

Photon pair production at next-to-next-to-next-to-leading order

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THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES



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the European Union



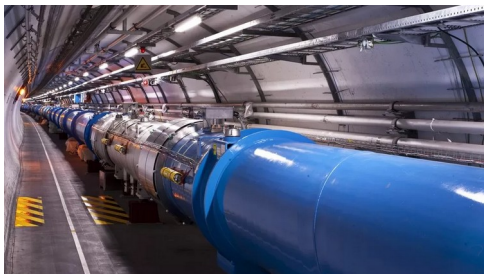
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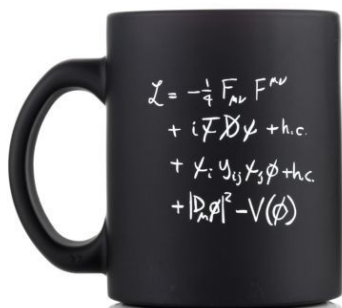
What are the fundamental building blocks of matter?

Scattering experiments

Large Hadron Collider (LHC)



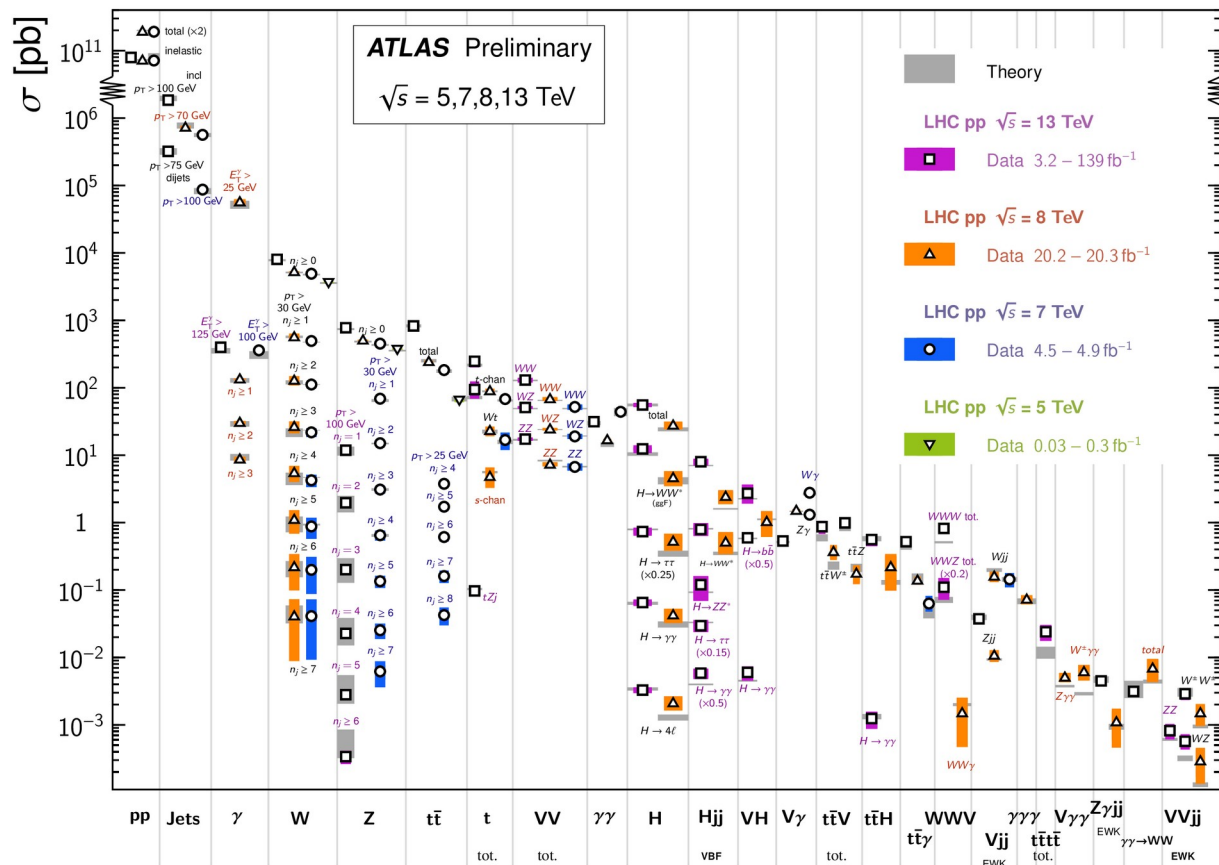
Credit: CERN



Theory/ Standard Model

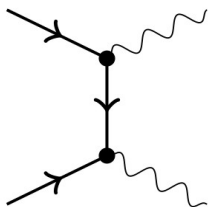
Standard Model Production Cross Section Measurements

Status: February 2022



Fixed-order QCD in collinear factorization

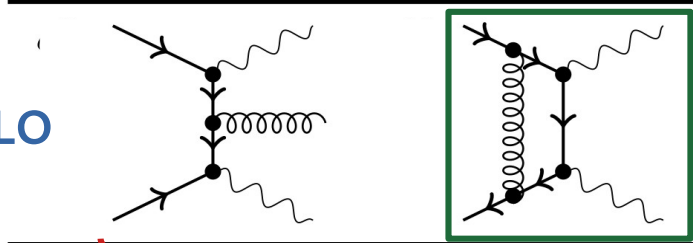
LO



$$\sigma_{h_1 h_2 \rightarrow X} = \sum_{ij} \int_0^1 \int_0^1 dx_1 dx_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j,h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

partonic cross section

NLO



$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \hat{\sigma}_{ab \rightarrow X}^{(1)} + \hat{\sigma}_{ab \rightarrow X}^{(2)} + \mathcal{O}(\alpha_s^3)$$

$$\hat{\sigma}^{(i)} \ni \frac{1}{\mathcal{F}} \int |\mathcal{M}_n|^2 d\Phi_n$$

IR-finite cross section

NNLO

Automated fixed-order @ NLO

- **oneloop-matrix elements**

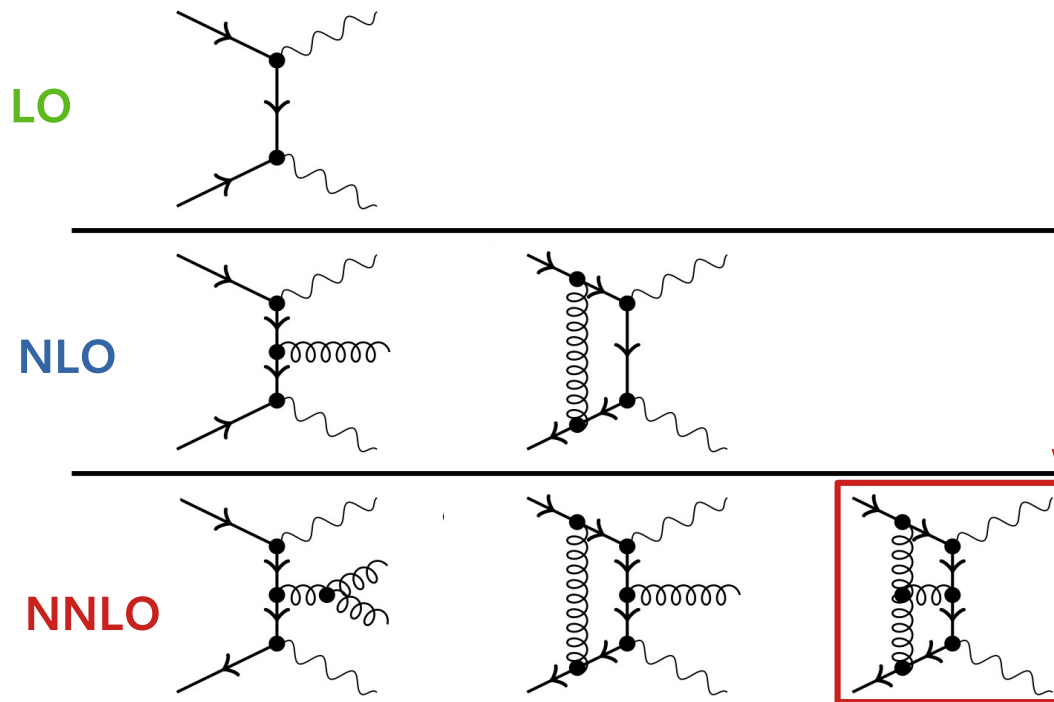
- Openloops, Recola, Helac, MadGraph, GoSam

- **local QCD/EW subtraction**

- FKS: MadGraph5_aMC@NLO, Powheg

- CS dipoles: Sherpa, Herwig

NNLO QCD challenges: two-loop amplitudes



How to compute

multi-scale two-loop QCD amplitudes?

- **fast growing complexity:**
rational coef. and **special functions**
- deeper understanding of the analytical properties
- refinement of computational tools

Two-loop 5-point

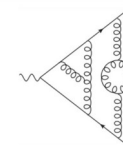
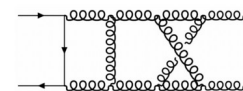
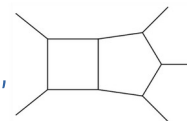
[Abreu, Agarwal, Badger, Buccioni, Chawdhry, Chicherin, Czakon, de Laurenties, Febres-Cordero, Gambuti, Gehrmann, Henn, Lo Presti, Manteuffel, Ma, Mitov, Page, Peraro, Poncelet, Schabinger, Sotnikov, Tancredi, Zhang,...]

Three-loop 4-point

[Bargiela, Dobadilla, Canko, Caola, Jakubcik, Gambuti, Gehrmann, Henn, Lim, Mella, Mistleberger, Wasser, Manteuffel, Syrrakow, Smirnov, Tancredi, ...]

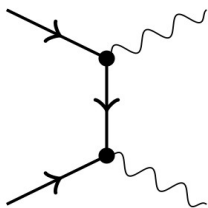
Four-loop 3-point

[Henn, Lee, Manteuffel, Schabinger, Smirnov, Steinhauser,...]

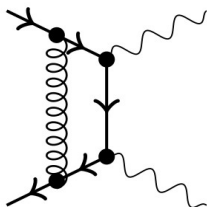
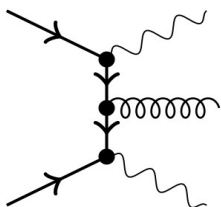


NNLO QCD challenges: subtraction

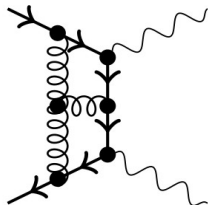
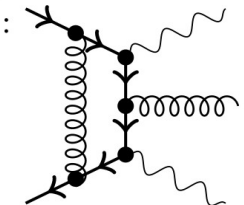
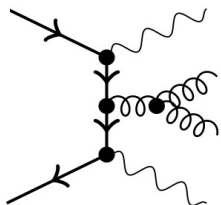
LO



NLO



NNLO



IR-finite cross section

How to achieve **infrared (IR) finite differential** cross sections at NNLO QCD?

~20 years to solve this problem

qT-slicing [Catani'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projettion [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Local Analytic [Magnea'18] Sector-improved residue subtraction [Czakon'10-'14,'19]

Automation of NNLO QCD subtraction:

- qT-slicing (massive/color singlet) in **MATRIX & MCFM** (public)
- Antenna subtraction in **NNLOJet** (public)
- Sector-improved residue subtraction in **Stripper** (private impl.)
- color singlets with nested-collinear in **history** (public)

NNLO QCD challenges: subtraction

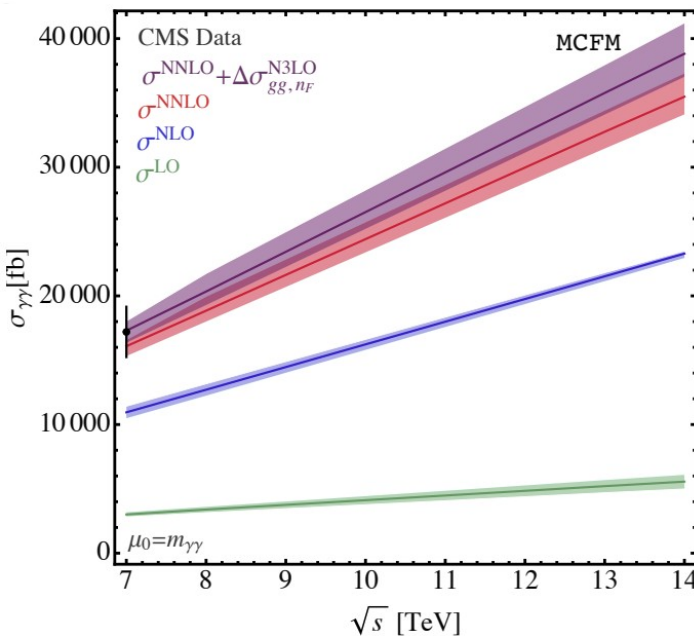
LO

NLO

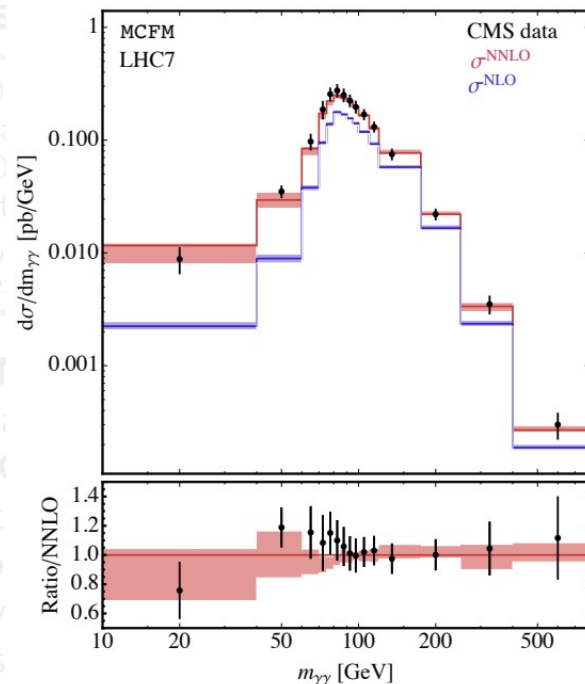
NNLO

NNLO QCD for diphton production [Catani, Cieri, de Florian, Ferrera, Grazzini, 1110.2375]

[Campbell, Ellis, Li, Williams, 1603.02663]

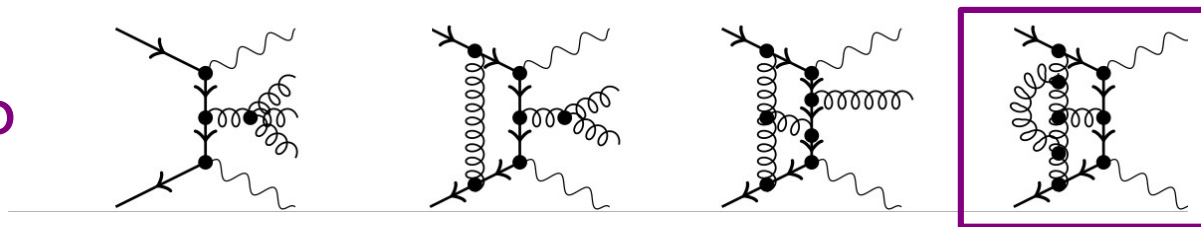


Large NNLO corrections & Loop induced sizeable



Ultimate precision for HL-LHC → towards N3LO QCD

N3LO

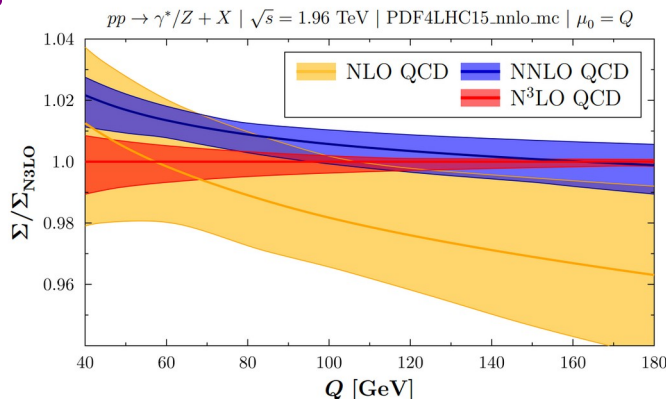


(Semi-)inclusive computations:

- Higgs/DY [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger, '15-'20]
- VH [Baglio, Duhr, Mistlberger, Szafron '22]
- (Di-)Higgs in VBF [Dreyer Karlberg'16'18]

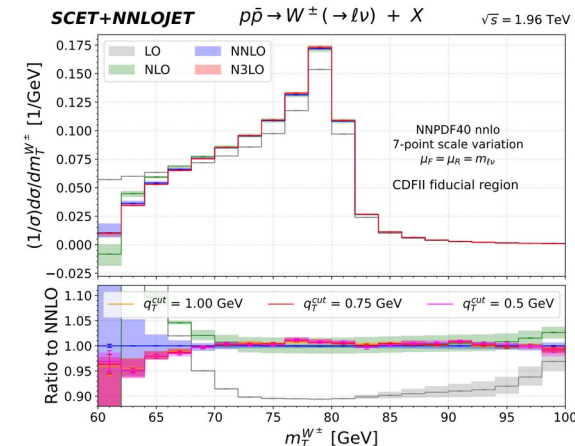
Fully differential cross sections

- Local e+e- → jets with Antenna [Chen, Jakubčík, Marcoli, Stagnitto'25]
- Projection-to-Born for Higgs in pp [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni]
- Local NNLO + qT slicing for hadron-hadron [Chen, Gehrmann, Glover, Huss, Yang, Zhu'21-'22]
- Double slicing in DIS [Czakon, Eschment, Generet, Poncelet'26]
- Double slicing in DIS [Dong, Fang, Gao, Li, Shao, Zhu, Zhu'26]



Three loop amplitudes:

- 2 → 2 partons/photons [Bargiela, Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi '20-'23]
- 2 → 2 with one massive leg:
 - H+jet [Chen, Guan, Mistlberger'25]
 - V+jet [Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi'23]



N3LO QCD with slicing methods

Combination of local subtraction
+ qT-slicing [Grazzini, Catani'07]

$$\sigma_{\text{N}^3\text{LO}}^{pp \rightarrow \gamma\gamma} = \int_0^{q_T^{\text{cut}}} dq_T \frac{d\sigma_{\text{N}^3\text{LO}}^{pp \rightarrow \gamma\gamma}}{dq_T} + \boxed{\int_{q_T^{\text{cut}}}^{\infty} dq_T \frac{d\sigma_{\text{NNLO}}^{pp \rightarrow \gamma\gamma j}}{dq_T}}$$

local subtraction:
sector-improved
residue subtraction
[Czakon'10' et al 14'19]

computational bottleneck for
 $q_T^{\text{cut}} \rightarrow 0$

QT-factorization →

N3LO TMDs [Ebert, Mistlberger, Vita '20] [Luo, Yang, Zhu, Zhu '20]

$$\frac{d\sigma^{pp \rightarrow \gamma\gamma}}{dq_T^2 dm_{\gamma\gamma} dy} = \sum_{a,b} H_{ab}(m_{\gamma\gamma}) \int \frac{d^2 \mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{q}_T \cdot \mathbf{b}_T} \times \tilde{f}_a(x_1, \mathbf{b}_T) \tilde{f}_b(x_2, \mathbf{b}_T) + \mathcal{O}(q_T/m_{\gamma\gamma})$$

$$\tilde{f}_i(x, \mathbf{b}_T) = \sum_j \int_x^1 \frac{dz}{z} \tilde{\mathcal{I}}_{ij}^{\text{TMD}}(z, \mathbf{b}_T) f_j\left(\frac{x}{z}\right) + \mathcal{O}(\mathbf{b}_T \Lambda_{\text{QCD}})$$

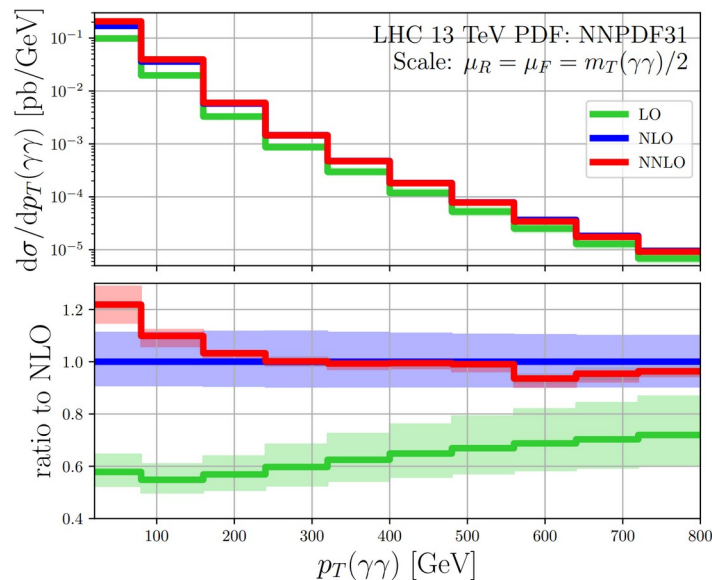
NNLO QCD for photon pair plus jet

Chawdhry, Czakon, Mitov, Poncelet [2103.04319]

with **leading color approx for two-loop**

Agarwal, Buccioni, von Manteuffel, Tancredi [2102.01820]

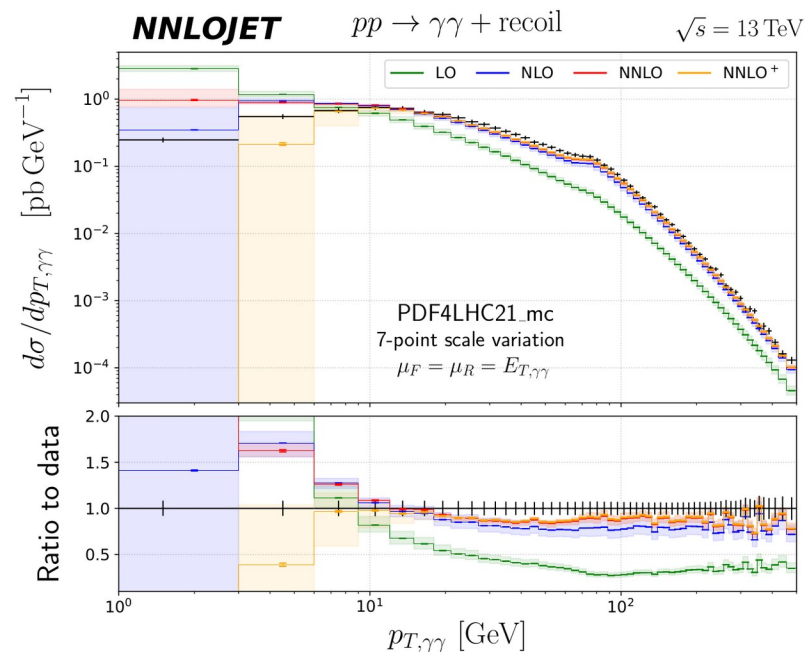
Chawdhry, Czakon, Mitov, Poncelet [2103.04319]



Buccioni, Chen, Feng, Gehrmann, Huss, Marcoli [2501.14021]

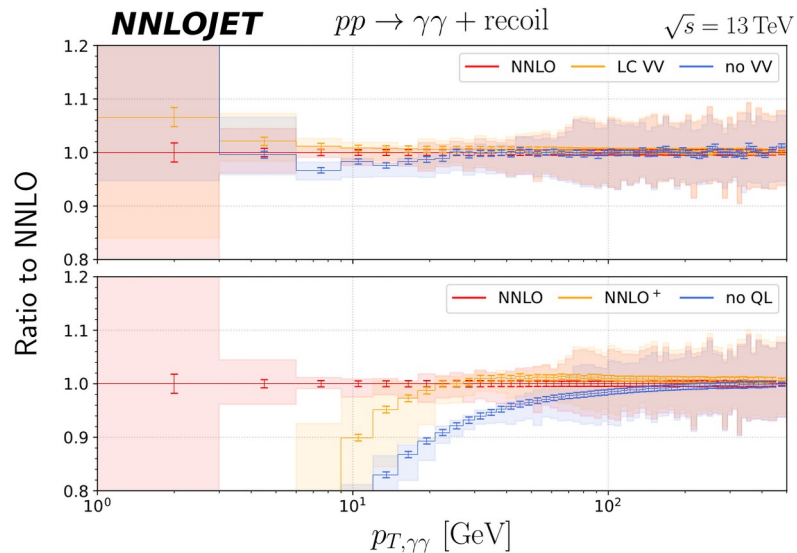
with **complete two-loop**

Agarwal, Buccioni, von Manteuffel, Tancredi [2105.04585]



Subleading color effects in di-photon + jet

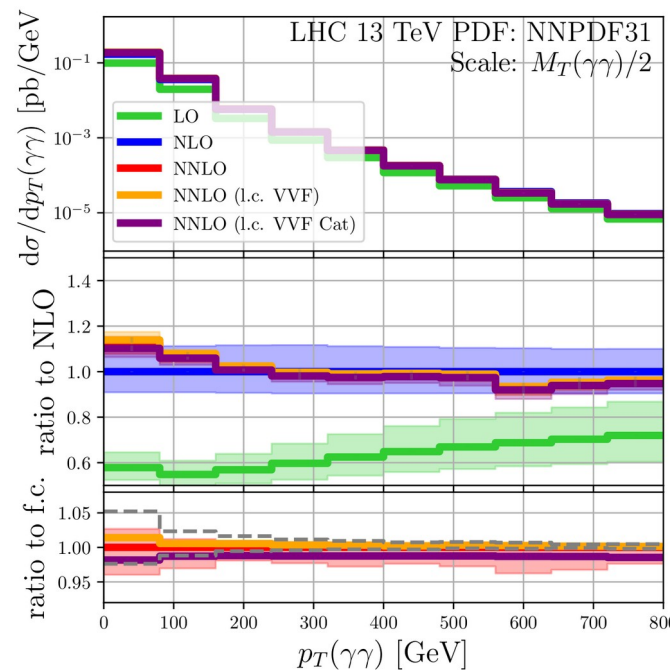
Buccioni, Chen, Feng, Gehrmann, Huss, Marcoli [2501.14021]



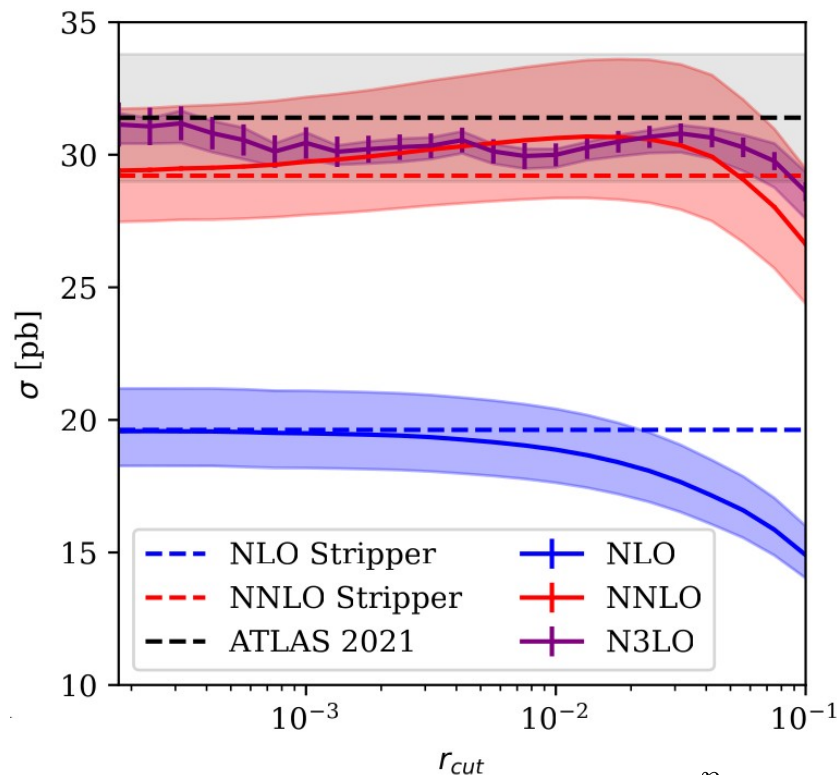
Observation:

two-loop / sub-leading color contribution indeed **small in the bulk**
but can be **important in the corners of phase space**
→ **full color crucial** for applications like qT subtraction

Czakon, Poncelet [2512.17591]



Diphoton total cross section



ATLAS fiducial phase space

$$p_{T,\gamma_{1(2)}} \geq 40(30) \text{ GeV}$$

$$\Delta R_{\gamma\gamma} \geq 0.4$$

$$|\eta_\gamma| < 2.37 \text{ excluding } 1.37 < |\eta_\gamma| < 1.52$$

hybrid photon isolation:

$$\text{soft cone: } E_T^{\text{max}}(r) = 0.15 E_{T,\gamma} \frac{1 - \cos(r)}{1 - \cos(R_{\text{max}}^s)}, \quad r < R_{\text{max}}^s$$

$$\text{hard cone: } E_T^{\text{max}}(r) = 0.09 E_{T,\gamma}, \quad r < R_{\text{max}}^h$$

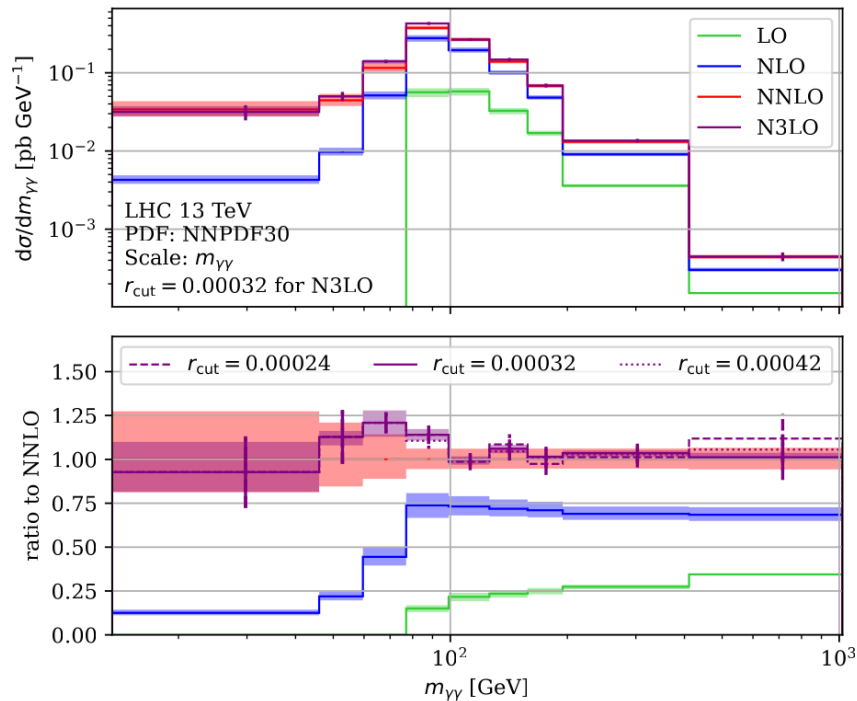
$$\sigma_{\text{N}^3\text{LO}}^{pp \rightarrow \gamma\gamma} = 31.2(6)^{+0.5}_{-0.7} \text{ pb } (\pm 0.6 \text{ pb power corrections})$$

$$\text{for reference: } \sigma_{\text{ATLAS}}^{pp \rightarrow \gamma\gamma} = 31.4 \pm 0.1 \text{ (stat.)} \pm 2.4 \text{ (syst.) pb}$$

$$\text{as evaluated @ } r_{\text{cut}} = 3.2 \cdot 10^{-4}$$

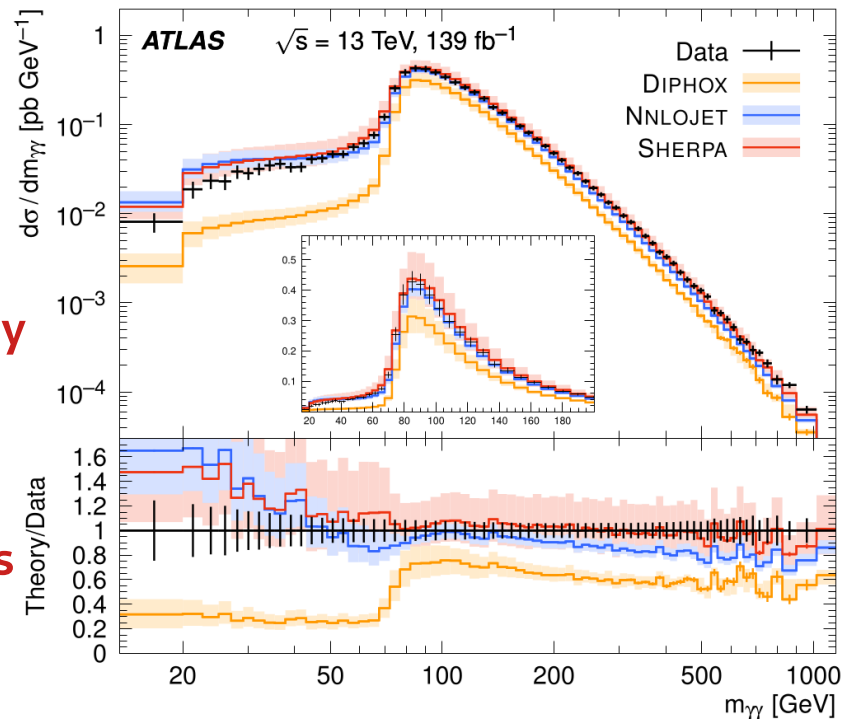
$$r_{\text{cut}} = \frac{p_{T,\gamma\gamma}}{m_{\gamma\gamma}} = 10^{-3} \rightarrow p_{T,\gamma\gamma} \approx 0.1 \text{ GeV (for } m_{\gamma\gamma} \approx 100 \text{ GeV @ PS threshold)}$$

Differential diphoton



Still a long way
to go...

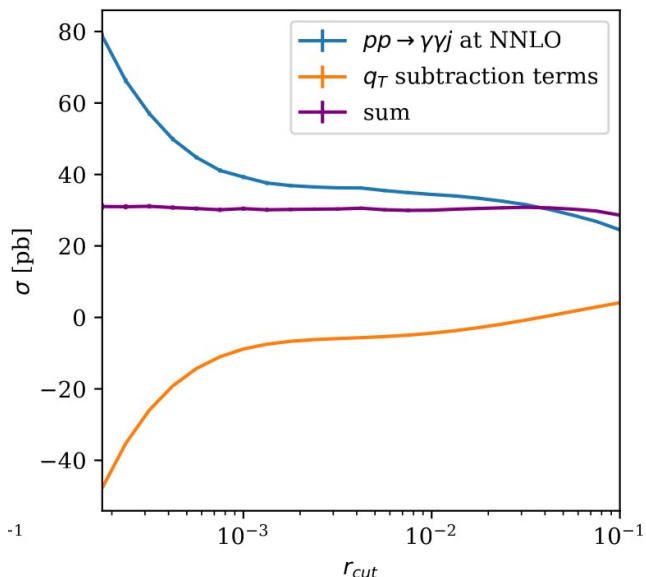
further eff.
improvements
necessary!



Numerical challenges

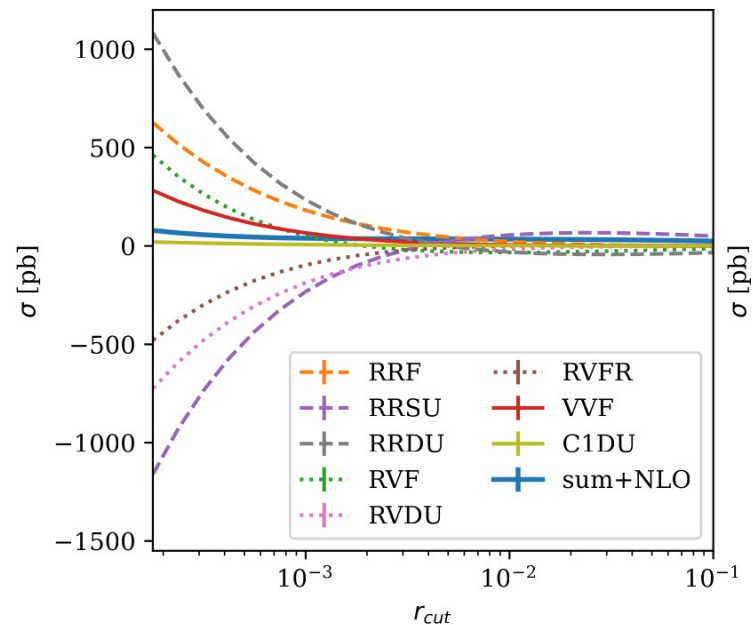
Large power corrections

- need to go very deep in q_T
- scheme needs to stay stable:
 - phase space
 - matrix elements
 - subtraction terms



Internal cancellations

- between different contributions
- large negative weight fraction



Stable six-point one loop matrix elements

usually we rely on **OpenLoops2** but

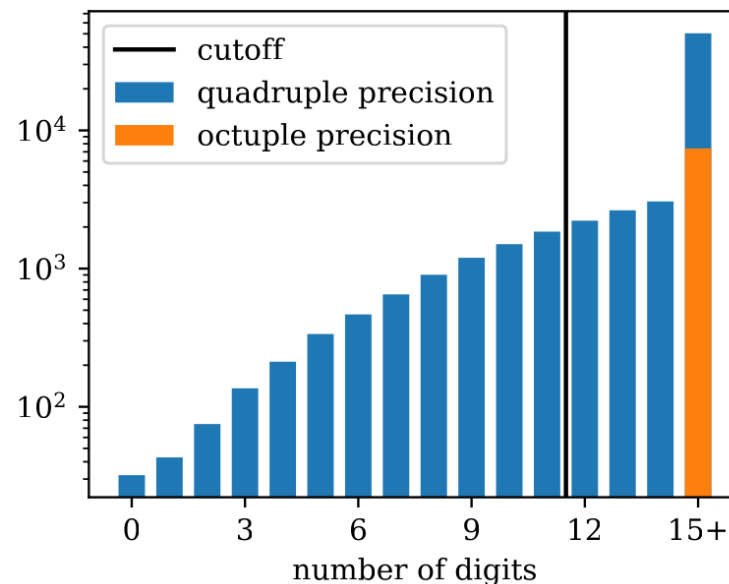
→ too inefficient in this case (qd stable but too slow)

→ **analytic expressions** in terms of logs, di-logs

Motivation: simplification in limits should be easier
to extract/implement
→ less high precision float evals

Finite field reconstructed amplitudes

- six-point kinematics is tricky:
 - vanishing Gram with Mandelstam var.
 - additional denominators and non transparent singularity structure with mom.-twistors
- following recipe in [De Laurentis, Page 2203.04269]
to reconstruct in terms of spinor products



0.5 secs per point on ave
(including checks and higher-precision)

Machine Learning for efficient Monte Carlo

Variance reduction through importance sampling:

$$I = \int_{\mathbf{H}(\mathbf{x}) \in \Omega} d\mathbf{H} \frac{f(\mathbf{x})}{h(\mathbf{x})} \quad h(\mathbf{x}) = \left| \det \left(\frac{\partial \mathbf{H}(\mathbf{x})}{\partial \mathbf{x}} \right) \right|$$

Task: find $h(\mathbf{x})$ adaptive MC techniques: VEGAS [Lepage'78], Parni [Hameren'14]

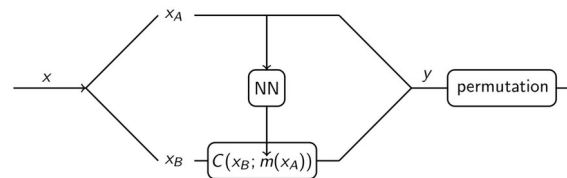
$$\hat{I} = \frac{1}{N} \sum_{i=1}^N f(\mathbf{x}_i), \quad \delta \hat{I} = \sqrt{\frac{1}{N-1} \left(\frac{1}{N} \sum_{i=1}^N f^2(\mathbf{x}_i) - \hat{I}^2 \right)}$$

Neural importance sampling

Normalising Flows in Sherpa [Bothmann'20], Madgraph [Heimel'22'25], STRIPPER [Jansen'25]

Coupling Layer Flows $\rightarrow$ series of simple bijections

Continuos Normalising Flows $\frac{d\psi_t(x)}{dt} = v_t(\psi_t(x))$



Non-positive definite integrands

- Non-definite integrands introduce new challenges
→ cancellation between +/- parts increase the variance
- Consider extreme case: $|f(x)|/h(x) = w = \text{const.}$

MC estimate:

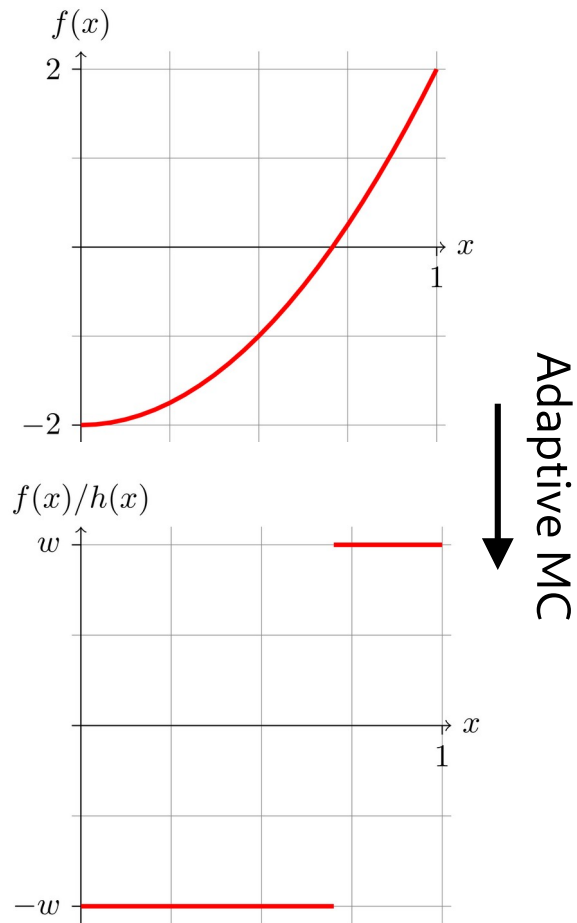
$$\hat{I} = w \frac{N_+ - N_-}{N} \equiv w(2\alpha - 1) \quad \alpha = N_+/N$$

- Lower bound on variance:

$$\text{Var}(\hat{I}) = w^2 - w^2(2\alpha - 1)^2 = w^2(4\alpha(1 - \alpha))$$

→ relative uncertainty: $\frac{\delta \hat{I}}{\hat{I}} = \frac{1}{\sqrt{N-1}} \frac{\sqrt{\alpha(1-\alpha)}}{\alpha - \frac{1}{2}}$

Rephrased: at some point it doesn't matter any more how good your adaptive MC is...



Stratification of signed integrands

$$f(\mathbf{x}) = f_+(\mathbf{x}) + f_-(\mathbf{x}), \quad \text{with} \quad f_{\pm}(\mathbf{x}) = \Theta(\pm f(\mathbf{x}))f(\mathbf{x})$$

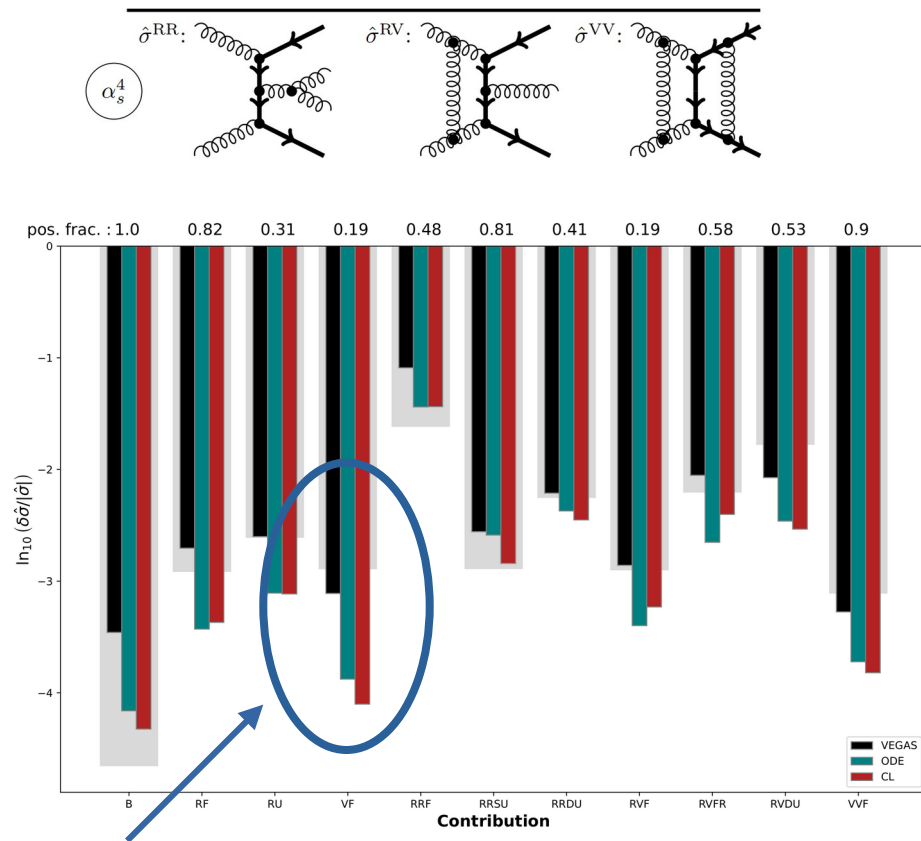
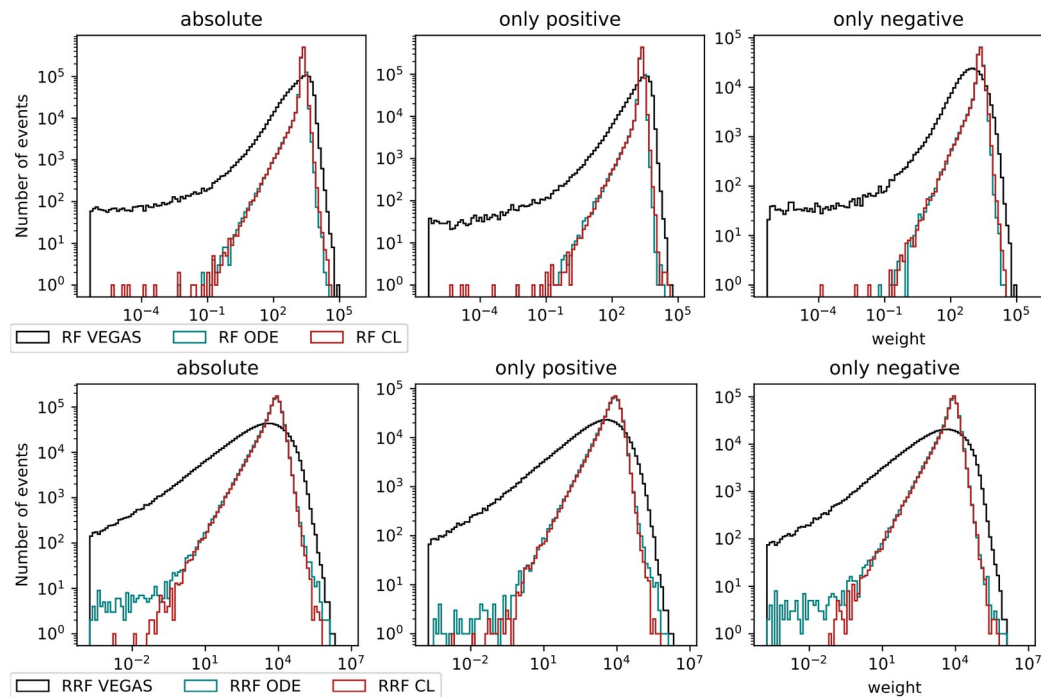

$$I = \int_{\mathbf{H}_+(\mathbf{x}) \in \Omega} d\mathbf{H}_+ \frac{f_+(\mathbf{x})}{h_+(\mathbf{x})} + \int_{\mathbf{H}_-(\mathbf{x}) \in \Omega} d\mathbf{H}_- \frac{f_-(\mathbf{x})}{h_-(\mathbf{x})}$$
 “two independent integrals”

$$\hat{I}_{\text{strat}} = \hat{I}_+ + \hat{I}_- = \frac{1}{N_+} \sum_{i=1}^{N_+} \frac{f_+(\mathbf{x}_i)}{h_+(\mathbf{x}_i)} + \frac{1}{N_-} \sum_{i=1}^{N_-} \frac{f_-(\mathbf{x}_i)}{h_-(\mathbf{x}_i)}$$
$$\delta \hat{I}_{\text{strat}} = \sqrt{\frac{1}{N-1} \left[\frac{N}{N_+} \text{Var}(\hat{I}_+) + \frac{N}{N_-} \text{Var}(\hat{I}_-) \right]}$$
$$\text{Var}(\hat{I}_{\pm}) = \frac{1}{N_{\pm}} \sum_{i=1}^{N_{\pm}} \left(\frac{f_{\pm}(\mathbf{x}_i)}{h_{\pm}(\mathbf{x}_i)} \right)^2 - \hat{I}_{\pm}^2$$

- + The total variance is now bounded by the individual variances
- The mappings are more complicated (need high phase space efficiency)

Normalizing Flows for NNLO QCD predictions

application to gluonic top-pair production:



potential for large performance gains

Take-home messages

- A first complete N3LO QCD computation for diphoton production
 - triggered many technical improvements in STRIPPER (hybrid precision, new selector function)
 - at this point still statistically limited...
- motivation for further advancements:
 - Local N3LO schemes might behave better
 - Improved numerical integration techniques: normalizing flows

Backup

Sub-leading colour in 2 → 3 processes

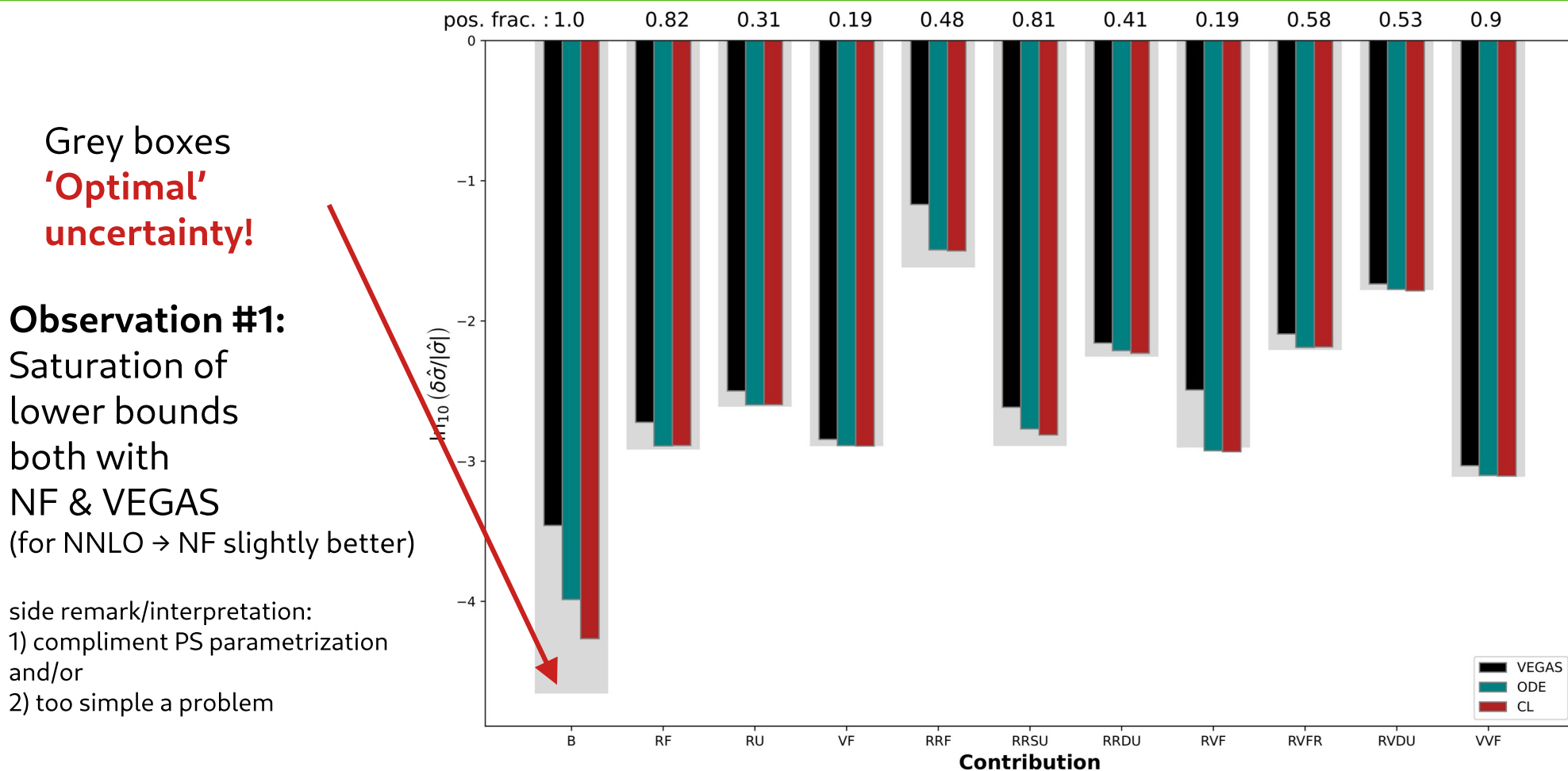
Since early 2023 all massless five point amplitudes are known with full color/charge dependence (for photon amplitudes “leading color” ~ “planar”)

	Leading colour amplitudes	Full colour amplitudes
$pp \rightarrow \gamma\gamma\gamma$	Chawdhry, Czakon, Mitov Poncelet [1911.00479] [2012.13553] Abreu, Page, Pascual, Sotnikov [2010.15834]	Abreu, De Laurentis, Ita, Klinkert, Page, Sotnikov [2305.17056]
$pp \rightarrow \gamma\gamma j$	Agarwal, Buccioni, von Manteuffel, Tancredi [2102.01820] Chawdhry, Czakon, Mitov Poncelet [2103.04319]	Agarwal, Buccioni, von Manteuffel, Tancredi [2105.04585]
$pp \rightarrow \gamma jj$		Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia [2304.06682]
$pp \rightarrow jjj$	Abreu, Dormans, Cordero, Ita, Page, Sotnikov [1904.00945] [2102.13609]	Agarwal, Buccioni, Devoto, Gambuti, von Manteuffel, Tancredi [2311.09870] De Laurentis, Ita, Klinkert, Sotnikov [2311.10086] [2311.18752]

all based in pentagon functions: Chicherin, Sotnikov [[2009.07803](#)]

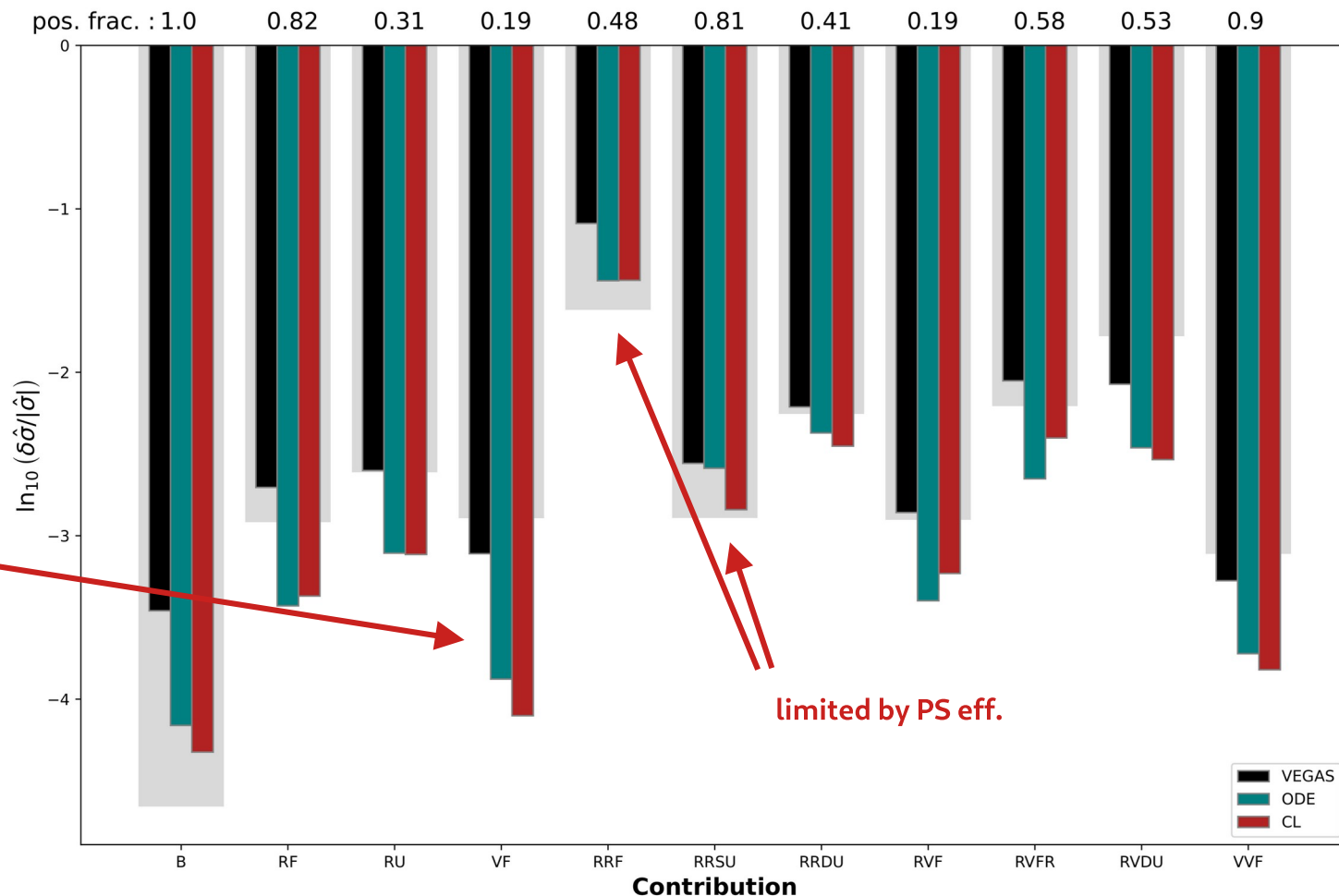
(except [[1904.00945](#), [1911.00479](#)] which use: Gehrmann, Henn, Presti [[1807.09812](#)])

NNLO cross section results → 'optimal' uncertainty

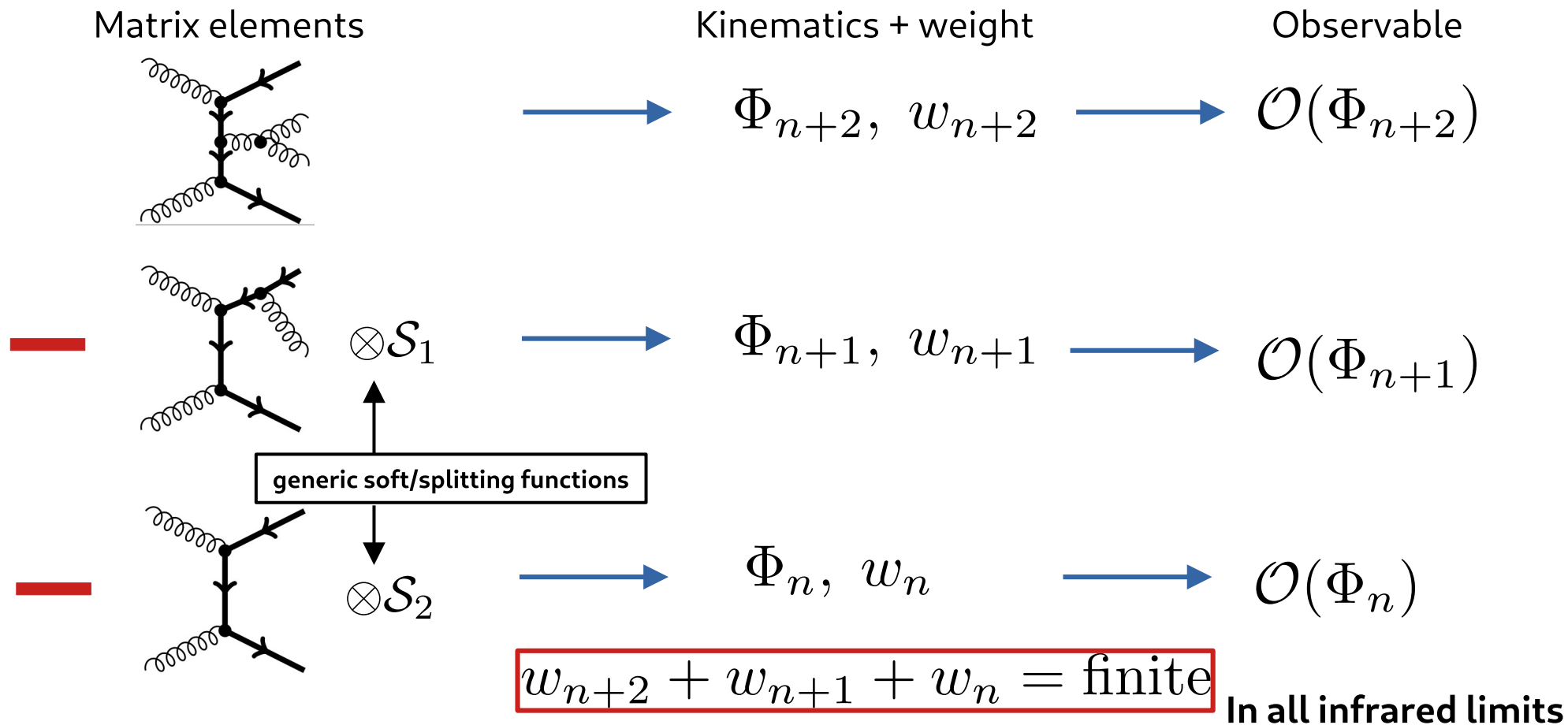


Relative uncertainties with positive/negative split

Observation #2:
Splitting can give a significant performance gains for flow based integrators



Anatomy of a fixed-order real emission event



Non-positive definite integrands

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- Consider extreme case: $|f(x)|/h(x) = w = \text{const.}$

MC estimate:

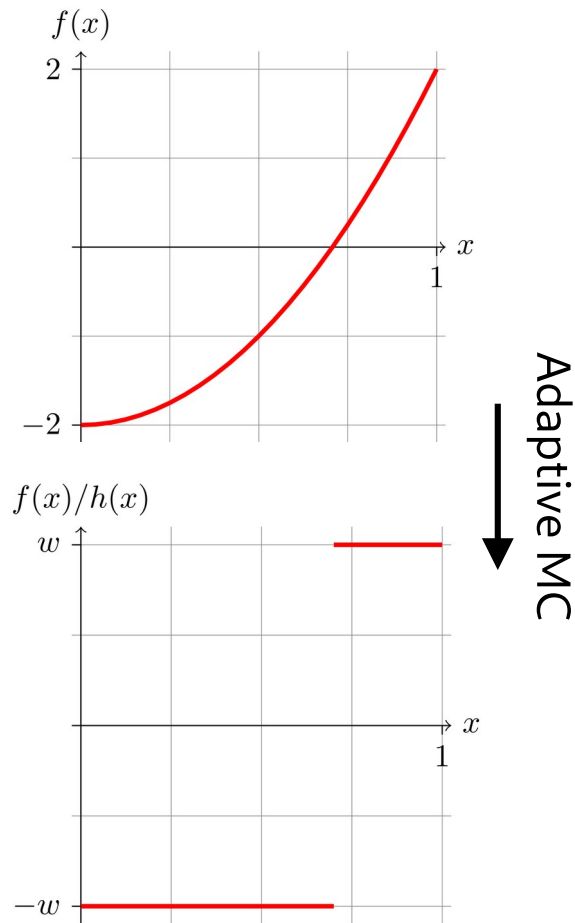
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→ relative uncertainty: $\frac{\delta \hat{I}}{\hat{I}} = \frac{1}{\sqrt{N-1}} \frac{\sqrt{\alpha(1-\alpha)}}{\alpha - \frac{1}{2}}$

Rephrased: at some point it doesn't matter any more how good your adaptive MC is...



Stratification of signed integrands

There are ways around:

1) Add a large constant

2) Stratification: $f(\mathbf{x}) = f_+(\mathbf{x}) + f_-(\mathbf{x})$, with $f_{\pm}(\mathbf{x}) = \Theta(\pm f(\mathbf{x}))f(\mathbf{x})$


$$I = \int_{\mathbf{H}_+(\mathbf{x}) \in \Omega} d\mathbf{H}_+ \frac{f_+(\mathbf{x})}{h_+(\mathbf{x})} + \int_{\mathbf{H}_-(\mathbf{x}) \in \Omega} d\mathbf{H}_- \frac{f_-(\mathbf{x})}{h_-(\mathbf{x})}$$
 “two independent integrals”

$$\hat{I}_{\text{strat}} = \hat{I}_+ + \hat{I}_- = \frac{1}{N_+} \sum_{i=1}^{N_+} \frac{f_+(\mathbf{x}_i)}{h_+(\mathbf{x}_i)} + \frac{1}{N_-} \sum_{i=1}^{N_-} \frac{f_-(\mathbf{x}_i)}{h_-(\mathbf{x}_i)}$$
$$\delta \hat{I}_{\text{strat}} = \sqrt{\frac{1}{N-1} \left[\frac{N}{N_+} \text{Var}(\hat{I}_+) + \frac{N}{N_-} \text{Var}(\hat{I}_-) \right]}$$
$$\text{Var}(\hat{I}_{\pm}) = \frac{1}{N_{\pm}} \sum_{i=1}^{N_{\pm}} \left(\frac{f_{\pm}(\mathbf{x}_i)}{h_{\pm}(\mathbf{x}_i)} \right)^2 - \hat{I}_{\pm}^2$$

- + The total variance is now bounded by the individual variances
- The mappings are more complicated (need high phase space efficiency)