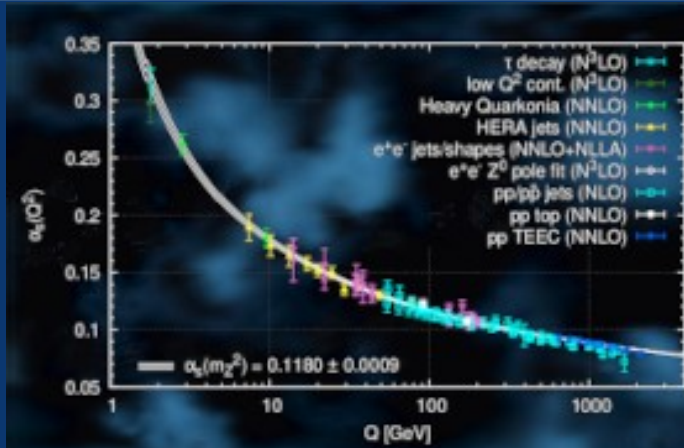




$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{q}_j (i \gamma^\mu D_\mu + m_j) q_j$$

where $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + if_{abc} A_\mu^b A_\nu^c$

and $D_\mu \equiv \partial_\mu + it^a A_\mu^a$

Panel discussion on α_s

QCD@LHC 2026

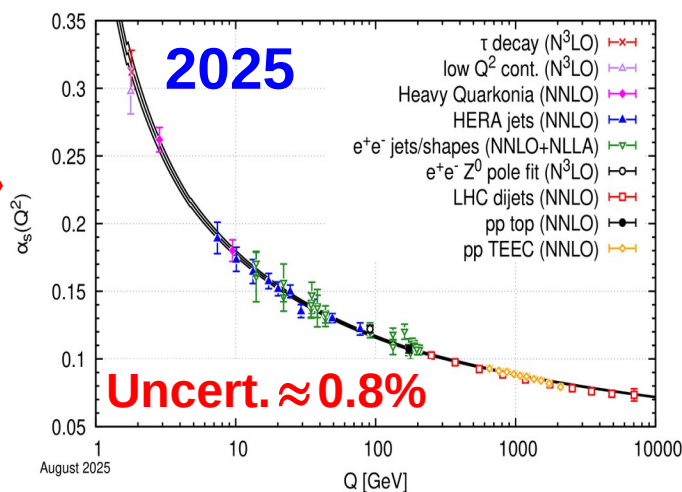
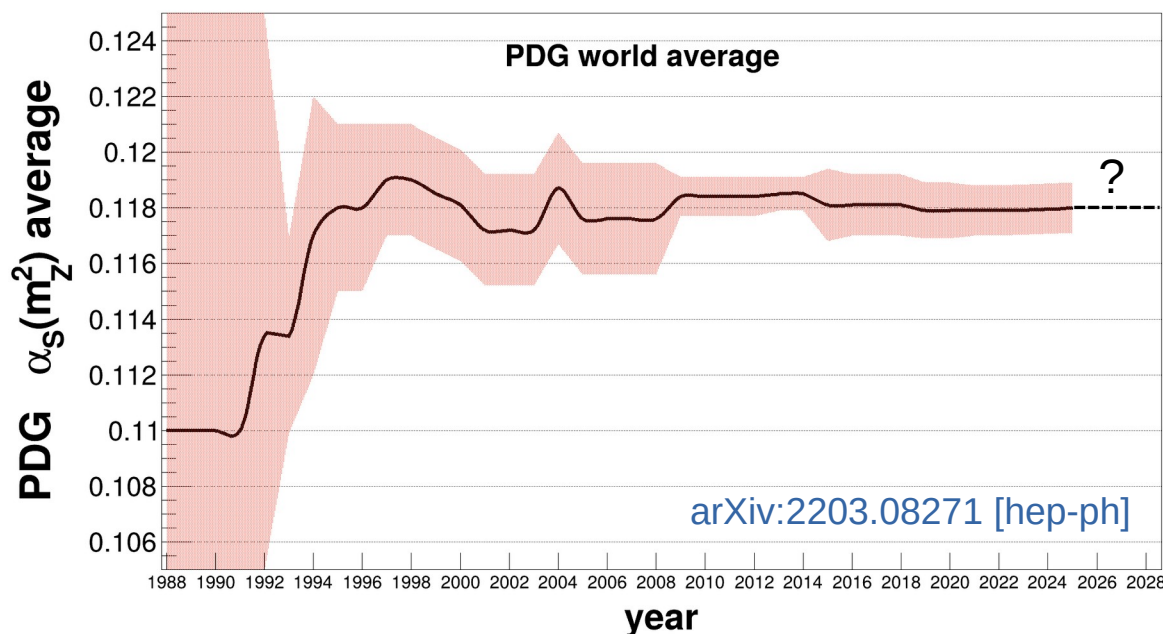
University of Oxford, England

29th Sept. 2026

D. d'Enterria (CERN), A. Ramos (IFIC-València)
G. Zanderighi (MPP & TUM, Munich)

The strong coupling α_s

- Determines **strength of the strong interaction** among quarks & gluons.
- **Single free parameter of QCD** in the $m_q = 0$ limit.
- Determined at a **ref. scale** ($Q=m_Z$), decreases as $\alpha_s \approx \ln(Q^2/\Lambda^2)^{-1}$, $\Lambda \approx 0.2$ GeV



$$\alpha_s(m_Z^2) = 0.1180 \pm 0.0009$$

♦ **Least precisely known** of all interaction **couplings** !

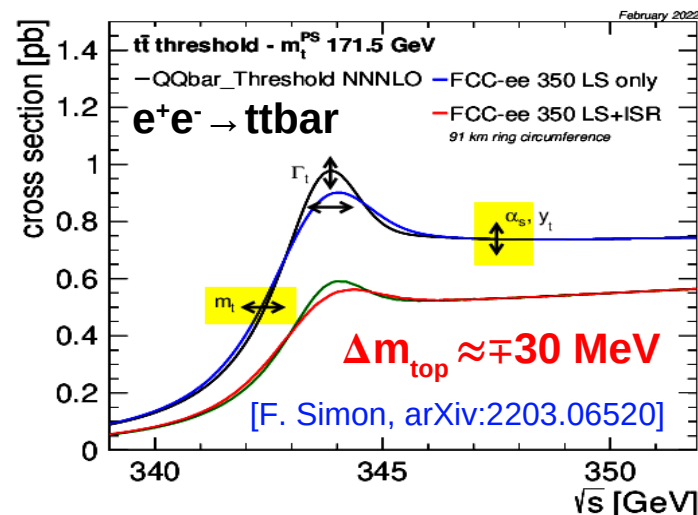
$$\Delta\alpha \approx 10^{-10} \ll \Delta G_F \approx 10^{-7} \ll \Delta G \approx 10^{-4} \ll \Delta\alpha_s \approx 0.8\%$$

α_s impact beyond QCD

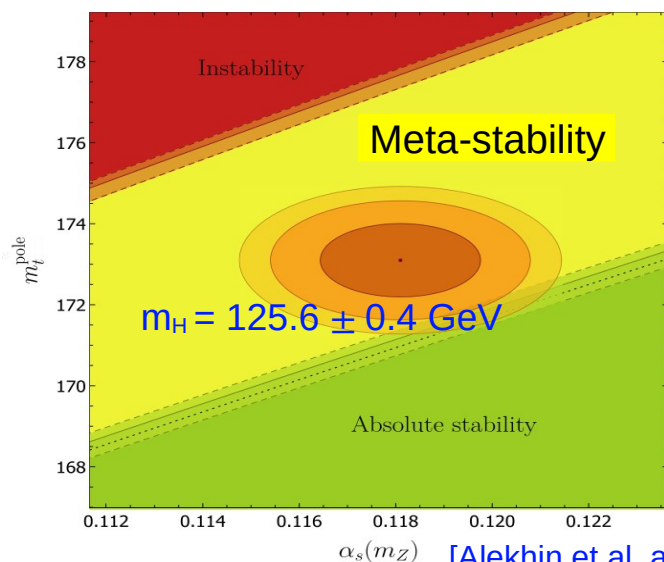
- Parametric uncertainty for key Higgs, EWPO, top quark calculations:

Process	σ (pb)	$\delta\alpha_s(\%)$	PDF + $\alpha_s(\%)$	Scale(%)
ggH	49.87	± 3.7	-6.2 +7.4	-2.61 + 0.32
ttH	0.611	± 3.0	± 8.9	-9.3 + 5.9

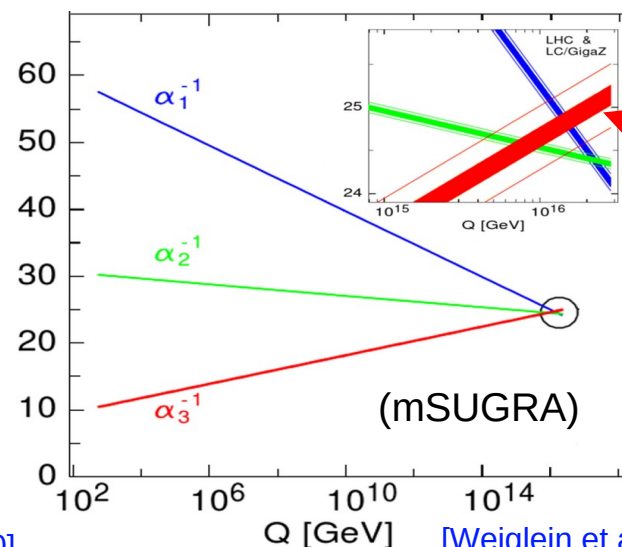
Partial width	intr. QCD	para. m_q	para. α_s
$H \rightarrow b\bar{b}$	$\sim 0.2\%$	1.4%	0.4%
$H \rightarrow c\bar{c}$	$\sim 0.2\%$	4.0%	0.4%
$H \rightarrow gg$	$\sim 3\%$	$< 0.2\%$	3.7%



- Impacts physics approaching Planck scale: EW vacuum stability, GUT:



[Alekhin et al. arXiv:1207.0980]

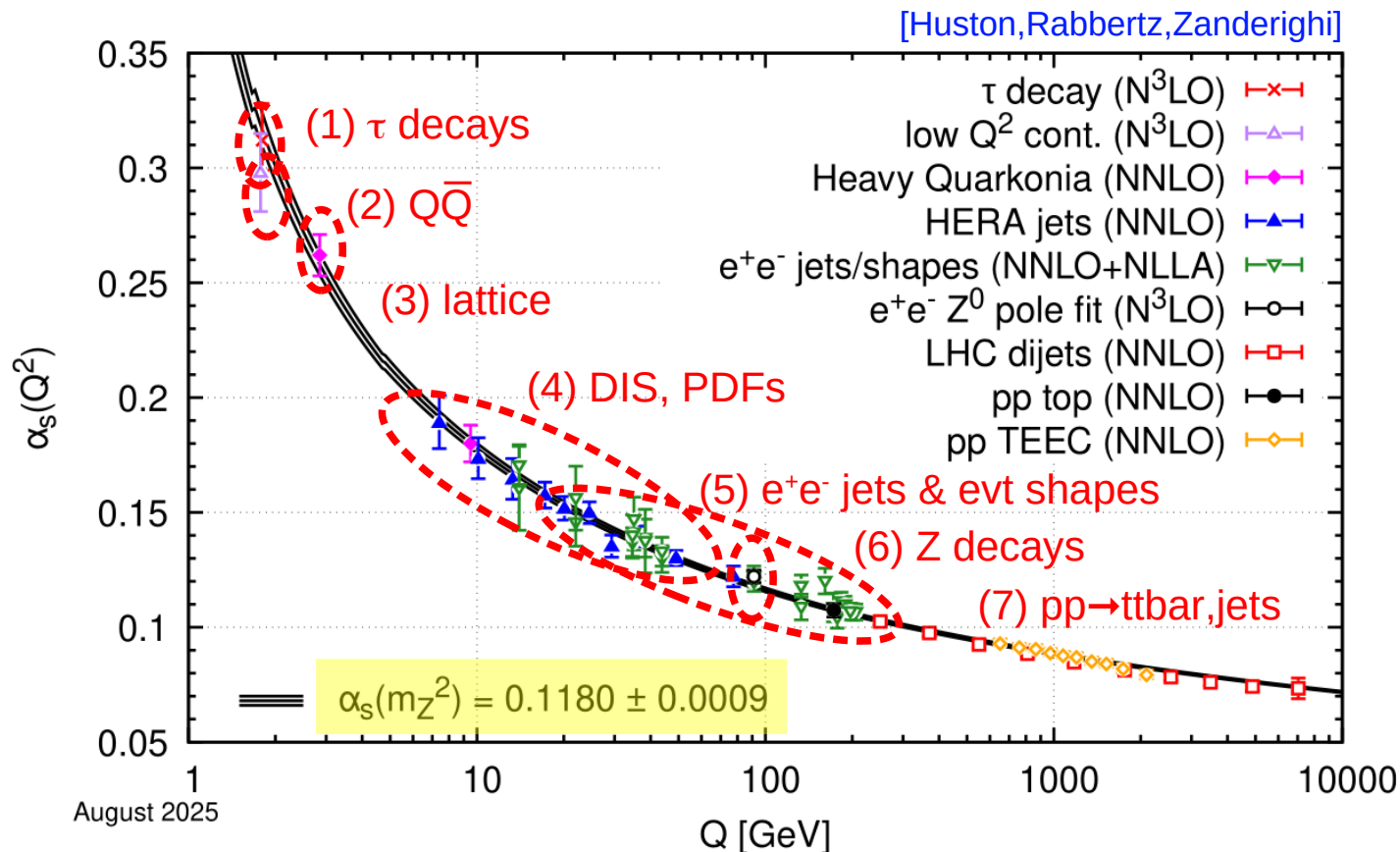


[Weiglein et al. hep-ph/0410364]

α_s determination (PDG 2025)

■ Current precision: $\Delta\alpha_s = \pm 0.8\%$

From combination of 7 categories (grouped exp. observables) compared to $N^{2,3}LO$ pQCD predictions:



■ Running coupling probed up to $Q \approx 7$ TeV at the LHC

World-average α_s (PDG 2025)

■ 6 unweighted pre-averages categories averaged with lattice-QCD result:

$$\alpha_s(M_z) = 0.1180 \pm 0.0009 \quad (\pm 0.8\%)$$

Hadronic tau decay (5 values):

$$\alpha_s(M_z) = 0.1171 \pm 0.0017 \quad (\pm 1.4\%)$$

Quarkonia properties (6 values):

$$\alpha_s(M_z) = 0.1181 \pm 0.037 \quad (\pm 3.1\%)$$

DIS & PDFs fits (7 values):

$$\alpha_s(M_z) = 0.1161 \pm 0.0022 \quad (\pm 1.9\%)$$

$e^+e^- \rightarrow$ hadrons final states (7 values):

$$\alpha_s(M_z) = 0.1189 \pm 0.0037 \quad (\pm 3.1\%)$$

Hadron collider measurements (10 values):

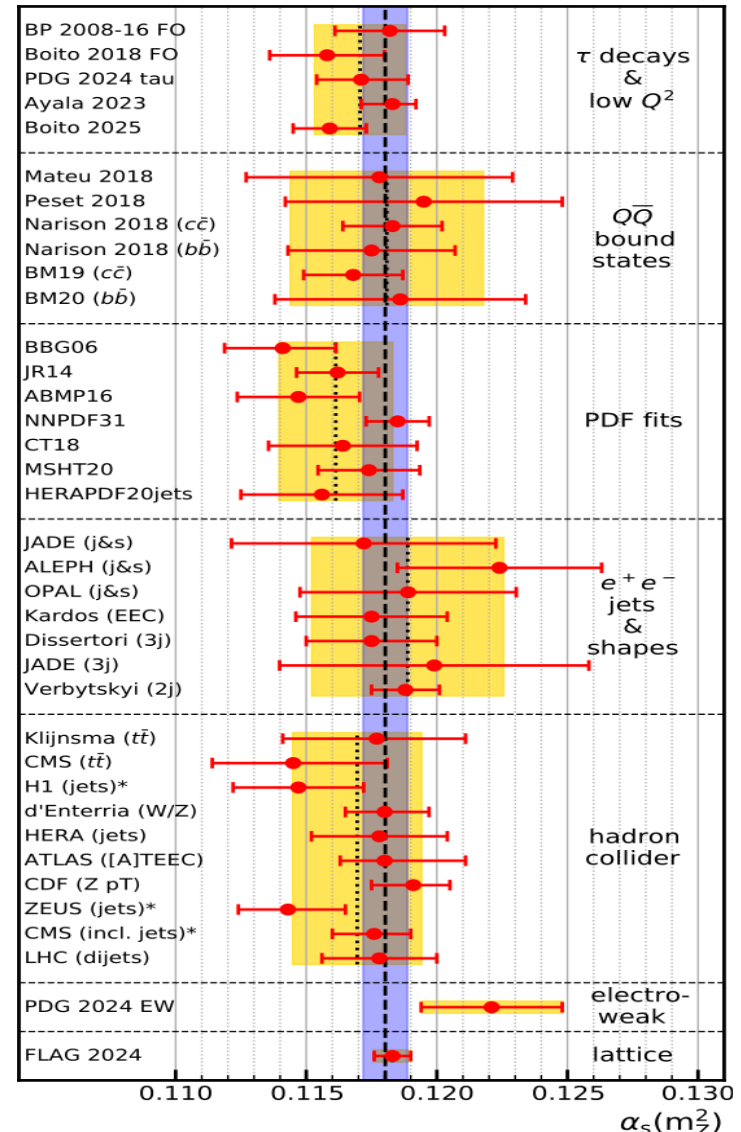
$$\alpha_s(M_z) = 0.1170 \pm 0.0025 \quad (\pm 2.1\%)$$

Electroweak precision fits (1 value):

$$\alpha_s(M_z) = 0.1221 \pm 0.0027 \quad (\pm 2.2\%)$$

Lattice QCD (1 FLAG average value):

$$\alpha_s(M_z) = 0.1183 \pm 0.0007 \quad (\pm 0.6\%)$$



August 2025

A few α_s topics for discussion

- (1) How much more & how fast can **lattice-QCD** improve its $\pm 0.6\%$ precision ?
- (2) Open points in α_s extractions from **e^+e^- event-shapes**.
- (3) Actual accuracy & precision of α_s from **Z boson low- p_T recoil in pp colls.** ?

Other possible items (and/or any other of your interest):

- (4) Correlation between **hadron-collider & PDF PDG categories** ?
 α_s from **pp collider** cross sections without **full global PDF refitting** ?
- (5) Non-perturbative and MHO uncertainties for **hadronic tau decays**.
- (6) Combination of **PDG world α_s** from **lattice & non-lattice averages** ?
- (7) **Ultimate α_s experimental precision** achievable ?

...

(1) α_s from lattice QCD: state-of-the-art

- **Comparison** of short-distance quantities (QCD static energy/force, light-q & heavy-Q currents, quarkonium,...) computed at $N^{2,3}LO$ in pQCD to **lattice** data with m_{had}, f_{had} fixed to exp. data:

$$K^{NP} = K^{PT} = \sum_{i=0}^n c_i \alpha_s^i$$

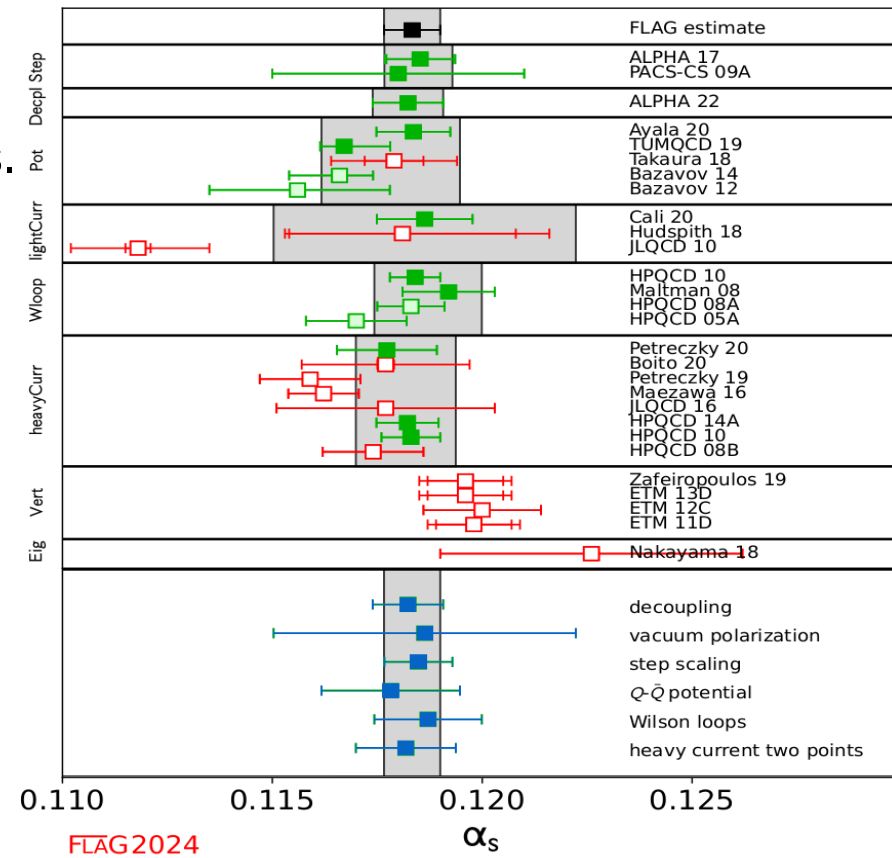
Plus **step-scaling & decoupling** methods.

- **Community-agreed (FLAG) requirements:**

- large renorm. scale for pQCD part: $\mu \gg \Lambda_{QCD}$
- continuum limit (spacing $a \rightarrow 0$): $\mu \ll 1/a$
- large latt. vol. ($L \rightarrow \infty$): $L/a \gg \mu/\Lambda_{QCD} \approx \mathcal{O}(10^3)$
- peer-reviewed results

- **Current uncertainties** driven mostly by **pQCD truncation/matching** at low scales, (except for state-of-the-art methods: Step-scaling & Decoupling) and lattice **statistics**.

[FLAG Collab. <http://flag.unibe.ch>]



$$\alpha_s = 0.1183 \pm 0.0007 (\pm 0.6\%)$$

(1) α_s from lattice QCD: state-of-the-art

■ Most precise methods today: **step scaling/decoupling**

Step scaling

- ▶ Non-perturbative finite volume approach

$$\mu_{\text{had}} \approx M_p : \alpha(\mu_{\text{had}}) \rightarrow \alpha(2\mu_{\text{had}}) \rightarrow \dots \alpha(2^n \mu_{\text{had}})$$

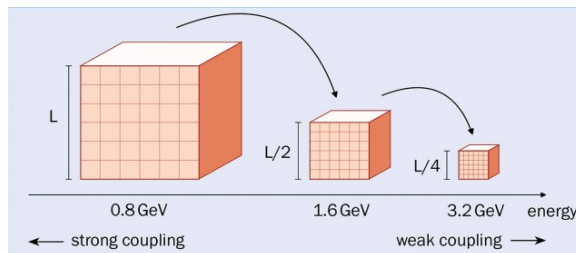
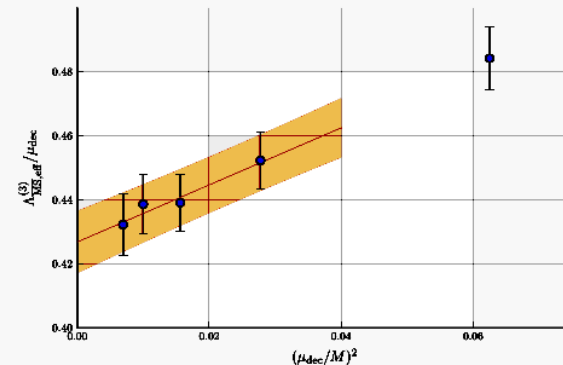
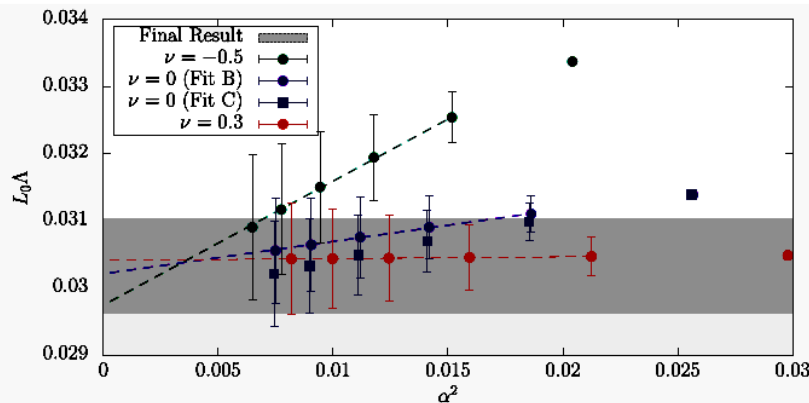
- ▶ PT only used at $2^n \mu_{\text{had}} \approx 100 \text{ GeV}$ (arbitrary)
 - ▶ Negligible PT uncertainties
- ▶ Systematic: continuum extrapolation $a \rightarrow 0$
 - ▶ Systematically improvable (larger lattices L/a)
 - ▶ Increasing computing power helps here!

Decoupling

- ▶ Exact NP relation between Λ_{QCD} and Λ_{YM}

$$\sqrt{t_0} \times \Lambda_{\text{QCD}} = \lim_{M \rightarrow \infty} \left[\frac{\sqrt{t_0}}{\sqrt{t_0(M)}} \right] \times \left[\sqrt{t_0^{\text{YM}}} \times \Lambda_{\text{YM}} \right] \times \frac{1}{P(\Lambda/M)}.$$

- ▶ Matching via QCD with **all quarks heavy** $M \gg \Lambda$



(1) α_s from lattice QCD: future progress

■ Perspectives for uncertainty reduction:

Uncertainty in last determination [Nature 652 no. 8109, (2026) 328–334.]

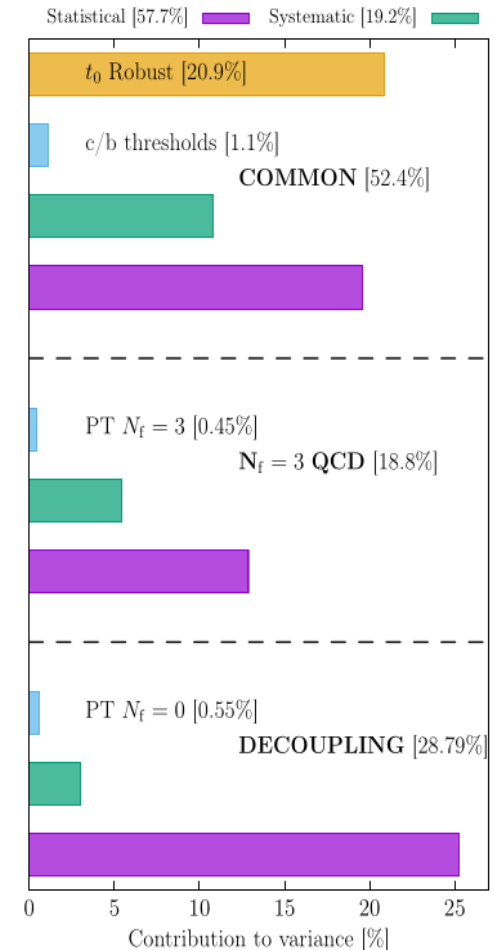
- ▶ t_0 : scale setting. Community average, reflecting some tensions.
 - ▶ Expectations (5y): 1.3% $\rightarrow$ 0.4%
- ▶ $N_f = 3 \rightarrow N_f = 4$: Systematic subdominant ($a \rightarrow 0$)
 - ▶ Can be reduced, but here $\mathcal{O}(a)$ unavoidable(?)
 - ▶ Expectation: 2.3% $\rightarrow$ 1% – 1.5%
- ▶ Decoupling: Statistics dominated with old computation
 - ▶ Better schemes: less systematic, more precision [A.R. JHEP 1411 (2014) 101.], faster decoupling [S. Sint, '14]
 - ▶ Dedicated pure gauge codes and more computing time
 - ▶ Better control of systematic (i.e. QED running “automatically included”)
 - ▶ Expectations: 1.6% $\rightarrow$ 0.3%

$\Delta\Lambda_{\text{QCD}} \approx 2.5\%$
 $(\Delta\alpha_s(m_Z) \text{ is } \sim 10 \text{ smaller})$

Perspectives

- ▶ Possible error reduction:
 - $\alpha_s = 0.11876(58) \quad 0.5\% \rightarrow 0.1\%$
- ▶ Still statistics dominated
- ▶ Mainly driven by decoupling approach

Timeframe: depends strongly on cpu/human time



(2) α_s from e^+e^- event shapes: state-of-the-art

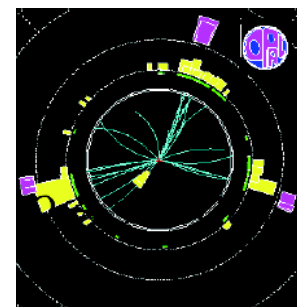
- Thrust, C-param, $y_3, \dots$ computed at $N^{2,3}\text{LO}+N^2\text{LL}$ accuracy, and measured at LEP:

- Results sensitive to NP (hadronization) corrections.

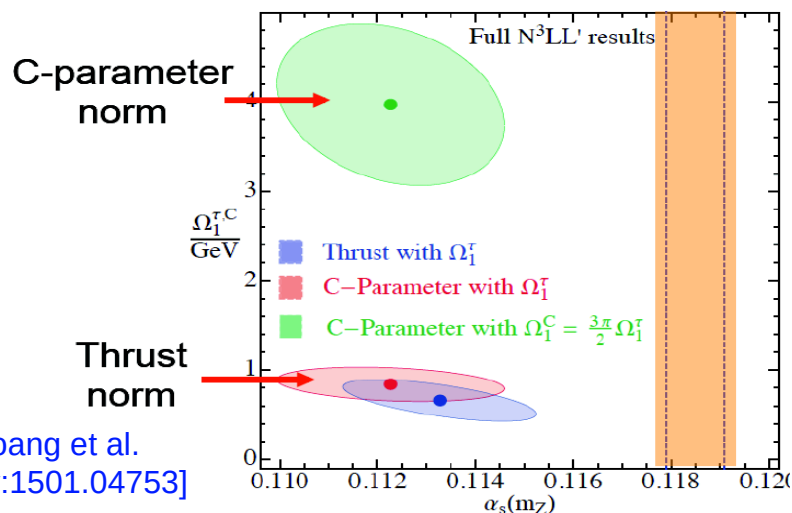
Large diffs. between MC & analytic corrections:

$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

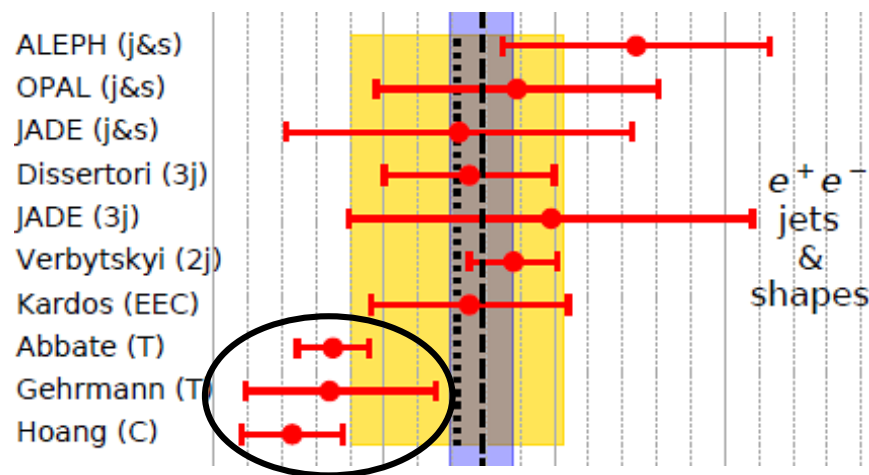
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



OPAL 3 jet event



[A.Hoang et al.
arXiv:1501.04753]



$$\alpha_s(M_Z) = 0.1171 \pm 0.0031 (\pm 2.6\%)$$



$$\alpha_s(M_Z) = 0.1189 \pm 0.0037 (\pm 3.1\%)$$

Analytic NP extractions removed from PDG world-average in 2023:

- Separation of 2 vs 3-jet pQCD regions?
- Hadron mass-scheme uncertainties?

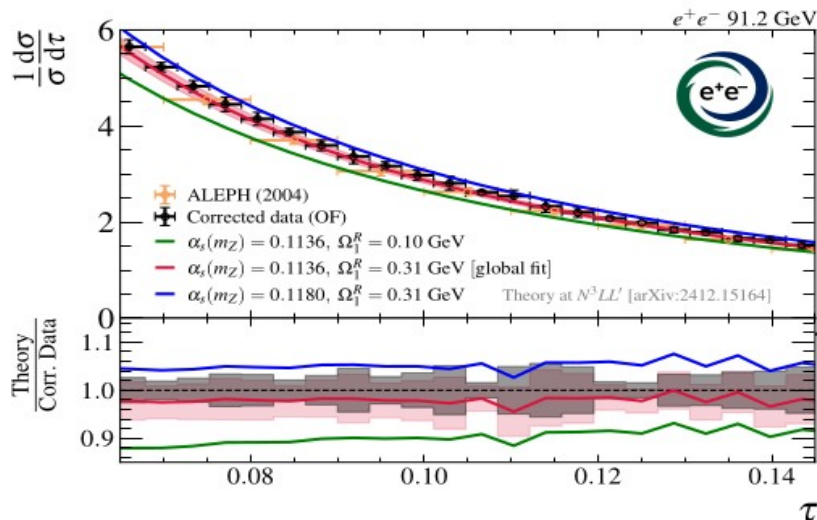
(2) α_s from e^+e^- event shapes: Open points

[A.Badea et al. arXiv:2609.03002]

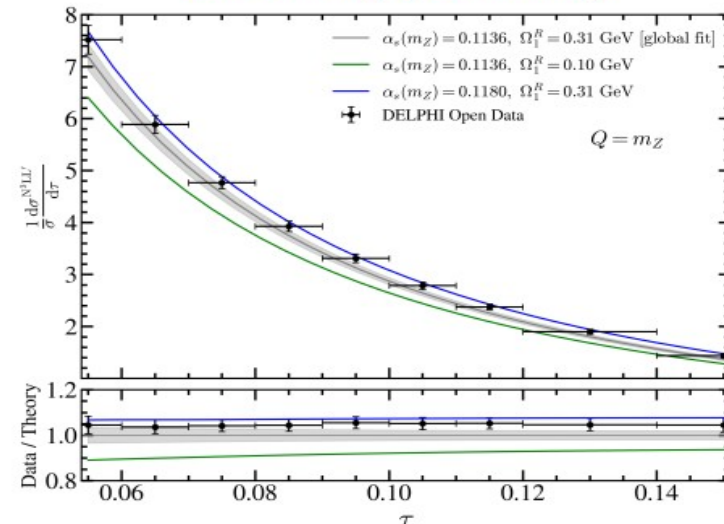
1. Data consistency (including recent reanalyses).

Degeneracy between pQCD (α_s) & NP (Ω^R) parameters.

ALEPH (archived data)



DELPHI (open data)

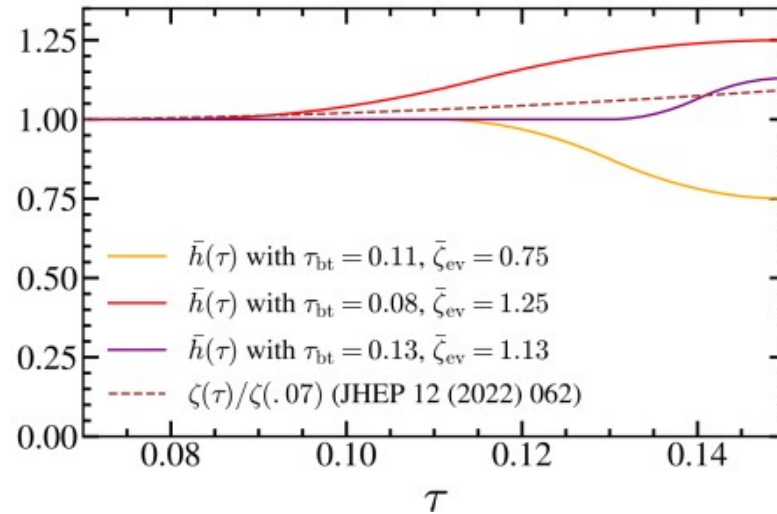
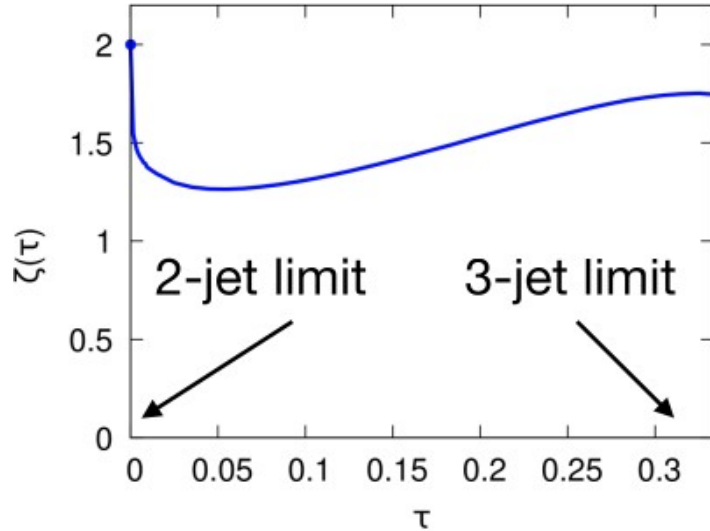


2. Hadron mass effects (exp/theory fits in E-scheme for diff. obs.):

	$\alpha_s(M_Z)$							
	CTy3		C		T		y_3	
Variation	$\zeta(v)$	$\zeta(0)$	$\zeta(v)$	$\zeta(0)$	$\zeta(v)$	$\zeta(0)$	$\zeta(v)$	$\zeta(0)$
E scheme	0.1181	0.1161	0.1169	0.1139	0.1168	0.1158	0.1155	0.1154
No Variation	0.1173	0.1153	0.1164	0.1118	0.1152	0.1148	0.1150	0.1149
p scheme	0.1160	0.1141	0.1164	0.1118	0.1152	0.1148	0.1137	0.1135
D scheme	0.1199	0.1173	0.1190	0.1153	0.1205	0.1170	0.1168	0.1166

(2) α_s from e^+e^- event shapes: Open points

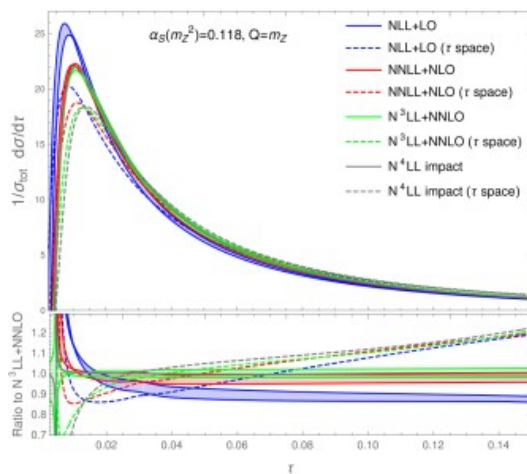
3. Separation of dijet vs trijet region even at the perturbative level



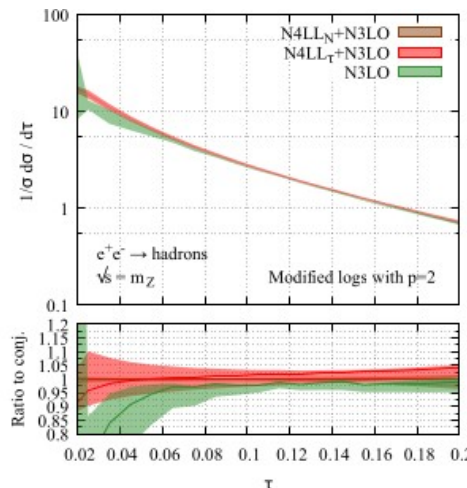
Thrust fits in resummation → fixed-order transition region

4. Mellin vs. direct-space resummation:

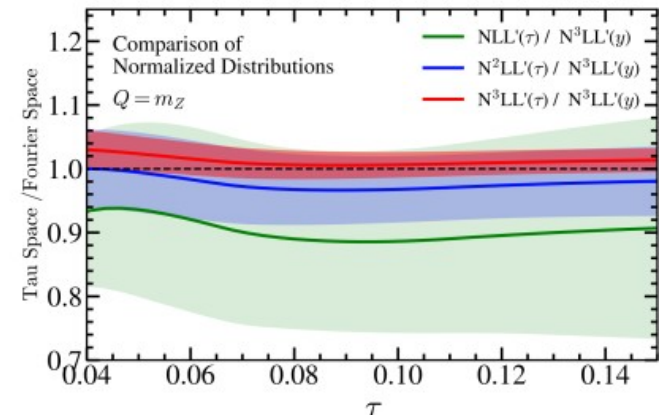
[A.Badea et al. arXiv:2609.03002]



Ref. [51]



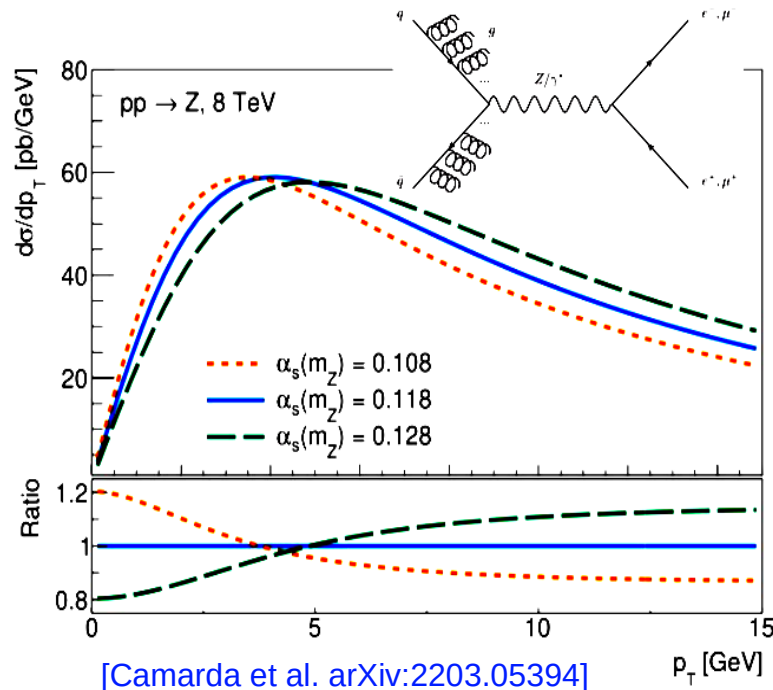
Ref. [55]



Ref. [103]

(3) α_s from Z boson p_T recoil

- Differential Z boson x-sections available at $N^{2,3}\text{LO}+N^{3,4}\text{LL}$
- Method: Compare $d\sigma/dp_T(\text{exp})$ to $d\sigma/dp_T(N^n\text{LO}+N^n\text{LL})$ for diff. PDFs/ α_s : Extract α_s
- + Advantages: $\mathcal{O}(<1\%)$ exp. uncerts, sensitivity to α_s via soft/coll. ISR gluons.
- Disadvantages: Sensitivity to NP effects, MHOU, HF PDFs, ...



- Just **one approx- $N^3\text{LO}$ PDF set** used
- Gaussian **NP model** under control?
- Treatment of **MHOU** ?

- ATLAS extraction with “**aggressive**” subpercent uncertainty: [\[arXiv:2309.12986\]](https://arxiv.org/abs/2309.12986)

$$\alpha_s = 0.11828 \pm 0.00088 (\pm 0.75\%)$$

Experimental uncertainty	± 0.44	
PDF uncertainty	± 0.51	
Scale variation uncertainties	± 0.42	
Matching to fixed order	0	-0.08
Non-perturbative model	+0.12	-0.20
Flavour model	+0.40	-0.29
QED ISR	± 0.14	
$N^4\text{LL}$ approximation	± 0.04	
Total	+0.91	-0.88

Backup slides

(5) α_s from hadronic τ -lepton decays

■ Computed at **N³LO**: $R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = S_{\text{EW}} N_C (1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\text{np}})$

■ Experimentally:

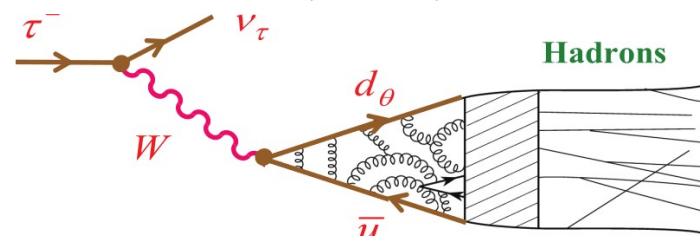
$$R_{\tau, \text{exp}} = 3.634 \pm 0.008 (\pm 0.2\%)$$

MHOU: $\pm 0.2\%$

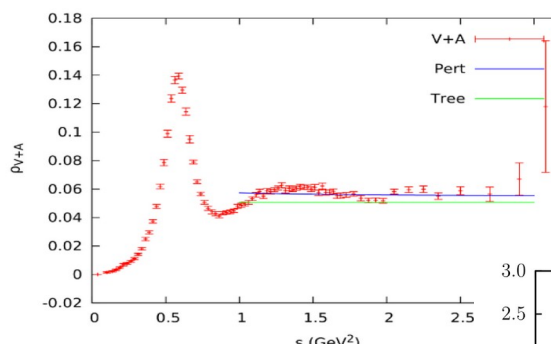
$$\Delta_{\text{HO}} \simeq (307.8 + c_{5,1}) \left(\frac{\alpha_s(m_\tau)}{\pi}\right)^5, \quad c_{5,1} = 283 \pm 142.$$

■ Theoretically:

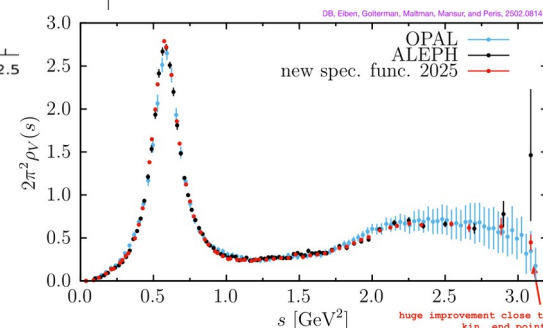
- “Historical” **diffs. in pQCD approaches** in **FOPT vs. CIPT truncations**.
- Solved: CIPT converges more rapidly but oscillatory. Not in PDG anymore



Hadronic τ spectral function:



$$\rho_{\text{DV}}(s) = e^{-\delta - \gamma s} \sin(\alpha + \beta s)$$



■ Uncertainty today

$$\alpha_s = 0.1173 \pm 0.0017 (\pm 1.4\%)$$

driven by **NP corrections**:
truncated-OPE vs.
duality violations:

$$\delta_{\text{OPE}}^{D \geq 4} = d_w^{(4)} \frac{C_4}{Q^4} + d_w^{(6)} \frac{C_6}{Q^6} + d_w^{(8)} \frac{C_8}{Q^8} + \dots$$

➔ **Future prospects:**

- **N⁴LO** corrections.
- **Better spectral functions** needed:
BELLE-II, STCF. Longer future: **$\mathcal{O}(10^{12})$ $Z \rightarrow \tau\tau$ at FCC-ee(90)**

Theory calculations “wish list” (ESPP’25)

■ Summary table for precision-QCD wish-list (pQCD, lattice, MC PS):

	Observable	Missing higher-order & power-suppressed corrections
$\alpha_s(m_Z)$ in e^+e^-	Hadronic Z width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_s\alpha^3)$, $\mathcal{O}(\alpha_s^2\alpha^2)$
	Hadronic W width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_s\alpha^2)$, $\mathcal{O}(\alpha_s\alpha^3)$, $\mathcal{O}(\alpha_s^2\alpha^2)$
	Hadronic τ width	$\mathcal{O}(\alpha_s^5)$
	Hadronic event shapes (Z, W, H decays)	N^3 LO differential, $N^{3,4}$ LL resummation, power corrections
$\alpha_s(m_Z)$ in latt.	Inclusive jet rates	3-jet cross sections at N^3 LO, 4-jets at N^2 LO, 5-jets at NLO
	Lattice QCD results	$\mathcal{O}(\alpha_s^6)$ β -function; $\mathcal{O}(\alpha_s^5)$ heavy quark decoupling; $\mathcal{O}(\alpha_s^4)$ static potential
	(α_s extractions; quark masses m_c, m_b)	$\mathcal{O}(\alpha_s^3)$ lattice perturbation theory matching (lattice coupling to $\alpha_s^{\overline{MS}}$ etc.)
m_W, m_{top} in e^+e^-	$\sigma(e^+e^- \rightarrow W^+W^-)$ vs. $\sqrt{s}$	EW N^2 LO: $\mathcal{O}(\alpha^2)$, Mixed EW-QCD: $\mathcal{O}(\alpha_s\alpha^2)$, $\mathcal{O}(\alpha_s^2\alpha)$
	$\sigma(e^+e^- \rightarrow t\bar{t})$ vs. $\sqrt{s}$	NRQCD: $\mathcal{O}(\alpha_s^5)$, Non-resonant: $\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^3)$ differential; QED: $\mathcal{O}(\alpha^3)$ at NNLL
QCD in Higgs e^+e^- , e-p, p-p PS	$H \rightarrow b\bar{b}$ width	N^4 LO (massive b-quark); N^4 LO differential (massless b-quark)
	$H \rightarrow gg$ width	N^5 LO (heavy-top limit), N^4 LO (massive top)
	MC simulations for $e^+e^- \rightarrow X$ processes	N^4 LO differential, N^3 LO differential (massive top)
$\alpha_s(m_Z)$ in DIS		$N^{2,3}$ LO matched to $N^{2,3}$ LL PS.
	ep $\rightarrow$ hadrons (PDF and α_s determination)	Per mille control of non-perturbative QCD effects (hadronization, CR,...)
	ep $\rightarrow$ jets (α_s determination)	$N^{3,4}$ LO evolution equations and inclusive cross sections N^3 LO cross sections

arXiv:2511.03883 [hep-ex]

■ Future Higgs/EW/top factories (in particular FCC-ee) impose very strong requirements on theory developments to match expected exp. precision.

Summary: Current & future α_s precision

- Well-defined exp./th. path towards $\alpha_s(m_Z)$ permil precision in coming years

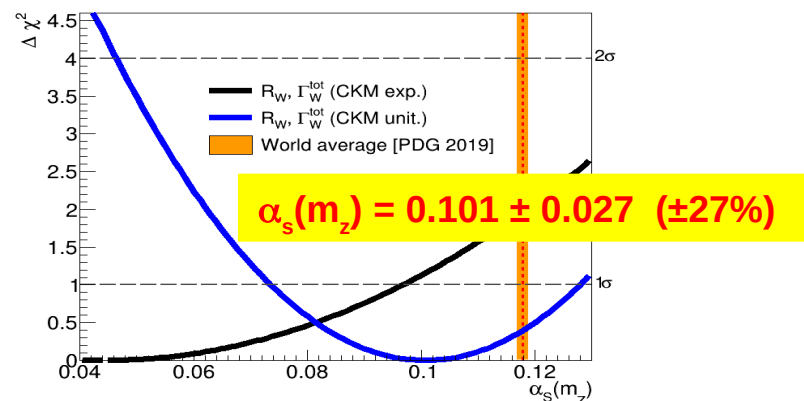
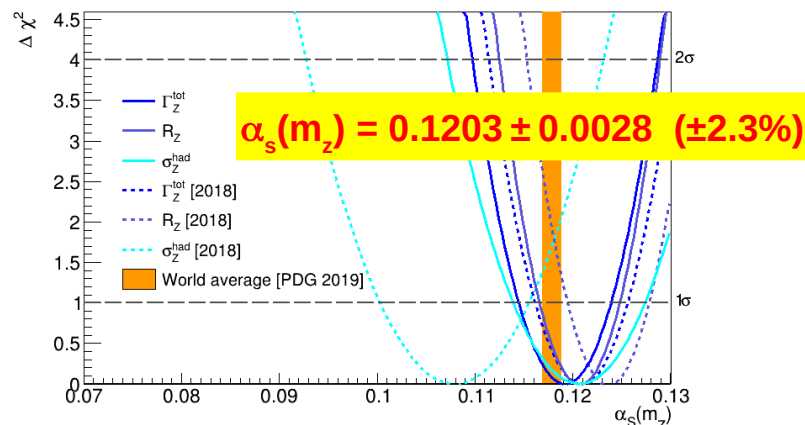
Method	Relative $\alpha_s(m_Z^2)$ uncertainty	
	Current theory & exp. uncertainties sources	Near (long-term) future theory & experimental progress
(1) Lattice	0.7% Finite lattice spacing & stats. N ^{2,3} LO pQCD truncation	$\approx 0.3\%$ (0.1%) Reduced latt. spacing. Add more observables Add N ^{3,4} LO, active charm (QED effects) Higher renorm. scale via step-scaling to more observ.
(2) τ decays	1.6% N ³ LO CIPT vs. FOPT diffs. Limited τ spectral data	$< 1\%$ Add N ⁴ LO terms. Solve trunc-OPE vs. DV Improved τ spectral functions at Belle II
(3) $Q\bar{Q}$ bound states	3.3% N ^{2,3} LO pQCD truncation $m_{c,b}$ uncertainties	$\approx 1.5\%$ Add N ^{3,4} LO & more ($c\bar{c}$), ($b\bar{b}$) bound states Combined $m_{c,b} + \alpha_s$ fits
(4) DIS & PDF fits	1.7% N ^{2,(3)} LO PDF (SF) fits Span of PDF-based results	$\approx 1\%$ (0.2%) N ³ LO fits. Add new SF fits: $F_2^{p,d}$, g_i (EIC) Better corr. matrices. More PDF data (LHeC/FCC-eh)
(5) e^+e^- jets & evt shapes	2.6% NNLO+N ^(1,2,3) LL truncation Different NP analytical & PS corrs. Limited datasets w/ old detectors	$\approx 1.5\%$ ($< 1\%$) Add N ^{2,3} LO+N ³ LL, power corrections Improved NP corrs. Hadron mass effects New improved data at B factories (FCC-ee)
(6) Electroweak fits	2.3% N ³ LO truncation Small LEP+SLD datasets	($\approx 0.1\%$) N ⁴ LO, reduced param. uncerts. ($m_{W,Z}$, α , CKM) Add W boson. Tera-Z, Oku-W datasets (FCC-ee)
(7) Hadron colliders	2.4% NNLO(+NNLL) truncation, PDF uncerts. Limited data sets ($t\bar{t}$, W, Z, e-p jets)	$\approx 1.5\%$ N ³ LO+NNLL (for color-singlets), improved PDFs Add more datasets: Z p_T , p-p jets, σ_i/σ_j ratios,...
World average	0.8%	$\approx 0.4\%$ (0.1%)

α_s from EW bosons hadronic decays

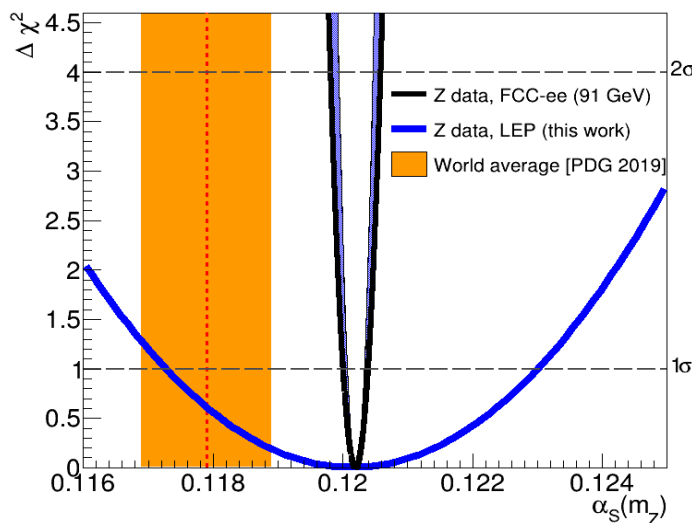
■ Updated Z,W-based $\alpha_s(m_Z)$ extractions:

[DdE&Jacobsen, 2005.04545 [hep-ph]]

New fit with **HO EW corrs. + corrected Z LEP** data. New **N³LO** fit to Γ_W , R_W

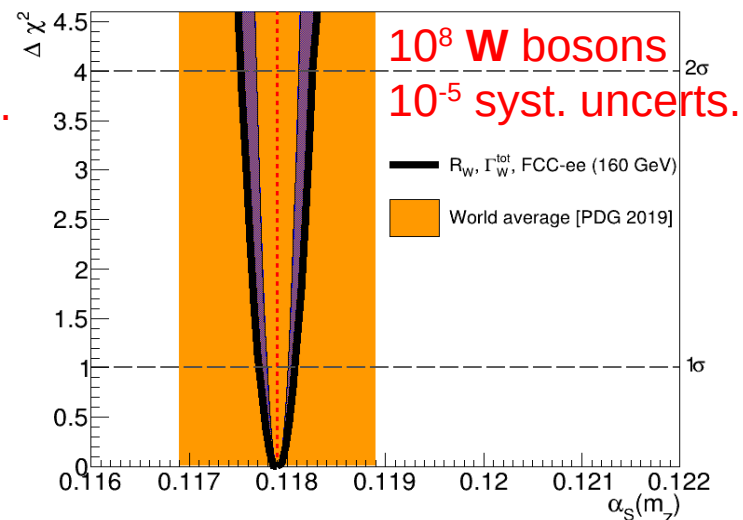


■ Future: Permil uncertainty possible only with a machine such as FCC-ee



10^{12} Z bosons
 10^{-5} syst. uncs.

Strong SM
"stress test"

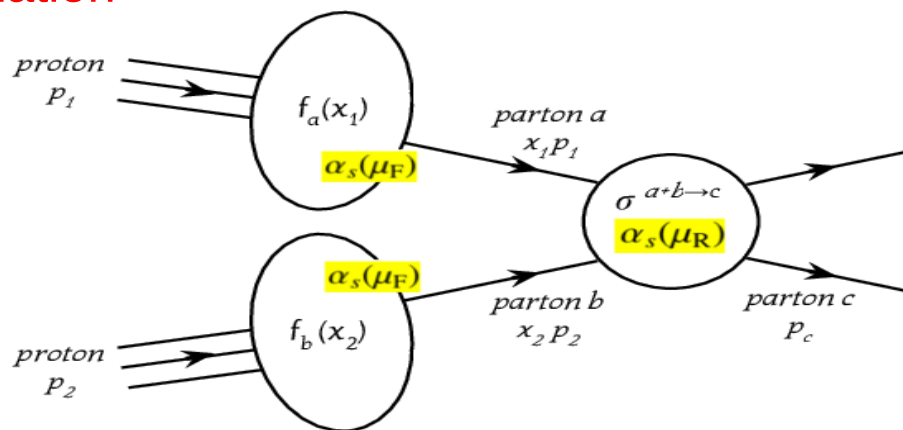


10^8 W bosons
 10^{-5} syst. uncs.

α_s extraction from LHC pQCD x-sections

- Method: (1) Compare $\sigma(\text{exp})$ to $\sigma(\text{NNLO})$ for varying PDFs+ α_s
- (2) Extract α_s via χ^2 minimization
- (3) Extra requirement:
Redo (partial) PDF fit

Gluon & α_s strongly
(anti)correlated at
high (low) x.



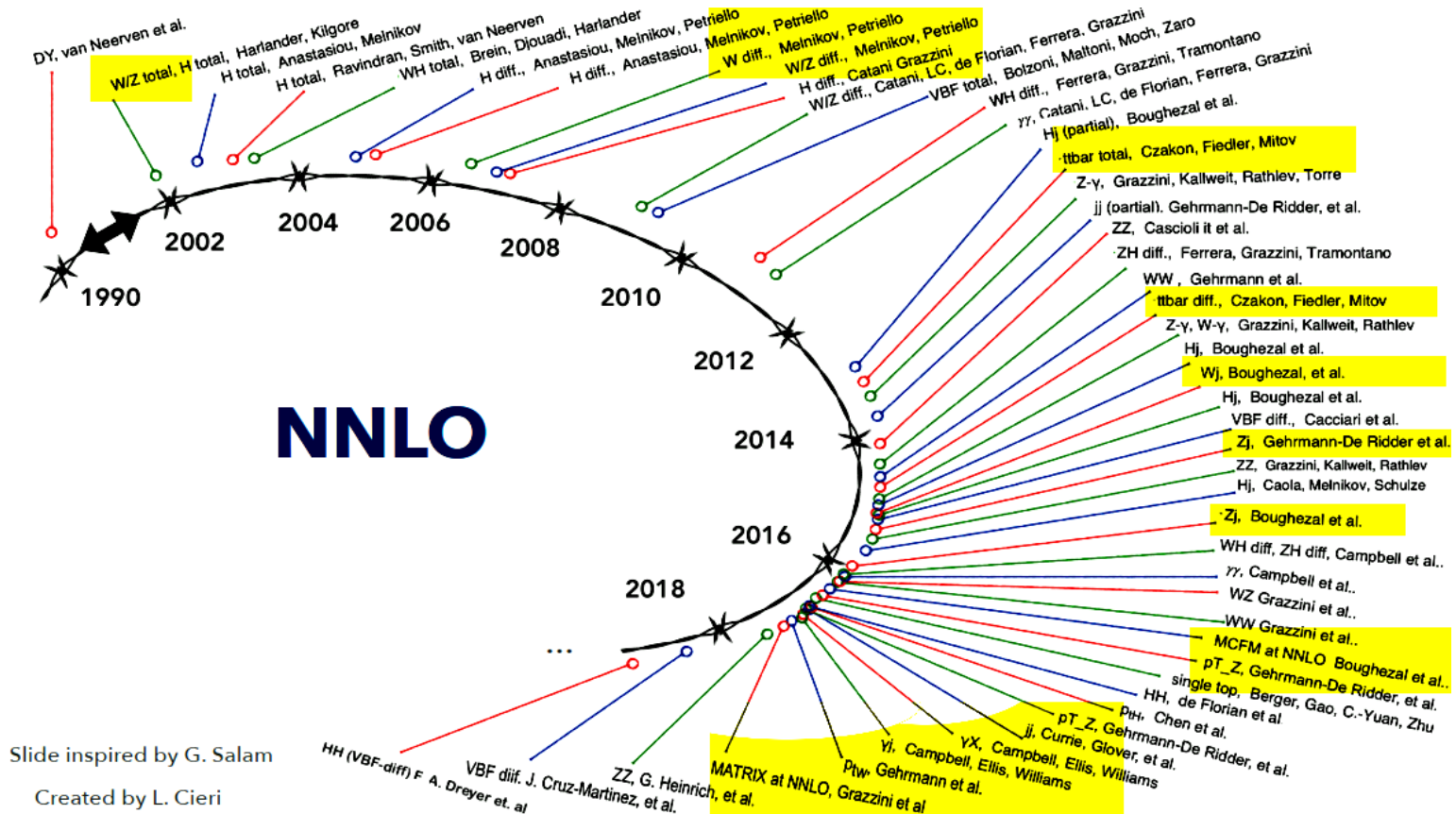
$$\sigma(\text{pp} \rightarrow \text{X}) = \sum_{\text{partons}} \int \int dx_1 dx_2 f_1(x_1, \alpha_s(\mu_F)) f_2(x_2, \alpha_s(\mu_F)) \left[\hat{\sigma}_{\text{LO}} + \alpha_s(\mu_R) \hat{\sigma}_{\text{NLO}} + \alpha_s^2(\mu_R) \hat{\sigma}_{\text{NNLO}} + \dots \right]$$

pQCD accuracy: $\mathcal{O}(50\%)$ $\mathcal{O}(20\%)$ $\mathcal{O}(2-5\%)$

- Global requirement:
 - x-section NOT incorporated in global-PDF fits (avoid “double counting”)
- Theory requirements:
 - NNLO prediction: Few % scale uncertainty
 - Low non-pQCD & parametric (e.g. m_q) uncertainties & controlled EW corrections
- Exp. requirements:
 - Few % stat. uncertainty: Process with large cross sections (say $\sigma \gtrsim 100$ pb).
 - Few % syst. uncertainties: Ratios (luminosity & other syst. cancel out) better?

α_s extraction from LHC pQCD x-sections

- Increasing number of NNLO processes, but: not yet in global-PDF-fits, large x-sections, small EW/non-pQCD corrections, small exp. uncerts,...



- First ones tried: $\sigma_{\text{tot}}(\text{ttbar})$, $\sigma_{\text{tot}}(W,Z)$, $d\sigma/dp_T(Z, \text{low } p_T)$,...
 - Not yet tried at NNLO: Ratios $R_{32}(p_T) = \sigma(3j)/\sigma(2j)$, $\sigma(Z+j)/\sigma(Z)$, $\sigma(W+j)/\sigma(W)$,...
- Note: Different TH prescriptions exist to construct “correct” NNLO R_x ratios...

Summary: α_s wish-list for the next ~10 years

■ Theoretical needs to approach (0.1%) precision:

(1) Lattice QCD: Sufficient dedicated **computing resources & person-power**:

- Develop pQCD **$N^{3,4}$ LO theory** for observables in a finite space-time volume
- **Extend** higher renormalization scales via **step-scaling to more observables**

(2) Other phenomenology efforts:

- **N^4 LO** for **tau** decays.
- **Advanced power corrections** for e^+e^- event shapes & **resummation** for jet rates
- **NNLL accuracy** parton showers matched to NNLO:
NNLO(+NNLL) MCs for complex final states in e^+e^- , e -p, p-p
- Differential **NNLO** predictions for **LHC & HERA multi-jet observables**,...
- **Z, W hadronic decays** at **N^4 LO & mixed QCD+EW** needed for longer term.

■ Experimental needs to approach (0.1%) precision:

(3) Extension of **$N^{2,3}$ LO** hadron-collider- and/or PDF-based extractions via:

- LHC: Add **new precision observables & datasets** (TEEC, Z p_T , σ_i/σ_j ratios,...)

Improved treatment of **exp. correl. matrices** uncertainties among measurements

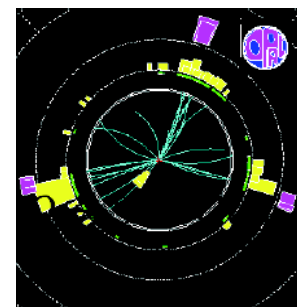
- **EIC**: **New DIS measurements** (also LHeC/FCC-eh): $F_2^{p,d}(x,Q^2)$, $PDF_{pol}(x,Q^2)$, Bj.SR
- FCC: Hadronic **Z (and W) decays** is the **only non-lattice method** known that can reach **permil precision**: Tera-Z machine needed.

(2) α_s from e^+e^- event shapes

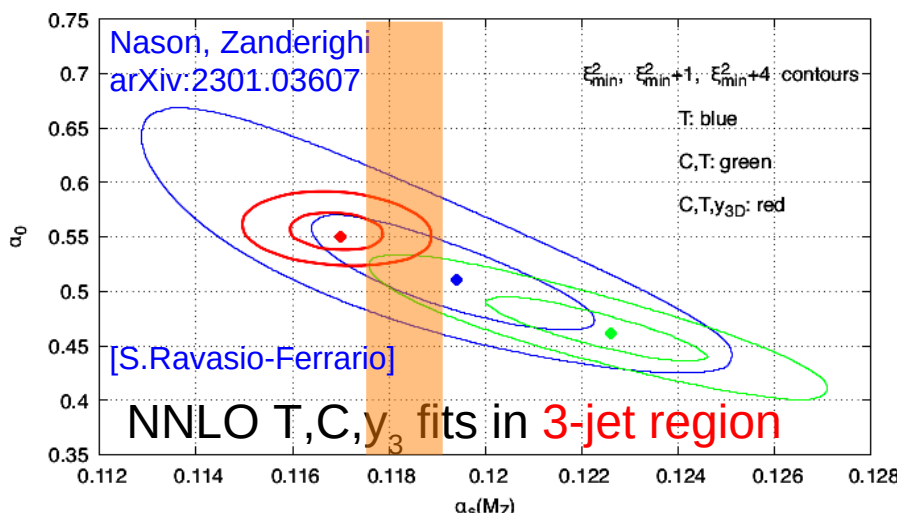
- Computed at $N^{2,3}LO+N^2LL$ accuracy.
- Experimentally (LEP):
Thrust, C-parameter, jet shapes
3-jet x-sections
- Results sensitive to non-pQCD (hadronization) corrections.
- Improved evt-shape power-corrs:

$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



OPAL 3 jet event



- Impact of power corrections & NNLL resummations in the 2-jet limit?
- Uncertainty of mass-scheme to extend evt. shapes to massive hadrons?
- Understand data (incl. reanalysis) diffs.

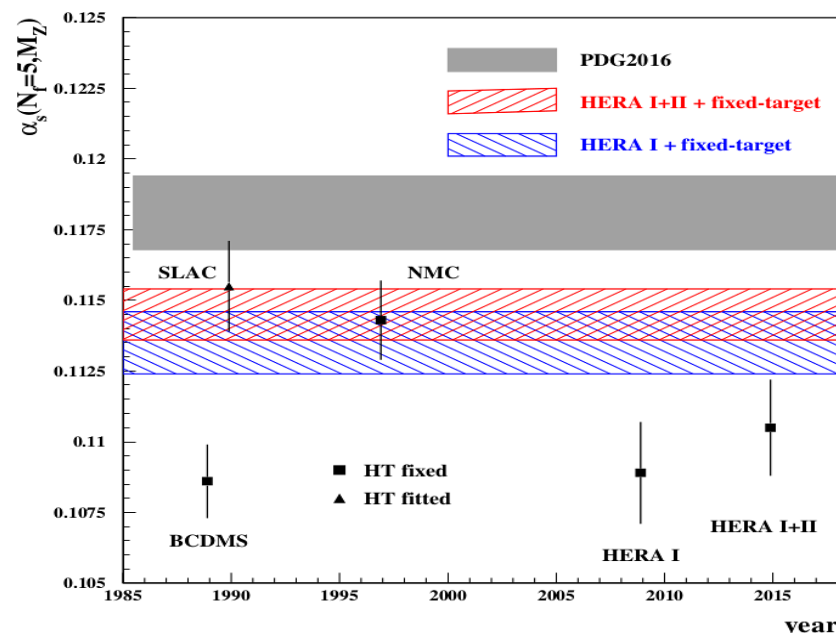
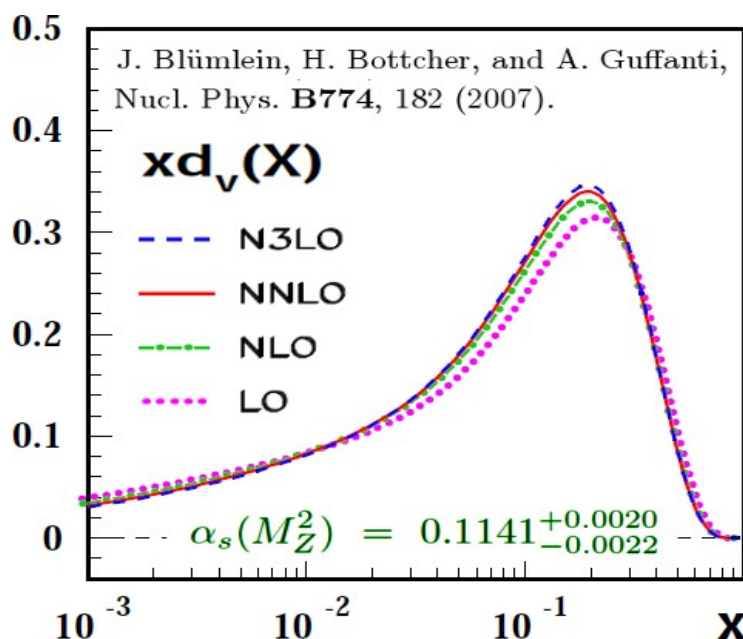
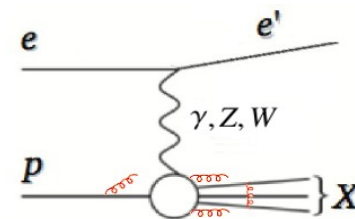
➤ Future prospects:

- Better power-corrs for shapes, ($N^{2,3}LL$) resummation for rates. Grooming?
- New e^+e^- data at lower- $\sqrt{s}$ (Belle-II) & higher- $\sqrt{s}$ (Higgs factories) needed.

α_s from DIS struct. functions

- N^{2,3}LO analysis of (non)singlet struct. functions (BBG, JR14) (and NNLO global PDF AMBP fit) tend to give “lowish” $\alpha_s(M_Z) \approx 0.1150$

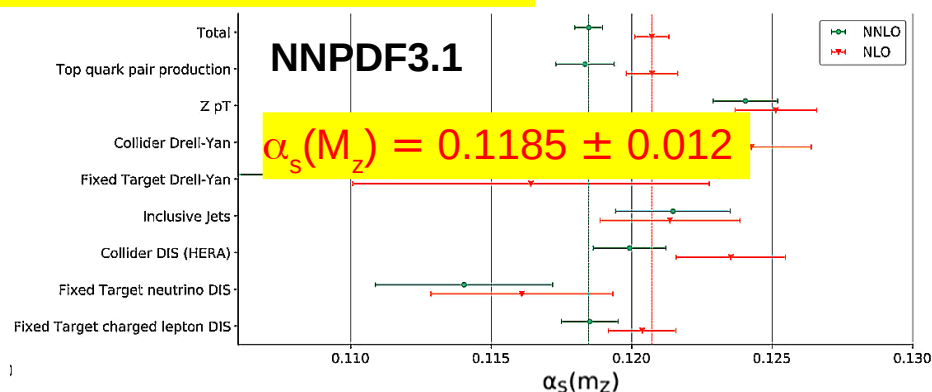
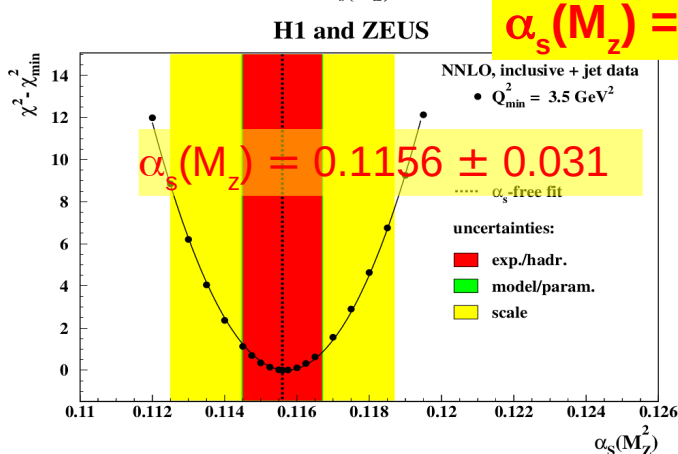
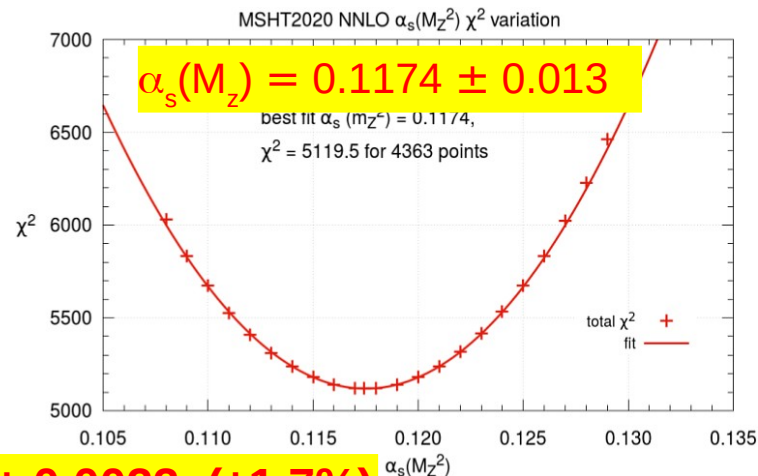
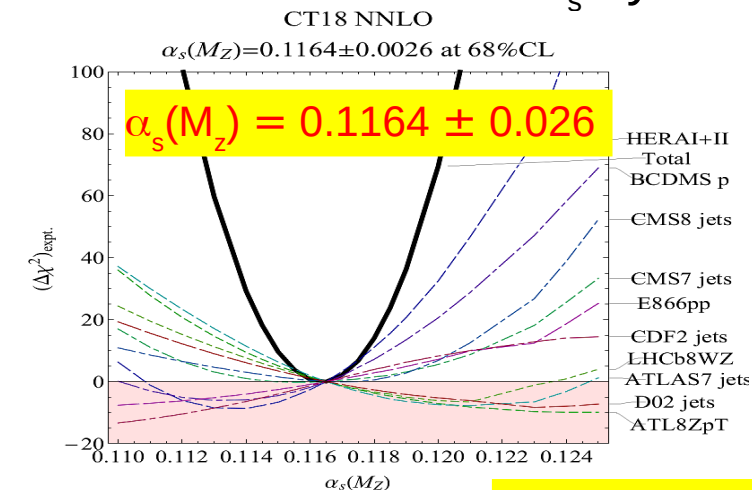
$$F_2(x, Q^2) = x \sum_{n=0}^{\infty} \frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n} \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_{2,i}^{(n)}(z, Q^2, \mu_R^2, \mu_F^2) f_{i/p}\left(\frac{x}{z}, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$



- Neglect of singlet contrihs. for $x > 0.3$ in NS fits? Size of higher-order corrs.?
- ➔ Future prospects: New high-precision $F_2(x, Q^2)$ at EIC.

α_s extraction from global-PDF fits

- **NNLO PDF fits** extract α_s by leaving it float in the global fit (PDF+ α_s fit):



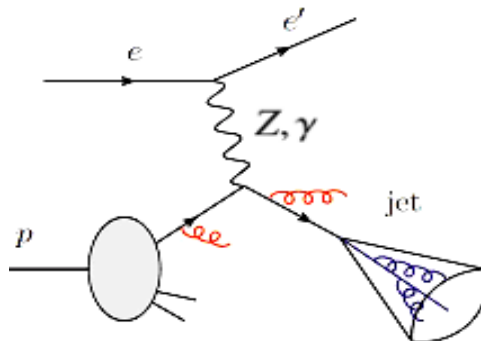
- **Diffs. across fits:** Tolerance factors, weights on specific data sets, parametriz. biases
- **Theory uncertainties** (size of missing HO corrections)? Full **N³LO** upcoming.
- ➡ **Future prospects:** New high-precision (polarized) PDF(x, Q²) at EIC.

α_s from hadron colliders: HERA jet x-sections

■ DIS HERA jet x-sections employed for α_s extractions via NNLOjet:

Double-differential HERA jet data in NC DIS (as function of p_T, Q^2) commonly used for α_s determinations

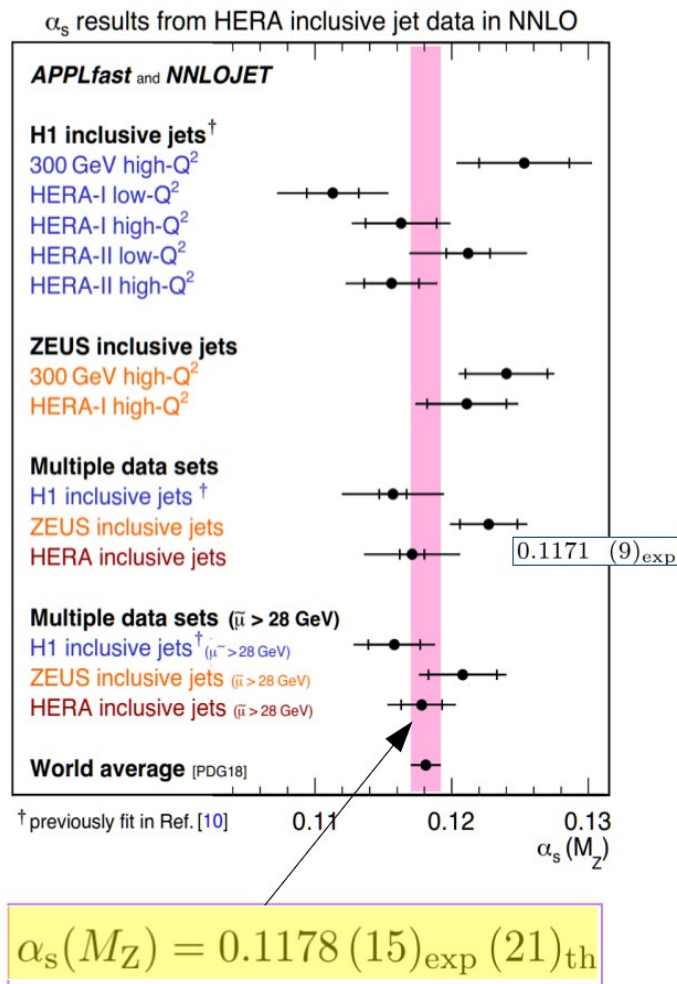
		H1	ZEUS	"Absolute" jet cross sections
				820GeV HERA-I HERA-II
Inclusive jet	low Q^2			
	high Q^2			
Dijet	low Q^2			
	high Q^2			
Three-jet	low Q^2			
	high Q^2			



- α_s from inclusive jet cross sections in NC DIS
- NNLO pQCD w/ non-pert. hadronisation corrections
- H1 and ZEUS consistent
- Sizeable scale uncertainties (MHOU) since data are at comparably low scales
- Highest precision obtained in fit to data with $\mu > 28$ GeV

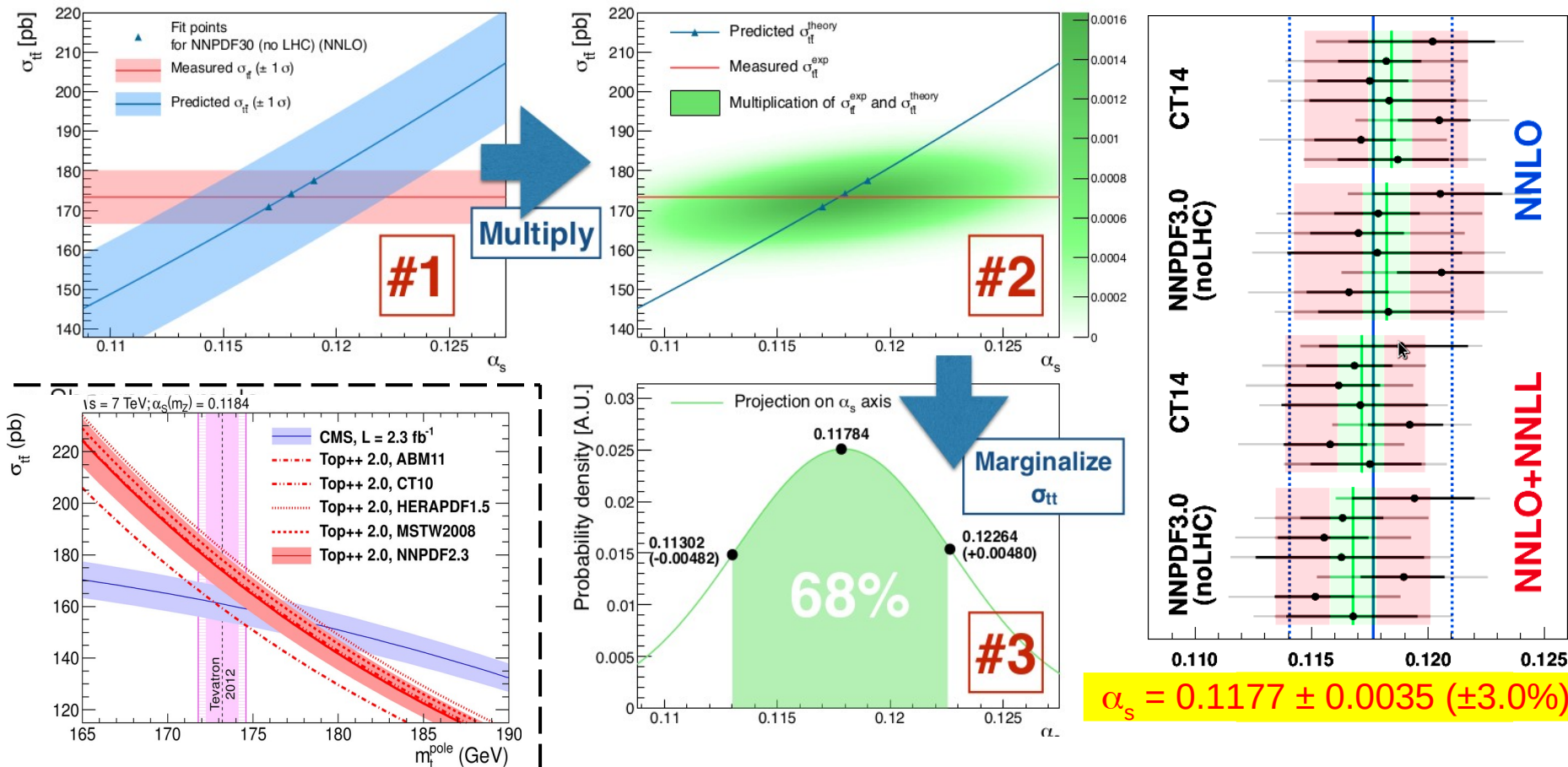
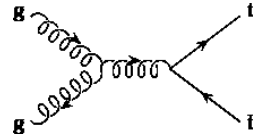
■ Largest uncertainty from missing HO corr.

➤ Future prospects: NNLO jet x-sections-based extractions at EIC



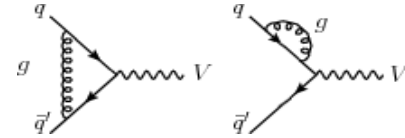
α_s from inclusive LHC ttbar x-sections

- Compare $\sigma(\text{exp}, \text{ttbar})$ to $\sigma(\text{NNLO}, \text{ttbar})$ for diff. PDFs + α_s
+ Advantages: Direct **LO sensitivity to α_s** (via $gg \rightarrow \text{ttbar}$)
– Disadvantages: **$\mathcal{O}(5\%)$ exp/th. uncertainties**, dependence on m_{top}
- First tried by **CMS (pp @ 7TeV)**. Extended to **8 LHC data sets** by Klijnsma et al.

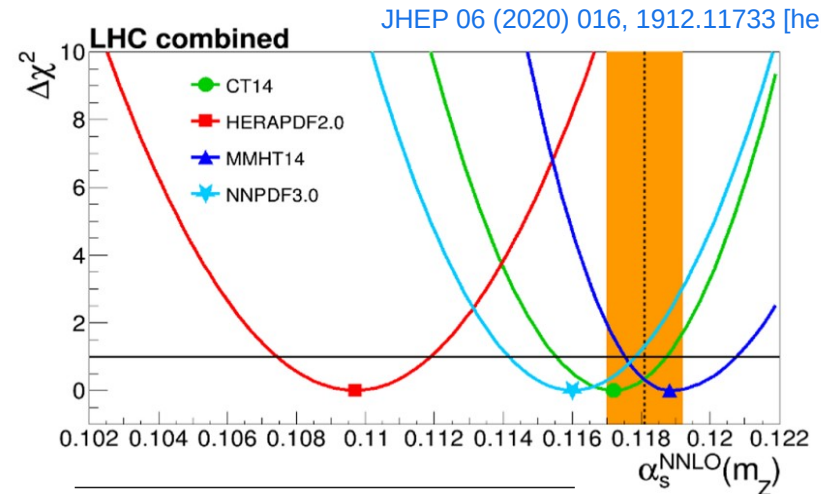
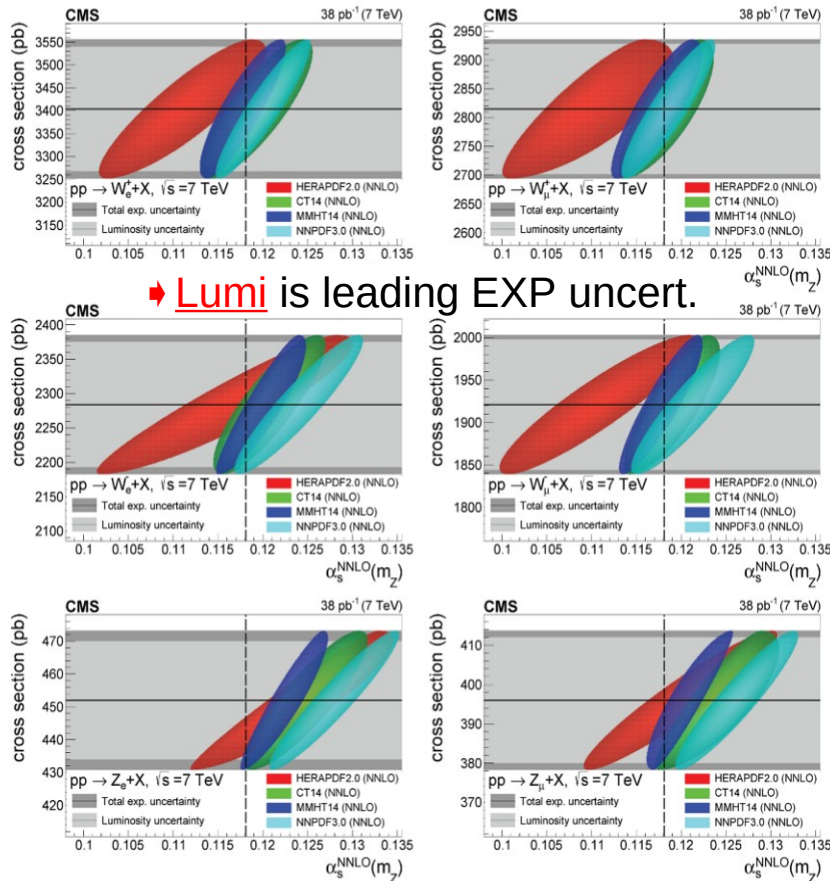


α_s from inclusive LHC W, Z x-sections

- Compare $\sigma(\text{exp}; W, Z)$ to $\sigma(\text{NNLO}; W, Z)$ for diff. PDFs + α_s
+ Advantages: $\mathcal{O}(1-2\%)$ exp/th. uncertainties
– Disadvantages: No LO sensitivity to α_s (only via $K_{\text{NNLO}} \sim 1.3$)



- First tried by CMS (pp @ 7,8 TeV). Extended to 28 LHC data sets by DdE&Poldaru



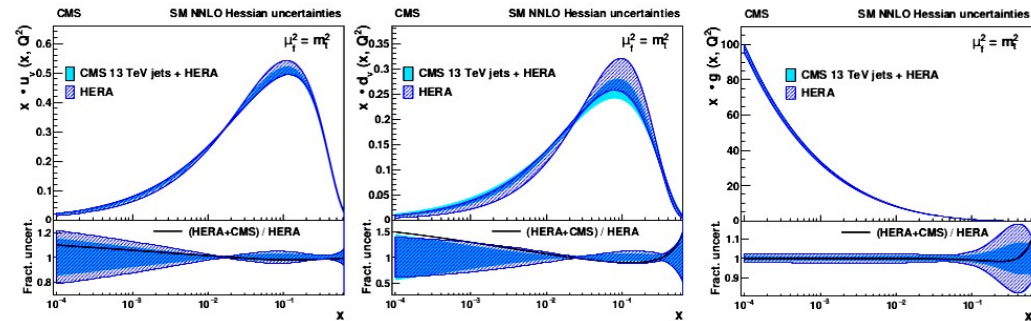
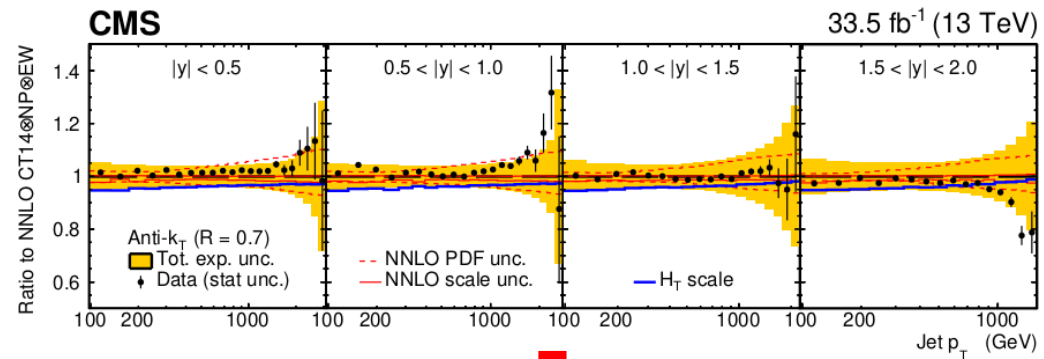
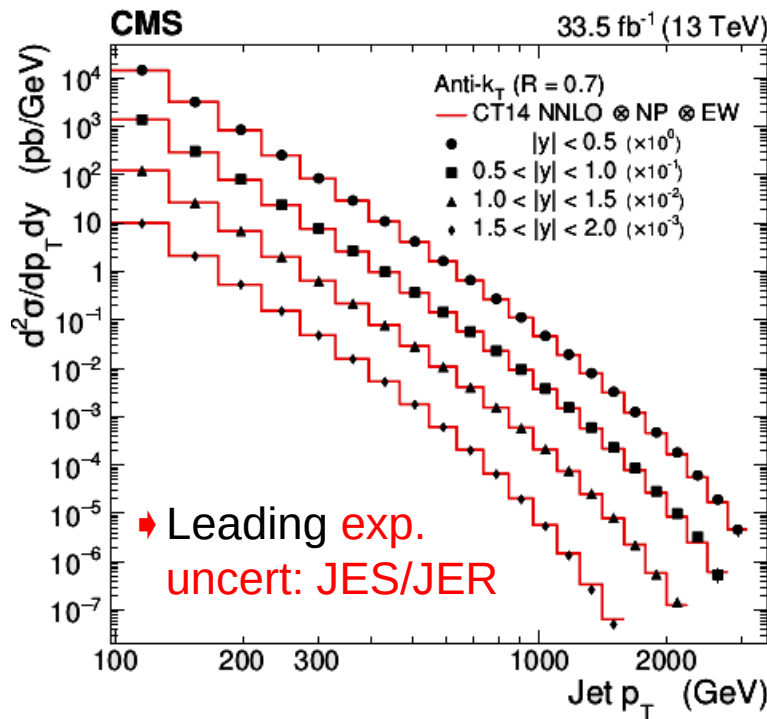
PDF	$\alpha_s(m_Z)$
CT14	$0.1172^{+0.0015}_{-0.0017}$
MMHT14	$0.1188^{+0.0019}_{-0.0013}$
HERAPDF2.0	$0.1097^{+0.0022}_{-0.0023}$
NNPDF3.0	0.1160 ± 0.0018

$$\alpha_s = 0.1180 \pm 0.0016 (\pm 1.3\%)$$

- ➔ **Future:** Add past-2020 data sets + redo (partial) aN³LO PDF fit.

α_s from inclusive jet x-sections (CMS)

- Combined xFitter **NNLO fit of PDF+ α_s** of incl. jet $d^2\sigma/dydp_T$ + HERA DIS data
+ Advantages: Correlation of **gluon PDF & α_s** accounted for.
- CMS 13 TeV [arXiv:2111.10431] **HERAPDF-like analysis**:



- Extraction with **small (mostly fit) uncertainty**: **$\alpha_s = 0.1166 \pm 0.0017$ ($\pm 1.5\%$)**
- Multiple jet spectra **incorporated into upcoming full global-PDF fits**.
(overall tolerance will decrease α_s sensitivity?)

α_s from EEC in multijet events (ATLAS)

■ Multijet energy-energy correlations available at NNLO

+ Advantages: **Small exp. uncertainties** (JES/JER). **PDF uncertainties subleading.**

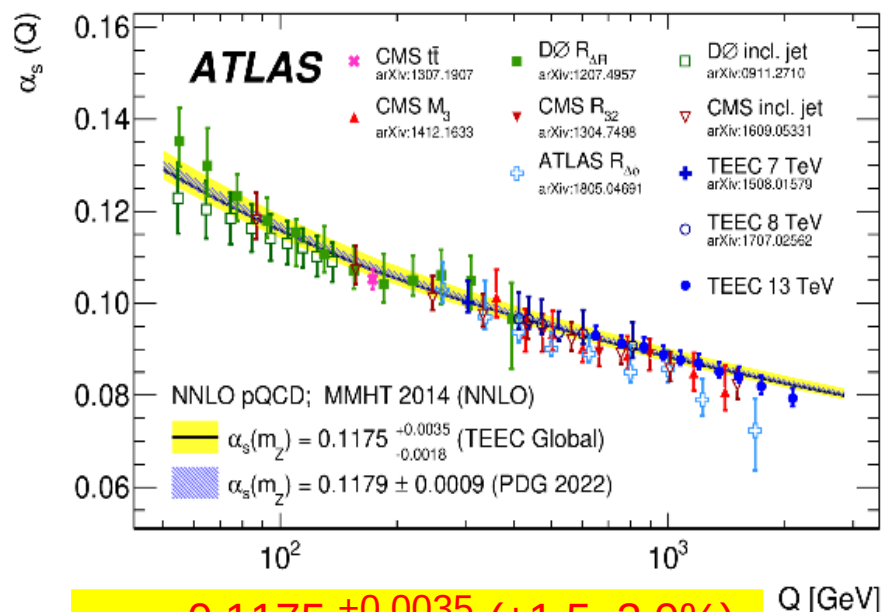
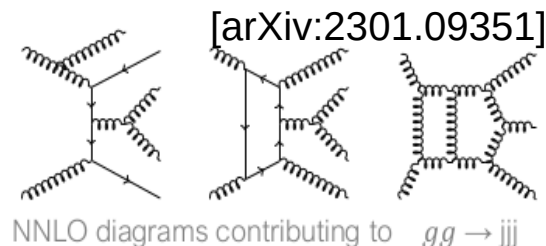
- Use multi-jets transverse energy–energy correlations (**TEEC**) and their associated azimuthal asymmetries (**ATEEC**) to perform the measurement
- The TEEC function is defined as the transverse-energy-weighted distribution of the azimuthal differences between jet pairs in the final state

$$\frac{1}{\sigma} \frac{d\Sigma}{d \cos \phi} = \frac{1}{N} \sum_{A=1}^N \sum_{ij} \frac{E_{Ti}^A E_{Tj}^A}{\left(\sum_k E_{Tk}^A \right)^2} \delta(\cos \phi - \cos \phi_{ij})$$

Also use associated azimuthal asymmetries (ATEEC) to cancel uncertainties symmetric in $\cos(\phi)$

$$\frac{1}{\sigma} \frac{d\Sigma}{d \cos \phi} = \frac{1}{\sigma} \frac{d\Sigma}{d \cos \phi} \Big|_{\phi} - \frac{1}{\sigma} \frac{d\Sigma}{d \cos \phi} \Big|_{\pi-\phi}$$

- Large theoretical improvement from NNLO correction to $gg \rightarrow 3\text{-jet}$ production in pp collisions
 - reduction of theoretical uncertainty by a factor of 3 (**TH scale** still leading uncertainty)



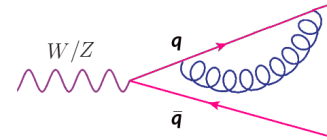
$$\alpha_s = 0.1175^{+0.0035}_{-0.0018} (\pm 1.5\text{--}3.0\%)$$

■ RGE running of QCD coupling tested up to scales $Q \approx H_T/2 \approx 2$ TeV

α_s from Z boson hadronic decays

- ♦ Z-boson decays known at N³LO, no NP uncs. (but only ~4% sensitivity to α_s):

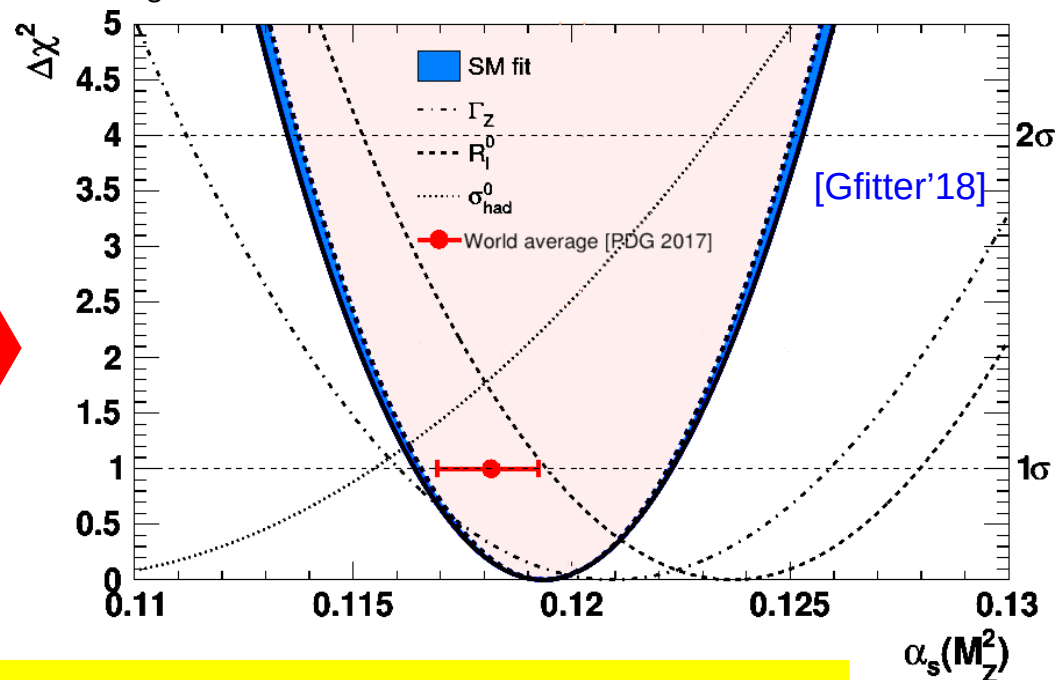
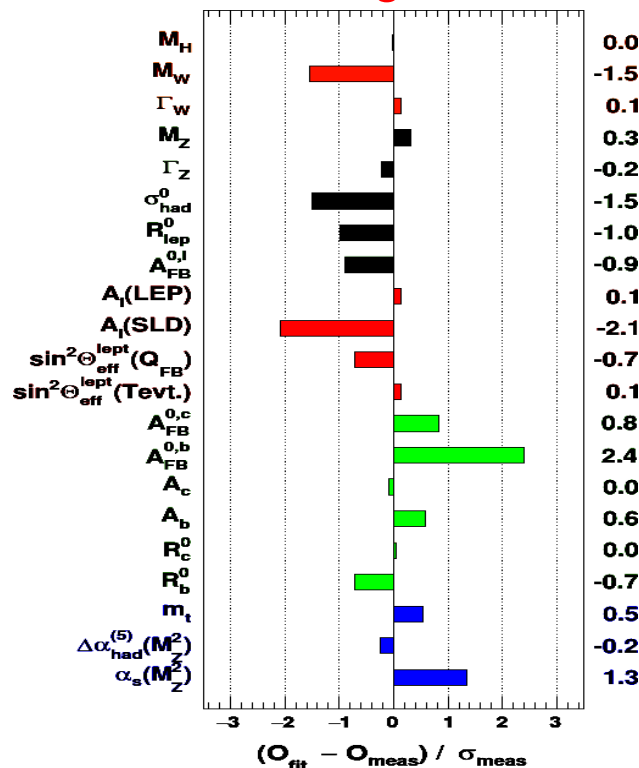
$$R_l^0 \equiv \frac{\Gamma(Z \rightarrow h)}{\Gamma(Z \rightarrow l)} = R_Z^{\text{EW}} N_C \left(1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi} \right)^n + \mathcal{O}(\alpha_s^5) + \delta_m + \delta_{\text{np}} \right)$$



- ♦ Extraction from three Z-peak pseudo-observables (LEP, SLC):

$$R_\ell^0 = \frac{\Gamma_{\text{had}}}{\Gamma_\ell}, \sigma_{\text{had}}^0 = \frac{12\pi}{m_Z} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}, \sigma_\ell^0 = \frac{12\pi}{m_Z} \frac{\Gamma_\ell^2}{\Gamma_Z^2} \Rightarrow \alpha_s(M_Z) = 0.1221 \pm 0.0027 \quad (\pm 2.3\%)$$

- ♦ Also from the global EW fit leaving α_s as single free parameter:



$$\alpha_s(M_Z) = 0.1203 \pm 0.0028 \quad (\pm 2.3\%)$$

QCD coupling at e^+e^- (FCC-ee)

EW boson pseudoobservables known at N³LO in pQCD:

- The W and Z hadronic widths :

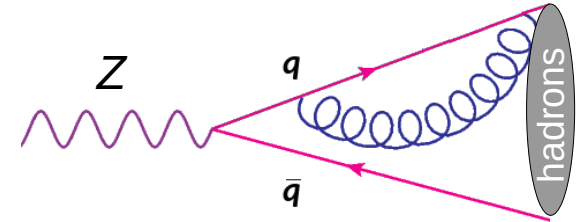
$$\Gamma_{W,Z}^{\text{had}}(Q) = \Gamma_{W,Z}^{\text{Born}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{EW}} + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

- The ratio of W, Z hadronic-to-leptonic widths :

$$R_{W,Z}(Q) = \frac{\Gamma_{W,Z}^{\text{had}}(Q)}{\Gamma_{W,Z}^{\text{lep}}(Q)} = R_{W,Z}^{\text{EW}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

- In the Z boson case, the hadronic cross section at the resonance peak in e^+e^- :

$$\sigma_Z^{\text{had}} = \frac{12\pi}{m_Z} \cdot \frac{\Gamma_Z^e \Gamma_Z^{\text{had}}}{(\Gamma_Z^{\text{tot}})^2}$$



Note: Sensitivity to $\alpha_s(m_Z)$ via $\mathcal{O}(4\%)$ higher-order corrs.

[arXiv:2005.04545]

FCC-ee will reach 0.1% precision on $\alpha_s(m_Z)$ ($\times 20$ better than LEP):

- Huge Z pole stats. ($\times 10^5$ LEP):
- Exquisite syst./parametric precision:

$$\Delta R_Z = 10^{-3}, \quad R_Z = 20.7500 \pm 0.0010$$

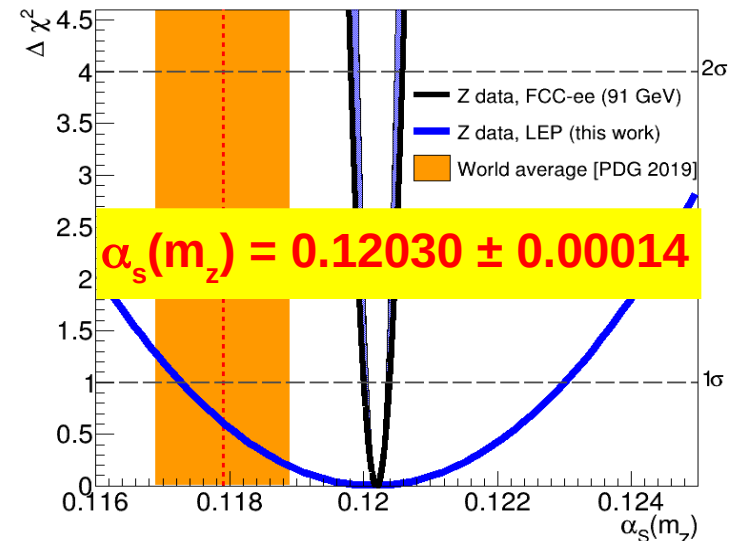
$$\Delta \Gamma_Z^{\text{tot}} = 0.1 \text{ MeV}, \quad \Gamma_Z^{\text{tot}} = 2495.2 \pm 0.1 \text{ MeV}$$

$$\Delta \sigma_Z^{\text{had}} = 4.0 \text{ pb}, \quad \sigma_Z^{\text{had}} = 41494 \pm 4 \text{ pb}$$

$$\Delta m_Z = 0.1 \text{ MeV}, \quad m_Z = 91.18760 \pm 0.00001 \text{ GeV}$$

$$\Delta \alpha = 3 \cdot 10^{-5}, \quad \Delta \alpha_{\text{had}}^{(5)}(m_Z) = 0.0275300 \pm 0.0000009$$

- TH uncertainty needs to be reduced by $\times 4$ from missing $\alpha_s^5, \alpha^3, \alpha\alpha_s^2, \alpha\alpha_s^2, \alpha^2\alpha_s$ terms



QCD coupling at e^+e^- (FCC-ee)

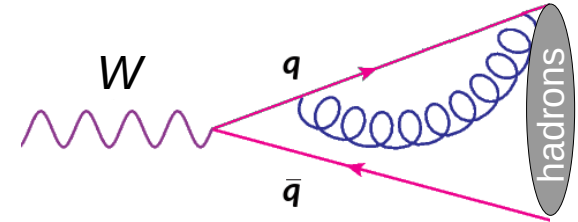
EW boson pseudoobservables known at $N^3\text{LO}$ in pQCD:

- The W and Z hadronic widths :

$$\Gamma_{W,Z}^{\text{had}}(Q) = \Gamma_{W,Z}^{\text{Born}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{EW}} + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

- The ratio of W, Z hadronic-to-leptonic widths :

$$R_{W,Z}(Q) = \frac{\Gamma_{W,Z}^{\text{had}}(Q)}{\Gamma_{W,Z}^{\text{lep}}(Q)} = R_{W,Z}^{\text{EW}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$



Note: Sensitivity to $\alpha_s(m_Z)$ via $\mathcal{O}(4\%)$ higher-order corr.

[arXiv:2005.04545]

FCC-ee will reach 0.2% precision on $\alpha_s(m_W)$ ($\times 300$ better than LEP):

- Huge W pole stats. ($\times 10^4$ LEP-2).
- Exquisite syst./parametric precision:

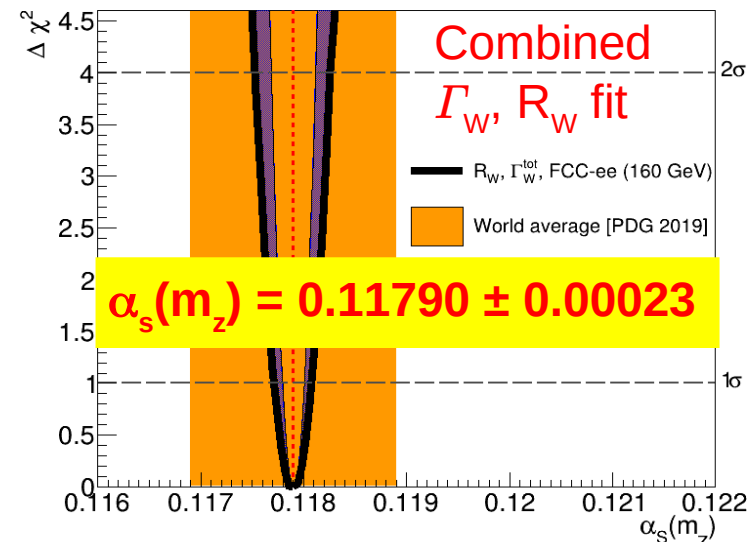
$$\Gamma_W^{\text{tot}} = 2088.0 \pm 1.2 \text{ MeV}$$

$$R_W = 2.08000 \pm 0.00008$$

$$m_W = 80.3800 \pm 0.0005 \text{ GeV}$$

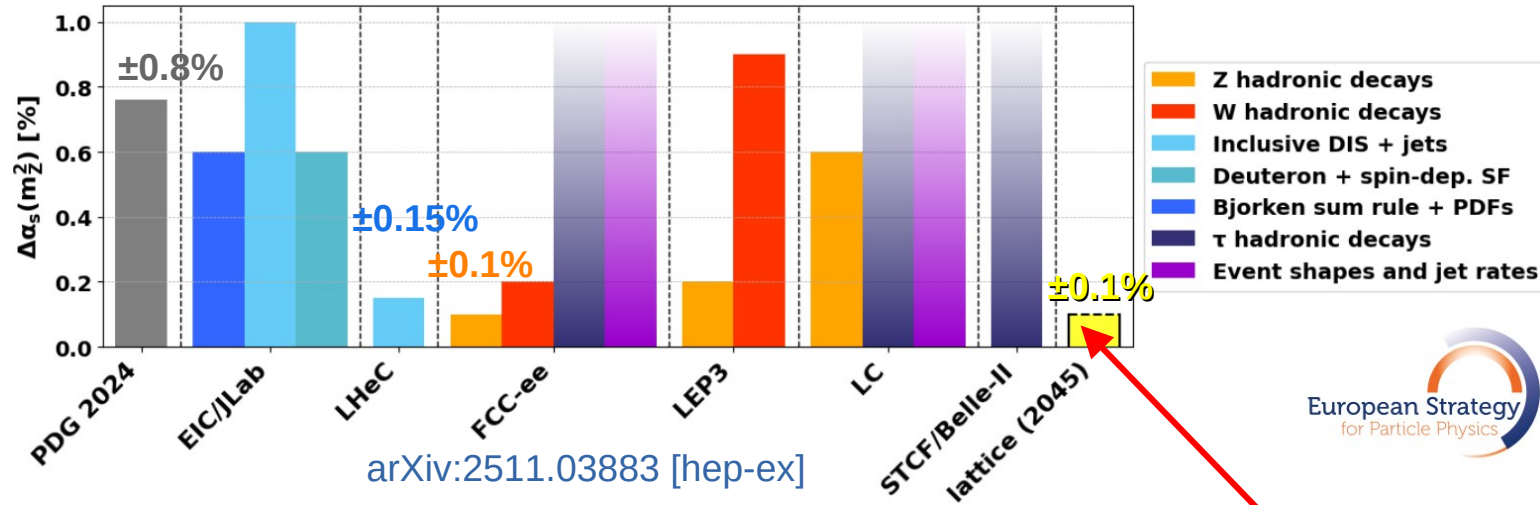
$$|V_{cs}| = 0.97359 \pm 0.00010 \quad \leftarrow \mathcal{O}(10^{12}) \text{ D mesons}$$

- TH uncertainty needs to be reduced by $\times 10$ from missing $\alpha_s^5, \alpha_s^2, \alpha_s^3, \alpha\alpha_s^2, \alpha\alpha_s^2, \alpha^2\alpha_s$ terms



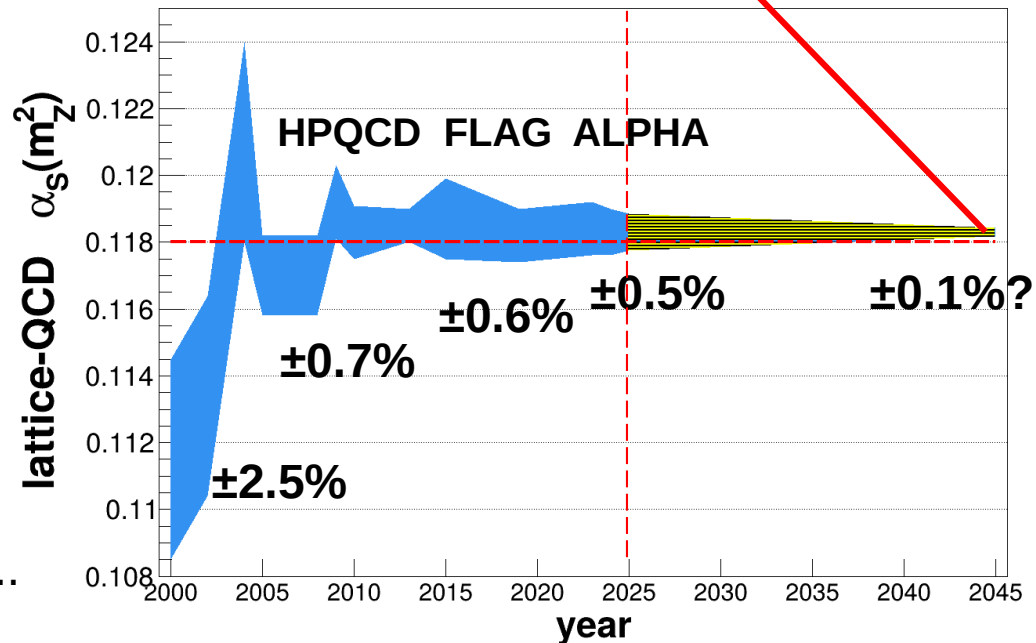
Summary: QCD coupling α_s

■ **Forecast**
precision
at future
facilities:



■ Lattice-QCD:

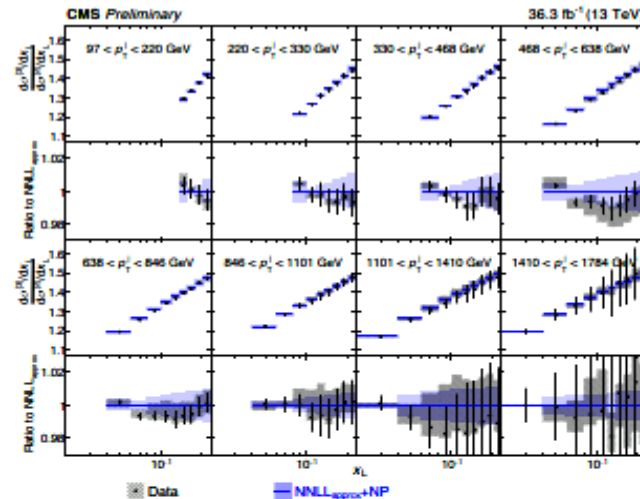
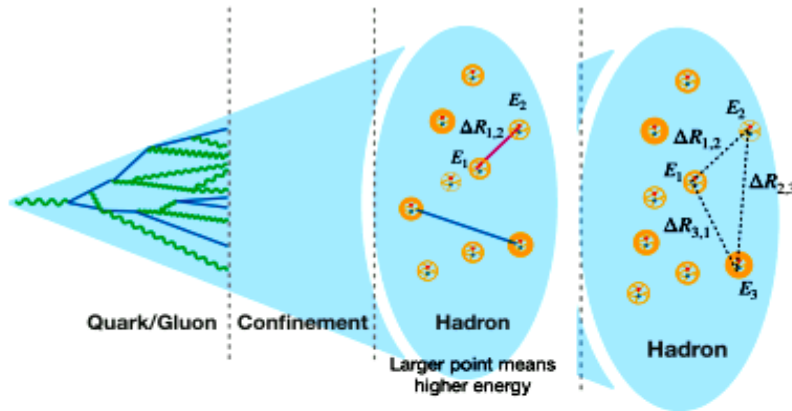
- **Step-scaling & decoupling** are state-of-the-art techniques.
- **Stat. uncert.** $\propto (\text{computing power})^2$ reducible by $\sim 1/2$ every 10 yrs
- **pQCD** observables needed at higher accuracy: α_s^n
- **Syst. uncerts.** will start playing role: QED, $m_{u,d}$, dynamical **charm mass**,...



α_s from energy correlations inside jets (CMS)

- 3-,2-particle **jet substructure energy correlators** (E3C and E2C) as a function of ang. separation (x_L) known **at NLO+NNLL**. Advantages: **Insensitive to PDFs**. Uncorrelated with other extractions

(NLO, not in PDG)

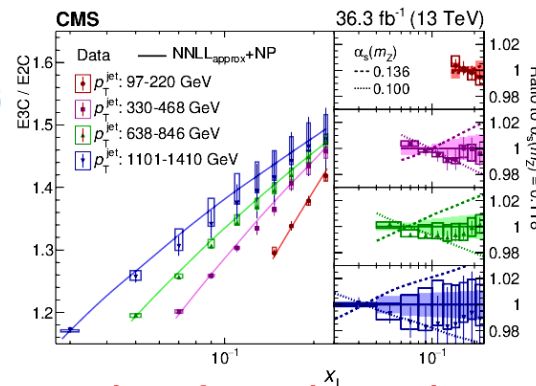


SMP-22-015

Unfolded
E3C/E2C

- In QFT at LL, E3C/E2C is linear function of α_s :

$$\Delta \propto \alpha_s(Q) \ln x_L + O(\alpha_s^2)$$



$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050} \quad (< 4.1 \% \text{ rel.})$$

PRL133 (2024) 071903,
arXiv:2402.13864

- New & **most precise** α_s extraction from **jet substructure** so far.

Does E3C/E2C $\propto \alpha_s$ hold at NⁿLL? **NNLO** theory soon?