

# Interpreting Parton Distribution Functions using Shapley Values

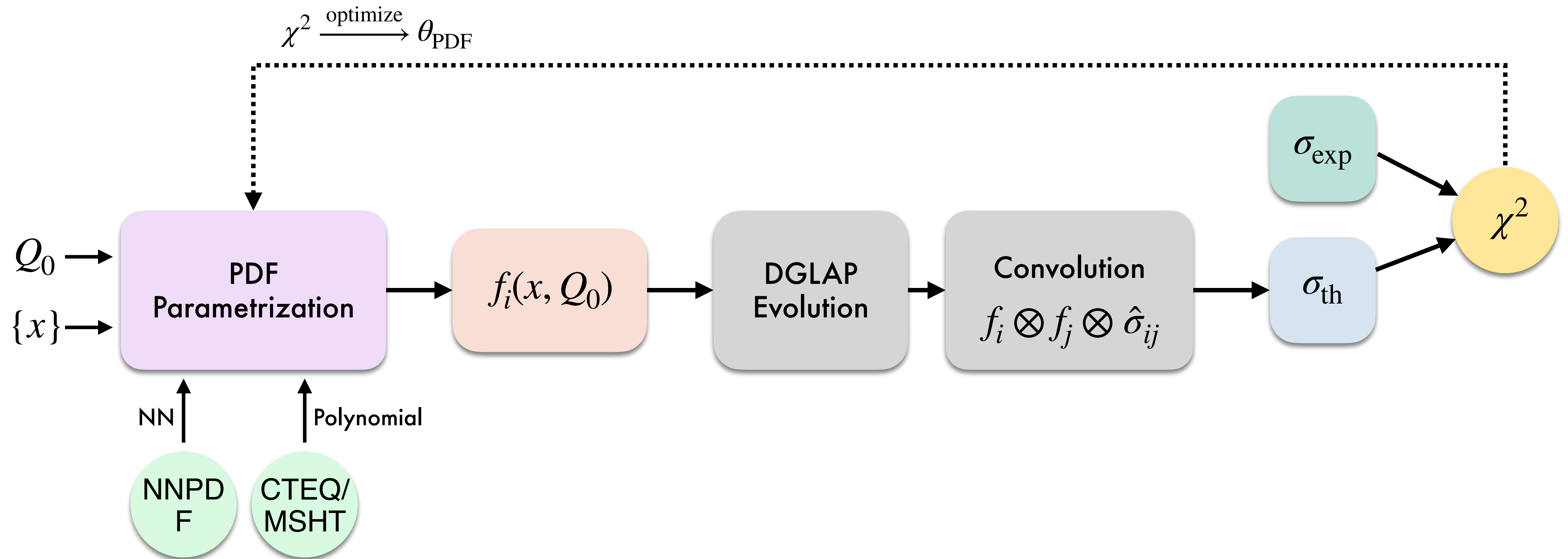
In collaboration with Raphaël Bonnet-Guerrini, Stefano Carrazza, Stefano Forte and Ramon Winterhalder

Eva Groenendijk, QCD@LHC 2026

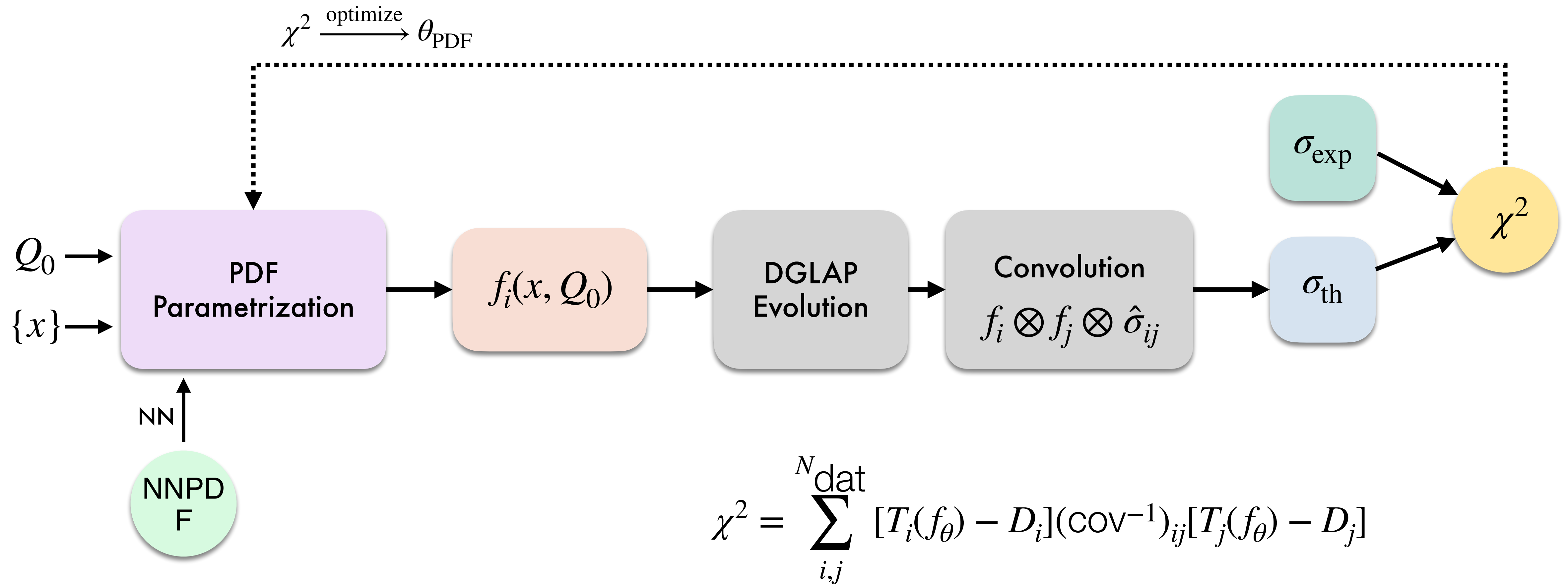


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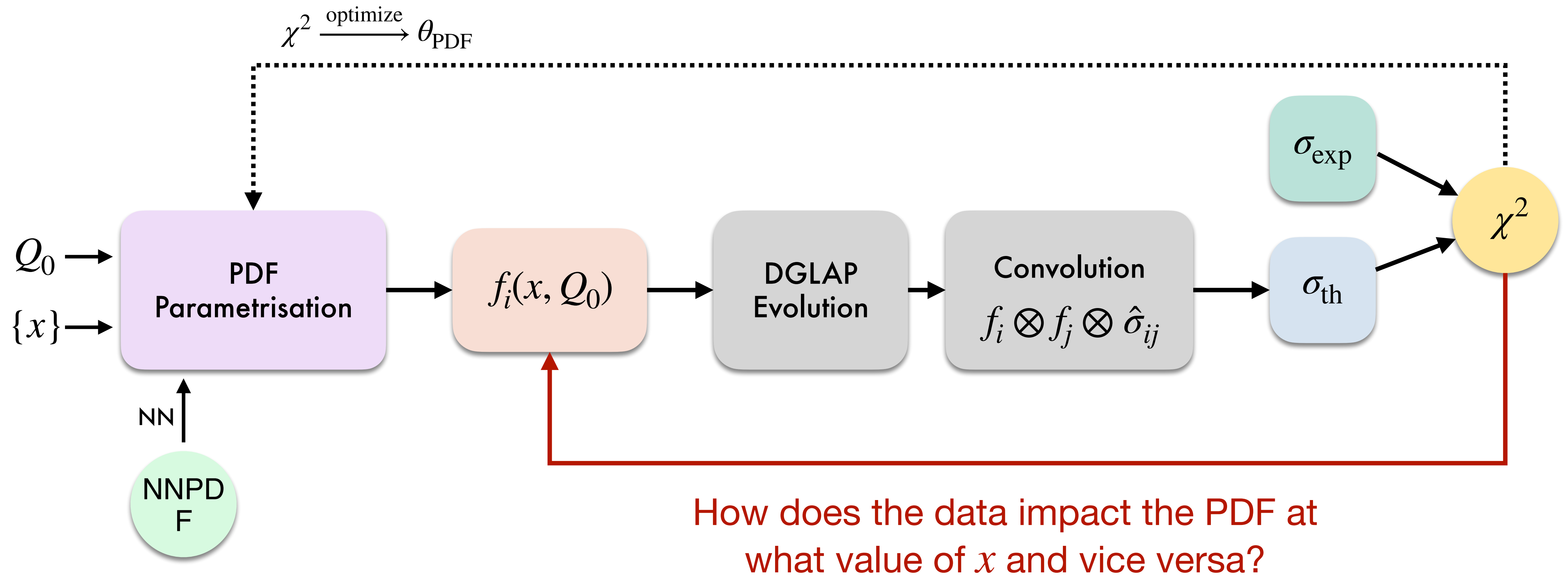
# Learning Parton Distribution functions



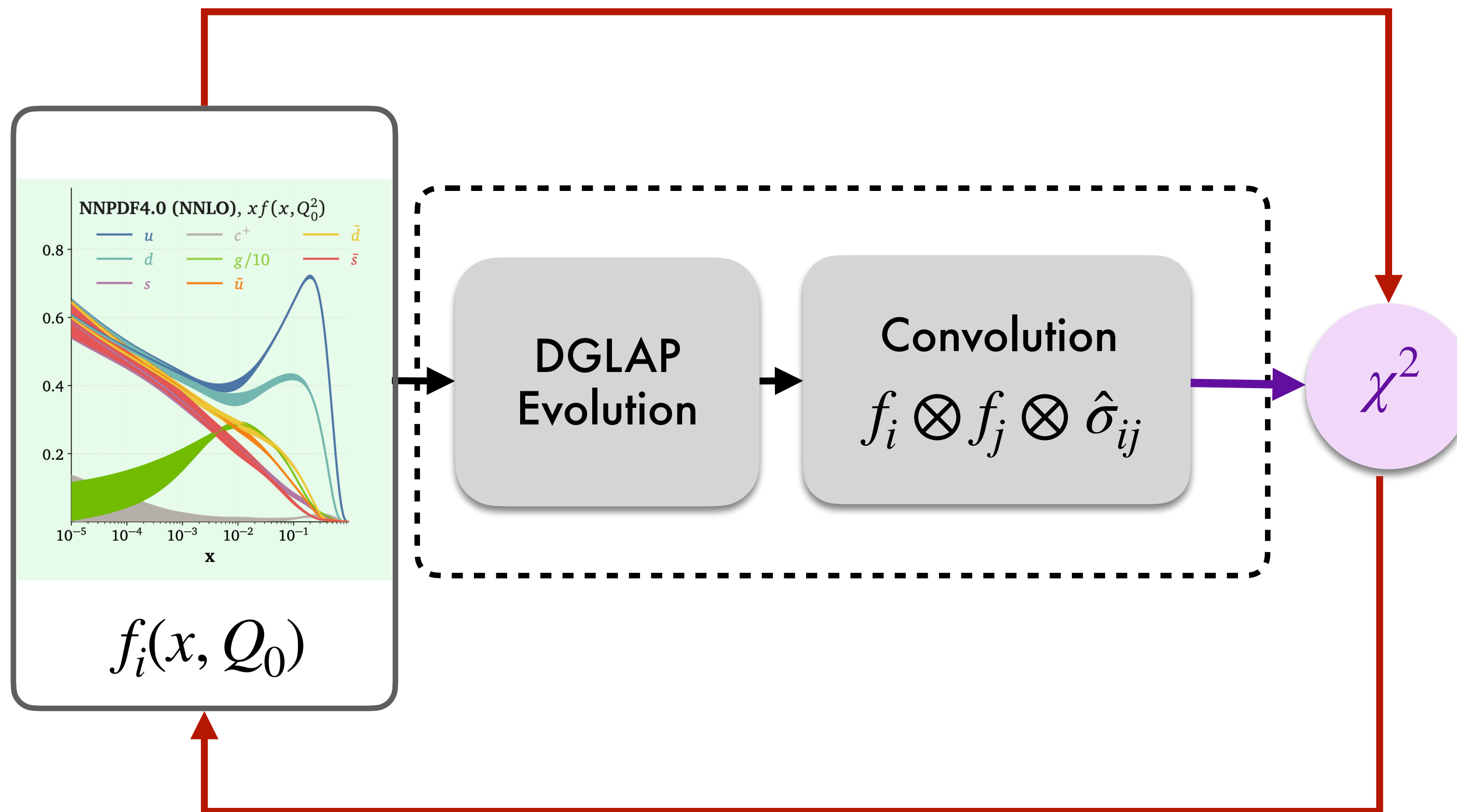
# Learning Parton Distribution Functions with NNPDF



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# What is the relation between data and PDF?



How does the shape of one PDF contribute to the overall prediction?

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## Shapley Values

The value of the marginal contribution  $\phi$  of a player  $i$  across all coalitions  $S$

$$\phi_i = \sum_{S \subset N \setminus i} w(S) [v(S \cup i) - v(S)]$$

- $N$  the set of all players
- $S$  a subset of players not including player  $i$
- $v(S)$  metric representing the value of coalition  $S$

How does the shape of one PDF contribute to the overall prediction?

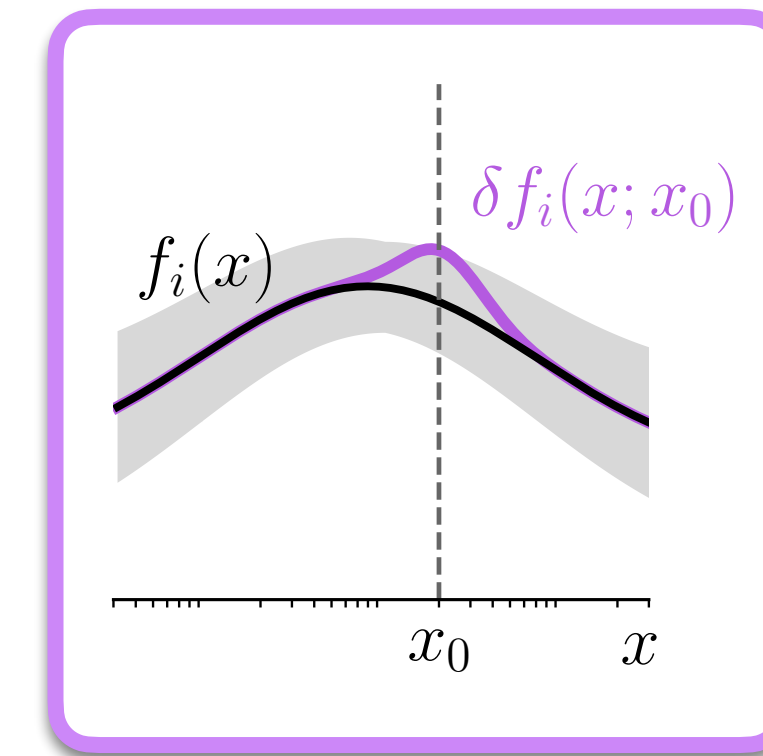
# What is the relation between data and PDF?

## Shapley Values for PDFs

The value of the marginal contribution  $\phi$  of a player  $i$  across all coalitions  $S$

$$\phi_i = \sum_{S \subset N \setminus i} w(S) [\nu(S \cup i) - \nu(S)]$$

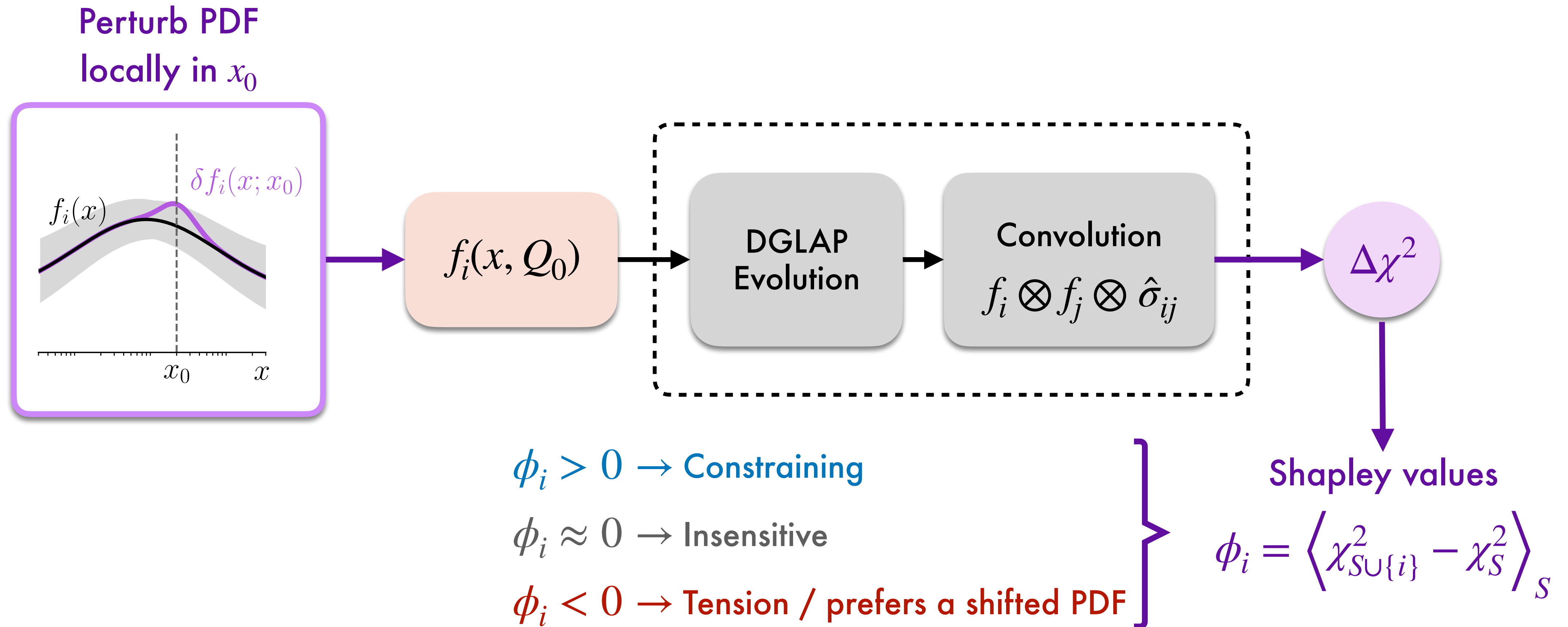
- $N$  the set of all players  $\rightarrow$  set of 8 independent PDFs  $\{f_j\}$
- $S$  a subset of players not including player  $i$
- $\nu(S)$  metric representing the value of coalition  $S \rightarrow \chi^2$



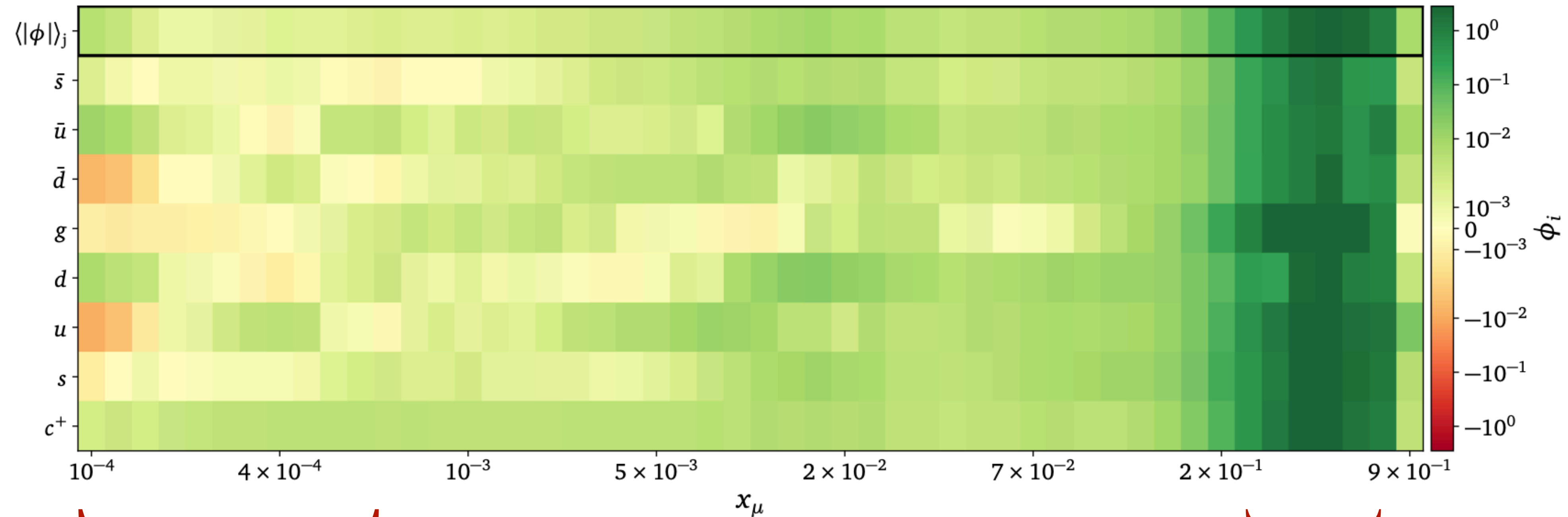
PDF perturbed in  $x_0$

$$f_j^{(S)} = \begin{cases} f_j^{\text{pert}}, & j \in S, \\ f_j^{(0)}, & j \notin S, \end{cases}$$

# Shapley values for PDFs



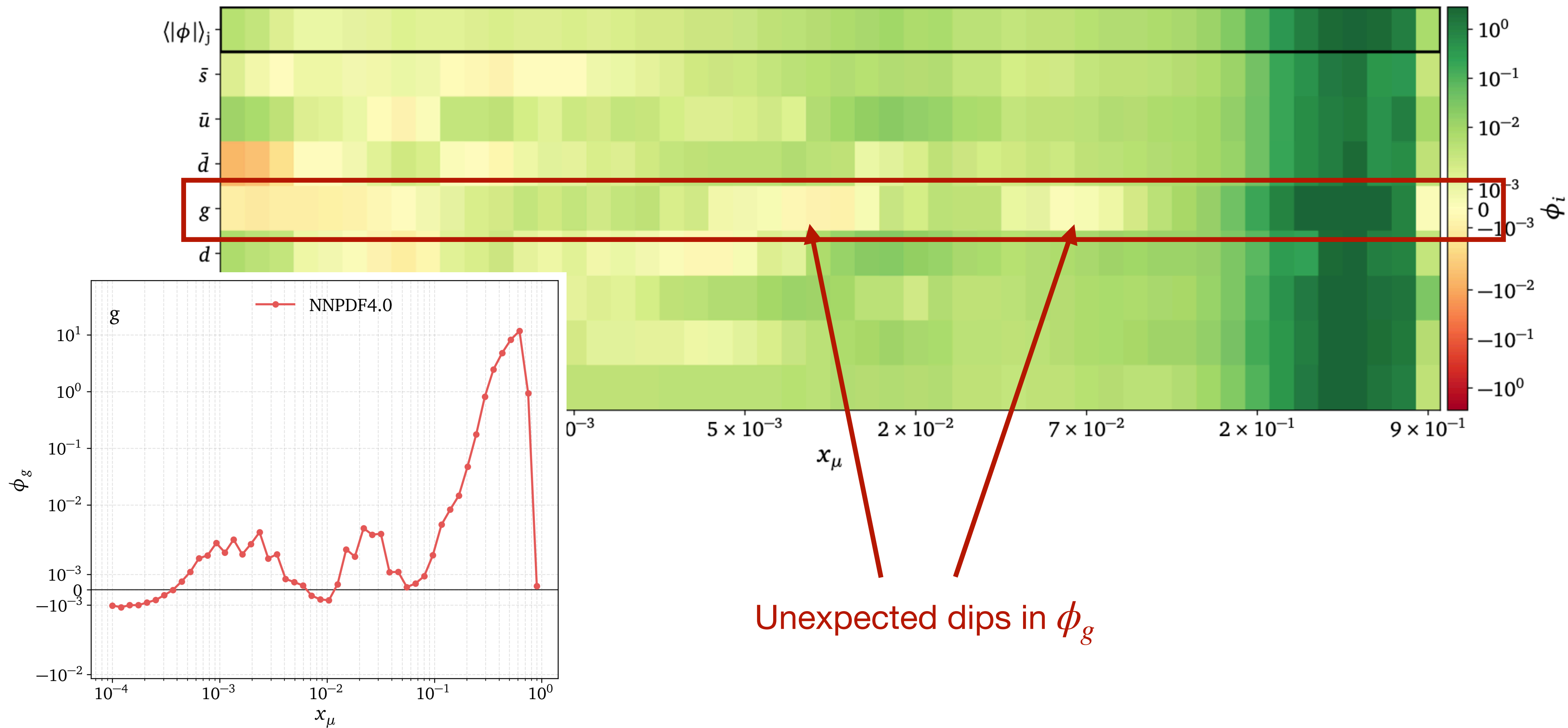
# Shapley values for all flavour basis PDFs



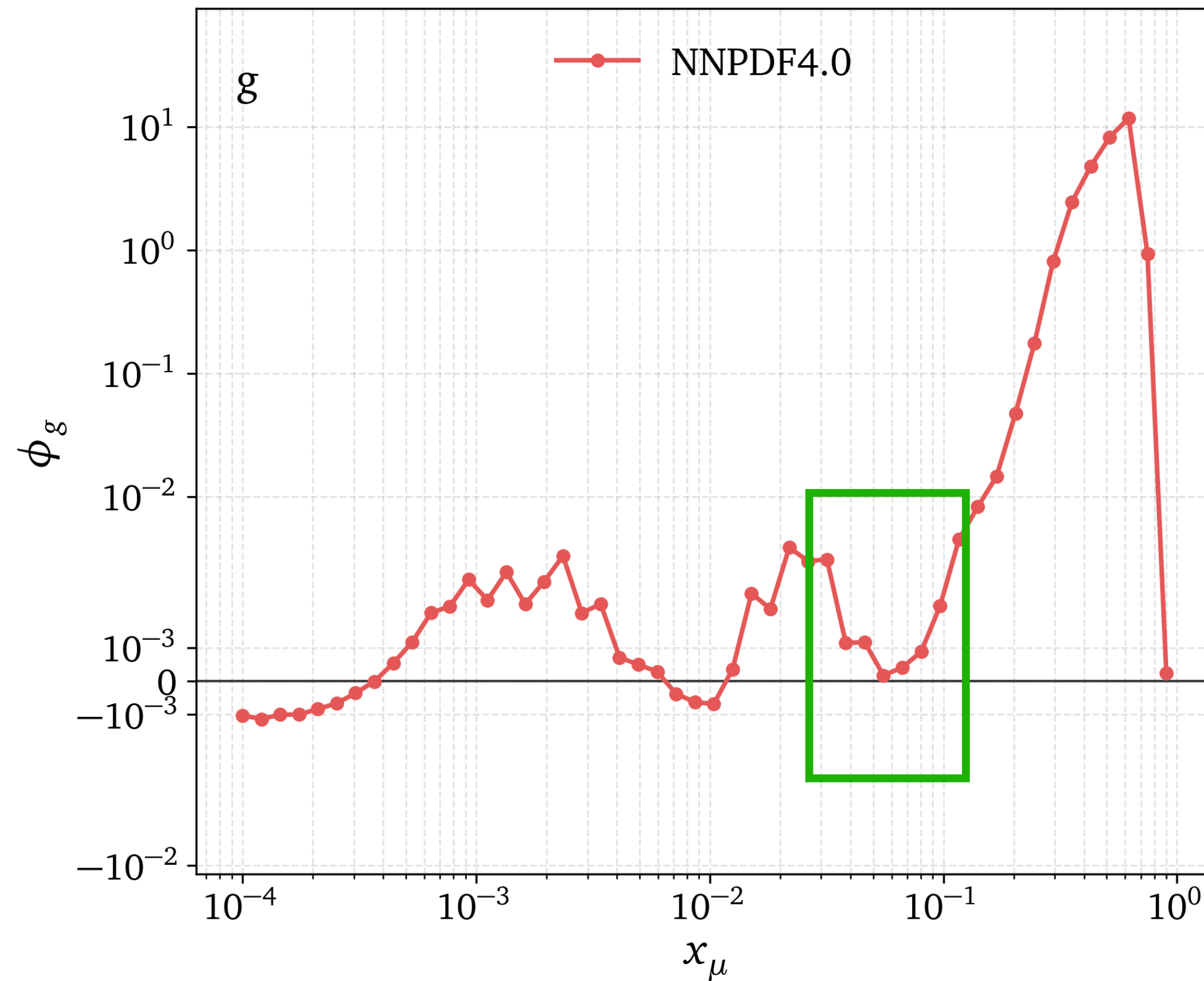
No constraints from  
data in extrapolation  
region

Data at intermediate  
 $x$  most constraining

# Shapley values for all flavour basis PDFs



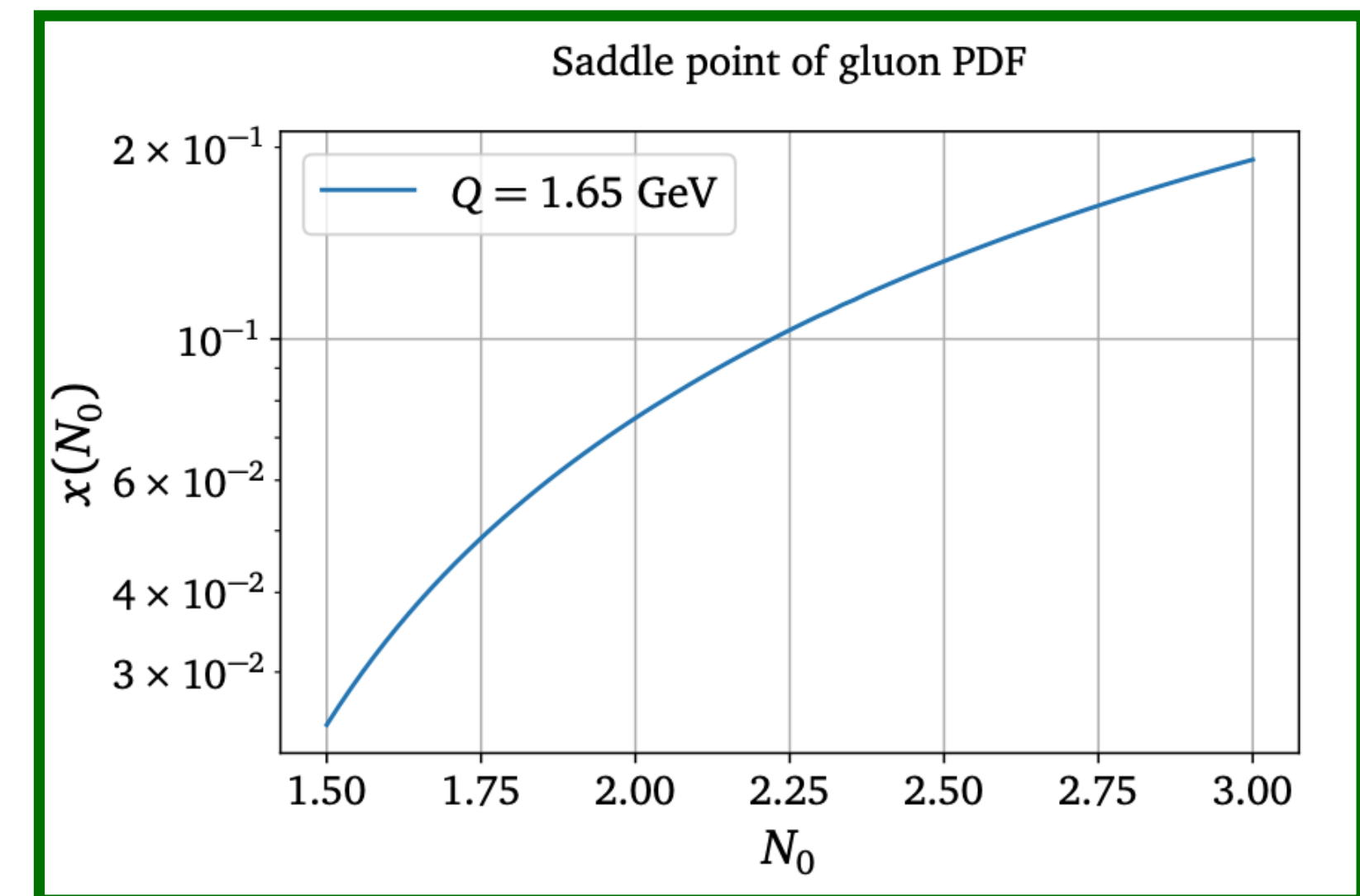
# The gluon Shapley Value



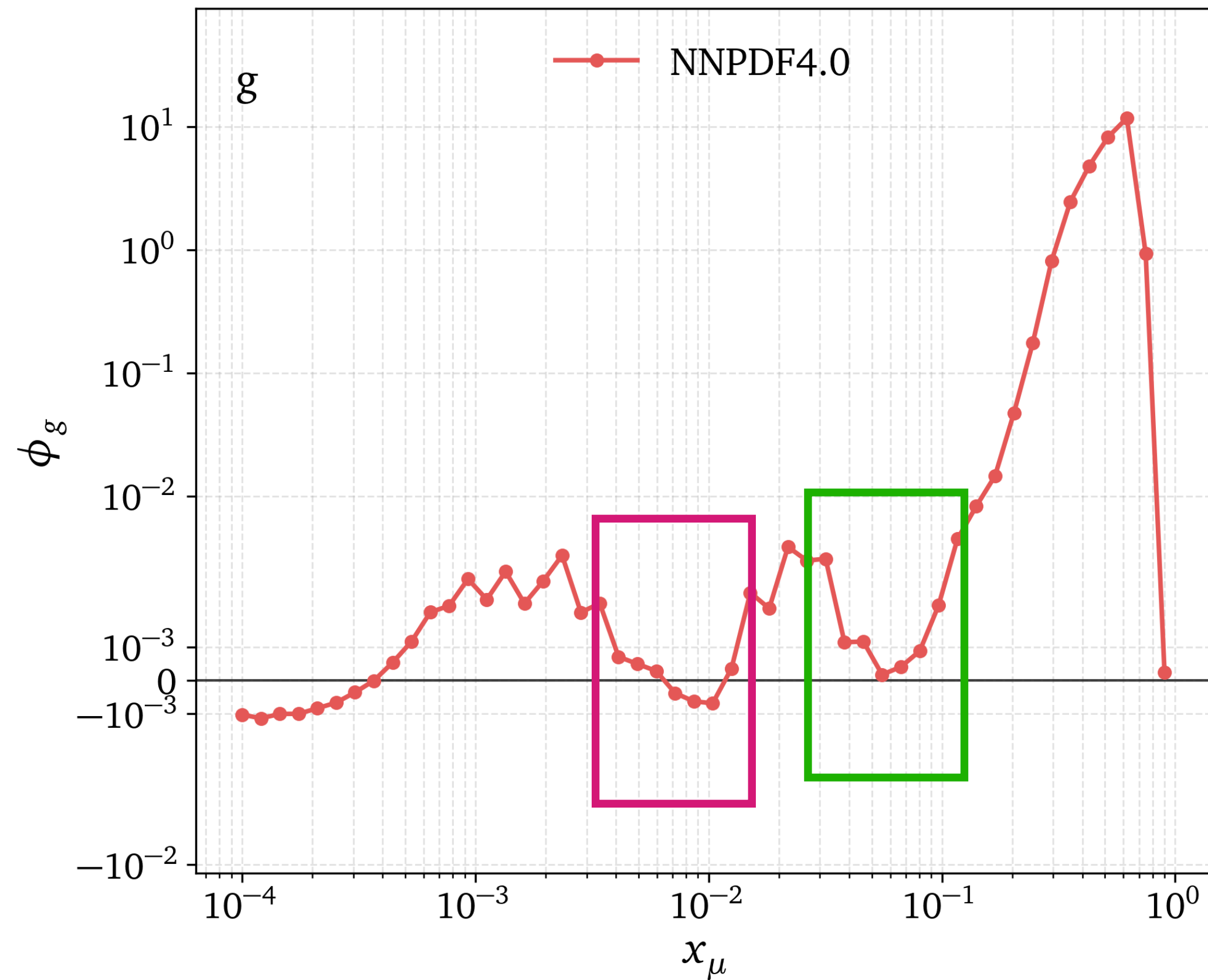
**Dip at  $x \sim 0.07$**

→  $\gamma_{ij}(N)$  has zero eigenvalue at  $N = 2$  due to momentum sum rule

→  $N = 2$  maps to  $x = 0.07$  from saddle-point



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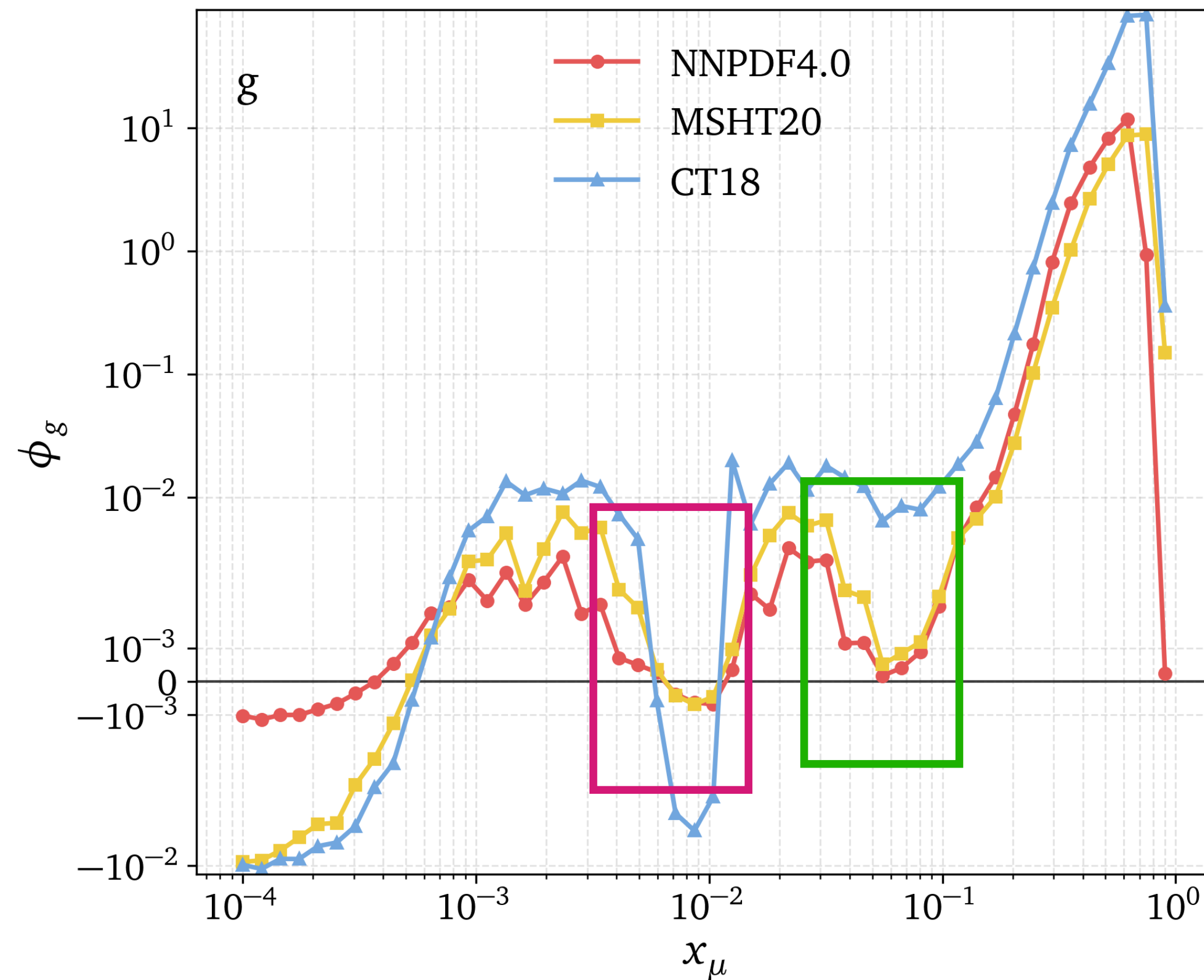
→  $N = 2$  maps to  $x = 0.07$  from saddle-point

**Dip at  $x \sim 0.01$**

→ region of  $gg \rightarrow H$

→ no clear origin

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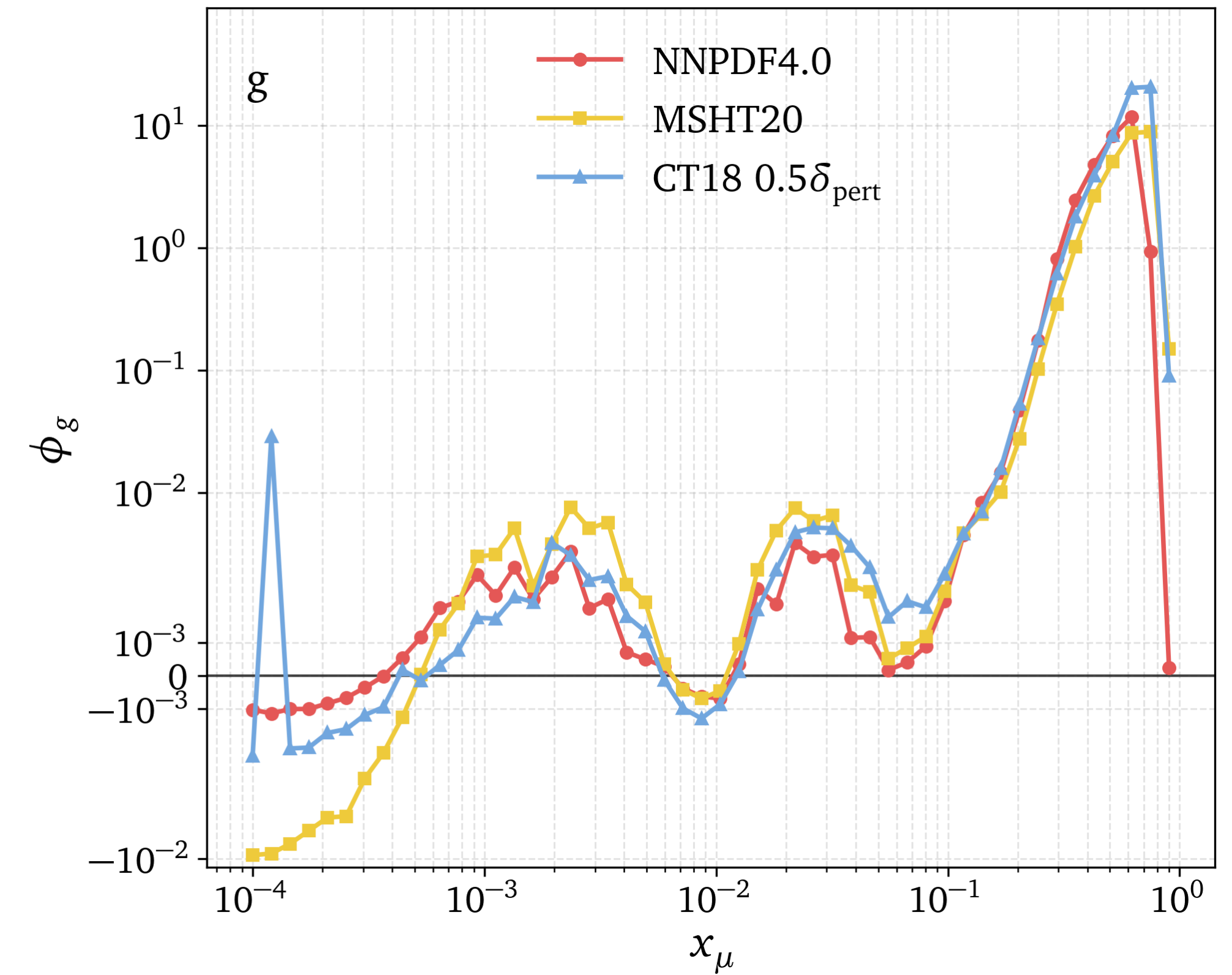
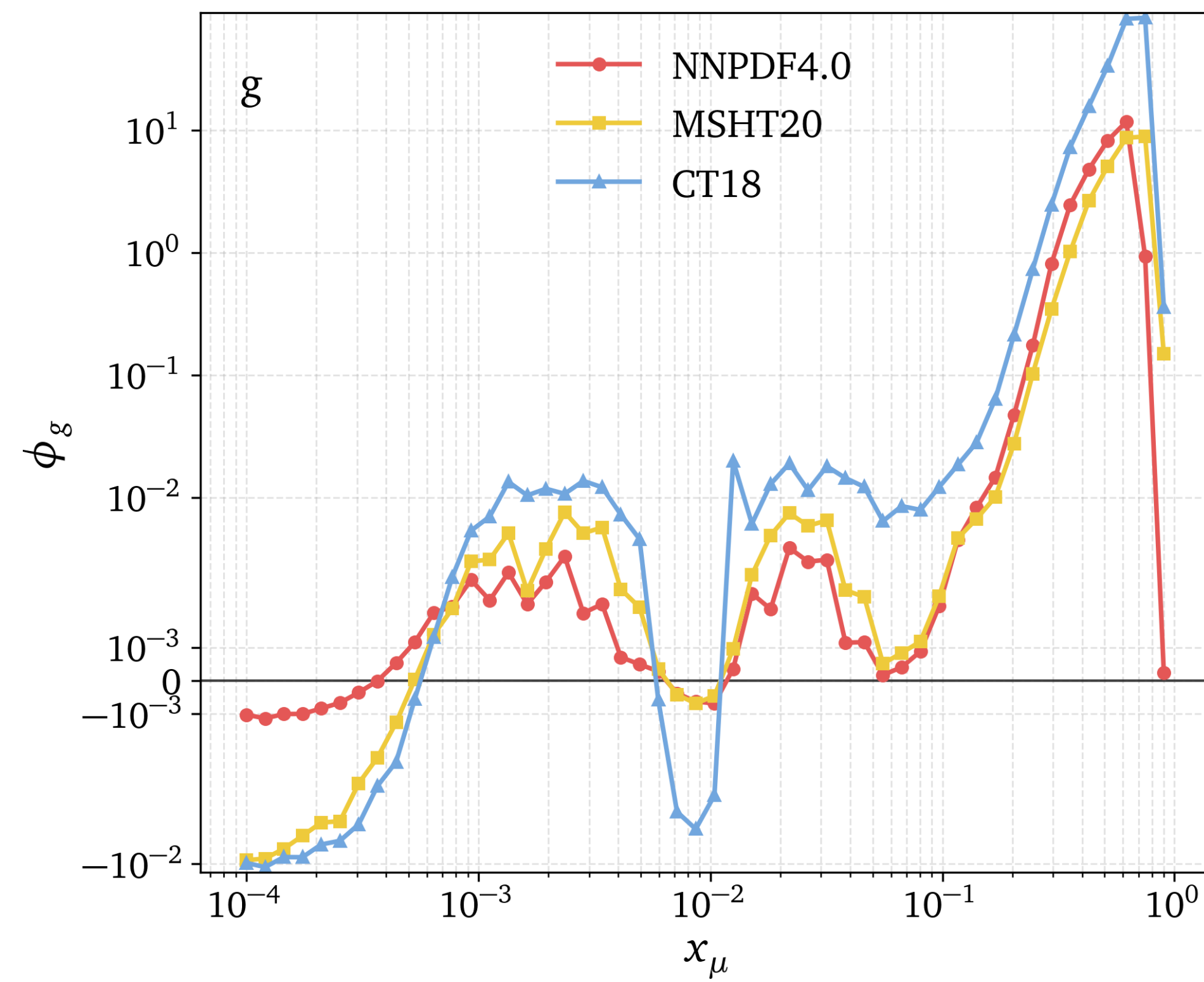
→ no clear origin

# Outlook

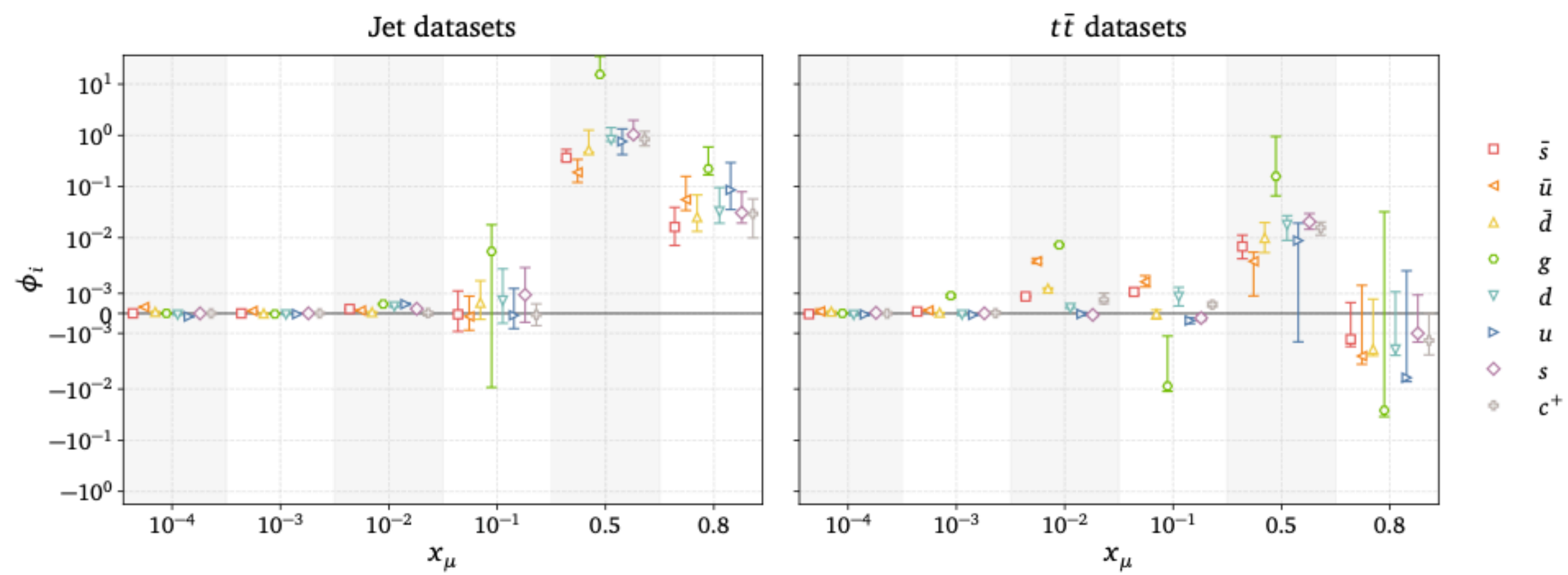
- We quantise which data have an impact on which PDFs and where, and vice versa
  - We recover the expected spectrum
  - And can show what usually gets fitted away
- Shapley values provide a diagnostic tool to assess the interplay between data and PDFs

# Backup slides

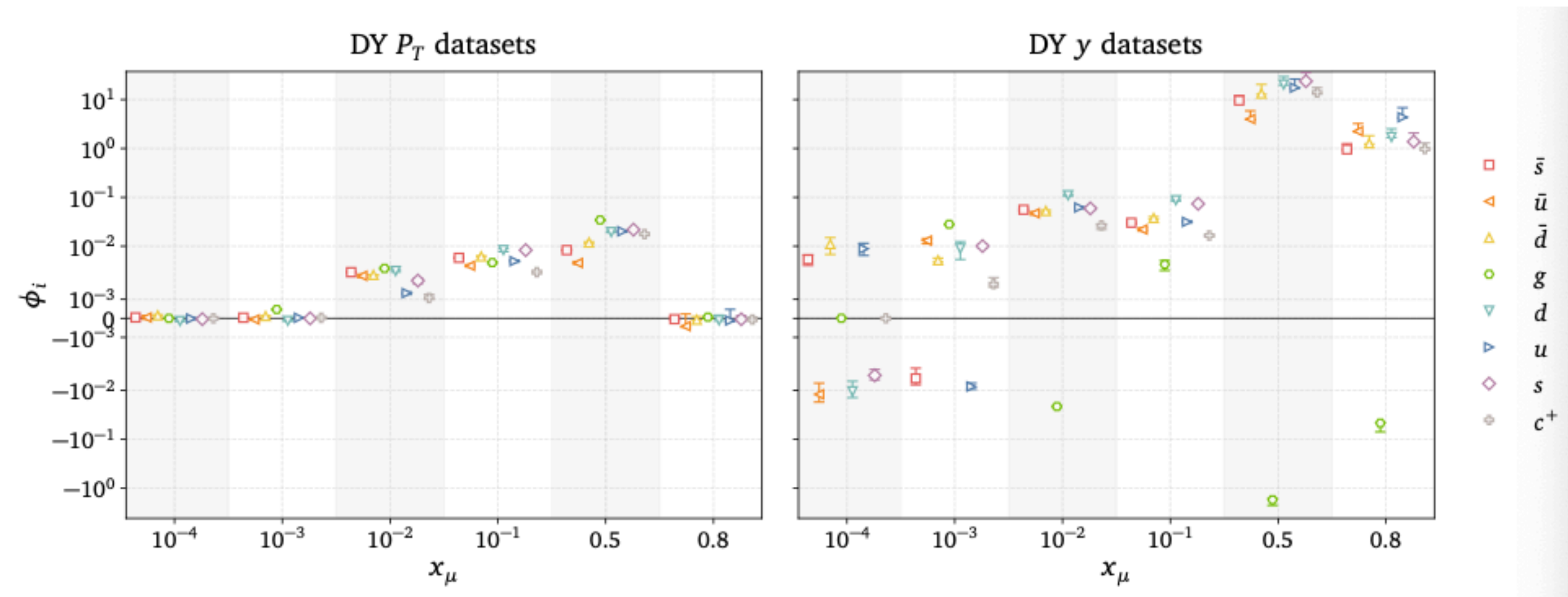
# CT18 with 0.5 bump



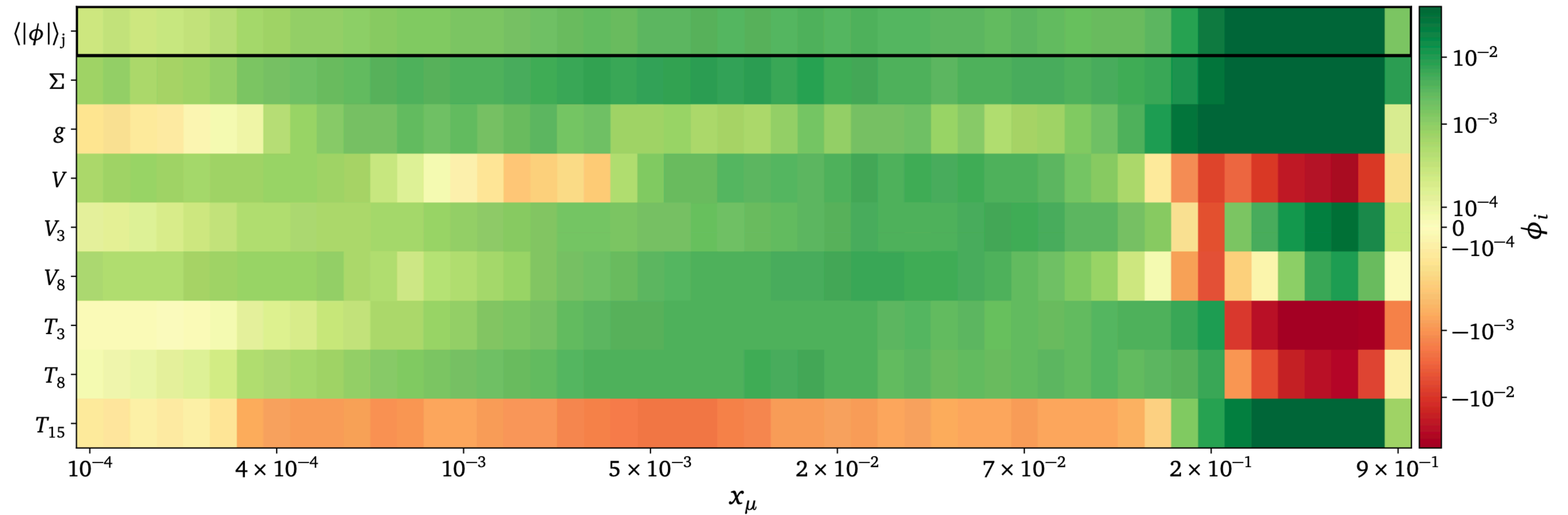
# Dataset impact: top and jets



# Dataset impact: Drell-Yan pt vs rapidity



# Evolution basis for full NNPDF4.0 dataset



# Weight of Shapley values

Weights for the importance of the coalition being tested

$$w(S) = \frac{|S|! (|N| - |S| - 1)!}{|N|!}$$

The number of different subsets S out of |N|-1 elements

- $|S|!$  # of ways to make the coalition
- $(|N|-|S|-1)!$  # of ways to make the not-coalition
- $|N|!$  # of ways to order all members

Each **coalition size** receives  $1/|N|=1/8$  of the total weight

# Calibrated perturbation

$$x f_j^{\text{pert}\pm}(x, Q_0^2) = x f_j(x, Q_0^2) + \delta_j^\pm(x),$$
$$\delta_j^\pm(x) = \pm \Delta_j^\pm(x_\mu) \exp \left[ -\frac{(\log_{10}(x) - \log_{10}(x_\mu))^2}{2w^2} \right],$$

$\delta^\pm$  within either positive or negative uncertainty band, either positive or negative applied to all PDFs in the coalition

→ then sum rules applied via normalisation  $A_i$

$$v(S) = \frac{1}{2} \left[ \chi^2 \left( f^{(S+)}; \mathcal{D} \right) + \chi^2 \left( f^{(S-)}; \mathcal{D} \right) \right],$$

Some next step might be to sample from negative and positive perturbation in one coalition

# Shapley value vs L2 sensitivity

| Shapley value                                                                                                                               | L2 sensitivity                                 |
|---------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------|
| Physical perturbation of PDF at localised x                                                                                                 | Change in fit parameters                       |
| Average marginal contribution to overall prediction of perturbation in PDF shape accross <b>all possible combinations of perturbed PDFs</b> | Gradient of $\chi^2$ wrt fit parameters of PDF |

→ Also capturing correlations between perturbed PDFs