

Energy Correlators & Track Functions

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Outline

1. Introduction
2. Two-point energy correlator
3. Energy Correlators & Track Functions
4. Higher-point correlators

Personal selection of recent developments

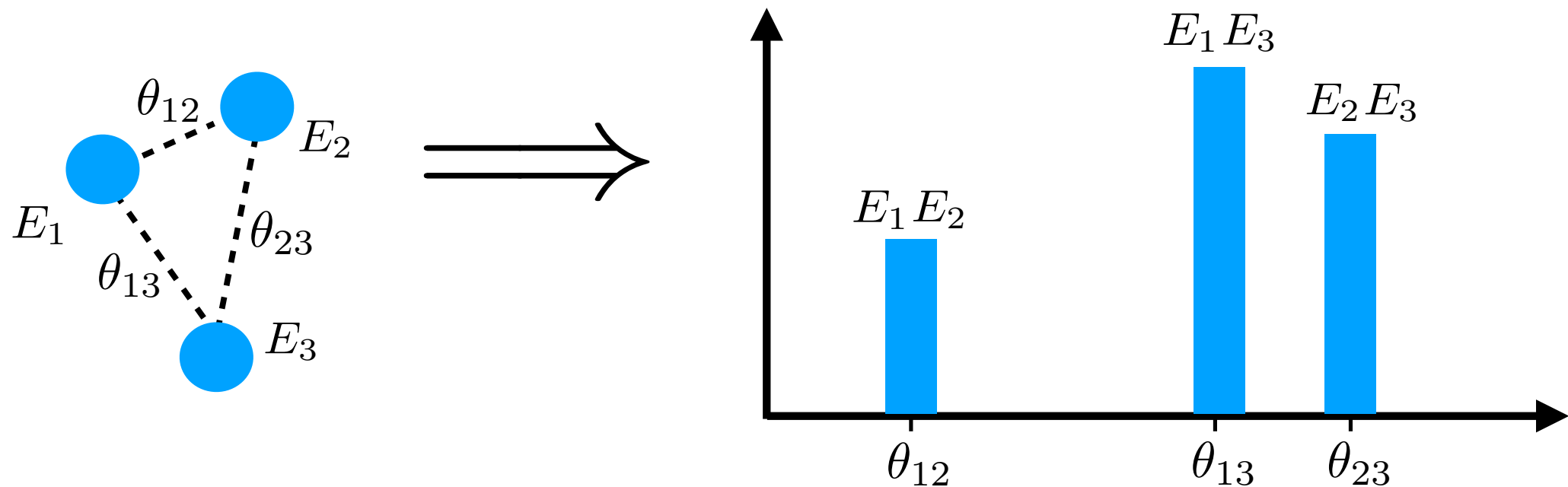
1. Introduction



Energy-Energy Correlator (EEC)

$$\frac{d\sigma}{d\theta} = \int d\sigma \sum_{i,j} \frac{E_i E_j}{(\sum_k E_k)^2} \delta(\theta - \theta_{ij})$$

[Basham, Brown, Ellis, Love]

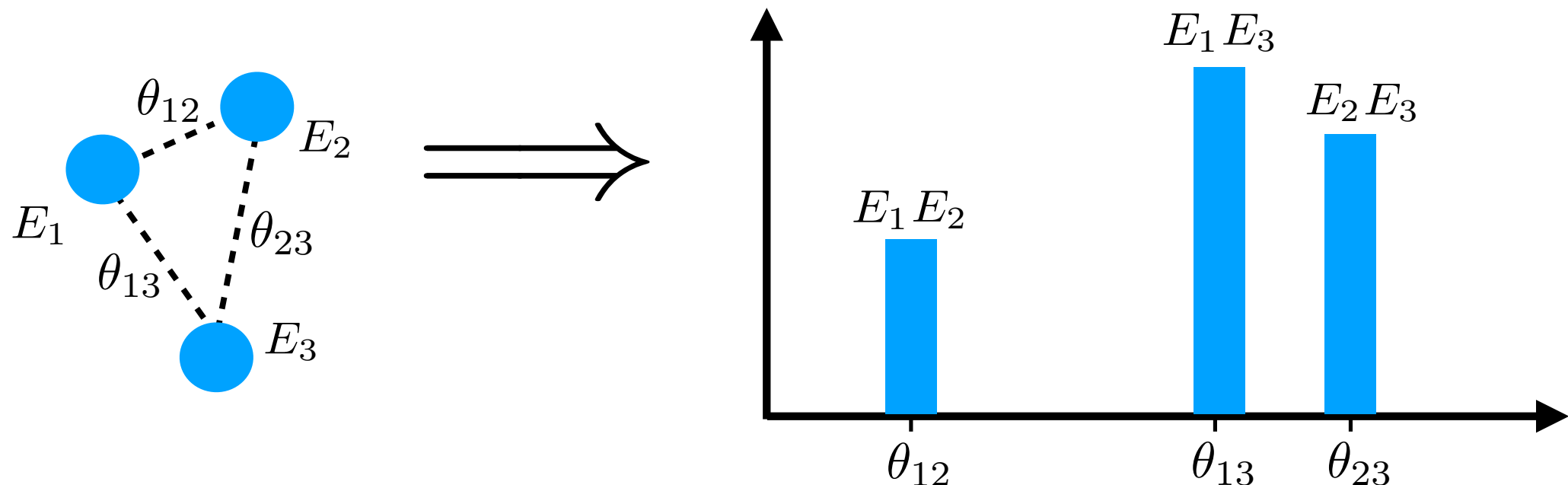


- A single event/jet yields a distribution, unlike most observables.

Energy-Energy Correlator (EEC)

$$\frac{d\sigma}{d\theta} = \int d\sigma \sum_{i,j} \frac{E_i E_j}{(\sum_k E_k)^2} \delta(\theta - \theta_{ij}) \sim \langle \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \rangle$$

[Basham, Brown, Ellis, Love]

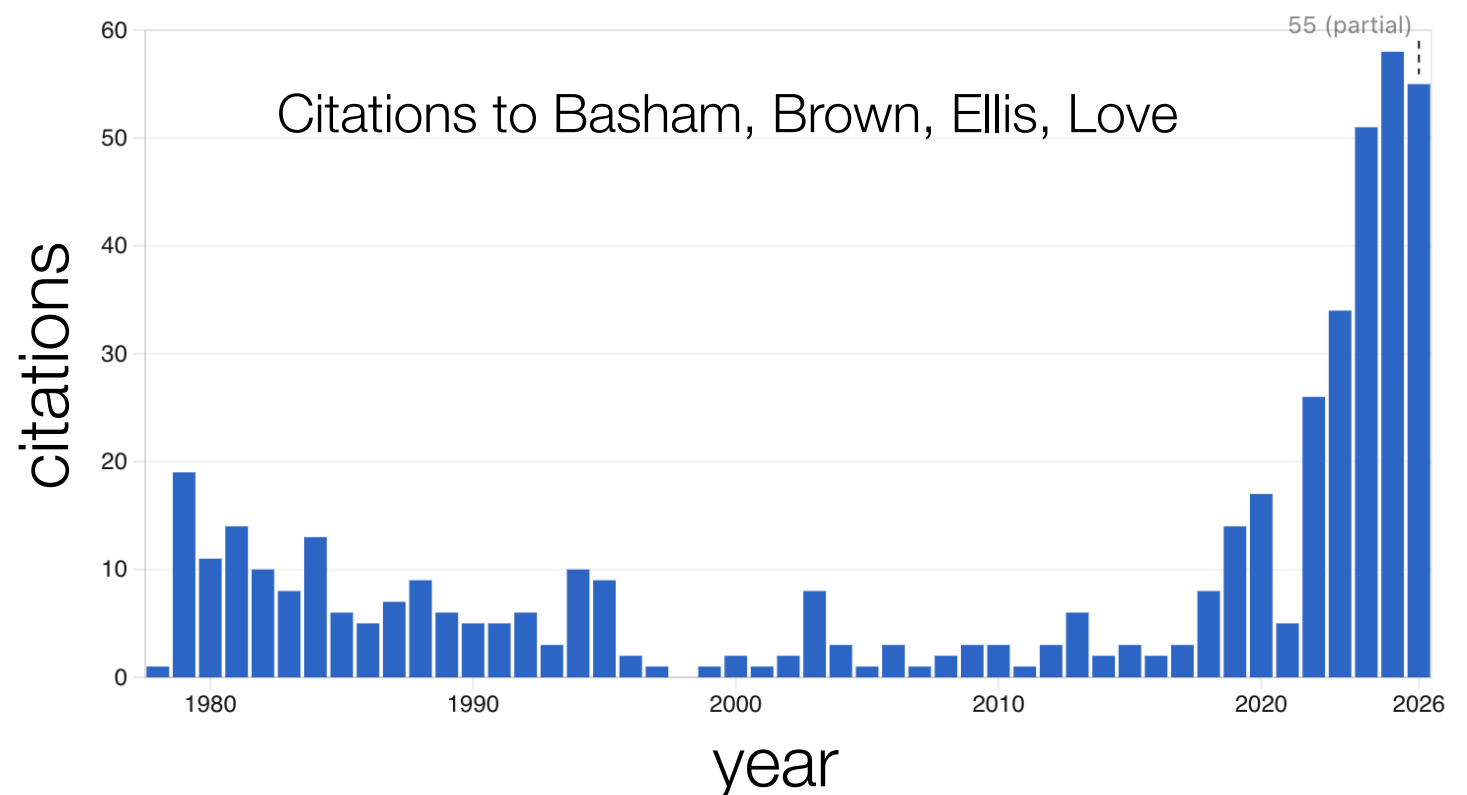


- A single event/jet yields a distribution, unlike most observables.
- Generalizes to higher-point correlators.
- Can be described using the light-ray OPE [Hofman, Maldacena]

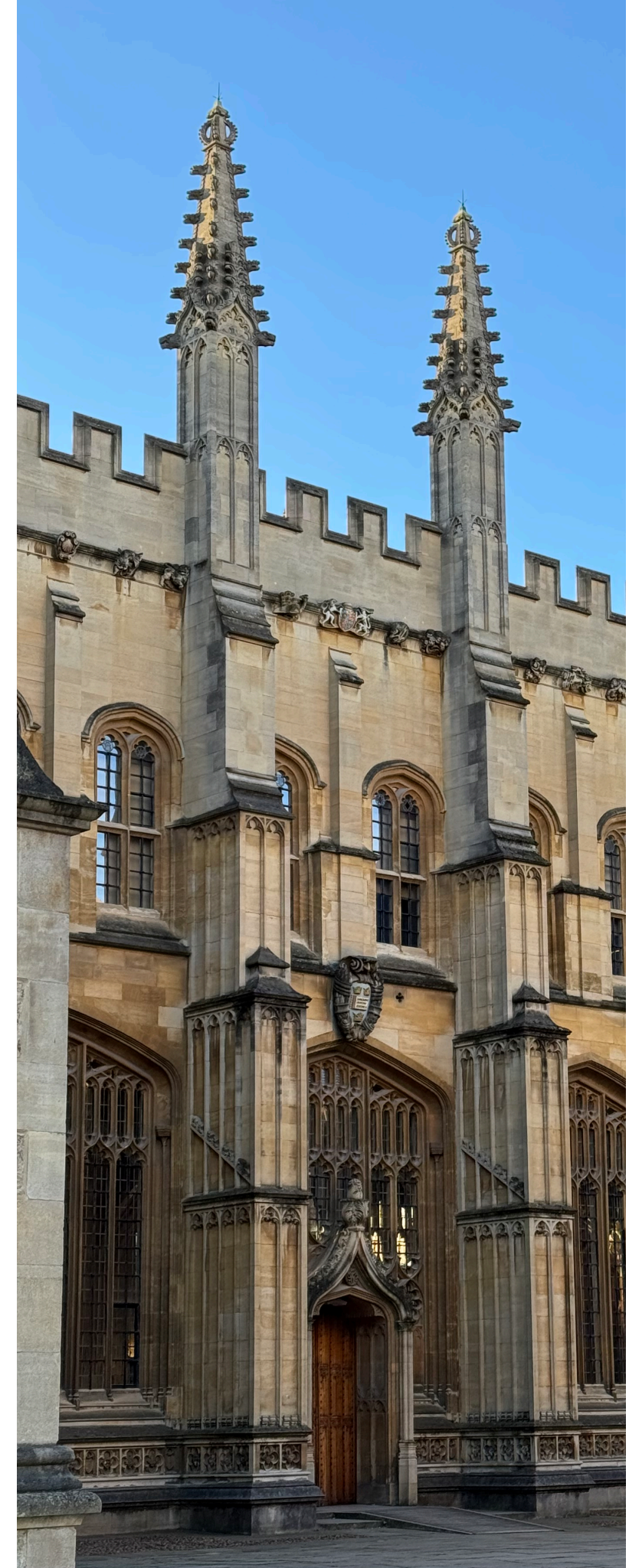
Why energy correlators?

Recent interest in energy correlators has been driven by:

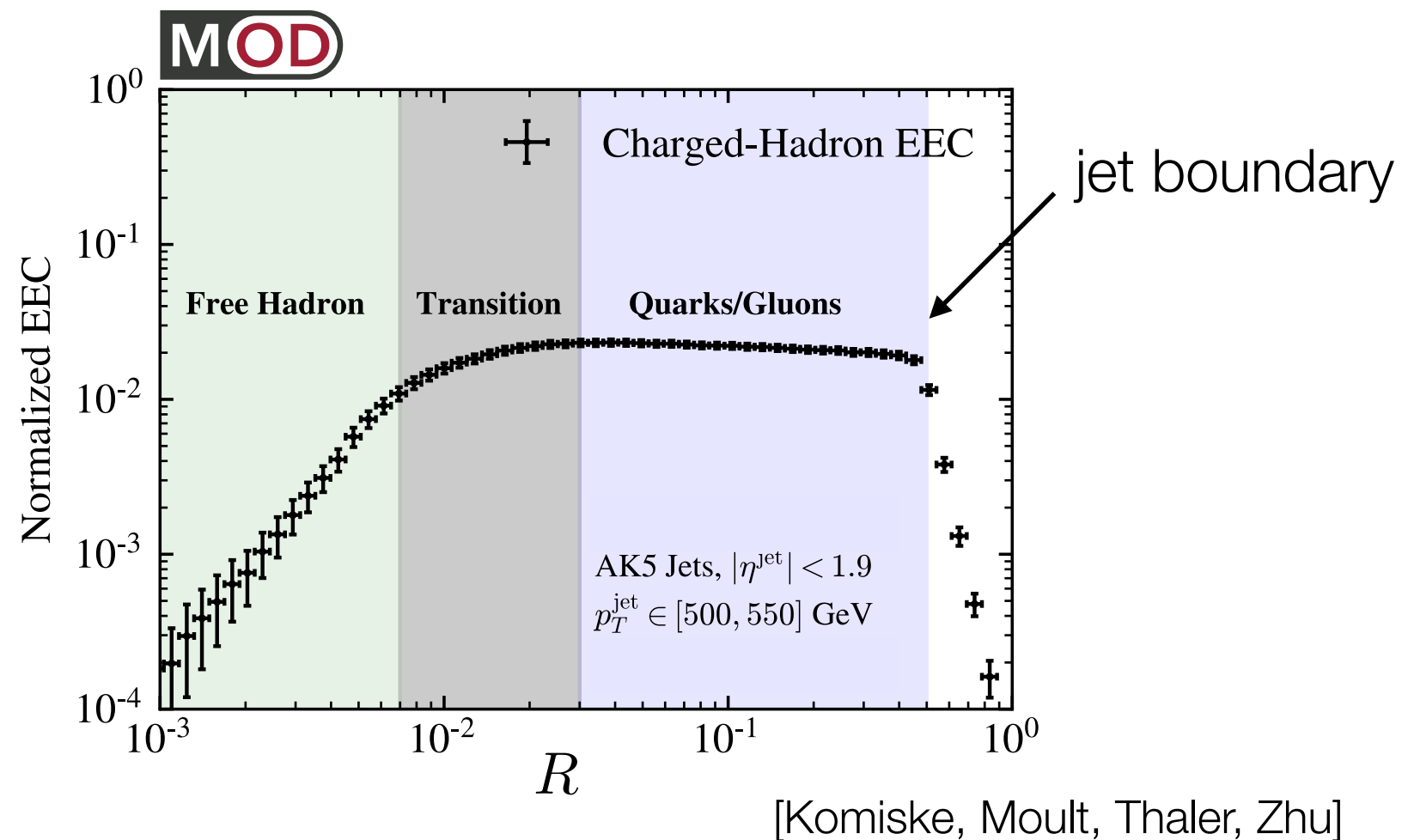
- ✓ Simpler theory $\rightarrow$ better description.
- ✓ Separation of physics at different scales $\rightarrow$ interpretability.
- ✓ Suppression of soft contamination.
- Wide range of applications: strong coupling, top quark mass, quark-gluon plasma, ...
- Partly hype: jet shape = 1-point correlator 😊



2. Two-point energy correlator



Power-law scaling in collinear regime



- Perturbative: $\sim R^\gamma$ with γ set by DGLAP:

$$\int^R dR' \frac{d\sigma}{dR'} = \int_0^1 dx x^2 \vec{H}\left(x, \frac{p_T}{\mu}\right) \cdot \vec{J}\left(\ln \frac{x p_T R}{\mu}\right)$$

[Dixon, Moul, Zhu]

- Nonperturbative: $\sim R^2$.

Scaling the plateau

- Free-hadron region is not calculable, but its evolution is.

[Lee, Stewart; Chang, Chen, Liu, Simmons-Duffin, Yuan, Zhu]

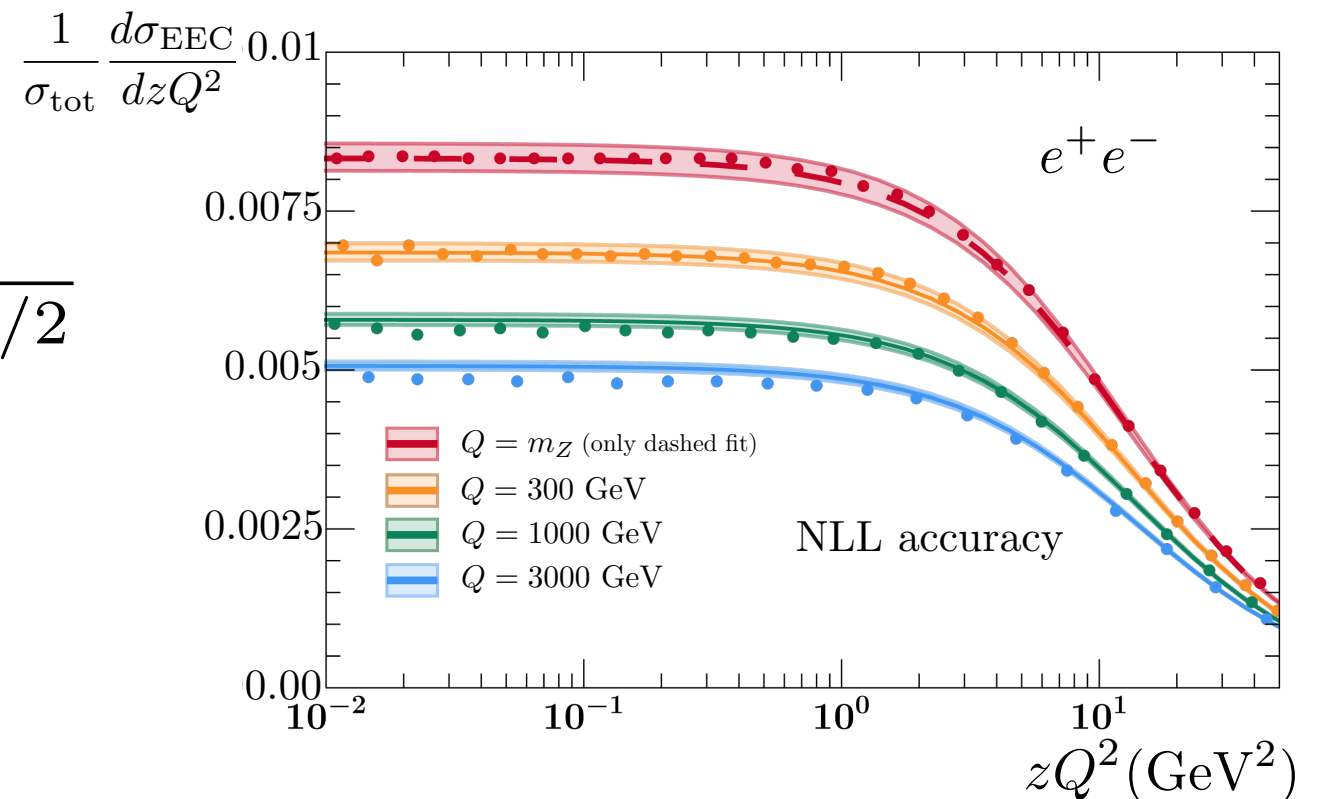
- EEC is now constant, because compared to previous slide:

$$R \frac{d\sigma}{dR} \rightarrow \frac{d\sigma}{dz} \quad z = \frac{1}{2}(1 - \cos \theta) \approx \frac{1}{4}\theta^2$$

- Fit ansatz and evolve:

[Lee, Stewart]

$$J_i(Q^2 z, \mu_0) = \frac{N_i}{1 + \left(\frac{Q^2 z}{\Lambda_i}\right)^{1-\gamma_i/2}}$$

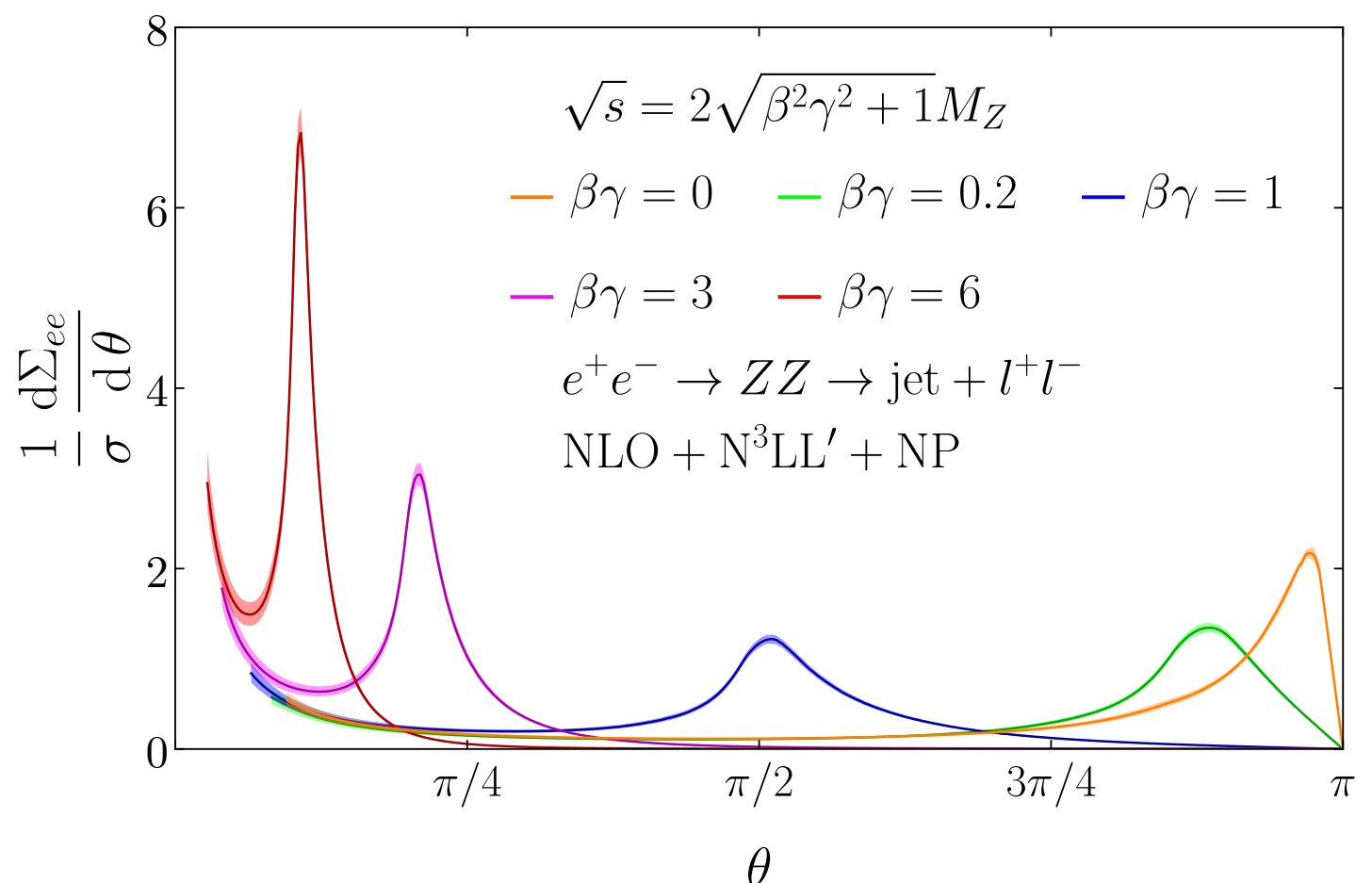


Hadronically decaying boosted Z bosons

- Boost Z pole predictions or data using:

$$\text{EEC}(n_1, n_2, q) = \frac{q^2}{4\pi^2 (n_1 \cdot n_2)^3} \mathcal{F}_{\varepsilon\varepsilon} \left(\frac{q^2 n_1 \cdot n_2}{2q \cdot n_1 q \cdot n_2} \right)$$

- Simplest for $e^+e^- \rightarrow ZZ$, power corrections for Z at LHC.

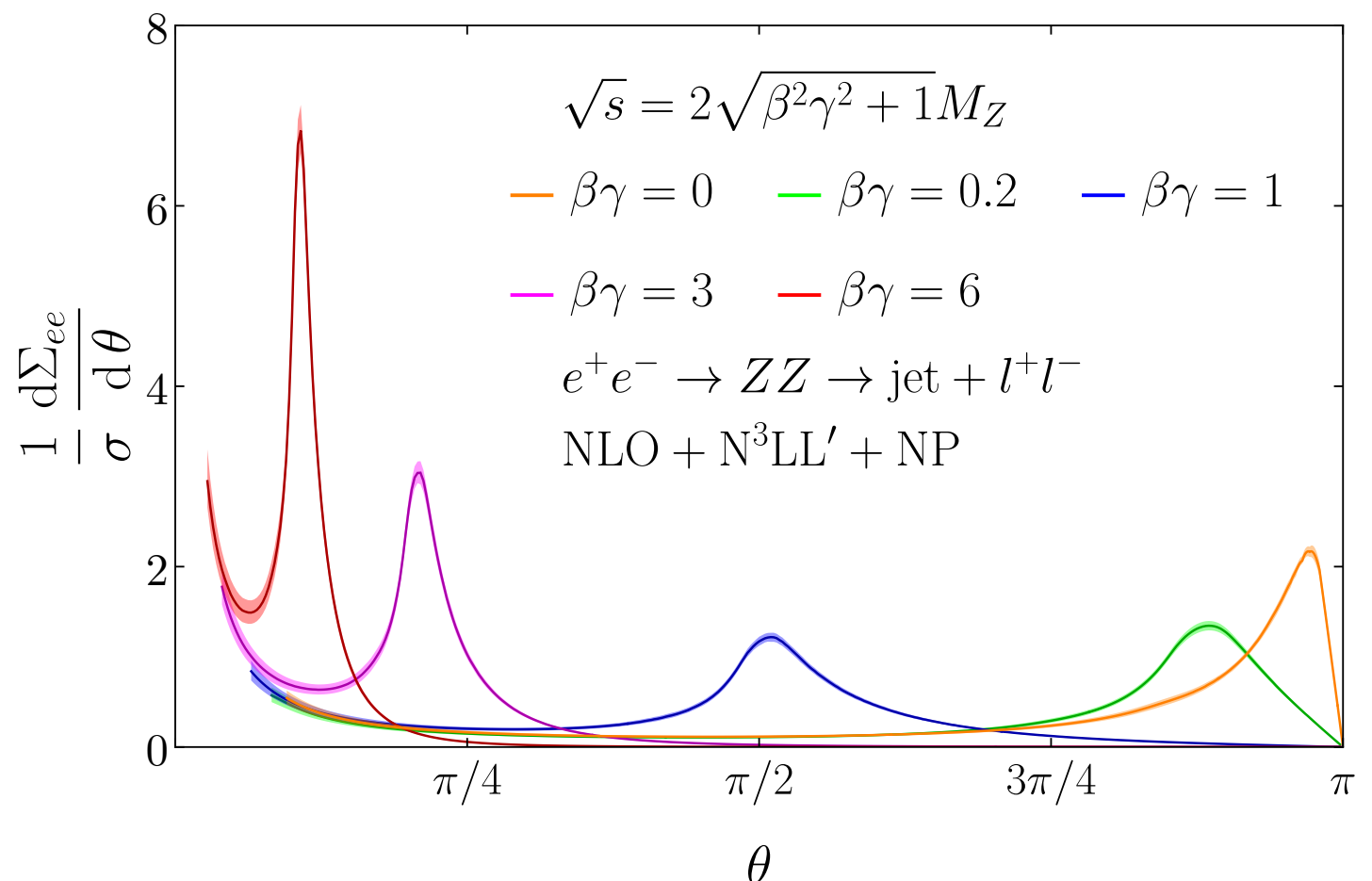


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- Simplest for $e^+e^- \rightarrow ZZ$, power corrections for Z at LHC.
- Structure function missing [Gao, ESI program '26]
- Talk on related topic by Zhen Xu.



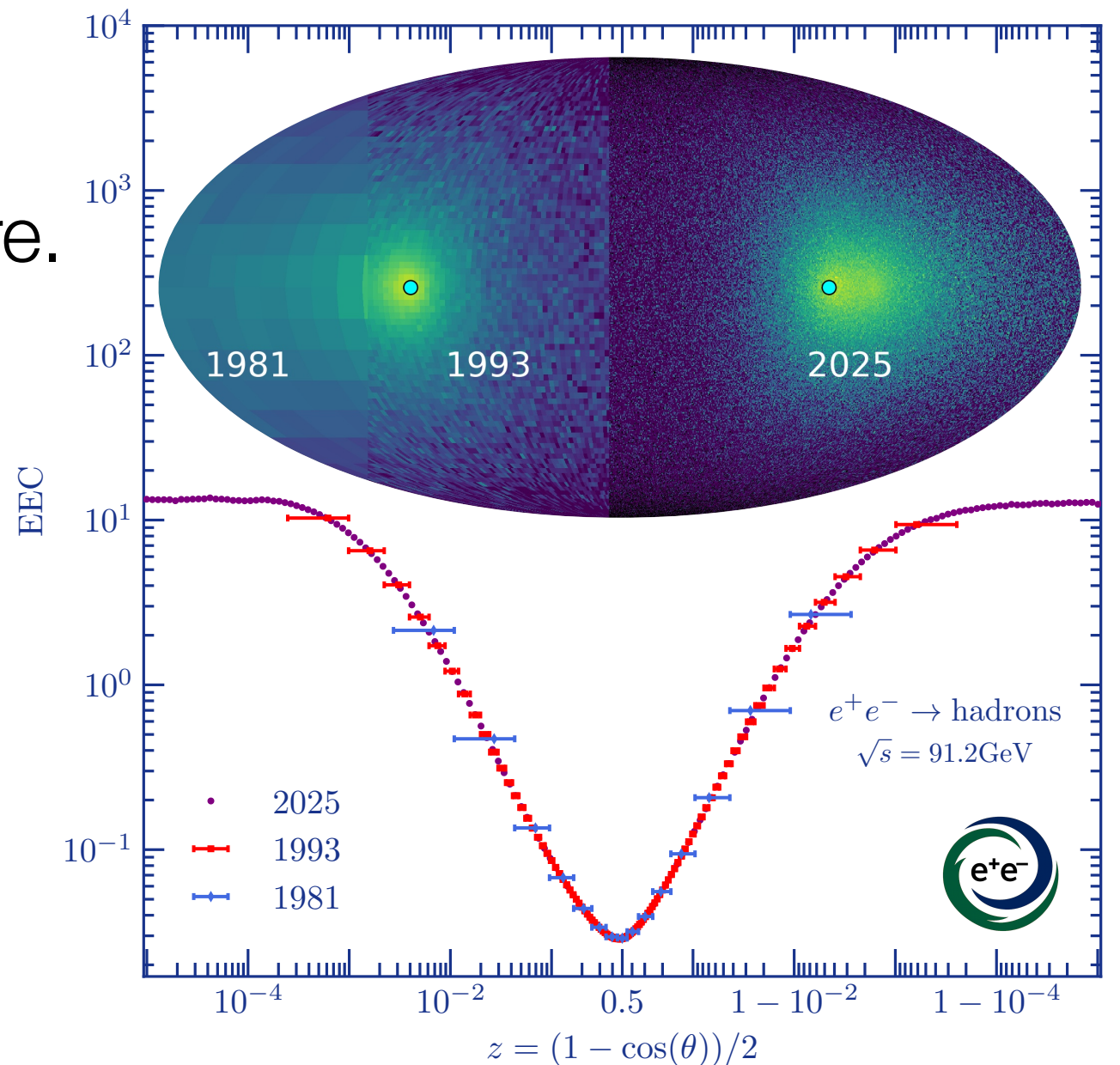
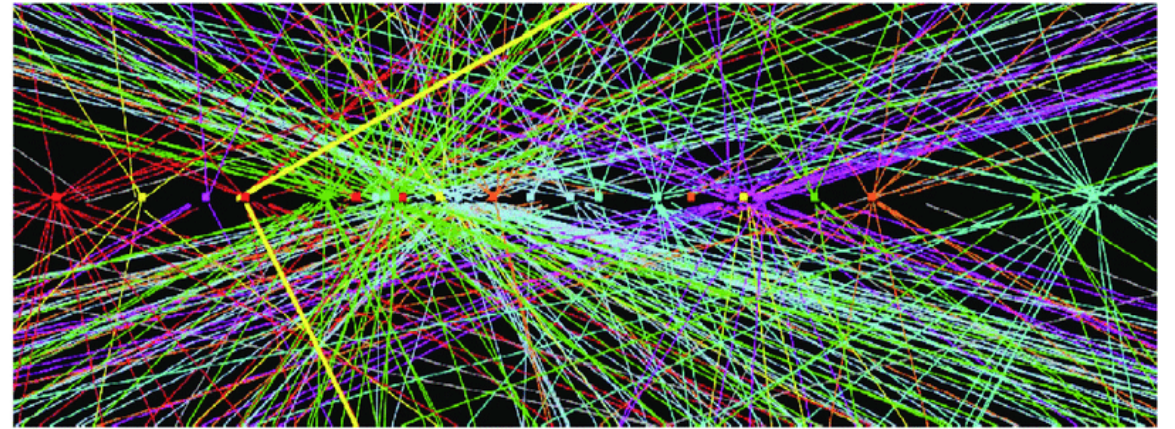
3. Energy Correlators & Tracks Functions



Experimental benefits of using tracks

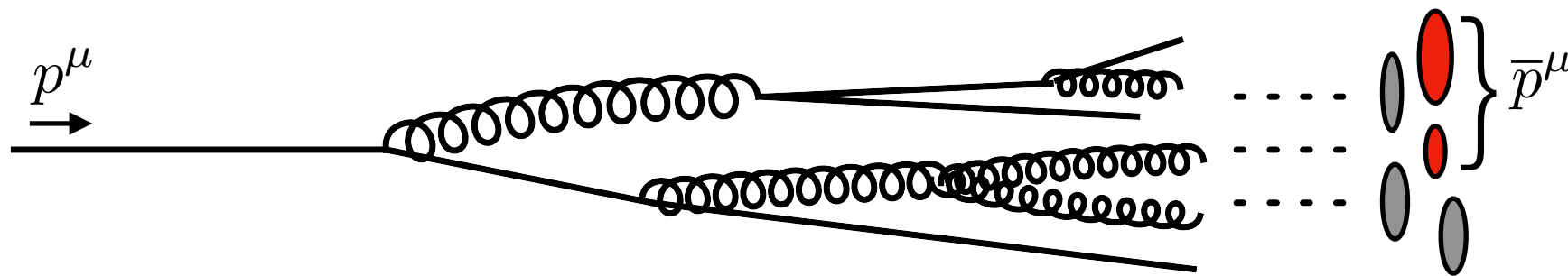
✓ Pile-up removal.

✓ Superior angular resolution
→ crucial for jet substructure.



Theoretical framework for tracks

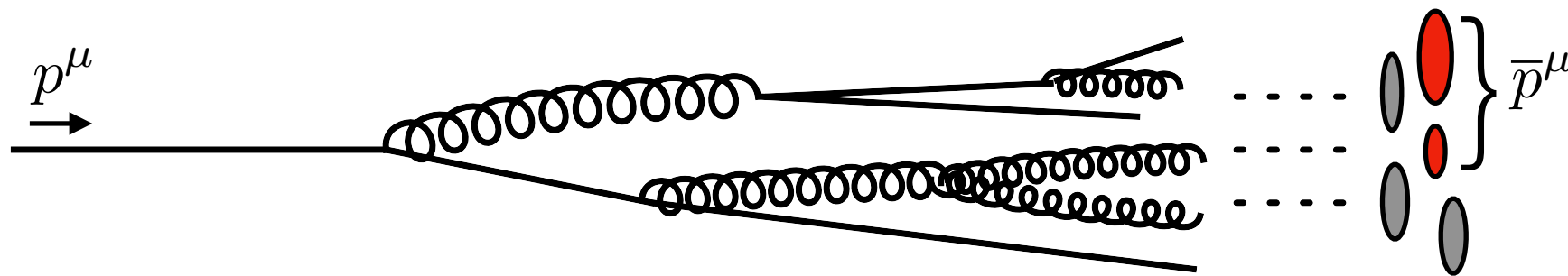
- Track-based observables are sensitive to hadronization.
- Track functions offer a systematically improvable framework:
[Chang, Procura, Thaler, WW]



- $T_i(x, \mu)$ describes **total** momentum fraction x of initial parton converted to **tracks**, i.e. $\bar{p}^\mu = x p^\mu + \mathcal{O}(\Lambda_{\text{QCD}})$

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- $T_i(x, \mu)$ describes **total** momentum fraction x of initial parton converted to **tracks**, i.e. $\bar{p}^\mu = x p^\mu + \mathcal{O}(\Lambda_{\text{QCD}})$
- Like fragmentation function, but with **nonlinear** evolution:

$$\frac{d}{d \ln \mu^2} T_i(x, \mu) = \sum_{j,k} \int dz \frac{\alpha_s}{4\pi} P_{ji}(z) \int dx_1 T_j(x_1, \mu) \int dx_2 T_k(x_2, \mu) \times \delta[x - zx_1 - (1-z)x_2] + \mathcal{O}(\alpha_s^2)$$

Track functions enable precision energy correlators

$$\frac{d\sigma}{d\theta} = \int d\sigma \sum_{i,j} \frac{E_i E_j}{Q^2} \delta(\theta - \theta_{ij})$$

- Conversion to tracks is simple, but requires flavor information:

$$E_i \rightarrow \int dx_i T_i(x_i) x_i E_i = T_i(1) E_i$$

- $T_i(2)$ enters contact term at $\theta = 0$ and resummation of $\ln \theta$.

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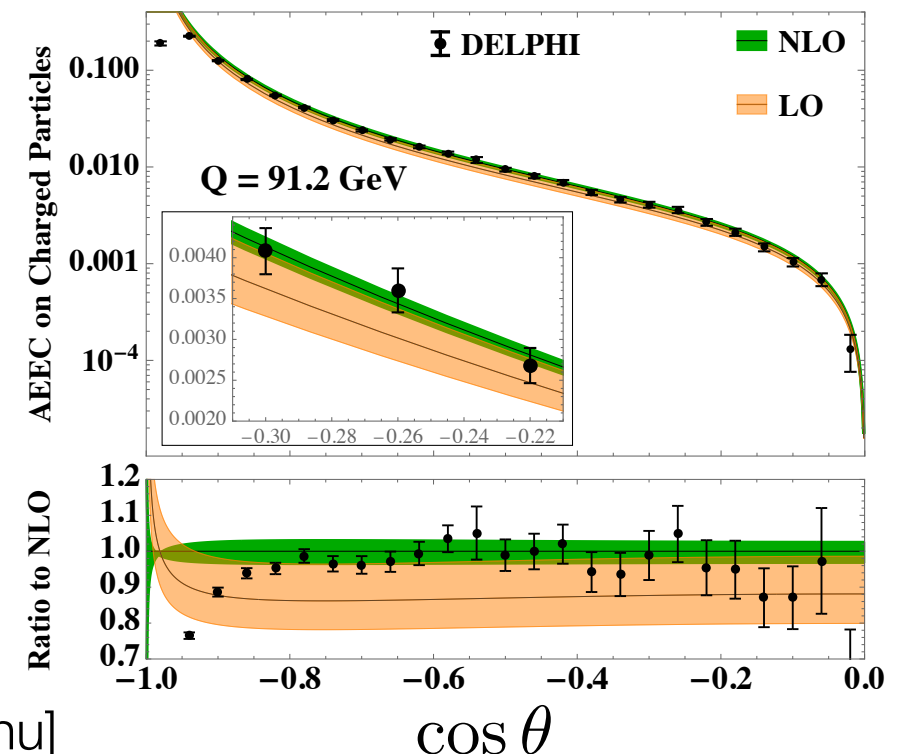
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- Track functions have been extended to $\mathcal{O}(\alpha_s^2)$ → **precision!**

[Chen, Jaarsma, Li, Moult, WW, Zhu]

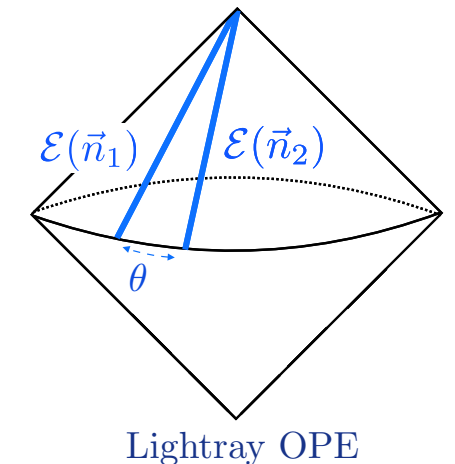


[Li, Moult, Schrijder van Velzen, WW, Zhu]

Ingredients for precision track-based EEC

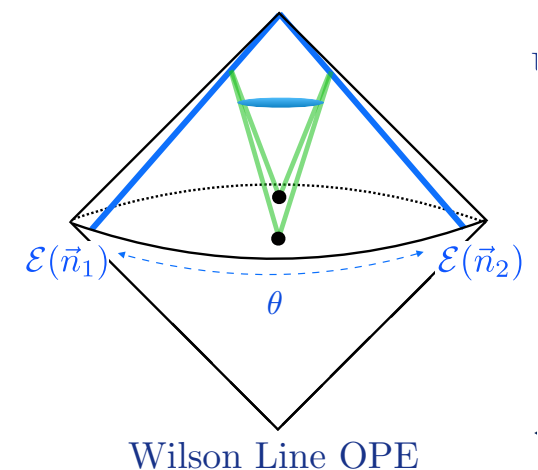
Collinear region ($z \rightarrow 0$)

- NNLL resummation of single logarithms of z .
- Jet function matched onto track functions.
- Nonperturbative plateau modelled.



Back-to-back region ($z \rightarrow 1$)

- NNNLL resummation of double logarithms of $1-z$.
- TMD factorization, nonperturbative Collins-Soper kernel.
- Jet function matched onto track functions, soft function only contributes through recoil.

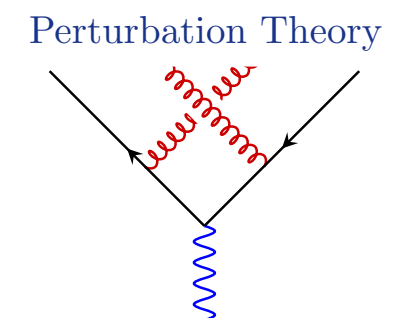


Fixed-order region

- Order α_s^3 from CoLoRFuNNLO.

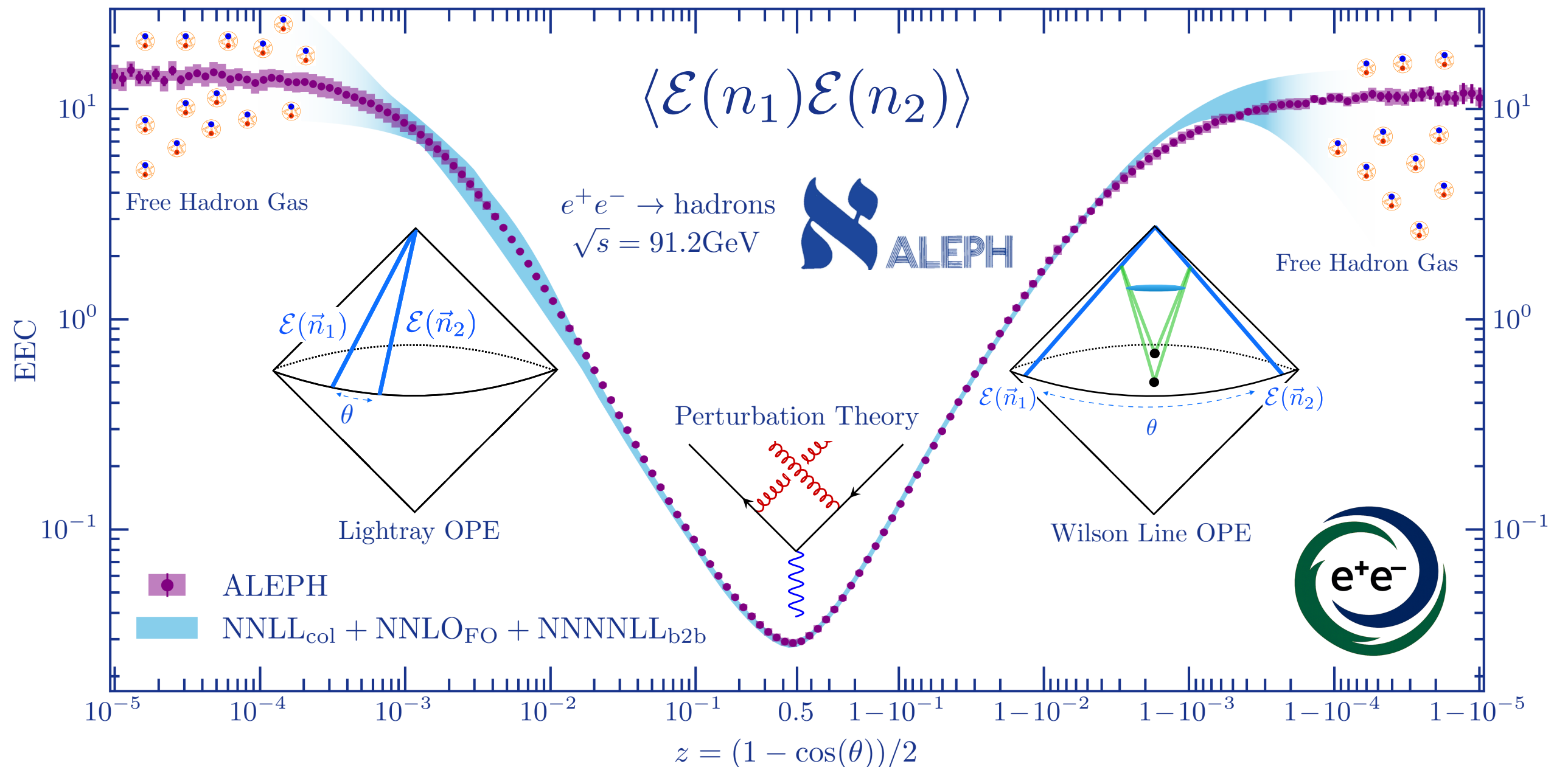
All regions:

- Leading nonperturbative correction described by parameter Ω_1 , rescaled by $T_g(1)$
- Transition between regions using profile functions.



Comparison to archival ALEPH data

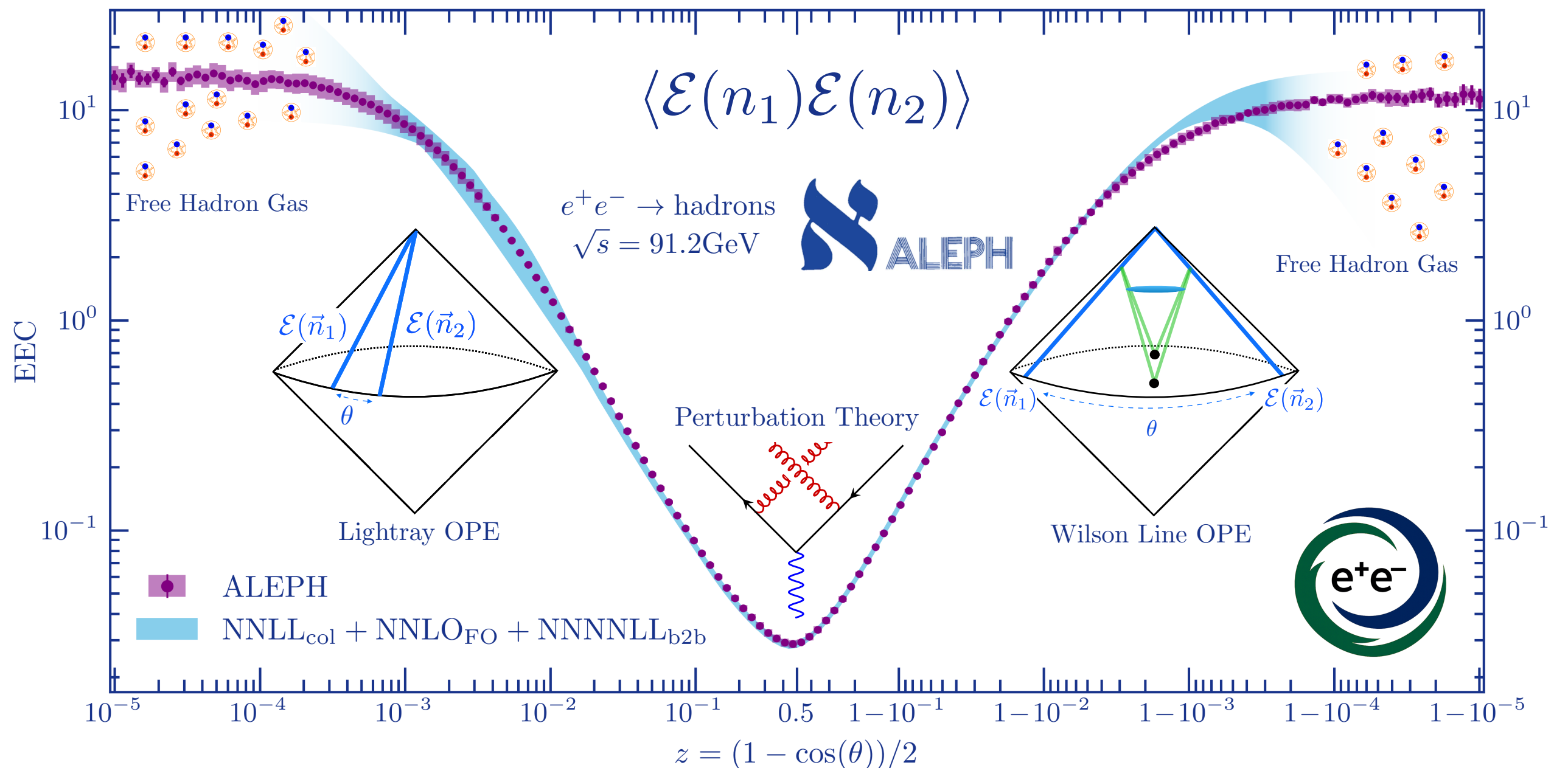
- Over order-of-magnitude gain in angular resolution with tracks:



[Lee et al; theory: Jaarsma, Li, Moult, WW, Zhu]

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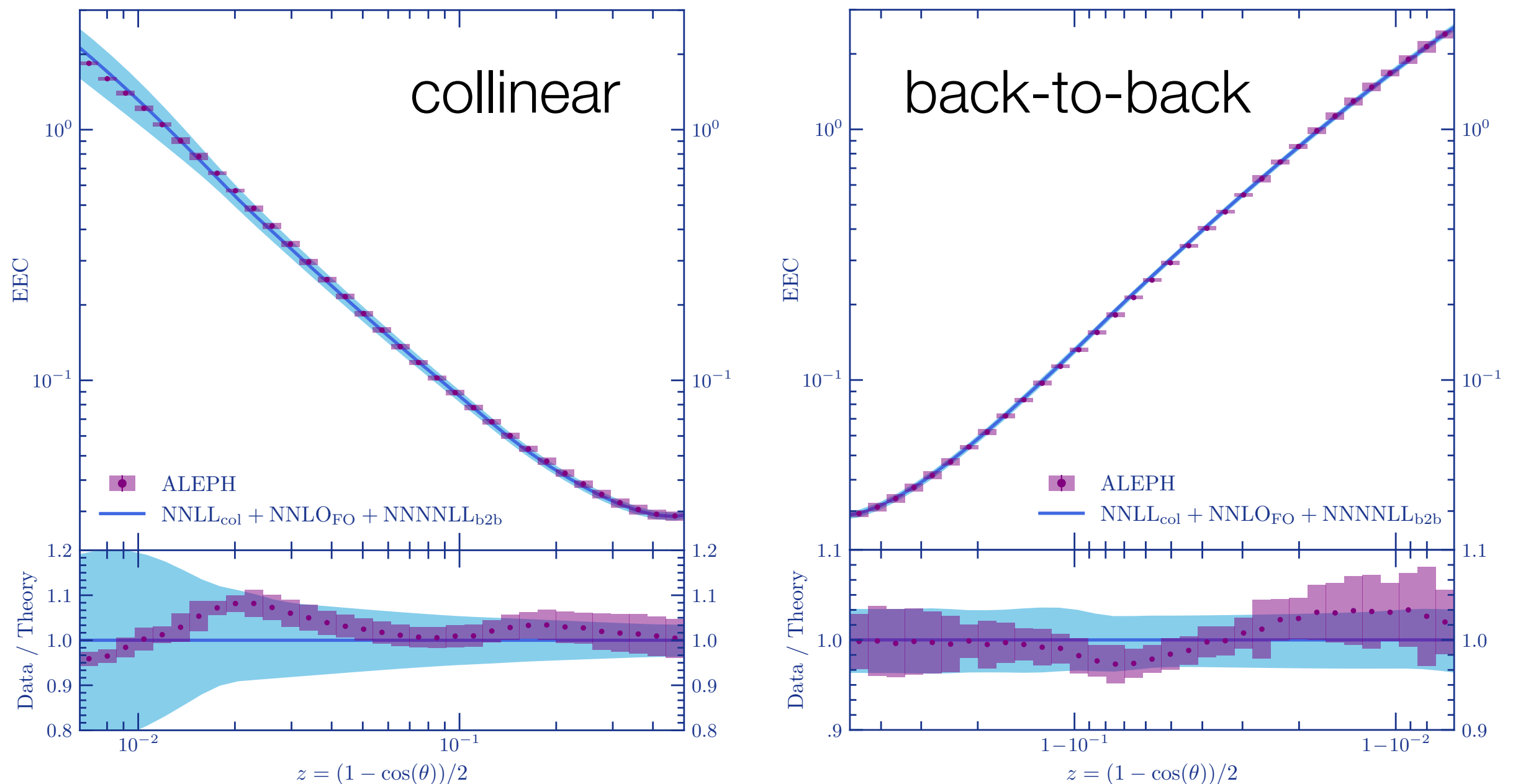
- Over order-of-magnitude gain in angular resolution with tracks:



[Lee et al; theory: Jaarsma, Li, Moult, WW, Zhu]

- More cool stuff with archival data: talk by Yi Chen.

Results for the resummation regions



- Theory uncertainty is $\sim 5\%$ for most of the range.
- Good agreement between theory and data.

[Lee et al; theory: Jaarsma, Li, Moult, WW, Zhu)]

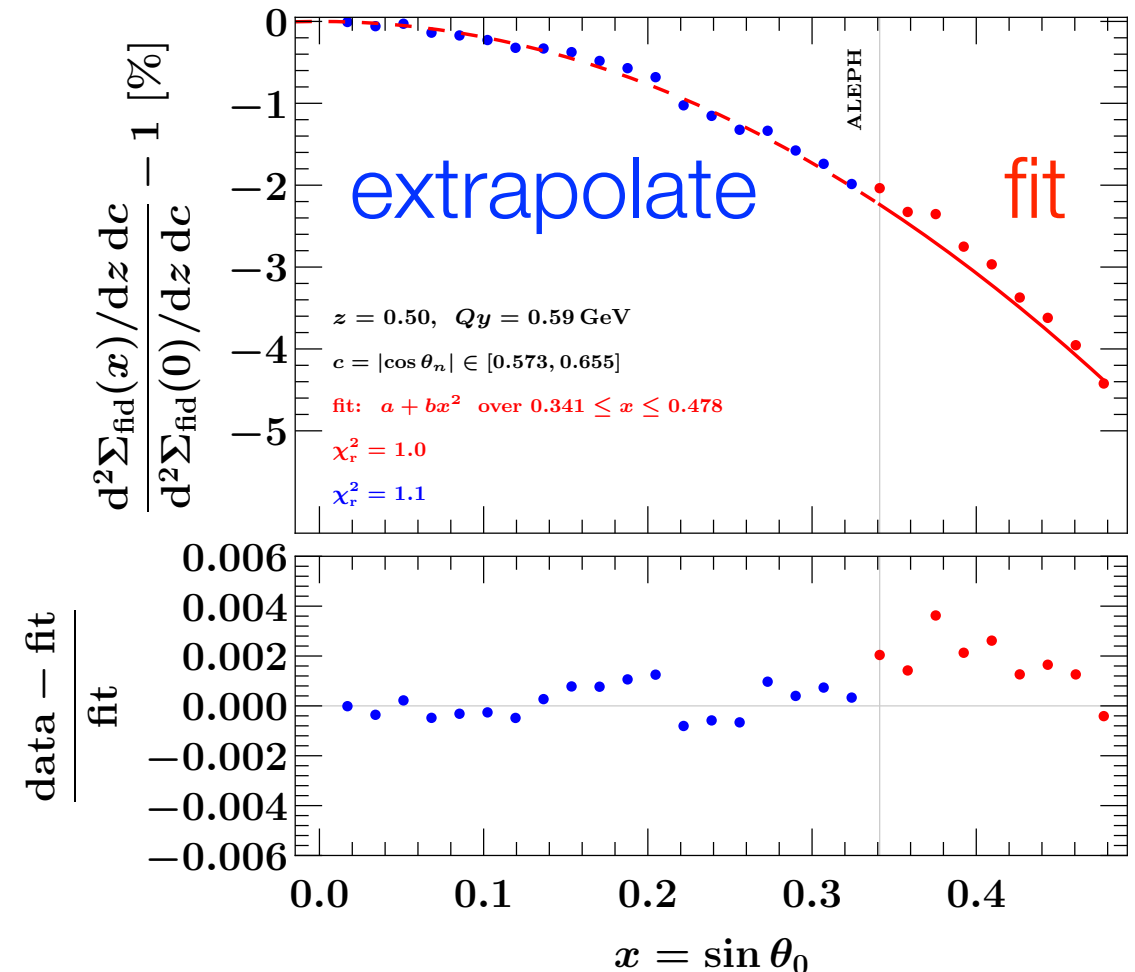
Fiducial cuts

- Cut on angle θ_n between beam and event axis.
→ Decompose in unoriented and oriented structure functions:

$$\frac{d^2\Sigma_{\text{fid}}}{dz d\cos\theta_n} = \frac{3}{8} (1 + \cos^2\theta_n) \frac{d\Sigma_U(Q, z)}{dz} + \frac{3}{16} (1 - 3\cos^2\theta_n) \frac{d\Sigma_O(Q, z)}{dz}$$

want this

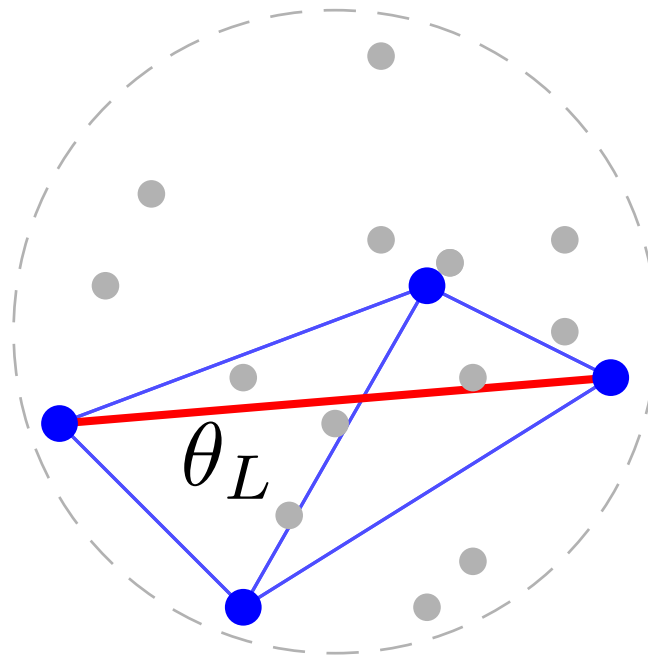
- Particle-level cuts in p_T and η
→ (quadratic) expansion.
- See talk by Alessio Vorona.



4. Higher-point correlators

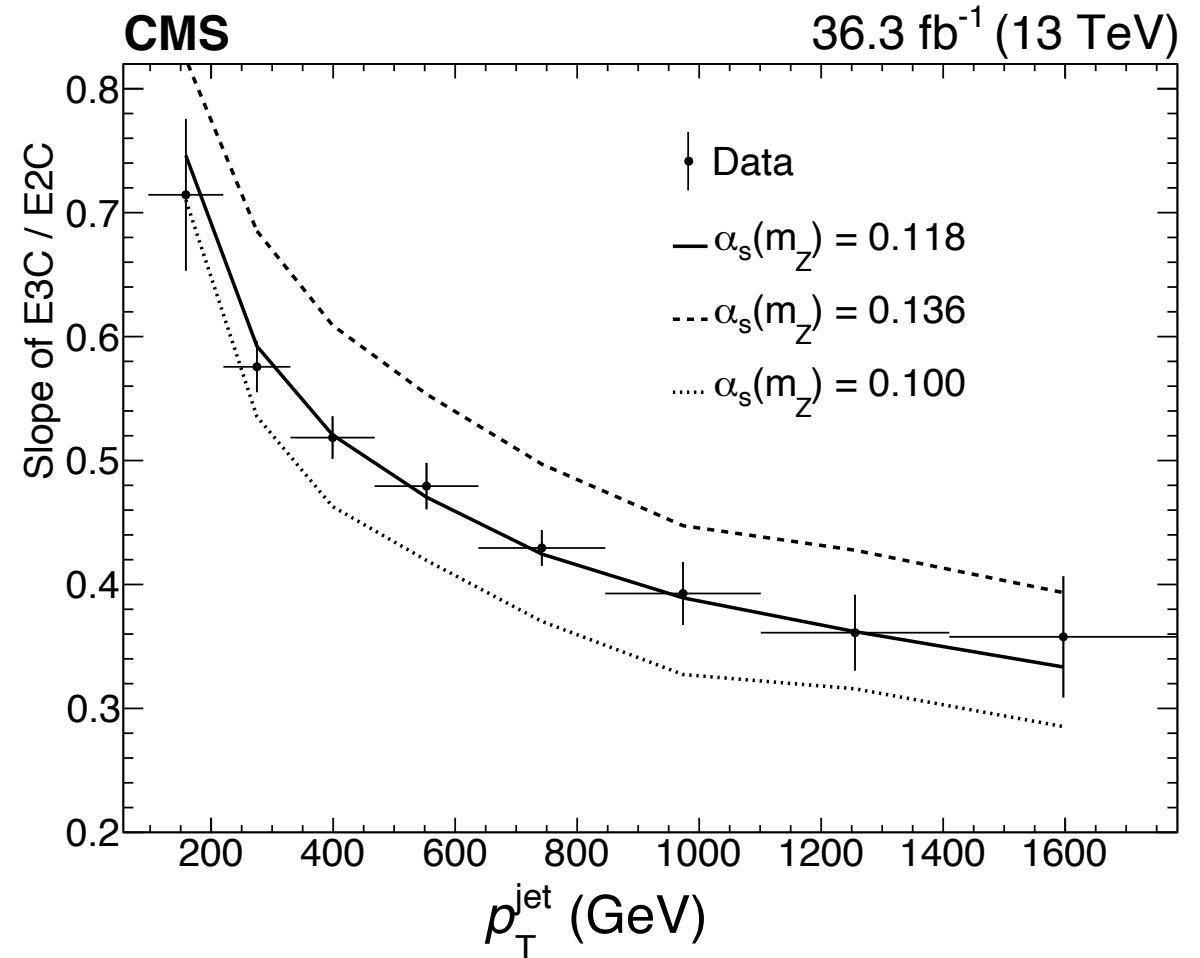
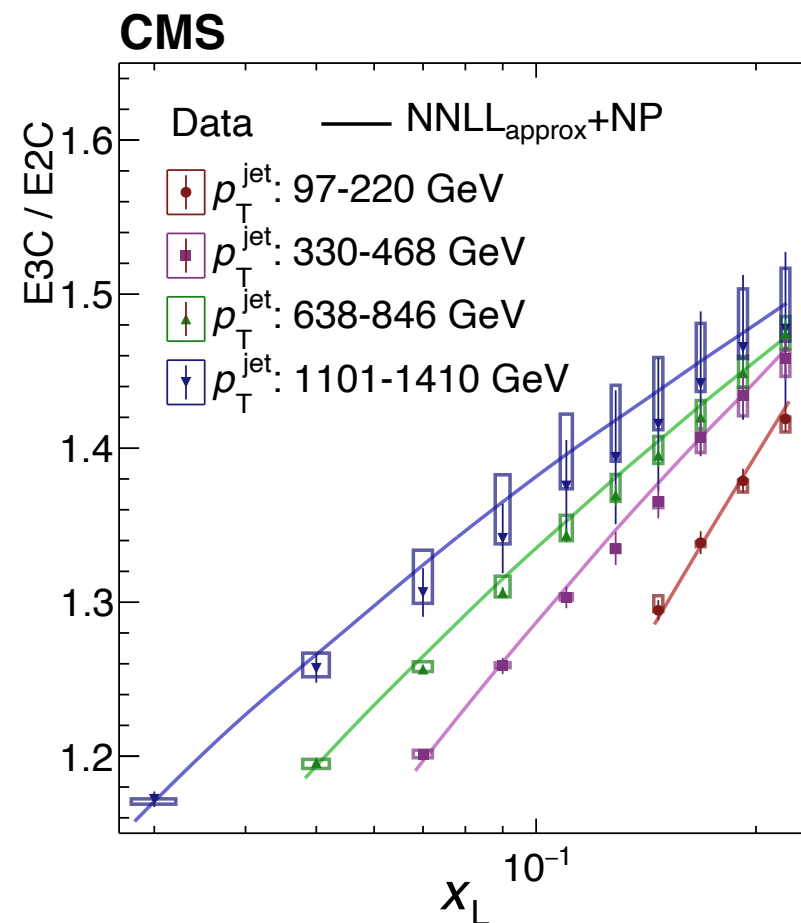


Higher-point correlators



- N -point energy correlators depend on all angles θ_{ij}
- The **largest** angle θ_L projection satisfies the power-law $\frac{d\sigma}{d\theta_L} \sim \theta_L^{-1+\gamma(N)}$ where $\gamma(N)$ is a moment of the splitting function.
- Analytic continuation to $N \rightarrow 0$ accesses small x physics

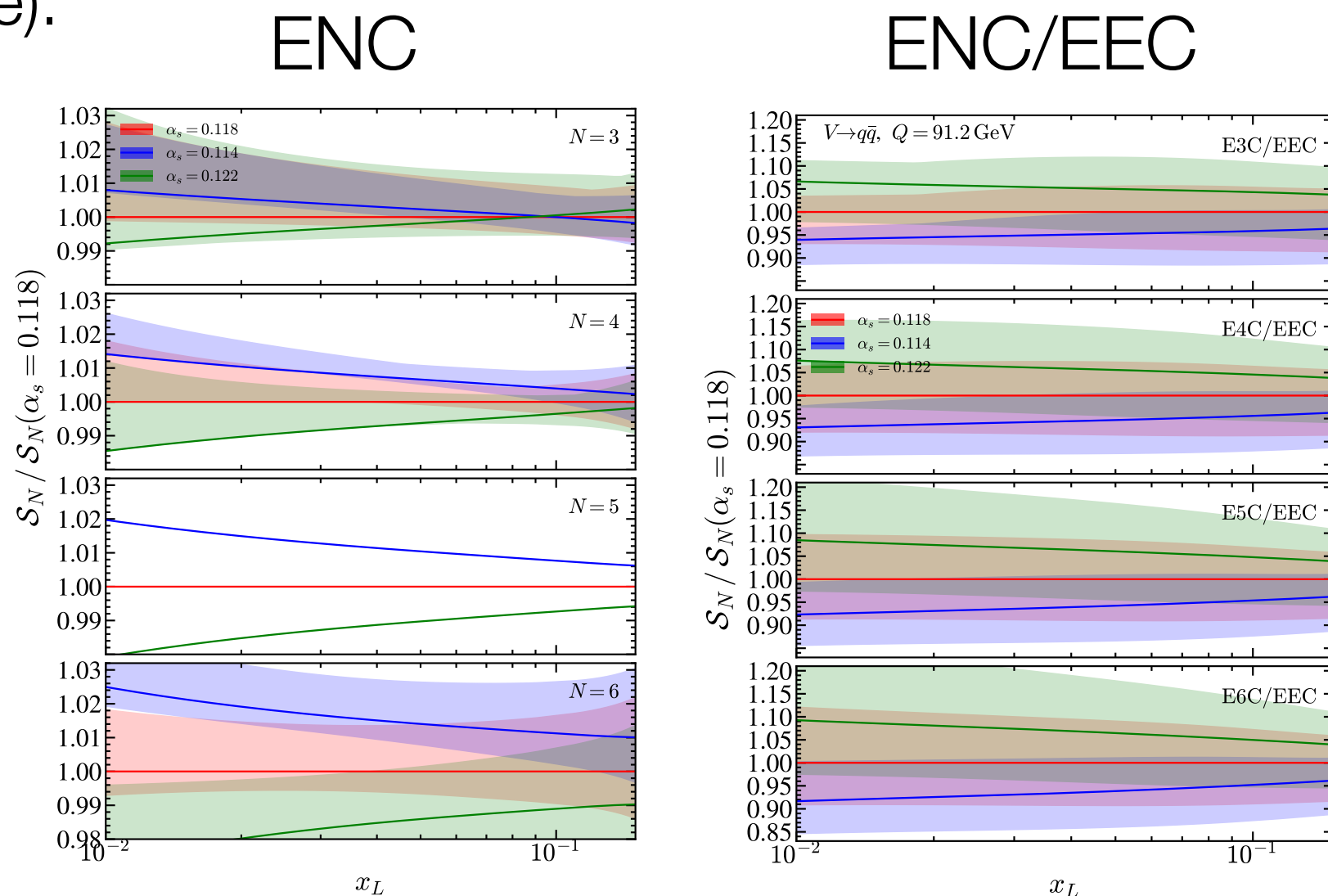
Extracting α_s from E3C/EEC ratio



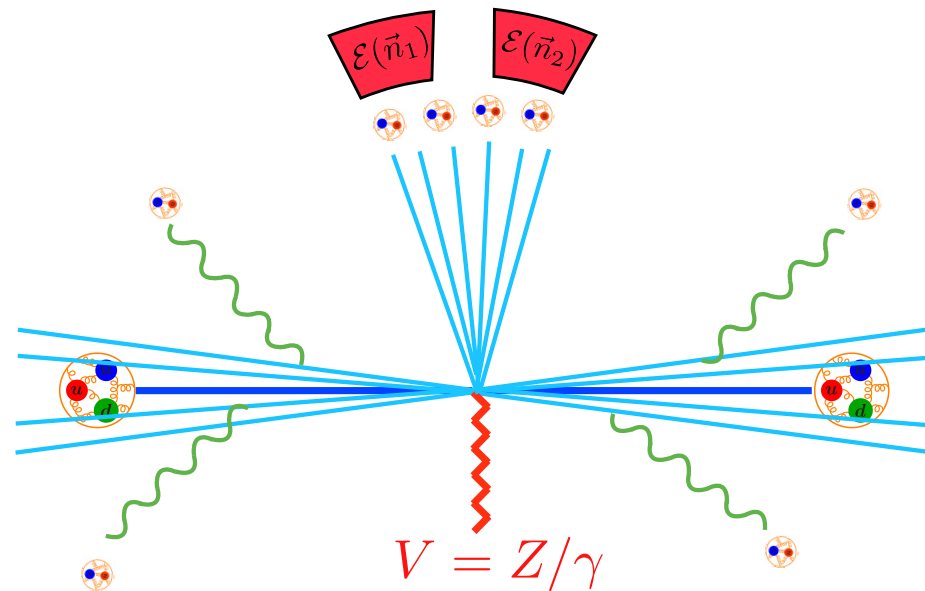
- Extract $\alpha_s(m_Z)$ from slope of E3C/EEC, compare to NLO+NNLL.
- Most precise* measurement of α_s from jet substructure.

NNLL accuracy for projected correlators

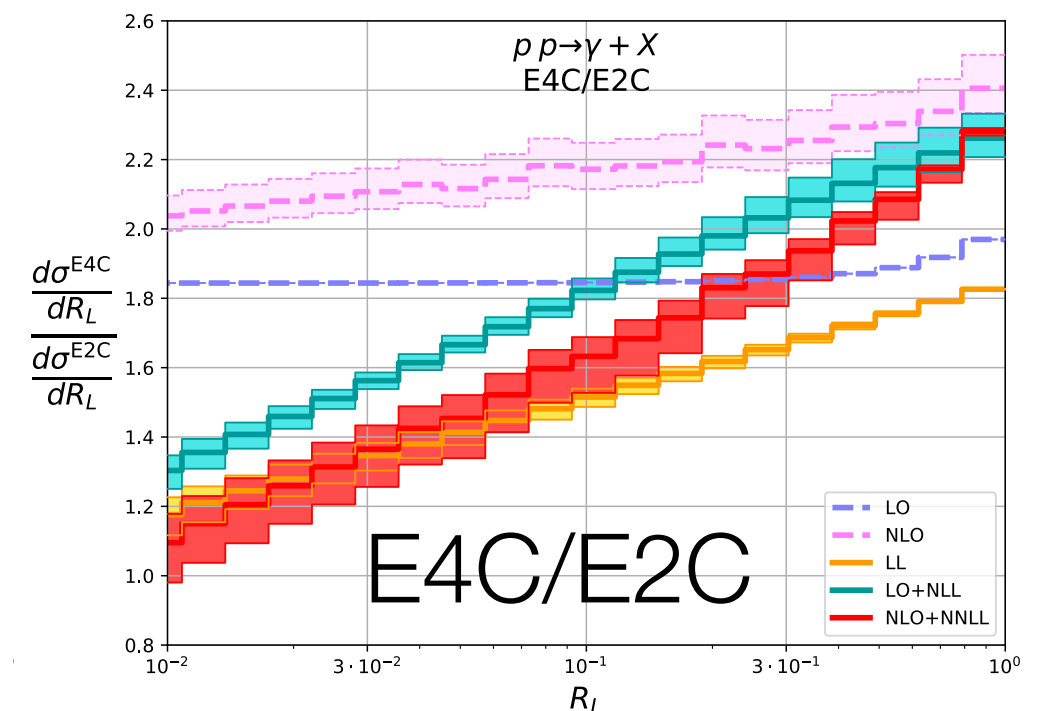
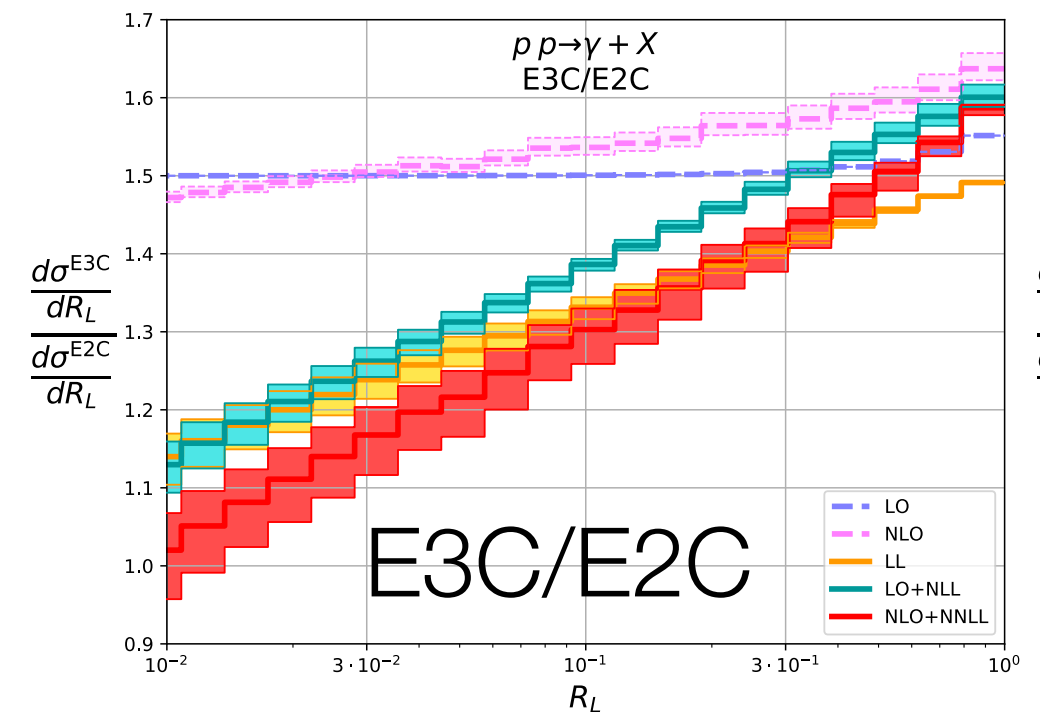
- Projected E3C known at NNLL [Chen, Luo, Moul, Yang, Zhang, Zhu].
E4C, E5C and E6C, specifically two-loop constants, are new.
- The slope of the ratio to the E2C is much more sensitive to α_s (includes nonperturbative).



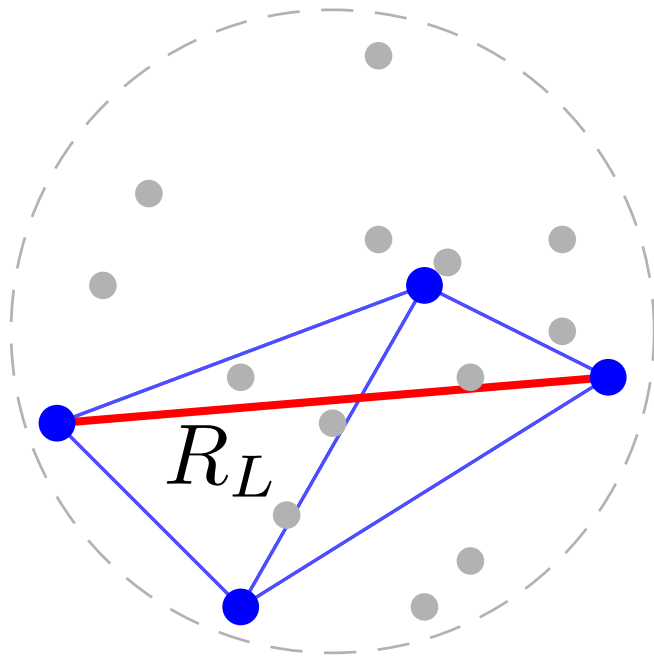
Vector-boson recoil removes need for a jet algorithm



- New ingredient: hard function for this process.
- Resummation essential to predict shape of distribution.



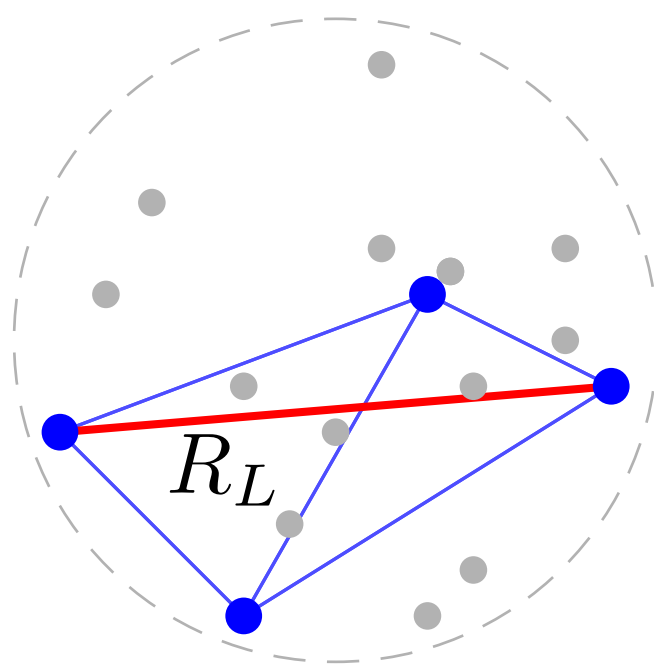
Higher-point correlators are computationally expensive



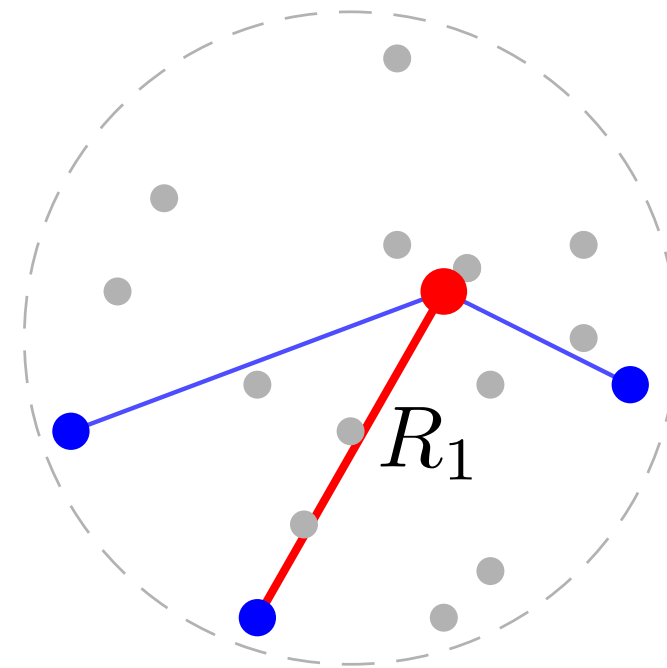
traditional

- Time scales like $\mathcal{O}(M^N)$ for an N -point correlator of M particles.

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traditional



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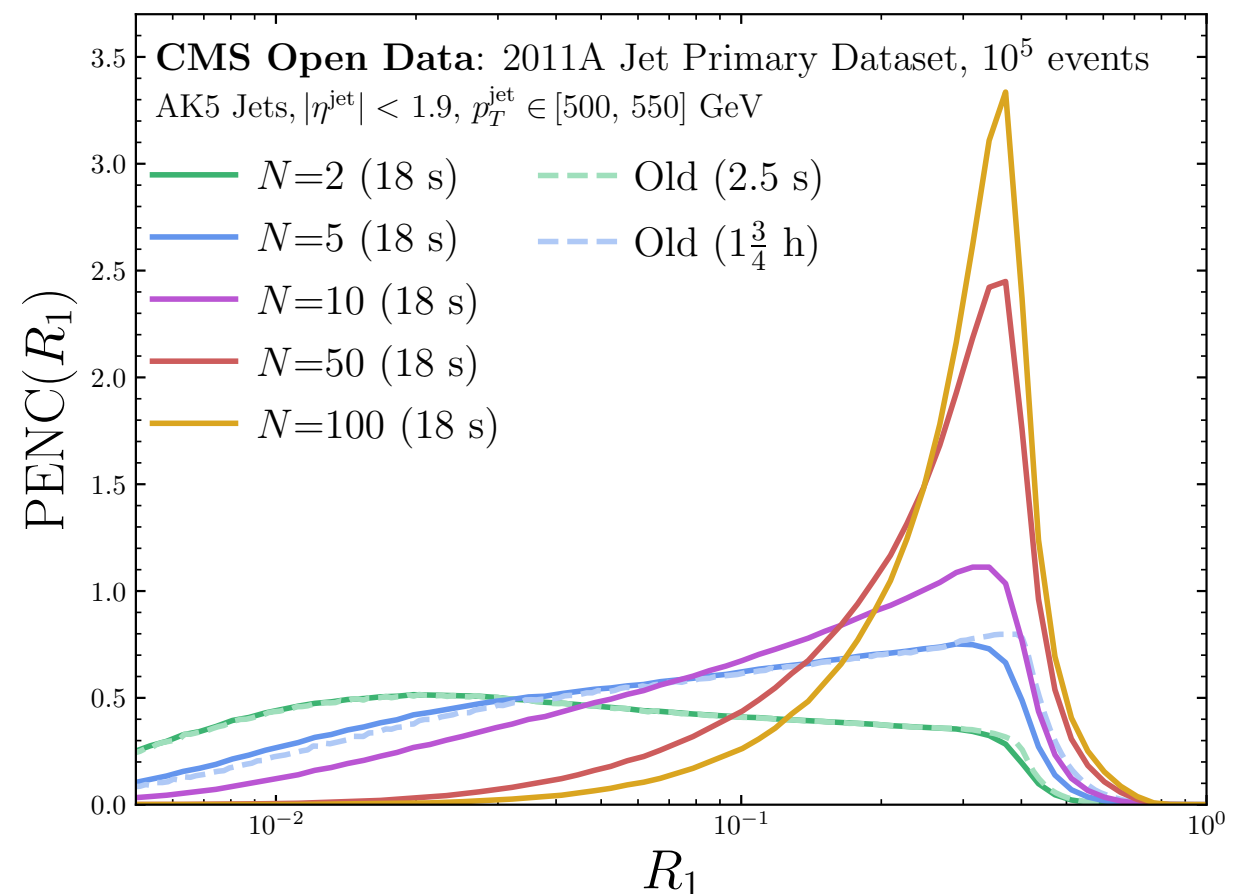
- Time scales like $\mathcal{O}(M^N)$ for an N -point correlator of M particles.
- Solution: measure distances to a **special** particle

$$\frac{d\sigma^{[N]}}{dR_1} = \int d\sigma \sum_{\mathbf{s}} z_{\mathbf{s}} \sum_{j,k,\dots} z_j z_k \cdots \delta(R_1 - \max\{R_{\mathbf{s}i}, R_{\mathbf{s}j}, \cdots\})$$

Fast N -point correlators, same physics

- Runtime is $\mathcal{O}(M^2 \ln M)$ for projected correlators for **all N** .
- This follows from the cumulative distribution:

$$\int^{R_1} dR'_1 \frac{d\sigma^{[N]}}{dR'_1} = \int d\sigma \sum_{\mathbf{s}} z_{\mathbf{s}} z_{\text{disk}}(\mathbf{s}, R_1)^{N-1}$$

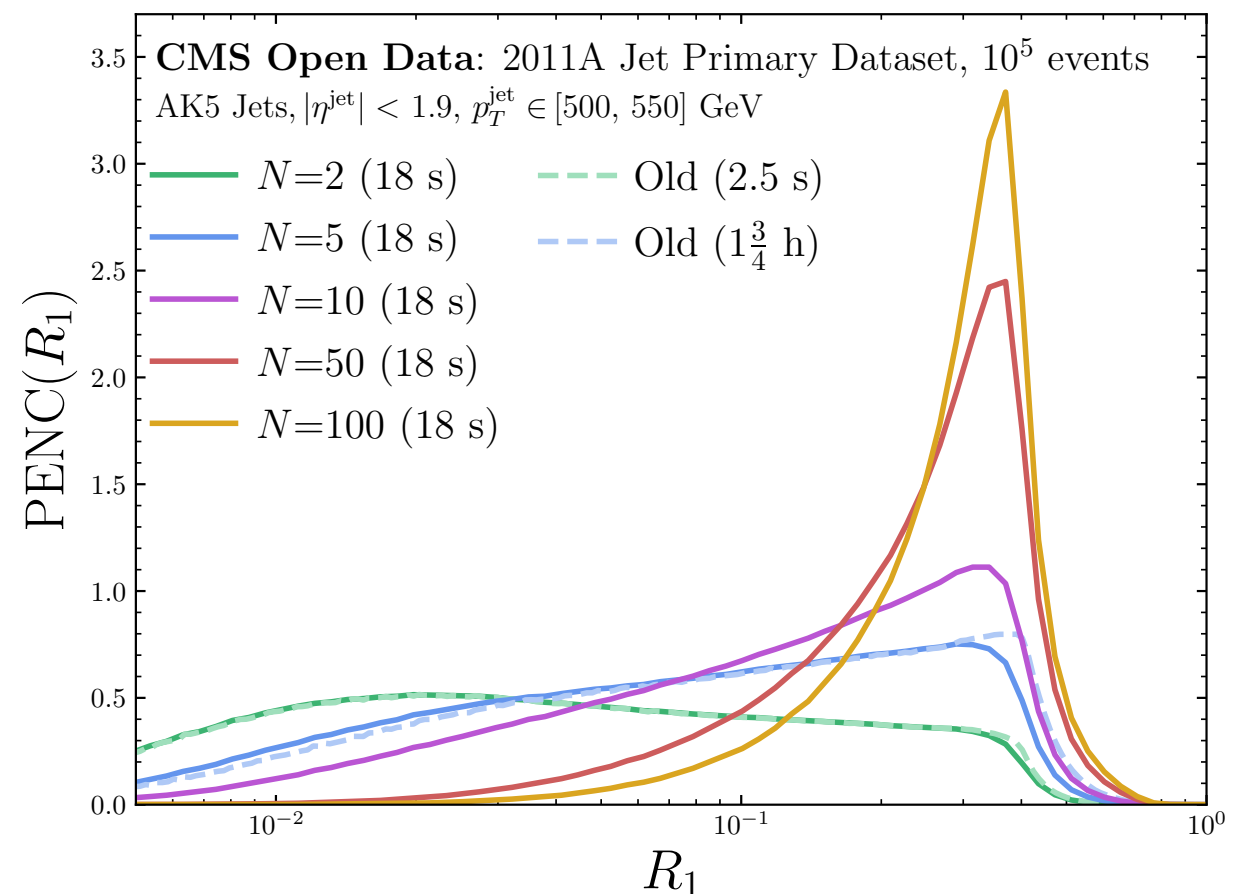


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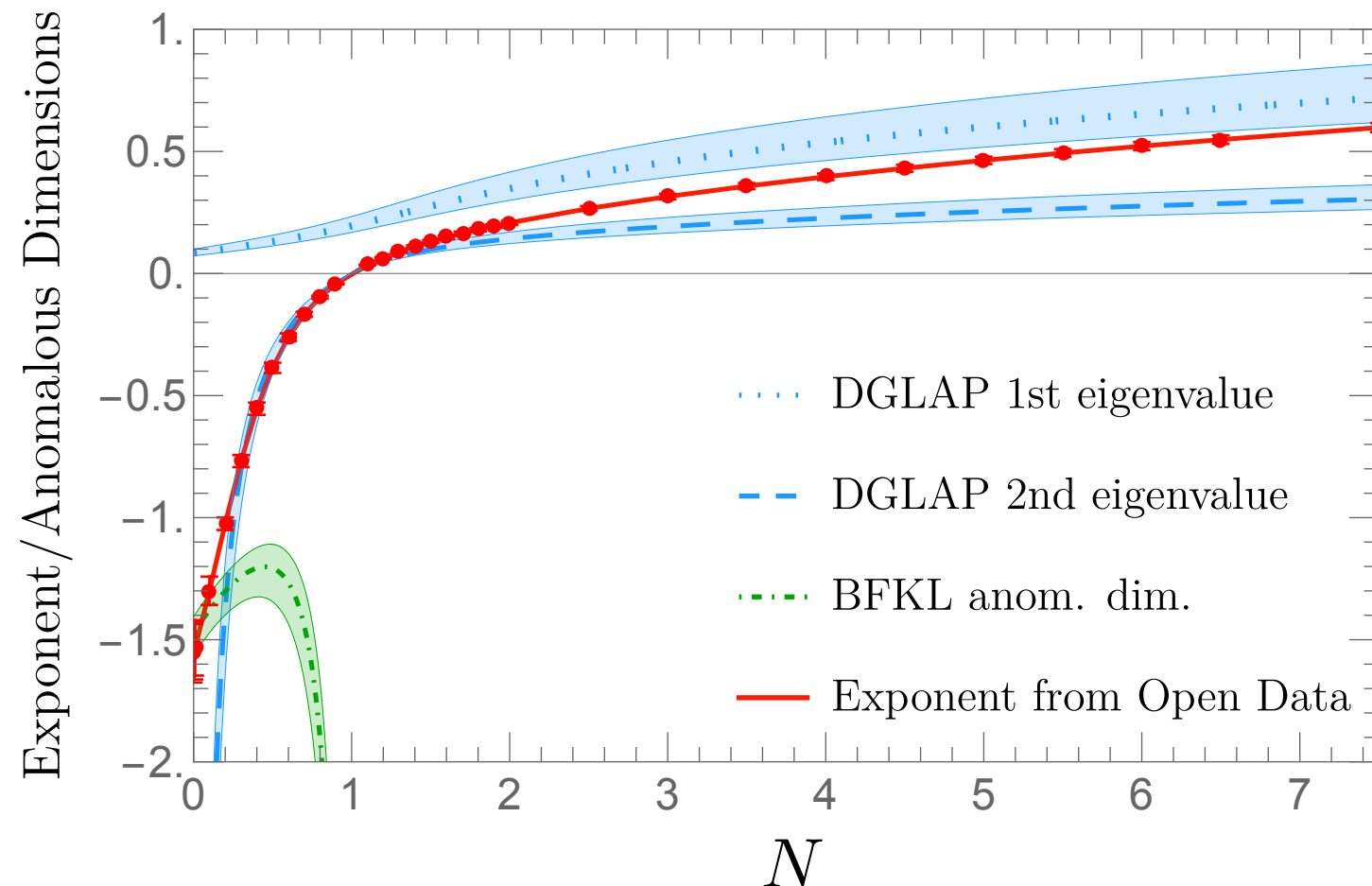
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- Same theory framework, with differences starting at NNLL.
- Analytic continuation in N is straightforward.

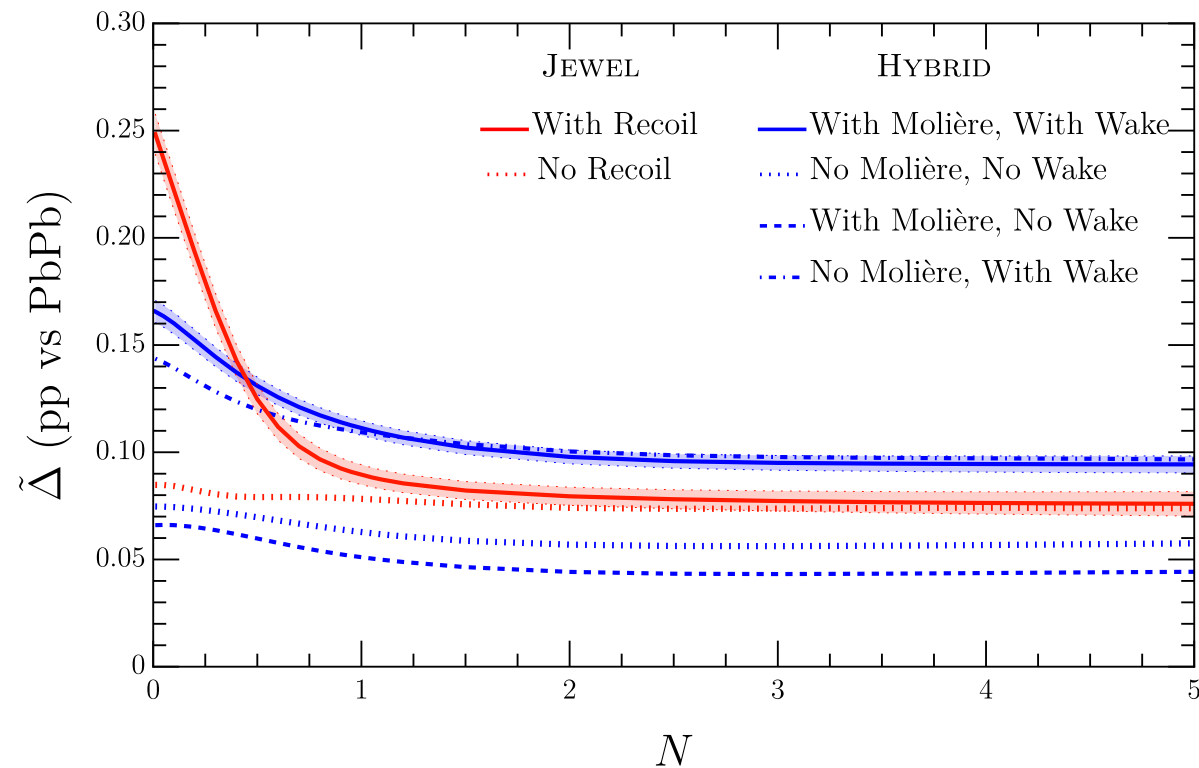


Power-law exponent for $N \rightarrow 0$ approaches BFKL

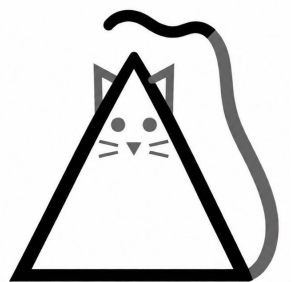


- Fit **CMS open data** to a power law.
- Quark/gluon mixing produces more than one exponent
→ plot both **DGLAP** eigenvalues.
- Interestingly, approaches **BFKL** for $N \rightarrow 0$.

JEWEL and Hybrid Model differ most at small N

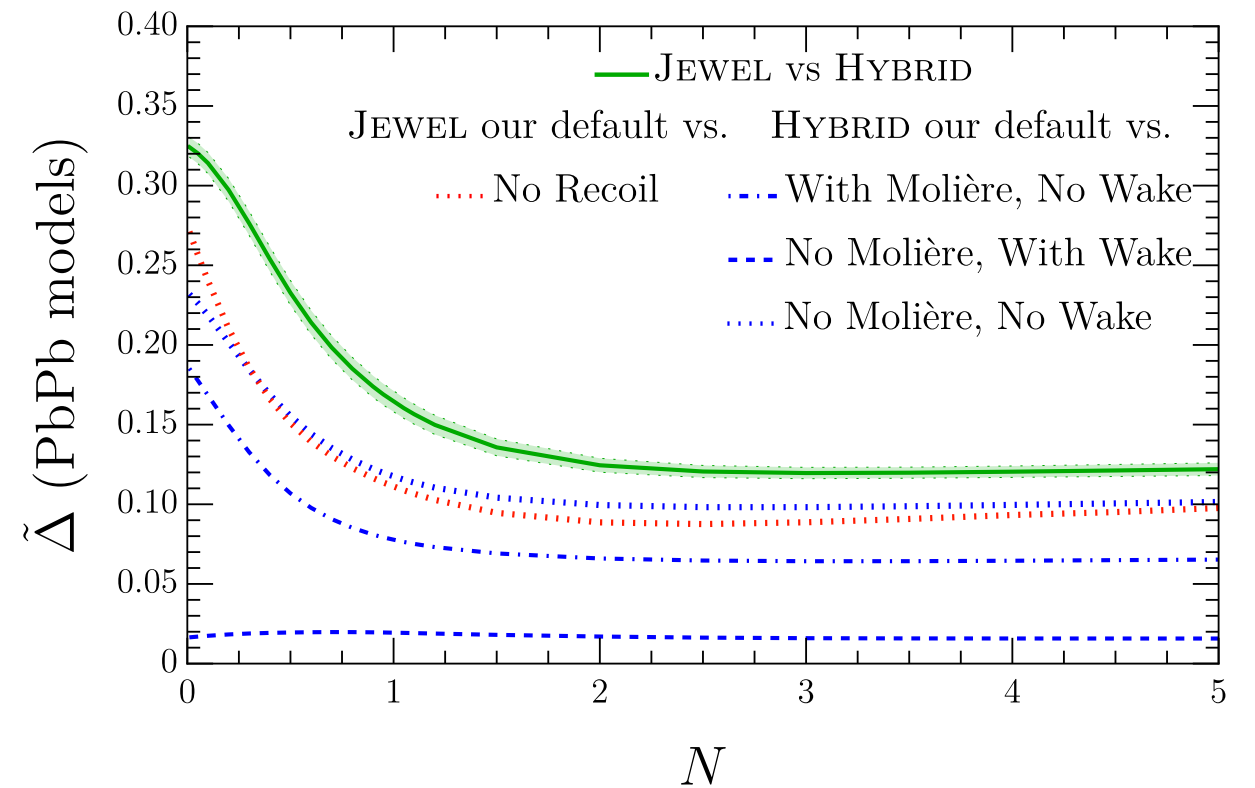
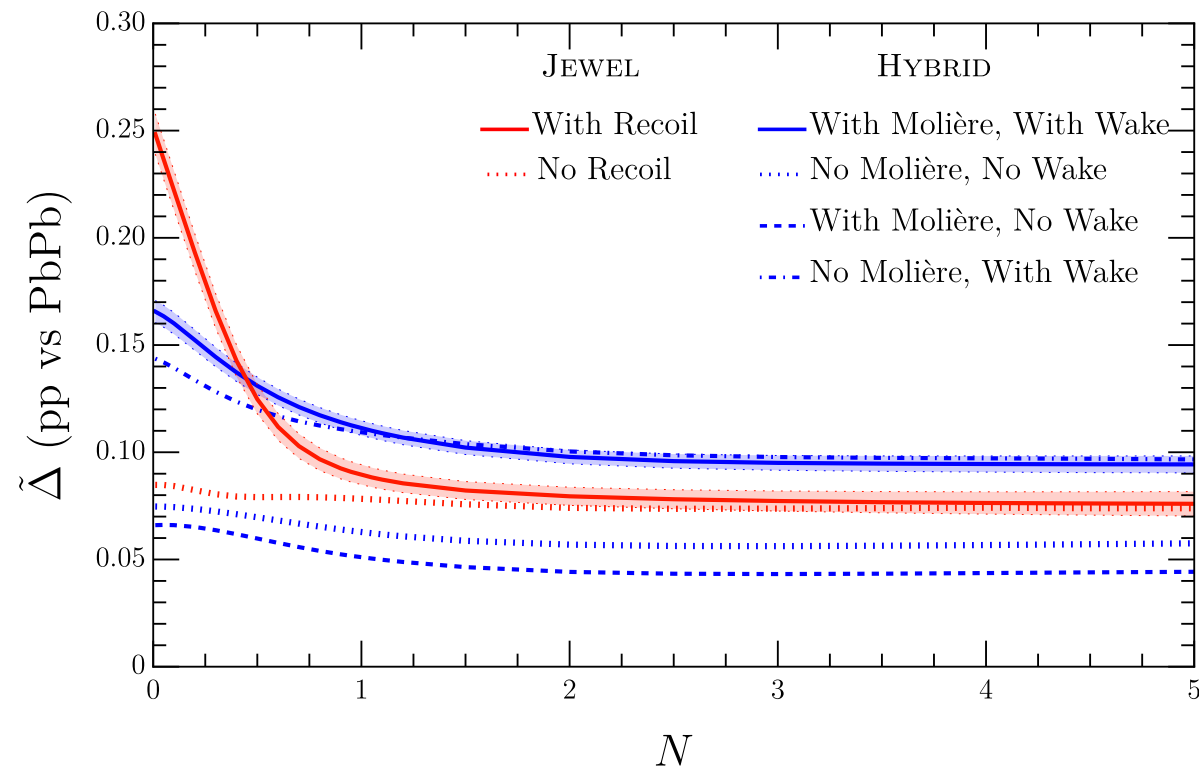


- Correlation-Aware Triangular (CAT) divergence $\tilde{\Delta}$ quantifies the difference between model predictions.
- Largest difference between PbPb and pp for $N \rightarrow 0$.

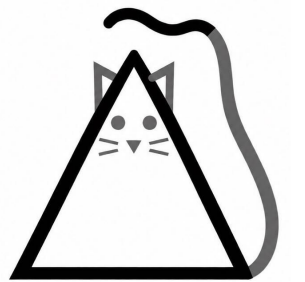


[Alipour-Fard, Budhraja, Kudinoor, Thaler, WW]

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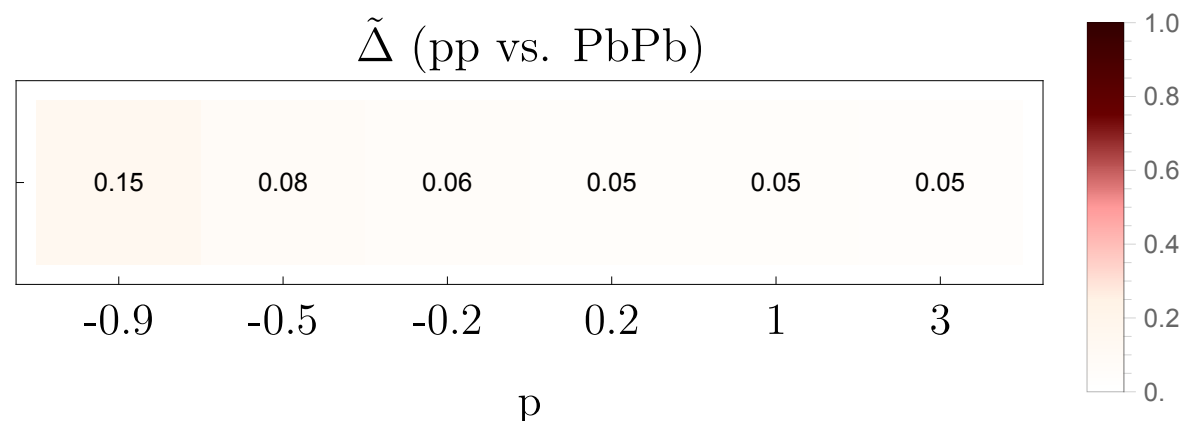
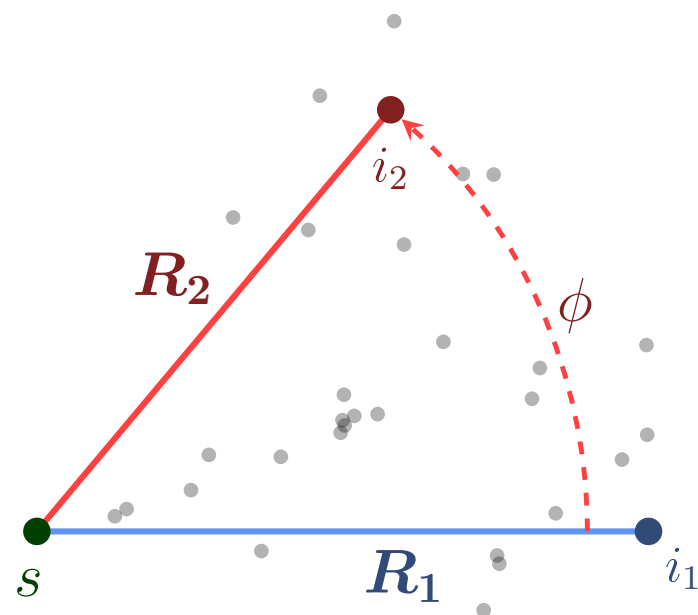
- Correlation-Aware Triangular (CAT) divergence $\tilde{\Delta}$ quantifies the difference between model predictions.
- Largest difference between PbPb and pp for $N \rightarrow 0$.
- Difference between Hybrid and JEWEL is even larger.



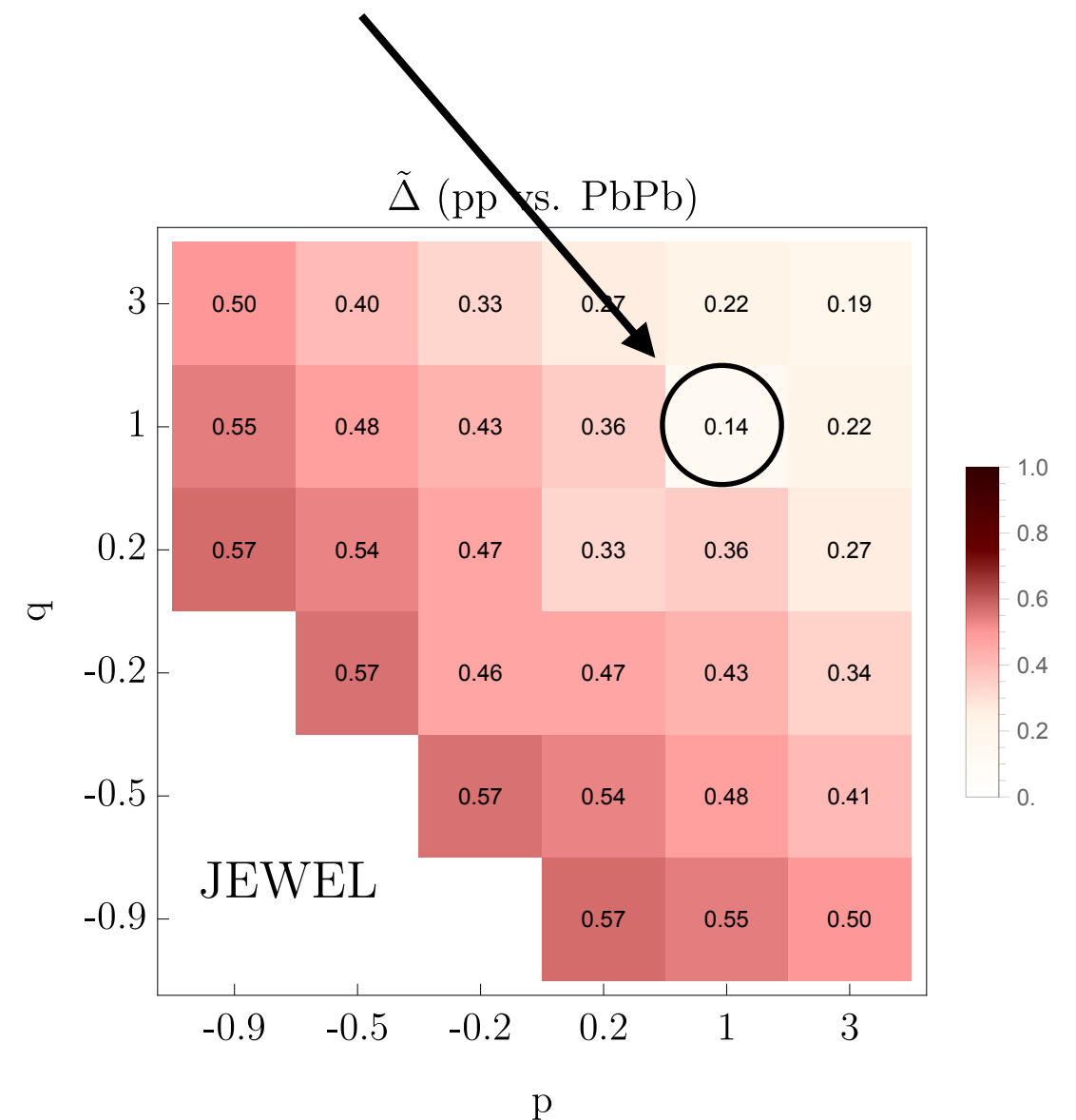
[Alipour-Fard, Budhraja, Kudinoor, Thaler, WW]

Resolved correlators enhance medium sensitivity

- More sensitivity from resolved structure & analytic continuation.
- Standard E3C has $p = q = 1$ and performs worst.



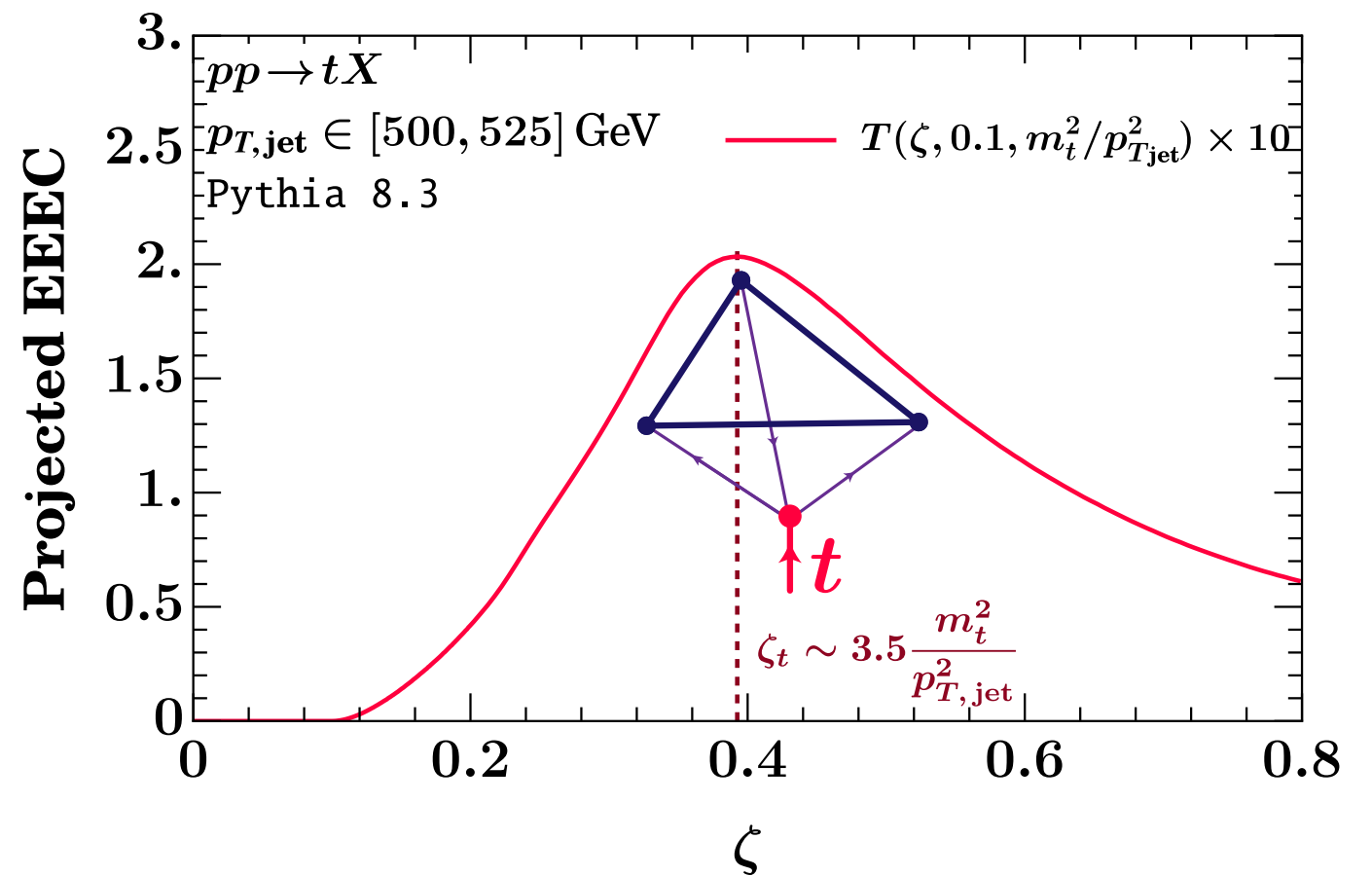
Projected



Resolved

Top quark mass from energy correlators

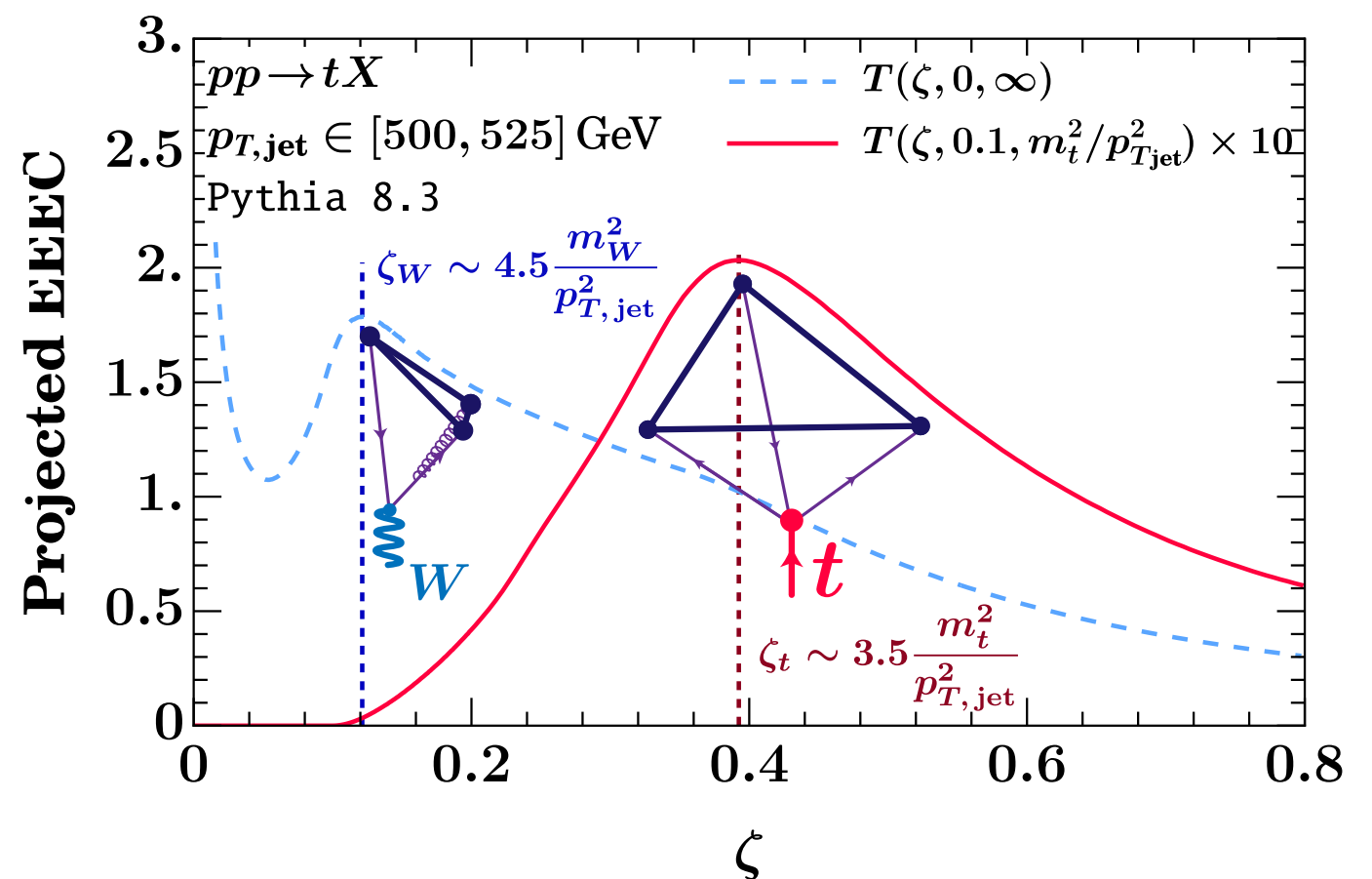
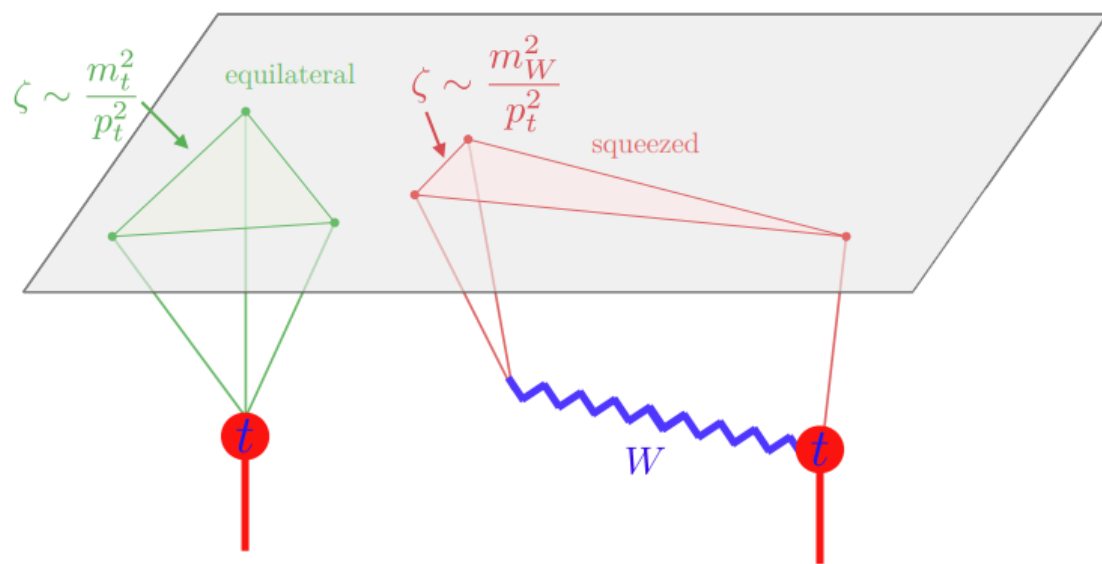
- Existing approaches offer **either** good theoretical control **or** good sensitivity to top quark mass $\rightarrow$ try energy correlators.



[Holguin, Moul, Pathak, Procura, Schofbeck, Schwarz]

Top quark mass from energy correlators

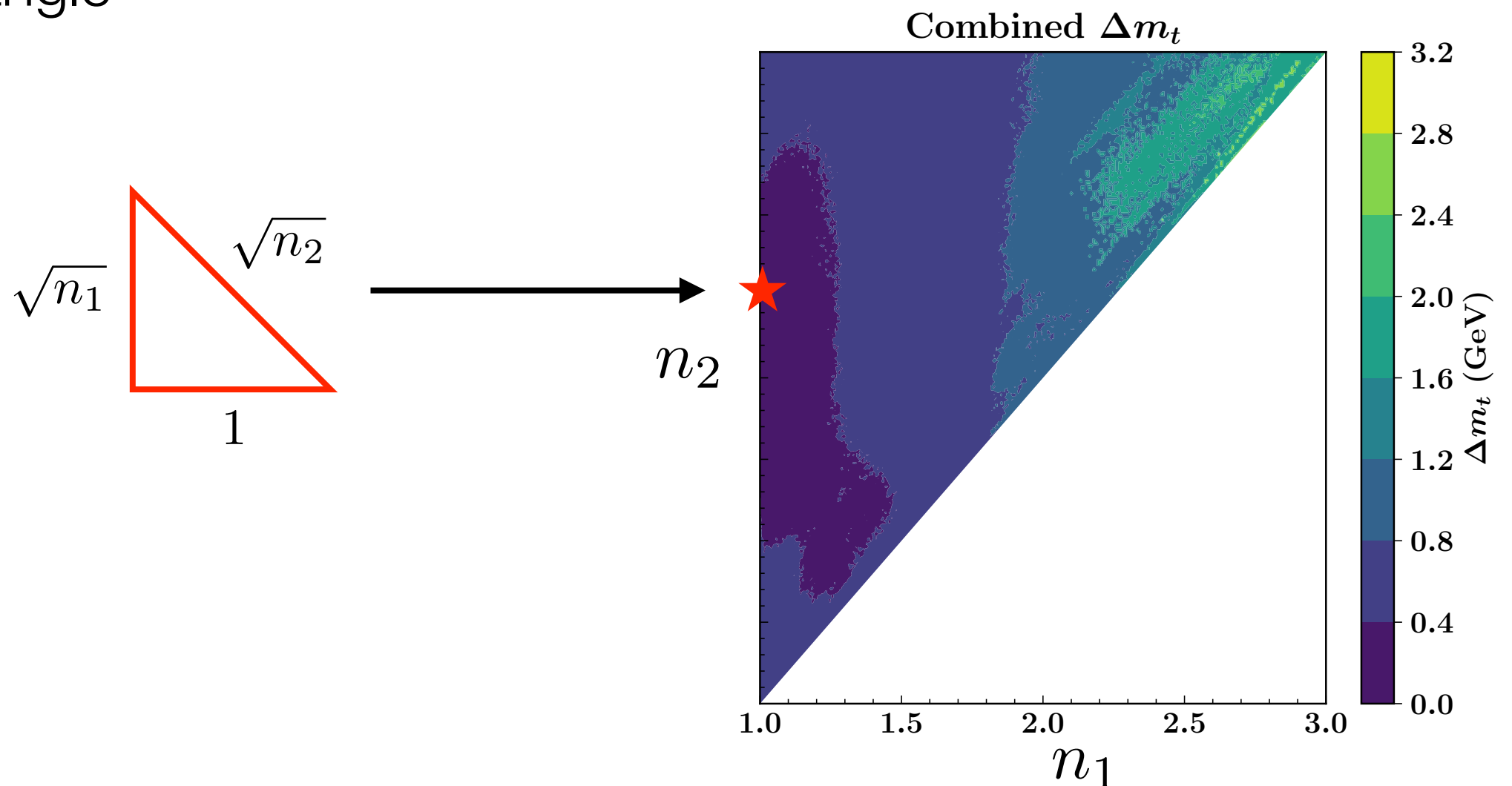
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- Convert the top quark peak position into a mass using W .



[Holguin, Moul, Pathak, Procura, Schofbeck, Schwarz]

ML-optimized triangle for top quark determination

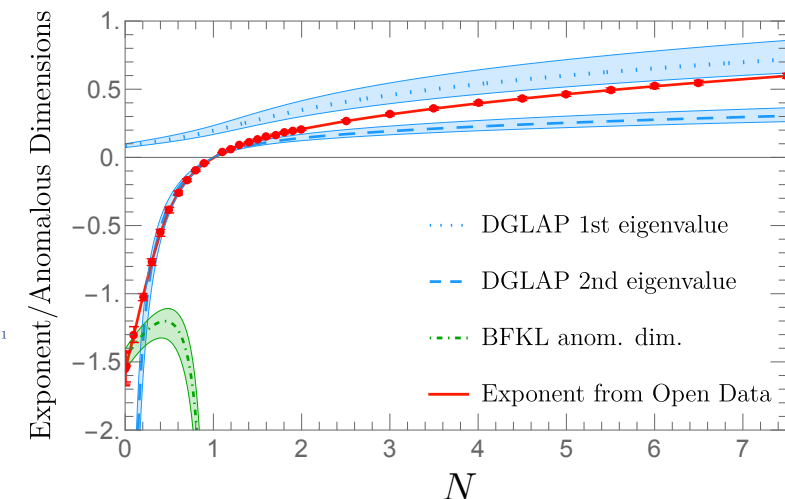
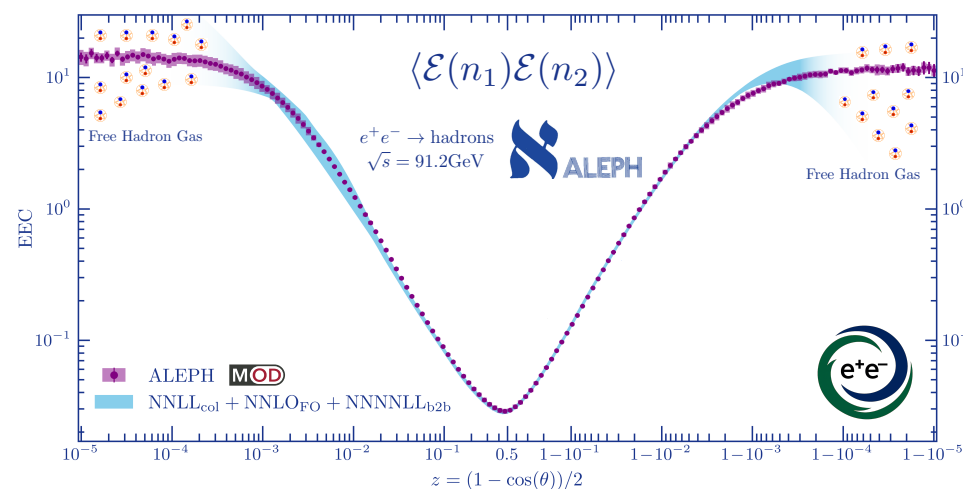
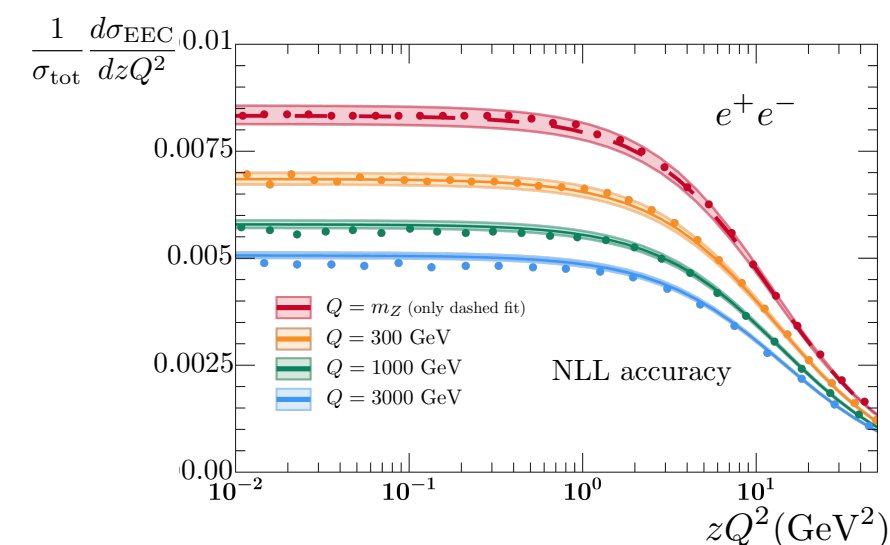
- Learn $(R_{12}, R_{13}, R_{23}, E_1 E_2 E_3, m_{\text{top}})$ using DNN.
- Find optimal configuration for identifying m_{top} :
the right triangle



[Bhattacharya, Fraser, Schwartz]

Conclusions

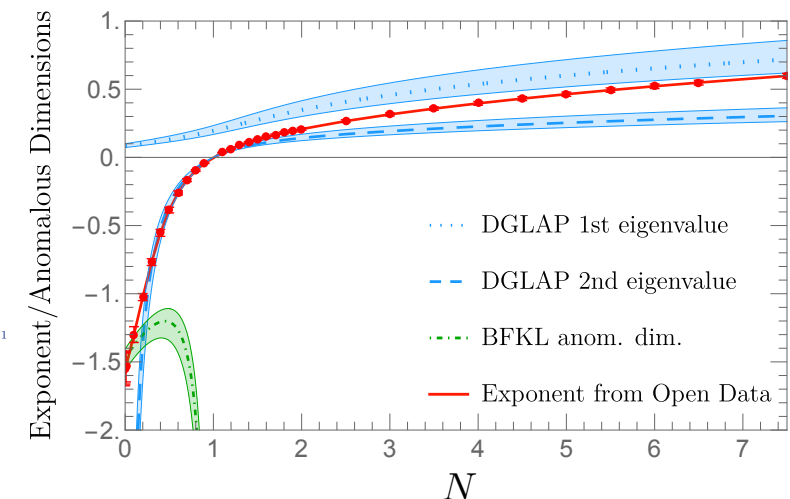
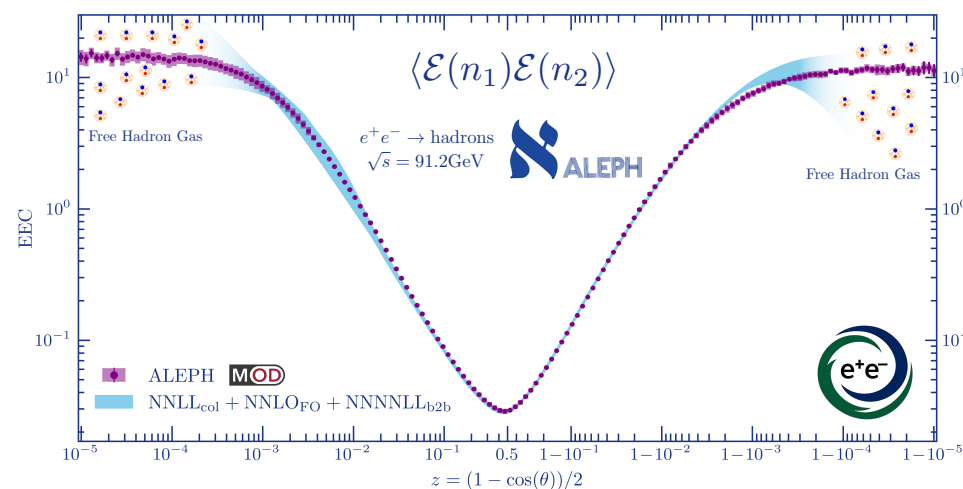
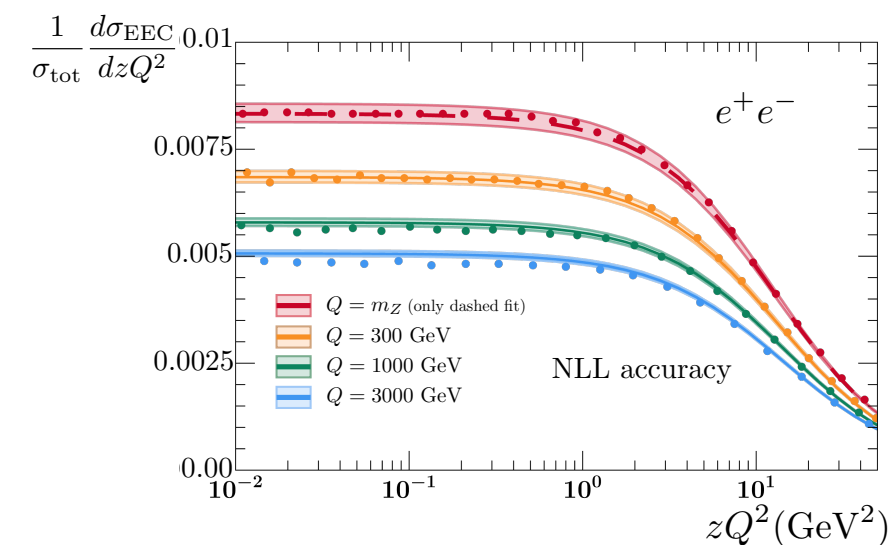
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- **Tracks:** superior angular resolution with precise theory.
- **Higher-point correlators:** NNLL predictions, ratios with EEC have enhanced sensitive to α_s .
- **Fast parametrization:** arbitrary N, including analytic continuation. New probes of e.g. BFKL and QGP.



Conclusions

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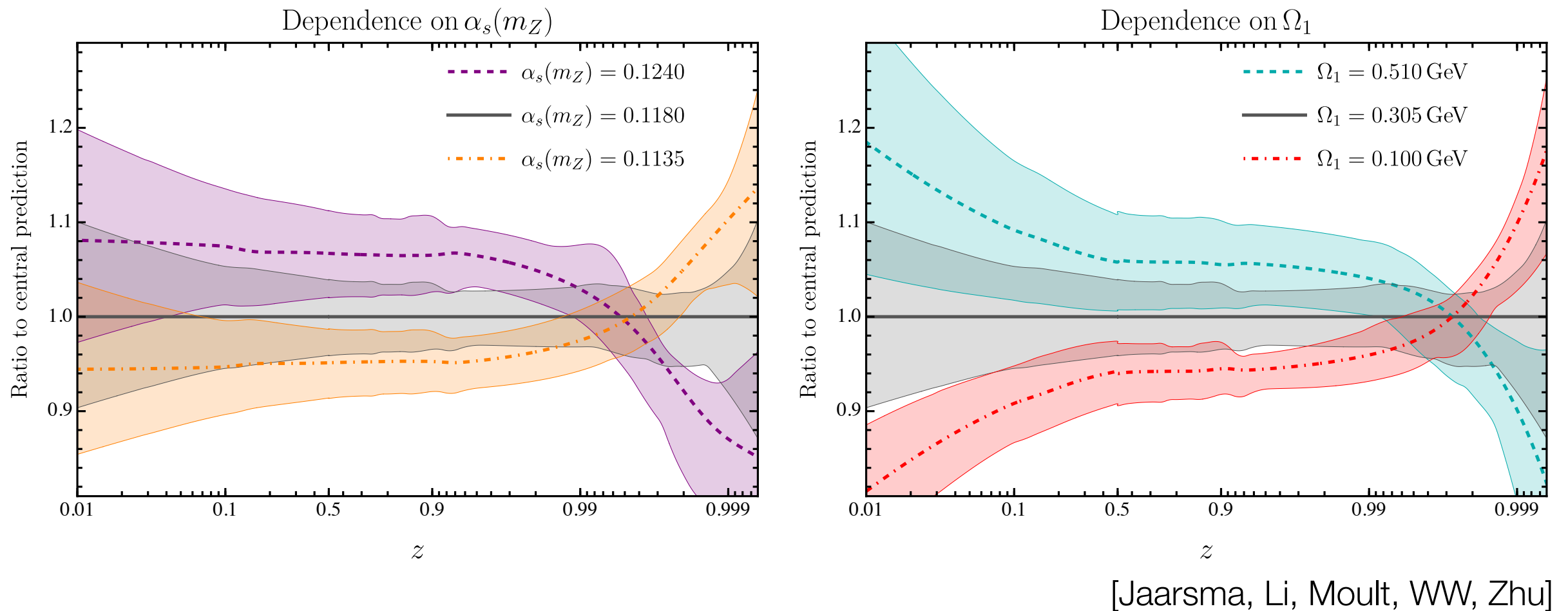
Thank you!



Backup



Breaking degeneracy between α_s and Ω_1



- Challenge: α_s from event shapes correlated with leading nonperturbative parameter Ω_1 .
- Potential solution: combine collinear and back-to-back region.