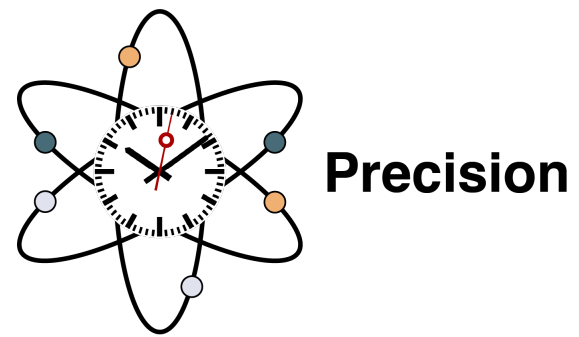




Recent developments and results in (precision) lattice QCD+QED calculations

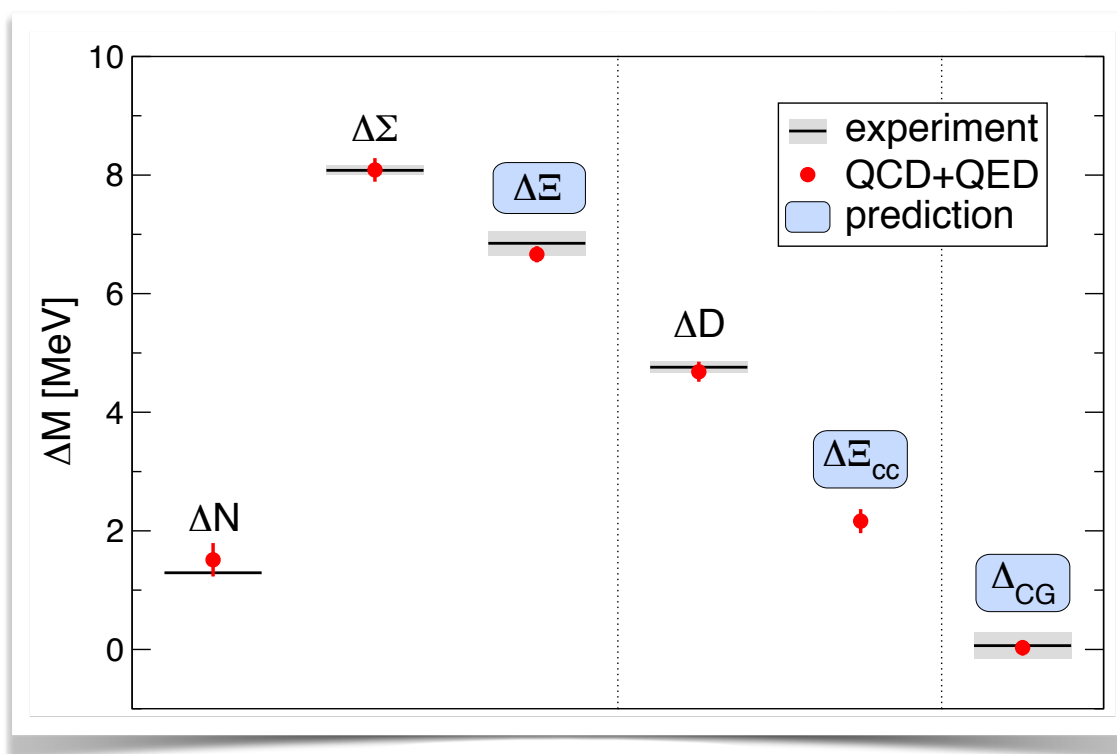
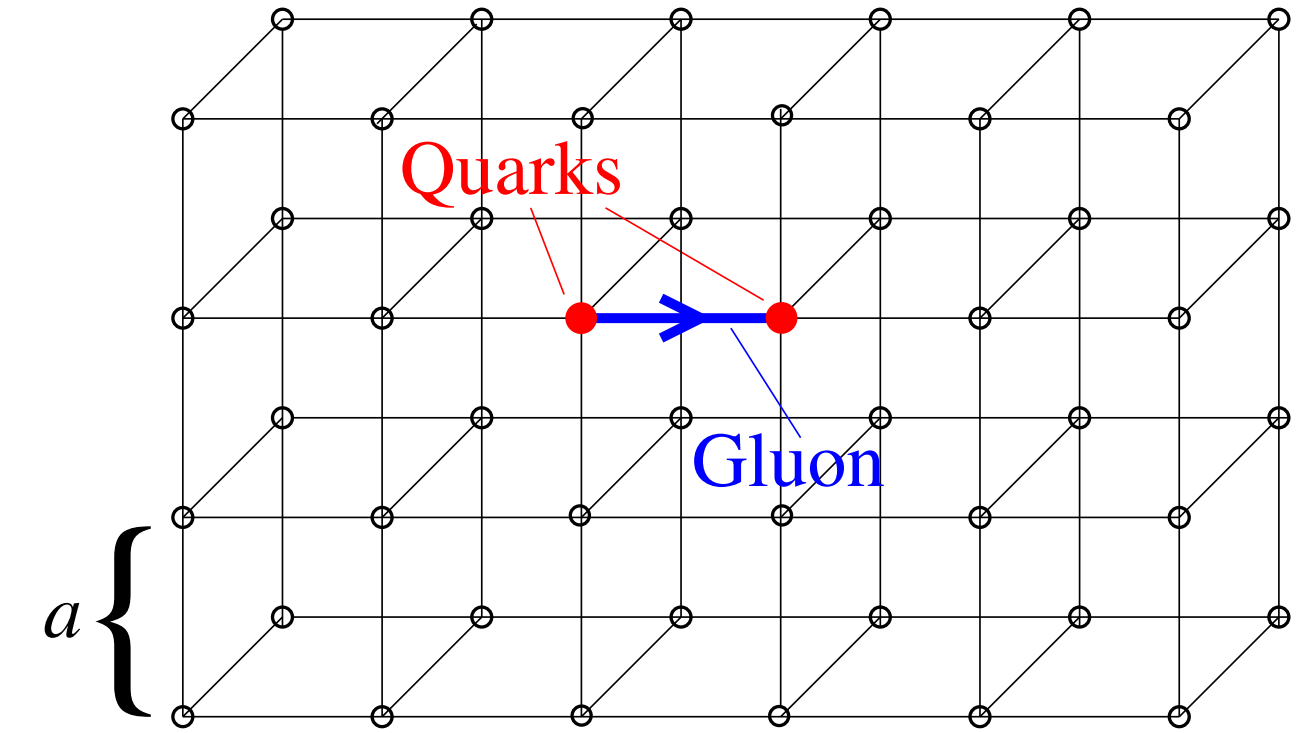
MARINA KRSTIC MARINKOVIC

marinama@ethz.ch

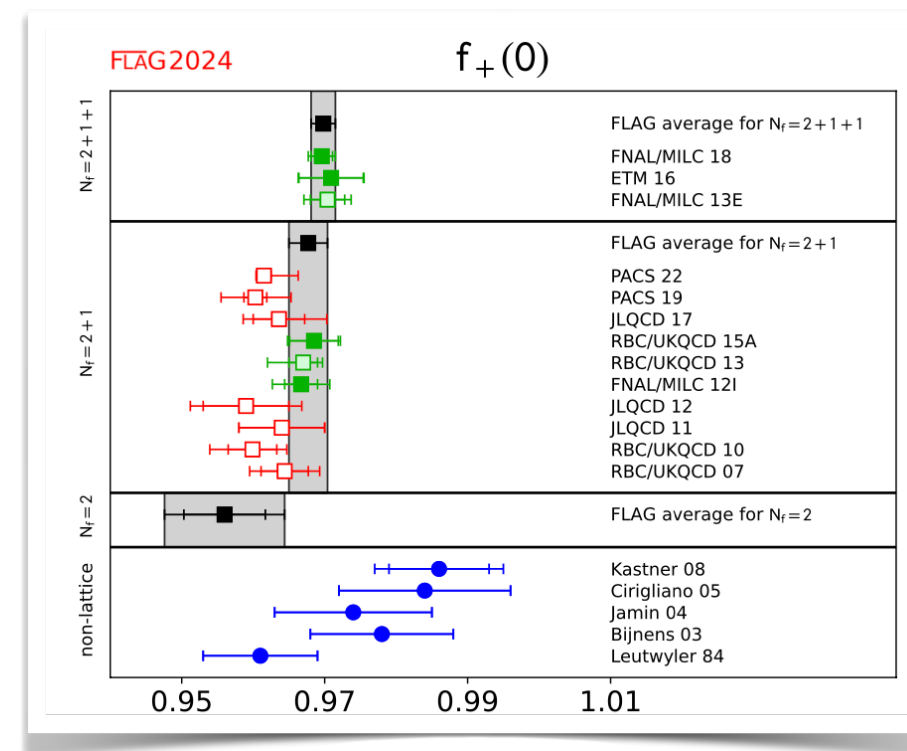
Lattice QCD as a precision tool over the past decade

OBSERVABLE ACCESSIBLE AT $O(1\%)$ PRECISION (or higher):

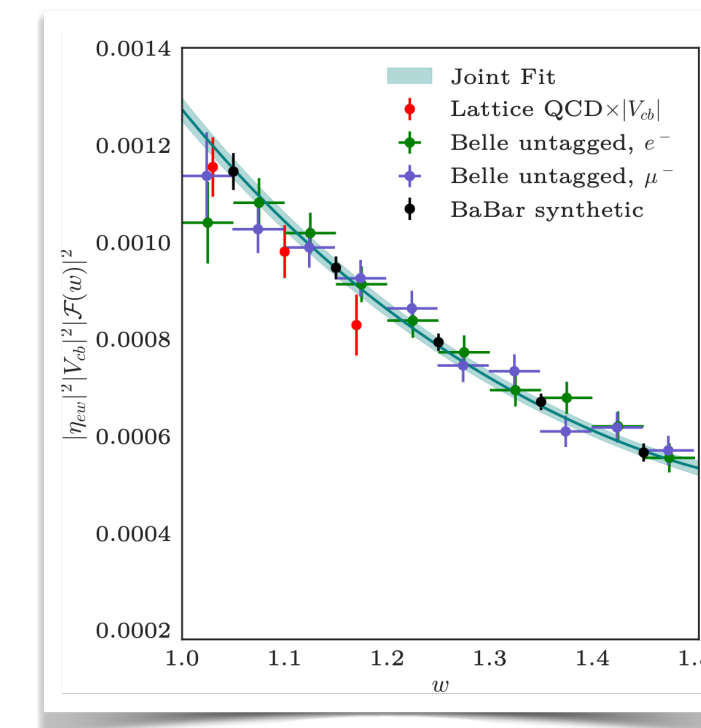
- ✦ From precision hadron spectroscopy to observables that are not ordinary Euclidean hadron matrix elements
- ✦ Heavy flavour matrix elements [PDG CKM lattice review + FLAG]
[ALPHA, Dalla Brida et al. Nature 652 (2026) 2501.06633]
- ✦ QCD parameters: $m_u, m_d, m_s, m_c, m_b, (m_t), \alpha_s$ [Panel after this talk, Alberto Ramos, Giulia Zanderighi, David d'Enterria, Tue. 11.30am]
[Avkhadiev et al. PRD 108, 114505 (2023) 2307.12359]
- ✦ Also relevant for LHC: PDFs, quasi-PDFs, other factorization inputs (e.g. Collins-Soper kernel)
- ✦ Prototypical precision observable: leading hadronic contribution to the muon g-2



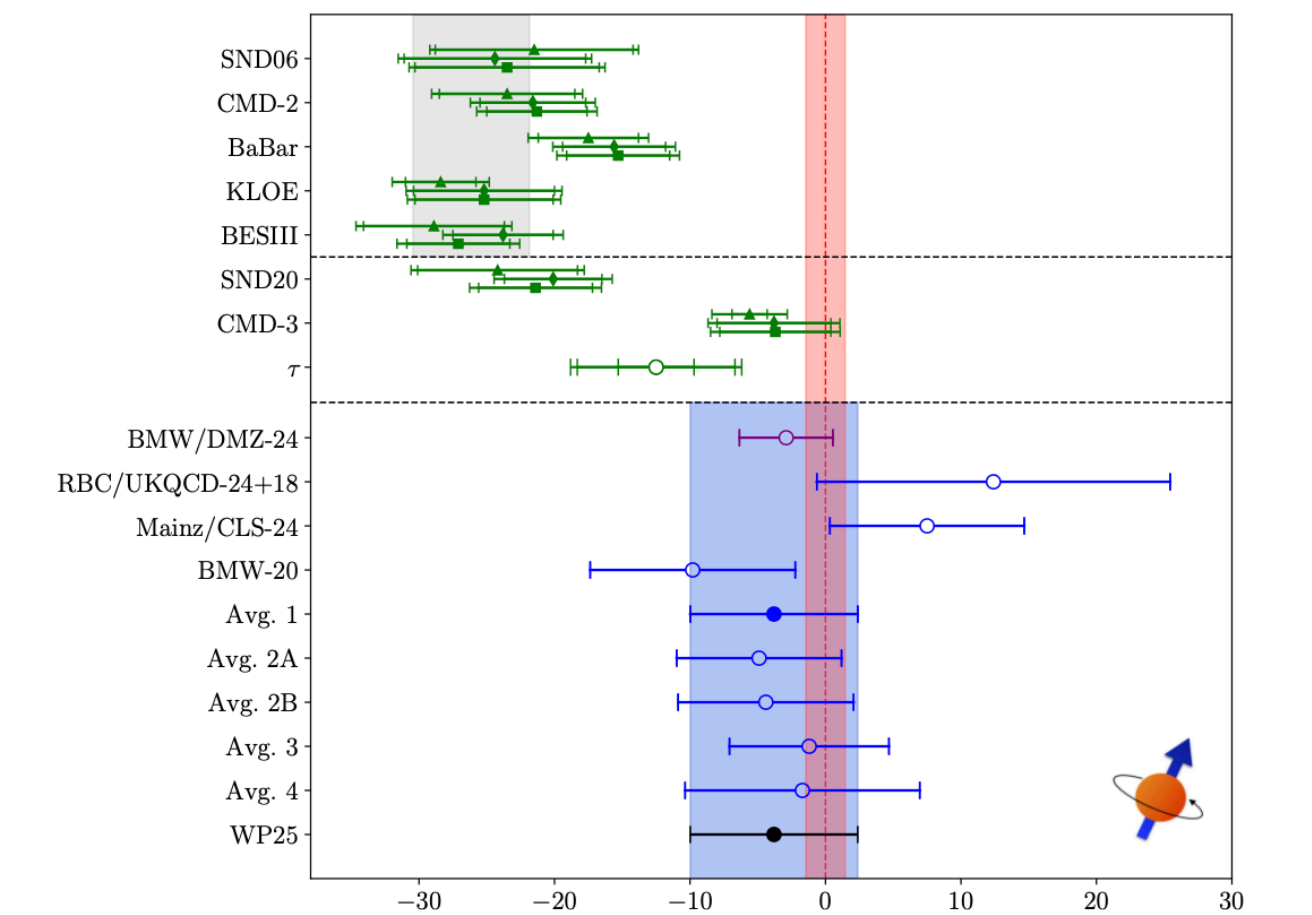
$M_p - M_n$ [Borsanyi et al. 1406.4088]



FLAG24 [Aoki et al. 2411.04268]



$|V_{cb}|$ from $B \rightarrow D^* l \nu$
[Bazavov et al. 2105.14019]

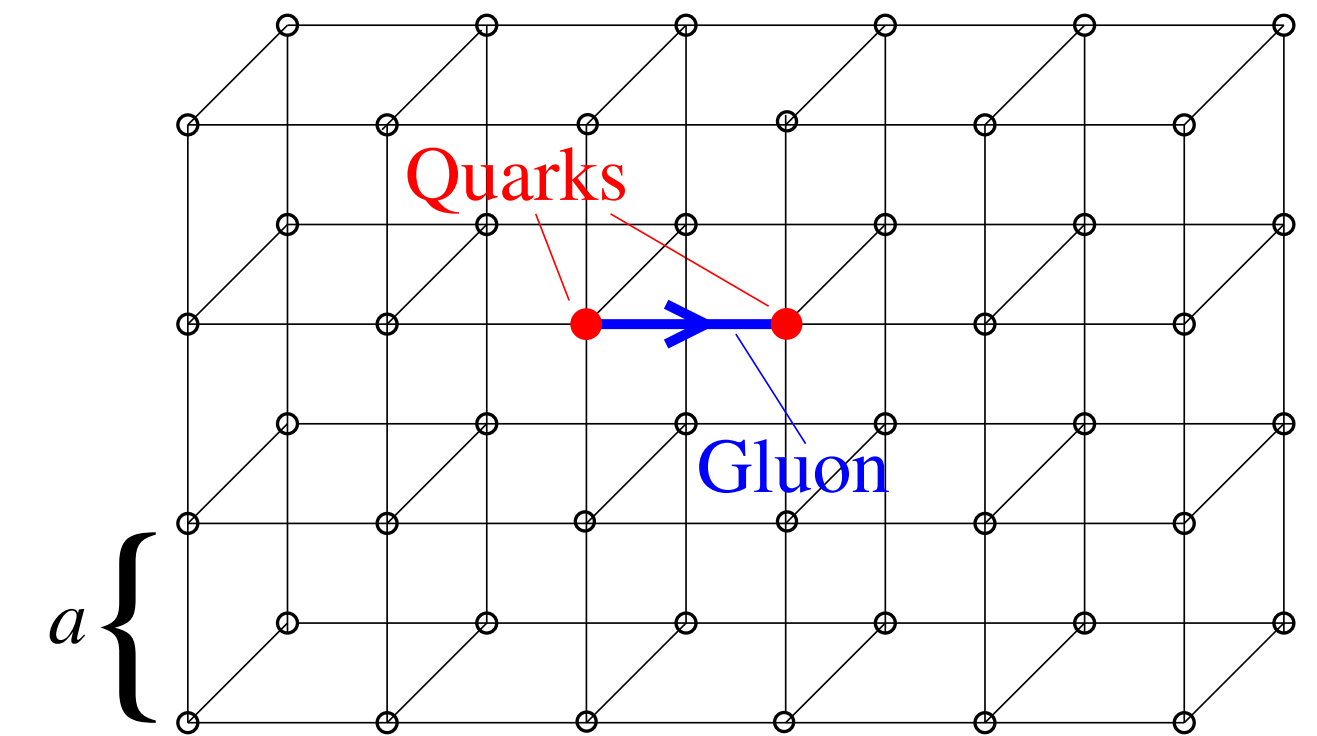


[Muon g-2 theory Initiative, White Paper 2025, 2505.21476 [hep-ph]]

Lattice QCD as a precision tool over the next decade

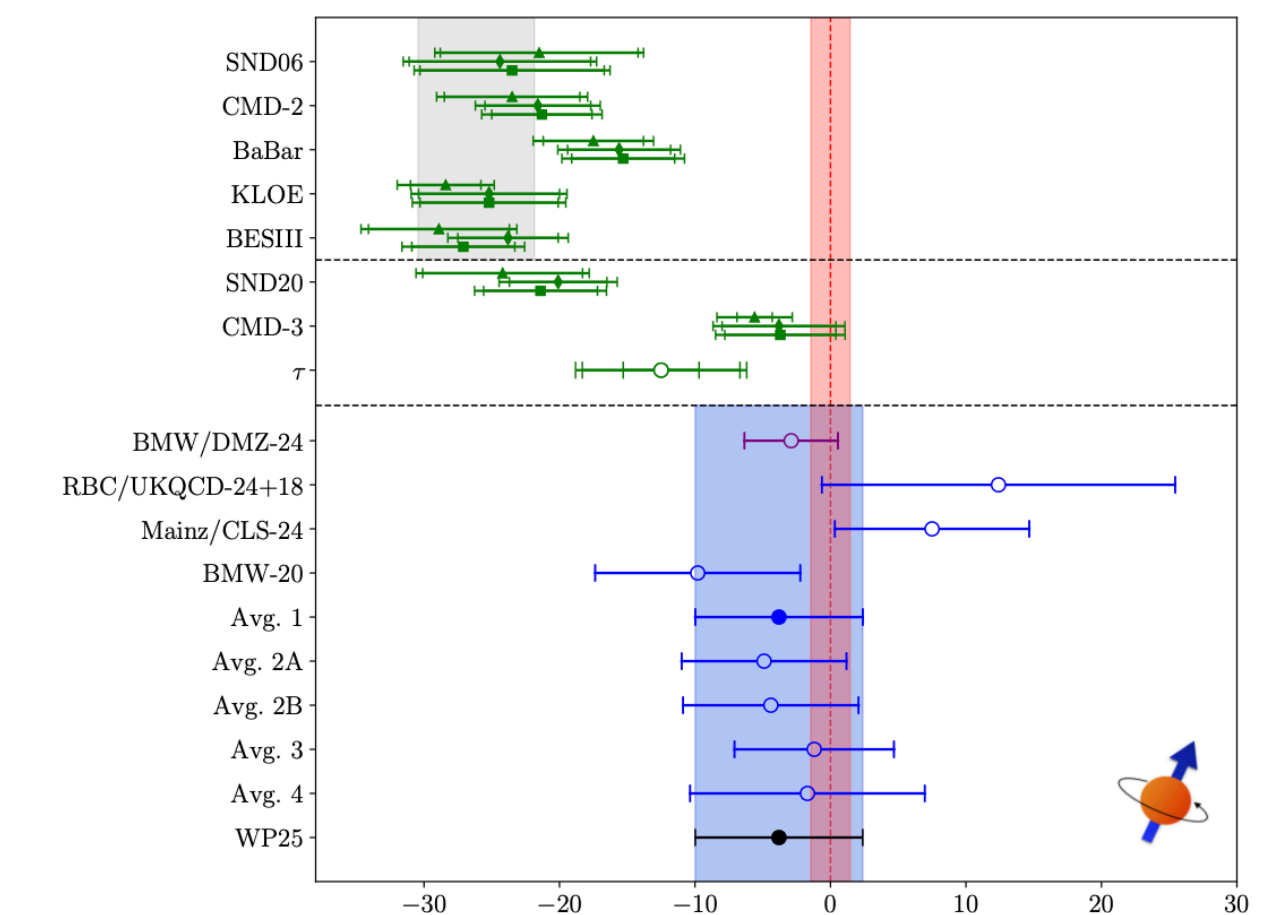
OBSERVABLE ACCESSIBLE AT $O(1\%)$ PRECISION (or higher):

- ✦ From precision hadron spectroscopy to observables that are not ordinary Euclidean hadron matrix elements
- ✦ Heavy flavour matrix elements [PDG CKM lattice review + FLAG]
[ALPHA, Dalla Brida et al. Nature 652 (2026) 2501.06633]
- ✦ QCD parameters: $m_u, m_d, m_s, m_c, m_b, (m_t), \alpha_s$ [Panel after this talk, Alberto Ramos, Giulia Zanderighi et al, Tue. 11.30am]
[Avkhadiev et al. PRD 108, 114505 (2023) 2307.12359]
- ✦ Also relevant for LHC: PDFs, quasi-PDFs, other factorization inputs (e.g. Collins-Soper kernel)
- ✦ Prototypical precision observable: leading hadronic contribution to the muon g-2



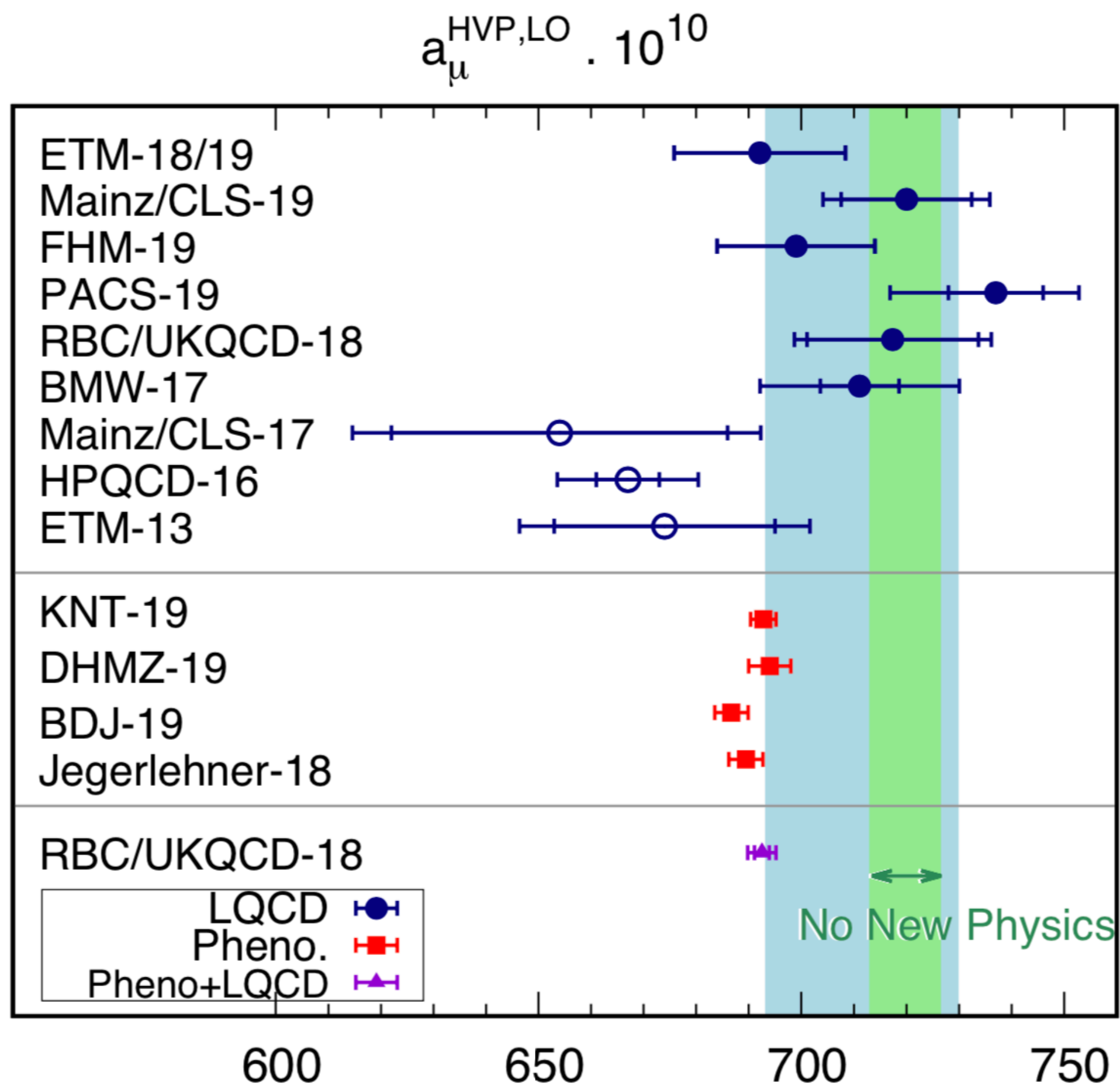
TECHNICAL CHALLENGES:

- ✦ At the current level of precision, not possible to ignore a range of systematic effects
 - ➔ Disconnected diagrams
 - ➔ Noise at long distance in Euclidean correlators
 - ➔ Isospin-breaking (IB) corrections
 - ➔ Multi-hadron states ...
- ✦ All these must be included in lattice computations of observables where sub-percent precision is a goal

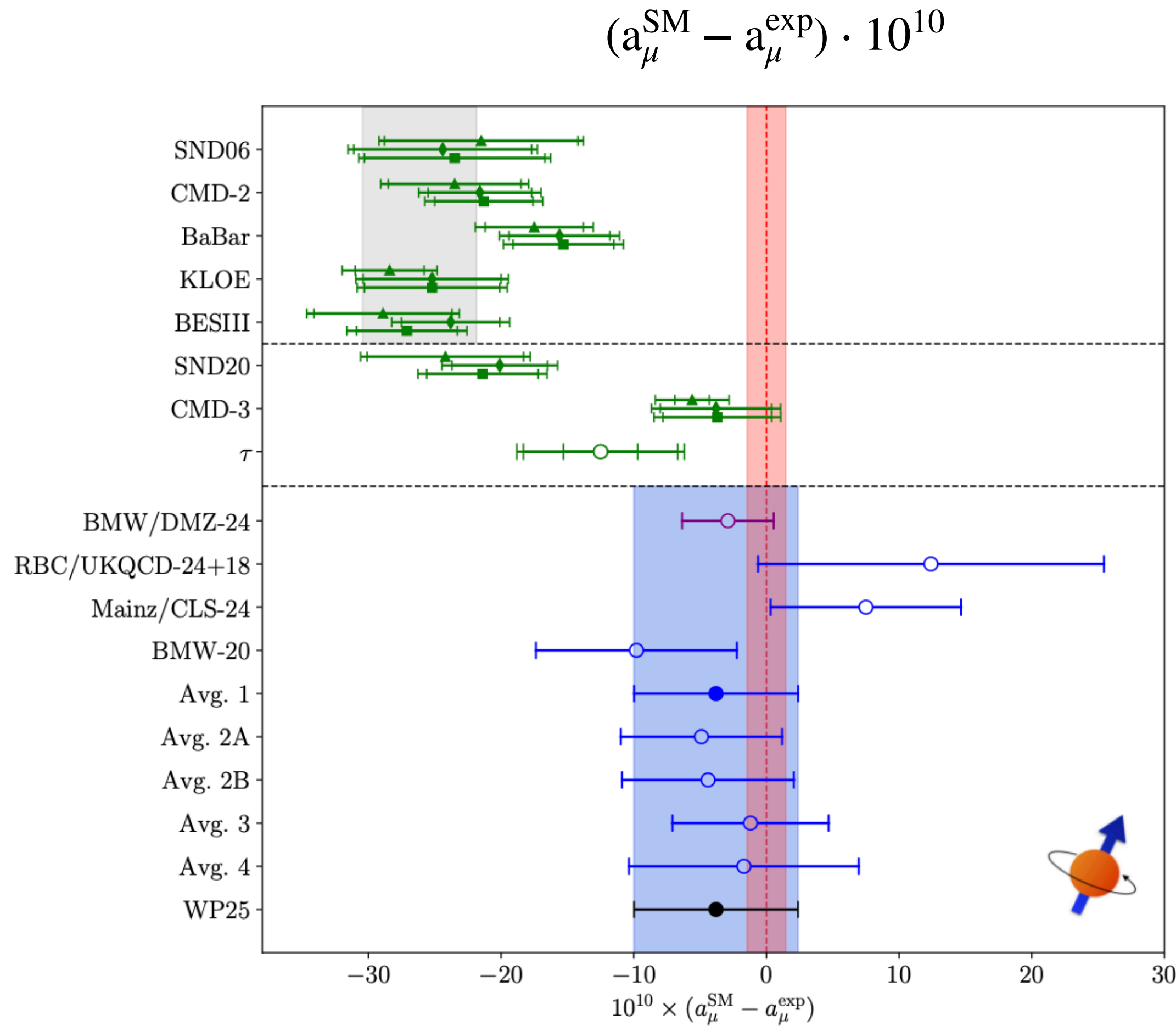


[Muon g-2 theory Initiative, White Paper 2025, 2505.21476 [hep-ph]]

Muon g-2: hadronic contributions to $a_\mu = \frac{g_\mu - 2}{2}$

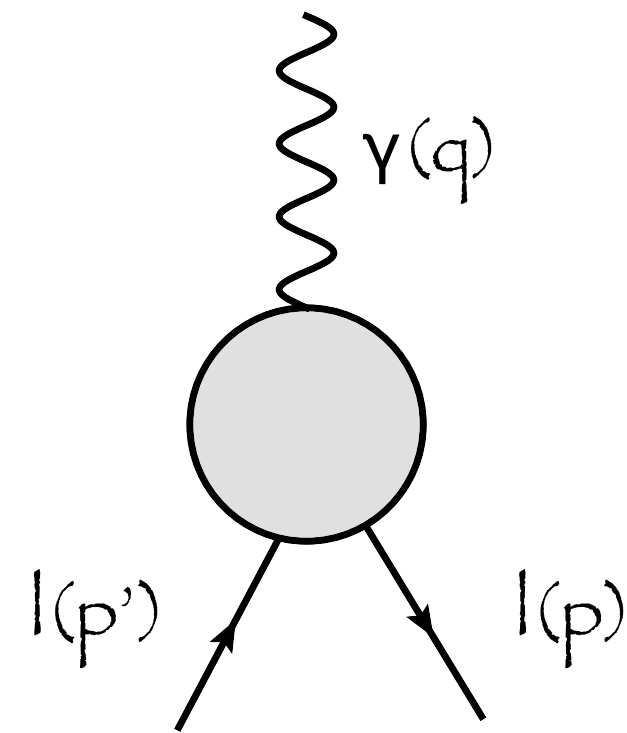


[WP20: Muon g-2 theory Initiative, Phys.Rept. 887 (2020), I-I66
arXiv:2006.04822 [hep-ph]]



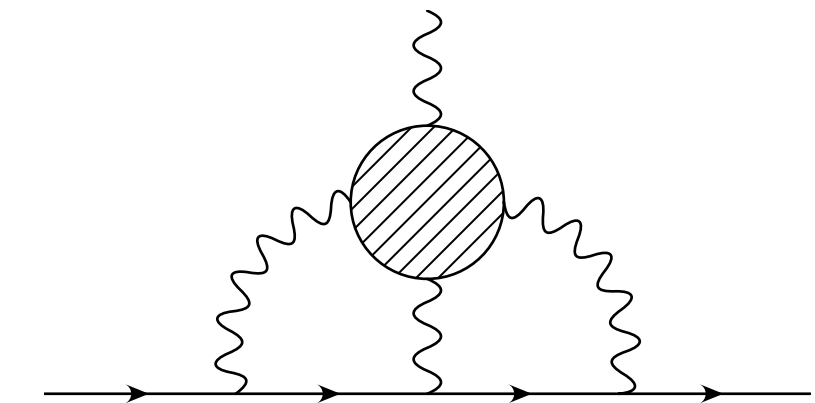
[WP25: Muon g-2 theory Initiative, Aliberti et al., Phys.Rept. 1143 (2025) I-I58 arXiv:2505.21476 [hep-ph]]

Non-perturbative calculations of hadronic contributions to $a_\mu = \frac{g_\mu - 2}{2}$



$$(-ie) \bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p)$$

$$F_1(0) = 1 \text{ and } g_l = 2 + 2F_2(0)$$



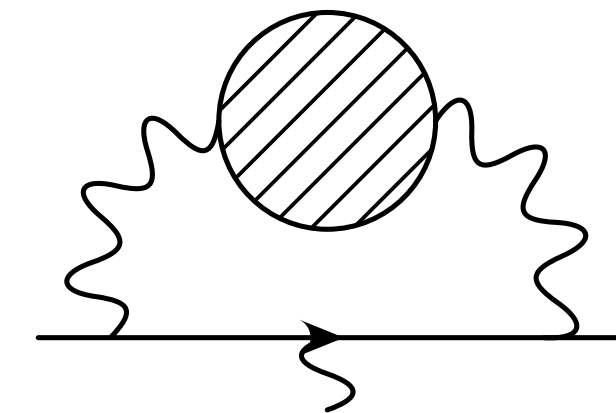
HLbL

- ♦ Hadronic contributions are obtained by integrating over all possible virtual photon momenta



$$\hat{\Pi}(q^2)$$

$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi} \right)^2 \int dq^2 \omega(q^2) \hat{\Pi}(q^2)$$

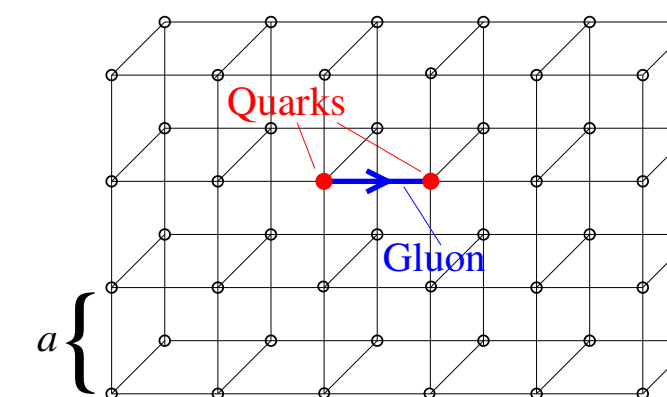


HVP

- ♦ P.T. cannot reliably compute full hadronic contribution
- ♦ Two-point & four-point correlation functions:

$$\text{HVP: } \langle 0 | T\{j_\mu j_\nu\} | 0 \rangle$$

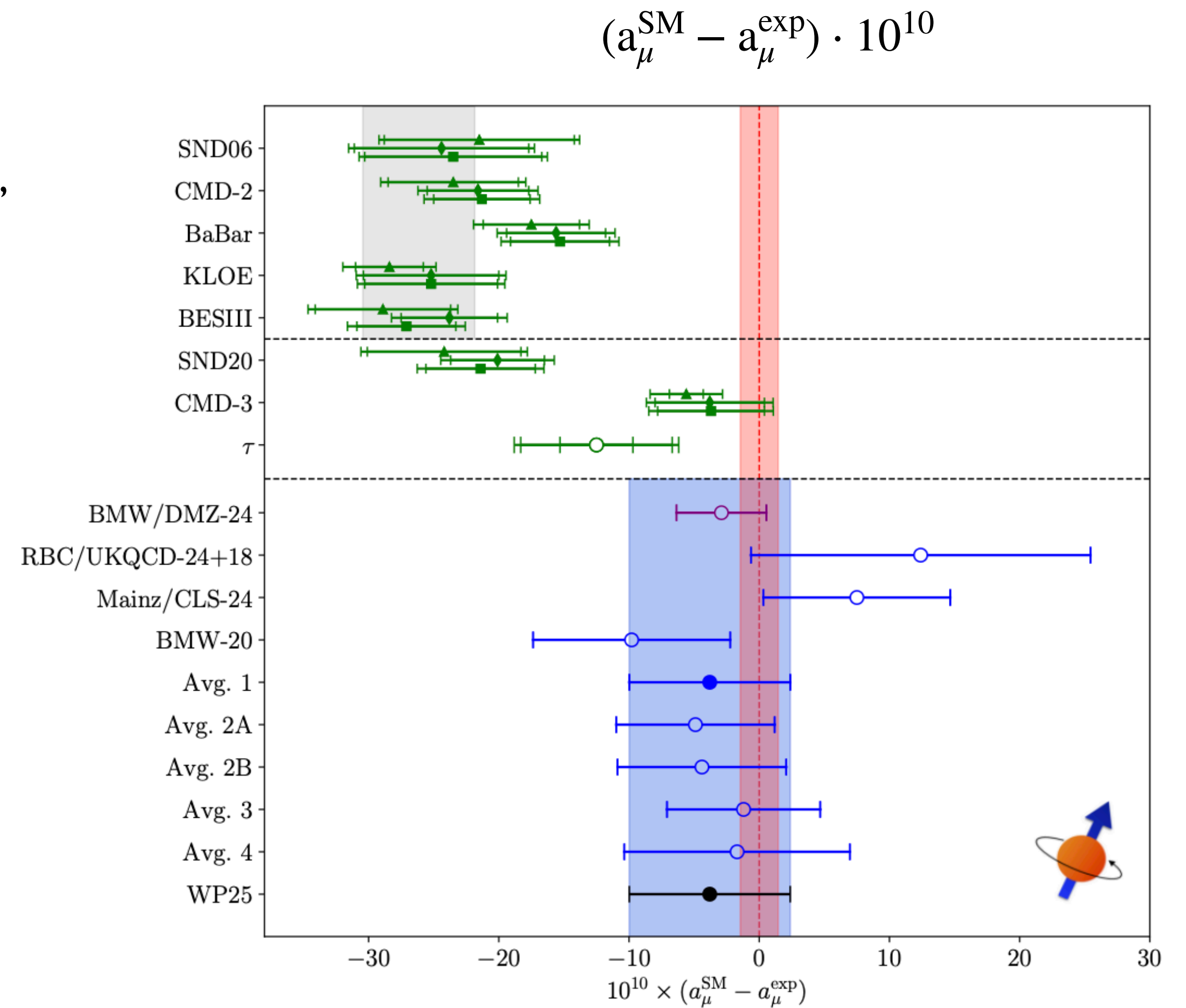
$$\text{HLbL: } \langle 0 | T\{j_\mu j_\nu j_\rho j_\sigma\} | 0 \rangle$$



[Muon g-2 theory Initiative, White Paper Update [WP25], Phys.Rept.1143 (2025), 2505.21476 [hep-ph]]

Muon g-2: hadronic contributions to a_μ

- ✦ [WVP25] identifies as future lattice HVP focus areas:
 - ➔ “more precise evaluations of **isospin-breaking effects**, and”
 - ➔ “the **noisy contributions** at long distances”
- ✦ Signal to noise at long distances challenge also in other lattice QCD observables [See talk by [Christopher Thomas, Thu. 4.40pm](#)]



[WVP25: Muon g-2 theory Initiative, Aliberti et al., Phys.Rept. 1143 (2025) 1-158 arXiv:2505.21476 [hep-ph]]

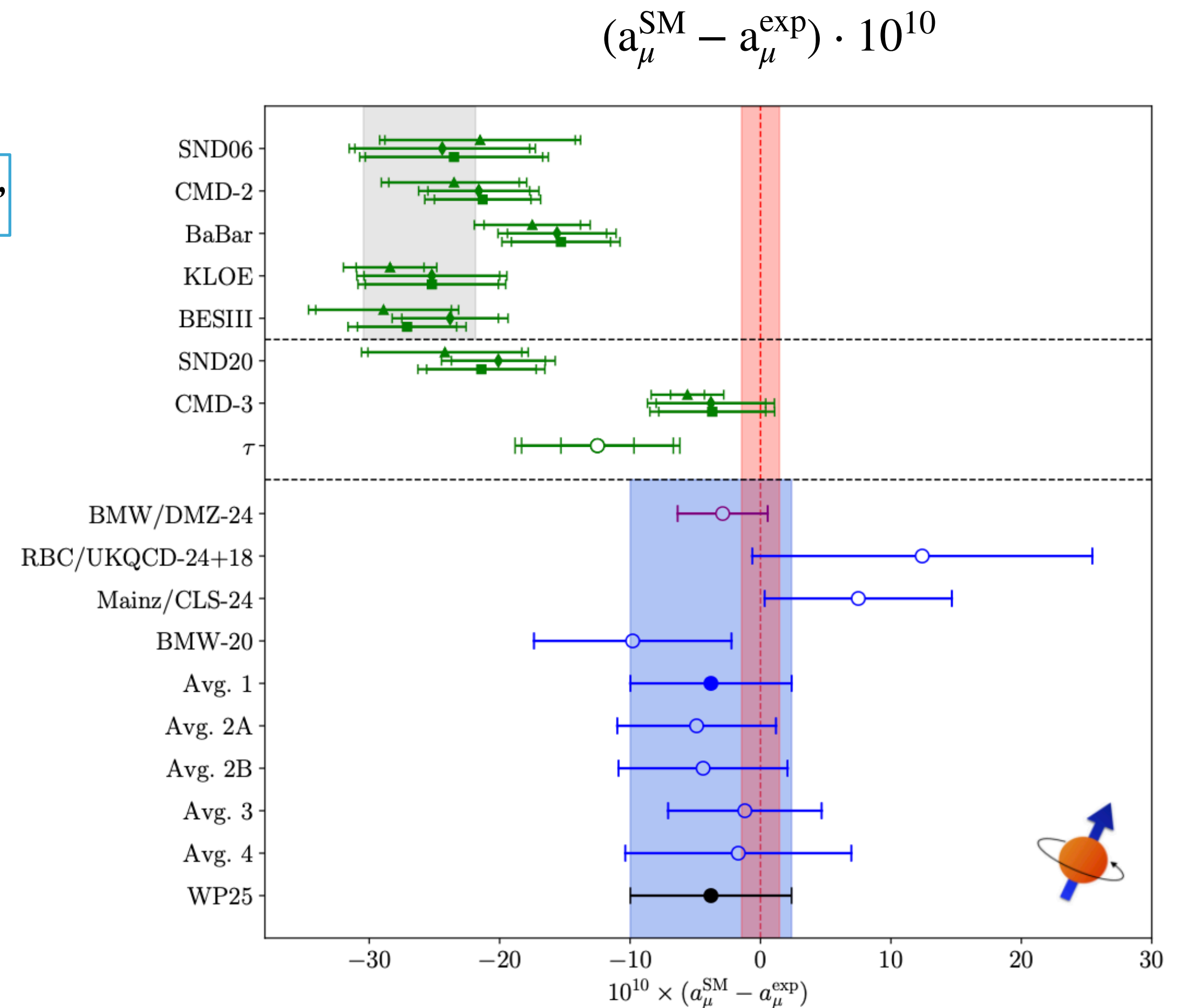
Muon g-2: hadronic contributions to a_μ

✦ [WVP25] identifies as future lattice HVP focus areas:

➔ “more precise evaluations of **isospin-breaking effects**, and”

➔ “the **noisy contributions** at long distances”

✦ Signal to noise at long distances challenge also in other lattice QCD observables [See talk by [Christopher Thomas, Thu. 4.40pm](#)]



[WVP25: Muon g-2 theory Initiative, Aliberti et al., Phys.Rept. 1143 (2025) 1-158 arXiv:2505.21476 [hep-ph]]

Periodic bc's: classical picture

♦ Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{j}$$

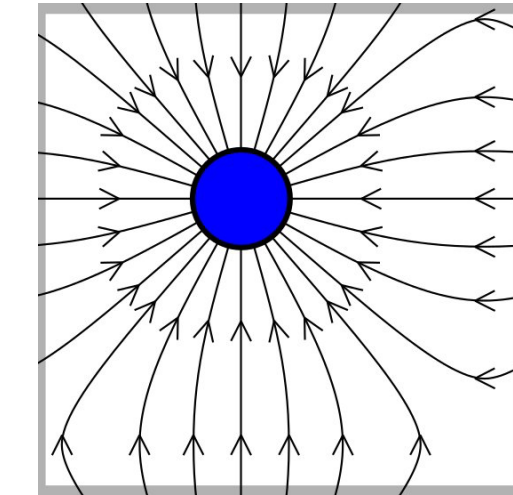


$Q(t)$

Finite (spacial) V:

$\mathbb{R}^4 \longrightarrow \mathbb{R} \times [0, L]^3$, or

$[0, \tau] \times [0, L]^3$



Periodic bc's: classical picture

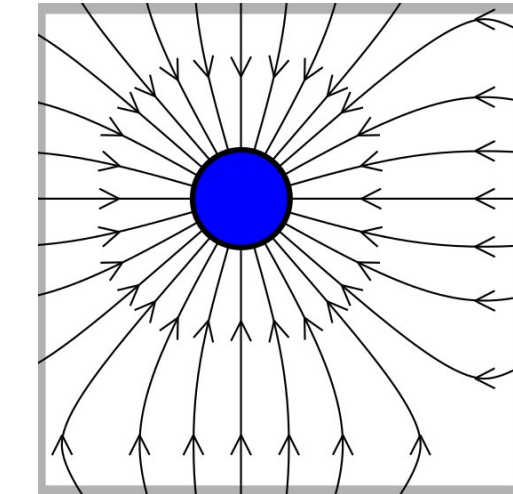
♦ Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{j}$$



$Q(t)$

Finite (spacial) V:

$\mathbb{R}^4 \longrightarrow \mathbb{R} \times [0, L]^3$, or

$[0, \tau] \times [0, L]^3$



$$Q(t) = \int_{[0, L]^3} d^3x \rho(t, \vec{x}) = \int_{[0, L]^3} d^3x \sum_{k=1}^3 \partial_k E_k(t, \vec{x}) = \sum_{k=1}^3 \int_{[0, L]^2} d^2x_{\perp}^{(k)} [E_k(t, \vec{x} + L\hat{k}) - E_k(t, \vec{x})]_{x_k=0}$$

Periodic bc's: classical picture

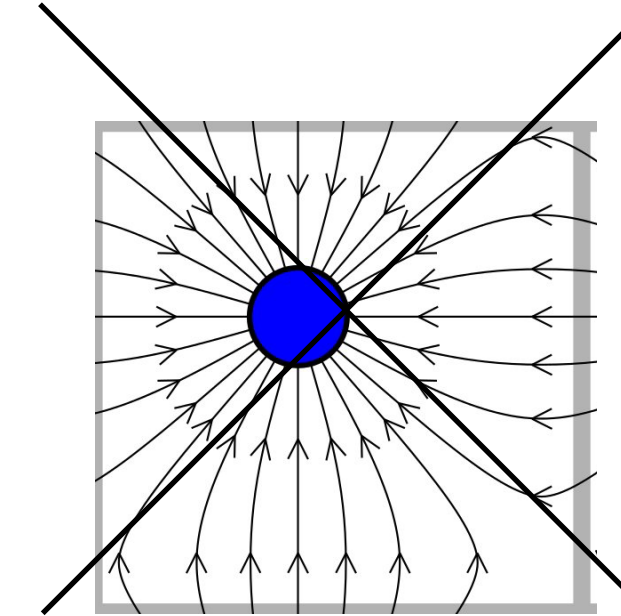
♦ Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{j}$$



$Q(t)$

Finite (spacial) V:

$$\mathbb{R}^4 \longrightarrow \mathbb{R} \times [0, L]^3, \text{ or}$$

$$[0, \tau] \times [0, L]^3$$

Periodic bc's for $\vec{E}$:

$$E_k(t, \vec{x} + L\hat{k}) = E_k(t, \vec{x}),$$

$$\implies Q(t) = 0$$



$$Q(t) = \int_{[0, L]^3} d^3x \rho(t, \vec{x}) = \int_{[0, L]^3} d^3x \sum_{k=1}^3 \partial_k E_k(t, \vec{x}) = \sum_{k=1}^3 \int_{[0, L]^2} d^2x_{\perp}^{(k)} [E_k(t, \vec{x} + L\hat{k}) - E_k(t, \vec{x})]_{x_k=0}$$

Alternative bc's: classical picture

♦ **Non-trivial boundary conditions:** $\varphi(x + L\hat{k}) = B_k(x, \varphi(x), \partial\varphi(x), \partial^2\varphi(x), \dots)$

♦ **Consistency relation (commutation of translations)**

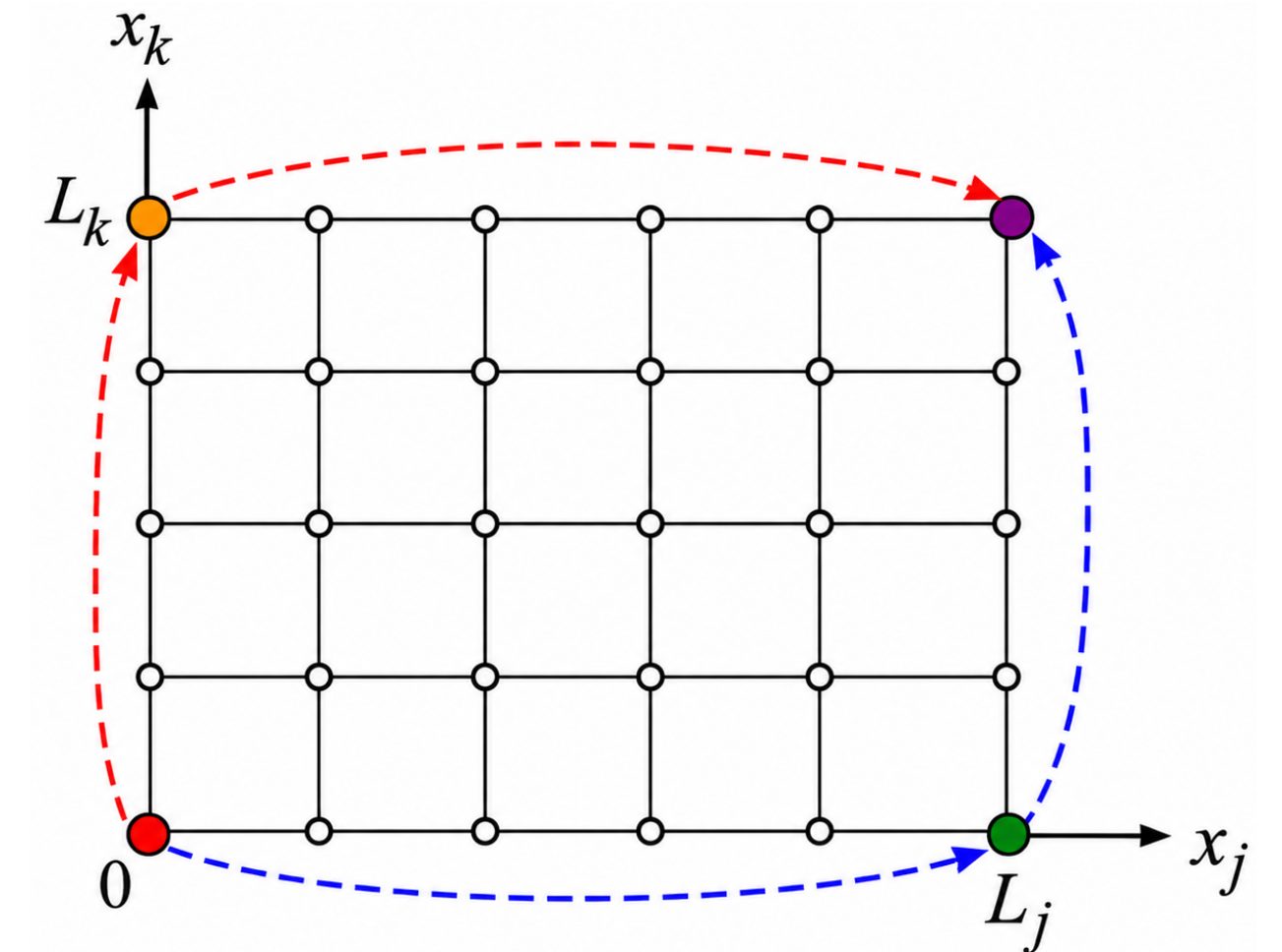
$$\varphi(x + L\hat{j} + L\hat{k}) = \varphi((x + L\hat{j}) + L\hat{k}) = \varphi((x + L\hat{k}) + L\hat{j}) \Rightarrow B_k \circ B_j = B_j \circ B_k$$

♦ **Translational invariance $\Rightarrow B_k$ can be**

- ➔ gauge transformation
- ➔ flavour transformation
- ➔ charge conjugation



E is periodic $\longrightarrow$ no charged particles in a finite box



Alternative bc's: classical picture

♦ **Non-trivial boundary conditions:** $\varphi(x + L\hat{k}) = B_k(x, \varphi(x), \partial\varphi(x), \partial^2\varphi(x), \dots)$

♦ Consistency relation (commutation of translations)

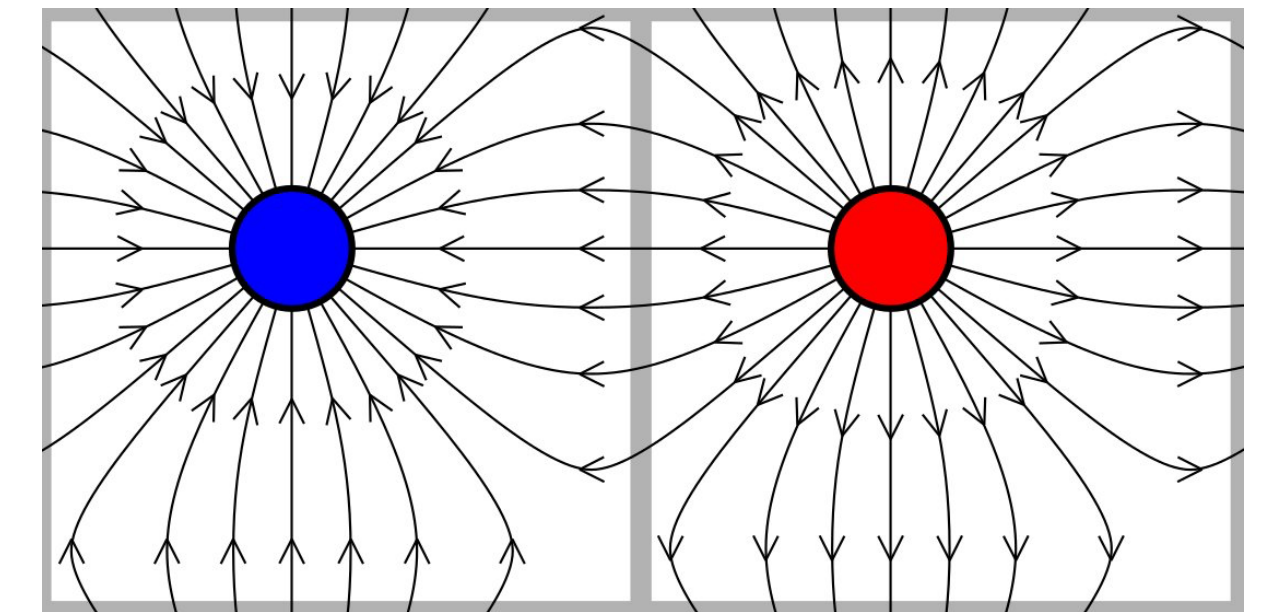
$$\varphi(x + L\hat{j} + L\hat{k}) = \varphi((x + L\hat{j}) + L\hat{k}) = \varphi((x + L\hat{k}) + L\hat{j}) \Rightarrow B_k \circ B_j = B_j \circ B_k$$

♦ Translational invariance $\Rightarrow B_k$ can be

- ⇒ gauge transformation
 - ⇒ flavour transformation
 - ⇒ charge conjugation (**E is antiperiodic**)
- } E is periodic

[Kronfeld and Wiese Nucl. Phys. B 357 (1991) 521,

Lucini et al. JHEP02(2016)076, 1509.01636]



The electric field flux is forced **not to vanish** at the boundary

Alternative bc's: classical picture

♦ **Non-trivial boundary conditions:** $\varphi(x + L\hat{k}) = B_k(x, \varphi(x), \partial\varphi(x), \partial^2\varphi(x), \dots)$

♦ Consistency relation (commutation of translations)

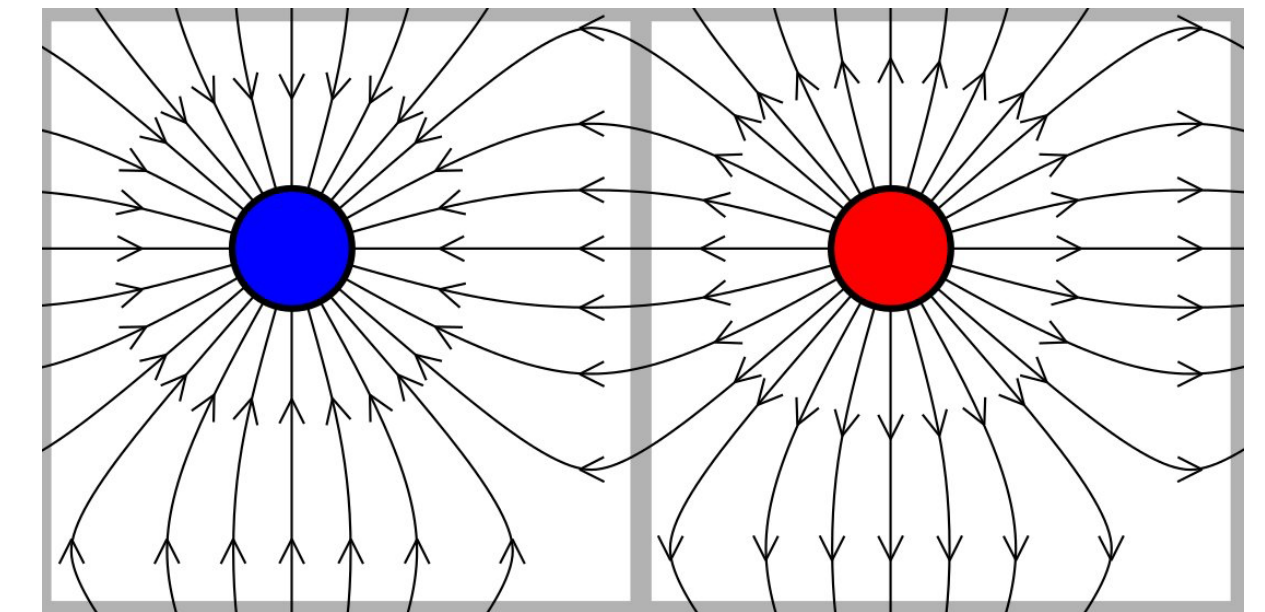
$$\varphi(x + L\hat{j} + L\hat{k}) = \varphi((x + L\hat{j}) + L\hat{k}) = \varphi((x + L\hat{k}) + L\hat{j}) \Rightarrow B_k \circ B_j = B_j \circ B_k$$

♦ Translational invariance $\Rightarrow B_k$ can be

- gauge tranformation
 - flavour transformation
 - charge conjugation (*E is antiperiodic*)
- } E is periodic

[Kronfeld and Wiese Nucl. Phys. B 357 (1991) 521,

Lucini et al. JHEP02(2016)076, 1509.01636]



The electric field flux is forced **not to vanish** at the boundary



Collaboration: compute hadronic observables in full QCD+QED using C* boundary conditions as FV QED prescription
[Campos et al. [1908.11673 EPJC 80 \(2020\) 3](#), Bushnaq et al. [2209.13183 JHEP03, 012 \(2023\)](#), Altherr et al., [2506.19770 JHEP10\(2025\)158](#)]

Many ways to QCD+QED

→ continuum:

$$\alpha_{em} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \cdots$$

→ lattice:

$$\frac{\alpha_{em}}{V} \sum_k \frac{1}{k^2} \cdots$$

♦ Several (in)finite volume (FV) formulations of QED

→ **Modify gauge field:** removing the global zero mode/spatial zero mode per timeslice (QED_{TL}/QED_L)

[PRL76(1996), Prog.Theor.Phys. 120(2008)413, Science 347 (2015) 1406.4088, ...]

→ **Massive photon** QED_m [PRL 117 (2016) 7, PoS LATTICE2021 (2022) 281]

→ **Infinite volume photon** QED_∞ [Phys. Rev. D 96 (2017), Phys. Rev. D 100 (2019) 094509]

→ **C* boundary conditions** [Phys. Rev. D 96 (2017), Phys. Rev. D 100 (2019) 094509]

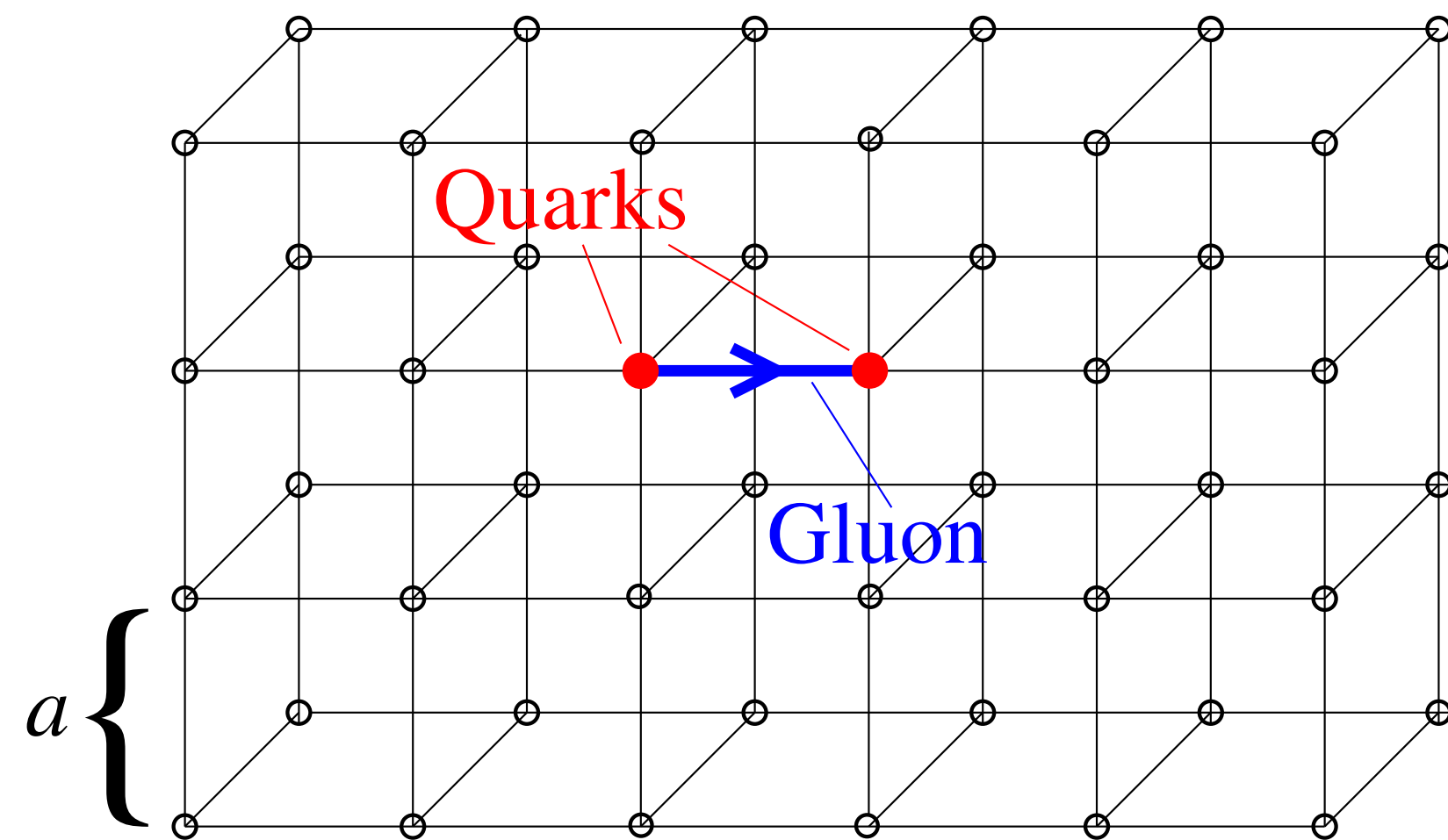
[Kronfeld and Wiese Nucl. Phys. B 357 (1991) 521, Lucini et al. JHEP02(2016)076]

→ **Other recent proposals** QED_r [JHEP 10 (2025) 026], ...



Two ways to get precision lattice QCD(+QED) predictions

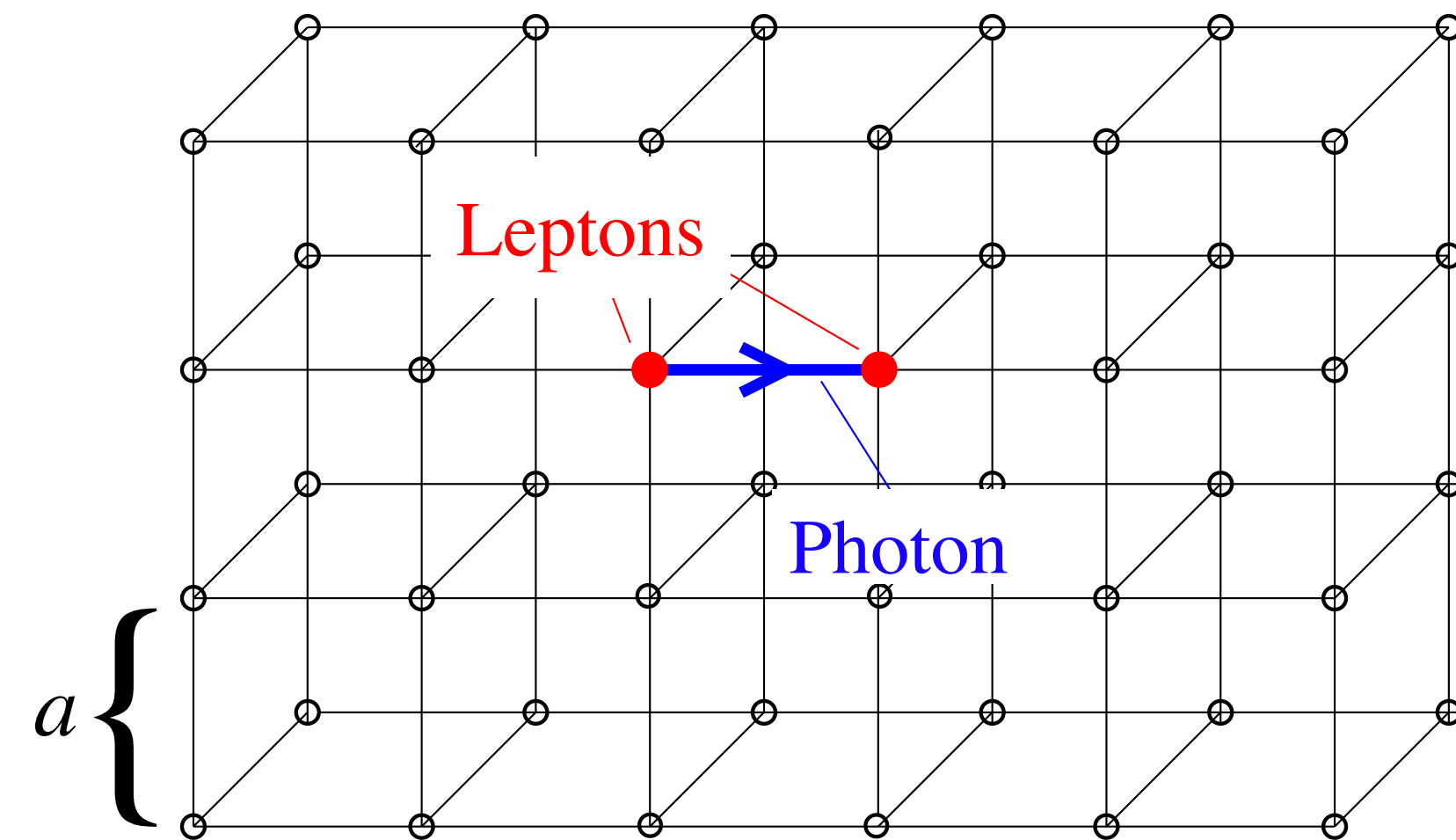
- ✦ Compare the two approaches to including IB corrections by computing a HVP-related observable [Altherr et al., 2506.19770 JHEP10(2025)158]
- ✦ Treat QED perturbatively and expand about iso-symmetric QCD observables [RM123: Divitiis et al. 1303.4896 PRD 87 (2013) 11]



Typically: $m_u = m_d$ and $\alpha_{em} = 0$

❖ IsoQCD + Perturbative IB (RM123)

➔ expand in α_{em} and $(m_d - m_u)$ about iso-symmetric theory
[RM123: Divitiis et al. 1303.4896 PRD 87 (2013) 11]

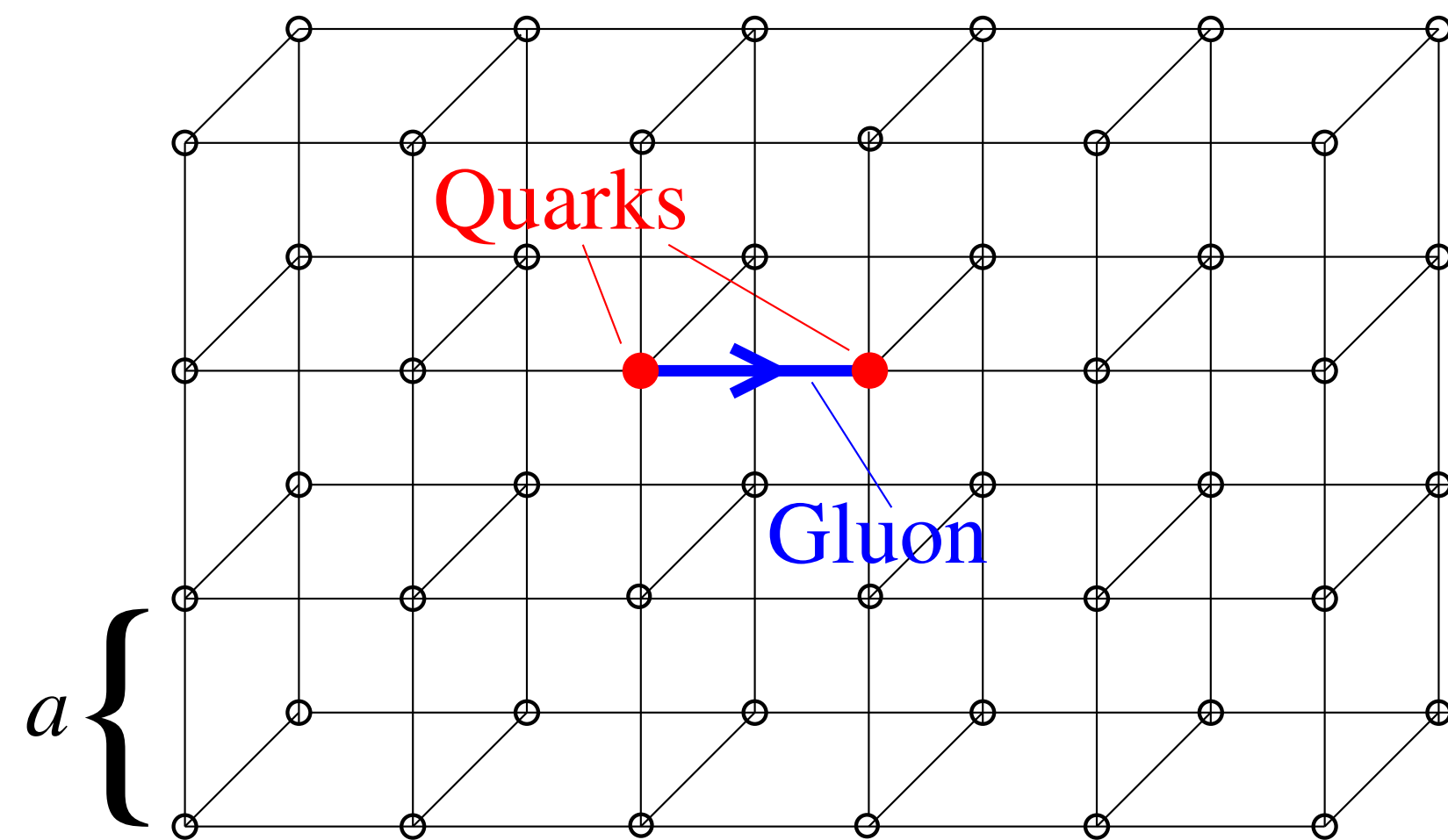


❖ non-perturbative (dynamical) QCD+QED

$$p(U, z) \propto \prod \text{Pf}(CKD_f[U, z]) e^{-S_{g, \text{SU}(3)}[U] - S_{g, \text{U}(1)}[z]}$$

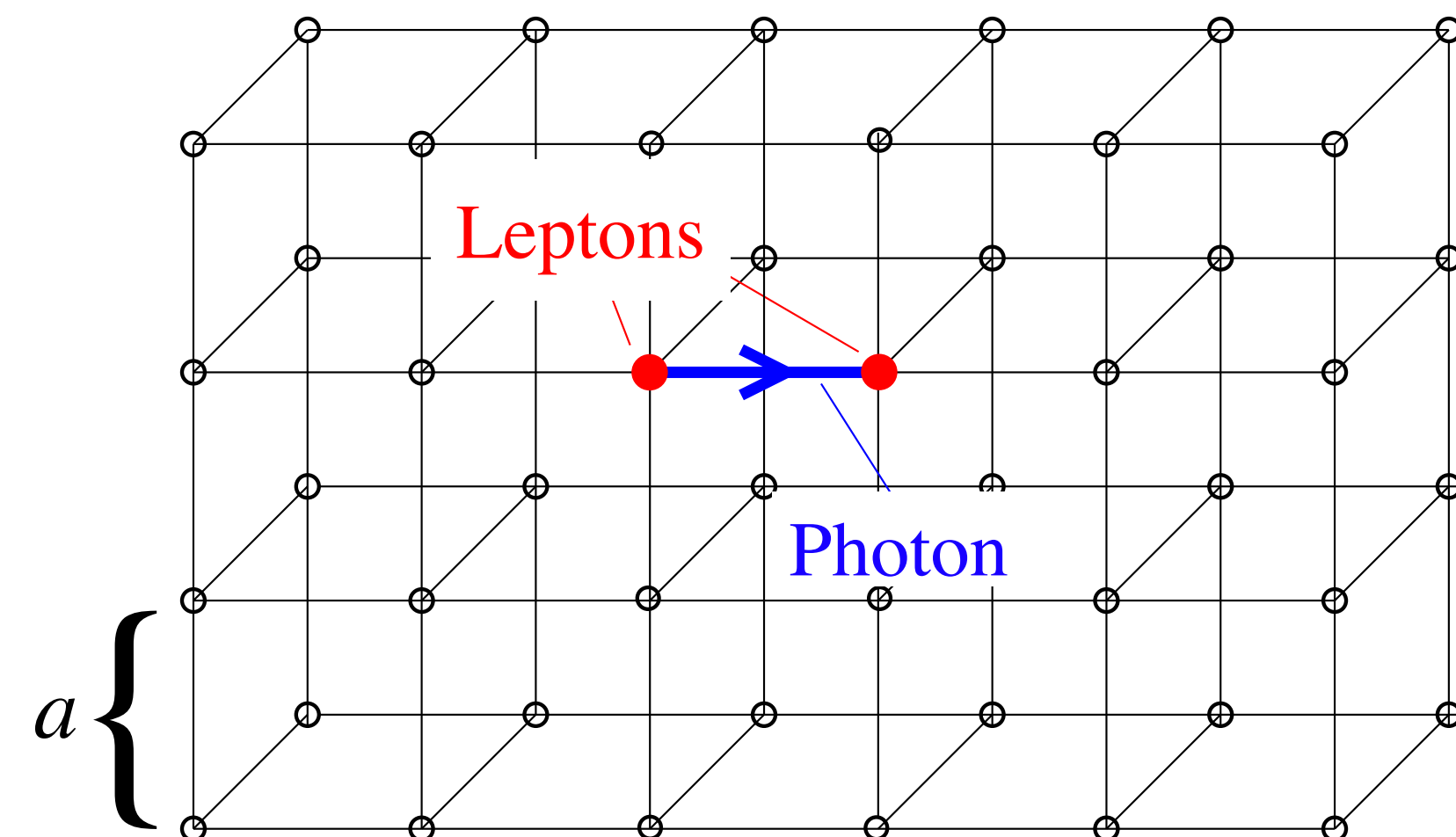
Two ways to get precision lattice QCD(+QED) predictions

- ✦ Compare the two approaches to including IB corrections by computing a HVP-related observable [Altherr et al., 2506.19770 JHEP10(2025)158]
- ✦ Use C* boundary conditions [Kronfeld and Wiese Nucl. Phys. B 357 (1991) 521, Lucini et al. JHEP02(2016)076]



❖ IsoQCD (C* bc's) + Perturbative IB

➡ expand in α_{em} and $(m_d - m_u)$ about iso-symmetric theory
[RM123: Divitiis et al. 1303.4896 PRD 87 (2013) 11]



❖ non-perturbative (dynamical) QCD+QED with C* bc's

~~RC*QCD~~ compute hadronic observables in full QCD+QED using C* boundary conditions as FV QED prescription [Lucini et al. 1509.01636 JHEP02(2016)076, Campos et al. 1908.11673 EPJC 80 (2020) 3, Bushnaq et al. 2209.13183 JHEP03, 012 (2023)]

Two ways to get precision lattice QCD(+QED) predictions

♦ Which approach gives better statistical precision (for HVP)?

♦ Perturbative IB effects or QCD + QED_q

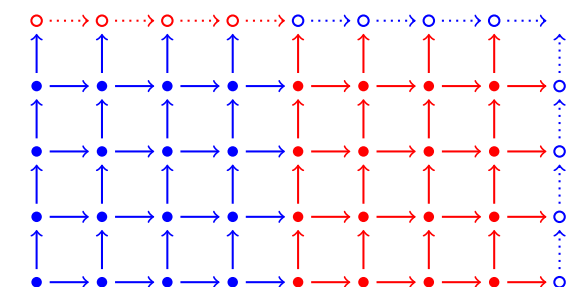
$$\Delta^{\text{stoch}} < \Delta^{\text{pert,summed-}\mu} < \Delta^{\text{pert,single-}\mu}$$

[RBC/UKQCD: Boyle et al, 1706.05293 JHEP 09 (2017) 153]

♦ Perturbative IB effects or non-perturbative QCD + QED

$a_\mu^{\text{U,w}} \times 10^{11}$	$a_\mu^{\text{U,w}} \text{ (LL)}$	$a_\mu^{\text{U,w}} \text{ (CL)}$
isoQCD+RM123 _{eq}	1084(5)	1087(5)
isoQCD+RM123	1093(20)	1094(21)
non-perturbative QCD+QED	1082(8)	1085(7)

[RC*: Alther et al. 2506.19770 JHEP10(2025)158]



		from action			from V_μ^c at sink
		IB type	mass	QED	QED
valence connected	vv				
	vs				
	ss				
valence disc.ed	vv				
	vs				
	ss				

[RC*: Paola Tavella @ 8th plenary workshop of the Muon g-2 TI Initiative; MKM@ LATTICE2025, PoS LATTICE2025 (2026) 198, arXiv:2605.01085]

U-spin symmetric hadronic vacuum polarization: $a_{\mu}^{U,w}$

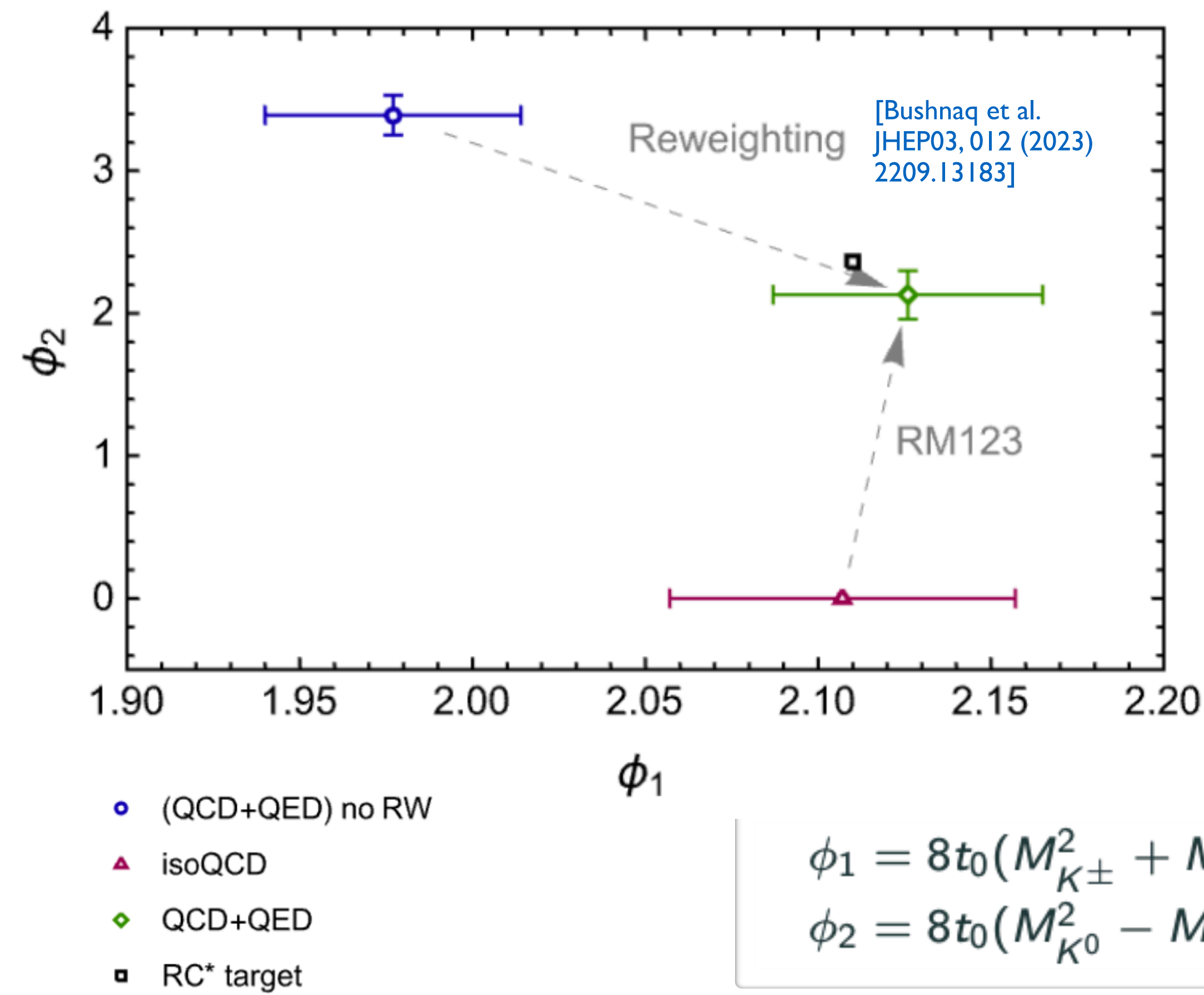
$$a_{\mu}^{U,w} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt G^U(t) \tilde{K}(t; m_{\mu}) w_I(t).$$

- $G^U(t)$ correlator of the non-singlet current $V_{\mu} = \frac{1}{2}(\bar{\psi}_s \gamma_{\mu} \psi_s - \bar{\psi}_d \gamma_{\mu} \psi_d)$
- $w_I(t)$ gives the intermediate window [\[Blum et al. PRL121, 022003 \(2018\)\]](#)
- disconnected-free due to SU(3)/U-spin symmetry in our setup

$$\begin{aligned} & \frac{1}{4} \left(\text{diagram with two s quarks in a loop} + \text{diagram with two s quarks in a loop} - \text{diagram with one s and one d quark in a loop} \right) + \\ & + \frac{1}{4} \left(\text{diagram with two d quarks in a loop} + \text{diagram with two d quarks in a loop} - \text{diagram with one d and one s quark in a loop} \right) = \frac{1}{2} C \end{aligned}$$

Note: in isoQCD with SU(3) symmetry $\rightarrow a_{\mu}^{U,w} = \frac{3}{4} a_{\mu}^{\text{HVP}, \text{uds}, w}$ [\[Gerardin, Harris, Meyer, PRD99, 014519 \(2019\)\]](#)

Matching isoQCD and QCD+QED [RC*: Alther et al. 2506.19770 JHEP10(2025)158] [RC*: Alther et al. 2605.01085 PoS(LATTICE2025)198]



$$\begin{aligned}\phi_1 &= 8t_0(M_{K^\pm}^2 + M_{\pi^\pm}^2 + M_{K^0}^2) \\ \phi_2 &= 8t_0(M_{K^0}^2 - M_{K^\pm}^2)/\alpha_R\end{aligned}$$

“Edinburgh Consensus” [FLAG Review 24 2411.04268 Aoki et al. PRD 113 (2026)]

	QCD	isoQCD
M_{π^+}	135.0 MeV	135.0 MeV
M_{K^+}	491.6 MeV	494.6 MeV
M_{K^0}	497.6 MeV	494.6 MeV
$M_{D_s^+}$	1967 MeV	1967 MeV
$M_{B_s^0}$	5367 MeV	5367 MeV
f_{π^+}	130.5 MeV	130.5 MeV

RC*: CLS-inspired separation [Bruno, Korzec, Schaefer 1608.08900 PRD 95 (2017) 074504]

Precision lattice QCD+QED observables: lessons from a_μ

- ✦ Sub-percent precision for hadronic observables requires inclusion of QED and strong IB corrections (also FV effects, disconnected, ...)
- ✦ Compared two approaches to lattice QCD+QED: sampling of full QCD+QED path integral and perturbative IB inclusion (RM123)
- ✦ Test observable: U-spin symmetric HVP intermediate window

$$a_\mu^{\text{U,w}} \times 10^{11} = \begin{cases} 1094(21) & \text{isoQCD + perturbative IB effects} \\ 1085(7) & \text{dynamical QCD + QED} \end{cases}$$



Paola Tavella



Letizia Parato



[RC*: Alther et al. 2506.19770
JHEP10(2025)158]

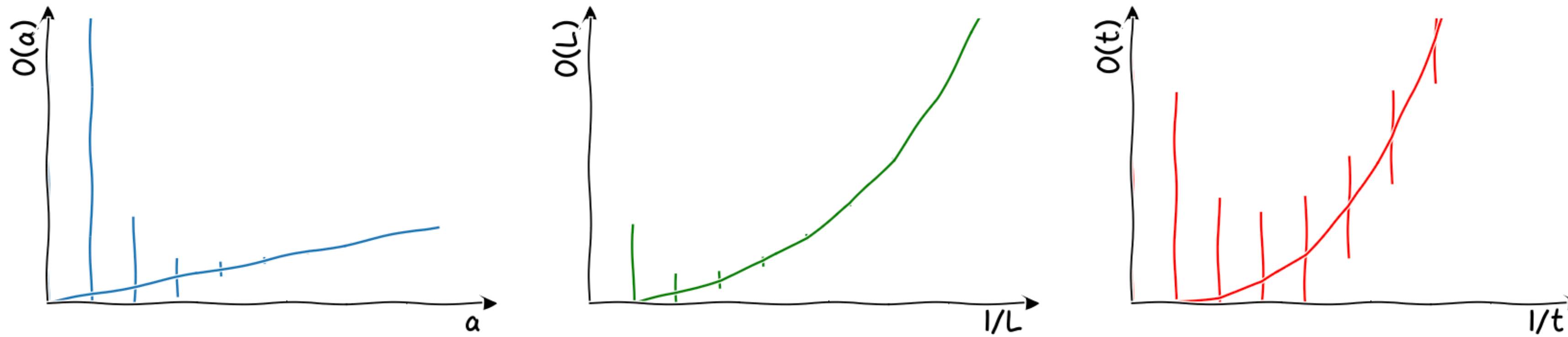
- ✦ Different QED prescriptions applied to HVP [Kuberski et al. JHEP 03 (2024) 172, Djukanovic et al. JHEP 04 (2025) 098, Bazavov et al. Phys.Rev.Lett. 135 (2025) 1, 011901, Blum et al. Phys.Rev.Lett. 134 (2025) 20, 201901, Clark et al. 2201.03251, Boccaletti et al. Nature 653 (2026) 8114, 373-377], decay constants
- ✦ Next steps: understand **volume dependence** [P.Tavella @ LATT26, K. Kersic @ LATT26], M_π dependence (here: $m_\pi \approx 400$ MeV), extend comparison to other observables [Rosso et al., PoS(LATTICE2025)080, 2602.23910], HLbL contribution [K. Phan @ LATT26]

Precision QCD(+QED) requires new algorithms: long distances

- ◆ Noise in Euclidean correlators: $\langle O \rangle \approx \bar{O} = \frac{1}{N_{cnfg}} \sum_{n=1}^{N_{cnfg}} O[U_i]$

Future lattice HVP focus areas [WP25]:

- ➔ “more precise evaluations of **IB effects**, and”
- ➔ “the **noisy contributions** at long distances”

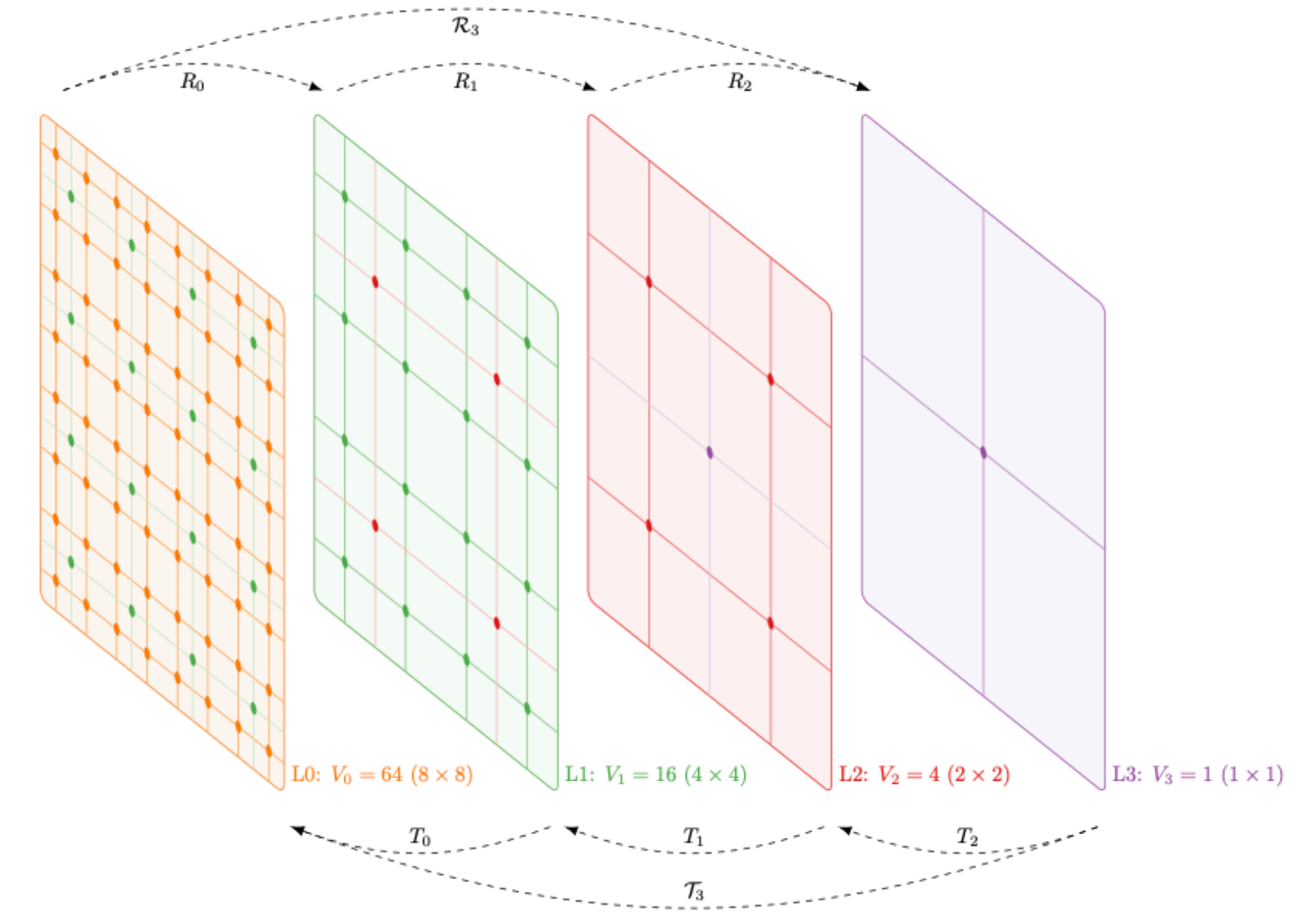
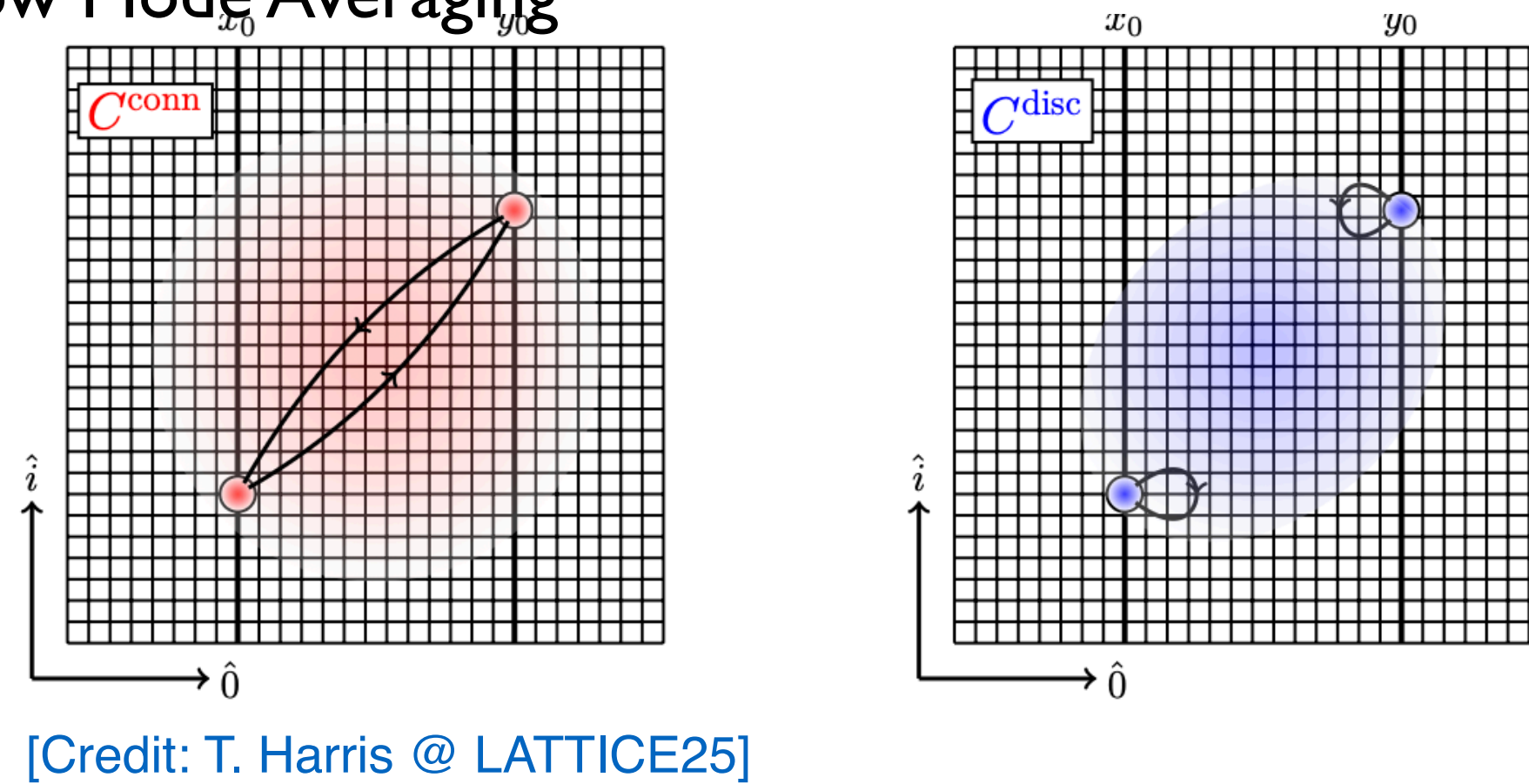


[Credit: T. Harris @ LATTICE25]

- ◆ Observable $O = \langle j_k j_k \rangle$ and a (stochastic) estimator of the observable $\bar{O}$ (with N_η random fields; $\eta_i \sim e^{-\frac{1}{2}\eta_i^\dagger \eta_i} = 1$; $i = 1 \dots N_\eta$)
- ◆ Total variance $Var[\bar{O}] = Var[O] + \frac{1}{N_\eta} Var[\eta]$ not necessarily finite as $a \rightarrow 0$, $L \rightarrow \infty$, ...

Precision QCD(+QED) requires new algorithms: long distances

- ◆ Multigrid-based variance reduction technique for connected correlators
- ◆ Multigrid Low Mode Averaging



- ◆ Current-current correlator $C_{\mu\nu} = \sum_{\vec{x}} \langle 0 | J_{\mu}(\vec{x}, t) J_{\nu}(\vec{0}, 0) | 0 \rangle = C^{conn.} + C^{disc.}$

- ◆ Applied in to speed up Monte Carlo gauge field configuration sampling over the past two decades

- ◆ Proposal and first application to HVP-like observables { [\[Gruber, Harris, MKM, PRD 111 \(2025\) 7, 074508, 2412.06347\]](#)
[\[Alexandrou et al. 2606.27072\]](#)

- ◆ Larger volumes for similar cost



Roman Gruber



Tim Harris

Observables inaccessible with Euclidean lattice QCD

Conventional MC approach:

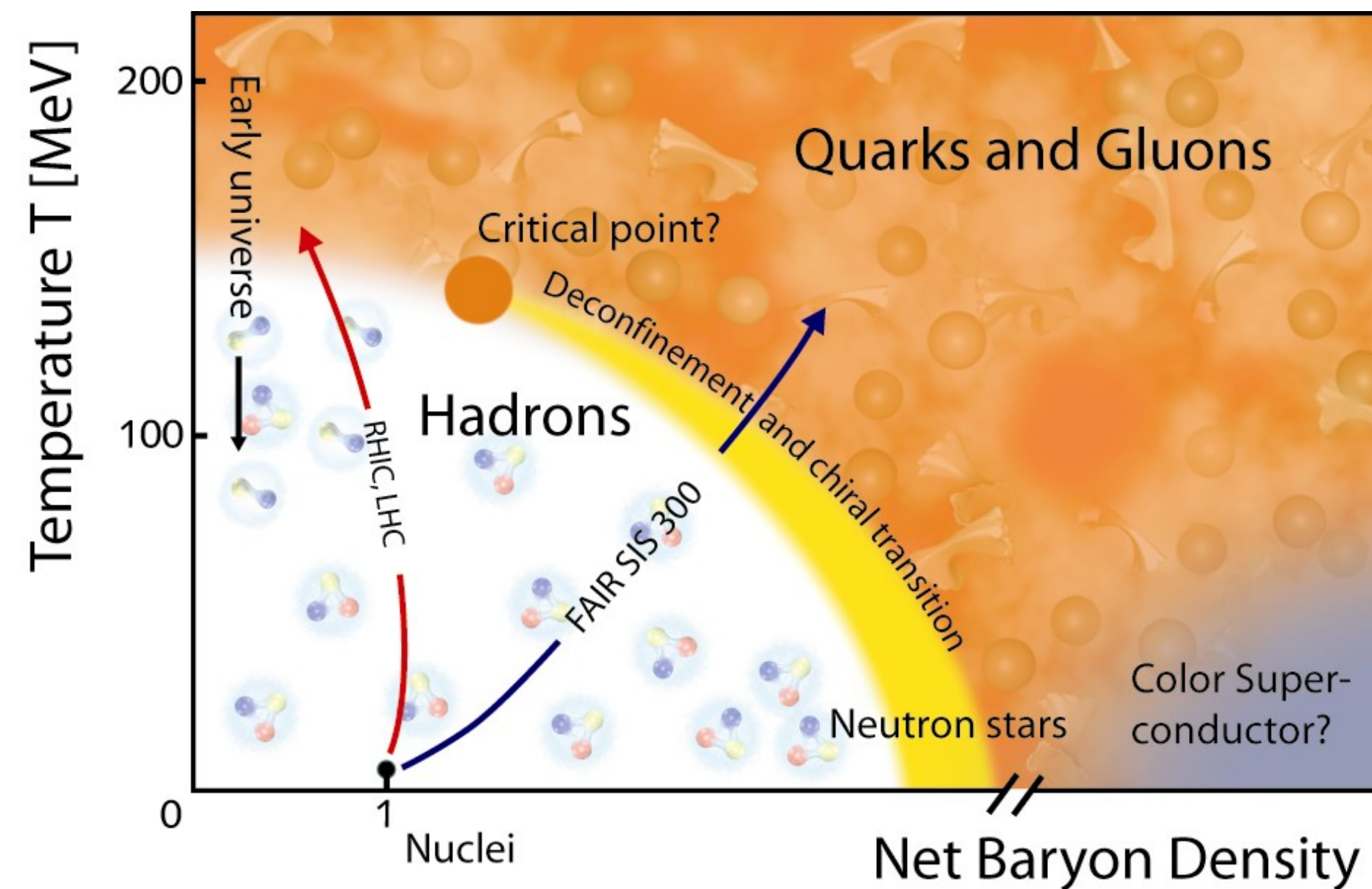
$$\langle O[A, \psi, \bar{\psi}] \rangle = \frac{1}{Z} \int \mathcal{D}A \det D[A] e^{-S_G[A]}$$

Observables inaccessible with Euclidean lattice QCD

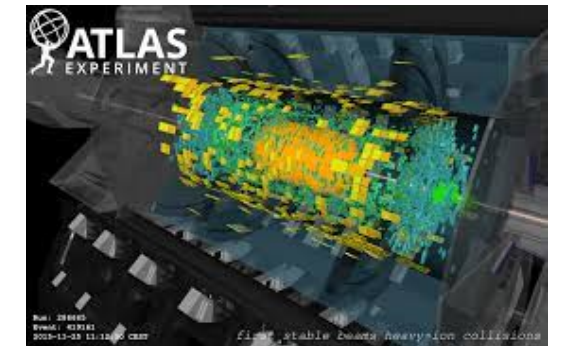
Experiments probing nonzero baryonic density

Conventional MC approach:

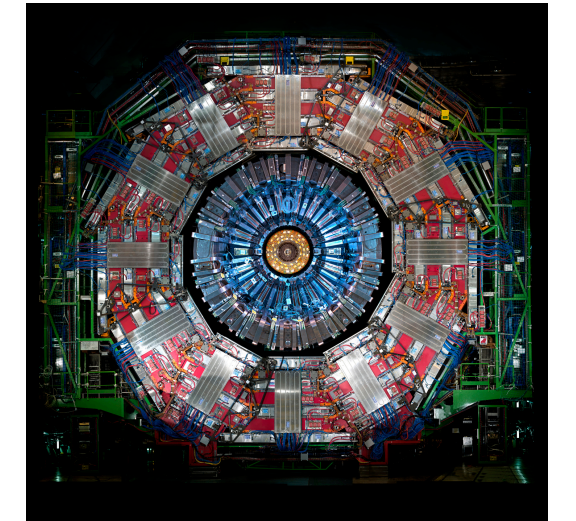
$$\langle O[A, \psi, \bar{\psi}] \rangle = \frac{1}{Z} \int \mathcal{D}A \det D[A] e^{-S_G[A]}$$



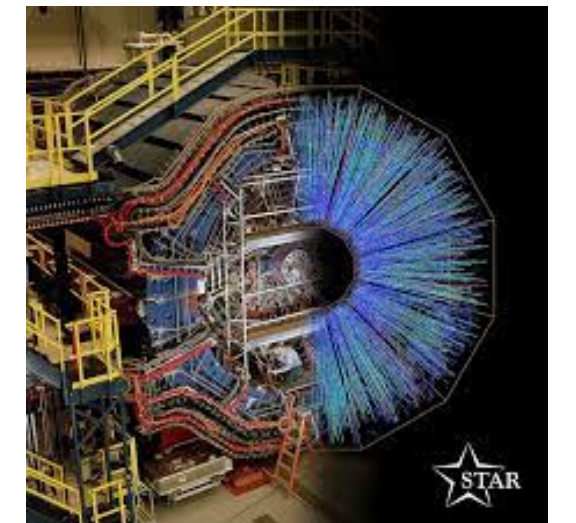
[Credit: Peter Senger (p.senger@gsi.de)]



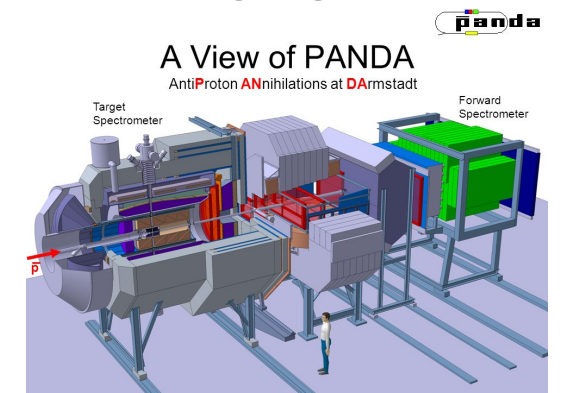
LHC: ATLAS



LHC: ALICE



RHIC: STAR



FAIR: PANDA

Observables inaccessible with Euclidean lattice QCD

Inaccessible with conventional MC approach:

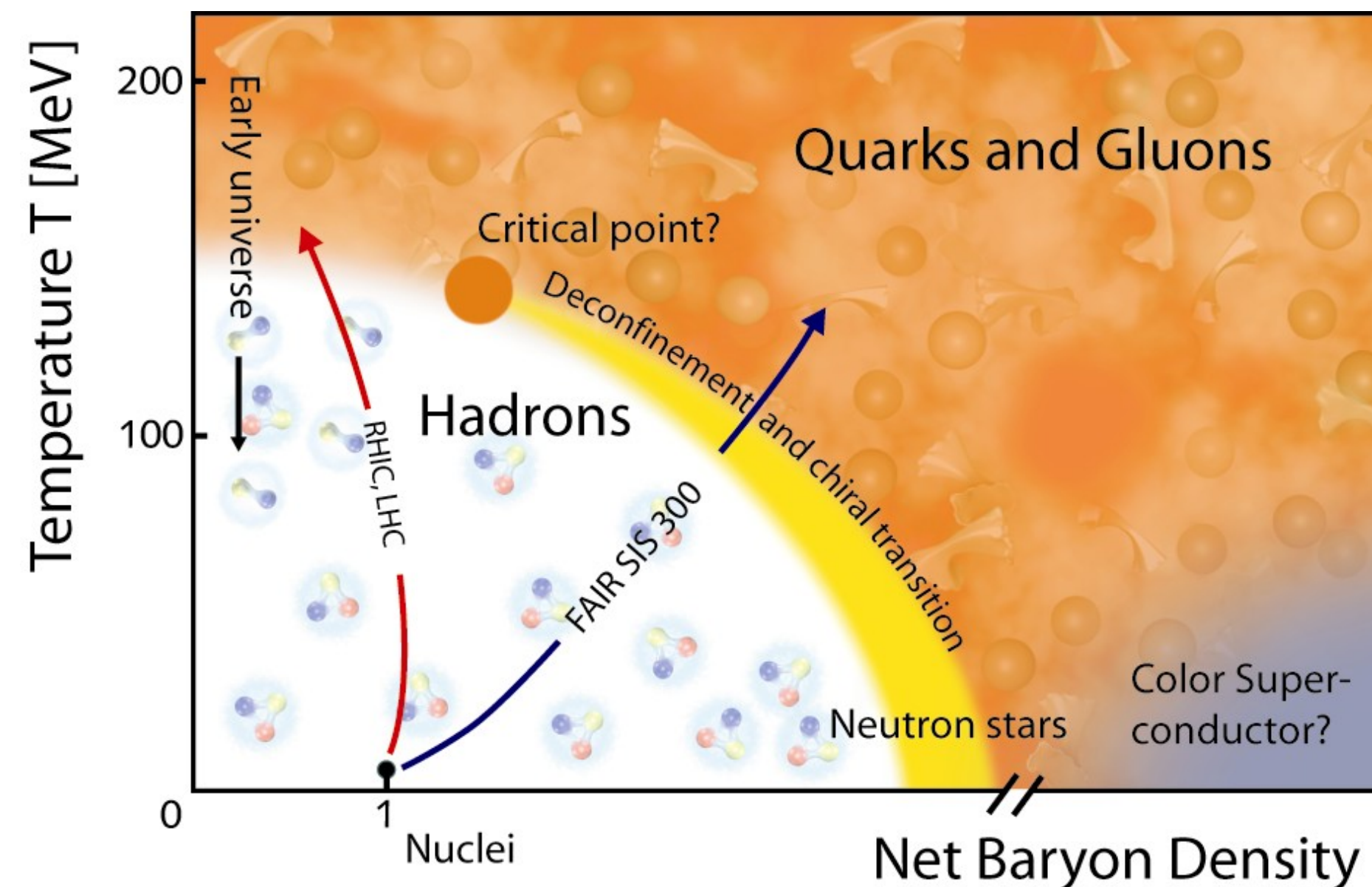
$$\langle O[A, \psi, \bar{\psi}] \rangle = \frac{1}{Z} \int \mathcal{D}A \det D[A] e^{-S_G[A]}$$

Complex Weight W

$$\begin{aligned} \langle O \rangle &= \frac{\sum_C O_C \cdot W_C}{\sum_C W_C} = \frac{\sum_C O_C \cdot \text{sign}(W_C) \cdot |W_C|}{\sum_C \text{sign}(W_C) \cdot |W_C|} \\ &= \frac{\langle \text{sign} \cdot O \rangle_{|W|}}{\langle \text{sign} \rangle_{|W|}} \end{aligned}$$

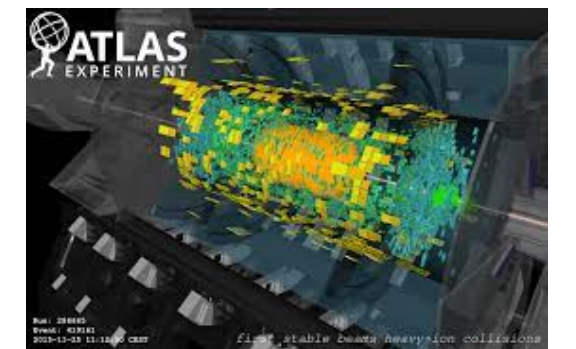
“Sign Problem”

[Troyer, Wiese PRL 94 170201(2005)]

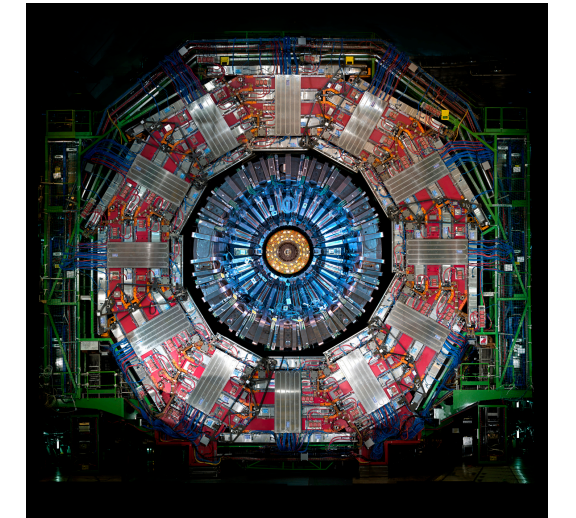


[Credit: Peter Senger (p.senger@gsi.de)]

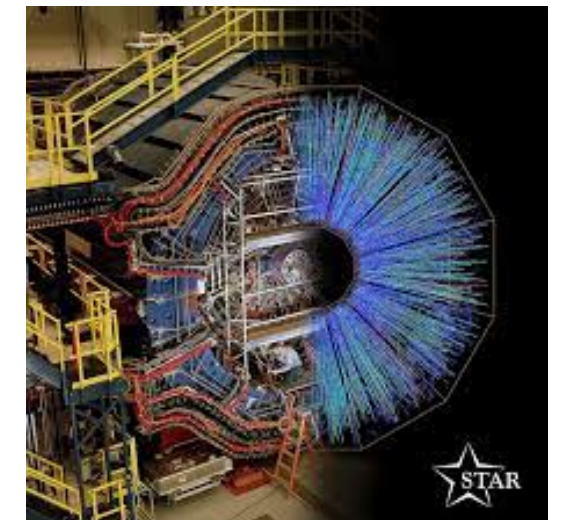
Experiments probing nonzero baryonic density



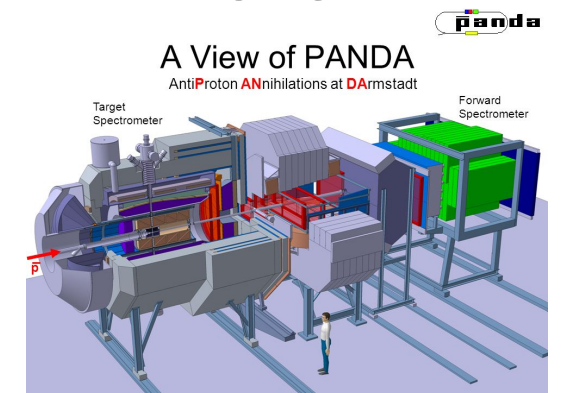
LHC: ATLAS



LHC: ALICE



RHIC: STAR



FAIR: PANDA

Observables inaccessible with Euclidean lattice QCD

Inaccessible with conventional MC approach:

$$\langle O[A, \psi, \bar{\psi}] \rangle = \frac{1}{Z} \int \mathcal{D}A \det D[A] e^{-S_G[A]}$$

Complex Weight W

- Real-time dynamics
- Topological θ -term
- Finite chemical potential
- ...

$$e^{-i H_{\text{QCD}} t}$$

$$\det D[A] e^{-S_G[A] + i \theta Q_{\text{top}}[A]}$$

$$\det D(\mu)^* = \det D(-\mu) \neq \det D(\mu)$$

Observables inaccessible with Euclidean lattice QCD

Inaccessible with conventional MC approach:

$$\langle O[A, \psi, \bar{\psi}] \rangle = \frac{1}{Z} \int \mathcal{D}A \det D[A] e^{-S_G[A]}$$

Complex Weight W

→ Real-time dynamics

$$e^{-i H_{\text{QCD}} t}$$

→ Topological θ -term

$$\det D[A] e^{-S_G[A] + i \theta Q_{\text{top}}[A]}$$

→ Finite chemical potential

$$\det D(\mu)^* = \det D(-\mu) \neq \det D(\mu)$$

→ ...

♦ Quantum simulation opens a route to first-principles real-time QCD dynamics

♦ So far: applications in low spacetime dimensions, 1+1D and 2+1D, mainly Abelian gauge theories, few non-Abelian, too

[Mukherjee et al. PRB(2021), Banerjee, Sen PRL (2021), Surace et al. PRX (2021), Delacretaz et al. JHEP (2022), Desaulles et al. PRB (2023), Srdinsek et al. PRL (2024), Su, Sun, Hudomal et al. PRR (2023), Calajo et al. PRR (2025). Hartse, Fidkowski, Mueller PRL 135, 060402 (2025), Froland et al 2608.02944, Froland et al 2607.24947, Hartse et al. 2607.14212...]

Beyond precision: hadronization from first principles

- ◆ Lund-string model (Pythia), cluster fragmentation (Herwig, SHERPA), tuned to LEP/LHC data

[See e.g. talk by [Steffen Schumann, Mon. 11.40am](#)]

$$f(z) \propto \frac{1}{z}(1-z)^a e^{-\frac{bm_T^2}{z}}; \quad m_T^2 = m_h^2 + p_T^2$$

	a	$b[\text{GeV}^{-2}]$	$\sigma_{p_T}[\text{GeV}]$
Pythia Monash	0.68	0.98	0.335
Herwig-LH	0.75	0.90	0.31

[Divisova et al., EPJC86(2026)3 2509.02348]

$$\sigma_{p_T} = \sqrt{\frac{\kappa}{\pi}}$$

- ◆ Successful phenomenological description, large number of parameters → is first principles description possible?

- ◆ Static flux tube

$$V_{q\bar{q}}(r) = \sigma r - \frac{\pi(d-2)}{24r} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

- ◆ Lattice gauge theory has a long history of precise predictions of σ

[M. Teper, [hep-th/9812187](#)]

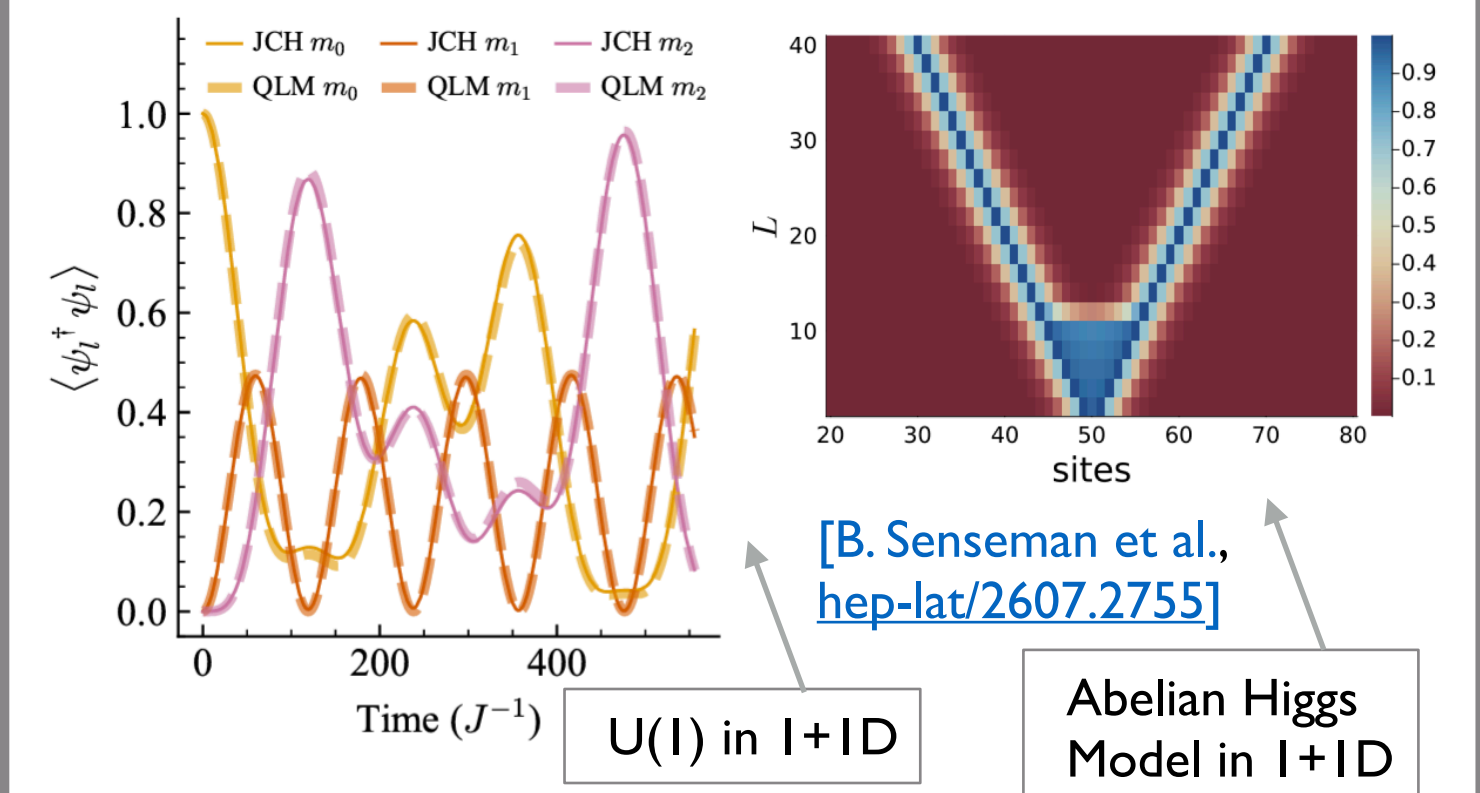
[G. Bali & K. Schilling, [hep-lat/9208028](#) PRD46(1992)2636]

[Bulava et al., [2403.00754](#) Phys.Lett.B854(2024)138754]

- ◆ From static to fragmenting string?
- ◆ Schwinger-like pair production

$$P_{q\bar{q}} \sim e^{-\frac{\pi m_T^2}{\kappa}}$$

- ◆ Real-time dynamics of string breaking



[B. Senseman et al., [hep-lat/2607.2755](#)]

[N. Gonzalez et al., [hep-lat/2606.02756](#)]

- ◆ Questions we might be able to answer:
 - ➔ Is a constant-field Schwinger ansatz justified?
 - ➔ Time-dependent field dynamics, inhomogeneous fields
 - ➔ Sauter pulses - dynamically assisted pair production?

Summary & Outlook

- ✦ Precision QCD $\longrightarrow$ precision QCD+QED
- ✦ Sub-percent precision requires control of IB corrections, FV effects, long-distance contributions, disconnected diagrams ...
- ✦ HVP-like benchmark $a_\mu^{\text{U,W}}$: two approaches agree, with $3 \times$ improved statistical precision in dynamical QCD+QED
[Albergo, Kanwar, Shanahan, PRD 100 (2019) 1904.12072], ...
[Wang et al. JHEP 05 (2024) 060 2309.17082 Vega et al., JHEP 07 (2026) 213 2510.26081]
- ✦ Hadronic contributions to muon g-2: driving algorithmic development, e.g. multigrid-LMA, but also machine-learning-based MC sampling
[Hermansson-Truedsson, Kanwar, 2605.22444]
- ✦ Next frontier: heavy flavour observables.PDFs/TMDs, ... [FLAG 2024, 2411.04268, Constantinou, Cichy, 2608.00251, Avkhadiev et al. PRD 109 (2024) 2307.12359, ...]
- ✦ New classical algorithms, tensor networks and **quantum simulation** open access to inaccessible regimes thus far, e.g. **hadronization**
- ✦ Which parts of collider phenomenology can we bring next under first principles control?

QCD@LHC 2026

September 28, 2026 to October 2, 2026
University of Oxford
Europe/London timezone

Enter your search term

