

Efficient calculations of dijet soft functions at higher orders

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Based on work done together with Emmet Byrne, Alexander Bennett, Elsa Lang, Samuel Abreu, Pier Monni, Robert Szafron, Luca Rottoli

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Oxford, 01/10/2026



OUTLINE

- What are dijet soft functions?
- Efficient technique to compute them via simplified reference measurements
- Application at NNLO: Generalized angularities
- Application at N3LO: Cross sections with a jet veto

RESUMMATION

Measurement/cut with scale $\mathcal{T}$ much smaller than hard process scale Q can yield large logs – fixed order PT breaks down, need to resum

$$\sigma(\mathcal{T}, Q) = \# + \# \alpha_s \log^2 \left(\frac{Q}{\mathcal{T}} \right) + \# \alpha_s^2 \log^4 \left(\frac{Q}{\mathcal{T}} \right) + \dots$$

$$\text{Soft log} \quad \int_{\mathcal{T}}^Q \frac{dk_T}{k_T} \int_{-\log(Q/k_T)}^{\log(Q/k_T)} d\eta \quad \text{Collinear log}$$

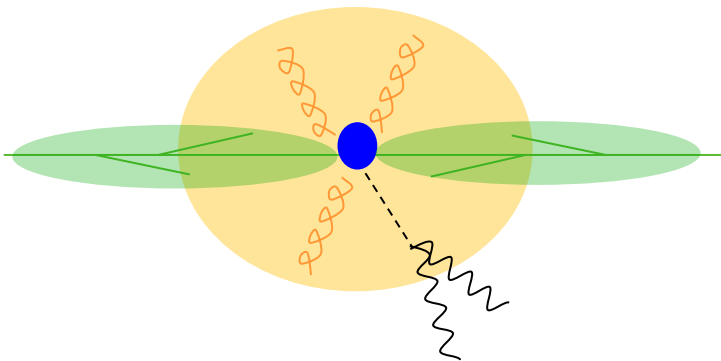
For some measurements, resummation can be achieved by a factorization formula:

$$\sigma(\mathcal{T}, Q) = H \times [B \otimes \dots \otimes J \otimes \dots \otimes S]$$

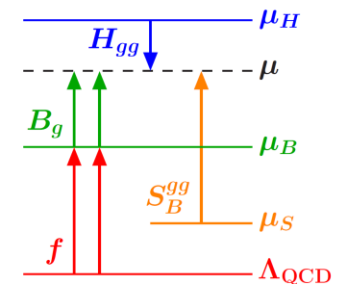
Initial-state collinear

Final-state collinear

Soft radiation



Functions 'live' at different scales, resum logs by RGE running

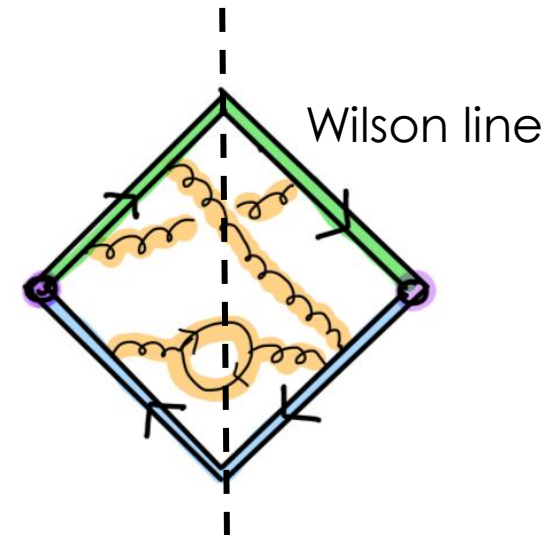


(For hadron colliders, most observables have issues with Coherence Violating Logs at some perturbative order: JHEP 09 (2008) 128, JHEP 07 (2014) 110, JHEP 10 (2015) 189, JHEP 12 (2021) 084, Phys. Rev. Lett. 136 (2026) 221901, Phys. Rev. Lett. 137 (2026) 111906, Phys. Rev. Lett. 137 (2026) 121902, etc. See talk by Alex Fraley)

DIJET SOFT FUNCTION

For processes with two jets (either initial- or final-state), we have the dijet soft function.

- $e^+e^- \rightarrow \text{dijets}$. Examples: thrust, C-parameter
- $e^+p \rightarrow \text{jet} + X$. Example: DIS 1-jettiness
- $pp \rightarrow \text{colourless } V + X$. Examples: p_T of V , $V + 0$ jets



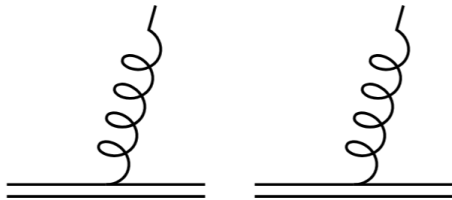
Higher precision resummation requires evaluating factorization ingredients to higher orders.

N^{LL} requires S, B, J to n loops + their (non-cusp) anomalous dimensions at $(n+1)$ loops

CORRELATED & INDEPENDENT EMISSIONS

Lots of IR + UV divergences in computation of soft function at higher orders, which complicates the calculation.

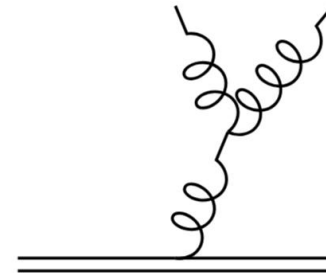
Two “types” of emissions:



Independent emissions.

Amplitude is simply product of single emissions.

Divergences when $k_t \rightarrow 0$ and/or $\eta \rightarrow \pm\infty$ for each emission



“Correlated” emissions

Amplitude heavily suppressed as $\Delta\eta \rightarrow \pm\infty$

Divergences when daughter particles become collinear to each other, or soft $k_t \rightarrow 0$. Also, when rapidity of entire system $\rightarrow \pm\infty$

SIMPLIFIED MEASUREMENTS

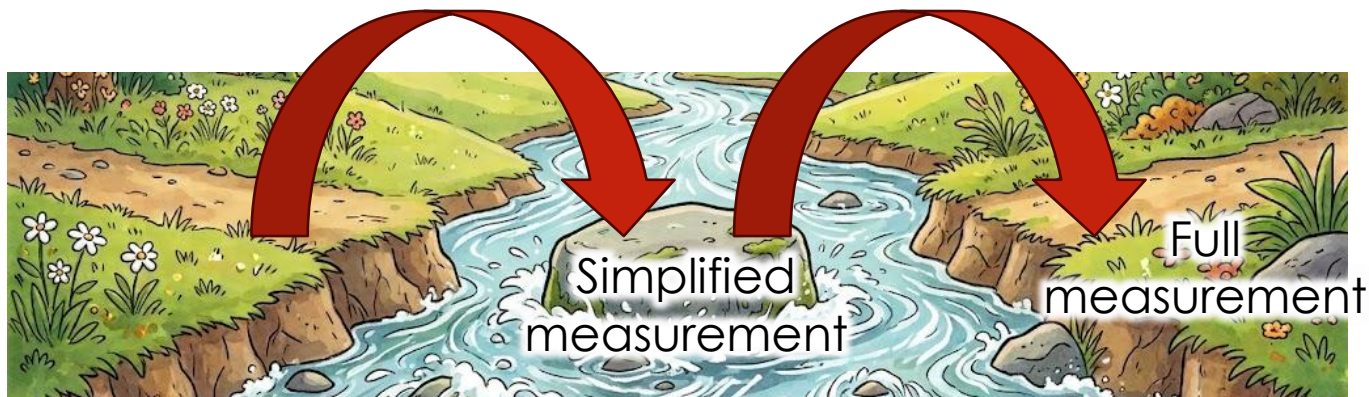
How to deal with all these divergences, especially if the measurement is complex?

Efficient method: Use some simplified measurement (doesn't have to be physical) as an intermediate step:

Jouttenus, Stewart, Tackmann, Waalewijn, Phys. Rev. D 83 (2011) 114030, Kasemets, Waalewijn, Zeune, JHEP 03 (2016) 153, Banfi, McAslan, Monni, Zanderighi, JHEP 05 (2015) 102, Gangal, JG, Stahlhofen, Tackmann, JHEP 02 (2017) 026, Bauer, Manohar, Monni, JHEP 07 (2021) 214, Abreu, JG, Monni, Szafron, JHEP 08 (2022) 268, Abreu, JG, Monni, Rottoli, Szafron, JHEP 04 (2023) 127, Buonocore, Delto, Melnikov, Monni, Pikelner, Vita, 2604.13167,...

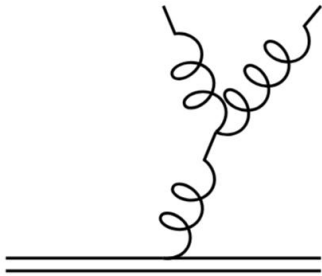
Easier to do (semi-)analytically, captures most of divergences

Difference ΔS has simpler divergence structure, can be done numerically



SIMPLIFIED MEASUREMENTS: CORRELATED

Focus on NNLO for now. Then have RR, RV (VV scaleless).



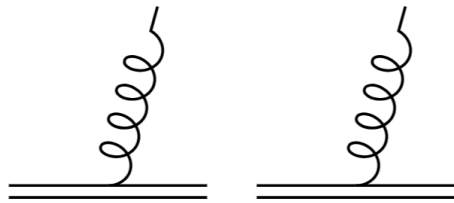
For correlated:

Key requirement: simplified = full for a single emission.

For RR: In ΔS soft + collinear regularized, only divergence as system rapidity $\rightarrow \pm\infty$

Complexity of calculation reduced essentially to tree-level.

SIMPLIFIED MEASUREMENTS: INDEPENDENT



For correlated:

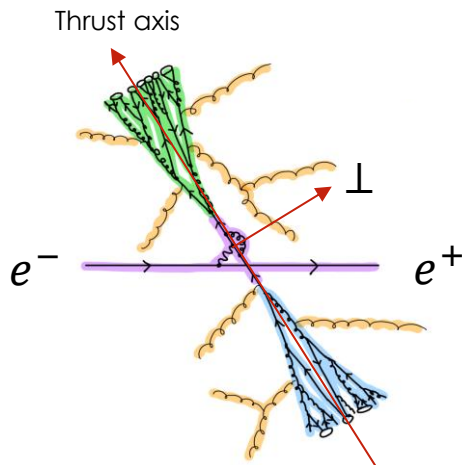
If there is some space where the measurement factorizes: non-Abelian exponentiation (“ $S^{(2)} = \frac{1}{2}(S^{(1)})^2$ ”)

Otherwise: usually useful to use a simplified measurement that corresponds to full in $\Delta\eta \rightarrow \pm\infty$ limit

NB: There is an existing automated, numerical code to compute soft functions at NNLO [SoftServe: Bell, Rahn, Talbert], but here our aim is to make NNLO as simple as possible, and in a manner which can be readily extended to higher orders.

GENERALIZED ANGULARITY

NNLO example: Generalised angularity event shapes Byrne, JG, arXiv:2609.10716



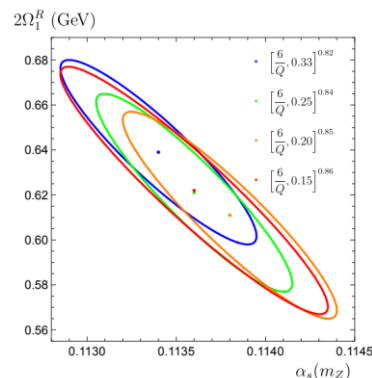
Motivation: α_s measurement via event shapes:

See talk by Yi Chen

$$\tau_e(X) = \frac{1}{Q} \sum_{i \in X} f_e(\eta_i) |\vec{k}_{i\perp}|$$

Can use “classic” thrust and/or C-parameter:

$$f_\tau(\eta) = e^{-|\eta|}, \quad f_C(\eta) = \frac{1}{2 \cosh \eta}$$



Benitez, Hoang, Mateu, Stewart,
Vita, *JHEP* 07 (2025) 249

Important complication: hadronization/NP corrections. Degeneracy between hadronization effects and α_s

(+ potential issues with 2/3 jet regions, hadron masses)

See panel discussion on α_s , Rene Poncelet's talk today

MOTIVATION

To break degeneracy, can:

- Use different Q values (see Rene's talk)
- Use family of different measurements.

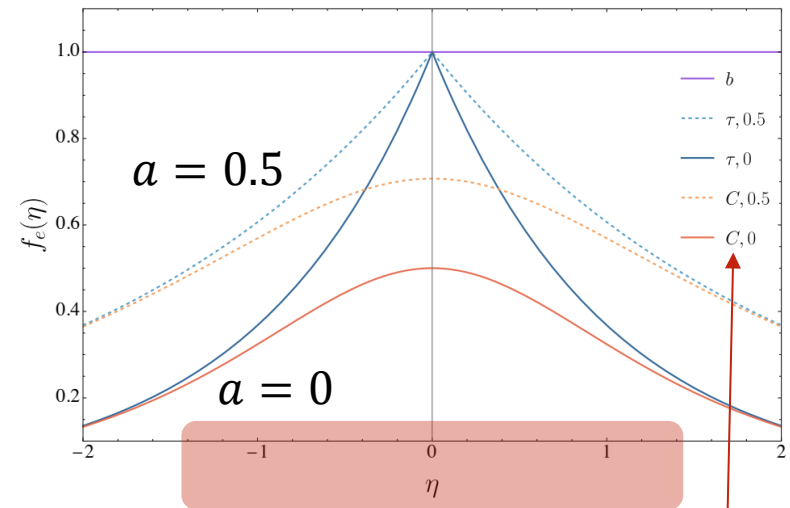
E.g. angularity

Bell, Hornig, Lee Talbert, JHEP 01 (2019) 147

$$f_{\tau,a}(\eta) = e^{-(1-a)|\eta|}$$

Size of NP corrections
determined by shape of $f_e(\eta)$
near $\eta \sim 0$

e.g. Lee, Sterman, Phys. Rev. D 75 (2007) 014022



C-angularity

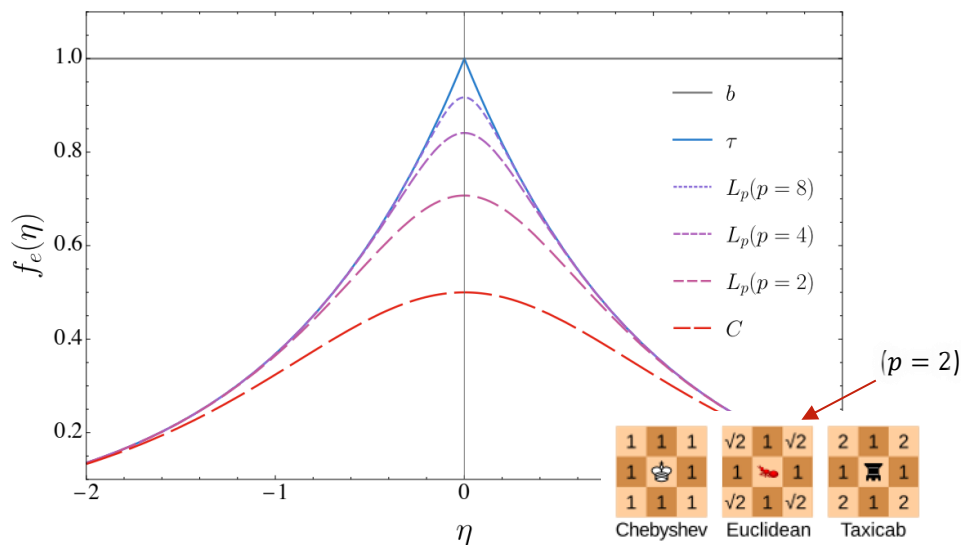
C. Lee and G.F. Sterman, Phys. Rev. D 75 (2007) 014022
Bennett, Byrne, JG, Lang, JHEP 05 (2026) 24

Changing a changes $f_e(\eta)$ everywhere: changes NP corrections but also changes perturbative structure!

MOTIVATION

Alternative idea: just change the shape in the soft region!

Byrne, JG, arXiv:2609.10716



“ L_p angularities”

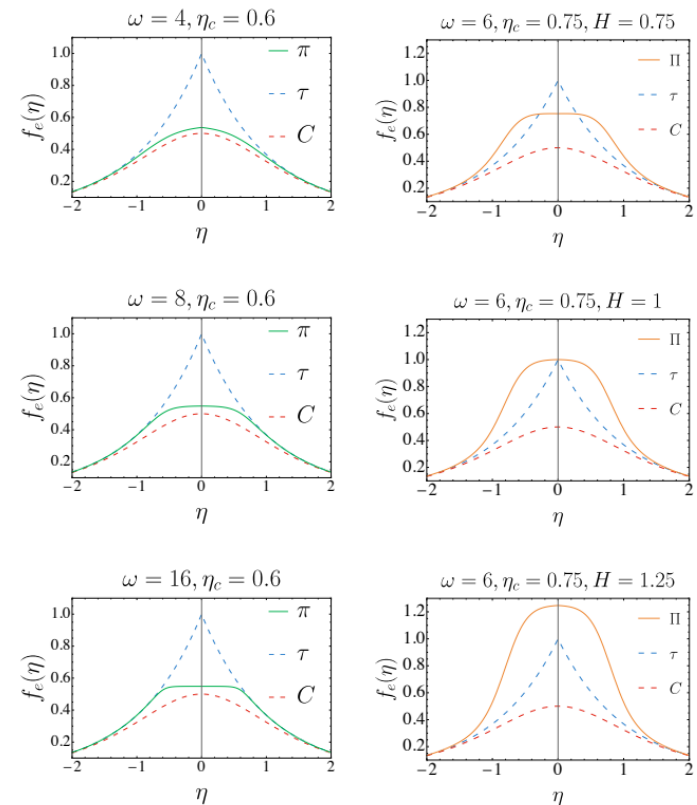
$$f_{p,a=0}(\eta) = \frac{k_T}{\|(k^+, k^-)\|_p}$$

“ L_p norm”

Thrust

C-parameter

“Plateau angularities”



Cmglee, CC BY-SA 4.0
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Wikimedia Commons

Useful to stress test our predictions for NP corrections & fit α_s

SIMPLIFIED MEASUREMENT

We recently constructed an efficient algorithm to compute the NNLO soft function for any generalized angularity event shape. Last ingredient for NNLL' resummation.

Byrne, JG, arXiv:2609.10716

Independent: use non-abelian exponentiation

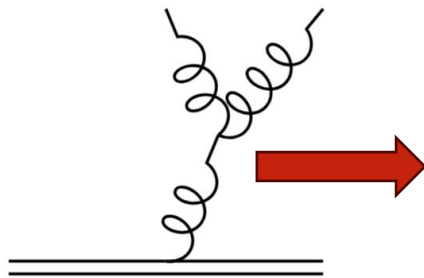
Correlated: Consider difference to a simplified measurement where we measure event shape on **total** radiation (using light-cone components). Symmetry + dim analysis $\rightarrow$ this has a very simple form at all loops

$$S_{e_I}^{\text{bare}}(\mathcal{T}) = \sum_{\ell=0} \left(\frac{\alpha_s^{\text{bare}}}{4\pi} \right)^\ell F_T^{(\ell)}(\epsilon, N_c, n_f) \int \frac{dq^+ dq^-}{q^+ q^-} \left(\frac{\mu^2}{q^+ q^-} \right)^{\ell\epsilon} \delta(\mathcal{T}_{e_I} - \mathcal{T})$$


Just a number, can be extracted from (e.g.) threshold soft function

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, et al., Phys. Lett. B 737 (2014) 325, Li, von Manteuffel, Schabinger, Zhu, Phys. Rev. D 91 (2015) 036008, Billis, Ebert, Michel, Tackmann, Eur. Phys. J. Plus 136 (2021) 214

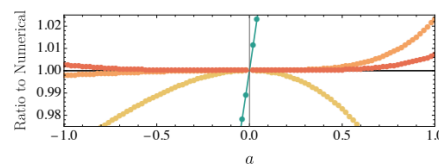
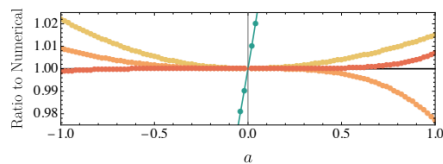
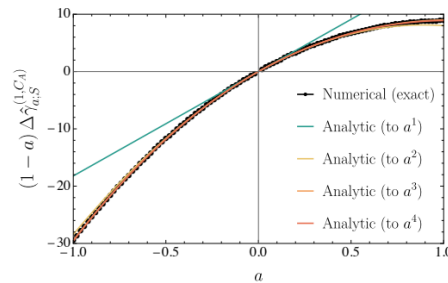
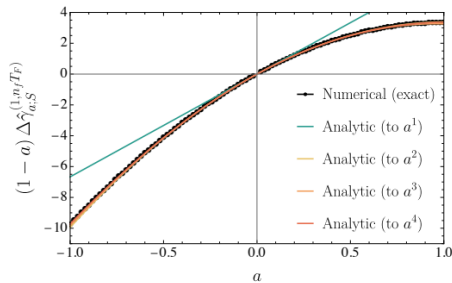
LARGE RAPIDITY BEHAVIOUR



One divergence left associated with $y_{tot} \rightarrow \pm\infty$.

But here the behaviour is the same as (C-)angularity. Recently computed this analytically to high precision as a series in a

Bennett, Byrne, JG, Lang, JHEP 05 (2026) 24



Subtract off, leaves finite remainder that can be computed numerically

N3LO: JET VETO ANOMALOUS DIMENSION

Can one use this kind of strategy at N3LO?

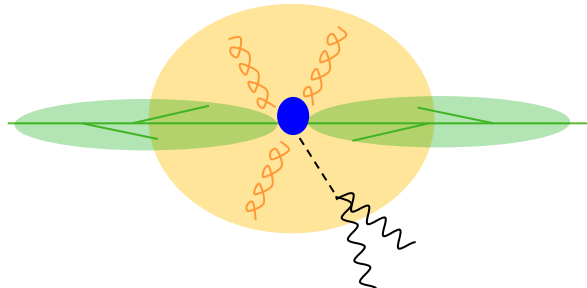
With this strategy, N3LO $\rightarrow$ “NLO-like” calculation. Will need to deal with soft + collinear divergences via counterterms.

First calculation of this kind: Three-loop rapidity anomalous dimension $\gamma_v^{(3)}$ for cross section with a jet veto

Abreu, JG, Monni, Rottoli, Szafron, to appear

Becher, Neubert, JHEP 07 (2012) 108, Becher, Neubert, Rothen, JHEP 10 (2013) 125, Tackmann, Walsh, Zuberi, Phys. Rev. D 86 (2012) 053011, Stewart, Tackmann, Walsh, Zuberi, Phys. Rev. D 89 (2014) 054001

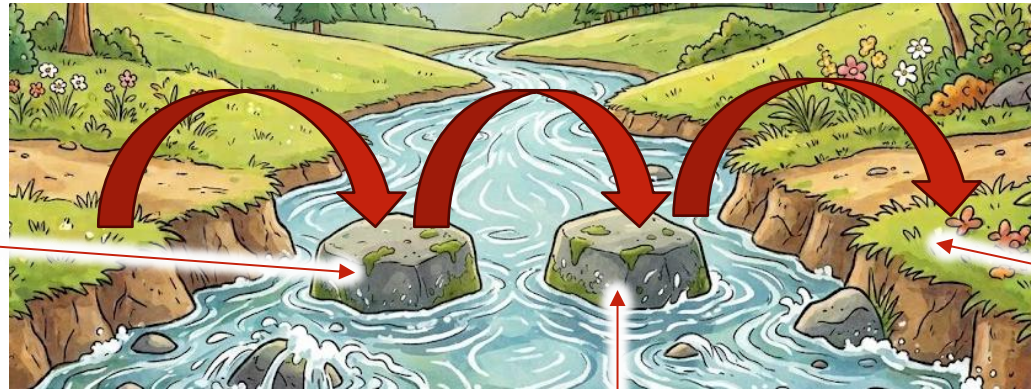
$$\frac{d\sigma_0}{dY}(p_{Tj} < p_T^{cut}) = \sigma_B H(Q, \mu) B_i(x_a, Q, p_T^{cut}, R, \mu, \nu) \times B_i(x_b, Q, p_T^{cut}, R, \mu, \nu) S(p_T^{cut}, R, \mu, \nu)$$



$\gamma_v^{(3)}$ corresponds to dependence of S on rapidity scale ν to 3 loops, last piece needed for N3LL resummation

STEPPING STONES FOR JET VETO

Two stepping stones:

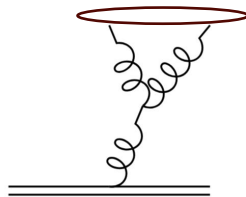


TMD soft function
(total transverse mtm).
Known analytically to
three loops

Li, Zhu, Phys. Rev. Lett. 118 (2017) 022004

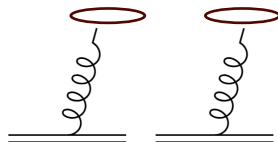
Jet veto
soft
function

One jet



“Reference simplified variable” RSV: Similar to jet veto soft function, but with veto applied to correlated clusters rather than jets (not physical!)

Two jets

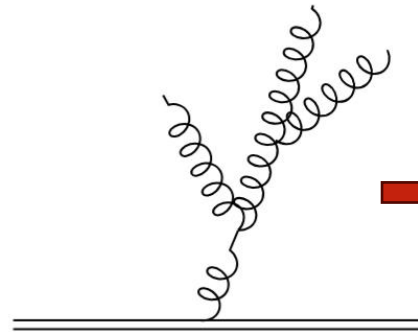


Helps with computation of III & CCI pieces.

Can obtain analytically from TMD soft at any order.

COMPUTATION: CCC

Demonstrate
how things work
for CCC (here
RSV = TMD)



$y_{tot} \rightarrow \pm\infty$ behaviour
corresponds to $\gamma_v^{(3)}$

$$\Sigma_{\text{CCC,RRR}}^{(3)} = \int [dp_1][dp_2][dp_3], \left(\overset{\text{CCC real emission amplitudes}}{\frac{1}{3!} \tilde{\mathcal{A}}_{ggg}^{(0)}(p_1, p_2, p_3) + \tilde{\mathcal{A}}_{gq\bar{q}}^{(0)}(p_1, p_2^q, p_3^{\bar{q}})} \right) \\ \times \left[\underset{\text{Jet veto measurement}}{\Theta(p_T^{\text{veto}} - p_t^{j_1}(p_1, p_2, p_3))} - \underset{\text{TMD measurement}}{\Theta(p_T^{\text{veto}} - |\vec{p}_{t1} + \vec{p}_{t2} + \vec{p}_{t3}|)} \right]$$

After TMD subtraction, only “NLO-like” divergences (two particles collinear and/or one soft)

COMPUTATION: CCC

Split integral via partition functions such that divergences are only associated with partons i & j :

$$\sigma_{ij} = \frac{\frac{1}{p_i \cdot p_j}}{\sum_{\{kl\}} \frac{1}{p_k \cdot p_l}}, \quad \text{with } \{ij\}, \{kl\} \in \{12, 13, 23\},$$

Construct a counterterm that captures IR divergences in ij

$$\begin{aligned} \text{CT}_{\text{CCC}}^{\{ij\}} = & \int [dp_1] [dp_2] [dp_3] \left(\frac{1}{3!} \tilde{\mathcal{A}}_{ggg}^{(0)}(p_1, p_2, p_3) + \tilde{\mathcal{A}}_{gq\bar{q}}^{(0)}(p_1, p_2^q, p_3^{\bar{q}}) \right) \times \sigma_{ij} \\ & \times \left[\Theta \left(p_T^{\text{veto}} - p_t^{j_1}(p_k, p_i + p_j) \right) - \Theta \left(p_T^{\text{veto}} - |\vec{p}_{ti} + \vec{p}_{tj} + \vec{p}_{tk}| \right) \right] \end{aligned}$$

Measurement only depends on total $p_i + p_j$ - facilitates extracting IR divergences (in $4-2\epsilon$ dims) and combining with virtuals

Difference between RRR & CT no longer has divergences, can integrate in 4D

$$\begin{aligned} \Sigma_{\text{CCC,RRR}}^{(3),\{ij\}} - \text{CT}_{\text{CCC}}^{\{ij\}} = & \int [dp_1] [dp_2] [dp_3], \left(\frac{1}{3!} \tilde{\mathcal{A}}_{ggg}^{(0)}(p_1, p_2, p_3) + \tilde{\mathcal{A}}_{gq\bar{q}}^{(0)}(p_1, p_2^q, p_3^{\bar{q}}) \right) \times \sigma_{ij} \\ & \times \left[\Theta \left(p_T^{\text{veto}} - p_t^{j_1}(p_1, p_2, p_3) \right) - \Theta \left(p_T^{\text{veto}} - p_t^{j_1}(p_k, p_i + p_j) \right) \right] \end{aligned}$$

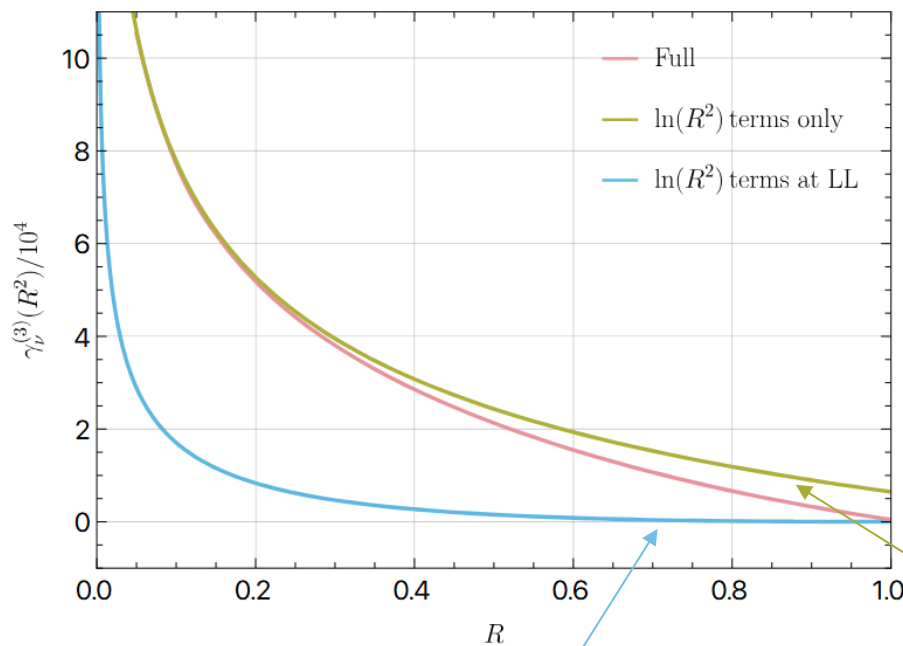
RESULTS: ANOMALOUS DIMENSION

Results:

$\gamma_v^{(3)}$ is quite large!

LL small-R term insufficient even for quite small R !

LL + (fitted) NLL seems quite a good approximation up to $R \sim 0.4$



$\log^2(R^2)$ terms + fitted $\log(R^2)$ terms

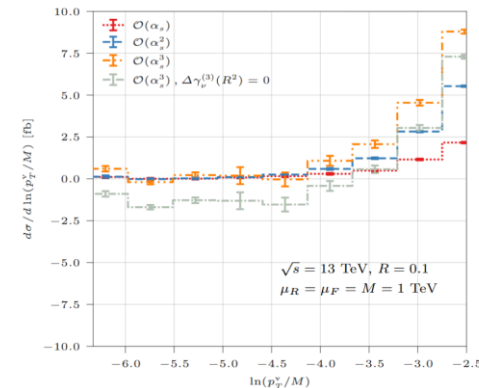
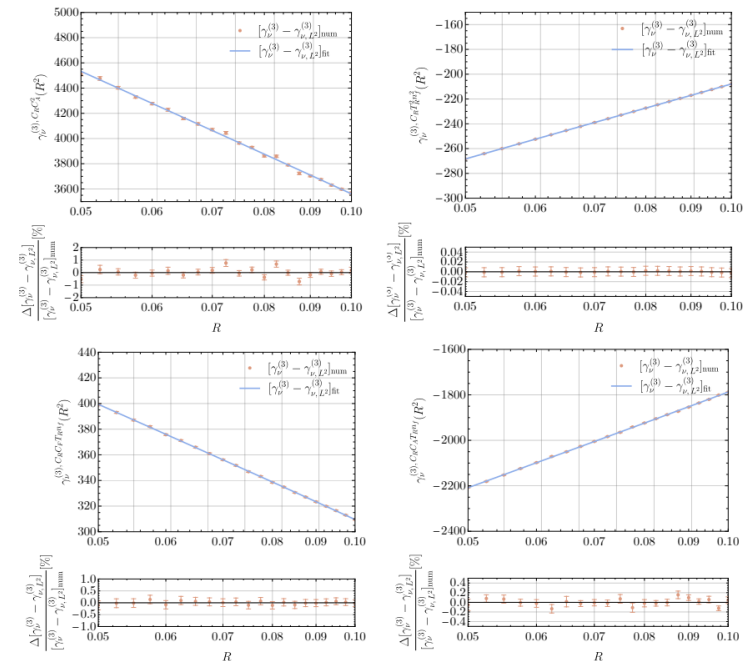
$\log^2(R^2)$ terms, known analytically

CHECKS

Checks:

- Pole cancellation ✓
- Check that small R dependence of our result is consistent with small R LL prediction ✓
- Compare leading-jet spectrum at small jet p_T with fixed order (NNLOjet) ✓

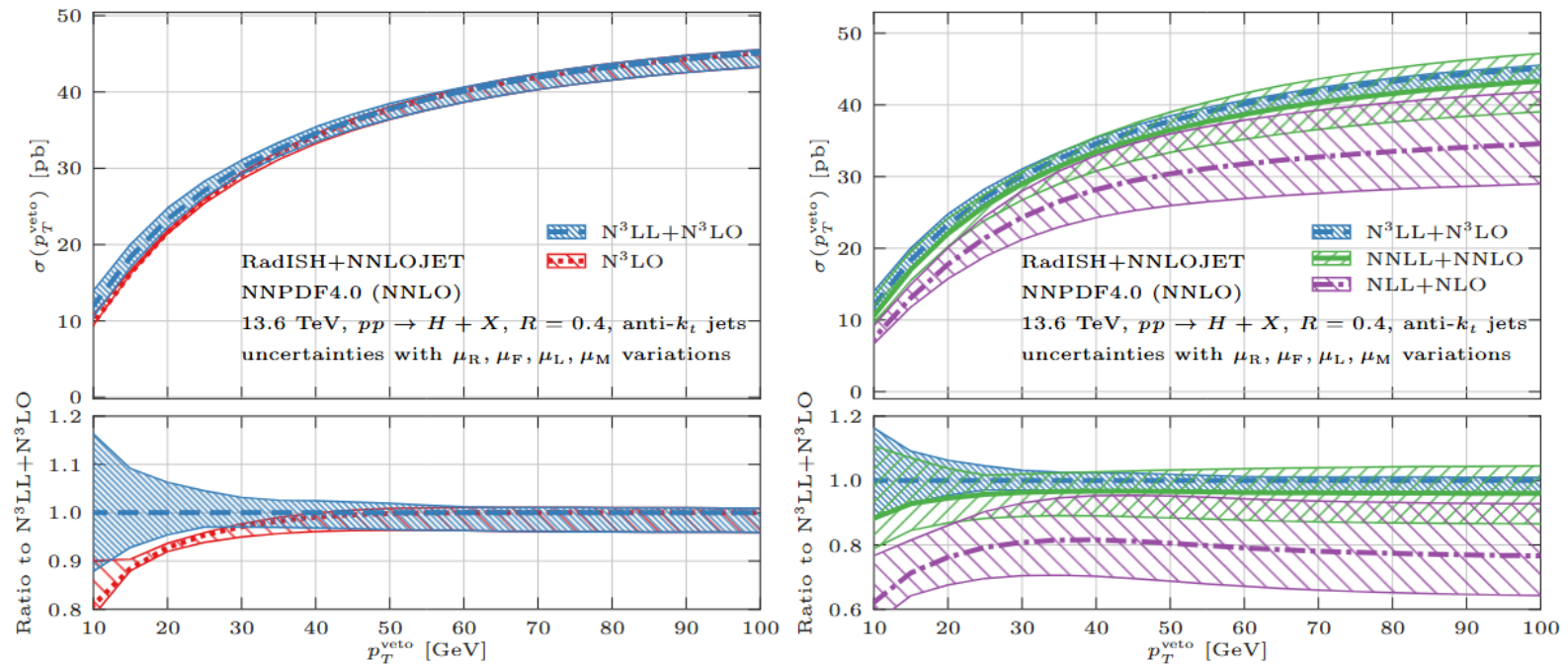
Huss, et al., SciPost Phys. Codeb. 69 (2026) 1



HIGGS VETO CROSS SECTION

Results at N³LL + N³LO for Higgs with a jet veto:

Abreu, JG, Monni, Rottoli, Szafron, to appear



SUMMARY

- Dijet soft functions required for resummation where two jet directions are involved.
- Efficient computation via simplified reference measurements.
Can also be used for beam, jet! E.g. Gangal, JG, Stahlhofen, Tackmann, JHEP 02 (2017) 026, Bauer, Manohar, Monni, JHEP 07 (2021) 214, Abreu, JG, Monni, Szafron, JHEP 08 (2022) 268, Abreu, JG, Monni, Rottoli, Szafron, JHEP 04 (2023) 127
- Generalized angularities: useful for studies of hadronization + α_s at lepton colliders. NNLO soft $\rightarrow$ last ingredient for NNLL' resummation
- Three loop rapidity anomalous dimension for jet veto resummation. Implemented in N3LL + N3LO Higgs + jet veto prediction. Appreciable impact of N3LL corrections.