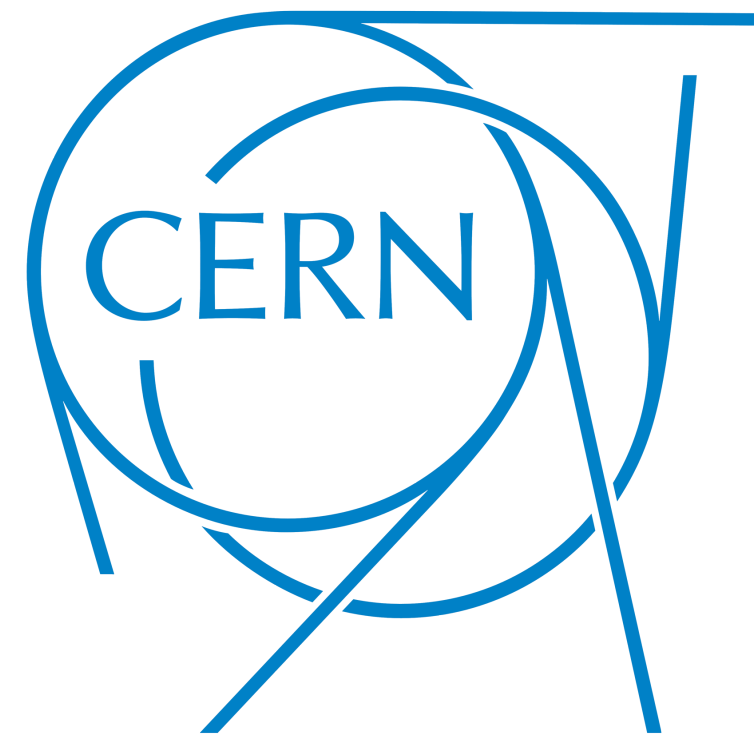




RECENT DEVELOPMENTS IN QCD PREDICTIONS FOR COLLIDER PHYSICS

CHIARA SIGNORILE-SIGNORILE

QCD@LHC

Oxford, 29.09.2026

DISCLAIMER

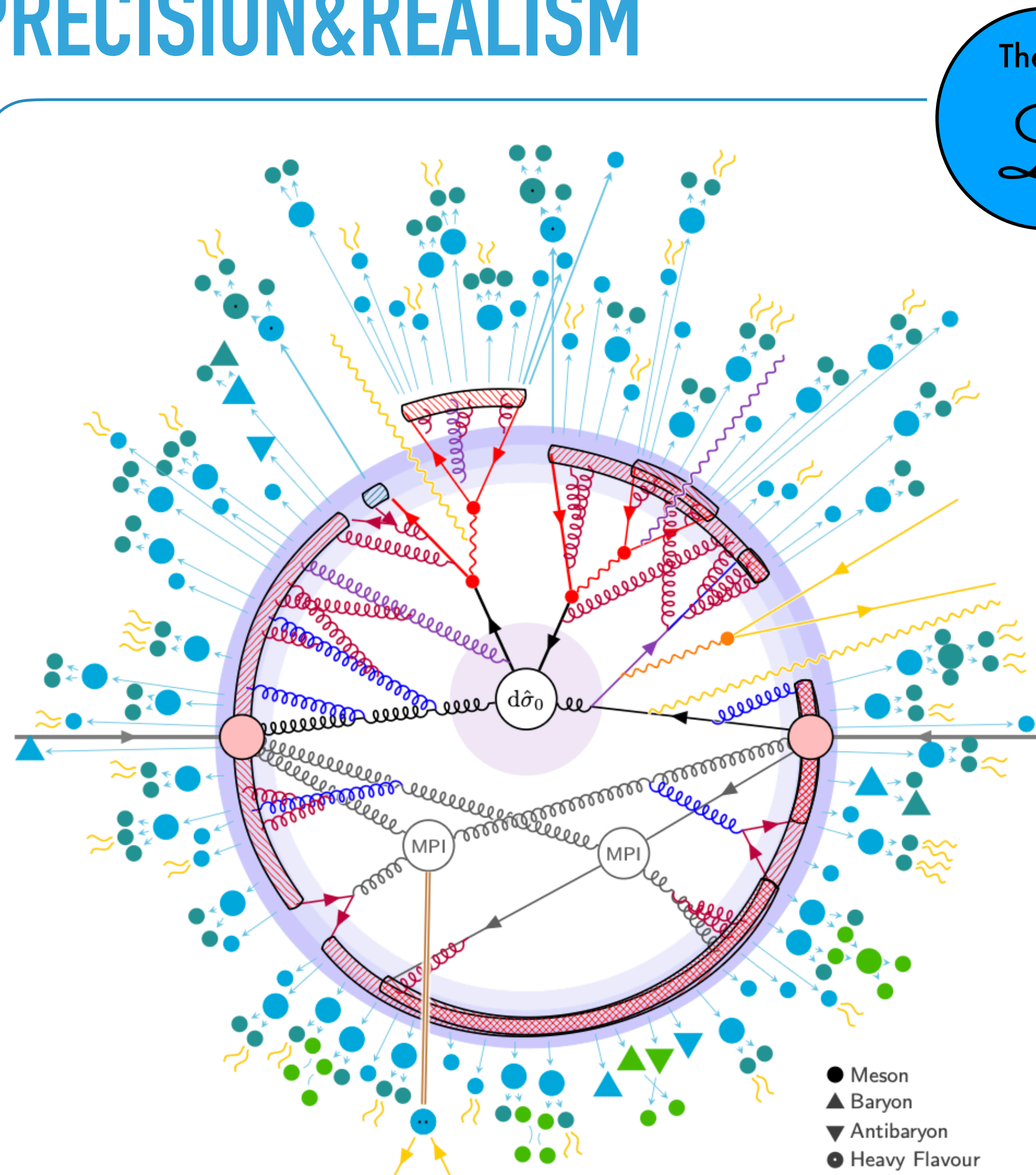
Given the title I was assigned, I had to make choices.

Such choices are based on my persona taste and experience

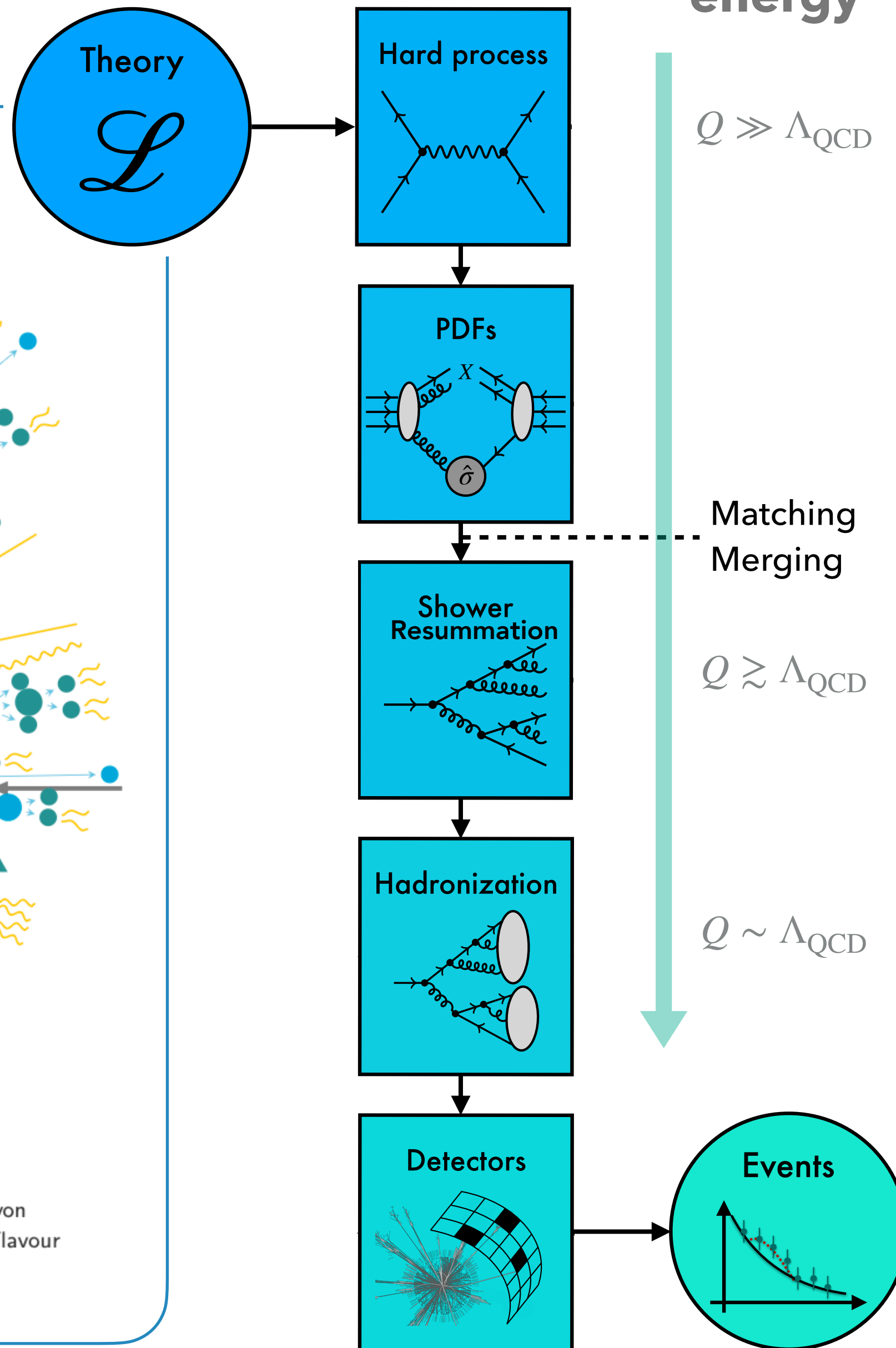


Both are debatable...

PRECISION&REALISM



[Pythia8.3 manual]



current efforts

F.O. perturbation theory: N^xLO

- ▶ Automate (N)NLO QCD
- ▶ EW and mixed QCD-EW

Matching: $N^xLO + PS_{N^yLL}$ Parton-shower/Resummation: PS_{N^yLL}

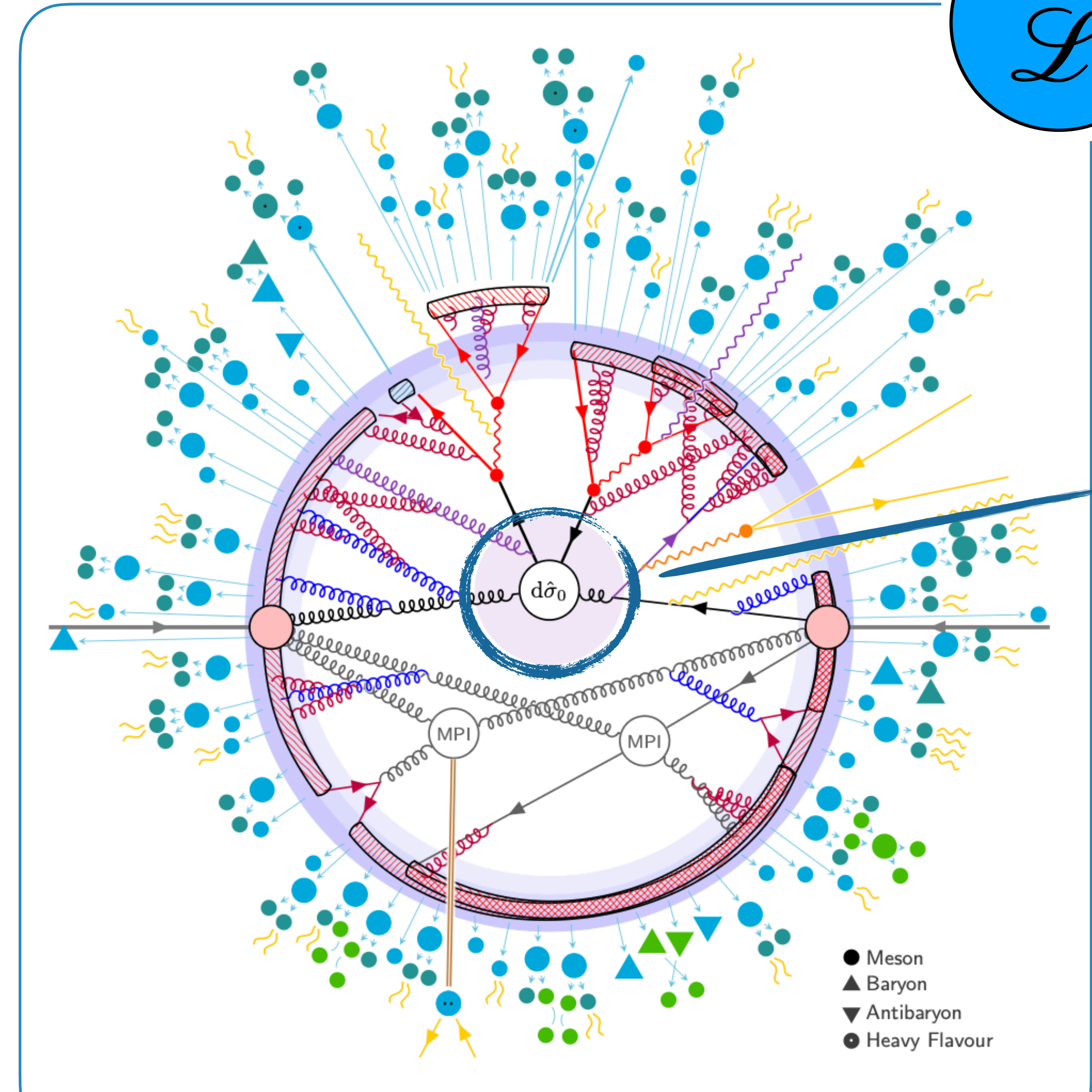
- ▶ Increasing PS accuracy
- ▶ (N)NLL accurate showers
- ▶ Sub-leading color effects

Hadronisation:

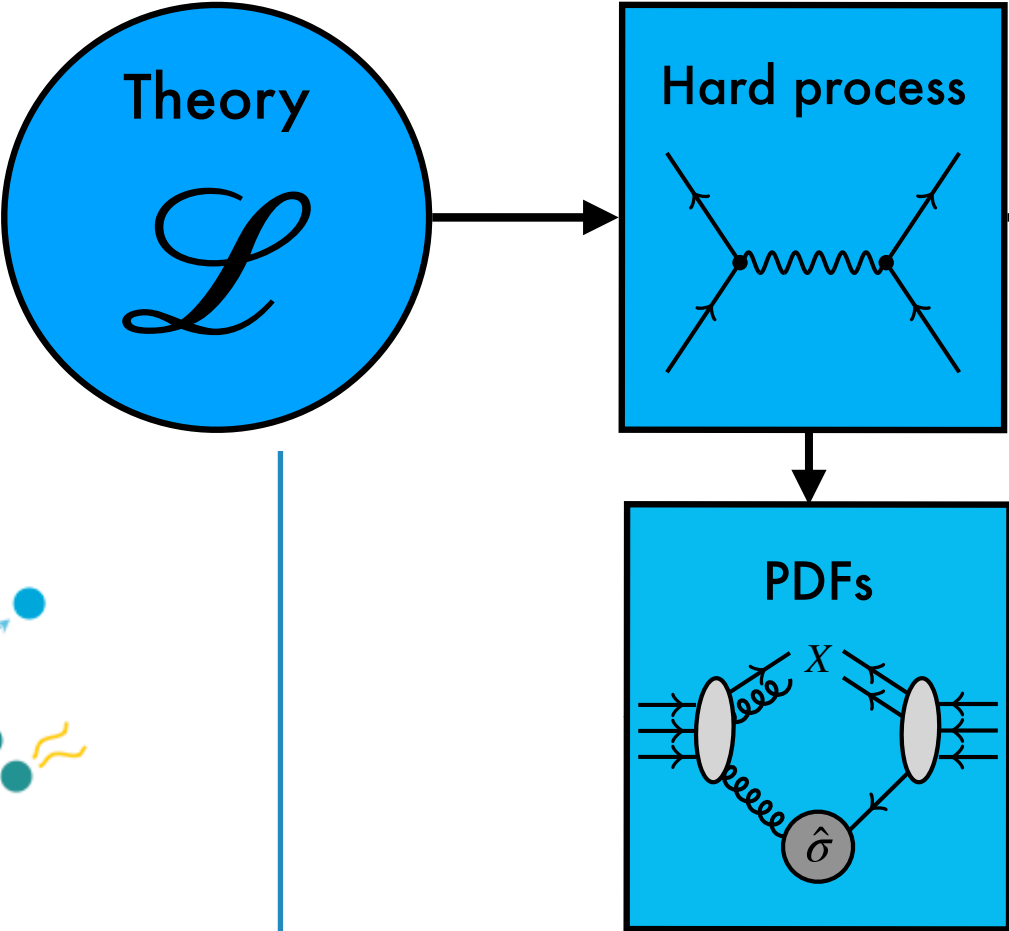
- ▶ Refinement of models
- ▶ Improving constraints

See S. Schumann [Mon]

PRECISION&REALISM



[Pythia8.3 manual]



energy current efforts

$Q \gg \Lambda_{\text{QCD}}$ F.O. perturbation theory: N^xLO

Collinear factorisation formula [Collins, Soper, Sterman '04]

$$\frac{d\sigma}{dX} = \sum_{ab} \int dx_1 dx_2 f_{a/A}(x_1, \mu) f_{b/B}(x_2, \mu) \frac{d\hat{\sigma}_{ab}(x_1, x_2, \mu)}{dX} \left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^p}{Q^p}\right) \right]$$

Parton distribution functions
(long-distance, non-perturbative)

See e.g. S. Forte,
A. Sarkar [Mon]

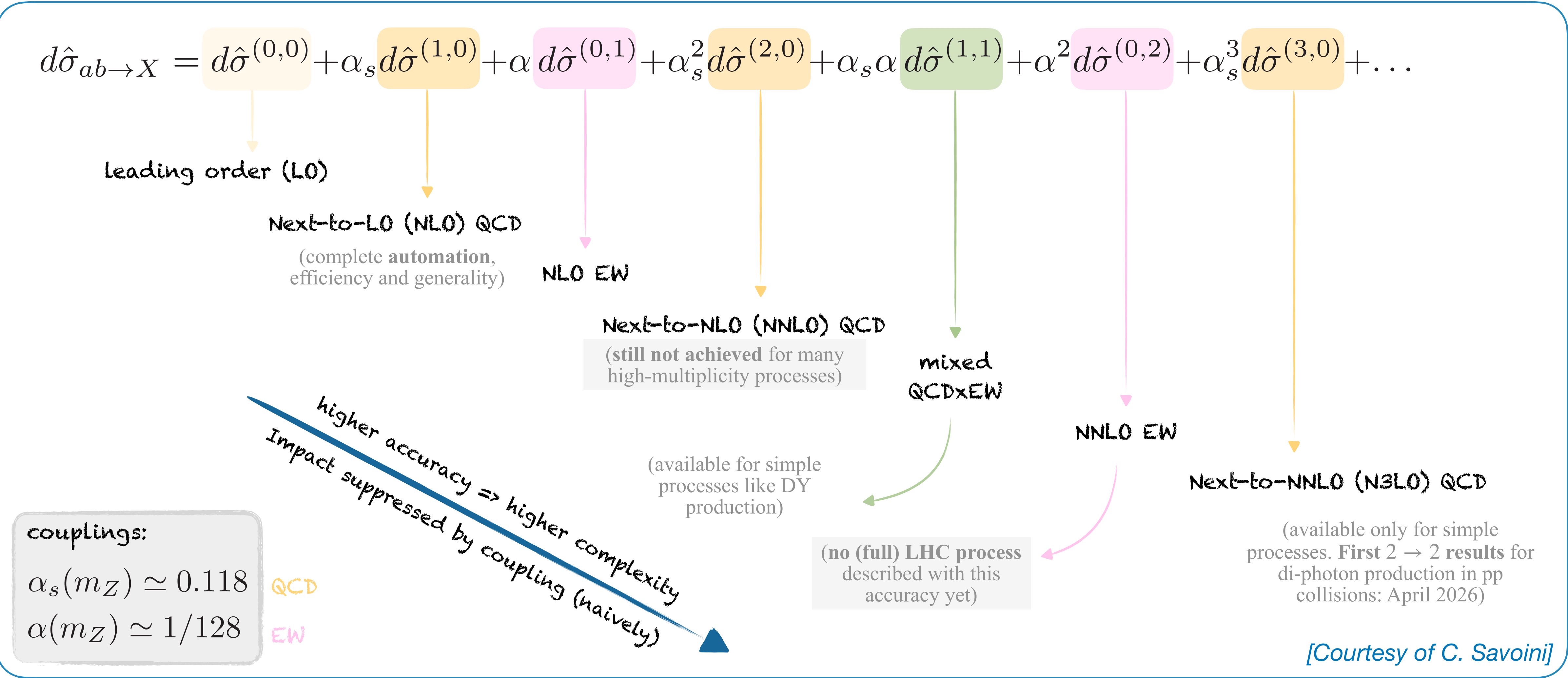
Hard scattering
(short-distance, perturbative regime)
Target:
 $\pm (\lesssim \text{a few percent})$

Non-perturbative effects
(Power corrections)
 $p \geq 1$

PRECISION&REALISM

At typical LHC energy scales: simultaneous expansion in QCD and EW couplings

energy current efforts
 $Q \gg \Lambda_{\text{QCD}}$ F.O. perturbation theory: N^xLO



PRECISION&REALISM

At typical LHC energy scales: simultaneous expansion in QCD and EW couplings

energy current efforts

$Q \gg \Lambda_{\text{QCD}}$ F.O. perturbation theory: N^xLO

$$d\hat{\sigma}_{ab\rightarrow X} = d\hat{\sigma}^{(0,0)} + \alpha_s d\hat{\sigma}^{(1,0)} + \alpha d\hat{\sigma}^{(0,1)} + \alpha_s^2 d\hat{\sigma}^{(2,0)} + \alpha_s \alpha d\hat{\sigma}^{(1,1)} + \alpha^2 d\hat{\sigma}^{(0,2)} + \alpha_s^3 d\hat{\sigma}^{(3,0)} + \dots$$

Not just an academic exercise:

Relevant observables show a poor perturbative convergence

Experience to consolidate in view of next flagship experiments

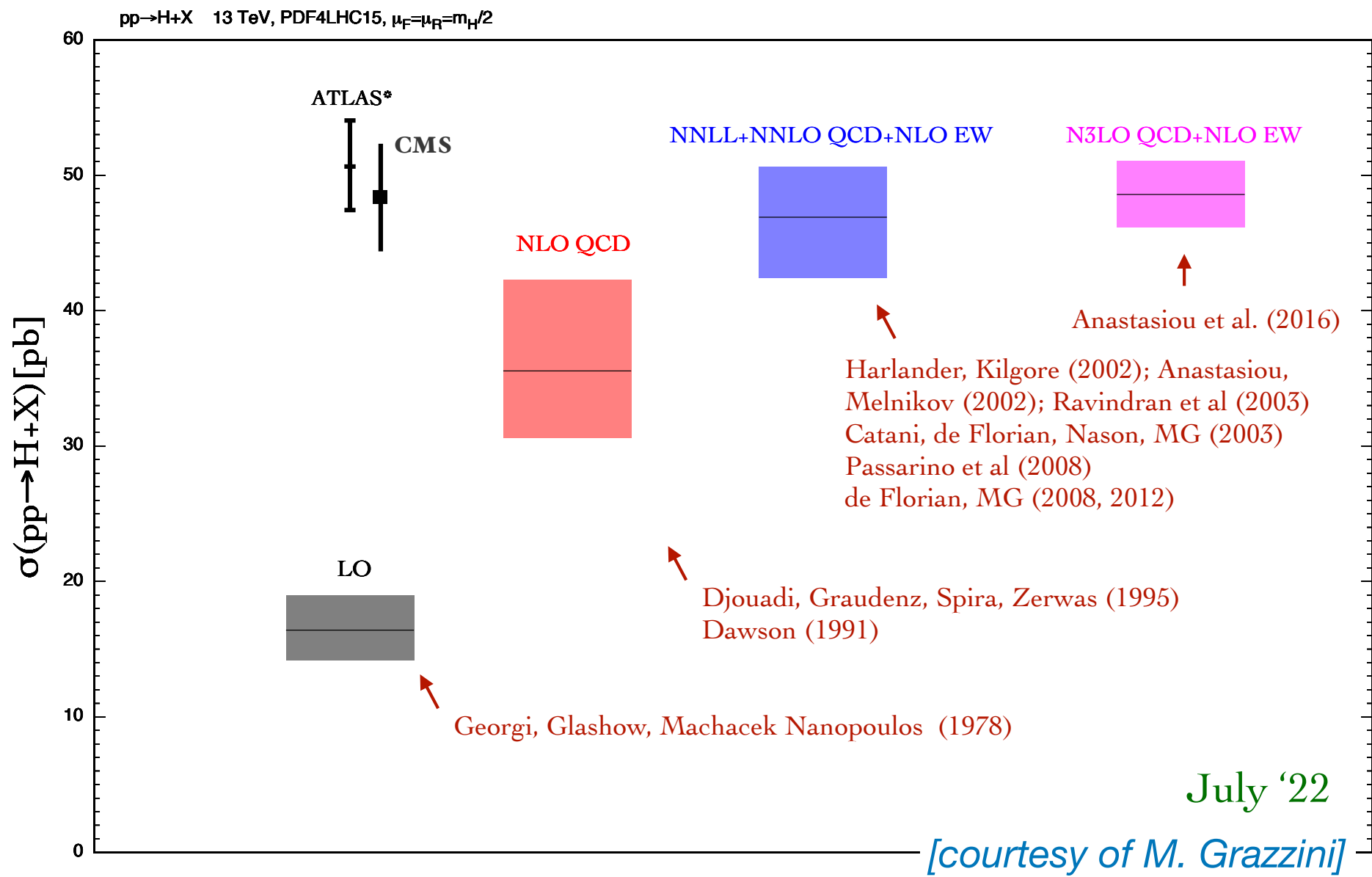


Table 4.2: Wish-list for calculations of missing higher-order perturbative QCD $\mathcal{O}(\alpha_s^n)$ and/or EW $\mathcal{O}(\alpha^n)$ to match the expected experimental uncertainty at future e^+e^- and ep colliders.

Observable	Missing higher-order & power-suppressed corrections
Hadronic Z width	$\mathcal{O}(\alpha_s^5), \mathcal{O}(\alpha_s^6), \mathcal{O}(\alpha^3), \mathcal{O}(\alpha_s \alpha^3), \mathcal{O}(\alpha_s^2 \alpha^2)$
Hadronic W width	$\mathcal{O}(\alpha_s^5), \mathcal{O}(\alpha^2), \mathcal{O}(\alpha_s^2 \alpha)$
Hadronic τ width	$\mathcal{O}(\alpha_s^5)$
Hadronic event shapes	N^3LO differential, $N^{3,4}LL$ resummation, power corrections
Inclusive jet rates	3-jet cross-sections at N^3LO , 4-jets at N^2LO , 5-jets at NLO
Lattice QCD results	$\mathcal{O}(\alpha_s^6)$ β -function; $\mathcal{O}(\alpha_s^5)$ heavy quark decoupling; $\mathcal{O}(\alpha_s^4)$ static potential
$(\alpha_s \text{ extr.; quark masses } m_c, m_b)$	$\mathcal{O}(\alpha_s^3)$ lattice perturbation theory matching (lattice coupling to $\alpha_s^{\overline{MS}}$ etc.)
$\sigma(e^+e^- \rightarrow W^+W^-)$ vs. $\sqrt{s}$	EW N^2LO : $\mathcal{O}(\alpha^2)$, Mixed EW-QCD: $\mathcal{O}(\alpha_s \alpha^2), \mathcal{O}(\alpha_s^2 \alpha)$
$\sigma(e^+e^- \rightarrow t\bar{t})$ vs. $\sqrt{s}$	NRQCD: $\mathcal{O}(\alpha_s^5)$, Non-resonant: $\mathcal{O}(\alpha_s^5), \mathcal{O}(\alpha_s^3)$ differential; QED: $\mathcal{O}(\alpha^3)$ at NNLL
$H \rightarrow b\bar{b}$ width	N^4LO ($m_b \neq 0$); N^4LO differential ($m_b = 0$)
$H \rightarrow gg$ width	N^5LO (heavy-top limit), N^4LO ($m_t \neq 0$); N^4LO differential, N^3LO differential ($m_t \neq 0$)

NNLO

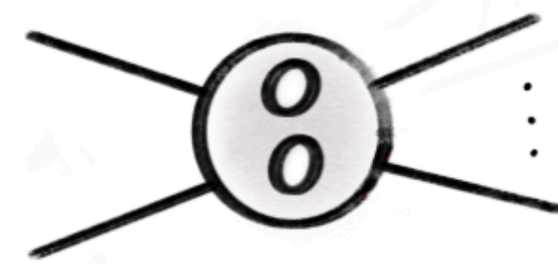
NNLO QCD calculations are being extended to an increasing variety of processes

Adding external particles and/or additional mass scales remains a challenge

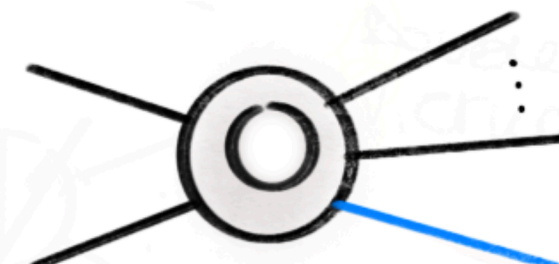
Schematic NNLO ingredients:

$$\frac{d\sigma_{\text{N}^2\text{LO}}}{dX} = \int d\Phi_n \textcolor{blue}{VV} \delta_{X_n} + \int d\Phi_{n+1} \textcolor{blue}{RV} \delta_{X_{n+1}} + \int d\Phi_{n+2} \textcolor{blue}{RR} \delta_{X_{n+2}}$$

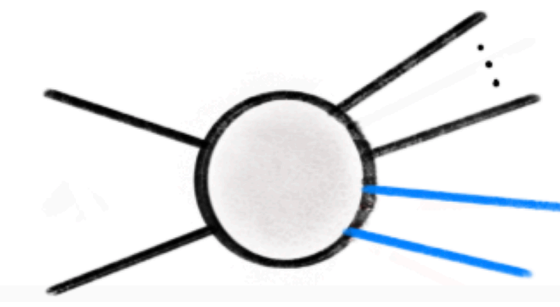
$X = \text{IRC-safe observable}$



Up to $1/\epsilon^4$
explicit poles



Up to $1/\epsilon^2$ explicit poles
Up to 2 PS singularities



up to 4 PS
singularities

$$d = 4 - 2\epsilon$$

Virtual corrections:

Algebraic and analytic complexity, increasing with the number of masses and external particles

Real radiation:

Systematic treatment of **soft and collinear singularities**
→ fully differential predictions

Ironically, in many cases the most complicated part, the double virtual, is also the “less relevant” (not always true!)

LOOP CORRECTIONS

- At his very core, the problem of computing loop amplitudes is the problem of **calculating divergent integrals of rational functions in Minkowski space**

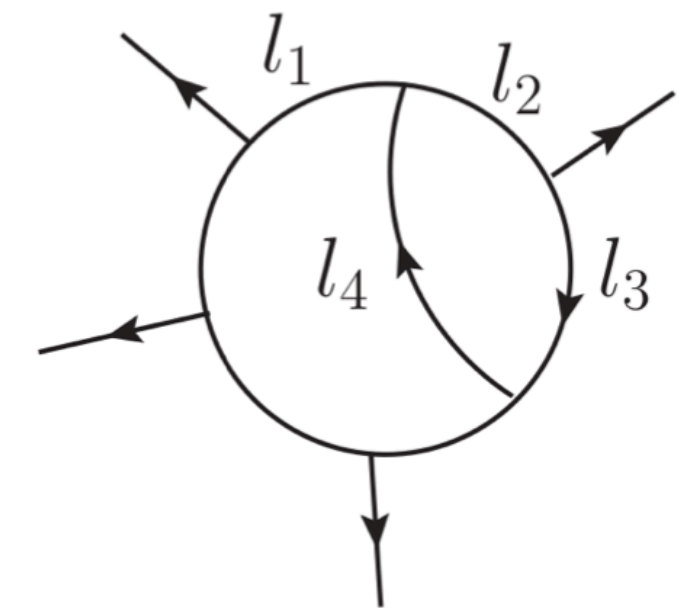
$$\mathcal{A} \sim \int_{-\infty}^{+\infty} \left(\prod_{i=1}^{\ell} d^d k_i \right) \frac{\mathcal{N}(k_i)}{\mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3 \dots}$$

Loop momenta: k_i = unresolved momenta of virtual particles (not constrained by mom. conservation)

Inverse propagator: $\mathcal{D}_k = l_j^2 - m_j^2$

l_j = linear combination of

- External (initial/final states) momenta p_j
- Loop momenta k_j



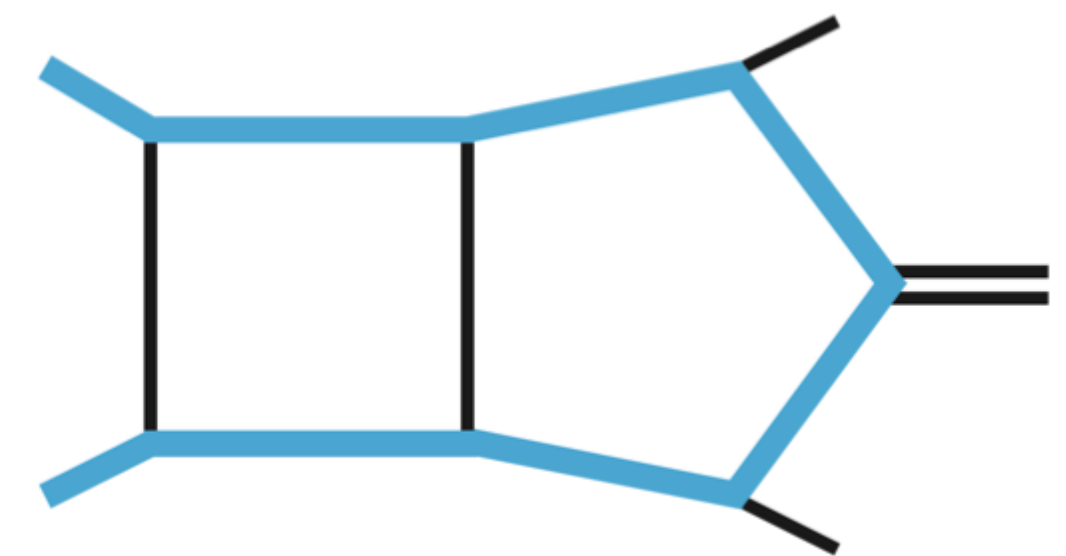
@1loop:

- fully automated* in public programs** [FormCalc, MadGraph+MadLoop, GoSam, BlackHat, OpenLoops, Recola, ...]
- Available tools for:
 - Systematic reduction to known integrals [CutTools, Samurai, Ninja, Golem, Collier, ...]
 - Evaluate such integrals [OneLoop, QCDLoop, LoopTools, ...]
- Current focus: performance, numerical stability, extensions to more models...

* In principle. In practice, it might still be very challenging

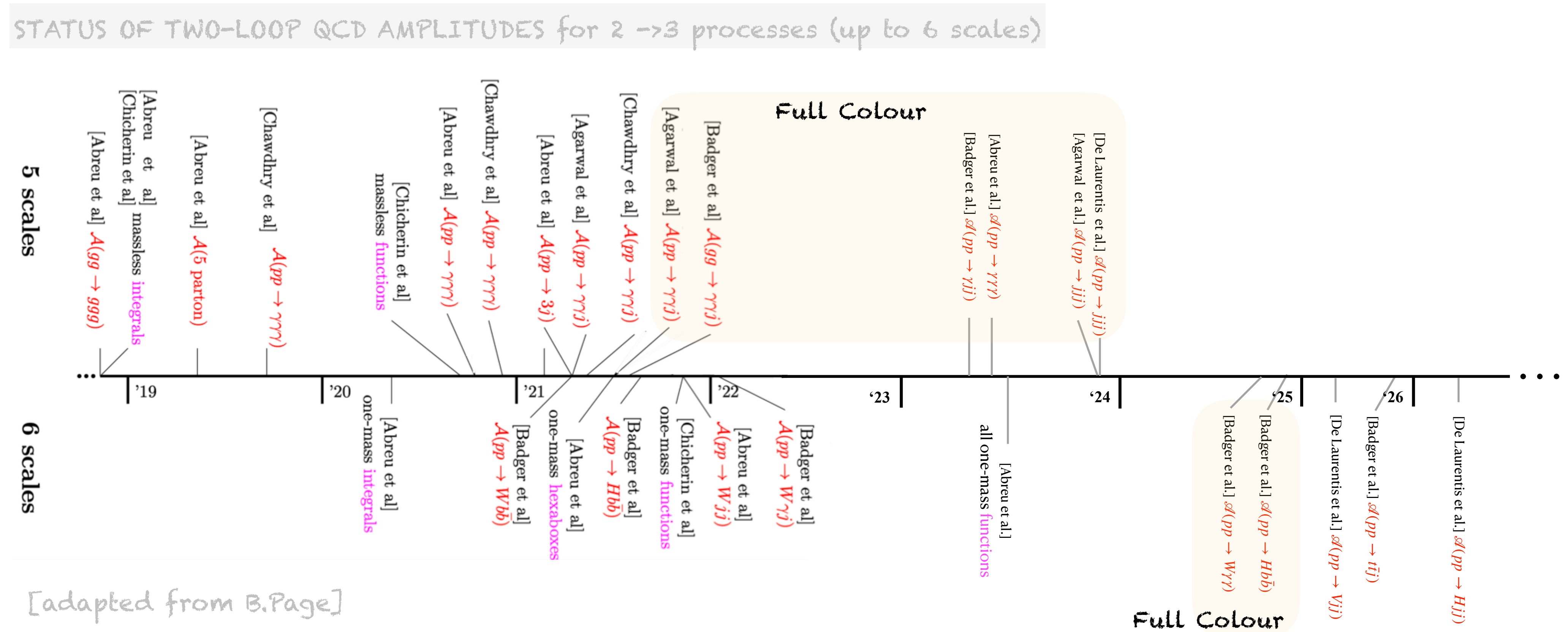
LOOP CORRECTIONS

- The **complexity increases** dramatically **with loop order, number of external particles and internal masses**
- **Beyond 1loop: still case-by-case calculation**
 - More diagrams, but generally not a bottleneck
 - Higher **algebraic complexity**: proliferation of scales + huge intermediate expression
 - Higher **analytic complexity**: new mathematical structures (special functions, elliptic and beyond)
 - Higher **numerical complexity**: representations suitable for efficient and stable amplitude evaluation
- **State of the art:**
 - Today in QCD: 2loop $2 \rightarrow 3$ scattering with masses, 3loop $2 \rightarrow 2$ with one external mass
 - Today in EW: 2loop $2 \rightarrow 2$ with massive particles (Z, W, H, t) everywhere
- **Various approaches***: **analytic** (representation dependent), **numerical** (might be unstable), **approximate** (non universal), ...



* Very rapidly growing field. Bare in mind: **progressively accurate results lead to amazing pheno studies!**

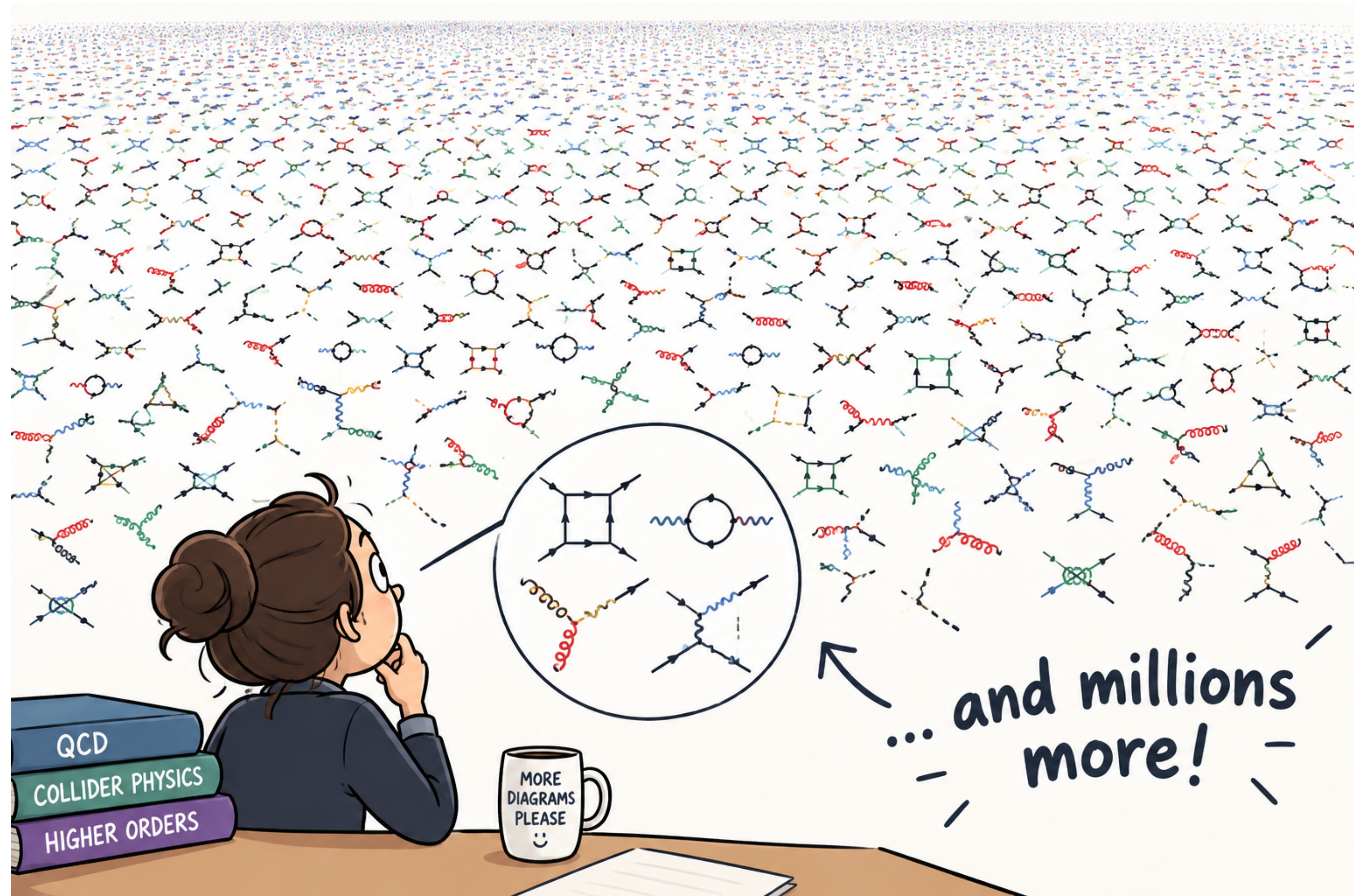
LOOP CORRECTIONS



LOOP CORRECTIONS

Feynman diagrams

- ▶ From the Lagrangian, one infer the Feynman diagrams allowed by the theory: typically $\mathcal{O}(10^2 - 10^4)$
- ▶ However, fully automated techniques available [[QGraph](#), [DiaGen](#), ...]
- ▶ **Direct evaluation not possible:**
 - ▶ (tensor-)loop integrals need regularisation (IR & UV): $d = 4 - 2\epsilon$



LOOP CORRECTIONS

Feynman diagrams

Scalar integrals

Projection

$$\mathcal{A} = \sum_{C,S} \mathcal{A}_{C,S} \mathbf{C}_C \mathbf{T}_S$$

Color structure

Spin structure

Scalar partial amplitudes

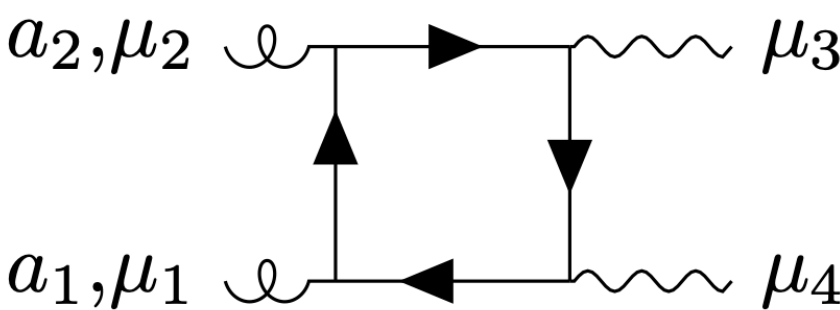
$$\mathcal{A}_{C,S} = \sum_j c_j(\{p,k\}, \epsilon) \, I_j(\{p,k\}, \epsilon)$$

Rational functions
(kinematics and ϵ)

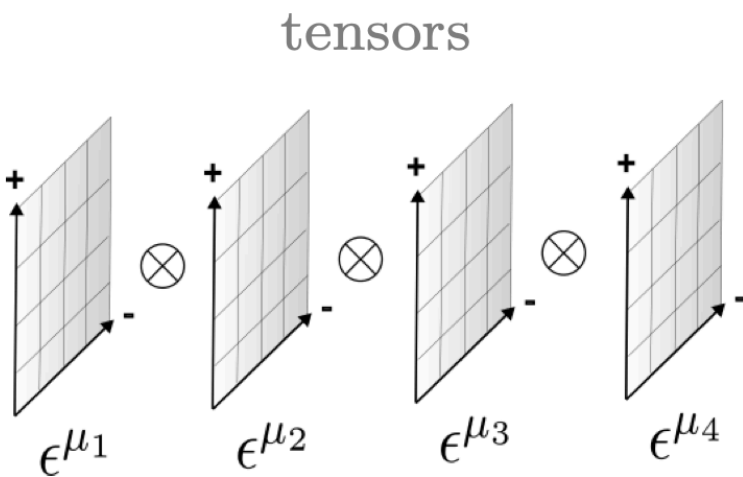
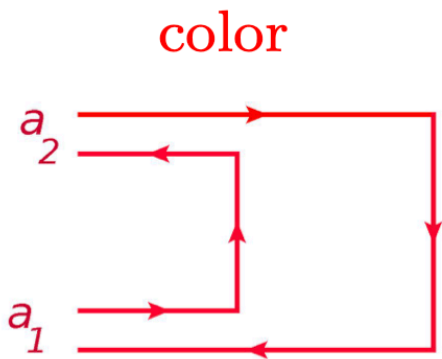
(scalar) Feynman integrals

$$I_i = \int \left(\prod_{i=1}^{\ell} d^d k_i \right) \prod_{k=1} \mathcal{D}_k^{-\alpha_{i,k}}$$

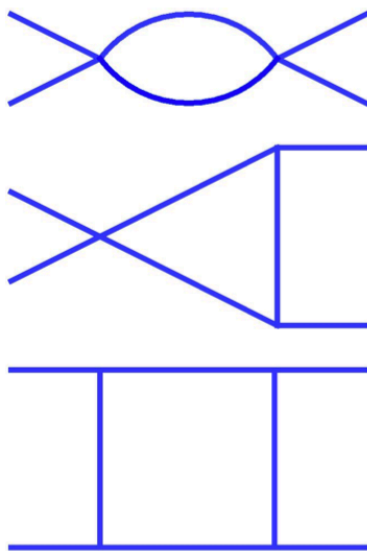
Integral family:
common set of denominators



$$= g_s^4 \text{Tr}\{T^{a_1} T^{a_2}\} \int \frac{d^d k}{(2\pi)^d} \frac{\text{tr} \left[(\not{k}) \not{\epsilon}_1 (\not{k} + \not{p}_1) \not{\epsilon}_2 (\not{k} + \not{p}_{12}) \not{\epsilon}_3 (\not{k} + \not{p}_{123}) \not{\epsilon}_4 \right]}{(k)^2 (k + p_1)^2 (k + p_{12})^2 (k + p_{123})^2}$$



integrals



LOOP CORRECTIONS

Feynman
diagrams

Scalar
integrals

Master
integrals

Projection

$$I_i = \int \left(\prod_{i=1}^{\ell} d^d k_i \right) \prod_{k=1} \mathcal{D}_k^{-\alpha_{i,k}}$$

IBPs

- Loop integrals obey **linear relations: Integration By Parts (IBPs)**
[Chetyrkin, Tkachov (1981), Laporta (2000)]

$$\int \left(\prod_{i=1}^{\ell} d^d k_i \right) \frac{\partial}{\partial k_j^\mu} v^\mu \prod_{k=1} \mathcal{D}_k^{-\alpha_{i,k}} = 0, \quad v^\mu \in \{p^\mu, k^\mu\}$$

- Very **large and sparse linear system** $\implies$ often a huge bottleneck
- Solution of linear system $\leftrightarrow$ reduction into a basis of **linearly independent master integrals (MIs)**: $\{M_j\} \subset \{I_j\}$

$$I_j = \sum c_{jk} \mathbf{M}_k$$

- Modern solvers: Reduze, Fire, Kira, LiteRed, FiniteFlow
- Simplified systems: NeatIBP, Blade, CALICO

LOOP CORRECTIONS

Feynman
diagrams

Scalar
integrals

Master
integrals

'Analytic'
functions

Projection

$$I_i = \int \left(\prod_{i=1}^{\ell} d^d k_i \right) \prod_{k=1} \mathcal{D}_k^{-\alpha_{i,k}}$$

IBPs

$$I_j = \sum c_{jk} \mathbf{M}_k$$

DEs

- Master integrals obey systems of DEs w.r.t. invariants x .
[Kotikov (1991), Gehrmann, Remiddi (2000)]

$$\partial_x \mathbf{M}_j = \sum_k A_{jk}(\{p\}, \epsilon) \mathbf{M}_k$$

- Solving DEs requires **boundary conditions**
 - e.g. systematic approach: **canonical form** [Henn 2013]
 - Alternative direct **numerical evaluation**, e.g. **sector decomposition** [Binoth, Heinrich] via **PySecDec** [Heinrich, Jones, Margery, Olsson et al.] **FIESTA** [Smirnov et al.]

LOOP CORRECTIONS

Feynman
diagrams

Scalar
integrals

Master
integrals

'Analytic'
functions



Successes of the standard "work chain" :

Three-Loop **QCD** Corrections to Scattering
Amplitudes of a Higgs Boson and Three Partons

[2607.12033]

Xin Guan, Bernhard Mistlberger, Michael S. Ruf

Alternatives to the standard "work chain" :

- ▶ generate the relevant **unitarity cuts**
- ▶ construct tree amplitudes on those cuts
- ▶ reconstruct the loop integrand.

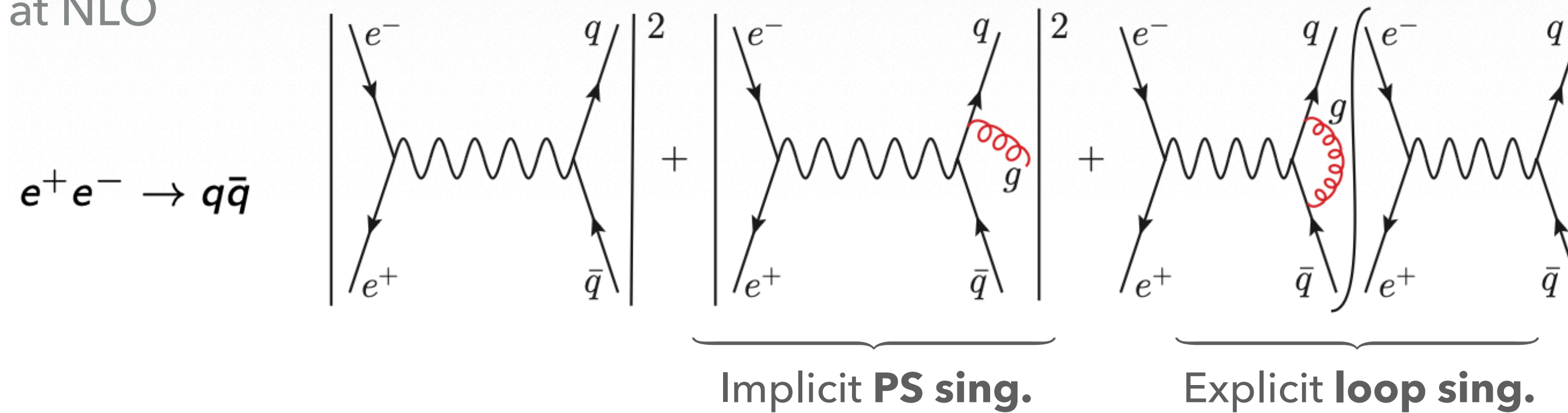
Two-loop leading-color QCD corrections for Higgs
plus two-jet production in the heavy-top limit

Giuseppe De Laurentis,¹ Harald Ita,^{2,3} Viktor Kuschke,^{2,3} Michael Ruf,⁴
Vasily Sotnikov⁵

[2605.04009]

REAL CORRECTIONS@NLO

- ▶ The problem emerges already at NLO



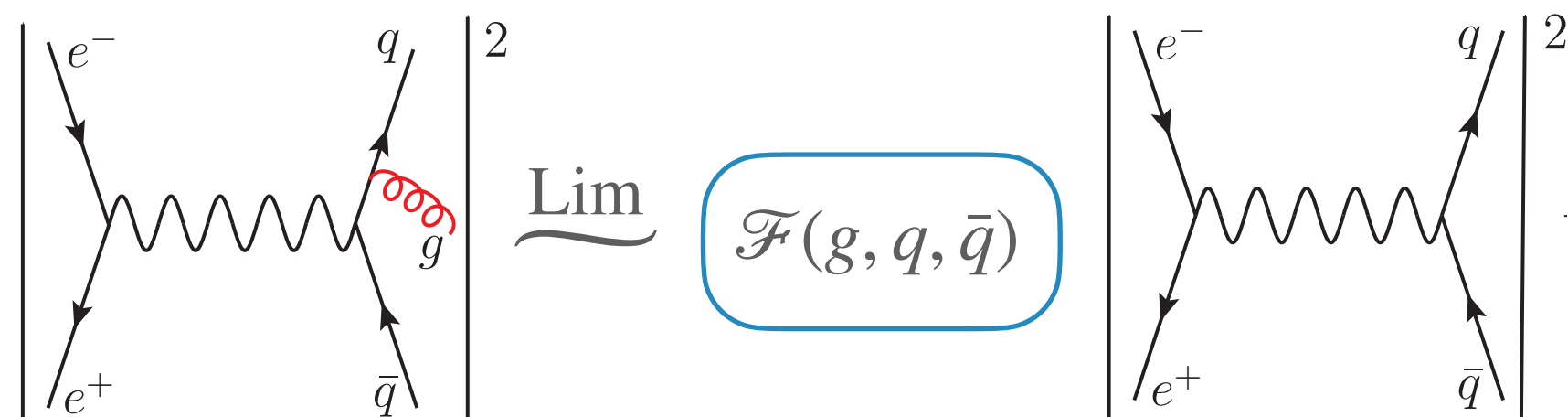
$$d = 4 - 2\epsilon$$

- ▶ The explicit poles of 1-loop amplitudes are known and universal

$$\mathcal{A}_{1\text{loop}} = \left[\sum_i \left(\frac{1}{\epsilon^2} + \frac{\gamma_i}{\mathbf{T}_i^2} \frac{1}{\epsilon} \right) \sum_{i \neq j} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \left(\frac{\mu^2}{-s_{ij}} \right)^\epsilon \right] \mathcal{A}_{\text{tree}} + \mathcal{A}_{1\text{loop}}^{\text{fin}}$$

Finite remainder requires **dedicated calculations on a process-by-process basis**. It enters directly **numerical implementations**

- ▶ Singular limits of tree-level amplitudes are known and universal



$\text{Lim} = E_g \rightarrow 0$: **soft limit** $\Rightarrow \mathcal{F}(g, q, \bar{q}) =$ eikonal function

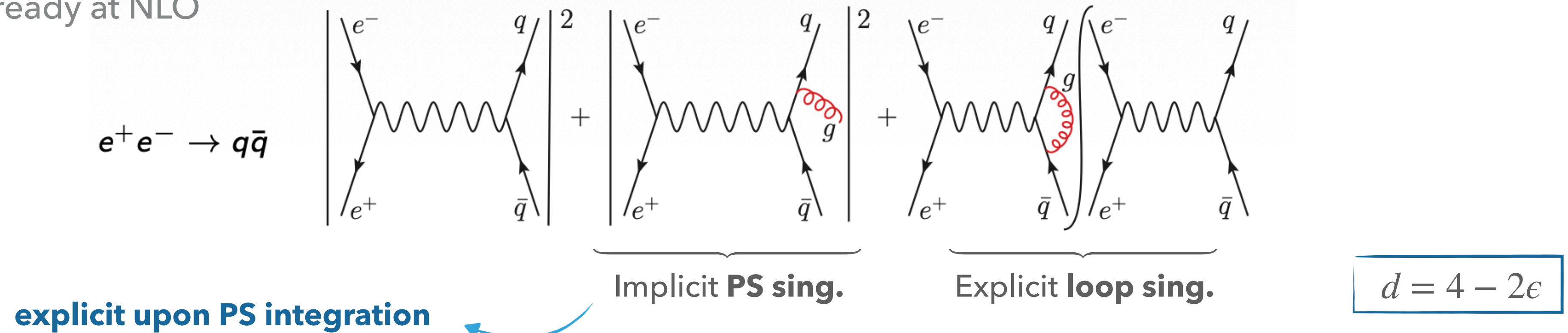
$\text{Lim} = \theta_{gq} \rightarrow 0$: **collinear limit** $\Rightarrow \mathcal{F}(g, q, \bar{q}) =$ splitting function

$\mathcal{F}(g, q, \bar{q})$ universal: depends only on external-parton momenta, spin, colour.

No explicit poles, unresolved PS integration generate $1/\epsilon$ poles.

REAL CORRECTIONS@NLO

- ▶ The problem emerges already at NLO



KLN

IR singularities vanish if all degenerated states with arbitrary multiplicities in the final state are combined
[Kinoshita '62; Lee, Nauenberg '64]

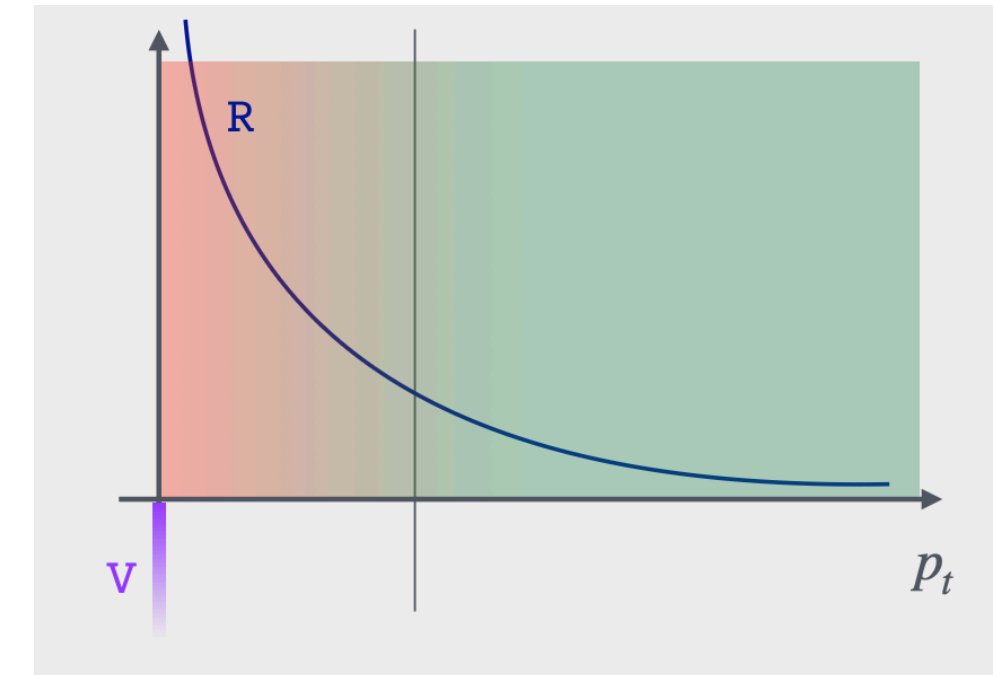
- ▶ However: need to **extract real-radiation IR poles without integrating** over PS → **retain fully differential predictions**
 - ▶ Regulate infrared singularities of the real emission
 - ▶ Extract infrared $1/\epsilon$ poles in d-dimension without integrating over the resolved phase space
 - ▶ Cancel the $1/\epsilon$ poles stemming from the phase space integration against the poles of the loop correction

SLICING/SUBTRACTION

▸ Slicing

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int_0^\delta [|\mathcal{M}|^2 F_J d\phi_d]_{\text{simp}} + \int_\delta^1 |\mathcal{M}|^2 F_J d\phi_d + \mathcal{O}(\delta)$$

- non-local in phase space
- conceptually simple, straightforward to implement (bulk is NLO)
- dependence on the slicing parameter and large cancellations between singular and regular terms
- "easy" generalization to N3LO



▸ Subtraction

$$\int \text{diagram} d\Phi_g = \underbrace{\int \left[\text{diagram} - \text{diagram} \right] d\Phi_g}_{\text{Finite in } d=4} + \underbrace{\int \text{diagram} d\Phi_g}_{1/\epsilon \text{ poles}}$$

Subtraction term

Integrated subtraction term

- more difficult conceptually
- local in phase space
- potentially, offers better numerical stability and scalability than slicing
- at least for now, every step to yet higher order (e.g. N2LO → N3LO) is a challenge

A lot of recent progress on NNLO formalisms and tools

Loop-Tree
Duality

Antenna
subtraction

Nested soft-collinear subtraction

Local analytic sector subtraction

NNLOJET: a parton-level event generator for jet cross sections at NNLO QCD accuracy

NNLOJET Collaboration

A. Huss^{1,*}, L. Bonino², O. Braun-White³, S. Caletti⁴, X. Chen⁵, J. Cruz-Martinez¹, J. Currie⁶, W. Feng⁷, G. Fontana⁸, E. Fox⁹, R. Gauld¹⁰, A. Gehrmann-De Ridder¹¹, T. Gehrmann¹², E.W.N. Glover¹³, M. Hofer¹⁴, P. Jakubek¹⁵, M. Jaquier¹⁶, M. Löchner¹⁷, E. Lorkowski¹⁸, I. Mjor¹⁹, M. Marcoli²⁰, H. Meisinger²¹, J. Mo²², T. Morgan²³, J. Niehues²⁴, J. Pires^{10,11}, C. T. Preuss^{2,13}, A. Rodríguez García⁴, K. Schönwald²⁵, R. Schürmann²⁶, V. Sotnikov²⁷, G. Stagnitto¹⁴, D. Walker²⁸, S. Wells²⁹, J. Whitehead³⁰, T.Z. Yang³¹ and H. Zhang¹⁶

Generalised antenna functions for higher-order calculations

Elliot Fox⁹, Nigel Glover¹³ and Matteo Marcoli²⁰

The colourful antenna subtraction method

T. Gehrmann¹², E.W.N. Glover¹³ and M. Marcoli^{20,a,b}

A general algorithm to build real-radiation antenna functions for higher-order calculations

Oscar Braun-White^a, Nigel Glover^a and Christian T Preuss^b

^aInstitute for Particle Physics Phenomenology, Department of Physics, Durham University, Leazes Road, DH1 1TA, Durham, UK

^bDepartment of Physics, University of Zurich, Winterthurerstrasse 190, CH-8057 Zurich, Switzerland

^coscar.f.braun-white@durham.ac.uk, e.w.n.glover@durham.ac.uk, ct.preuss@phys.ethz.ch

The antenna subtraction method has been successfully applied to a wide range of processes relevant for the Large Hadron Collider at next-to-next-to-leading order in α_s . We propose an algorithm for building antenna functions for any number of real and virtual emissions. We use the algorithm to explicitly build all single- and double-real emission contributions and compare them to the previous antenna functions, which were constructed using the subtraction method. The improved antenna functions should be more easily integrated into the subtraction method. Finally, we match the integration of the antenna subtraction terms to the previous incarnation, serving as a final unresolved phase space to the previous incarnation, serving as

Integrated subtraction terms and finite remainders for arbitrary processes with massless partons at colliders in the nested soft-collinear subtraction scheme

Federica Devoto,^a Kirill Melnikov,^b Raoul Röntsch,^c Chiara Signorile-Signorile,^d Davide Maria Tagliabue,^b Matteo Tressoldi^b

Towards a general subtraction formula for NNLO QCD corrections to processes at hadron colliders: final states with quarks and gluons

Federica Devoto,^a Kirill Melnikov,^b Raoul Röntsch,^c Chiara Signorile-Signorile,^d Davide Maria Tagliabue,^{b,c} Matteo Tressoldi^b

^aSLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA

A fresh look at the nested soft-collinear subtraction scheme: NNLO QCD corrections to N -gluon final states in $q\bar{q}$ annihilation

Federica Devoto^a, Kirill Melnikov^b, Raoul Röntsch^c, Chiara Signorile-Signorile^{b,d,e} and Davide Maria Tagliabue^c

history: A tool for fully-differential cross sections at next-to-next-to-leading order

Sven Yannick Klein^{1,*} and Lukas Simon^{1,2†}

¹ Institute for Theoretical Particle Physics and Cosmology, RWTH Aachen University, Sommerfeldstraße 16, 52074 Aachen, Germany

² Laboratoire de Physique Théorique et Hautes Energies (LPTHE), UMR 7589, Sorbonne Université et CNRS, 4 place Jussieu, 75252 Paris Cedex 05, France

* yklein@physik.rwth-aachen.de, † lsimon@lpthe.jussieu.fr

Abstract

The software history is designed to calculate fully-differential cross sections for colour-singlet production processes in hadronic collision up to next-to-next-to-leading order

Advances in Local Analytic Sector Subtraction: massive NLO and elements of NNLO automation

Gloria Bertolotti,^a Giovanni Limatola,^{b,c,d} Paolo Torrielli,^d and Sandro Uccirati^d

^aDipartimento di Fisica, Università di Torino and INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy

^bMax-Planck-Institut für Physik, Föhringer Ring 6, D-80805 München, Germany

^cInstitut für Theoretische Teilchenphysik, Karlsruher Institut für Technologie (KIT), D-76128 Karlsruhe, Germany

^dInstitut für Astroteilchenphysik, Karlsruher Institut für Technologie (KIT), D-76021 Karlsruhe, Germany

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ABSTRACT: We use the Local Analytic Sector Subtraction scheme to construct a completely analytic set of expressions implementing a fully local infrared subtraction at NNLO for generic coloured massless final states. The cancellation of all explicit infrared poles appearing in the double-virtual contribution, in the real-virtual correction and in the integrated local infrared counterterms is explicitly verified, and all finite contributions arising from integrated local counterterms are analytically evaluated in terms of ordinary polylogarithms up to weight three. The resulting subtraction formula can readily be implemented in any numerical framework containing the relevant matrix elements up to NNLO.

Local finiteness for real-virtual corrections to electroweak production in partonic collisions

Charalampos Anastasiou^{a,*}, Julia Karlen^{a,*}, Yao Ma^{a,*}, George Sterman^{a,b}

NNLO QCD corrections to $\gamma\gamma \rightarrow Q\bar{Q}$ from Local Unitarity combined with Coulomb resummation and NLO EW effects

Zeno Capatti^a, Mathijs Fraaije^a, Valentin Hirschi^a, Lucien Huber^a, Ben Ruijl^b, Hua-Sheng Shao^c

^aInstitute for Theoretical Physics, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland

Numerical integration of loop integrals through local cancellation of threshold singularities

D. Kermanschah

Vacuum amplitudes and time-like causal unitarity in the loop-tree duality

The LTD Collaboration, Selomit Ramirez-Urbe,^a Andrés E. Rentería-Olivo,^a David F. Rentería-Estrada,^a Jorge J. Martínez de Lejarza,^a Prasanna K. Dhani,^a Leandro Cleri,^a Roger J. Hernández-Pinto,^a German F. R. Sborlini,^a William J. Torres Bobadilla,^a and Germán Rodrigo^a

^aDepartamento de Física, Universidad de los Andes, Bogotá, Colombia

Towards an automated generator based on OpenLoops + LTD

Stefano Pozzorini

based on ongoing work with

Gloria Bertolotti, Nicolò Giraudo, Florian Herren and Jonas Lindert

see also talk by Gloria Bertolotti today at 11 AM

HP2, Torino, September 10–13 2024

Slicing with jets

q_T -slicing with multiple jets

Rong-Jun Fu,¹ Rudi Rahn,² Ding Yu Shao,^{1,3,4} Wouter J. Waalewijn,^{5,6} and Bin Wu⁷

¹Department of Physics and Center for Field Theory and Particle Physics, Fudan University, Shanghai, 200433, China

NNLO predictions with nonlocal subtractions and fiducial power corrections in GENEVA

Simone Alioli,^a Georgios Billis,^{a,b} Alessandro Broggio^c and Giovanni Stagnitto^a

^aUniversità degli Studi di Milano-Bicocca # INFN Sezione di Milano-Bicocca

Jet Production at NNLO: Exploring a New Scheme

Luca Buonocore^(a), Massimiliano Grazzini^(b), Flavio Guadagni^(b), Jürg Haag^(c), Stefan Kallweit^(b) and Luca Rottoli^(d)

NNLO grids with STRIPPER

HighTEA: high energy theory event analyser

Michał Czakon¹, Zahari Kassabov², Alexander Mitov^{3,*}, Rene Poncelet⁴ and Andrei Popescu⁴

¹ Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany

² DAMTP, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, United Kingdom

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E-mail: mczakon@physik.rwth-aachen.de, zk261@cam.ac.uk, adm74@cam.ac.uk, poncelet@hep.phy.cam.ac.uk and andrei.popescu@cantab.net

CoLoRFul subtraction

NNLOCAL: completely local subtractions for color-singlet production in hadron collisions

V. Del Duca^{a,*}, C. Duhr^b, L. Fekesházy^{c,d}, F. Guadagni^e, P. Mukherjee^e, G. Somogyi^f, F. Tramontano^g and S. Van Thurenhout^h

CoLoRFul for hadron collisions: Integrating the counterterms^{*}

S. Van Thurenhout^{a,i}, V. Del Duca^{b,c,d}, C. Duhr^e, L. Fekesházy^{a,f,g}, F. Guadagni^e, P. Mukherjee^f, G. Somogyi^a and F. Tramontano^{h,i}

CoLoRFulNNLO for hadron collisions: regularizing initial-state double real emissions

V. Del Duca^a, G. Somogyi^b, F. Tramontano^c

qT slicing, NNLO grids with MATRIX

Soft contributions to heavy quark production in arbitrary kinematics

Simone Devoto,^a Javier Mazzitelli^b

^aDepartment of Physics and Astronomy, Ghent University, 9000 Ghent, Belgium

^bInstitut für Theoretische Physik, Universität Münster, Wilhelm-Klemm-Straße 9, 48149 Münster, Germany

^cPhysik Institut, Universität Zürich, CH-8057 Zürich, Switzerland

^dUniversität Würzburg, Institut für Theoretische Physik und Astronomie, 97071 Würzburg, Germany

MATRIX HAWAII: PineAPPL interpolation grids with MATRIX

S. Devoto,^a T. Ježo,^b S. Kallweit,^c C. Schwan^d

^aDepartment of Physics and Astronomy, Ghent University, 9000 Ghent, Belgium

^bInstitut für Theoretische Physik, Universität Münster, Wilhelm-Klemm-Straße 9, 48149 Münster, Germany

^cPhysik Institut, Universität Zürich, CH-8057 Zürich, Switzerland

^dUniversität Würzburg, Institut für Theoretische Physik und Astronomie, 97071 Würzburg, Germany

NNLO

NNLO currently under investigation for a variety of processes. Adding more legs and/or more masses is a non-trivial challenge. In the following I present some representative examples

- $e^+e^- \rightarrow 4j@NNLO$
- $pp \rightarrow t\bar{t}j@NNLO$
- $pp \rightarrow t\bar{t}W@NNLO$

Given the current status of the loop and subtraction/slicing technology many new processes can be studied and the relevant phenomenological applications are becoming accessible

Strong control over all the aspects of the calculation allows for further extensions (N3LO, EW corrections, matching, ...)

- $pp \rightarrow \ell\bar{\ell}@QCD \times EW$
- $e^+e^- \rightarrow jj@N3LO$
- $pp \rightarrow \gamma\gamma@N3LO$
- $pp \rightarrow t\bar{t}H@NNLO(+PS)$
- $pp \rightarrow HH@NNLO(+PS)$

N2LO: INCREASING LEGS (LEPTON COLLIDER)

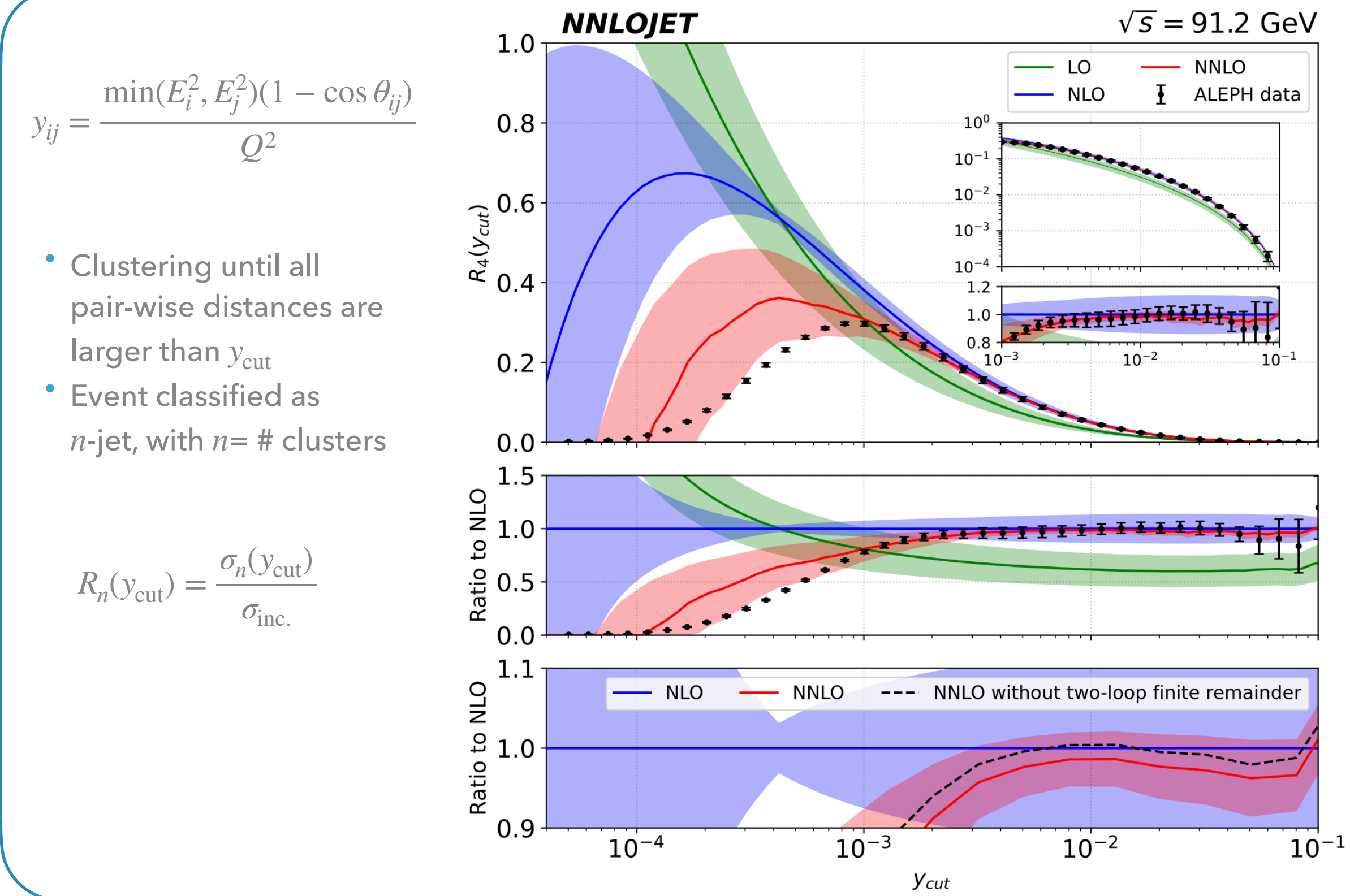
[CERN Yellow Rep. Monogr. 8 (2025)]

$e^+e^- \rightarrow 4j$

- ▶ e^+e^- colliders offers a very clean exp. environment
 - discovery of gluon in planar 3jet events, measurement of its spin
- ▶ 4j rate provides stringent test of non-abelian gauge theory structure
- ▶ α_s extraction

Table 4.2: Wish-list for calculations of missing higher-order perturbative QCD $\mathcal{O}(\alpha_s^n)$ and/or EW $\mathcal{O}(\alpha^n)$ to match the expected experimental uncertainty at future e^+e^- and $e p$ colliders.

Observable	Missing higher-order & power-suppressed corrections
Hadronic Z width	$\mathcal{O}(\alpha_s^5), \mathcal{O}(\alpha_s^6), \mathcal{O}(\alpha^3), \mathcal{O}(\alpha_s \alpha^3), \mathcal{O}(\alpha_s^2 \alpha^2)$
Hadronic W width	$\mathcal{O}(\alpha_s^5), \mathcal{O}(\alpha^2), \mathcal{O}(\alpha_s^2 \alpha)$
Hadronic τ width	$\mathcal{O}(\alpha_s^5)$
Hadronic event shapes (Z, W, H decays)	N ³ LO differential, N ^{3,4} LL resummation, power corrections
Inclusive jet rates	3-jet cross-sections at N ³ LO, 4-jets at N ² LO, 5-jets at NLO
Lattice QCD results	$\mathcal{O}(\alpha_s^6)$ β -function; $\mathcal{O}(\alpha_s^5)$ heavy quark decoupling; $\mathcal{O}(\alpha_s^4)$ static potential



- ▶ Shape of NNLO predictions in better agreement with data
- ▶ Theory uncertainty reduced by $\sim 1/3$
- ▶ Significant discrepancies at low y_{cut} : need for resummation
- ▶ In the perturbative region ($y_{\text{cut}} \sim 10^{-2}$) NNLO correction vanishes if $VV = 0$
- ▶ In this case: $VV \sim \text{NNLO} \sim 2 - 5 \%$

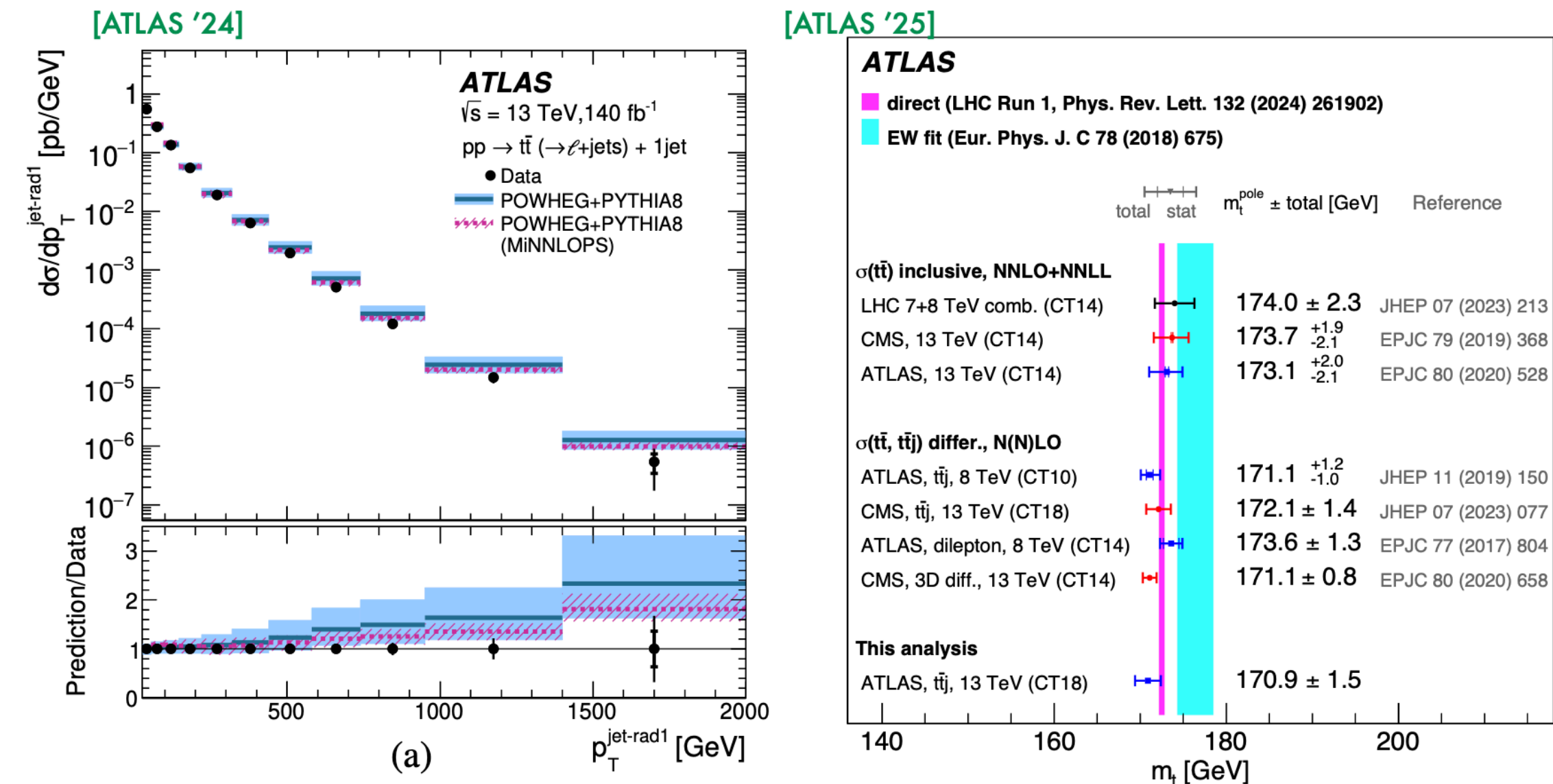
This pattern is different from what observed in hadronic processes, where the two-loop finite remainder typically represents a smaller contribution to the NNLO correction.
[Chen, Chicherin, Fox, Glover, Marcoli, Sotnikov, Sun, Zhang, Zoia 2602.18185]

- * VV : leading color approx. + no vector coupling to closed fermion loops [De Laurentis et al. 2503.10595].
- * RR : (generalised) antenna subtraction [Fox et al. 2410.12904]

N2LO: INCREASING LEGS AND MASSES

$pp \rightarrow t\bar{t}j$

- ▶ ~ 40 % of $t\bar{t}$ events at LHC are comes with an extra jet
→ NNLO QCD corrections needed to reduce theo. uncertainties
- ▶ Sensitivity to top-quark mass: $\rho = 2m_o/m(t\bar{t}j)$
- ▶ Impact of higher orders on xsection ratios: $R^{3/2}(X) = d\sigma^{t\bar{t}j}(X)/d\sigma^{t\bar{t}}(X)$
- ▶ NNLO $t\bar{t}j$ input for N3LO QCD corrections to $t\bar{t}$

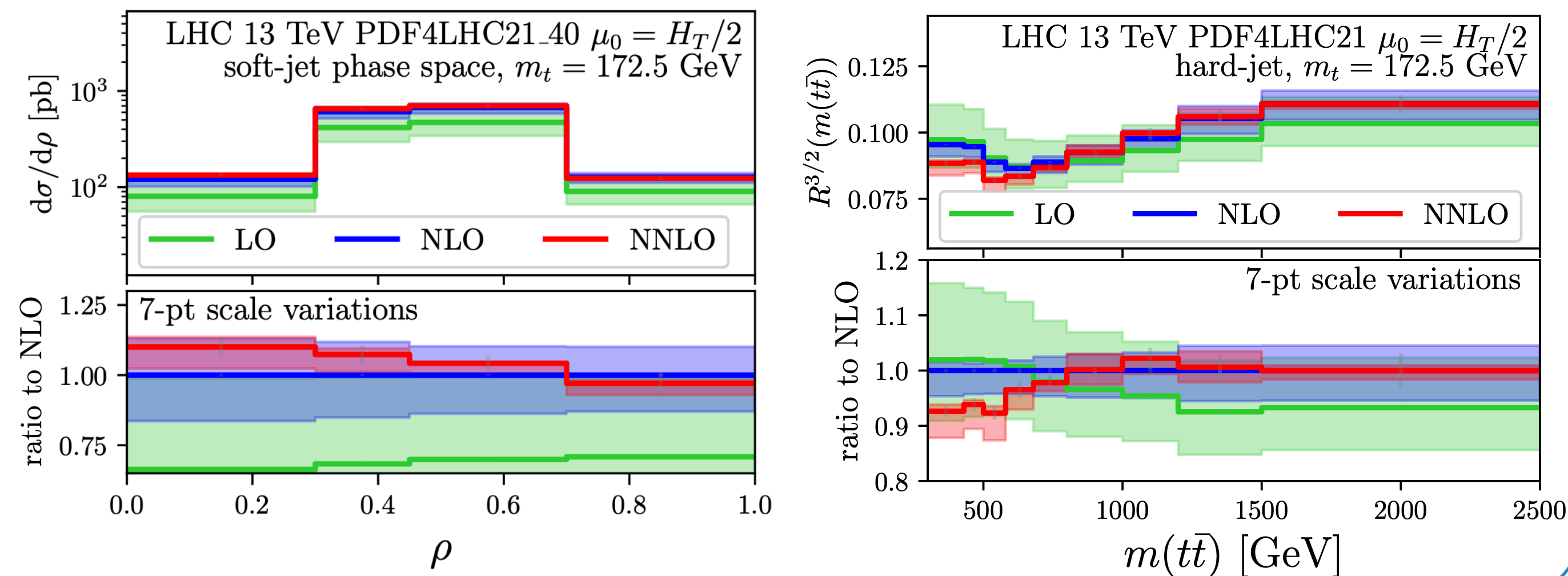


	σ^{LO} [pb]	σ^{NLO} [pb]	σ^{NNLO} [pb]
$\mu_R = \mu_F = H_T/4$			
soft	$341.9(0.2)^{+48.0\%}_{-30.2\%}$	$383.0(1.3)^{+0.984\%}_{-9.9\%}$	$362.9(10.1)^{+0.73\%}_{-4.27\%}$
hard	$60.8(0.0)^{+49.0\%}_{-30.6\%}$	$74.1(0.2)^{+4.27\%}_{-12.3\%}$	$70.4(0.6)^{+0.593\%}_{-6.47\%}$

~ + 12 %
~ + 22 %

~ - 5 %

[Badger, Becchetti, Brancaccio, Czakon, Hartanto, Poncelet, Zoia 2511.11431]



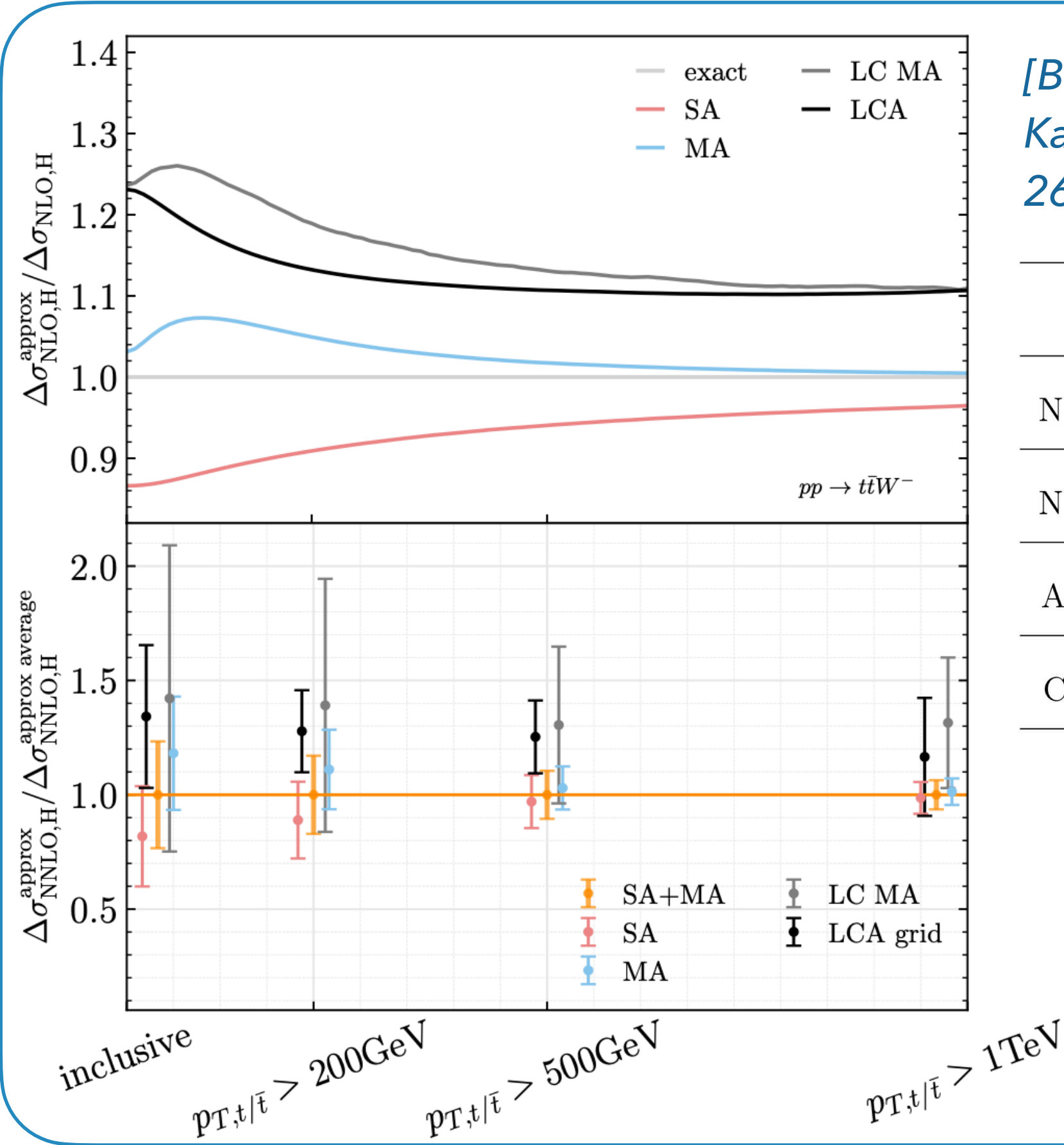
* VV: leading color approx. [Badger et al. 2511.11424]
(6 – 10 % of total xsection). Simplified amplitude expression for efficient numerical evaluation.

* RR: STRIPPER

N2LO: INCREASING LEGS AND MASSES

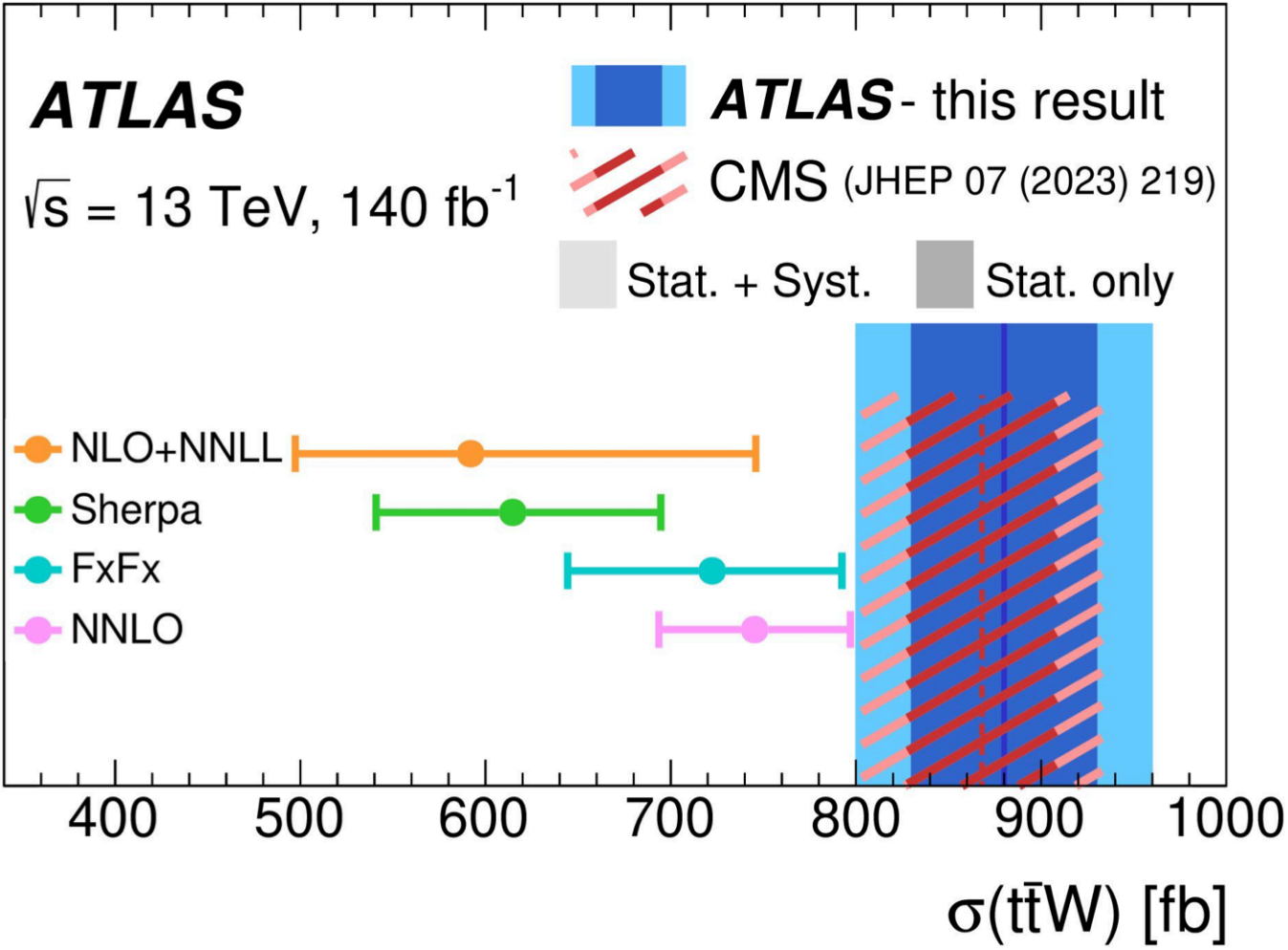
$pp \rightarrow t\bar{t}W$

- Background for key SM processes: $t\bar{t}H$ and $t\bar{t}t\bar{t}$ production
- Relevant for physics beyond the SM searches
- Theory predictions underestimate measurements



[Becchetti, Canko, Chen, Chestnov, Delto, Ditsch, Grazzini, Kallweit, Peraro, Pozzoli, Savoini, Tancredi, Zoia 2606.09503]

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]
NNLO ^{LC} _{QCD} + NLO _{EW}	$512.17^{+6.7\%}_{-7.38\%} \pm 2.51\%$	$254.49^{+6.94\%}_{-7.64\%} \pm 2.23\%$
NNLO [*] _{QCD} + NLO _{EW}	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$
ATLAS [2401.05299]	$585^{+6.0\%+8.0\%}_{-5.8\%-7.5\%}$	$301^{+9.3\%+11.6\%}_{-9.0\%-10.3\%}$
CMS [2011.04652]	$533^{+5.4\%+5.4\%}_{-5.4\%-5.4\%}$	$343^{+7.6\%+7.3\%}_{-7.6\%-7.3\%}$

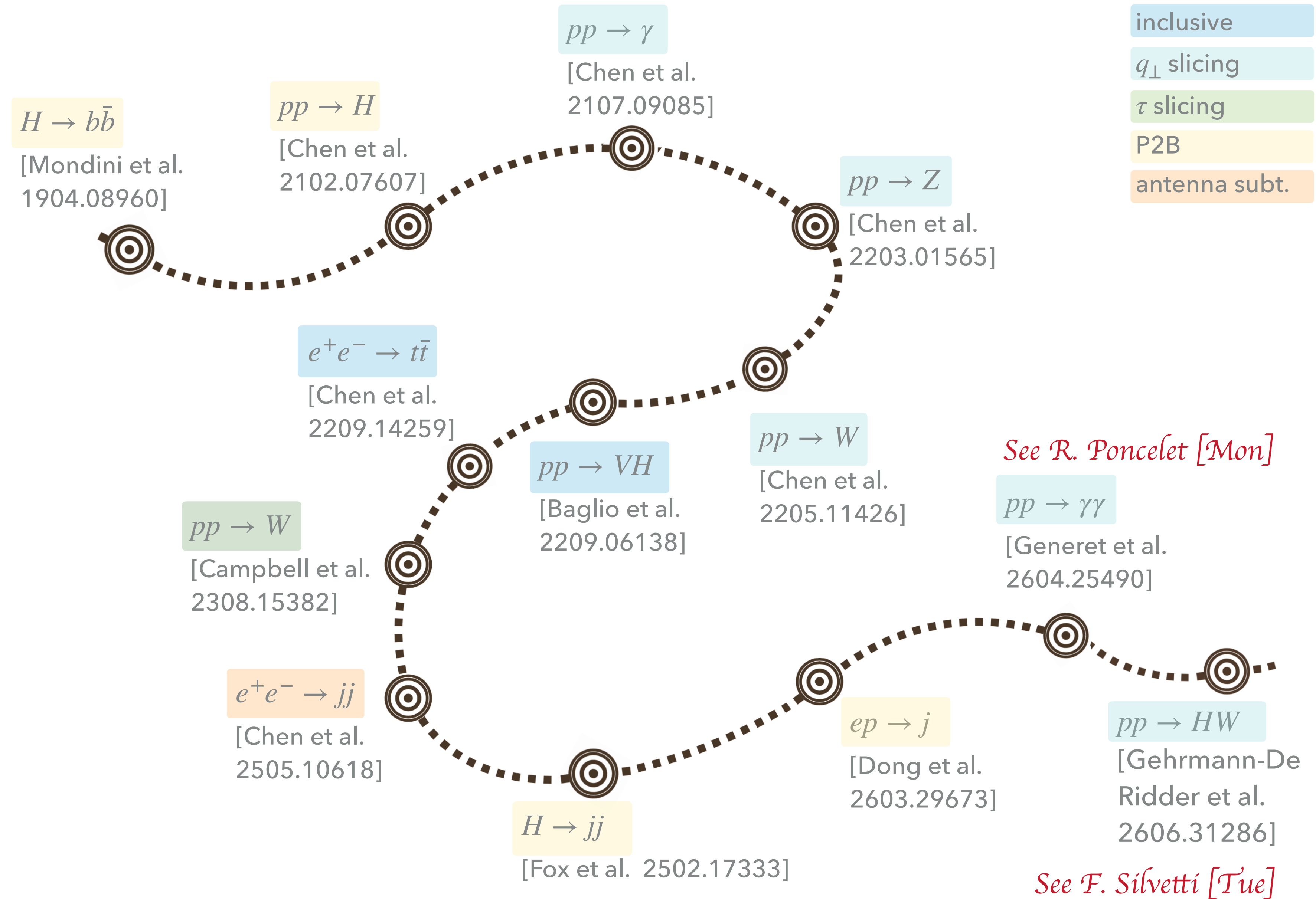


- Confirmed agreement with approximate 2loop (soft-W approx. [SA] + massification [MA]) [Buonocore, Devoto, Grazzini et al. 2306.16311]
- Confirmed tension with experimental measurements

- * VV: impressive calculation of LC VV amplitude (previously approximated):
 - Special function evaluation $\sim 20\text{-}120$ min
 - Amplitude coefficients evaluated in ~ 20 min
- * RR: $q_{\perp}$ subtraction [Catani, Grazzini 0703012]

GOING TO N3LO...

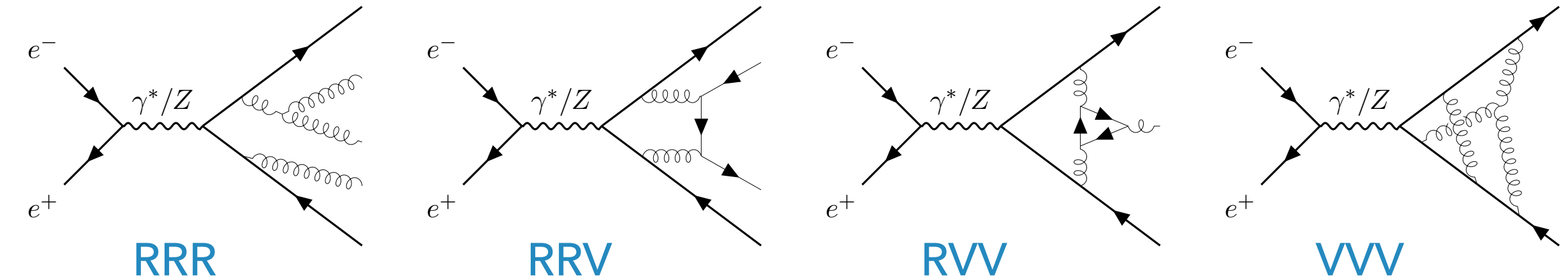
- ▶ Intense efforts on **advancing streamlined workflows**
- ▶ **NNLO EW** remains even more elusive
- ▶ The techniques and issues discussed will become more pressing when moving at N3LO
- ▶ **Tricks and tips learned at NNLO will come at hand**
- ▶ Nowadays the state of the art of N3LO calculation is largely dominated by slicing approaches and combinations with N2LO+j subtractions
- ▶ The theory community has plenty of space for developing methods and technologies to be onboard on such endeavour



N3LO: FEW RECENT ACHIEVEMENTS

[Chen, Jakubčík, Marcoli, Stagnitto (2025)]

- First pheno results for $2 \rightarrow 2$ massless processes
 - $e^+e^- \rightarrow jj$ via (partial) extension of antenna subtraction
 - First fully differential F.O. calculation for jet production with a local infrared subtraction
 - numerical stability of RRV and RVV is a major challenge: rescue systems (quadruple-precision) + Taylor exp. of special functions



$$d\sigma_{N^3LO} = \int_n [d\sigma^{VVV} - d\sigma^W] + \int_n [d\sigma^{RVV} - d\sigma^U] + \int_{n+1} [d\sigma^{RRV} - d\sigma^T] + \int_{n+2} [d\sigma^{RRR} - d\sigma^S]$$

triple-virtual subtraction term
 double-virtual real subtraction term
 double-real-virtual subtraction term
 triple-real subtraction term

- Inclusive hadronic cross section:

$$\sigma^{(3)} = \sigma_0 \left(\frac{\alpha_s}{2\pi} \right)^3 (-105 \pm 11)$$

Large MC error, due to large cancellations occurring in very inclusive set up

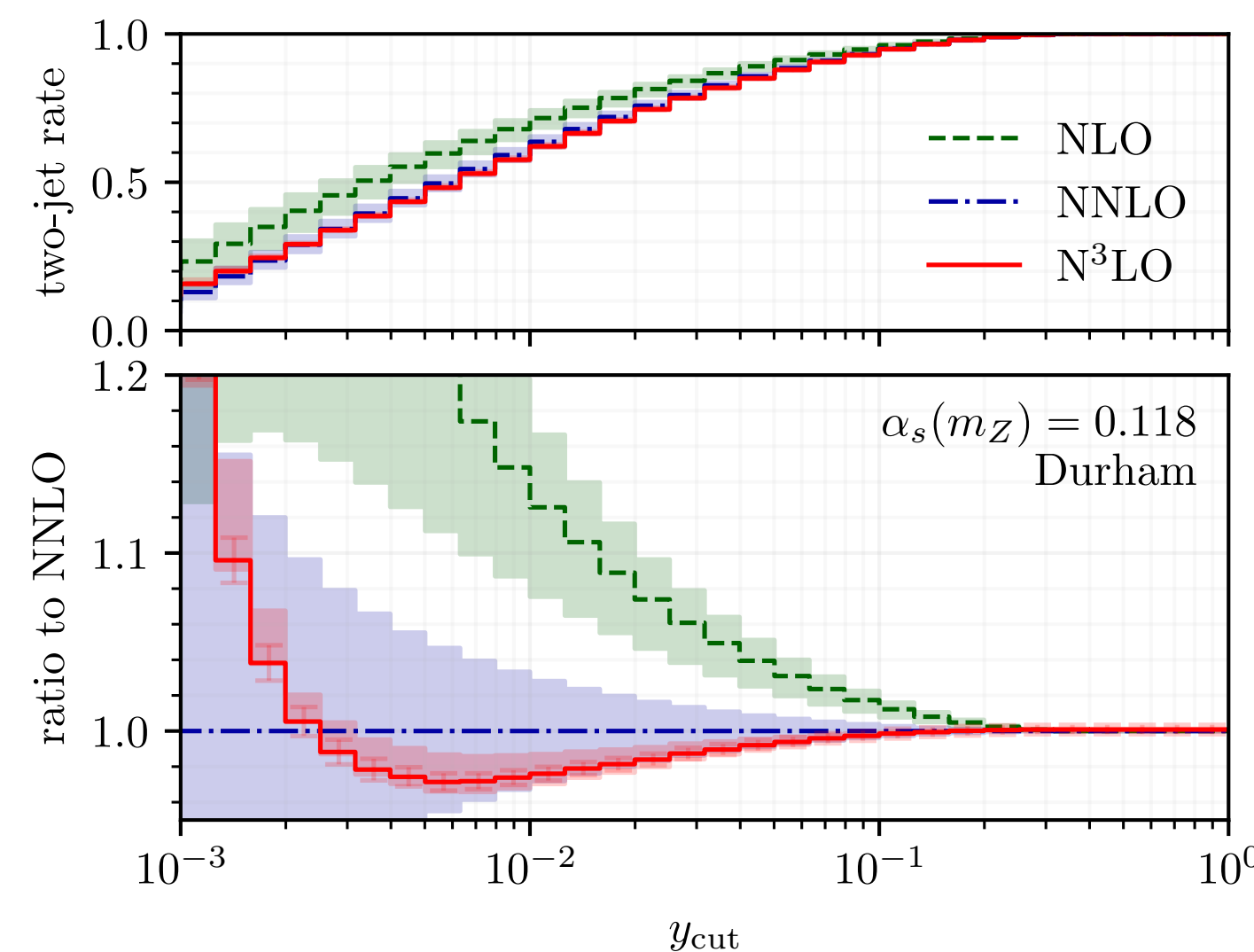
$$\sigma_{\text{exact}}^{(3)} = \sigma_0 \left(\frac{\alpha_s}{2\pi} \right)^3 (-102.14\dots)$$

[Chetyrkin, Kühn, Kwiatkowski '95]

Crucial validation benchmark, but not the point =)

- Differential distributions:

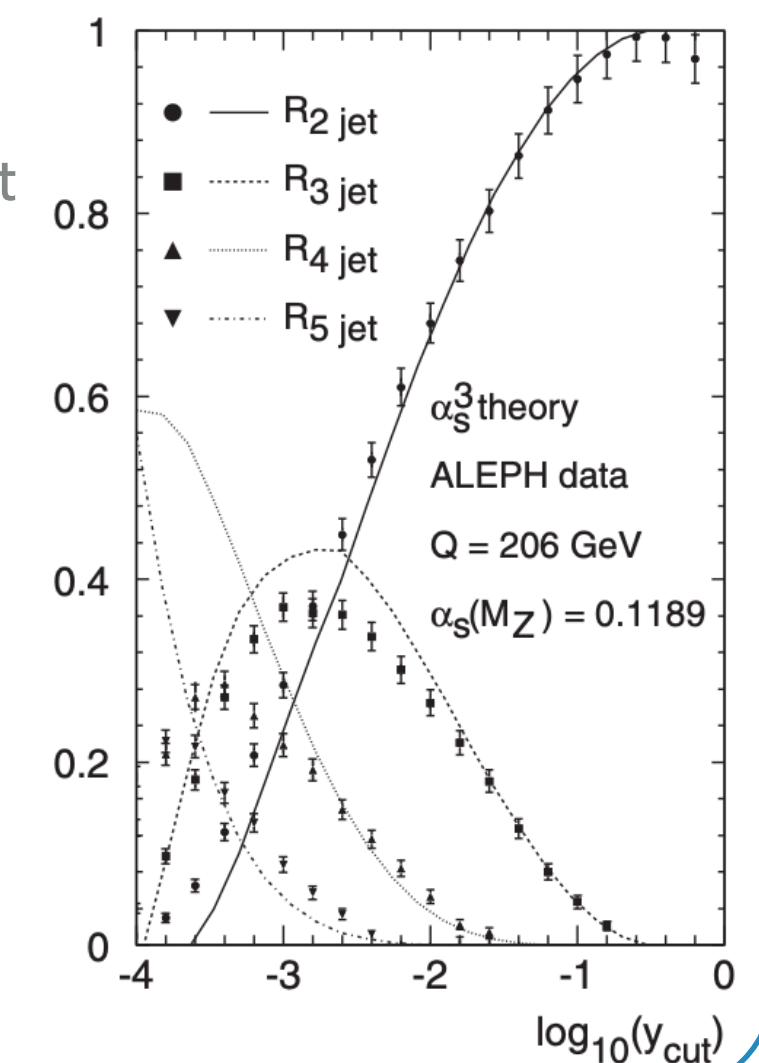
2jet production rate in resolution parameter y_{cut} (Durham algorithm) : direct calculation



Full agreement with indirect calculation [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich]

$$R_n^{(3)}(y_{\text{cut}}) = \frac{\Gamma_{\gamma^* \rightarrow n \text{ jets}}^{(3)}(y_{\text{cut}})}{\Gamma_{\gamma^* \rightarrow \text{hadrons}}^{(3)}}$$

$$R_2^{(3)}(y_{\text{cut}}) = 1 - \sum_{n=3}^5 R_n^{(3)}(y_{\text{cut}})$$



N3LO: FEW RECENT ACHIEVEMENTS

See R. Poncelet [Mon]

[Czakon, Eschment, Generet, Poncelet (2026)]

First pheno results for $2 \rightarrow 2$ massless processes

- ▶ $e^+e^- \rightarrow jj$ via (partial) extension of antenna subtraction
- ▶ $pp \rightarrow \gamma\gamma$ via $q_\perp$ subtraction + STRIPPER @(N2LO+j)

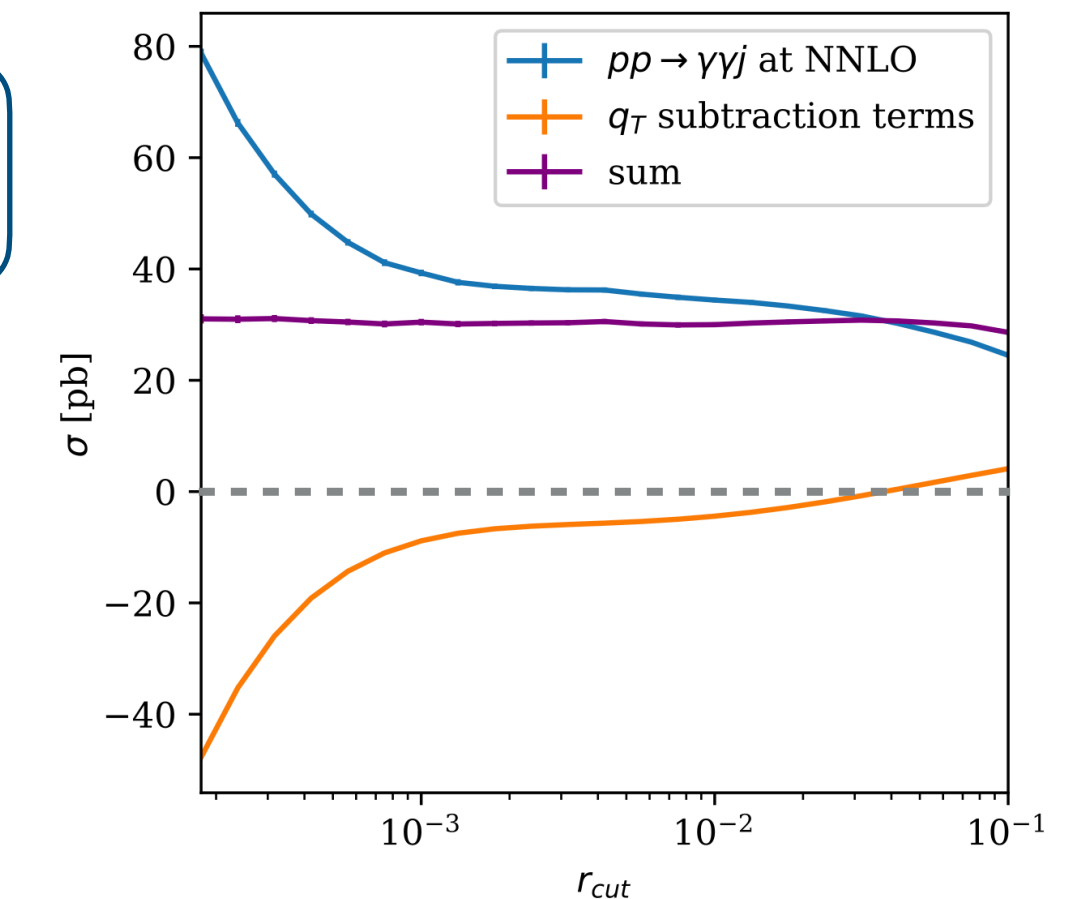
These various cancellations are responsible for the substantial computational resources used—in units of million CPUh: 2.7 for the real-virtual-finite (RVF), 2.3 for the double-real-finite (RRF), 2.4 for the double-real-single-unresolved (RRSU), 0.5 for the double-real-double-unresolved (RRDU), and 0.6 for the double-virtual-finite (VVF) contribution (other contributions required significantly less).

$$\sigma_{N^3LO}^{pp \rightarrow \gamma\gamma} = \int_0^{q_T^{\text{cut}}} dq_T \frac{d\sigma_{N^3LO}^{pp \rightarrow \gamma\gamma}}{dq_T} + \int_{q_T^{\text{cut}}}^\infty dq_T \frac{d\sigma_{NNLO}^{pp \rightarrow \gamma\gamma j}}{dq_T}$$

$q_\perp$ factorisation: need N3LO TMD [Ebert, Mistlberger, Vita '20] [Luo, Yang, Zhu, Zhu '20]

N2LO+j: STRIPPER

Result mostly driven by NNLO+j: the separate divergence of the terms is clearly visible for $r_{\text{cut}} < 10^{-3}$

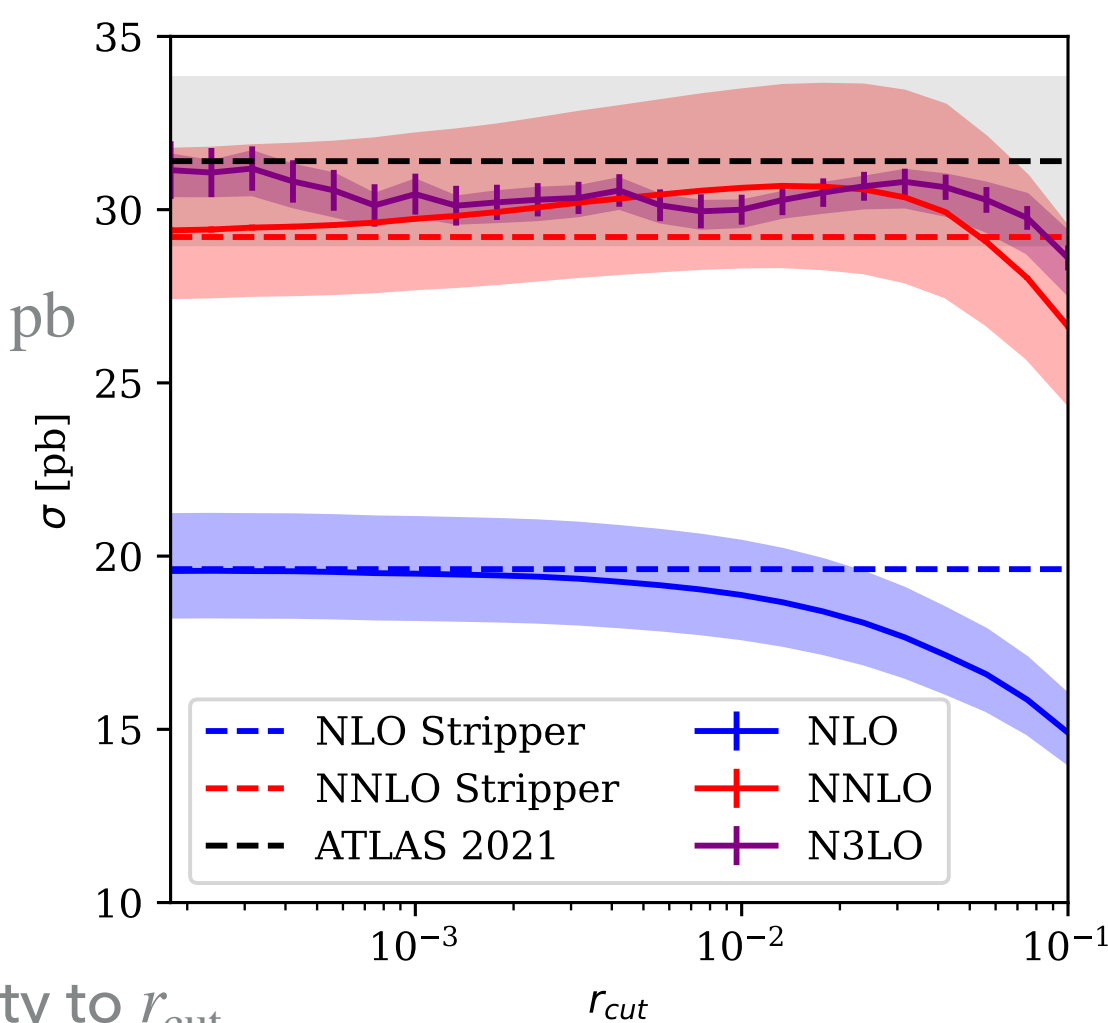


Fiducial xcross section:

$$\sigma_{N^3LO}^{pp \rightarrow \gamma\gamma} = 31.2(6)^{+0.5}_{-0.7} \text{ pb}$$

$$\sigma_{\text{ATLAS}}^{pp \rightarrow \gamma\gamma} = 31.6 \pm 0.1(\text{stat.}) \pm 2.4(\text{syst.}) \text{ pb}$$

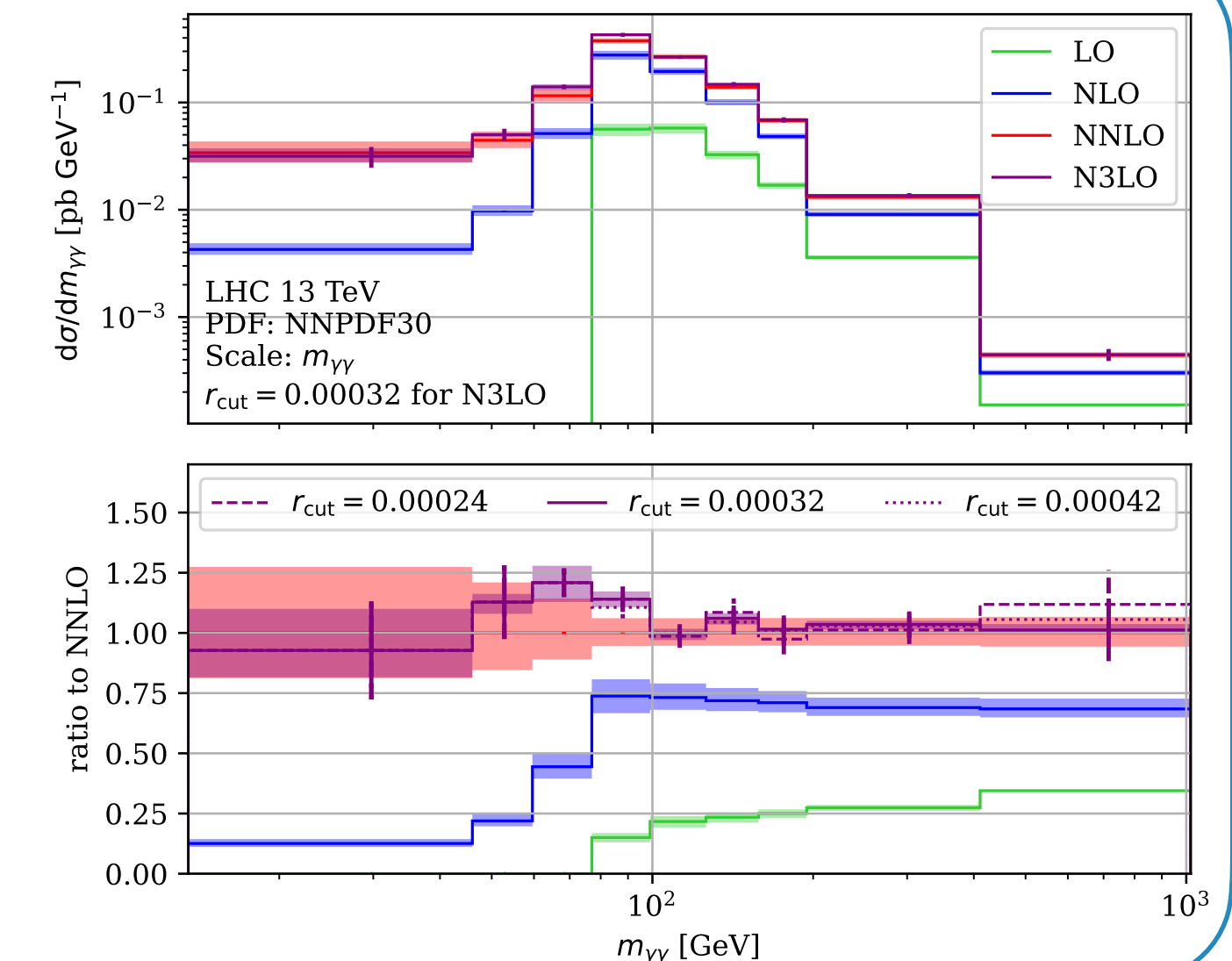
Phase space from [ATLAS 2107.09330] with hybrid-cone photon-isolation [Siebert '16, Chen et al. '19]



Solid line: $q_\perp$ slicing $\rightarrow$ clear sensitivity to r_{cut}

Differential predictions:

- ▶ Optimised phase space integration $\rightarrow$ minimise statistical unc.
- ▶ Statistical uncertainty comparable with scale uncertainty at NNLO
- ▶ Good perturbative convergence
- ▶ Stable behaviour in r_{cut}



MIXED QCDXEW CORRECTIONS

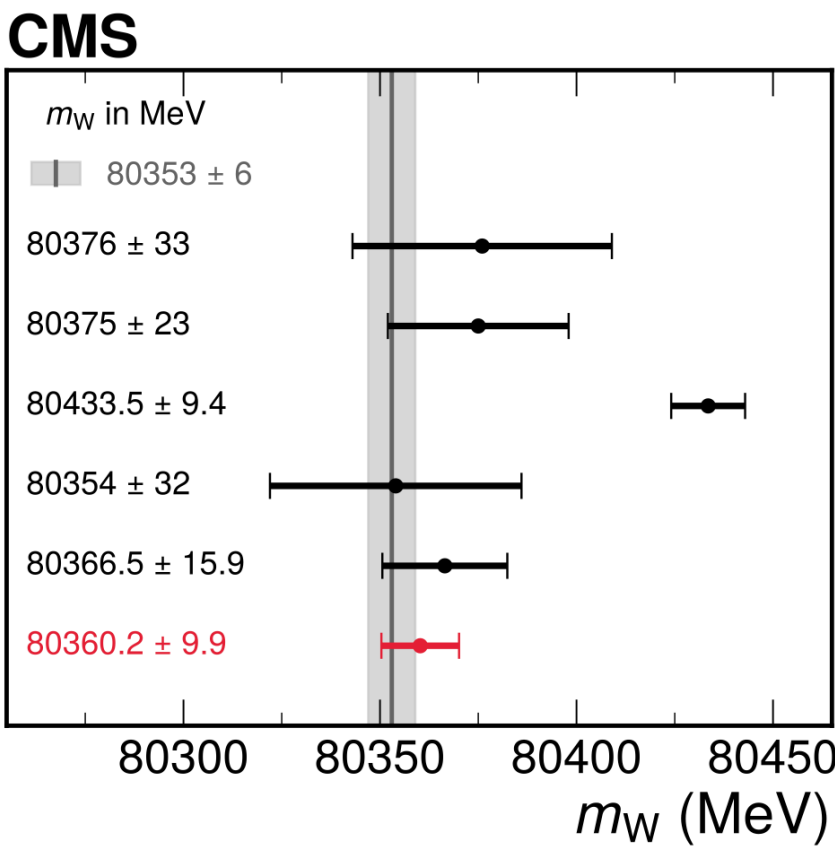
► Mixed QCD×EW corrections to the Drell-Yan process : increase our control on SM parameters and search for new physics

Resonance region:
EW parameters determination

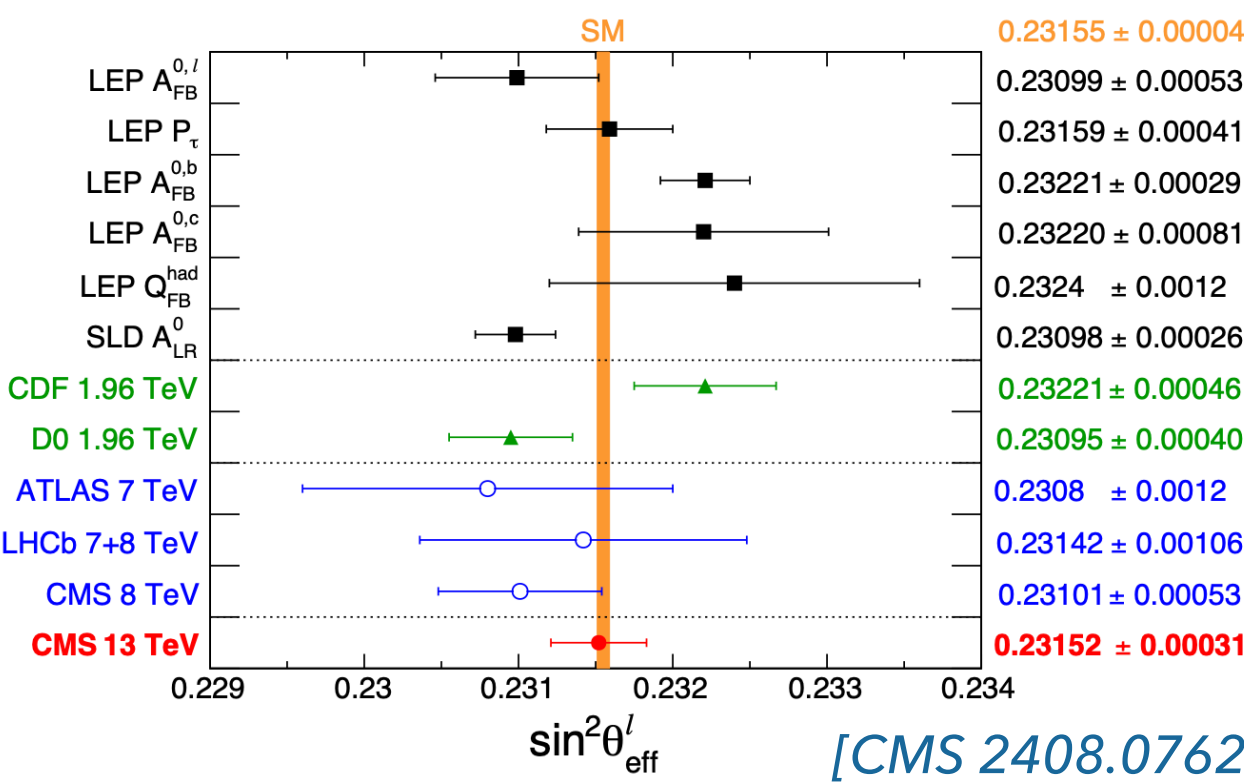
► m_W

Electroweak fit
PRD 110 (2024) 030001
LEP combination
Phys. Rep. 532 (2013) 119
D0
PRL 108 (2012) 151804
CDF
Science 376 (2022) 6589
LHCb
JHEP 01 (2022) 036
ATLAS
arXiv:2403.15085
CMS
This work

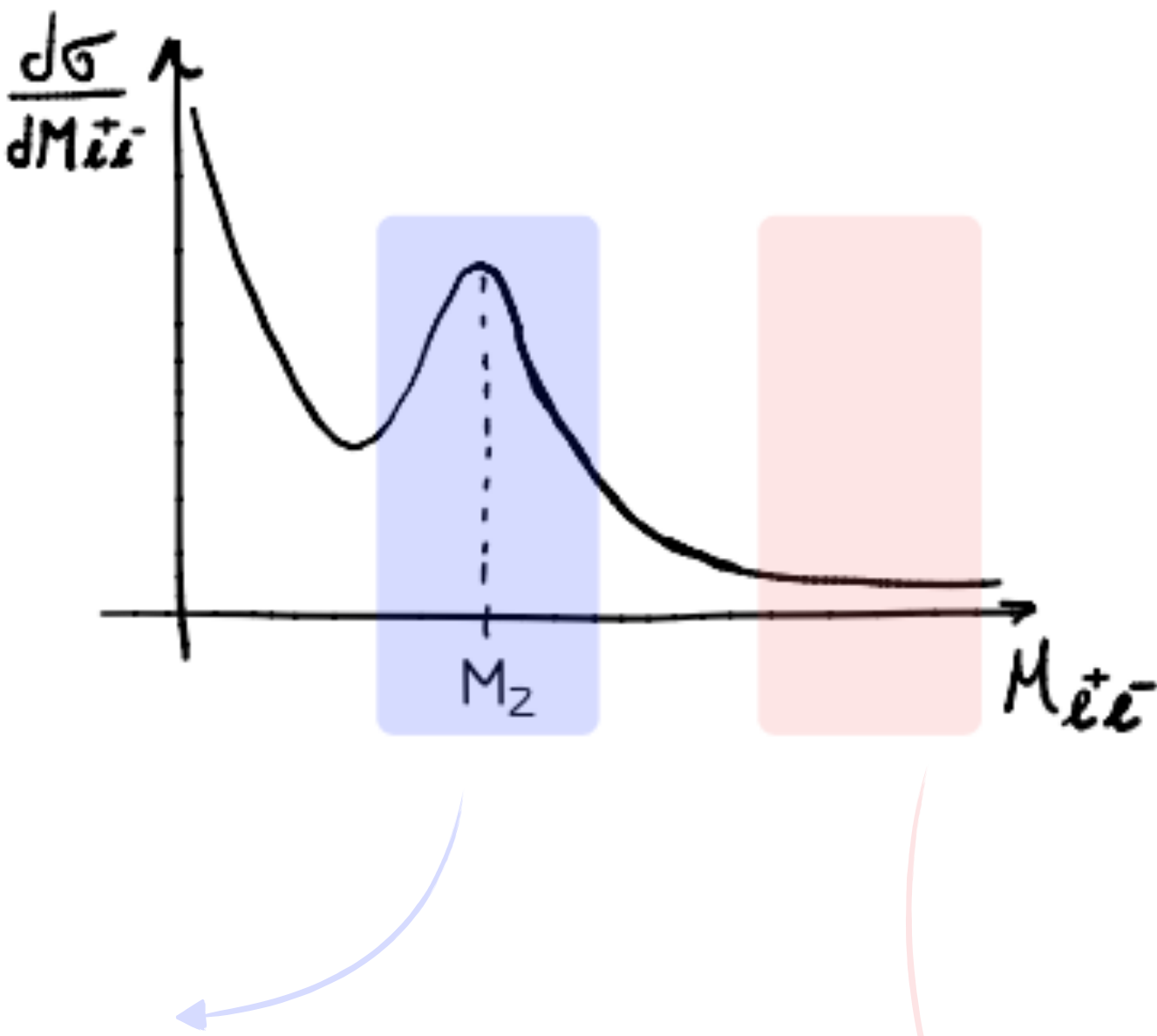
[2412.13872]



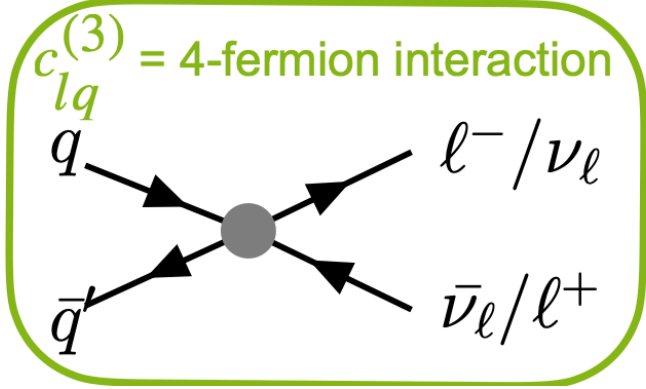
► $\sin^2 \theta_W$



[CMS 2408.07622]

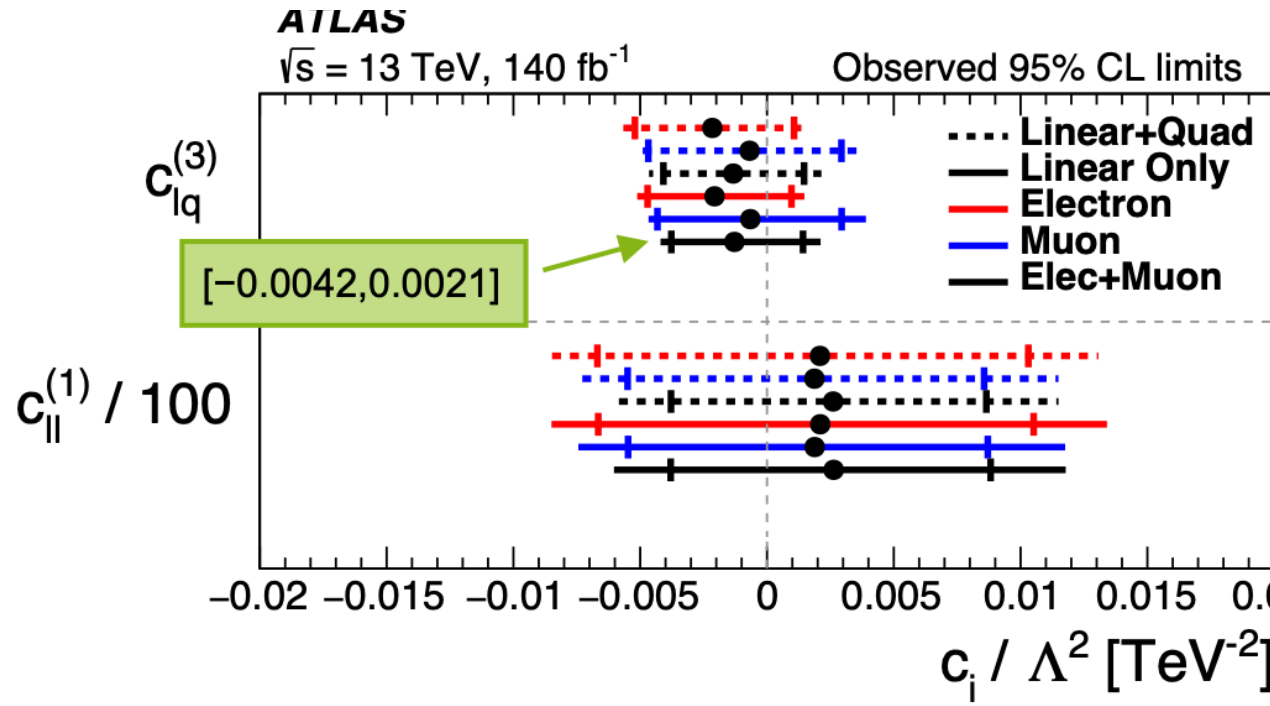
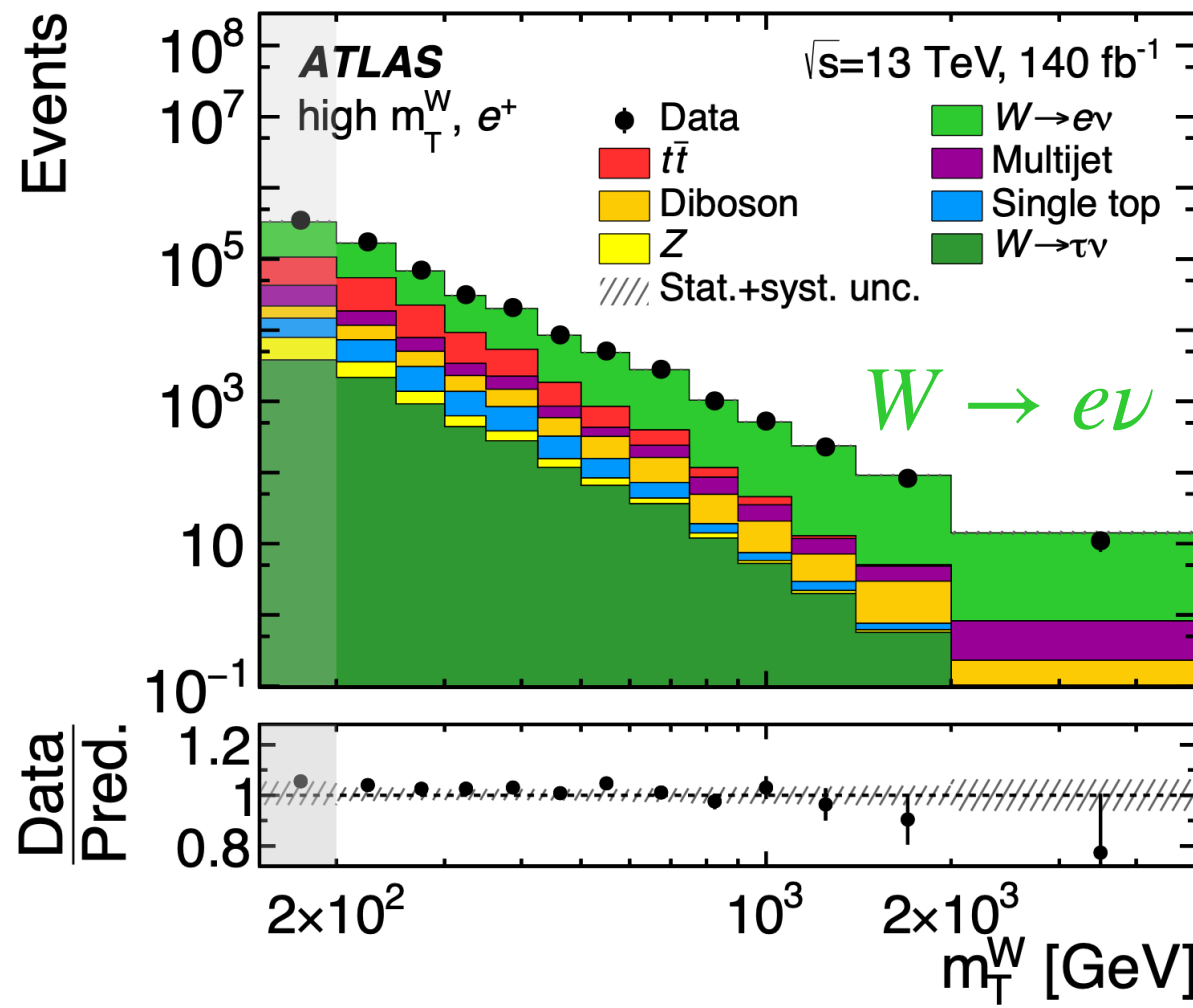


Constrain heavy NP in
a model-independent
way using **SMEFT**



Off-shell region:
New physics searches

[2502.21088]



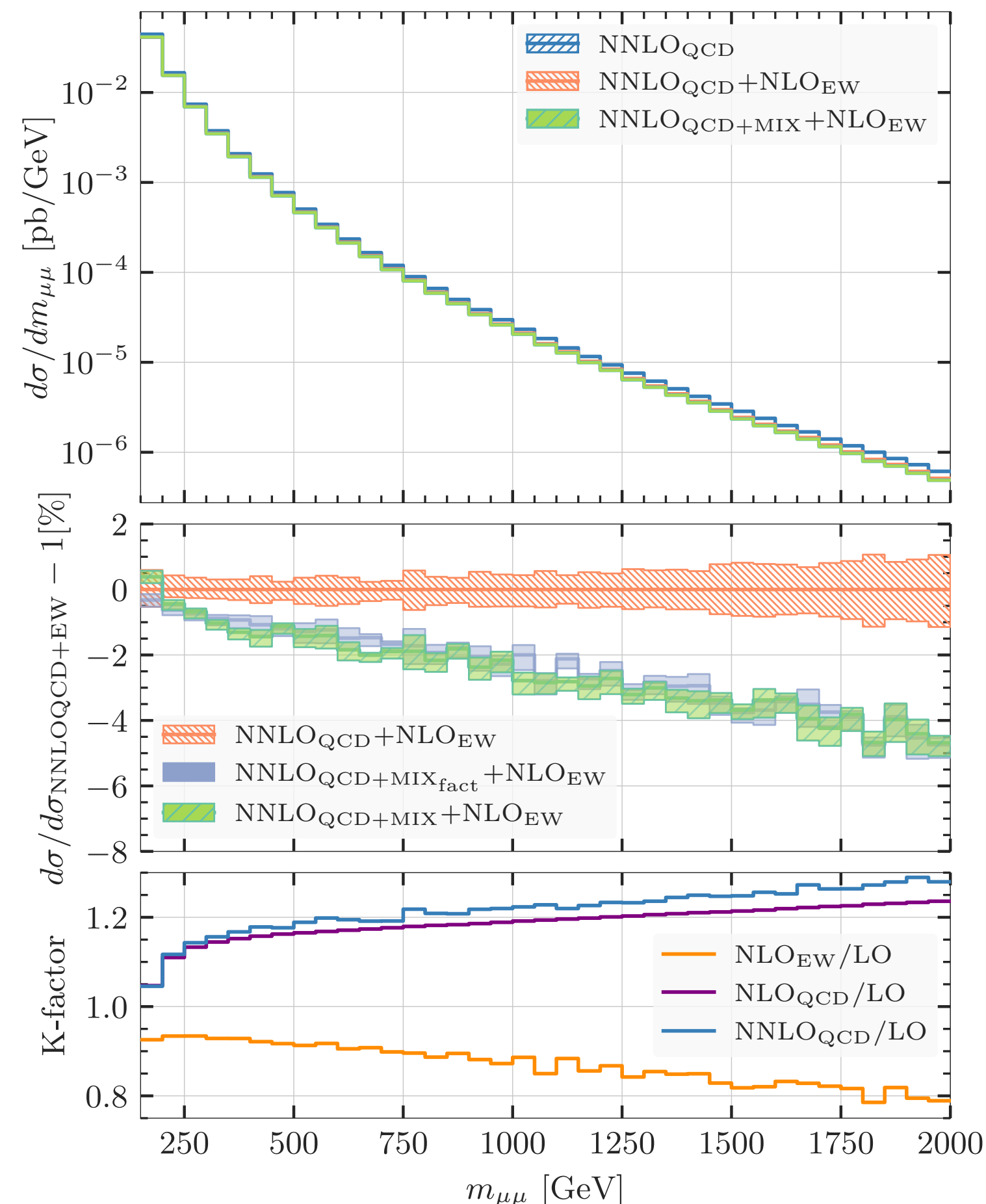
MIXED QCDXEW CORRECTIONS

► Differential predictions for NCDY:

[Armadillo, Bonciani, Buonocore, Devoto, Grazzini, Kallweit, Rana, Vicini (2025) + previous studies by Buccioni & friends + WIP on CCDY (also by C. Biello & friends)]

Massive muons as collinear- divergences regulator:

$$m_{\mu\mu} > 150\text{GeV}, p_{\perp,\mu^\pm} > 53\text{GeV}, |y_\mu| < 2.4, \mu_R = \mu_F = m_{\mu\mu}, \sqrt{s_{\text{had}}} = 13\text{TeV}$$



► Visible effects of EW effects wrt to **NNLO QCD** in the entire spectrum

► Impact of **exact mixed** ($d\sigma_{\text{exact}}^{\text{mix}} / (d\sigma_{\text{NNLO}}^{\text{QCD}} + d\sigma_{\text{NLO}}^{\text{EW}})$):

- Non-flat shape
- Large at low invariant mass: $\mathcal{O}(1\%)$ @ 250GeV
- Reaches $\mathcal{O}(4.5\%)$ @ 2TeV

► Impact of **factorised mixed** ($d\sigma_{\text{fact}}^{\text{mix}} = d\sigma_{\text{NLO}}^{\text{QCD}} \times d\sigma_{\text{NLO}}^{\text{EW}}$):

- Very good agreement with the exact calculation
- QCD and EW effects largely factorises at high invariant mass

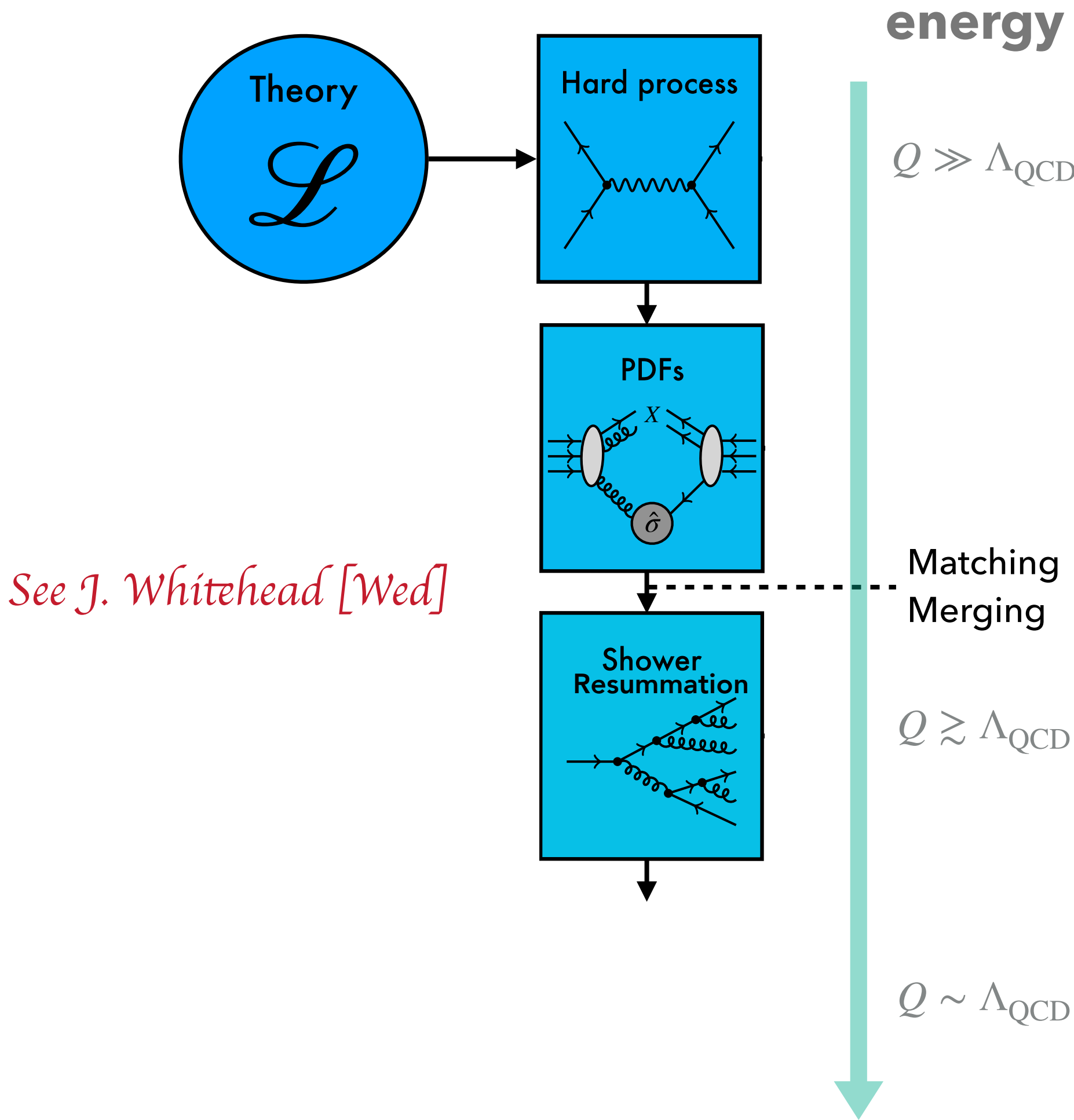
► **NLO EW/LO**:

- Large corrections growing from $\mathcal{O}(-2\%)$ to $\mathcal{O}(-20\%)$
- Shape compatible with Sudakov effects

► **NLO QCD/LO** and **NNLO QCD/LO**:

- **NLO QCD** pretty flat correction $\mathcal{O}(-20\%)$ in $m_{\mu\mu} \in [5 - 20]\text{GeV}$, and **NNLO QCD** being consistently $\mathcal{O}(+2\%)$ wrt **NLO** in the same range

QUICK DETOUR BEYOND
FIXED-ORDER PREDICTIONS

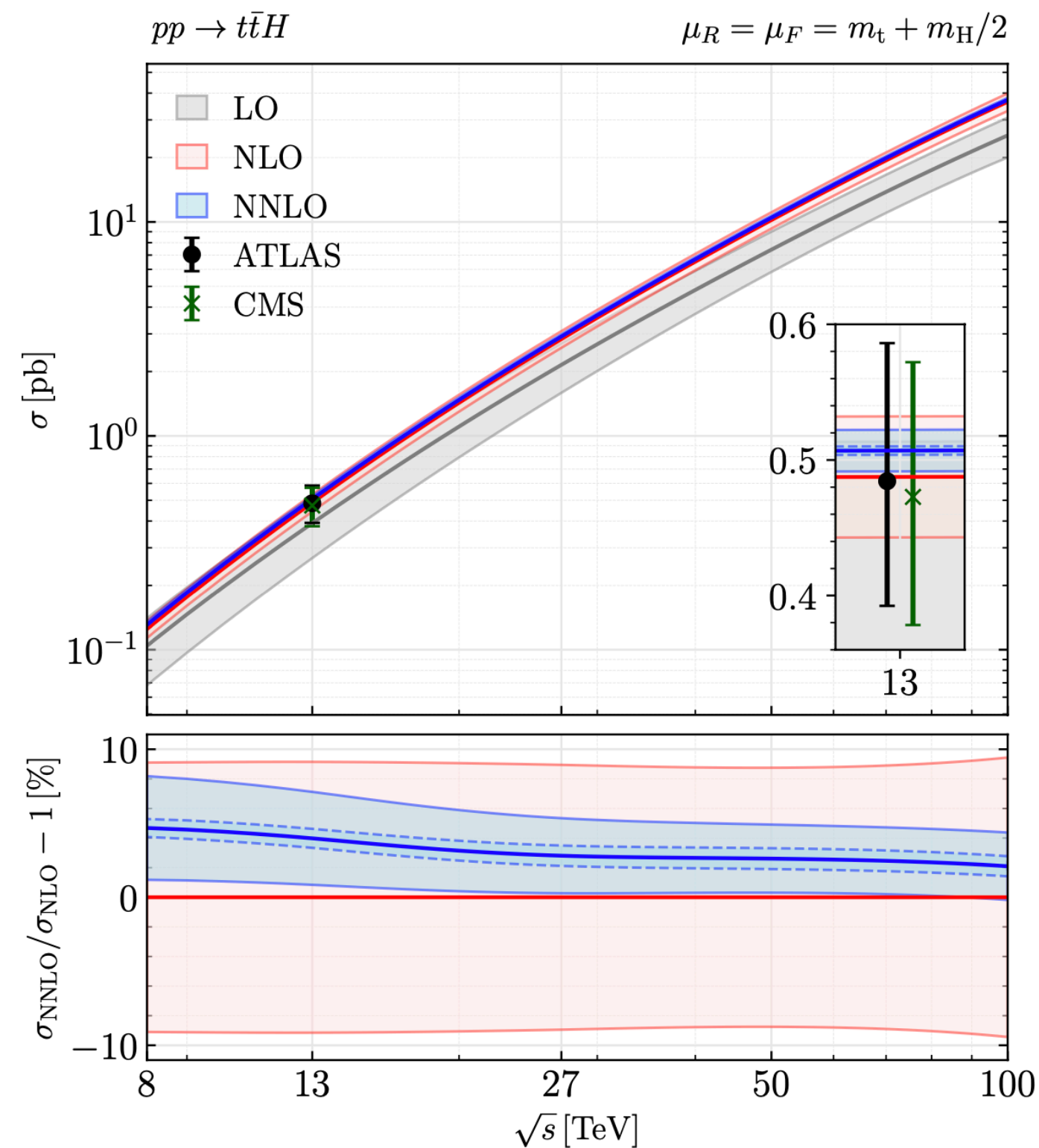


PRECISION&REALISM

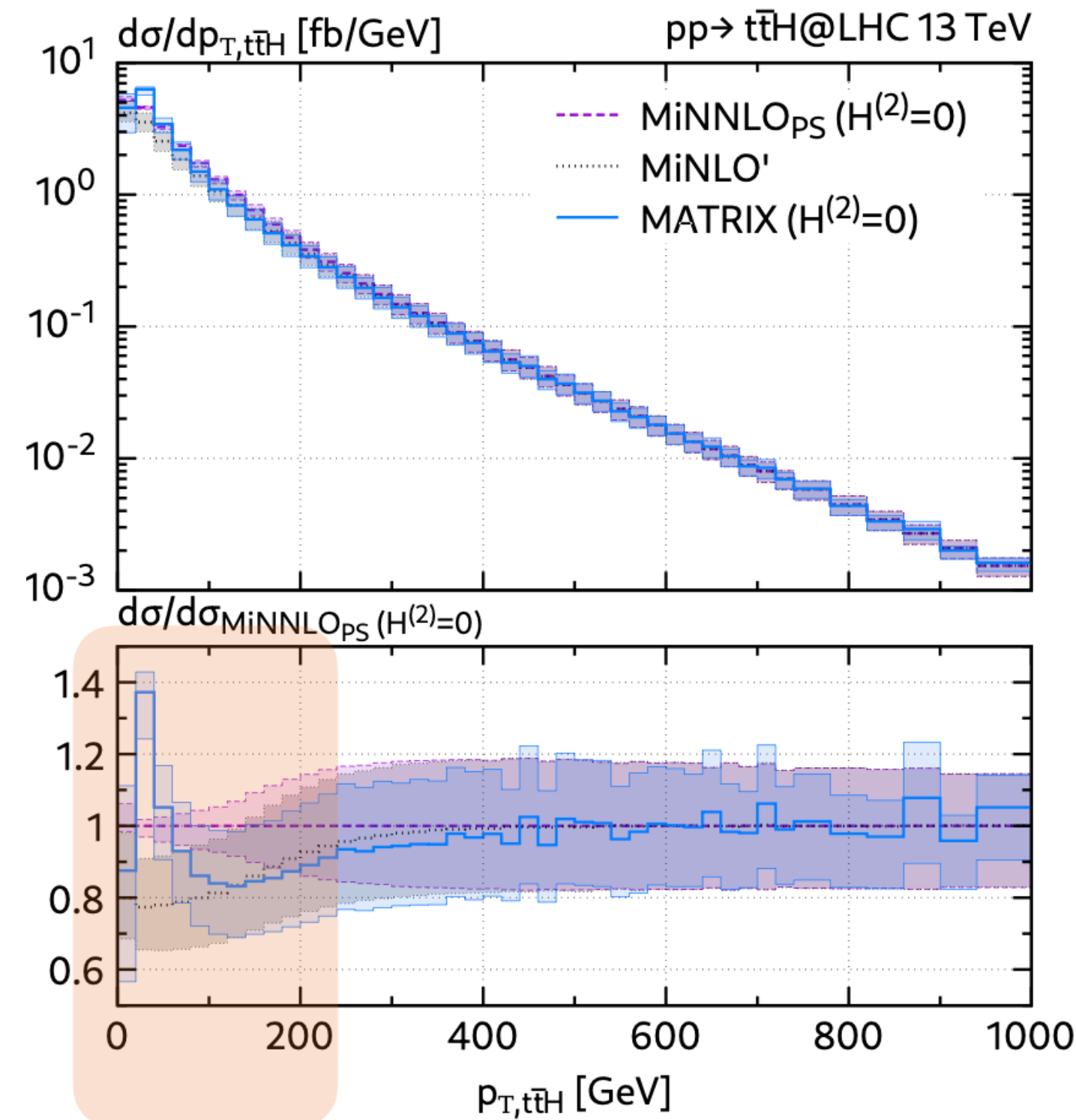
$$pp \rightarrow t\bar{t}H$$

[Biello, Savoini, CSS,
Wiesemann 2603.06143]

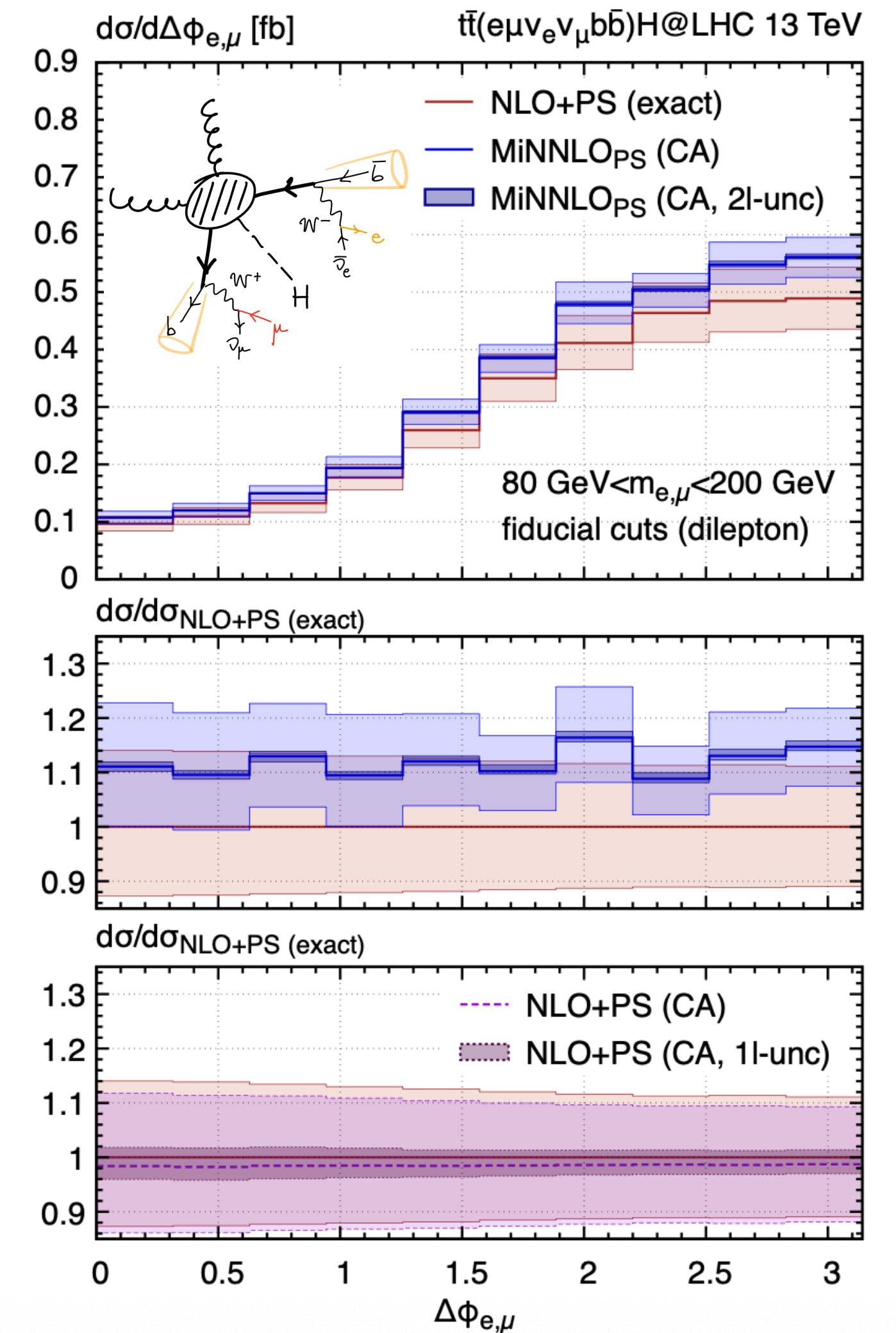
[(Catani), Devoto et al. 2210.07846, 2411.15340]



- NNLO large at xsection level, and larger at fiducial level:
 $\sim +10\%$ NLO @ $p_{\perp,H} = 200\text{GeV}$,
 uncertainty $\sim 1/3$ NLO



- Fixed-order predictions fails in the low- $p_{\perp}$ region $\rightarrow$ need for matching and resummation
- MiNNLOPS framework



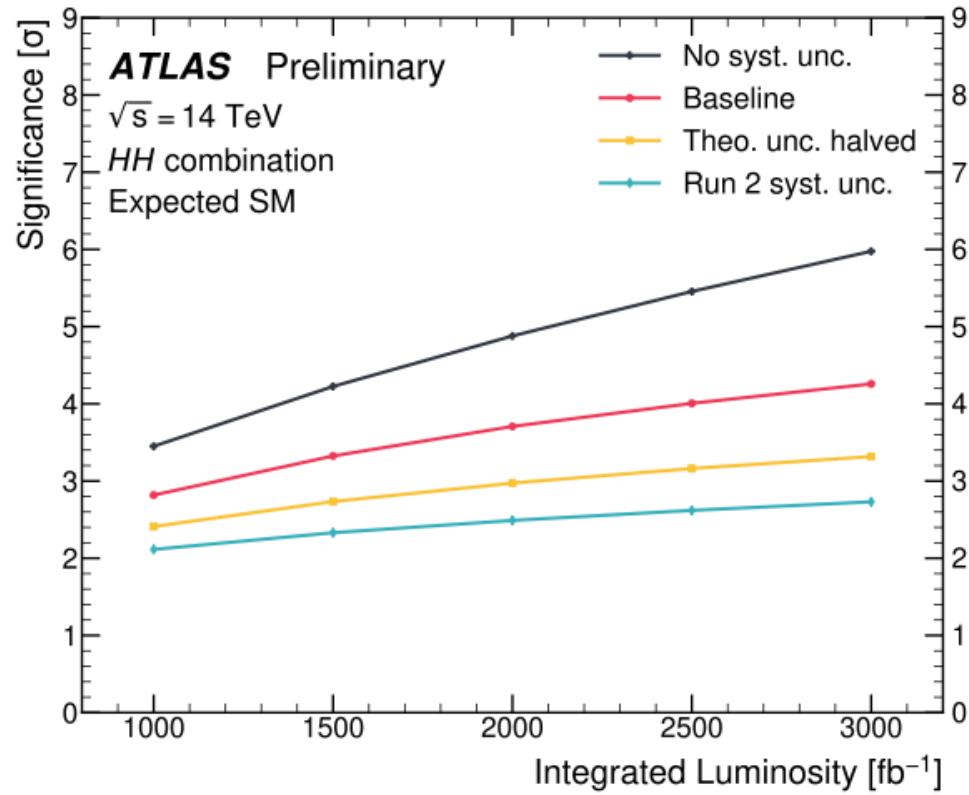
- VV via combined (CA) approximation (soft Higgs and massification)
- Public code for exp. analysis

PRECISION&REALISM $pp \rightarrow HH$

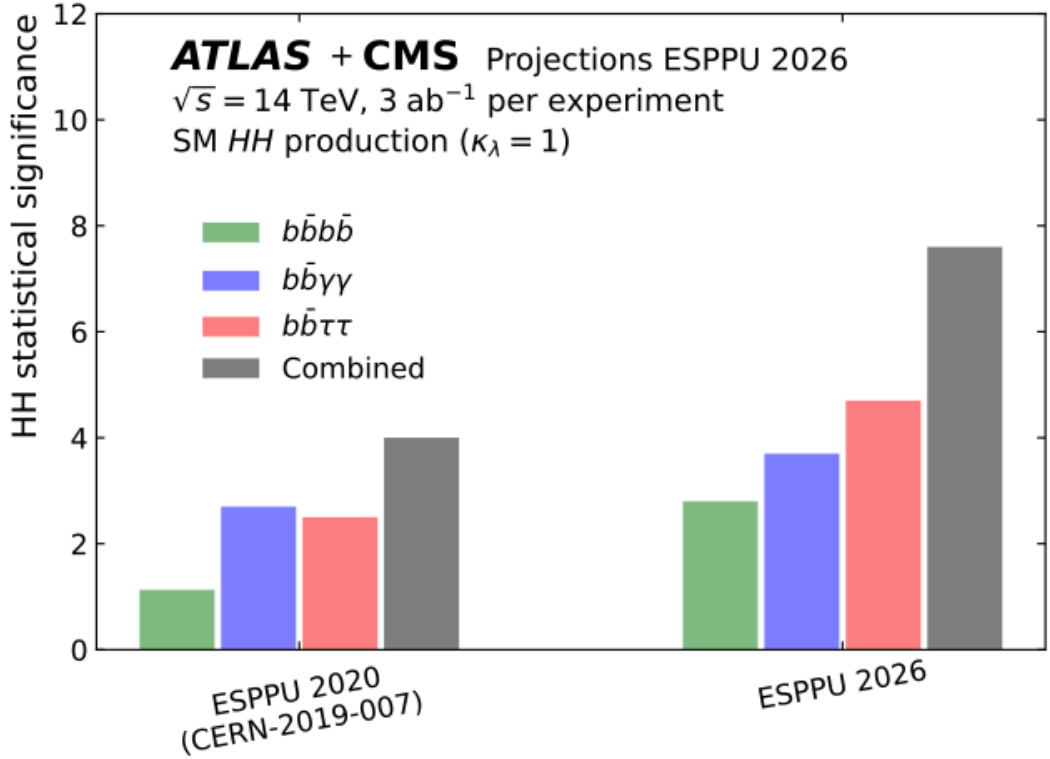
ATLAS and CMS are increasingly optimistic about HH at HL-LHC:
• order of magnitude more data, improved analysis techniques

Projections:

ATLAS: HH obs. $4.26 - 5.98\sigma$, $\kappa_\lambda = 1.0^{+0.48}_{-0.42}$
[ATL-PHYS-PUB-2025-006]



ATLAS+CMS: $>5\sigma$, $\kappa_\lambda = 1.0^{+0.31}_{-0.27}$ w/ $2ab^{-1}$,
 $>7\sigma$, $\kappa_\lambda = 1.0^{+0.29}_{-0.26}$ w/ $3ab^{-1}$ [ATL-PHYS-PUB-2025-018]



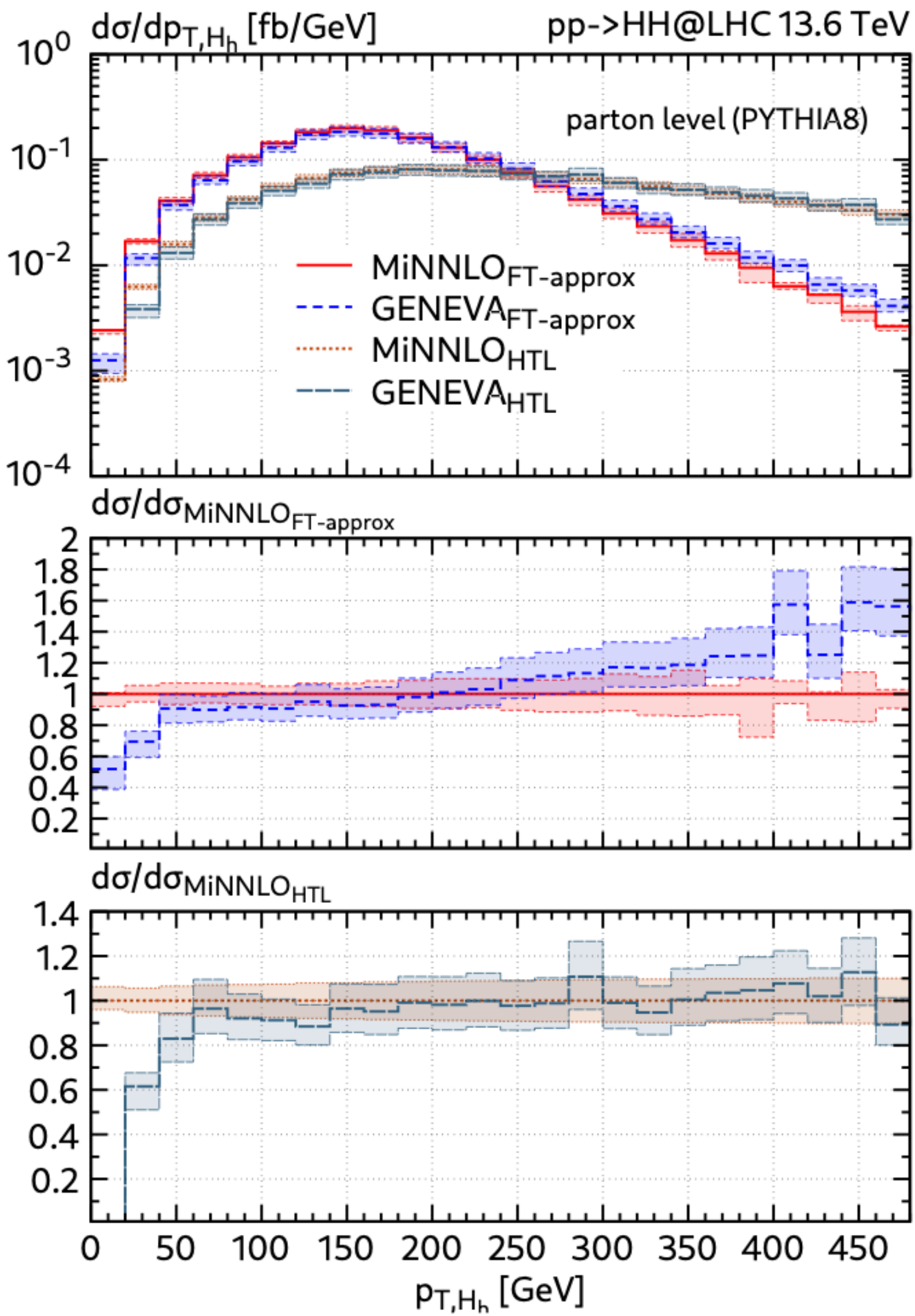
ggxy: public C++ library for NLO ggF amplitudes

- [JD, Schönwald, Steinhauser, Stremmer '25]
- user input: variation of masses, λ , m_t scheme/scale
- implements real radiation (Recola)
- fast: NLO $gg \rightarrow HH$ XS and dist. in 30 mins (1 CPU)
- also $ggZH$ (future: $ggZZ$, $gg\gamma\gamma$, EW corrections)

See K. Schönwald [Tue]

σ (fb)	HTL	Born-I	FT-approx
MINNLO _{PS}	$31.32^{+9.2\%}_{-9.4\%}$	$37.92^{+9.3\%}_{-9.2\%}$	$32.72^{+7.7\%}_{-8.5\%}$
GENEVA	$30.69^{+7.1\%}_{-8.5\%}$	$37.65^{+7.3\%}_{-8.4\%}$	$32.34^{+6.8\%}_{-5.3\%}$

[Alioli, Marinelli, Napoletano 2507.08558]
[Garosi, Wieseemann, Zanderighi 2606.05300]



CONCLUSIONS

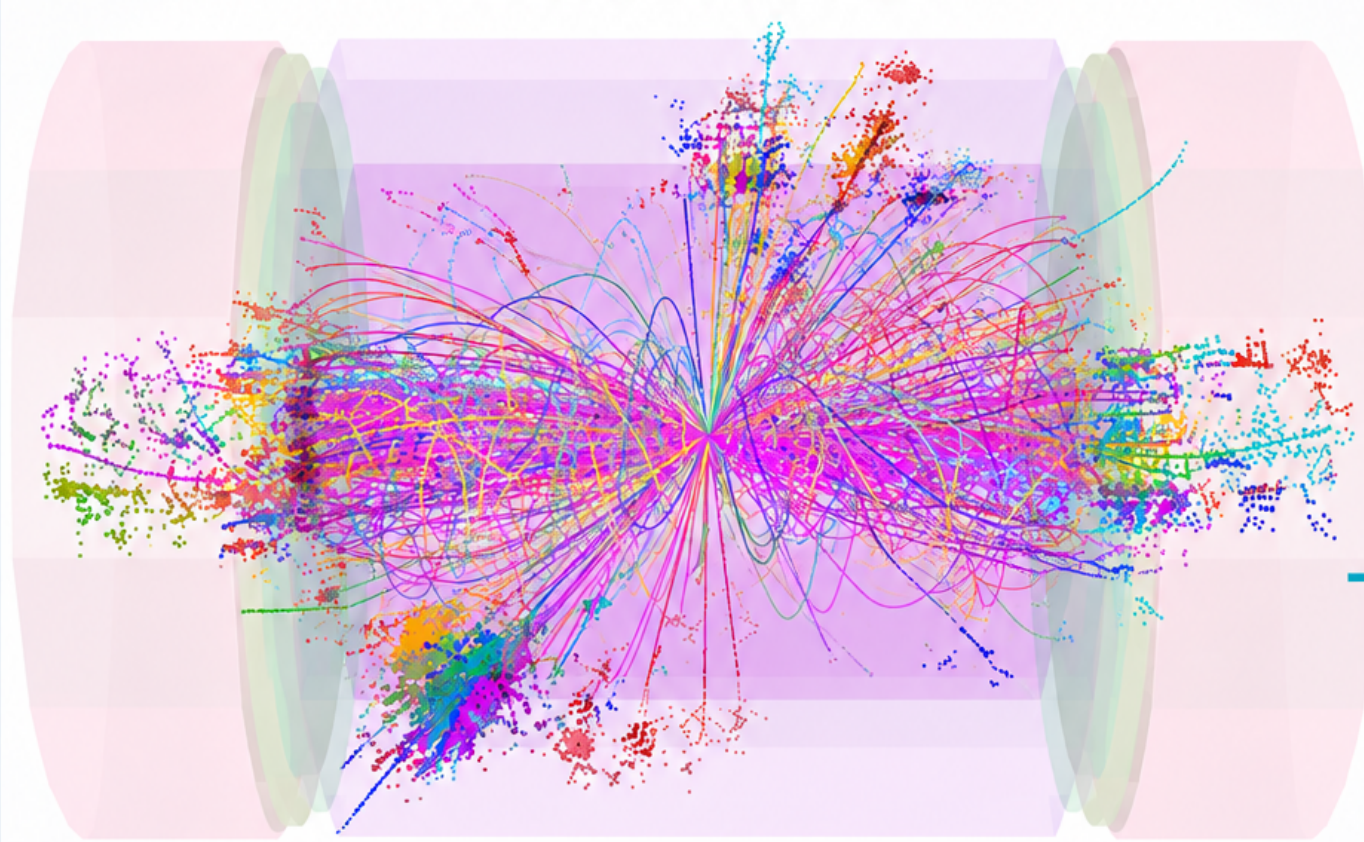
THE STANDARD MODEL LAGRANGIAN

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i \bar{\psi} \not{D} \psi \\ & + Y_i \bar{\psi}_i Y_{ij} \psi_j \phi + \text{h.c.} \\ & + |D_\mu \phi|^2 - V(\phi)\end{aligned}$$

● Gauge fields
 ● Fermions
 ● Yukawa interactions
 ● Higgs



A COLLISION AT THE LHC



The Standard Model

- ❖ Standard Model of particle physics gives us the “code of the Universe” through a **tiny formula**.
- ❖ It explains outcomes of most current terrestrial experiments and many aspects of the evolution of the Universe (add a grain of gravity!)
- ❖ However, **many questions remain unanswered**: origin of Higgs, EWSB, hierarchy puzzle, neutrino masses, matter/antimatter asymmetry...
- ❖ **Test the Standard Model as thoroughly as possible** stands as one of the main goal of the particle physics community

*“We are not only searching for new physics;
we are searching for something new in physics”*

THANK YOU FOR LISTENING

