

Recent analytic progress for di-Higgs production

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Outline

- **Introduction**
- **Our Methods**
- **New Results**
- **Conclusions and Outlook**

Introduction



Why HH production?

- Standard Model Higgs potential:

$$V(H) = \frac{1}{2}m_h^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4, \text{ where } \lambda = m_h^2/(2v^2) \approx 0.13.$$

- Want to measure λ , to determine if $V(H)$ is consistent with nature.

► Challenging! Cross-section $\approx 10^{-3} \times H$ prod.

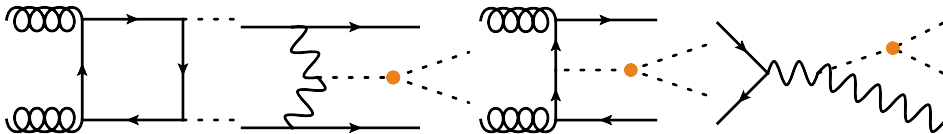
$$-0.71 < \lambda_{obs}/\lambda < 6.1$$

[ATLAS+CMS Run2 '26]

$$-1.7 < \lambda_{obs}/\lambda < 6.6$$

[ATLAS Run3 $b\bar{b}\gamma\gamma$ '25]

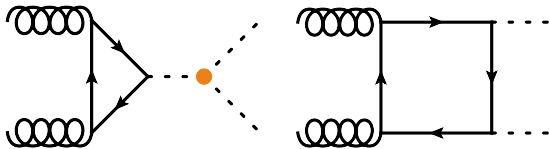
- λ appears in various production channels:



- Gluon fusion – dominant, 10x
- VBF

- $t\bar{t}$ associated production
- H -strahlung

Gluon Fusion



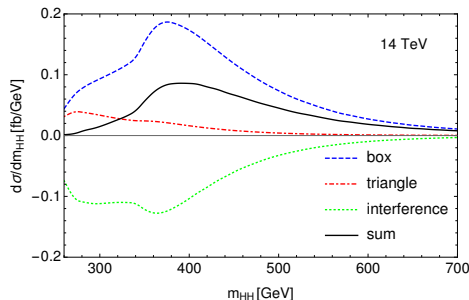
“Triangle contributions”

- depend on λ
- easier to compute: fn. of s, m_t

“Box contributions”

- harder to compute: fn. of s, t, m_t, m_H
- exact LO result: [Glover, van der Bij '88]
 - ▶ still no fully exact, analytic, NLO!

Destructive interference between triangle- and box-type contributions at LO: **small cross-section** and large **higher-order corrections**.



[LHCHSWG-2019-005]

NLO: need also $1L \text{ } gg \rightarrow HHg$

NNLO: $1L \text{ } gg \rightarrow HHgg$ and $2L \text{ } gg \rightarrow HHg$

Current WG recommendations

[LHCHWG-2026-010]

“NNLO FTapprox”:

[Grazzini et al '18]

- numeric NLO, reweighted HTL NNLO, exact RR

$\sqrt{s}$ [TeV]	13	13.6	14
σ [fb]	30.75	34.01	36.27
$\pm \text{PDF} + \alpha_s$ unc. [%]	2.47	2.43	2.39
QCD scale unc. [%]	+2.11 -4.98	+2.07 -4.85	+2.01 -4.78
$\pm \text{TH}_{\text{num}}$ [%]	0.14	0.19	0.21

Then, re-weight with K factors:

$$\sigma_{\text{rec}} = \sigma_{\text{NNLO}_{\text{FTapprox}}} \times K_3 \times K_{EW},$$

where

$$K_3 = \frac{\text{N}^3\text{LO} + \text{N}^3\text{LL HTL}}{\text{NNLO HTL}}, \quad K_{EW} = \frac{\sigma_{\text{NLO EW}}}{\sigma_{\text{LO}}}.$$

Final uncertainties:

- scale: from NNLO FTapprox
- PDF + α_s : from NNLO FTapprox
- top quark mass scheme, from NLO
 - bin-by-bin envelope of OS and $\overline{\text{MS}}$ with $\mu_t = m_t, \mu_t = m_{hh}, \mu_t = m_{hh}/4$, integrate

$\sqrt{s}$ [TeV]	13	13.6	14
$\sigma_{\text{rec.}}$ [fb]	30.40	33.62	35.81
$\pm \text{PDF} + \alpha_s$ unc. [%]	2.47	2.43	2.39
QCD scale unc. [%]	+2.11 -4.98	+2.07 -4.85	+2.01 -4.78
$\pm \text{TH}_{\text{num}}$ [%]	0.14	0.19	0.21
m_t scheme unc. [%]	+5 -16	+5 -16	+5 -16
Combined [%]	+7.25 -21.12	+7.26 -21.04	+7.22 -20.99

$gg \rightarrow HH$ Beyond LO

NLO QCD:

- large- m_t
- numeric
- large- m_t + threshold exp. Padé
- high-energy expansion
- small- p_T expansion
+ high-energy expansion

[Dawson, Dittmaier, Spira '98] [Grigo, Hoff, Melnikov, Steinhauser '13]

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16]
[Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]

[Gröber, Maier, Rauh '17]

[Davies, Mishima, Steinhauser, Wellmann '18,'19]

[Bonciani, Degrassi, Giardino, Gröber '18]
[Bagnaschi, Degrassi, Gröber '23]

NNLO QCD:

- large- m_t virtuals
- HTL+numeric real ("FTapprox")
matched to parton showers in the GENEVA framework
NEW: matched to parton showers in the MiNNLOPS framework [talk by Francesco Garosi]
- large- m_t reals
- $p_T = 0$ (n_l and leading color n_h contributions)

[de Florian, Mazzitelli '13] [Grigo, Hoff, Steinhauser '15] [Davies, Steinhauser '19]

[Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
[Aioli, Marinelli, Napoletano '25]

[Garosi, Wieseemann, Zanderighi '26]

[Davies, Herren, Mishima, Steinhauser '19 '21]

[Davies, KS, Steinhauser '23-'25]

N3LO QCD:

- Wilson coefficient C_{HH}
- HTL

[Spira '16] [Gerlach, Herren, Steinhauser '18]

[Chen, Li, Shao, Wang '19]

$gg \rightarrow HH$ Beyond LO

NLO EW:

- heavy top expansion
- high-energy expansion
- see also Yukawa corrections in (partial) HTL
- full (numerical) EW corrections
- numerical Yukawa- and Higgs self-coupling corrections
- light quark corrections
- light quark initiated

[Davies, Schönwald, Steinhauser, Zhang '23]

[Davies, Mishima, Schönwald, Steinhauser, Zhang '22,'25]

[Mühlleitner, Schlenk, Spira '22]

[Bi, Huang, Huang, Ma, Yu '23]

[Heinrich, Jones, Kerner, Stone, Vestner '24][Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, Spira '25]

[Bonetti, Rendler, Bobadilla '25]

[Bonetti, Heinrich, Bobadilla '26]

Our Methods



Expansions

Simplify the calculation by expanding in different kinematic regimes:

1. Large mass expansion $m_t^2 \gg s, |t|, m_H^2$

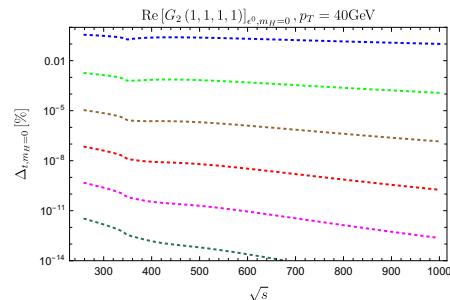
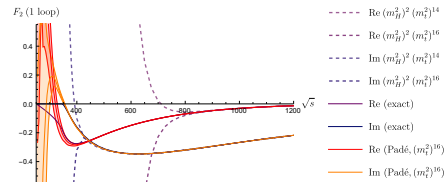
- ▶ Simplest expansion to perform.
- ▶ Very limited radius of convergence: $s < 2m_t^2$.

2. High Energy expansion $s, |t| \gg m_t^2 \gg m_H^2$

- ▶ Analytic results with simple analytic structure.
- ▶ Covers kinematic region which is challenging for numeric methods.
- ▶ Using Padé techniques can be used down to $p_T \sim 150 \text{ GeV}$.

3. Forward expansion $m_H^2, |t| \ll s, m_t^2$

- ▶ Expansion on amplitude level possible (simplifies calculation).
- ▶ Master integrals elliptic, but only one scale (efficient solutions through series expansions).
- ▶ Covers the remaining phase space.



Keep parametric dependence on all input parameters!

High energy expansion

Consider the hierarchy of scales: $s, |t| \gg m_t^2 \gg m_H^2$

- Form factors written in term of Feynman integrals:
- Taylor expand in $m_H^2 \rightarrow 0$:
- IBP reduction to master integrals:
- Expand master integrals in $m_t \rightarrow 0$:

$$I(m_H, m_t, s, t, \epsilon)$$

$$I(m_H, \dots) = I(0, \dots) + m_H^2 I'(0, \dots)$$

$$J(m_t, s, t, \epsilon)$$

$$J(m_t, s, t, \epsilon) = \sum_{i,j,k} c_{ijk}(s, t) \epsilon^i (m_t)^j \ln^k(m_t)$$

- Improve convergence with Padé approximation:

$$\sum_{k=0}^N c_k(m_t^2)^k = \frac{a_0 + a_1(m_t^2) + \dots + a_r(m_t^2)^r}{1 + b_1(m_t^2) + \dots + b_s(m_t^2)^s} = [r/s](m_t^2), \quad r + s = N$$

Forward expansion

Consider the hierarchy of scales: $s, m_t^2 \gg |t|, m_H^2$

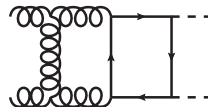
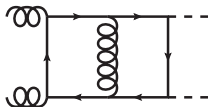
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$$J(m_t, s, t, \epsilon)$$

$$J(m_t, s, t, \epsilon) = \sum_{i,j} c_{ij}(s, m_t^2) \epsilon^i t^j$$

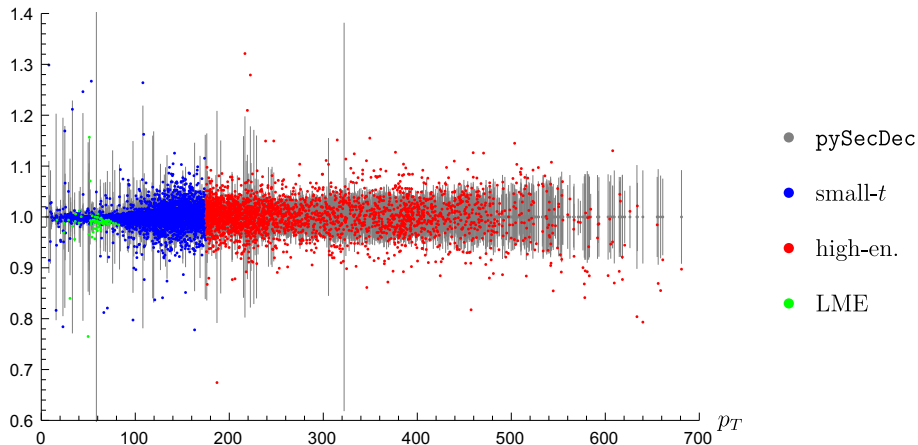


Validation – HH production

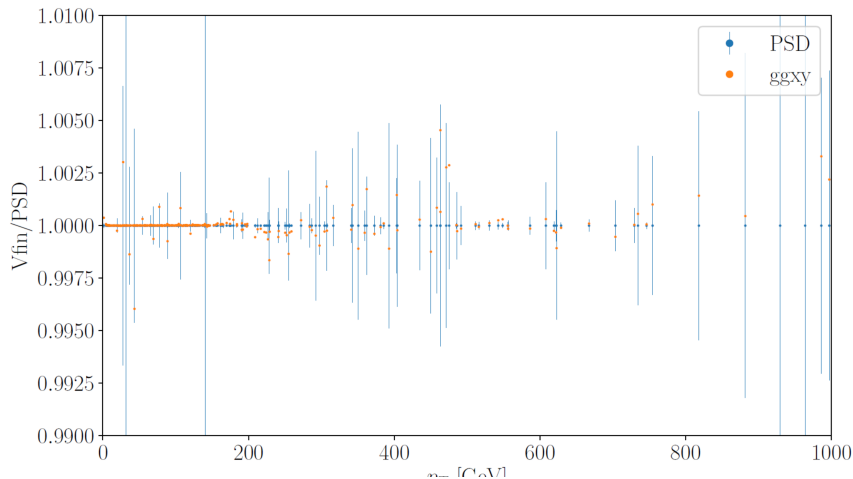
Comparison with hhgrid:

- interpolation grid of 6320 points evaluated numerically by pySecDec
- grid points normalized to hhgrid as function of p_T :

[<https://github.com/mppmu/hhgrid>]

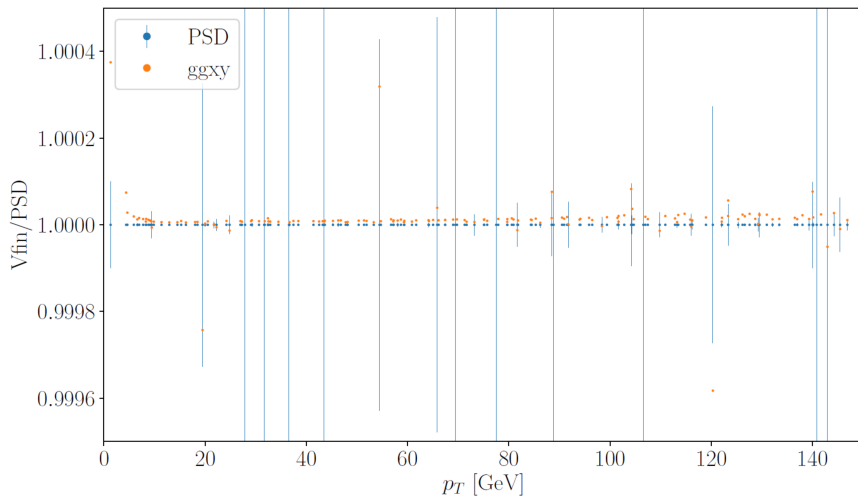


Validation – ZH production



PSD date from: [\[Chen, Heinrich, Jones, Kerner, Klappert, Schlenk '21\]](#)

Validation – ZH production

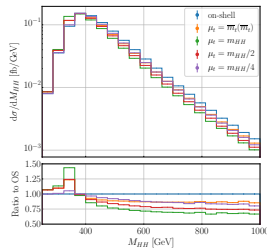
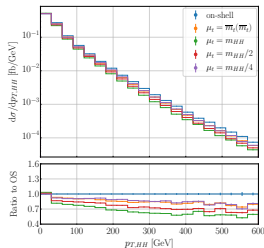


PSD date from: [\[Chen, Heinrich, Jones, Kerner, Klappert, Schlenk '21\]](#)

ggxy Implementation

ggxy: [<https://gitlab.com/ggxy/ggxy-release>]

- Public c++ library for the computation of LO and NLO amplitudes and cross sections for **gluon initiated top-quark loop** induced processes.
- Contains all necessary functions for:
 - $gg \rightarrow HH$ at NLO QCD [Davies, Stremmer, KS, Steinhauser '25]
 - $gg \rightarrow ZH$ at NLO QCD (also including leptonic Z decays) [Davies, Grau, KS, Steinhauser, Stremmer, Vitti '25][Davies, Stremmer, KS, Steinhauser '26]
 - NNLO large- m_t expansion of virtual $gg \rightarrow ZH$ amplitudes [Davies, Grau, KS, Steinhauser, Stremmer '25]
- linking to various packages developed for automated one loop computations (see backup for details).
- Analytic expansions allow to **modify** m_t , m_H and top-quark mass renormalization scheme ($\overline{\text{OS}}, \overline{\text{MS}}$).
- Supports variation of $\kappa_\lambda = \lambda_3/\lambda_3^{\text{SM}}$.
- Also [POWHEG] implementations available (ggxy_ggHH and ggxy_ggZH).

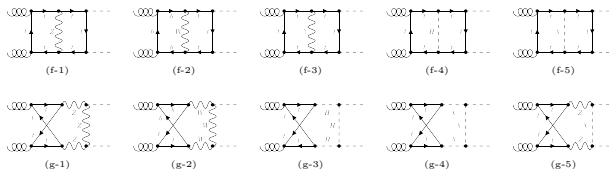


New Results



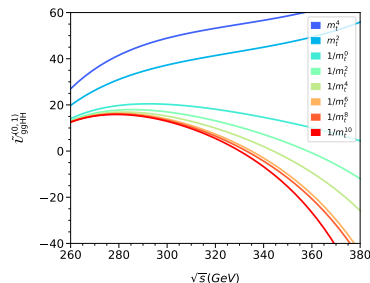
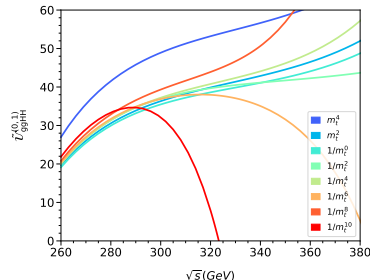
EW High Energy Expansion

[JD, Schönwald, Steinhauser, Zhang '23]



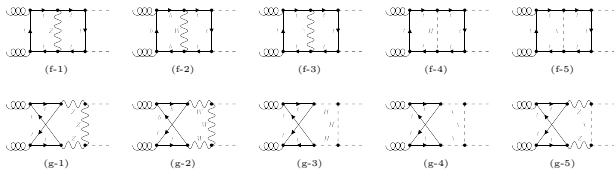
Large- m_t expansion of full NLO EW $gg \rightarrow HH$ amplitude

- exp. terms to $1/m_t^{10}$
- good description of amplitude for $\sqrt{s} \lesssim 320$ GeV
- (up to convergence issue in diagrams w/ $m_t + m_W$ cut)
- R_ξ gauge parameters ξ_Z, ξ_W, ξ_γ
 - ▶ expansion hierarchy: $m_t^2 > \xi_W m_W^2, \xi_Z m_Z^2 > s, t, m_W^2, m_Z^2, m_H^2$
 - ▶ gauge-independent result after renormalization ✓
- Used to check numerical results [Bi, Huang, Huang, Ma, Yu '23]



EW High Energy Expansion

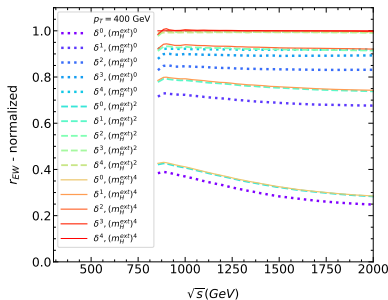
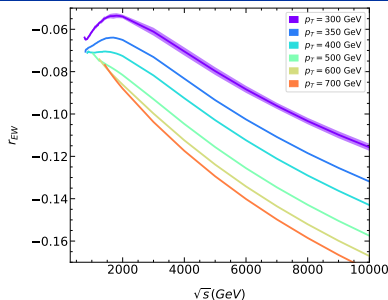
[JD, Schönwald, Steinhauser, Zhang '26]



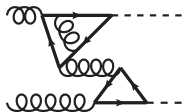
HE expansion of full NLO EW $gg \rightarrow HH$ amplitude

- expansion in:
 - ▶ $m_H < m_t < s, |t|$
 - ▶ $\delta_X = 1 - m_X/m_t$ for Higgs, W , Z propagator masses
- percent-level agreement with numerical approach (pySecDec) for $p_T > 300\text{GeV}$ (Yukawa, λ diagram set)

Work-in-progress: forward expansion

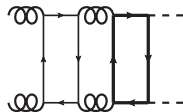


Towards NNLO Corrections



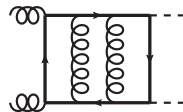
1PR

[Davies, KS, Steinhauser, Vitti '24]



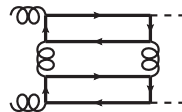
$n_l n_h \{C_A, C_F\}$
 $p_T = 0$

[Davies, Schönwald, Steinhauser '23]



$n_h \{C_A^2, C_A C_F, C_F^2\}$
 $p_T = 0$ and large- N_c

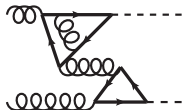
[Davies, Schönwald, Steinhauser '25]



$n_h^2 \{C_A, C_F\}$

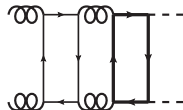
in progress

Towards NNLO Corrections



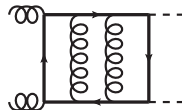
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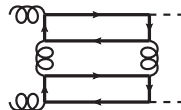
$n_l n_h \{C_A, C_F\}$
 $p_T = 0$

[Davies, Schönwald, Steinhauser '23]



$n_h \{C_A^2, C_A C_F, C_F^2\}$
 $p_T = 0$ and large- N_c

[Davies, Schönwald, Steinhauser '25]



$n_h^2 \{C_A, C_F\}$

in progress

n_h^2 **contribution needs new calculational strategy:**

- Direct asymptotic expansion in parameter space with Asy [Jantzen, Smirnov² '12].
- IPB reduction in parameter space with Calico [Bertolini, Fontana, Perraro '25] and FiniteFlow [Perraro '19].
- New strategy suitable for many more applications.

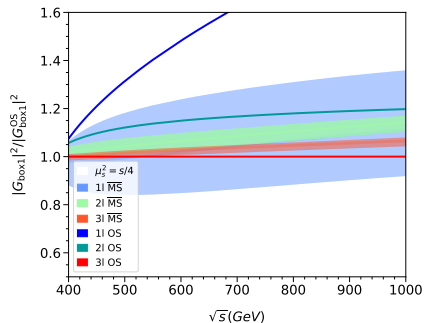
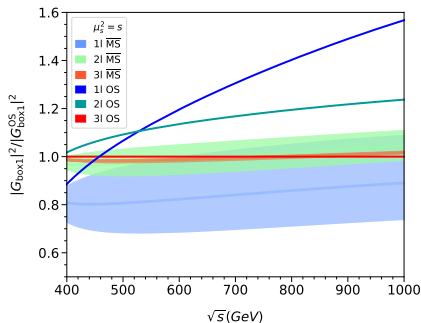
Impact of NNLO Corrections: n_l + large- n_c : $t^0 m_h^0$

We can produce a renormalized, IR-subtracted, virtual amplitude:

- in large- m_t region, the large- n_c approx induces a $\sim 10\%$ error
- **caveat:** not physical, require 2L $gg \rightarrow HHg$ and 1L $gg \rightarrow HHgg...$

This contribution **greatly reduces the m_t scheme dependence** in the squared virtual amplitude:

- $\mu_s^2 = s$ (L), $\mu_s^2 = s/4$ (R), $\mu_t^2 \in (s/16, s)$:



Conclusions and Outlook



Conclusions and Outlook

Conslusions:

- Multi-scale, multi-loop Feynman integrals are hard to evaluate:
→ Reduce the complexity by expanding in physically relevant regions.
 1. **large- m_t** : $s, |t|, m_h^2 \ll m_t^2$
 2. **small- t** : $|t|, m_h^2 \ll s, m_t^2$
 3. **high-energy**: $m_t^2 \ll m_h^2 \ll s, |t|$
- Expansions give a good description for $gg \rightarrow HH$, $gg \rightarrow ZH$ and $gg \rightarrow ZZ$ at NLO.
- Expansions are simple enough to be implemented into fast numerical codes:
<https://gitlab.com/ggxy/ggxy-release>
- We made first steps toward NNLO, with new challenges which require new tools.

Conclusions and Outlook

Outlook:

- Implement more processes into ggxy:
 - ▶ EW corrections to $gg \rightarrow HH$
 - ▶ $gg \rightarrow ZZ$ with off-shell Z bosons
- Finish full virtual **NNLO QCD** corrections:
 - ▶ Explore further the amplitude expansion in parameter space.
 - ▶ How far can we push the expansion depth?
- Which expansions are needed for more complicated kinematics, like $2 \rightarrow 3$?

Thank You!

Backup

High-Energy Expansion: Padé approximants

The expansion diverges for $\sqrt{s} \lesssim 750$ GeV.

The convergence can be improved by making use of **Padé approximants**:

- Approximate a function using a rational polynomial:

$$f(x) \approx [n/m](x) = \frac{a_0 + a_1x + a_2x^2 + \cdots + a_nx^n}{1 + b_1x + b_2x^2 + \cdots + b_mx^m},$$

where the coefficients a_i, b_j are fixed by the series expansion of $f(x)$.

Compute a set of approximants (various choices of n, m):

- combine to give a **central value** and **error estimate**
- deeper expansions $\Rightarrow$ larger $n + m \Rightarrow$ smaller error
- expansions to m_t^{120} allows for very high-order approximants

ggxy Implementation

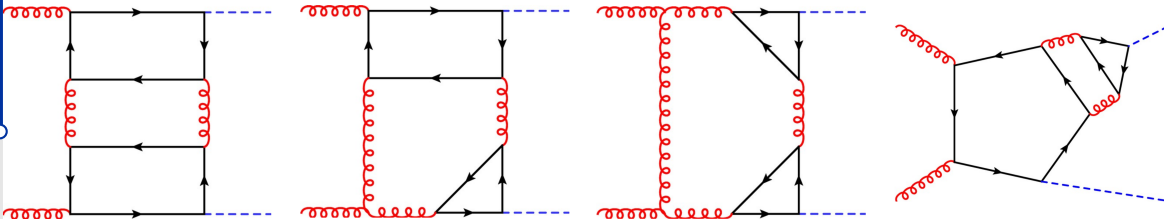
ggxy: [\[https://gitlab.com/ggxy/ggxy-release\]](https://gitlab.com/ggxy/ggxy-release)

- Public c++ library for the computation of LO and NLO amplitudes and cross sections for **gluon initiated top-quark loop** induced processes.
- Contains all necessary functions for $gg \rightarrow HH$ at NLO QCD.
- Analytic expansions allow to **modify** m_t , m_H and top-quark mass renormalization scheme ($\overline{OS}, \overline{MS}$).
- Supports variation of $\kappa_\lambda = \lambda_3 / \lambda_3^{\text{SM}}$.

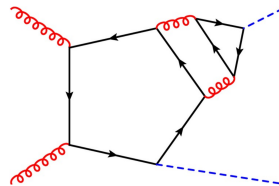
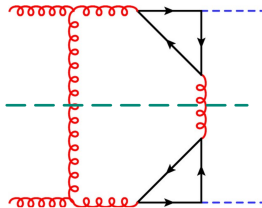
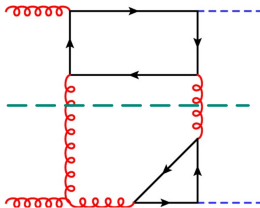
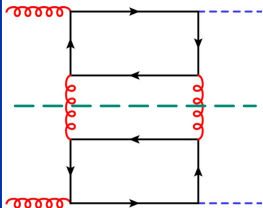
Installation:

- External dependencies: **Boost**, **CMake**, **Eigen**
- Path to **LHAPDF** installation needed for hadronic cross sections.
- The following libraries are directly installed and linked to **ggxy**:
 - **Collier** [Denner, Dittmaier, Hofer '17]
 - **CutTools** [Ossola, Papadopoulos, Pittau '09]
 - **OneLOop** [van Hameren '11]
 - **RecoLa** [Actis, Denner, Hofer, Lang, Scharf, Uccirati '17]
 - **avhlib/Kaleu** [van Hameren '10]
 - **CRunDec** [Herren, Steinhauser '18]
 - **lievaluate** [Frellesvig, Tommasini, Wever '16]

$gg \rightarrow HH$ at NNLO QCD: n_h^2 part, new challenges

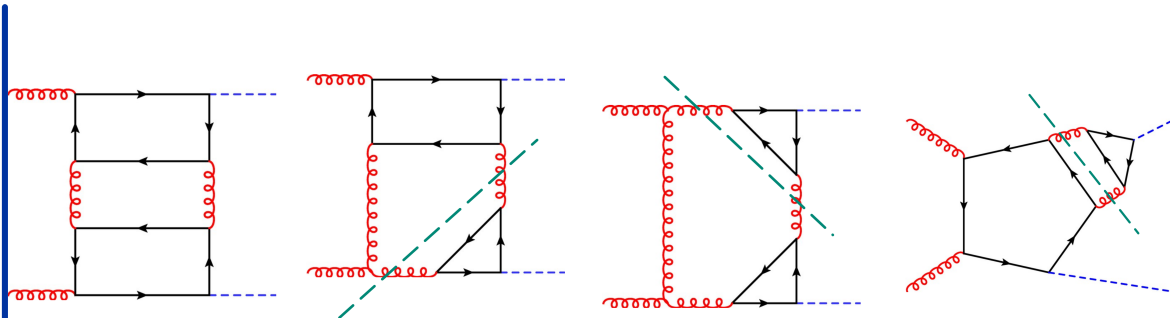


$gg \rightarrow HH$ at NNLO QCD: n_h^2 part, new challenges



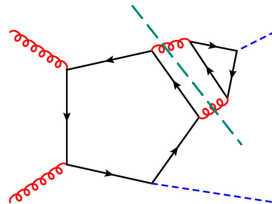
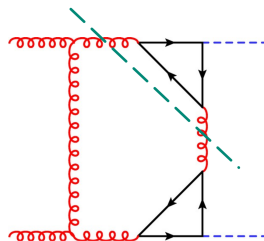
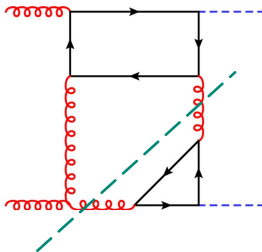
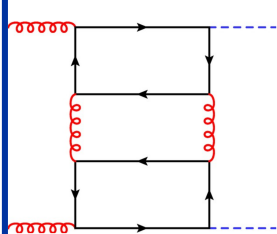
Cuts leading to: $\ln(t)$

$gg \rightarrow HH$ at NNLO QCD: n_h^2 part, new challenges



Cuts leading to: $\ln(m_h^2)$

$gg \rightarrow HH$ at NNLO QCD: n_h^2 part, new challenges



Cuts leading to: $\ln(m_h^2)$

Simple Taylor expansion in vector $\delta q = q_1 + q_3$ is not sufficient!

Asymptotic expansion for the n_h^2 part

- The asymptotic expansion can be best implemented by the **expansion-by-region**.
- For **euclidean kinematics**, i.e. positive definite F polynomial, regions can be found algorithmically in parameter space (Schwinger):

$$I(\alpha_1, \dots, \alpha_n) = \int_0^\infty d^n \mathbf{z} \prod_j^n \frac{z_j^{\alpha_j-1}}{\Gamma(\alpha_j)} e^{-\frac{\mathcal{F}(\mathbf{z})}{\mathcal{U}(\mathbf{z})}} \mathcal{U}(\mathbf{z})^{-d/2}$$

- We use `asy.m` to find the relevant scaling $z_j \rightarrow \lambda^{n_j} z_j$ for the expansion:

$$m_h^2 \rightarrow \lambda m_h^2,$$

$$t \rightarrow \lambda t,$$

$$\lambda \rightarrow 0$$