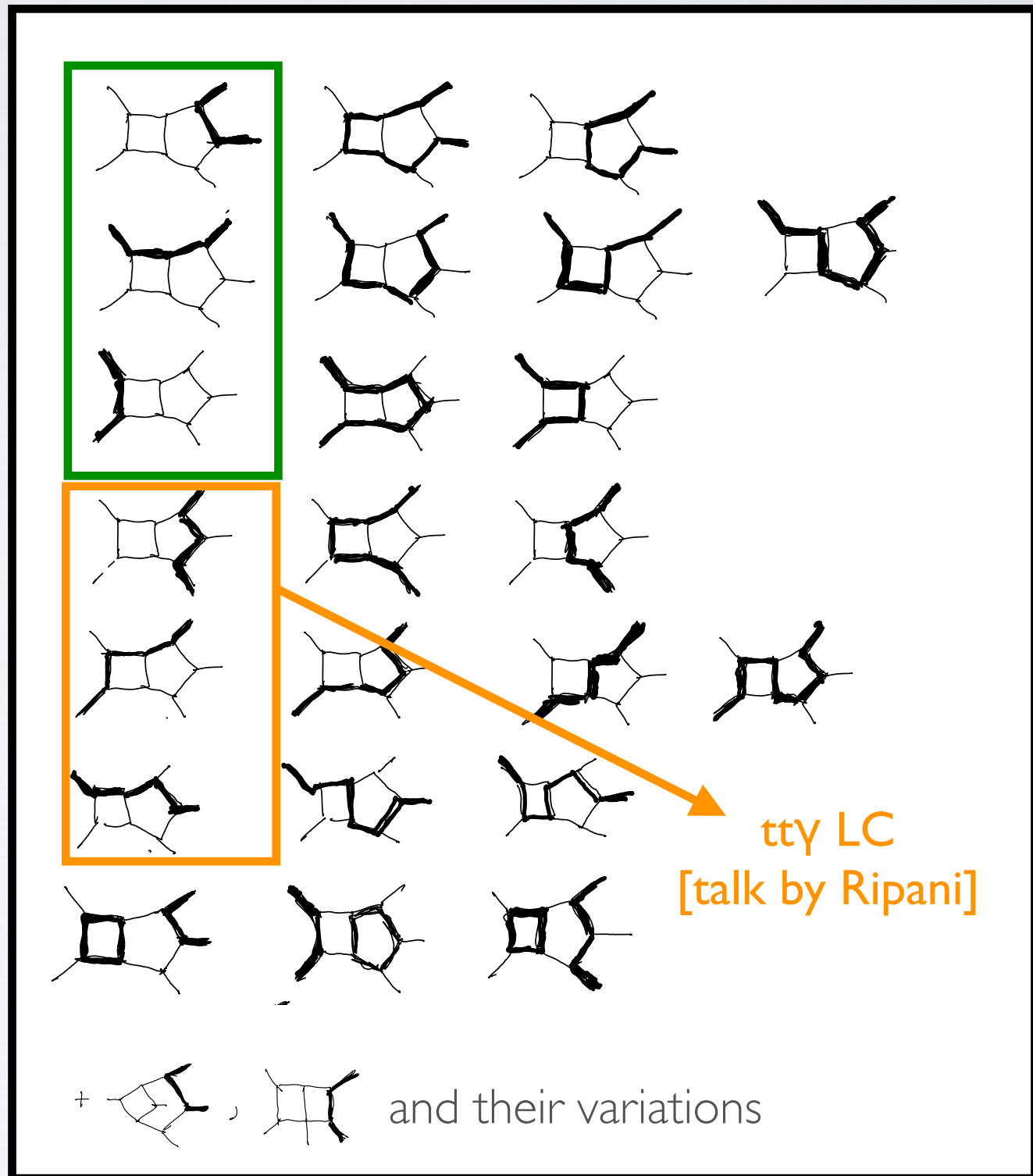
easy to square amplitude or attach decays

Two-loop corrections to $t\bar{t}$ +jet

The full colour decomposition would require a large number of integral topologies each with 'complicated geometries'

Leading colour requires only three

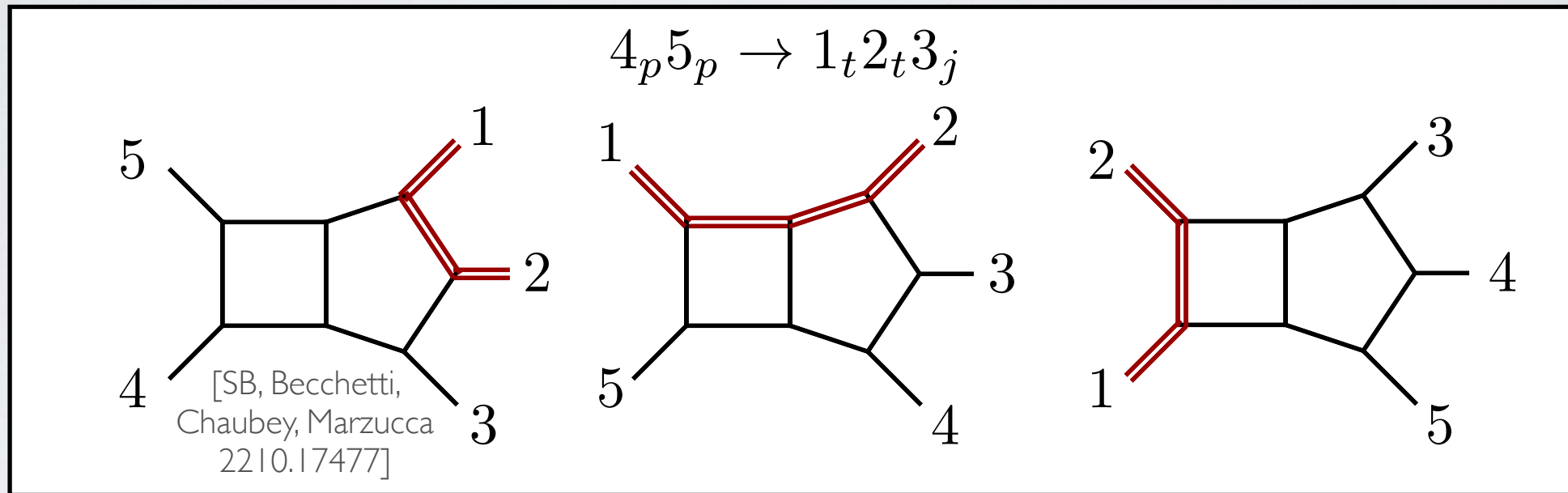
c.f. 5-point massless with 1 external scale (H_{jj}) has 8 for full colour



Two-loop corrections to $t\bar{t}$ +jet

Integrals and DEQs @ LC

SB, Becchetti, Giraudo, Zoia [2404.12325](#)



88 MIs

dlog ✓

74 letters

121 MIs

dlog ✗

109 MIs

dlog ✓

79 letters

high precision boundary values with Auxiliary Mass Flow (AMFLOW Liu '22)

Two-loop corrections to $t\bar{t}$ +jet

Identifying a special function basis for the finite remainder

SB, Brancaccio, Becchetti, Hartanto Zoia [JHEP 03 \(2025\) 070](#)

$$k_{\max}=2$$
$$dA^{(B)}(\vec{x}, \varepsilon) = \sum_{k=0}^{k_{\max}} \sum_i \varepsilon^k \omega_i(\vec{x}) c_{k,i}^{(B)}$$

while not ε -factorised, the non dlog element only appear from $O(\varepsilon)$ in the expansion of the master integrals

In the end only 12 master integral components are associated with non dlog forms and we obtain an (over-complete) **basis of special functions defined by their differential equation**

find non dlog ‘one-forms’

further rotation to ε -factorised form
introduces elliptic kernels

Becchetti, Dlapa, Zoia [Phys.Rev.D 112 \(2025\)](#)

New ideas coming out about ε -factorised forms for general geometries, e.g.

Duhr et al. [\[2602.04371\]](#) [talk by Forner]

Forner et al. [\[2604.25270\]](#)

Applications to analytic amplitudes: heavy quark loops in di-photons, di-jets and γ +j

Becchetti et al. [\[2502.00118\]](#),
Coro et. al [\[2509.15315\]](#)

Two-loop corrections to $t\bar{t}$ +jet

Efficient numerical evaluation of the function basis?



generalized series expansions

e.g. Liu, Ma, Wang [1711.09572] Moriello [1907.13234]

automated codes

DiffExp, AMFlow, LINE, SeaSyde

General but not always efficient

Chebyshev Approximations

Liu et al. [2606.26691]

Abreu et al. [2607.02411]

Runge-Kutta methods (e.g.
ODEINT Boost Library)

Bulirsch-Stoer shows
promising performance

Czakon, Niggetiedt 2001.03008

Czakon, Harlander, Klappert, Niggetiedt 2105.04436

Rosàs, Bobadilla 2507.12548

$t\bar{t}j$ (square roots) $\longrightarrow$ Czakon et al. [2511.11424]

arbitrary geometries $\longrightarrow$ Czakon, Tancredi [2606.30354]

$\searrow$ Baur, Duhr 2606.02744

Two-loop corrections to $t\bar{t}$ +jet

analytic reconstruction

	1	2	3	4
master integral coefficients of mass-renormalised amplitude (full ϵ dependence)	404/393	398/389	411/402	421/411
special function coefficients of finite remainder	314/303	305/296	321/312	326/317
linear relations	291/280	287/278	299/293	304/299
denominator matching #1	291/0	287/0	299/0	304/0
partial fraction decomposition in x_{5123}	44/40	55/51	57/54	58/54
denominator matching #2	44/0	54/0	54/0	56/0
number of sample points (1 prime field)	137076	89624	161482	179838

sequence of steps
implemented as a
FiniteFlow graph

partial fractioning in
momentum twistor variables
gave substantial reduction of
polynomial degree

~ overall computing time 2-3 months, memory intensive

Two-loop corrections to tt+jet

float type boundary	requested $ \Delta Q $	achieved $ \Delta Q $	maximal $ \Delta f_i/f_i $	number of steps	number of calls	call time	time
double $\vec{d}_{01}$	10^{-10}	8.2×10^{-10}	1.4×10^{-6}	17	716	0.77 ms	0.56 s
double $\vec{d}_{02}$	10^{-10}	8.2×10^{-10}	1.4×10^{-6}	28	1184	0.73 ms	0.89 s
dd_real $\vec{d}_{01}$	10^{-10}	7.7×10^{-11}	1.3×10^{-6}	16	709	9.3 ms	6.7 s
dd_real $\vec{d}_{02}$	10^{-10}	7.7×10^{-11}	1.3×10^{-6}	27	1166	9.3 ms	11 s
dd_real $\vec{d}_{01}$	10^{-20}	3.8×10^{-21}	1.2×10^{-16}	42	3002	9.0 ms	27 s
dd_real $\vec{d}_{02}$	10^{-20}	3.8×10^{-21}	1.2×10^{-16}	68	4873	9.0 ms	44 s

special function basis evaluation performs well using Bulirsch-Stoer approach

good control of numerical accuracy, high precision rescue system

stable enough BUT rational coefficients are extremely complicated

quadruple precision often necessary for the coefficients - basis functions much quicker

Channel	Functions [s]	Coefficients [s]	Assembly [s]	total [s]
$gg \rightarrow \bar{t}tg$	2.69	30.90	1.58	35.17
$\bar{q}g \rightarrow \bar{t}t\bar{q}$	2.16	9.40	0.18	11.74
$qg \rightarrow \bar{t}tq$	2.50	9.62	0.21	12.33
$q\bar{q} \rightarrow \bar{t}tg$	2.12	9.30	0.18	11.60

code available at DOI [10.5281/zenodo.17541754](https://doi.org/10.5281/zenodo.17541754)

tt+jet @ NNLO

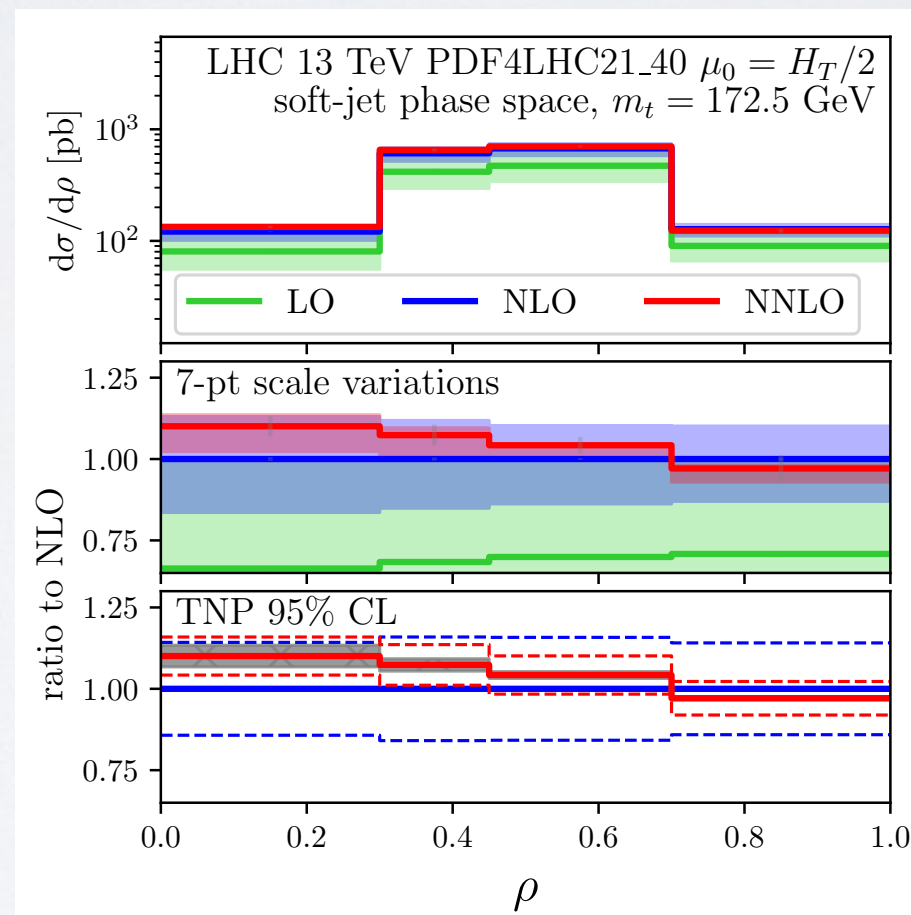
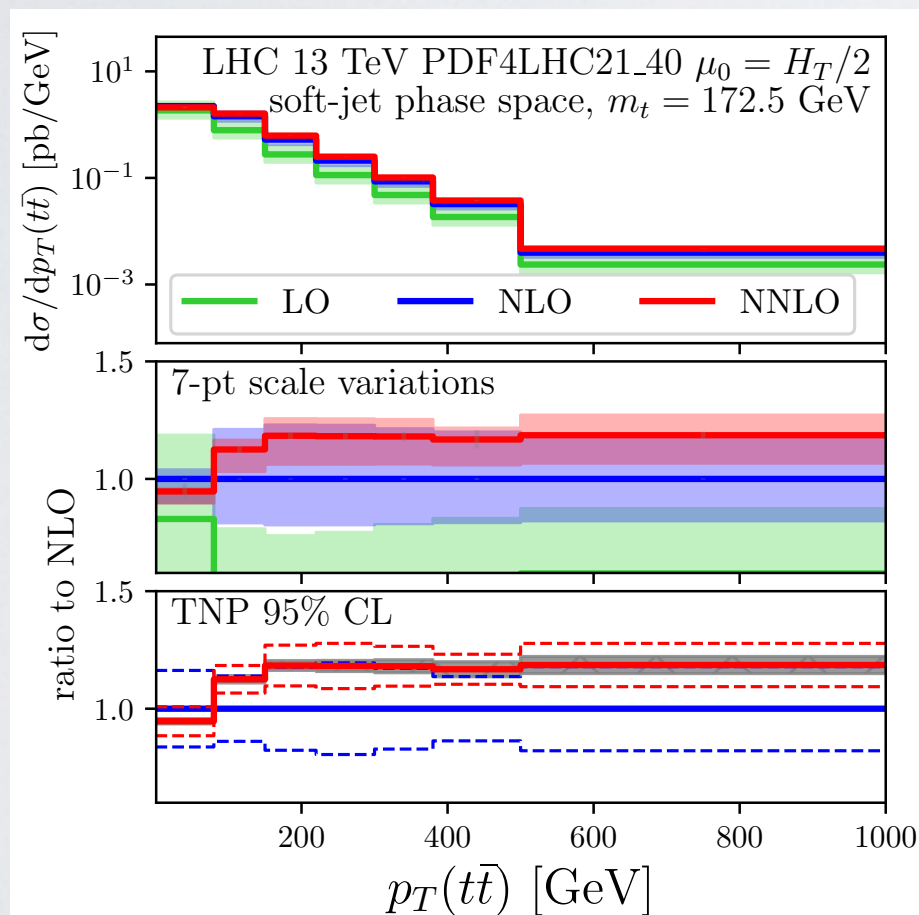
1st full simulation: on-shell tops

IR subtraction using STRIPPER

OpenLoops2 and AvH amplitude libraries

$$F_{\text{l.c. resc.}}^{(2)}(\mu^2) = \frac{F^{(0)}}{F_{\text{l.c.}}^{(0)}} F_{\text{l.c.}}^{(2)}(Q^2) + \sum_{i=0}^4 c_i \log^i(\mu^2/Q^2)$$

~VV 6-10% of σ



theory-nuisance-parameter analysis (TNP)

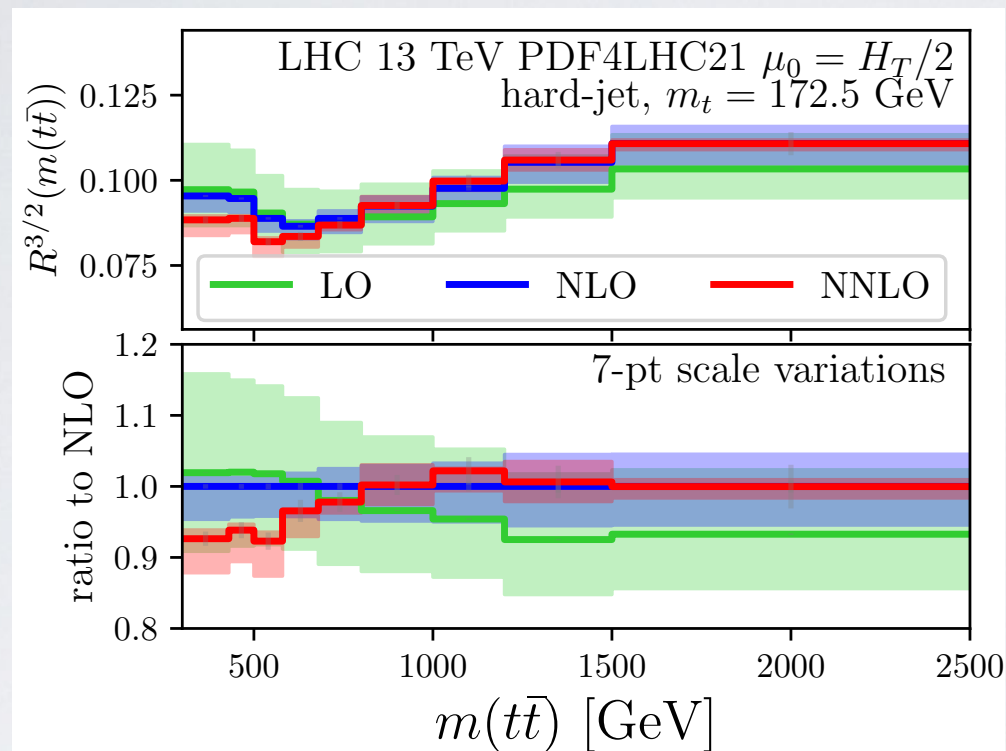
[Tackmann '25]
[Lim, Poncelet '25]

for missing higher order uncertainty (MHOU) and sub-leading colour

$$\frac{F_{\text{l.c.}}^{(2)}}{F_{\text{l.c.}}^{(0)}} \rightarrow \frac{F_{\text{l.c.}}^{(2)}}{F_{\text{l.c.}}^{(0)}} + \frac{\theta}{N_c^2} \frac{F_{\text{l.c.}}^{(2)}}{F_{\text{l.c.}}^{(0)}}$$

$$p_T(j) \geq 30 \text{ GeV}, |y(j)| \leq 2.5$$

tt+jet @ NNLO



$$R^{3/2}(X) = \frac{d\sigma^{t\bar{t}j}(X)}{d\sigma^{t\bar{t}}(X)}$$

$$p_T(j) \geq 120 \text{ GeV}, |y(j)| \leq 2.4, p_T(j) \geq 0.4(p_T(t) + p_T(\bar{t}))$$

next steps

- comparison with data
- decays in the narrow width approximation (2L $t \rightarrow Wb$ needed [[Brancaccio TOP25](#)])
- subleading colour?

Two-loop corrections to $t\bar{t}+W$

NNLO total cross sections using approximate 2-loop amplitudes

- soft boson limit
- massification

NEW! Leading colour amplitudes

Becchetti, Canko, Chen, Chestnov, Delto, Ditsch, Peraro, Pozzoli, Savoini, Zoia [2608.03746]

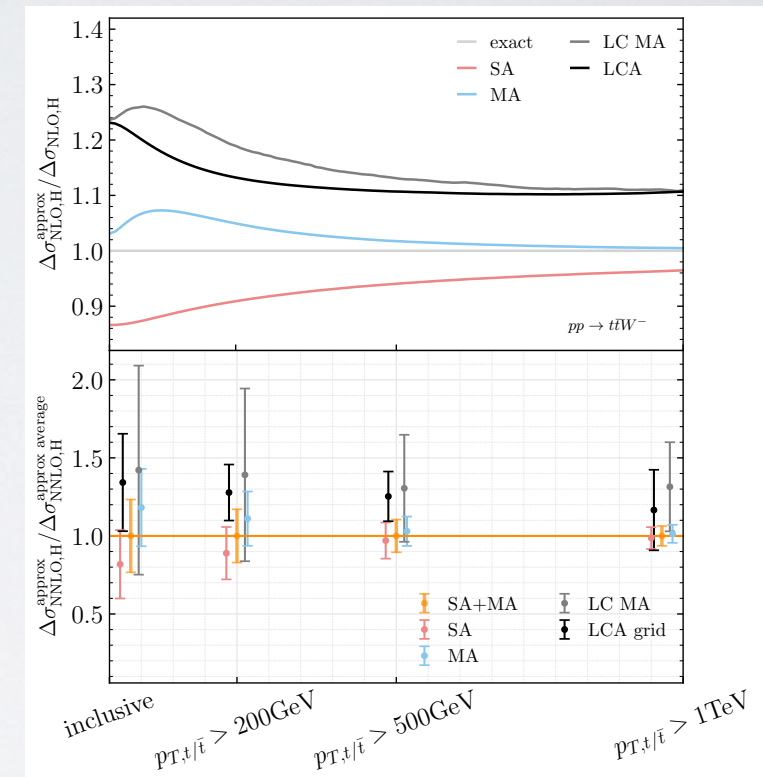
challenging amplitude computation and numerical evaluation

a global analytic reconstruction was impractical; coefficients reconstructed pointwise (worst point needed 400 prime evaluations)

200K points, interpolation grid, integrals via AMFlow

integrals $\sim$ 20-120 mins
coeffs. $\sim$ 20 mins

Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rotoli, Savoini Phys.Rev.Lett. 131 (2023)



$\Delta\sigma_{\text{NNLO,H}}[\text{fb}]$	$t\bar{t}W^-$	$t\bar{t}W^+$
SA	15.67 (0) $\pm$ 4.21	34.59 (0) $\pm$ 9.18
MA	22.62(15) $\pm$ 4.78	49.87(40) $\pm$ 8.74
SA+MA	19.14 (8) $\pm$ 4.49	42.23(20) $\pm$ 8.96
LCA	25.70 (0) $\pm$ 5.68	56.76 (0) $\pm$ 12.03

Becchetti, Canko, Chen, Chestnov, Delto, Ditsch, Grazzini, Kallweit, Peraro, Pozzoli, Savoini, Tancredi, Zoia [2606.09503]

Outlook

Techniques for multiscale amplitudes are improving a lot

- numerical integration of DEQs looks like it might provide a generic solution suitable for phenomenology

New precision predictions amplitudes with masses

- m_t mass extraction from $t\bar{t}j$ to be updated [joint CMS/ATLAS]
- improvements for $t\bar{t}H$ soon?

Simulations are still very expensive

- improvements will come for sure

Rational function reconstruction may not always be possible

- what are the prospects for purely numerical methods?

[talk by Gambuti]

Finding reliable approximations is very important

- e.g. massification is great when it works, can combining different limits avoid enormous computational effort?