

Gauge-Invariant Longitudinal Modes in



Electroweak Parton Shower

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EW Parton Shower

- One of the key components of all multi-purpose event generators: **process-independent parton shower**.
- Current meta: **QCD+QED** showers $\rightarrow$ satisfactory for most present analyses.
- At $\sqrt{s} \gg m_W$, W/Z , Higgs, and heavy quarks behave quasi-massless $\Rightarrow$ **partonic degrees of freedom**.
- Expectation supported by LHC observations of EW radiation.
[arXiv:1507.04548, arXiv:1807.08639]
- EW virtual corrections grow $\sim \alpha_W \ln^2(s/m_W^2)$ and are negative. [arXiv:hep-ph/0005316]
- Motivates **process-independent EW PS** $\rightarrow$ upgrade to **QCD+QED+EW** scheme.
- Few existing implementations:
[arXiv:1305.6837, arXiv:1401.5238, arXiv:1403.4788, arXiv:2002.09248, arXiv:2108.10786]
- In Herwig since v7.3.0. [arXiv:2312.05175]

Parton Shower Kinematics in Herwig7

- A shower branching is denoted as $\widetilde{ij} \rightarrow i + j$ with.
- Differential cross-section for the production of the children:

$$d\sigma_{i+j} \simeq \frac{\alpha_{\text{int}}(\tilde{q}^2)}{2\pi} \frac{d\tilde{q}^2}{\tilde{q}^2} dz P_{\widetilde{ij} \rightarrow i+j}(z, \tilde{q}) d\sigma_{\widetilde{ij}}$$

- The **helicity-dependent splitting function** can be written as

$$P_{\widetilde{ij} \rightarrow i+j}(z, \tilde{q}) = \sum_{\lambda_{\widetilde{ij}}, \lambda_i, \lambda_j} \left| H_{\widetilde{ij} \rightarrow i+j}(z, \tilde{q}; \lambda_{\widetilde{ij}}, \lambda_i, \lambda_j) \right|^2$$

- The **helicity amplitude** for the splitting: $H_{\widetilde{ij} \rightarrow i+j}(z, \tilde{q}; \lambda_{\widetilde{ij}}, \lambda_i, \lambda_j) = g_{\text{int}} F_{\lambda_{\widetilde{ij}}, \lambda_i, \lambda_j}^{\widetilde{ij} \rightarrow i+j}$
- The **vertex function** $F_{\lambda_{\widetilde{ij}}, \lambda_i, \lambda_j}^{\widetilde{ij} \rightarrow i+j}$ explicitly depends on Feynman rules of the splitting legs.
- g_{int} is the relevant coupling (QCD, QED, EW, BSM).
- *Quasi*-collinear approximation: $\tilde{q}_{\widetilde{ij} \rightarrow i+j}^2 \gg m_i^2, p_{\perp}^2, \quad m_i \rightarrow 0, \quad p_{\perp} \rightarrow 0, \quad \frac{m_i}{p_{\perp}} = \text{const.}$
- Rescaling: $m_i \rightarrow \lambda m_i, \quad p_{\perp} \rightarrow \lambda p_{\perp}.$

EW Splitting Functions

- Implementing IS and FS **EW parton shower** in Herwig 7.

[arXiv:2108.10817]

- ▷ Quark splittings (IS and FS)

$$q \rightarrow q' W^\pm, \quad q \rightarrow q Z^0, \quad q \rightarrow q H$$

- ▷ Gauge boson splittings (FS only)

$$W^\pm \rightarrow W^\pm Z^0, \quad W^\pm \rightarrow W^\pm \gamma, \quad Z^0 \rightarrow W^+ W^-, \quad \gamma \rightarrow W^+ W^-, \\ W^\pm \rightarrow W^\pm H, \quad Z^0 \rightarrow Z^0 H$$

- ▷ EW decays (already included in Herwig)

$$W^\pm \rightarrow f \bar{f}', \quad Z^0 \rightarrow f \bar{f}, \quad H \rightarrow f \bar{f}$$

- ▷ **Generalised AO EW PS**: Updated SM splittings / Included BSM Splittings; accommodating CP-even/CP-odd couplings and FCNCs splittings. [arXiv:2312.13125]

$$\phi \rightarrow \phi' \phi'', \quad f \rightarrow f' \phi, \quad V \rightarrow V' \phi \\ \phi \rightarrow \phi' V, \quad f \rightarrow f' V, \quad V \rightarrow V' V''$$

- Going to a full IS+FS EW PS requires incorporating EW PDFs and folding QCD and EW effects into a unified set of evolution equations → only relevant in the massless EW theory.

Helicity-dependent $q \rightarrow q'V$ Splitting Functions

– Incoming/Outgoing fermion spinors:

[arXiv:2108.10817]

$$u_{\pm\frac{1}{2}}(p) = \begin{pmatrix} \delta_{+\frac{1}{2}} \frac{m_0}{\sqrt{2p}} \lambda \\ \delta_{-\frac{1}{2}} \sqrt{2p} \left(1 + \frac{m_0^2 \lambda^2}{8p^2} \right) \\ \delta_{+\frac{1}{2}} \sqrt{2p} \left(1 + \frac{m_0^2 \lambda^2}{8p^2} \right) \\ \delta_{-\frac{1}{2}} \frac{m_0}{\sqrt{2p}} \lambda \end{pmatrix}, \quad \bar{u}_{\pm\frac{1}{2}}(q_1) = [A_{\pm}, B_{\pm}, C_{\pm}, D_{\pm}]$$

$$A_+ = D_- = \sqrt{2zp} \left(1 + \frac{m_0^2 \lambda^2}{8p^2} \right), \quad B_+ = -C_-^* = \frac{e^{-i\phi} p_{\perp} \lambda}{\sqrt{2zp}}, \quad C_+ = B_- = \frac{m_1}{\sqrt{2zp}} \lambda, \quad D_+ = -A_-^* = \frac{e^{-i\phi} p_{\perp} m_1 \lambda^2}{(2zp)^{3/2}}$$

– Gauge boson polarisation vectors:

$$\epsilon_{\lambda_2=\pm 1}^{\mu}(q_2) = \left[0; -\frac{\lambda_2}{\sqrt{2}} \left(1 - \frac{p_{\perp}^2 \lambda^2 e^{i\lambda_2 \phi} \cos \phi}{2p^2(1-z)^2} \right), -\frac{i}{\sqrt{2}} + \frac{\lambda_2 p_{\perp}^2 \lambda^2 e^{i\phi} \sin \phi}{2\sqrt{2}p^2(1-z)^2}, -\frac{\lambda_2 p_{\perp} \lambda e^{i\lambda_2 \phi}}{\sqrt{2}(1-z)p} \right]$$

$$\epsilon_{\lambda_2=0}^{\mu}(q_2) = \left[\frac{p(1-z)}{\lambda m_2} + \frac{p_{\perp}^2 + m_0^2(1-z)^2 - m_2^2}{4p(1-z)m_2} \lambda; \cos \phi \left(\frac{p_{\perp}}{m_2} - \frac{m_2 p_{\perp} \lambda^2}{2p^2(1-z)^2} \right), \right.$$

$$\left. -\sin \phi \left(\frac{p_{\perp}}{m_2} - \frac{m_2 p_{\perp} \lambda^2}{2p^2(1-z)^2} \right), \frac{p(1-z)}{\lambda m_2} - \frac{p_{\perp}^2 - m_0^2(1-z)^2 - m_2^2}{4p(1-z)m_2} \lambda \right]$$

Longitudinal EW Modes: SL vs GI

- The longitudinal polarisation vector contains a formally large p^μ/m_V component:

$$\epsilon_0^\mu(p) = \frac{p^\mu}{m_V} - \frac{m_V}{p \cdot n} n^\mu.$$

- In the broken EW theory, the corresponding Ward identity is

$$p_\mu \mathcal{M}^\mu(\dots V(p) \dots) = m_V \mathcal{M}(\dots G_V(p) \dots),$$

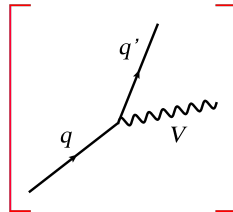
i.e. the p^μ/m_V contribution is related to the associated would-be Goldstone mode.

Subtraction-Longitudinal

- Dawson's prescription subtracts the p^μ/m_V component: [\[Nucl.Phys.B249\(1985\)42\]](#)

$$\epsilon_{0*}^\mu(p) = \epsilon_0^\mu(p) - \frac{p^\mu}{m_V}.$$

- Gives a finite longitudinal state suitable for the parton shower. [\[arXiv:2108.10817\]](#)
- Default longitudinal prescription used in Herwig 7 EW parton shower.



Longitudinal EW Modes: SL vs GI

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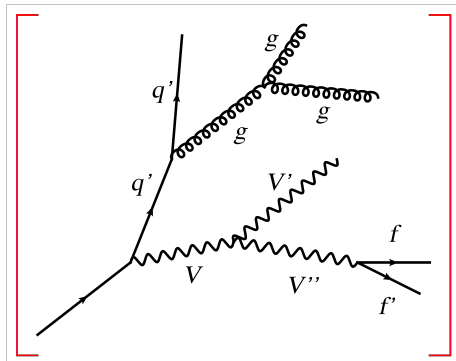
i.e. the p^μ/m_V contribution is related to the associated would-be Goldstone mode.

Gauge-Invariant

- Subtraction alone does not guarantee the reproduction of the full Ward identity.
- Complete the SL remainder by the corresponding Goldstone contribution: [\[arXiv:2602.19927\]](#)

$$\hat{H}_{0*}^{\text{GI}} = \hat{H}_{0*}^{\text{SL}} + c_G \hat{H}_{0*}^{G_V}, \quad c_G = 1$$

- Transverse modes are unchanged.



Goldstone Matching for $q \rightarrow q'V$

- Start from the vector current $J_V^\mu = \bar{u}_{\lambda_1}(q_1)\gamma^\mu (g_L P_L + g_R P_R) u_{\lambda_0}(q_0)$

and the decomposition $\epsilon_0^\mu(q_2) = \epsilon_{0*}^\mu(q_2) + q_2^\mu/m_2$

- The piece removed in the SL prescription therefore gives

$$\Delta \hat{H}_{\lambda_0 \lambda_1} = \frac{1}{\mathcal{D}} \frac{q_{2\mu}}{m_2} J_V^\mu, \quad \mathcal{D} = \sqrt{2(q_0^2 - m_0^2)}$$

- Using the broken-theory Ward identity,

$$q_{2\mu} J_V^\mu = m_2 J_{G_V}, \quad J_{G_V} = \bar{u}_{\lambda_1}(q_1) (y_L P_L + y_R P_R) u_{\lambda_0}(q_0)$$

the missing contribution is obtained directly from the corresponding Goldstone current.

- Inserting the quasi-collinear spinors gives

$$\hat{H}_{--0*}^{(G_V)} = \frac{\sqrt{2}(zm_0 y_R + m_1 y_L)}{\sqrt{z} \mathcal{D}}, \quad \hat{H}_{++0*}^{(G_V)} = \frac{\sqrt{2}(zm_0 y_L + m_1 y_R)}{\sqrt{z} \mathcal{D}},$$

$$\hat{H}_{-+0*}^{(G_V)} = -\frac{\sqrt{2} y_L p_\perp}{\sqrt{z} \mathcal{D}} e^{-i\phi}, \quad \hat{H}_{+-0*}^{(G_V)} = -\frac{\sqrt{2} y_R p_\perp}{\sqrt{z} \mathcal{D}} e^{i\phi}$$

- These are added to the SL longitudinal amplitudes:

[arXiv:2602.19927]

$$\hat{H}_{\lambda_0 \lambda_1 0*}^{\text{GI}} = \hat{H}_{\lambda_0 \lambda_1 0*}^{\text{SL}} + c_G \hat{H}_{\lambda_0 \lambda_1 0*}^{(G_V)} \quad \Rightarrow \quad F^{\text{GI}} = F^{\text{SL}} + 2 c_G \text{Re}(H_{\text{SL}} H_G^*) + c_G^2 |H_G|^2$$

Helicity-dependent $q \rightarrow q'V$ Splitting Functions

- The transverse contribution is identical in the two prescriptions:

$$F_T^{\text{SL}} = F_T^{\text{GI}}.$$

- The **SL** contribution is

$$F_{L;\text{SL}}^{q \rightarrow q'V} = \frac{m_2^2}{\tilde{q}^2(1-z)^3} \left(g_R^2 \rho_{++} + g_L^2 \rho_{--} \right).$$

- The **GI** longitudinal contribution is

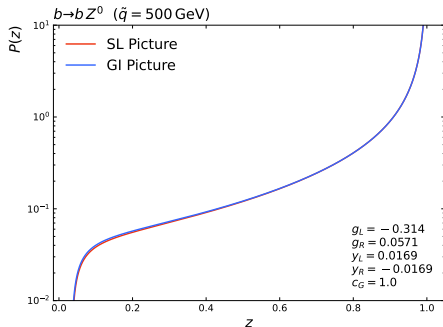
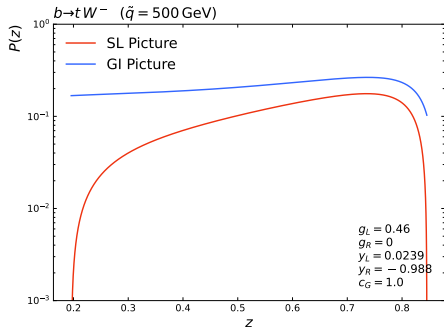
[arXiv:2602.19927]

$$\begin{aligned} F_{L;\text{GI}}^{q \rightarrow q'V} = & \frac{m_2^2}{\tilde{q}^2(1-z)^3} \left(g_R^2 \rho_{++} + g_L^2 \rho_{--} \right) \\ & - \frac{\sqrt{2} c_G m_2}{\tilde{q}^2 z (1-z)^2} \left[(g_R \rho_{++} y_L + g_L \rho_{--} y_R) m_0 z + (g_R \rho_{++} y_R + g_L \rho_{--} y_L) m_1 \right] \\ & + \frac{c_G^2}{2\tilde{q}^2 z (1-z)} \left[\left(\rho_{--} y_L^2 + \rho_{++} y_R^2 \right) \left(m_0^2 + m_1^2 - m_2^2 + \tilde{q}^2 z (1-z)^2 \right) \right. \\ & \left. + 4m_0 m_1 y_R y_L (\rho_{--} + \rho_{++}) \right]. \end{aligned}$$

- $c_G = 1$: gauge-invariant SM prescription; $c_G = 0$: pure SL prescription.
- $\text{GI} = \text{SL} + \text{Goldstone-vector interference} + \text{pure Goldstone contribution}.$

SL vs GI: $q \rightarrow q'V$ Splitting Functions

- Spin-averaged quasi-collinear splitting functions at fixed scale $\tilde{q} = 500$ GeV. [arXiv:2602.19927]



- Goldstone matching is enhanced by the heavy-quark Yukawa couplings.
 - Visible difference between the SL and GI splitting functions.
 - Goldstone couplings are suppressed by m_b/m_W .
 - SL and GI are practically indistinguishable.
- The corrections are largest where **EW symmetry-breaking and Yukawa effects** are important.

Helicity-dependent $V \rightarrow V'V''$ Splitting Functions

- The purely transverse contribution is identical in the two prescriptions:

$$F_{TTT}^{\text{SL}} = F_{TTT}^{\text{GI}}.$$

- The sectors containing a single longitudinal state differ between SL and GI:

$$F_{TTL}^{\text{SL}} = 2m_2^2(\rho_{--} + \rho_{++}) \frac{z}{\tilde{q}^2(1-z)^3},$$

$$F_{TTL}^{\text{GI}} = 2m_2^2(\rho_{--} + \rho_{++}) \left[\frac{z}{\tilde{q}^2(1-z)^3} + \frac{c_G^2}{\tilde{q}^2(1-z)z} \right],$$

$$F_{TLT}^{\text{SL}} = 2m_1^2(\rho_{--} + \rho_{++}) \frac{1-z}{\tilde{q}^2 z^3},$$

$$F_{TLT}^{\text{GI}} = 2m_1^2(\rho_{--} + \rho_{++}) \left[\frac{1-z}{\tilde{q}^2 z^3} + \frac{c_G^2}{\tilde{q}^2(1-z)z} \right],$$

$$F_{LTT}^{\text{SL}} = 4m_0^2 \rho_{00} \frac{1-z}{\tilde{q}^2 z},$$

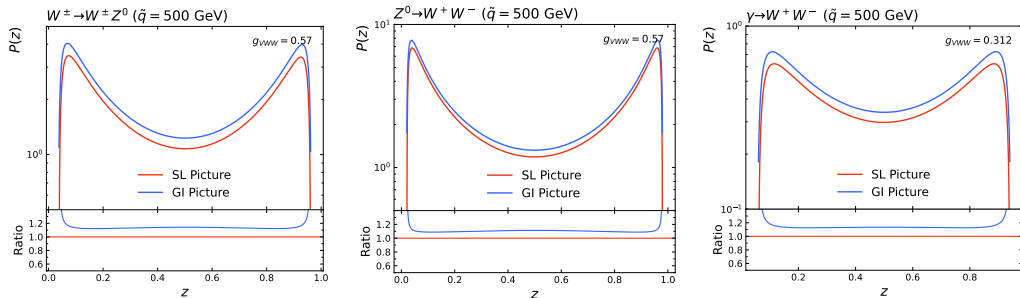
$$F_{LTT}^{\text{GI}} = 4m_0^2 \rho_{00} \left[\frac{1-z}{\tilde{q}^2 z} + \frac{c_G(c_G + 2z - 2)}{\tilde{q}^2(1-z)z} \right].$$

- TLL , LTL , LLT and LLL vanish at leading quasi-collinear order.
- Differences are **EW symmetry-breaking mass effects** $\sim m_i^2/\tilde{q}^2$ and vanish at high scales.

[arXiv:2602.19927]

SL vs GI: $V \rightarrow V'V''$ Splitting Functions

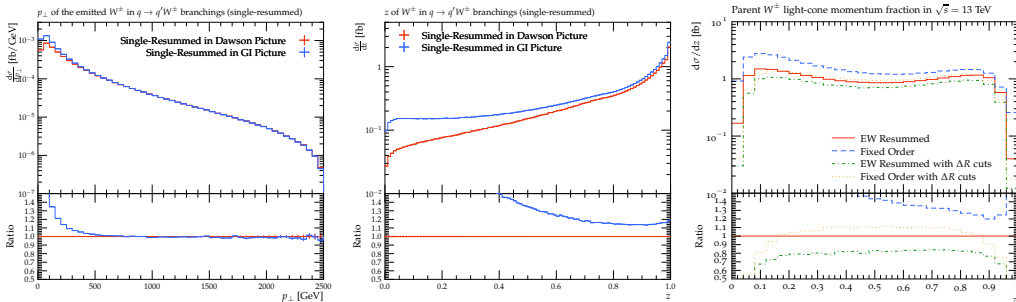
- Spin-averaged quasi-collinear splitting functions at fixed scale $\tilde{q} = 500$ GeV. [[arXiv:2602.19927](#)]



- ▷ The transverse contribution is identical in SL and GI.
- ▷ The GI completion modifies the single-longitudinal sectors through mass-dependent terms.
- ▷ A smooth, channel-dependent enhancement is visible, with the largest differences towards the edges of the allowed z range.
- The effect is suppressed for $\tilde{q} \gg m_{EW}$, where the SL and GI prescriptions converge.

Validation

- The original **SL implementation** was validated against fixed-order predictions and phenomenological observables. [\[arXiv:2108.10817, arXiv:2012.14746, arXiv:2112.15487\]](#)
- The **GI implementation** uses the same shower framework. [\[arXiv:2602.19927\]](#)



- Single-resummed $q \rightarrow q'W^\pm$ evolution: SL and GI agree at high $p_\perp$, while longitudinal differences appear in the low- $p_\perp$ and mapped- z regions.

Implementation

- The implementation has been rigorously validated.

[arXiv:2108.10817]

- To set up the $\text{QCD} \oplus \text{QED} \oplus \text{EW}$ PS:

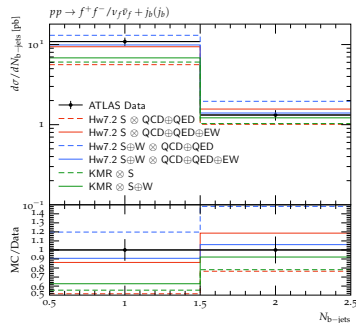
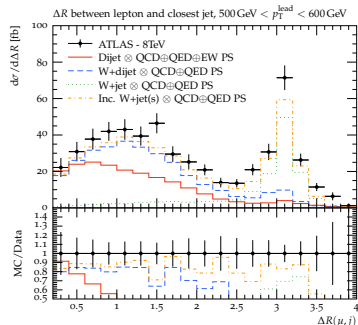
```
cd Herwig/Shower/  
set ShowerHandler:Interactions ALL
```

- Options: QEDQCD, QCD, QED and EWOnly.

- Longitudinal prescription:

[arXiv:2602.19927]

```
cd Herwig/Shower/  
set <Split class>:LongitudinalEWScheme Subtraction  
with GaugeInvariant now being a selectable option.
```



Summary

EW Parton Shower in Herwig 7:

- Developed a process-independent, angular-ordered EW parton shower.
- Helicity-dependent quasi-collinear splitting functions for fermions, gauge bosons and scalars, preserving spin correlations and QCD coherence.
- Generalised the shower framework to arbitrary $1 \rightarrow 2$ splittings from UFO models, providing direct support for BSM shower evolution.

Longitudinal EW Modes:

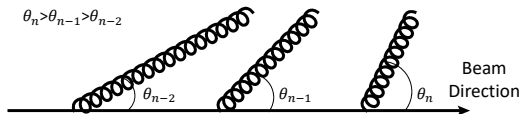
- EW shower uses the **SL**/**GI** prescriptions to ensure finite longitudinal splitting functions.
- The new **GI** prescription completes the **SL** current using the broken-theory Ward identity and the corresponding Goldstone contribution.
- Differences arise only in the longitudinal sector and are controlled by **EW symmetry-breaking and Yukawa effects**.
- SL and GI coincide at high evolution scales, while the largest differences occur in longitudinally sensitive low-scale and heavy-flavour configurations.
- The GI implementation shows controlled behaviour at both kernel and shower level and can be selected independently within the existing Herwig 7 EW shower framework.

For more details, please visit:
herwig.hepforge.org

Thank You!

Backup Slides!

Angular Ordering from QCD Coherence



- In the soft limit, emission amplitude from a colour dipole (ij):

$$\mathcal{M}_{ij}(k) \propto g_s \left(\frac{p_i}{p_i \cdot k} - \frac{p_j}{p_j \cdot k} \right); \quad |\mathcal{M}(k)|^2 \propto \underbrace{[\cos \phi_{ik} - \cos \phi_{jk}]}_{\text{interference term}} \frac{\theta_k > \theta_{ij}}{\phi_{ik} \approx \phi_{jk}} \rightarrow 0$$

- ϕ_{ik} is the polar opening angle between the emitted gluon k and leg i ,

$$\cos \phi_{ik} = (\vec{p}_i \cdot \vec{k}) / (|\vec{p}_i| |\vec{k}|)$$

- Large-angle soft gluons only see the *total colour charge* of the dipole $\Rightarrow$ destructive interference outside θ_{ij} :

$$\text{Quasi-collinear soft limit} \quad \Rightarrow \quad p_{T,n}^2 \simeq z_n(1 - z_n) E^2 \theta_n^2$$

- Successive emissions (in backward ISR evolution) must satisfy:

$$\theta_n > \theta_{n-1} > \theta_{n-2} > \dots \quad \equiv \quad y_n > y_{n-1} > y_{n-2} > \dots \quad \text{with} \quad y_k = \frac{1}{2} \ln \frac{E_k + p_{k,\parallel}}{E_k - p_{k,\parallel}}$$

Angular Ordering Evolution Scales

- **p_T-preserving** (HERWIG, Herwig++): [\[NPB 310 \(1988\) 461-526; arXiv:hep-ph/0311208\]](#)

$$\tilde{q}_{ij \rightarrow i,j}^2 = \frac{|p_T|^2 - zm_2^2}{(1-z)^2} \rightarrow p_T^2 = z^2(1-z)^2\tilde{q}^2 + m_0^2z(1-z) - m_1^2(1-z) - m_2^2z$$

- **q²-preserving** (Herwig-7.0): [\[arXiv:1708.01491\]](#)

$$\tilde{q}_{ij \rightarrow i,j}^2 = \frac{q_0^2 - m_0^2}{z(1-z)} \rightarrow p_T^2 = z^2(1-z)^2\tilde{q}^2 + m_0^2z(1-z) - q_1^2(1-z) - q_2^2z$$

- **dot-product preserving** (Herwig-7.3): [\[arXiv:1904.11866, arXiv:2107.04051\]](#)

$$\tilde{q}_{ij \rightarrow i,j}^2 = \frac{2q_1 \cdot q_2 + m_1^2 + m_2^2 - m_0^2}{z(1-z)}, \quad \tilde{q}^2 > 2 \max \left(\frac{m_1^2}{z^2}, \frac{m_2^2}{(1-z)^2} \right)$$
$$p_T^2 = z^2(1-z)^2\tilde{q}^2 - q_1^2(1-z)^2 - q_2^2z^2 + z(1-z) [m_0^2 - m_1^2 - m_2^2]$$

- For heavy quark radiation,

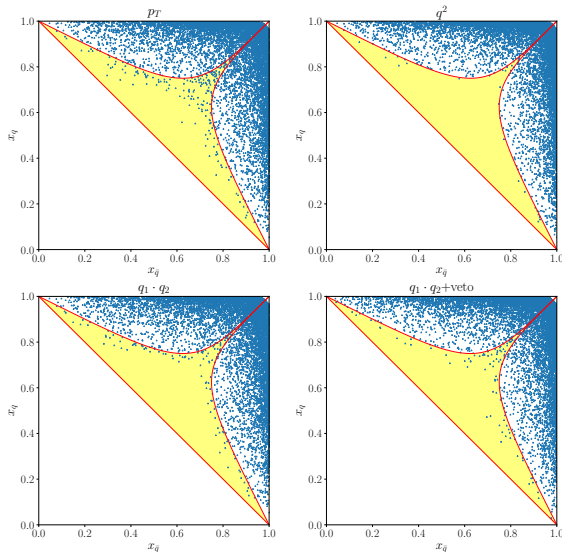
$$p_T \geq (1-z)m_Q \rightarrow \theta > m_Q/E \rightarrow p_T^{\text{cutoff}} \sim \mathcal{O}(m_c)$$

For 3rd generation quarks, a small fraction of the events end up in the dead zone any way, but these are subleading and don't impact the logarithmic accuracy.

Angular Ordering from QCD Coherence

The AOC is:

- A *physical consequence* of **gauge invariance** & **colour coherence**.
- Is realised in PS by ordering in an evolution variable $\tilde{q}$ that is *monotonic with emission angle* in the soft limit.
- The choice of $\tilde{q}^2$ definition:
 - Mapping between $\tilde{q}$ and θ for hard/massive splittings.
 - Phase-space coverage (dead zones).
 - Logarithmic accuracy.
- Built-in **LL+NLL accuracy**.
- Only required calculating the helicity-dependent splitting kernels.



[arXiv:1904.11866]