

NNLO QCD corrections to $t\bar{t}W$ production at the LHC

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Based on: [2504.13011] [2606.09503] [2608.03746]

QCD@LHC

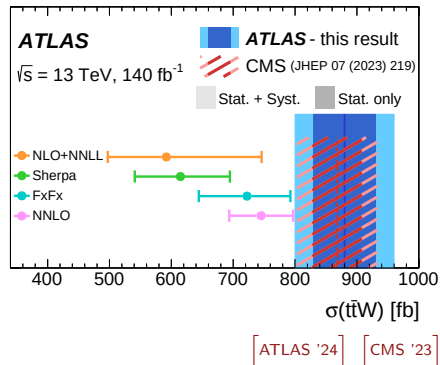
Oxford, 1 October 2026

Motivation

Why precise $t\bar{t}W$ predictions?

$$\bar{u}(p_1) + d(p_2) \longrightarrow t(p_3) + \bar{t}(p_4) + W^-(p_5)$$

- Important background to $t\bar{t}H$ and $t\bar{t}t\bar{t}$ production
- Sensitive probe of physics beyond the Standard Model
- Measured rates lie above SM predictions



Prior work: two-loop approximations

- NNLO cross section requires two-loop amplitude

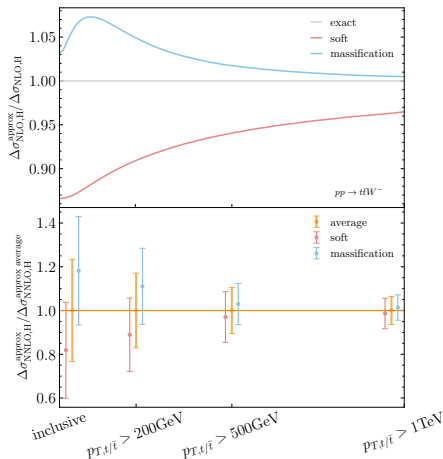
$$d\sigma_{\text{NNLO}} = d\sigma_{\text{VV}} + d\sigma_{\text{RV}} + d\sigma_{\text{RR}}$$

$\swarrow$
 $|\mathcal{A}^{(1)}|^2 + 2 \operatorname{Re}(\mathcal{A}^{(2)} \mathcal{A}^{(0)*})$

- First NNLO QCD prediction used two *dynamical* approximations
 - soft-boson approximation (SA)
 - massification (MA)

[Buonocore, Devoto, Grazzini, Kallweit
Mazzitelli, Rottoli, Savoini '23]

- Direct calculation needed to:
 - validate dynamical approximations
 - enable NNLO differential distributions



Two-loop amplitude

Leading-colour approximation (LCA) and full kinematics

- Two-loop amplitude decomposes into colour factors and kinematic functions:

$$\mathcal{M}_n^{(2)} = N_C^2 \mathcal{A}_{\{2,0\}}^{(2)} + n_f^2 \mathcal{A}_{\{0,2\}}^{(2)} + N_C n_f \mathcal{A}_{\{1,1\}}^{(2)} + N_C \mathcal{A}_{\{1,0\}}^{(2)} + n_f \mathcal{A}_{\{0,1\}}^{(2)} + \dots$$

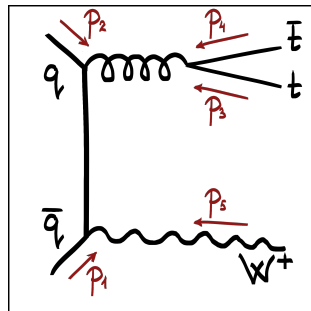
LCA

- LCA keeps terms $\propto \{N_C^2, N_C n_f, n_f^2\}$ with *full* kinematic dependence on

$$\vec{x} = \{s_{13}, s_{34}, s_{24}, s_{25}, s_{15}, m_t^2, m_W^2\}$$

- Only planar diagrams remain, with at most two massive propagators

722 diagrams $\longrightarrow$ 210 planar diagrams



Physical projectors and scalar form factors

- Physical projectors in the 't Hooft–Veltman scheme turn the tensor amplitude into 24 scalar form factors

[Peraro '19] [Peraro '20]
[Tancredi] [Tancredi]

$$\mathcal{A}^{(2)} = \sum_{i=1}^{24} T_i F_i^{(2)} .$$

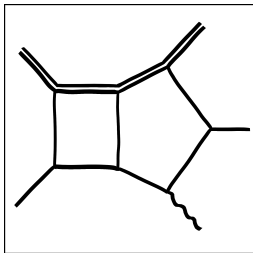
- Form factors contain 8959 scalar Feynman integrals with rational coefficients

$$F_i^{(2)}(\vec{x}; \varepsilon) = \sum_j c_{ij}(\vec{x}; \varepsilon) I_j(\vec{x}; \varepsilon) .$$

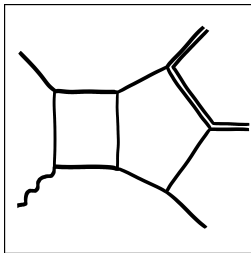
- UV renormalisation and IR subtraction remove poles from $1/\varepsilon^4$ to $1/\varepsilon$, leaving the finite remainder $\mathcal{A}_{\text{fin}}^{(2)}$

Three two-loop integral families

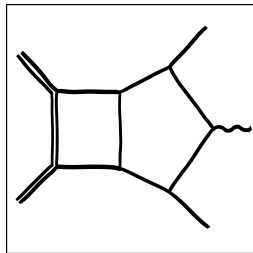
- Three genuine two-loop families, up to permutations, plus products of one-loop integrals



F1



F2



F3

$$I_{\vec{a}}(\vec{x}) = \int d^d k_1 d^d k_2 \frac{1}{D_1^{a_1} \dots D_{11}^{a_{11}}}, \quad \vec{a} = (a_1, \dots, a_{11}) \in \mathbb{Z}^{11}.$$

$D_i = \ell_i^2 - m_i^2$: inverse propagators

IBP reduction to master integrals

- Feynman integrals obey linear relations: integration-by-parts identities

$$0 = \int d^d k_1 d^d k_2 \frac{\partial}{\partial k_l^\mu} \frac{v^\mu}{D_1^{a_1} \cdots D_{11}^{a_{11}}}, \quad v^\mu \in \{k_j^\mu, p_j^\mu\}.$$

[Tkachov '81] [Chetyrkin
Tkachov '81] [Laporta '00]

- All integrals reduce to a finite set of master integrals $\vec{I}$

8959 scalar integrals

amplitude

IBP relations
NEATIBP+FINITEFLOW

330 master integrals

independent

[Wu, Boehm, Ma
Xu, Zhang '23] [Peraro '19]

Differential equations

- IBP relations yield differential equations for the MIs

[Kotikov '91] [Bern, Dixon, Kosower '94] [Gehrmann, Remiddi '00]

$$\partial_x \vec{I}(\vec{x}; \varepsilon) = B_x(\vec{x}; \varepsilon) \vec{I}(\vec{x}; \varepsilon).$$

- Canonical form decouples $\vec{x}$ and ε and avoids spurious singularities

[Henn '13]

$$d\vec{I} = \varepsilon dA \vec{I}, \quad dA = \sum_i a_i d \log W_i(\vec{x}).$$

- No general algorithm; conjectured criterion: simple poles and constant leading singularities (LS)

[Arkani-Hamed, Bourjaily, Cachazo, Trnka '12]

$$I = \frac{1}{\text{LS}(\vec{x})} \int d \log(\dots) \wedge \dots \wedge d \log(\dots) \implies \text{good choice: } \text{LS}(\vec{x}) I$$

LS need not be rational: square roots and more complicated structures may appear

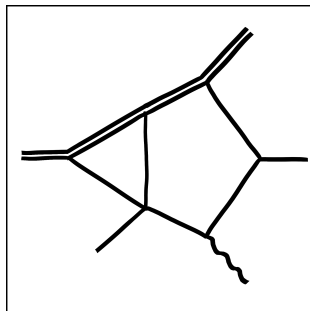
Challenge: nested square roots

- Leading singularities of some MIs contain nested square roots, also found in $t\bar{t}H$ and $t\bar{t}j$

[Cordero, Figueiredo] [Badger, Becchetti] [Becchetti]
[Kraus, Page, Reina '23] [Giraud, Zoia '24] [Dlapa, Zoia '25]

$$\text{NR}_{\pm} = \sqrt{q_1(\vec{x}) \pm q_2(\vec{x}) \sqrt{\text{Gram}(p_1, p_2, p_3, p_4)}}$$

- Series-expansion DE solvers do not support bases with nested roots
- Strategy:** keep nested roots out of basis; keep corresponding DE blocks linear in ε

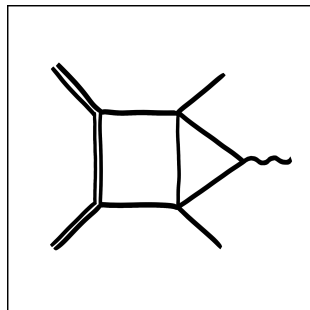


Challenge: elliptic curves

- Some MIs involve elliptic curves

$$I = \int \frac{dz}{\sqrt{\mathcal{P}_4(z)}} \wedge d \log(\dots) \wedge \dots \wedge d \log(\dots)$$

$\downarrow$
 $(z - a_1)(z - a_2)(z - a_3)(z - a_4)$



- Elliptic canonical DEs require transcendental functions and, among five-point integrals, have been constructed only for $t\bar{t}j$
[Görge, Nega '23] [Tancredi, Wagner] [Bechetti, Dlaga, Zoia '25]
- For $t\bar{t}W$, algebraic complexity makes canonical-basis construction very difficult; numerical benefit remains unclear
- Strategy:** avoid elliptic functions in DEs; keep corresponding DE blocks at most quadratic in ε and choose elliptic MIs finite as $\varepsilon \rightarrow 0$

Special-function basis for the finite remainder

- Expand MIs to order needed for finite remainder

$$I_j(\vec{x}; \varepsilon) = \sum_{w=0}^4 \varepsilon^w I_j^{(w)}(\vec{x}) \quad \text{for } j = 1, \dots, 330$$

- Coefficients $I_j^{(w)}$ satisfy algebraic relations: choose an independent special-function basis

[Gehrmann '18] [Chicherin] [Chicherin '21]
[Henn, Lo Presti] [Sotnikov '20] [Sotnikov, Zoia]

- Benefits: analytic pole subtraction, sparser ε -independent DEs, and separation of polylogarithmic and elliptic functions

[Badger, Becchetti, Brancaccio]
[Hartanto, Zoia '24]

	$w = 1$	$w = 2$	$w = 3$	$w = 4$	$w = 4^*$	total
amplitude	7	12	63	212	29	323
finite remainder	7	12	63	187	29	298

Numerical evaluation of the special functions

$$\mathcal{A}_{\text{fin}}^{(2)}(\vec{x}) = \sum_j q_j(\vec{x}) I_j^{(w_j)}(\vec{x})$$

- Transport boundary values along a path

$$\gamma(\eta) = (1 - \eta)\vec{x}_i + \eta\vec{x}_f$$

- On each segment, match generalised series expansions

[Pozzorini
Remiddi '06] [Moriello '20]

$$\vec{I}(\eta) \sim \vec{I}_0 + \sum_{k_1, k_2} \vec{c}_{k_1, k_2} \log^{k_1}(\eta - \tau)(\eta - \tau)^{k_2}$$

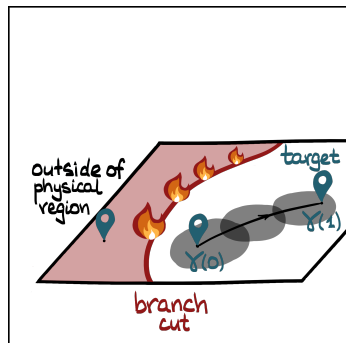
- Solvers:

- › DIFFEXP for benchmarks
- › AMFLOW for boundary values and production

[Hidding '20]

[Liu
Ma '22]

Timing: 20–120 CPU min per phase-space point



Rational coefficients from finite fields

$$\mathcal{A}_{\text{fin}}^{(2)}(\vec{x}) = \sum_j q_j(\vec{x}) I_j^{(w_j)}(\vec{x})$$

- **Strategy:** replace expensive analytic reconstruction with pointwise finite-field evaluation

[von Manteuffel] [Peraro '16]
[Schabinger '14]

- Rationalise kinematics and evaluate modulo primes

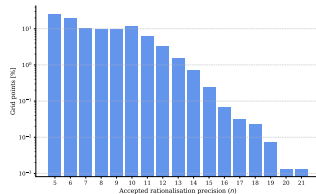
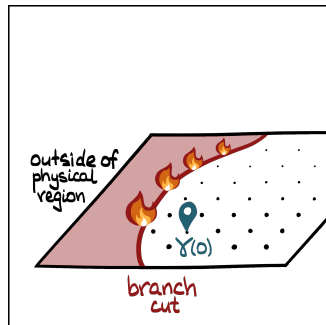
$$m_t^2 = \frac{749956}{25} \text{ GeV}^2, \quad m_W^2 = \frac{258469929}{40000} \text{ GeV}^2$$

$$\text{FFPrimeNo}[0] = 9223372036854775783$$

- Reconstruct exact coefficients $q_j(\vec{x})$ over $\mathbb{Q}$

- **Timing:** FINITEFLOW ~ 10 min per point

[Peraro '19]



Challenge: high-dimensional interpolation grid

- On-the-fly evaluation not viable
- Strategy:** interpolate finite remainder on a fixed-mass grid

β^2	$\text{frac}_{s_{t\bar{t}}}$	$\cos \theta_W$	$\cos \theta_t$	ϕ_t	total
$[0, 1]$	$[0, 1]$	$[-1, 1]$	$[-1, 1]$	$[0, 2\pi]$	
20	18	13	8	6	224 640

- Validate at NLO against:

➤ OPENLOOPS

[Buccioni, Lang, Lindert, Maierhöfer
Pozzorini, Zhang, Zoller '19]

➤ RECOLA

[Actis, Denner, Hofer '16
Lang, Scharf, Uccirati]

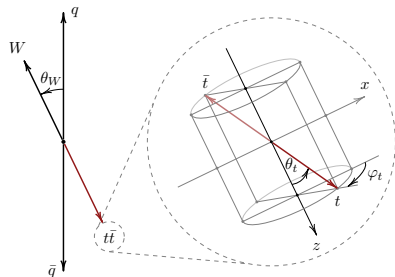
➤ analytic result

[Becchetti, Delto, Ditsch
Kreer, Pozzoli, Tancredi '25]

- For the two-loop hard-virtual contribution:

➤ Interpolation uncertainty $\sim 1\%$

➤ Estimated subleading-colour uncertainty $\sim 22\%$



Adapted from

[Agarwal, Heinrich, Jones, Kerner
Klein, Lang, Magerya, Olsson '24]

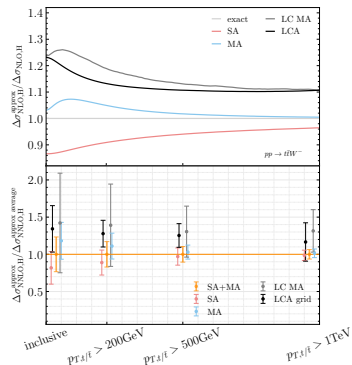
Results

NNLO QCD results at 13 TeV

- Cross sections computed with MATRIX using q_T subtraction

[Grazzini, Kallweit] [Catani '07]
[Wieseemann '18] [Grazzini]

- LCA: about 3% above SA+MA; compatible within stated uncertainties
- Two-loop hard-virtual contribution is large: about 11% of the NNLO cross section



σ_{NNLO} [fb]	$t\bar{t}W^-$	$t\bar{t}W^+$
SA+MA	235.4(0) ^{+5.1%} _{-6.6%} ±1.9%	474.9(2) ^{+4.8%} _{-6.4%} ±1.9%
LCA	241.9(0) ^{+6.4%} _{-7.3%} ±2.3%	489.4(1) ^{+6.3%} _{-7.2%} ±2.5%
parentheses: MC error scale hard-virtual uncertainty		

Conclusions

- Numerical computation of the two-loop $t\bar{t}W$ amplitude in the generalised leading-colour approximation
 - special-function basis for direct access to the finite remainder
 - differential equations and generalised series for the special functions
 - finite-field arithmetic and reconstruction of rational coefficients
- NNLO QCD cross sections using grid interpolation in `MATRIX`
 - agreement with the earlier dynamical approximations
 - theory–data tension remains
 - subleading colour is the next missing ingredient

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Thank you