



Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Tropical Monte Carlo Integration

FELIX TELLANDER

*School of Mathematics
Trinity College Dublin*

Oxford, October 2026

MA $\int$ CAMP



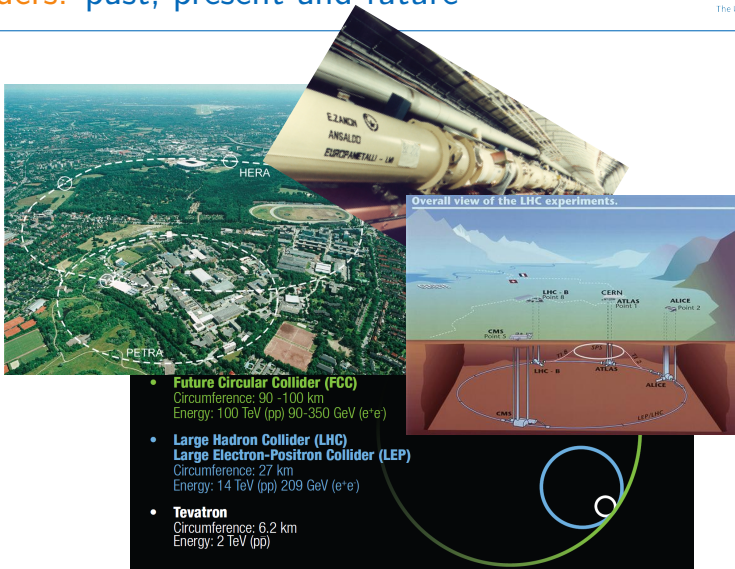
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the European Union

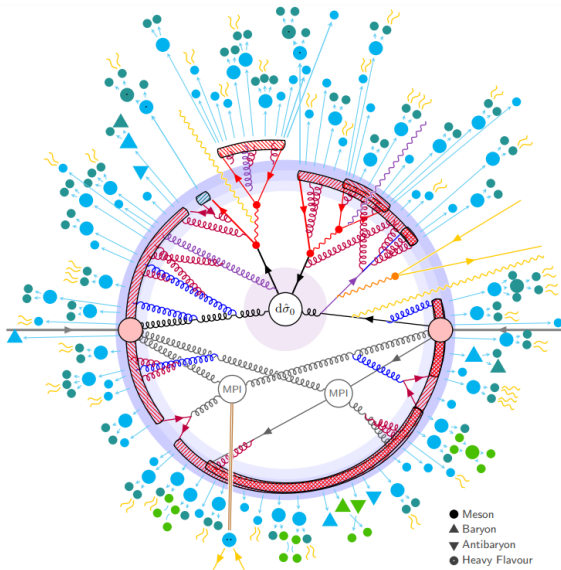


European Research Council
Established by the European Commission

Colliders: past, present and future

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- Hard Interaction
- Resonance Decays
- MECs, Matching & Merging
- FSR
- ISR*
- QED
- Weak Showers
- Hard Onium
- Multiparton Interactions
- Beam Remnants*
- Strings
- Ministrings / Clusters
- Colour Reconnections
- String Interactions
- Bose-Einstein & Fermi-Dirac
- Primary Hadrons
- Secondary Hadrons
- Hadronic Reinteractions
- (*: incoming lines are crossed)

Collinear factorization:

$$\sigma_{h_1 h_2 \rightarrow n} = \sum_{a,b} \int_0^1 dx_a dx_b f_{a/h_1}(x_a, \mu_F) f_{b/h_2}(x_b, \mu_F) \hat{\sigma}_{h_1 h_2 \rightarrow n}(\mu_F, \mu_R) + \mathcal{O}(\Lambda_{\text{QCD}}/Q)$$

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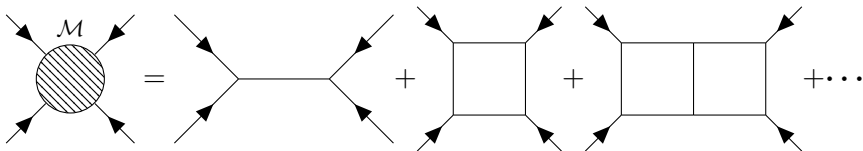
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$\mathcal{M}$ is the **matrix element** and is calculated as a sum of **Feynman diagrams**:



$$\mathcal{I} = \int \frac{d^D k_1 \cdots d^D k_L}{\prod_{e \in E} D_e}, \quad D_e = q_e^2 - m_e^2 + i\varepsilon$$

Direct integration

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Why not

$$\text{NIntegrate} \left[\int \frac{d^D k_1 \cdots d^D k_L}{\prod_{e \in E} D_e} \right] ?$$

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We assume $\mathcal{I}$ is (quasi-)finite.

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- ▶ $\mathbb{P}_+^{|E|-1}$: the positive projective simplex.
 - ▶ Ω : projective volume form.
 - ▶ ω : superficial degree of divergence.
 - ▶ $\mathcal{U}, \mathcal{F}$: the two Symanzik polynomials.
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Compact integration domain and D is a parameter in the integrand.

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~ Details and Options

- Multiple integrals use a variant of the standard iterator notation. The first variable given corresponds to the outermost integral and is done last.
- `NIntegrate` by default tests for singularities at the boundaries of the integration region and at the boundaries of regions specified by settings for the `Exclusions` option.
- `NIntegrate` [$f, \{x, x_1, x_2, \dots, x_d\}$] tests for singularities in a one-dimensional integral at each of the intermediate points x_i . If there are no singularities, the result is equivalent to an integral from x_1 to x_2 . You can use complex numbers x_i to specify an integration contour in the complex plane.
- The following options can be given:

AccuracyGoal	Infinity	digits of absolute accuracy sought
EvaluationMonitor	None	expression to evaluate whenever <code>Expr</code> is evaluated
Exclusions	None	parts of the integration region to exclude
MaxPoints	Automatic	maximum total number of sample points
MaxRecursion	Automatic	maximum number of recursive subdivisions
Method	Automatic	method to use
MinRecursion	0	minimum number of recursive subdivisions
PrecisionGoal	Automatic	digits of precision sought
WorkingPrecision	MachinePrecision	the precision used in internal computations

`NIntegrate` [$f, \{x, x_{\text{min}}, x_{\text{max}}\}$]
gives a numerical approximation to the integral, $\int_{x_{\text{min}}}^{x_{\text{max}}} f \, dx$.

`NIntegrate` [$f, \{x, x_{\text{min}}, x_{\text{max}}\}, \{y, y_{\text{min}}, y_{\text{max}}\}, \dots\}$
gives a numerical approximation to the multiple integral, $\int_{x_{\text{min}}}^{x_{\text{max}}} dx \int_{y_{\text{min}}}^{y_{\text{max}}} dy \dots f$.

`NIntegrate` [$f, \{x, x_1, \dots, x_{\text{reg}}\}$]
integrates over the geometric region `reg`.

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No $i\epsilon$ option!

Embedding $\iota_\lambda : \mathbb{P}_+^{|E|-1} \hookrightarrow \mathbb{CP}^{|E|-1}$:

$$\iota_\lambda : x_e \mapsto x_e \exp \left(-i\lambda \frac{\partial \mathcal{V}}{\partial x_e}(\mathbf{x}) \right)$$

where $\mathcal{V} = \mathcal{F}/\mathcal{U}$ and $\lambda > 0$.

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Computer Programs in Physics Computer Physics Communications 292 (2023) 108874

Tropical Feynman integration in the Minkowski regime ^{☆, ☆☆}

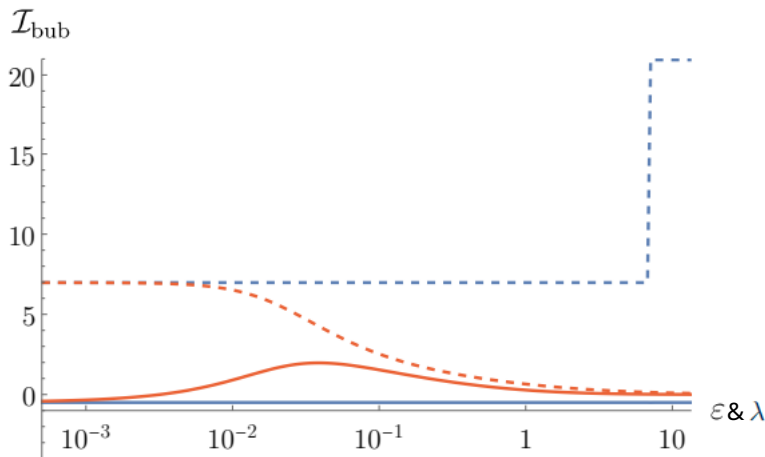
Michael Borinsky ^{a,*}, Henrik J. Munch ^b, Felix Tellander ^c

*i*ε **free parametric representation:**

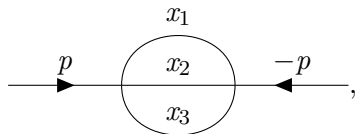
$$\int_{\mathbb{P}_+^{|E|-1}} \frac{J_\lambda}{\tilde{\mathcal{U}}^{D/2} \tilde{\mathcal{V}}^\omega} \Omega$$

where $\tilde{\mathcal{U}}$ and $\tilde{\mathcal{V}}$ are the deformed version of $\mathcal{U}$ and $\mathcal{V}$.

Comparison with direct numerics on the Feynman parameterization with $i\varepsilon$ and with deformation:

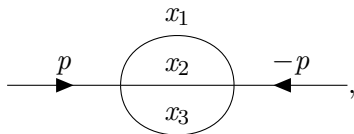


Problem 3: The integrand has (integrable) poles on the boundary.



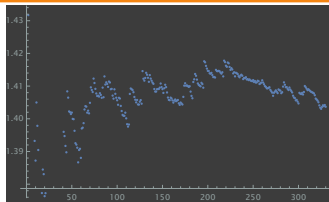
$$\mathcal{U} = x_1 x_2 + x_1 x_3 + x_2 x_3 \rightarrow 0 \text{ if } x_1, x_2 \rightarrow 0$$

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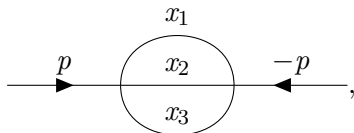


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$$\int_0^1 \int_0^1 \frac{1}{x+y} dx dy = 2 \log 2 \approx 1.3863$$

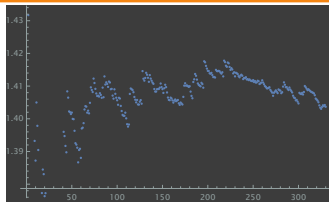


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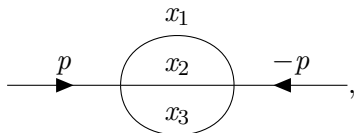
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```
In[] NIntegrate[1/(x+y), {x,0,1}, {y,0,1},
Method->{"MonteCarlo","RandomSeed"->19950309}]
```

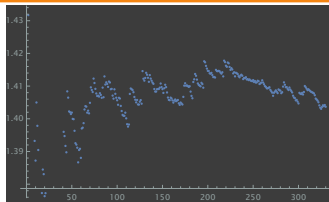
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The integral is **convergent** but the variance is **divergent**!

The **tropical approximation** of a polynomial

$$p(\mathbf{x}) = \sum_{\alpha \in \text{supp}(p)} c_{\alpha} \mathbf{x}^{\alpha}:$$

$$p^{\text{tr}}(\mathbf{x}) = \max_{\alpha \in \text{supp}(p)} \{\mathbf{x}^{\alpha}\}.$$

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Example

$$p(\mathbf{x}) = x_1 + x_2 + (5 + 2i)x_1x_2 - 3x_1^2 + \pi x_2^2$$

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Theorem (Borinsky)

For a homogeneous polynomial $p \in \mathbb{C}[x_1, \dots, x_n]$ that is **completely non-vanishing** in $\mathbb{P}_+^{n-1}$ there exists constants $C_1, C_2 > 0$ s.t.

$$C_1 \leq \frac{|p(\mathbf{x})|}{p^{\text{tr}}(\mathbf{x})} \leq C_2 \quad \text{for all } \mathbf{x} \in \mathbb{P}_+^{n-1}$$

$$\begin{aligned}
\mathcal{I} &= \int_{\mathbb{P}_+^{|E|-1}} \frac{J_\lambda}{\tilde{\mathcal{U}}(x)^{D/2} \tilde{\mathcal{V}}(x)^\omega} \Omega \\
&= \int_{\mathbb{P}_+^{|E|-1}} \frac{\Omega}{\tilde{\mathcal{U}}^{\text{tr}}(x)^{D/2} \tilde{\mathcal{V}}^{\text{tr}}(x)^\omega} \cdot J_\lambda \frac{\tilde{\mathcal{U}}^{\text{tr}}(x)^{D/2} \tilde{\mathcal{V}}^{\text{tr}}(x)^\omega}{\tilde{\mathcal{U}}(x)^{D/2} \tilde{\mathcal{V}}(x)^\omega} \\
&= I^{\text{tr}} \int_{\mathbb{P}_+^{|E|-1}} \mu^{\text{tr}} \cdot J_\lambda \frac{\tilde{\mathcal{U}}^{\text{tr}}(x)^{D/2} \tilde{\mathcal{V}}^{\text{tr}}(x)^\omega}{\tilde{\mathcal{U}}(x)^{D/2} \tilde{\mathcal{V}}(x)^\omega}
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where

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Bounded integrand and easy to sample measure.

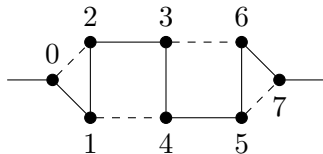
The program `feyntrop`

- ▶ Tabletop evaluation of *quasi-finite* Feynman integral with many scales.
 - ▶ Available at <https://github.com/michibo/feyntrop>, a C++ program with Python and JSON interface.
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Example: Four loop QED propagator (remember F.Forner's talk?)

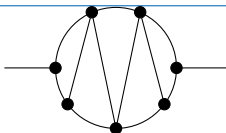


Setting $\lambda = 1.3$, $N = 10^7$, $m^2 = 1$, $p^2 = 2$, $D_0 = 4$,
we get in **31.6 seconds**:

Prefactor: $\gamma(4\epsilon + 3)$.
(Effective) kinematic regime: Minkowski (generic).
Generalized permutahedron property seems fulfilled.
Analytic continuation: activated. $\Lambda = 1.3$.
Started integrating using 4 threads and $N = 1e+07$ points.
Finished in 31.5622 seconds = 0.00876729 hours.

```
-- eps^0: [0.5129 +/- 0.0016] + i * [0.3577 +/- 0.0016]
-- eps^1: [-1.262 +/- 0.011 ] + i * [-0.529 +/- 0.011 ]
-- eps^2: [4.840 +/- 0.043 ] + i * [3.911 +/- 0.044 ]
-- eps^3: [-19.50 +/- 0.12 ] + i * [-1.44 +/- 0.13 ]
-- eps^4: [32.72 +/- 0.33 ] + i * [-24.09 +/- 0.28 ]
```

Euclidean region: where feynthrop flies



12 scales, $m_i^2 = i$, $D_0 = 3$, $\lambda = 0.02$, $N = 10^7$

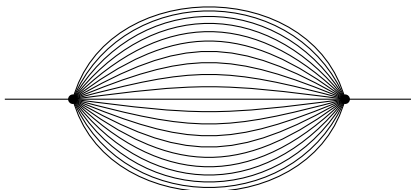
$p^2 = 100$ **Minkowski, 33 sec.**

$p^2 = -100$ **Euclidean, 9 s**

eps^0 :	$1.958(48) \times 10^{-4}$	+	$1.476(49) \times 10^{-4}$	i
eps^1 :	$-4.967(66) \times 10^{-3}$	-	$8.84(72) \times 10^{-4}$	i
eps^2 :	$4.968(49) \times 10^{-2}$	-	$1.504(55) \times 10^{-2}$	i
eps^3 :	$-0.2568(27)$	+	$0.2464(29)$	i
eps^4 :	$0.599(12)$	-	$1.726(12)$	i
eps^5 :	$1.034(40)$	+	$7.426(43)$	i
eps^6 :	$-14.69(12)$	-	$20.98(14)$	i
eps^7 :	$66.14(37)$	+	$35.15(36)$	i
eps^8 :	$-191.5(1.1)$	-	$3.89(93)$	i
eps^9 :	$393.4(2.6)$	-	$186.5(2.8)$	i
eps^{10} :	$-554.8(5.0)$	+	$696.3(8.9)$	i

eps^0 :	$9.03(10) \times 10^{-6}$
eps^1 :	$-1.941(20) \times 10^{-4}$
eps^2 :	$2.102(20) \times 10^{-3}$
eps^3 :	$-1.529(14) \times 10^{-2}$
eps^4 :	$8.416(72) \times 10^{-2}$
eps^5 :	$-0.3739(31)$
eps^6 :	$1.397(11)$
eps^7 :	$-4.522(36)$
eps^8 :	$12.95(11)$
eps^9 :	$-33.34(28)$
eps^{10} :	$78.29(70)$

20-loop banana:

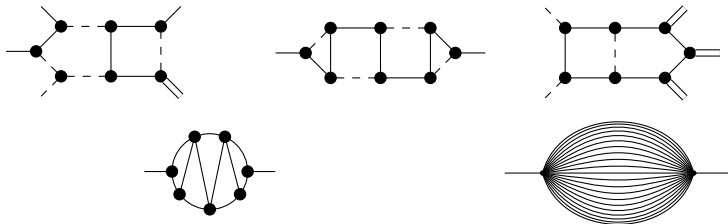


$$m^2 = 1, \quad p^2 = -1, \quad D_0 = 2, \quad \lambda = 0.1, \quad N = 10^8$$

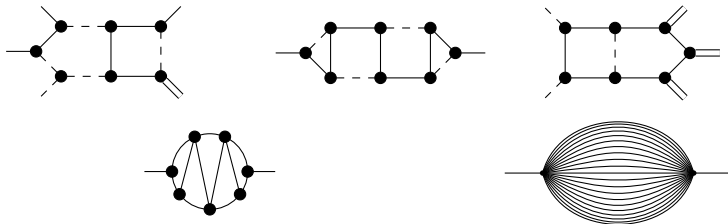
Time: 2 min. and 40 sec.

```
-- eps^0:  
(1.610477 +/- 0.000078) × 10^19  
-- eps^1:  
(-3.36580 +/- 0.00021) × 10^21  
-- eps^2:  
(3.69787 +/- 0.00031) × 10^23
```

- ▶ Using tropical Monte Carlo integration, Feynman integrals with **many scales** and **many legs** can be evaluated on a laptop:



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No free lunch:

- ▶ Might be hard to find quasi-finite basis
- ▶ Only Monte Carlo accuracy
- ▶ Works best for generalized permutohedra