



Double-real corrections to color singlet decay in a parton-shower inspired scheme

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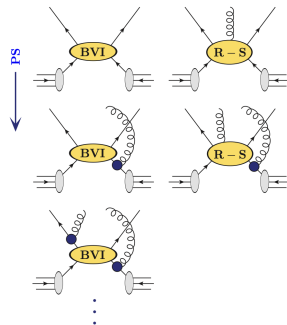
Based on:

J. Campbell, S. Höche, MK, C. Preuss, D. Reichelt Phys. Rev. D 113, 054031 (2026)

J. Campbell, S. Höche, MK arXiv:2606.26247

Towards MC@NLO-type matching at NNLO

- General-purpose MCs: NLO+PS automated; NNLO as a parton-level K -factor
- MC@NLO: **counterterm = shower kernel**
- NNLO schemes: built for a fixed-order *number*
 - $3 \rightarrow 2$ or $4 \rightarrow 2$ maps, sectorised or not
 - ↪ no* one-to-one link to shower branchings



Overlap: first shower branching at $\mathcal{O}(\alpha_s)$

MC@NLO [Frixione, Webber (2002)]

POWHEG [Nason (2004)]

Aim: NNLO counterterms with the structure of shower kernels.

First step: the double-real correction to $e^+e^- \rightarrow q\bar{q}$.

MC@NNLO requirements on the counterterms

Prerequisites for MC@NLO-type matching at NNLO without loss of logarithmic accuracy:

❶ **Coincide with the shower splitting kernels**

- known $1 \rightarrow 3$ splittings defined only in the collinear limit
- ↪ **exact spin-blind radiation**: scalar radiators + pure remainders

❷ **NLL-safe momentum mapping**

- ↪ ALARIC recoil, applied **iteratively**

❸ **Collinear limits within one eikonal disentangled**

- ↪ **partial fractioning** in angles, **no sectorisation**

The scheme presented here is designed to satisfy all three.

Scalar QCD from the Gordon decomposition

- **Exact rewriting** of the quark-gluon vertex [Gordon (1928)]

$$\frac{\not{p} + \not{q}}{(p+q)^2} T_{ij}^a \gamma^\mu = T_{ij}^a \left[S^\mu(p, q) + \frac{i\sigma^{\nu\mu} q_\nu}{(p+q)^2} - \frac{\gamma^\mu \not{p}}{(p+q)^2} \right], \quad S^\mu = \frac{(2p+q)^\mu}{(p+q)^2}$$

- **scalar current**: spinless charge; **magnetic term**: spin
- same decomposition for the triple-gluon vertex; physical **axial gauge**

- **Scalar radiator**: **full amplitude**, no kinematic limit taken

↪ double-soft current *and* spin-averaged collinear splitting

$$S_2 = \underbrace{\sum_{(i,k)} \hat{\mathbf{T}}_i \hat{\mathbf{T}}_k S_{i;k}^{(2 \rightarrow 4)}(q_1, q_2)}_{\text{scalar radiators, one per dipole}} + \underbrace{\sum_{\text{splittings}} \langle P_{1 \rightarrow 3}^{(p)} \rangle}_{\text{pure splitting remainders}}$$

Double-unresolved counterterm:

Scalar radiators: $q\bar{q}$ and two-gluon emission

- $q\bar{q}$ **emission**: complete radiator, **physical** propagators [Campbell, Glover (1997); Catani, Grazzini (1999)]

$$\mathcal{S}_{i;k}^{(q\bar{q})} = 4 \frac{s_{i1}s_{k2} + s_{i2}s_{k1} - s_{ik}s_{12}}{s_{12}^2 s_{i12} s_{k12}}$$

- same numerator, **eikonal** propagators $s_{i1}+s_{i2}$, $s_{k1}+s_{k2}$

- **Two gluons**, leading abelian term: **squared one-gluon current**

$$\mathcal{S}_{i,k;l,m}^{(ab)} = \frac{1}{4} \mathcal{S}_{i;l}(q_1) \mathcal{S}_{k;m}(q_2) , \quad \mathcal{S}_{i;k}(q_1) = -\frac{4p_i p_k}{p_{i1}^2 p_{k1}^2}$$

- the structure a shower iterates
- non-abelian part: reproduces Catani-Grazzini in the double-soft limit; sub-leading terms required for the ϵ -poles

All $1 \rightarrow 3$ splittings decomposed, quark *and* gluon initial states.

What remains: turn the ingredients into a subtraction scheme.

Structure of the double-real subtraction

$$\mathcal{M}_{RR} = \mathcal{M}_{n+2} - \sum \mathcal{M}_n \otimes \mathcal{S}_2 - \sum \left[\mathcal{M}_{n+1} - \mathcal{M}_n \otimes \mathcal{S}_1 \right] \otimes \mathcal{S}_1$$

$\mathcal{M}_k$: **squared** tree-level matrix element, k partons. $\otimes$: colour and spin correlations.

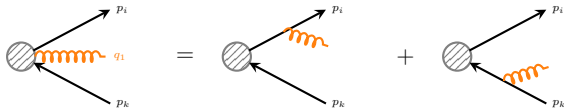
- $\mathcal{S}_2$: **double-unresolved** divergences; $\mathcal{S}_1$: **single-unresolved**
- Nested bracket: the **overlap** of iterated NLO with pure NLO [Gehrmann-De Ridder, et al. (2004)]
 $\rightsquigarrow \mathcal{S}_2$ **need only supply what iterated NLO does not**
- **Intermediate** polarisation tensor: consistent between the two clusterings

Open question: **to which emitter does each counterterm belong?**

Answered here by partial fractioning, not by sectorisation.

Kinematic assignment at NLO

$\mathcal{S}_{i;k}(q_1) = -4p_i p_k / (p_{i1}^2 p_{k1}^2)$ is singular for $q_1 \parallel p_i$ **and** $q_1 \parallel p_k$: two collinear directions in one object, no unique emitter.



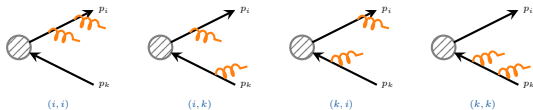
$$\mathcal{S}_{i;k}(q_1) = \mathcal{S}_{i;k}^{(i)}(q_1; n) + \mathcal{S}_{i;k}^{(k)}(q_1; n), \quad \mathcal{S}_{i;k}^{(i)}(q_1; n) = \frac{\eta_{k1}}{\eta_{k1} + \eta_{i1}} \mathcal{S}_{i;k}(q_1)$$

$$\eta_{ij} = \frac{2p_i p_j}{\zeta_i \zeta_j n^2}, \quad \zeta_i = \frac{2p_i n}{n^2}$$

- An **angular** weight $\rightsquigarrow$ a unique emitter, a well-defined branching
- **A smooth reweighting of the integrand, not a partition of phase space**

$\Rightarrow$ the ALARIC shower uses **the same construction for its kernels** [Herren, et al. (2022)]

Kinematic assignment at NNLO



Two emissions off one dipole: **four** emitter assignments

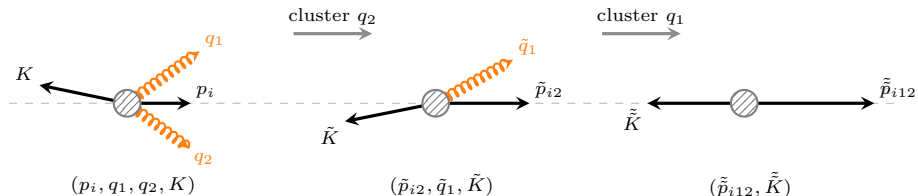
- Leading double-soft radiator **fully factorised**: each factor partial fractioned **exactly as at NLO**

$$\mathcal{S}_{i,k;l,m}^{(\text{ab}), (i,k)} = \underbrace{\frac{\eta_{l1}}{\eta_{l1} + \eta_{i1}} \mathcal{S}_{i;l}(q_1)}_{\mathcal{S}_{i;l}^{(i)}(q_1;n)} \underbrace{\frac{\eta_{m2}}{\eta_{m2} + \eta_{k2}} \mathcal{S}_{k;m}(q_2)}_{\mathcal{S}_{k;m}^{(k)}(q_2;n)} .$$

- the product of the two weights selects one of the four assignments
- $[\mathcal{M}_{n+1} - \mathcal{M}_n \otimes S_1] \otimes S_1$ needs a **hierarchy** of the two emissions [Dulat, et al. (2018)]
 - a **smooth weight** in transverse momentum, not a Θ -function
 - no region excluded from ordered evolution

⇒ a unique branching topology for every counterterm, without sectorisation:
 each counterterm $\leftrightarrow$ one parton-shower branching

Kinematics: iterated NLO mappings



The emitter keeps its direction $\hat{n}$ and is only **rescaled**; the spectator system absorbs the recoil.

- The identified-particle mapping [Catani, Seymour (1997)] applied **twice**, on shell at every step
- **Towards matching:** the **same iterated $1 \rightarrow 2$ kinematics** as the ALARIC shower
 $\rightsquigarrow$ recoil of an NLL-safe shower [Herren, et al. (2022)]
- Other schemes: a $3 \rightarrow 2$ antenna or a direct $4 \rightarrow 2$ map
 - fine for a fixed-order result, but **neither splits into two successive $1 \rightarrow 2$ branchings**
- **Prerequisite for MC@NLO-type NNLO+PS**

Integration of the counterterms

$\int d\Phi_{+2} S_2$ exposes the $1/\epsilon$ poles that cancel against the real-virtual and double-virtual contributions. For the **scalar radiators** in **back-to-back kinematics**, two independent routes:

1. Analytic

- Reverse unitarity
[Gehrmann-De Ridder, et al. (2003)]
- IBP reduction with KIRA
- Closed forms for all radiators, e.g.

$$\mathcal{I}_{i,i;k,k}^{(\text{ab})} = \frac{1}{4\epsilon^4} + \frac{1}{\epsilon^3} + \frac{1}{\epsilon^2} \left(\frac{9}{2} - \frac{3\pi^2}{8} \right) + \frac{1}{\epsilon} \left(19 - \frac{37}{6}\zeta_3 - \frac{3\pi^2}{2} \right) + \dots$$

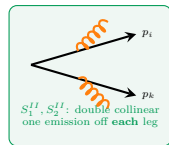
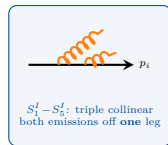
2. Numerical

- Laurent expansion in the iterated two-body phase-space variables
- Sector decomposition [Czakon (2010, 2011)], with PYSECDEC
- Only the *sector variables* adopted: the Born phase space stays factorised
- **Agreement order by order in ϵ**

Partial-fractioned terms integrate to the **expected fractions at the leading poles**.
Outstanding: the pure remainders, and general kinematics.

Pole structure after partial fractioning

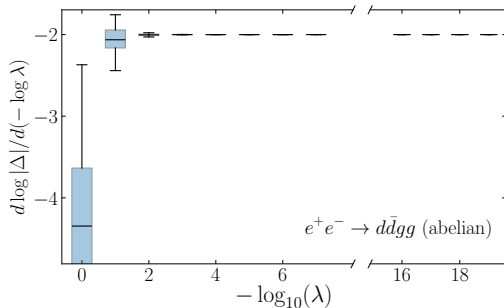
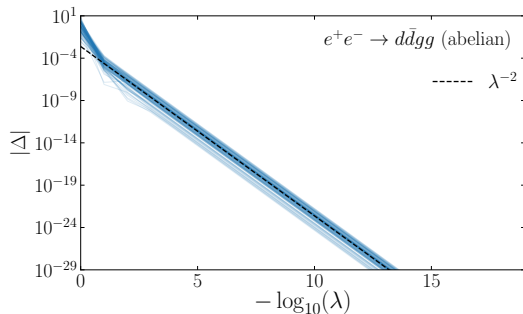
- Partial fractioning relocates singularities, it does not remove them
- Each term keeps the poles of a single branching topology
- Sectors labelled by **collinear topology**
 - an integration device; the scheme is not sectorised
- Abelian two-gluon radiator at ϵ^{-4} :
 (i, i) **exactly zero** in S^{II} , (i, k) in S^I



coeff. of ϵ^{-4}	S_1^I	S_2^I	S_3^I	S_1^{II}	S_2^{II}	ratio
$\mathcal{I}_{i,i;k,k}^{(ab)}$	$\frac{1}{16}$	$\frac{1}{48}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{12}$	1
$\mathcal{I}_{i,i;k,k}^{(ab),(i,i)}$	$\frac{1}{32}$	$\frac{1}{96}$	$\frac{1}{48}$	0	0	$\frac{1}{4}$
$\mathcal{I}_{i,i;k,k}^{(ab),(i,k)}$	0	0	0	$\frac{1}{48}$	$\frac{1}{24}$	$\frac{1}{4}$

S_4^I, S_5^I vanish at this order; the ratio is taken with respect to the *unpartitioned* integral.

Validation: convergence in the double-soft limit



$\Delta(\lambda) = |\mathcal{M}_{RR}|/\mathcal{M}_{n+2}$, phase-space point scaled into a singular limit by $\lambda \rightsquigarrow$ local subtraction:
 $\Delta \rightarrow 0$

- Abelian $e^+e^- \rightarrow q\bar{q}gg$, scaled into the **double-soft** region
- **Left:** $|\Delta|$ for 250 points, each vanishing for $\lambda \rightarrow 0$
- **Right:** local slope over 10^4 points, **all stabilising at -2** $\rightsquigarrow$ leading *and* sub-leading subtracted; other channels reach -1

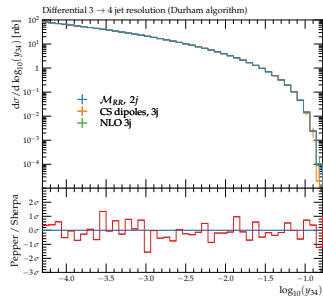
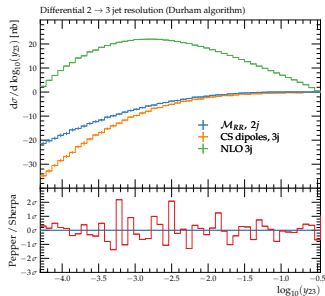
Convergence in all limits and channels

Slope of $\log |\Delta|$ against $\log(1/\lambda)$ at $-\log_{10} \lambda = 20$, over 10^4 points:

Limit	$q\bar{q}Q\bar{Q}$	$q\bar{q}q\bar{q}$	$q\bar{q}gg$ (ab)	$q\bar{q}gg$ (nab)
Single Soft	–	–	-1 ± 10^{-17}	-1 ± 10^{-18}
Double Collinear	-1 ± 10^{-17}	-1 ± 10^{-18}	-1 ± 10^{-17}	-1 ± 10^{-17}
Double Soft	-1 ± 10^{-18}	-1 ± 10^{-19}	-2 ± 10^{-18}	-1 ± 10^{-17}
Triple Collinear	-1 ± 10^{-19}	-1 ± 10^{-19}	-1 ± 10^{-19}	-1 ± 10^{-19}
Soft $\times$ Collinear	–	–	-2 ± 10^{-18}	–
Double Collinear $\times 2$	–	–	-2 ± 10^{-19}	–

- **Slope -1 or steeper in every limit:** leading power subtracted throughout; -2 where the sub-leading power is captured as well
- Variation over 10^4 phase-space points: the convergence uniform
- **Performance:** $5.1 \cdot 10^8$ points $\rightarrow 1\sigma$ of **654 fb**, below $2 \cdot 10^{-5}$ of the Born

Differential results: Durham jet rates



- Durham 2 $\rightarrow$ 3, 3 $\rightarrow$ 4 rates at $\sqrt{s} = 91.2$ GeV, against NLO Catani-Seymour for $e^+e^- \rightarrow 3$ jets
- **Caveat: double-real contribution only**
 - real-virtual and double-virtual still missing $\rightsquigarrow$ **not yet physical**
- Shown as evidence of a stable subtraction and a converged integral

Summary and outlook

- All $1 \rightarrow 3$ splittings = **scalar radiators** + spin-dependent remainders
[Campbell, Höche, MK, Preuss, Reichelt, Phys. Rev. D 113, 054031 (2026)]
- **Partial fractioning replaces sectorisation**; iterated ALARIC mapping
 $\rightsquigarrow$ each counterterm $\leftrightarrow$ one shower branching [Campbell, Höche, MK, arXiv:2606.26247 (2026)]
- Double-real $e^+e^- \rightarrow q\bar{q}$: correct scaling in all limits

Outlook:

- integrated remainders, general kinematics, real-virtual
- complete NNLO for $e^+e^- \rightarrow \text{jets}$, then MC@NLO-type matching with ALARIC

Thank you!