



A first glimpse of matching beyond NLO with logarithmically accurate showers

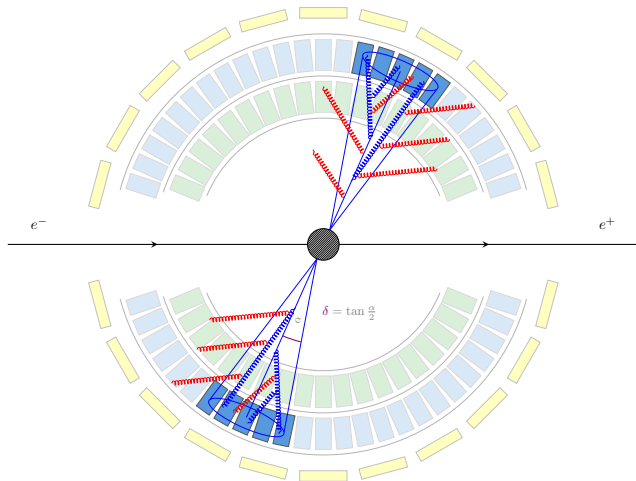
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with Melissa van Beekveld, Keith Hamilton, Gavin Salam, Gregory Soyez and Silvia Zanolì

Rudolf Peierls Centre for Theoretical Physics & Kellogg College

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Why start with e^+e^- ?



A clean first step towards LHC

applications: no PDFs or initial-state radiation.

PanScales has achieved **NNLL accuracy** for global event shapes.

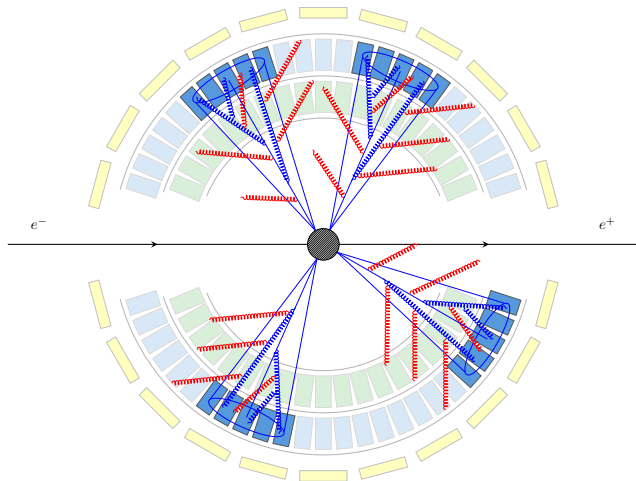
[2406.02661; see Keith's talk!]

This required both **NLO dijet matching** and **correlated double-soft radiation**.

[PanScales, 2301.09645; 2307.11142]

**Next: higher-order matching
for hard multi-jet radiation**

This talk: (double-)real contributions



From the **double-soft limit** to the **full double-real matrix element**

$$e^+e^- \rightarrow \gamma^* \rightarrow 4\text{jets}$$

The full set of double-real $\mathcal{O}(\alpha_s^2)$ contributions at **leading colour**.

Key requirement: preserve the shower's double-soft limit and logarithmic accuracy.

Multiplicative matching

⇒ positive event weights

How the shower builds an event

Starting from $q\bar{q}$, the shower proposes successive branchings.



$q\bar{q}$

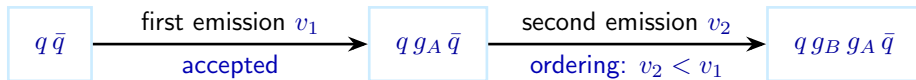
How the shower builds an event

Starting from $q\bar{q}$, the shower proposes successive branchings.



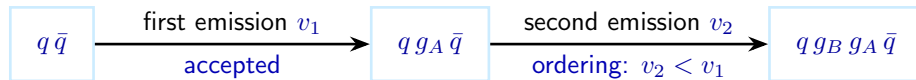
How the shower builds an event

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Starting from $q\bar{q}$, the shower proposes successive branchings.



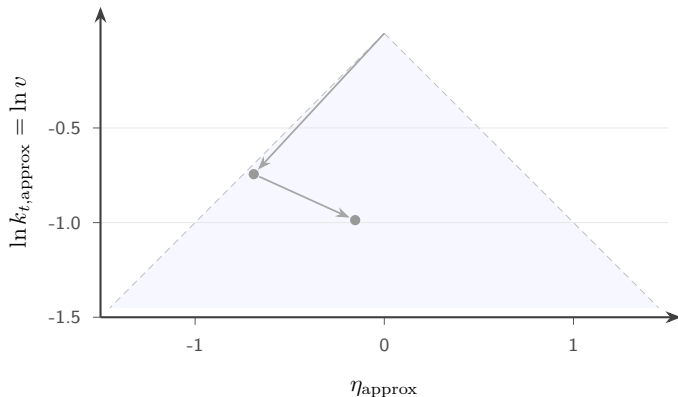
Accept the second emission with the **multiplicative correction**

$$P_{\text{acc}}^{(2)}(\Phi_4) = \frac{w^{\text{exact}}(\Phi_4)}{\Omega \sum_{h \in \mathcal{H}(\Phi_4)} w_h^{\text{shower}}(\Phi_4)} .$$

A **history** h is one route to the final momenta Φ_4 ; the denominator includes **all** such routes.

PanGlobal: four ordered histories, one final configuration

One point, four ordered histories



The same four momenta Φ_4

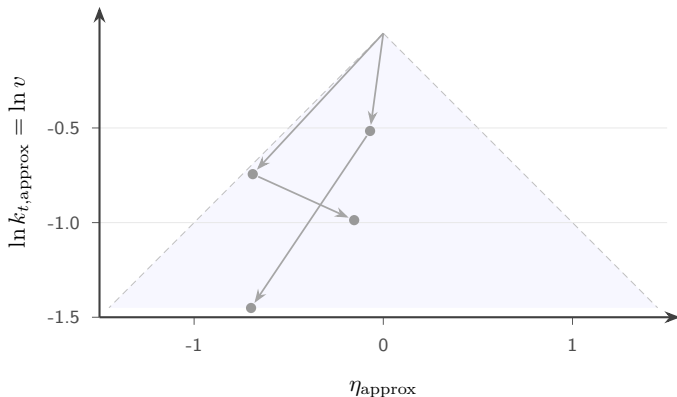
	p_x	p_y	p_z	E
q	0.35	-0.12	-0.10	0.38
g_A	-0.18	-0.15	0.15	0.28
g_B	-0.16	0.25	-0.02	0.30
$\bar{q}$	-0.01	0.02	-0.03	0.04

h_1

Each added route leads to **these same four-vectors**, through a different intermediate state.

PanGlobal: four ordered histories, one final configuration

One point, four ordered histories



The same four momenta Φ_4

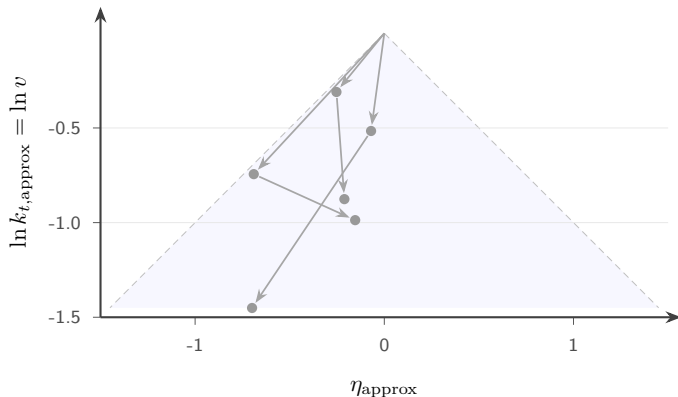
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$\bar{q}$	-0.01	0.02	-0.03	0.04

h_1 h_2

Each added route leads to **these same four-vectors**, through a different intermediate state.

PanGlobal: four ordered histories, one final configuration

One point, four ordered histories



The same four momenta Φ_4

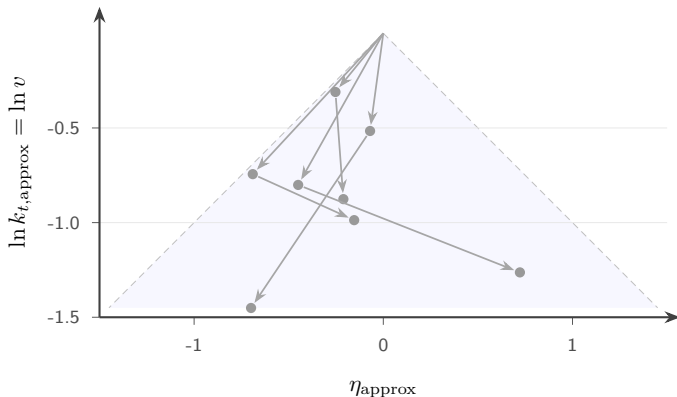
	p_x	p_y	p_z	E
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h_1 h_2 h_3

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PanGlobal: four ordered histories, one final configuration

One point, four ordered histories



The same four momenta Φ_4

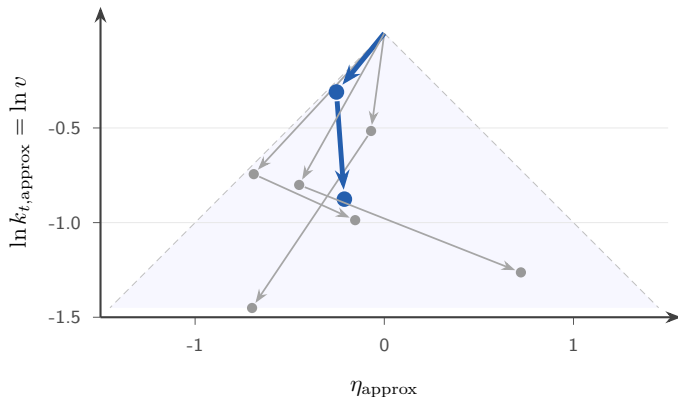
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Each added route leads to **these same four-vectors**, through a different intermediate state.

PanGlobal: four ordered histories, one final configuration

One point, four ordered histories



The same four momenta Φ_4

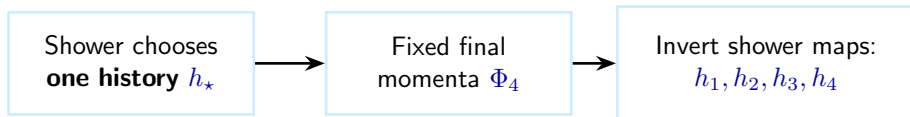
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h_1 h_2 h_3 h_4

One route is chosen; the other three reach the same Φ_4 .

Partitioning the matrix element over histories

The shower realises **one route**; matching needs **all routes** to the same Φ_4 .



The chosen history receives its share of the exact weight:

$$w_{h_\star}^{\text{matched}}(\Phi_4) = w_{h_\star}^{\text{shower}}(\Phi_4) \frac{w^{\text{exact}}(\Phi_4)}{\sum_{h \in \mathcal{H}(\Phi_4)} w_h^{\text{shower}}(\Phi_4)}.$$

Can an ordered shower reach every Φ_4 ?

Multiplicative matching can only reweight points the shower can propose.

A RAMBO point

	p_x	p_y	p_z	E
q	0.06	-0.09	-0.05	0.12
g_A	-0.12	-0.34	0.18	0.40
g_B	0.07	0.39	-0.16	0.43
$\bar{q}$	-0.01	0.04	0.03	0.05

Can an ordered shower reach every Φ_4 ?

Multiplicative matching can only reweight points the shower can propose.

A RAMBO point

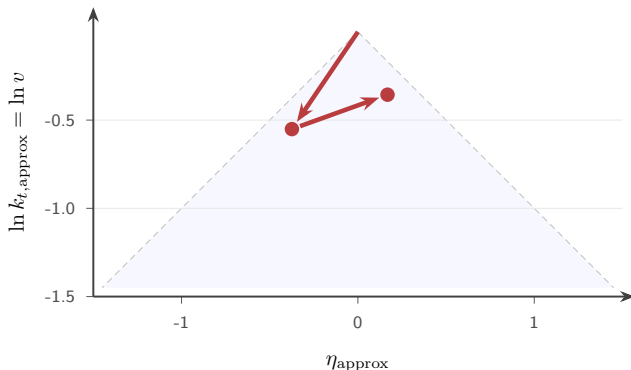
	p_x	p_y	p_z	E
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No ordered history reaches this point.

A strictly ordered shower cannot propose it, so reweighting cannot fill the hole.

One disordered history reaches the same Φ_4

One disordered history



The same RAMBO point

	p_x	p_y	p_z	E
q	0.06	-0.09	-0.05	0.12
g_A	-0.12	-0.34	0.18	0.40
g_B	0.07	0.39	-0.16	0.43
$\bar{q}$	-0.01	0.04	0.03	0.05

[Higher logarithmic accuracy
with disordered parton showers;
PanScales -- to appear]

This point is now reachable. Challenge: avoid double counting.

Multiplicative matching: three challenges

Correct the shower's **acceptance probability** at the same Φ_4 :

$$P_{\text{acc}}^{(2)}(\Phi_4) = \frac{w^{\text{exact}}(\Phi_4)}{\Omega \sum_{h \in \mathcal{H}(\Phi_4)} w_h^{\text{shower}}(\Phi_4)} .$$

1. Coverage

Each Φ_4 needs a route;
disordering fills gaps.

2. Shower weight

Sum contributing
routes; avoid double
counting.

3. Acceptance bound

Choose Ω so that
 $P_{\text{acc}}^{(2)} \leq 1$ everywhere.

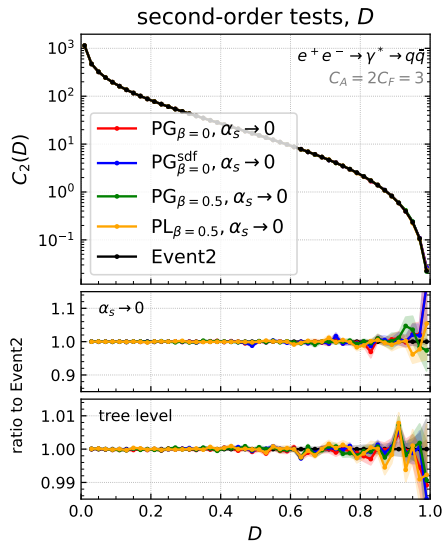
From matching procedure to validation

We can now generate and reweight a four-parton point.

Does this preserve the accuracy we need?

- ① **Fixed order:** reproduce the $\mathcal{O}(\alpha_s^2)$ result against Event2.
- ② **Infrared limits:** agree with the shower's double-soft route point by point.
- ③ **All orders:** leave the NNLL residual unchanged.

Fixed-order comparison against Event2



Shower mode

upper two panels

- Stop after **two emissions**
- Repeat at decreasing α_s and **extrapolate** $\alpha_s \rightarrow 0$

$$\frac{1}{\sigma_0} \frac{d\sigma}{d\mathcal{O}} = \left(\frac{\alpha_s}{2\pi}\right)^2 C_{\text{shower}}^{(2)} + \mathcal{O}(\alpha_s^3).$$

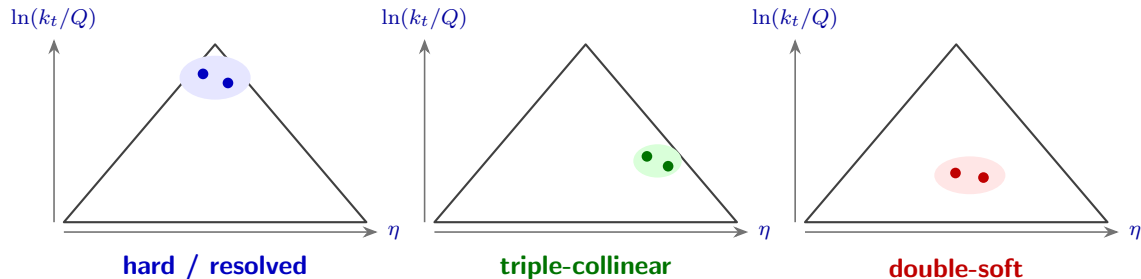
Compare with Event2 (black line).

Tree-level mode (bottom panel)

Weighted events;
no Sudakov, no extrapolation.

Consistency with unresolved limits

The four-parton matrix element spans hard and double-unresolved regions:

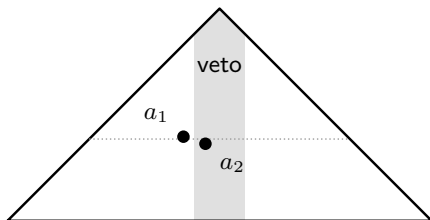


Matching must preserve the shower's infrared behaviour.

Otherwise, logarithmic accuracy can be lost.

Spurious SLLs from an infrared mismatch

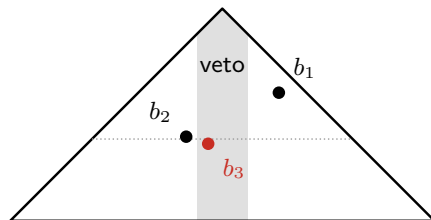
Double-soft pair from matching



No earlier emission

Double-soft pair completed by shower

$\ln k_t$
 η



Earlier soft-collinear emission

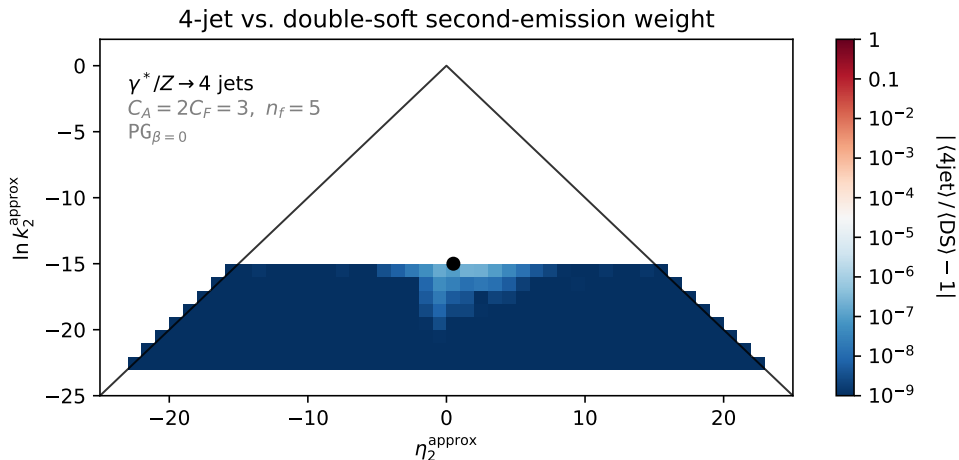
● matching

● shower

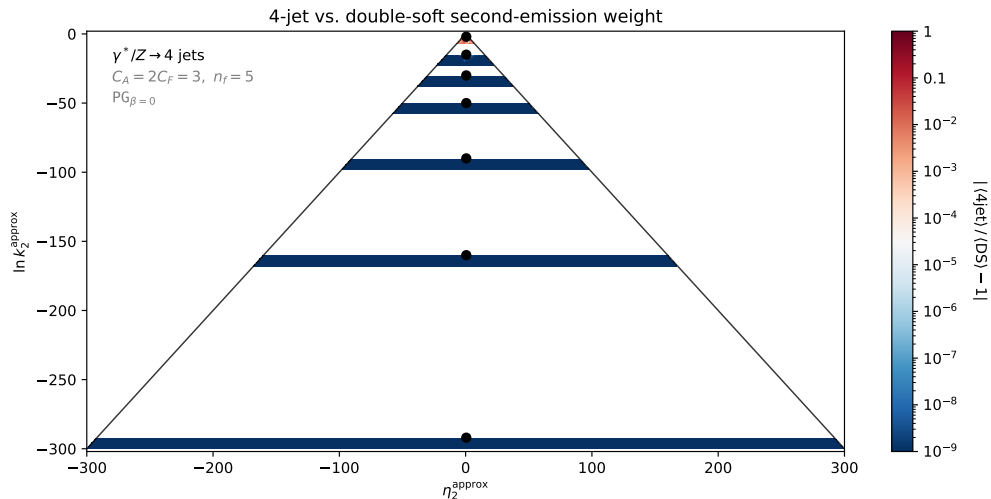
The two routes must agree differentially in the infrared.

Otherwise, spurious SLLs arise — even at leading colour.

Numerical check: matching approaches the double-soft limit

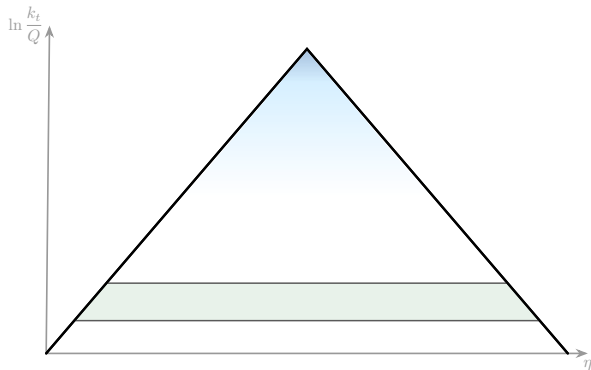


Acceptance probabilities agree to the edge of double precision



NNLL accuracy tests: overview

[2312.13275, PanScales]



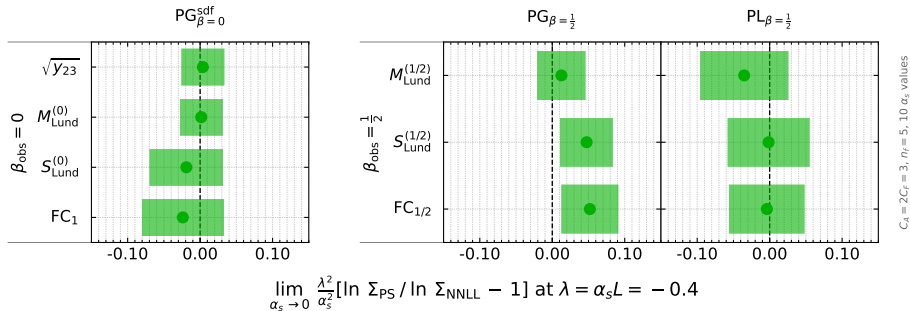
Extract the **logarithmic correction** by taking

$$\alpha_s \rightarrow 0 \quad \text{at fixed} \quad \lambda = \alpha_s L.$$

Compare the shower with the **NNLL prediction** in this limit.

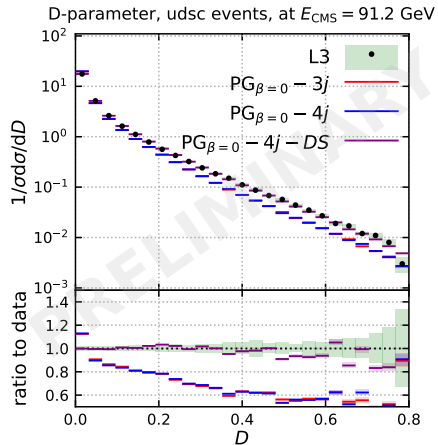
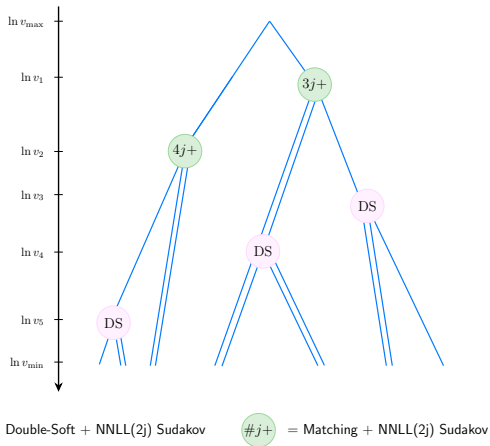
Matching must leave the NNLL residual unchanged.

NNLL accuracy tests

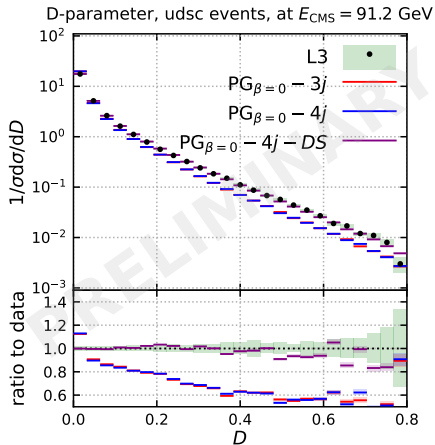


PRELIMINARY PHENO 4-JET SHAPES

Preliminary Pheno in PG [PanScales;2024]



**$4j$ matching must not spoil
the shower's logarithmic accuracy.**



Conclusions & Outlook

A significant step towards **higher-order matching** in **logarithmically accurate showers**.

- **Matched double-real contributions** at leading colour while retaining logarithmic accuracy

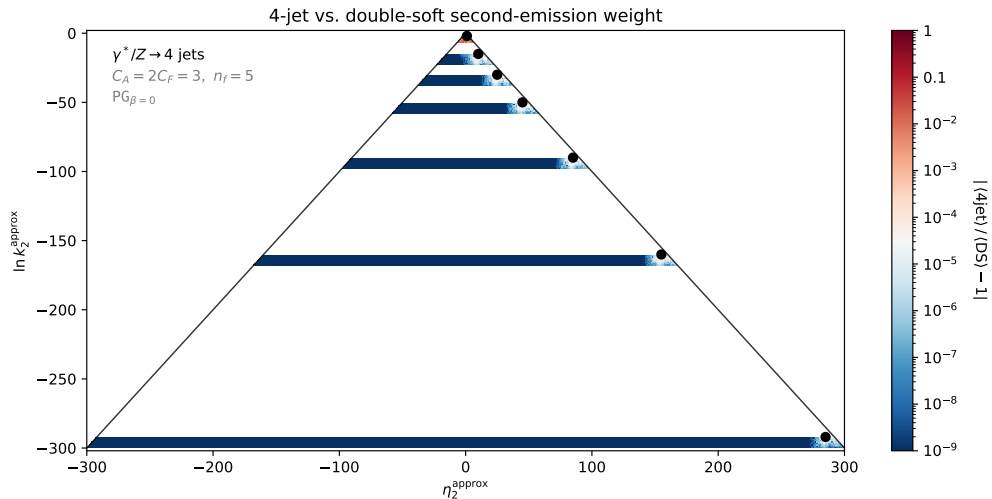
❗ Matching must align with subsequent showering in unresolved limits

- **Next steps**

- ① Complete NLO corrections for $e^+e^- \rightarrow \gamma^* \rightarrow 3\text{jets}$
- ② Fully differential triple-collinear contributions for quark legs

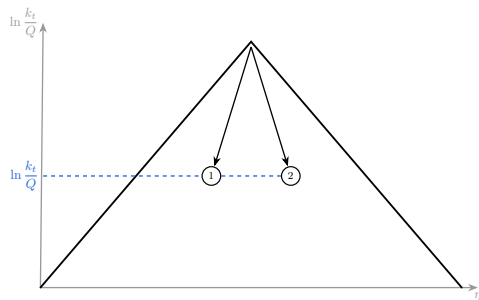
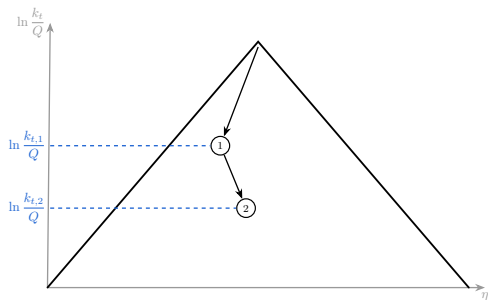
BACKUP

The triple-collinear (3C) route



Ordered vs. Unordered Emissions in Vincia

[Vincia 1611.00013,2507.00111]



Instead of using disordered emissions, Vincia includes direct $2 \rightarrow 4$ branchings to cover phase space [Vincia 1611.00013,2507.00111]

Swaps

