

Analytic Computation of the Two-Loop Penguin Jet Function

Riccardo Bartocci, Philipp Böer, Kevin Brune, Tobias Hurth



Kevin Brune (JGU Mainz)



QCD@LHC 2026

September 28, 2026

Motivation

- B-meson decays provide an important constraint on BSM
- One prominent decay $\bar{B} \rightarrow X_s \gamma$
- CP- and isospin-averaged branching ratio $\mathcal{B}_{s\gamma}$

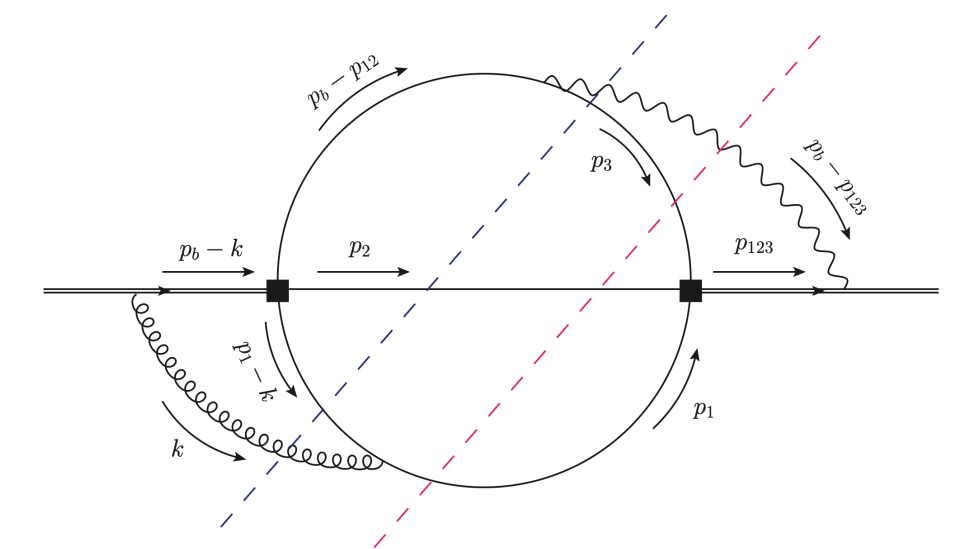
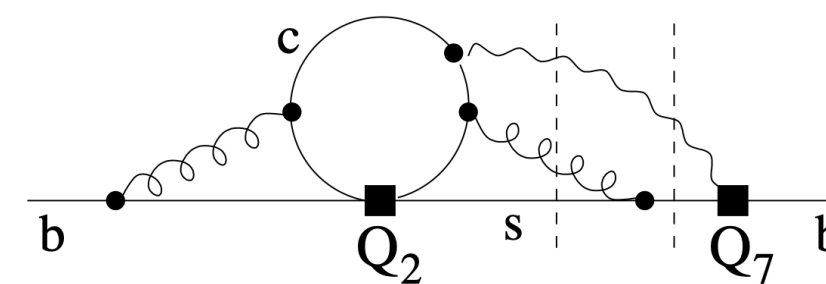
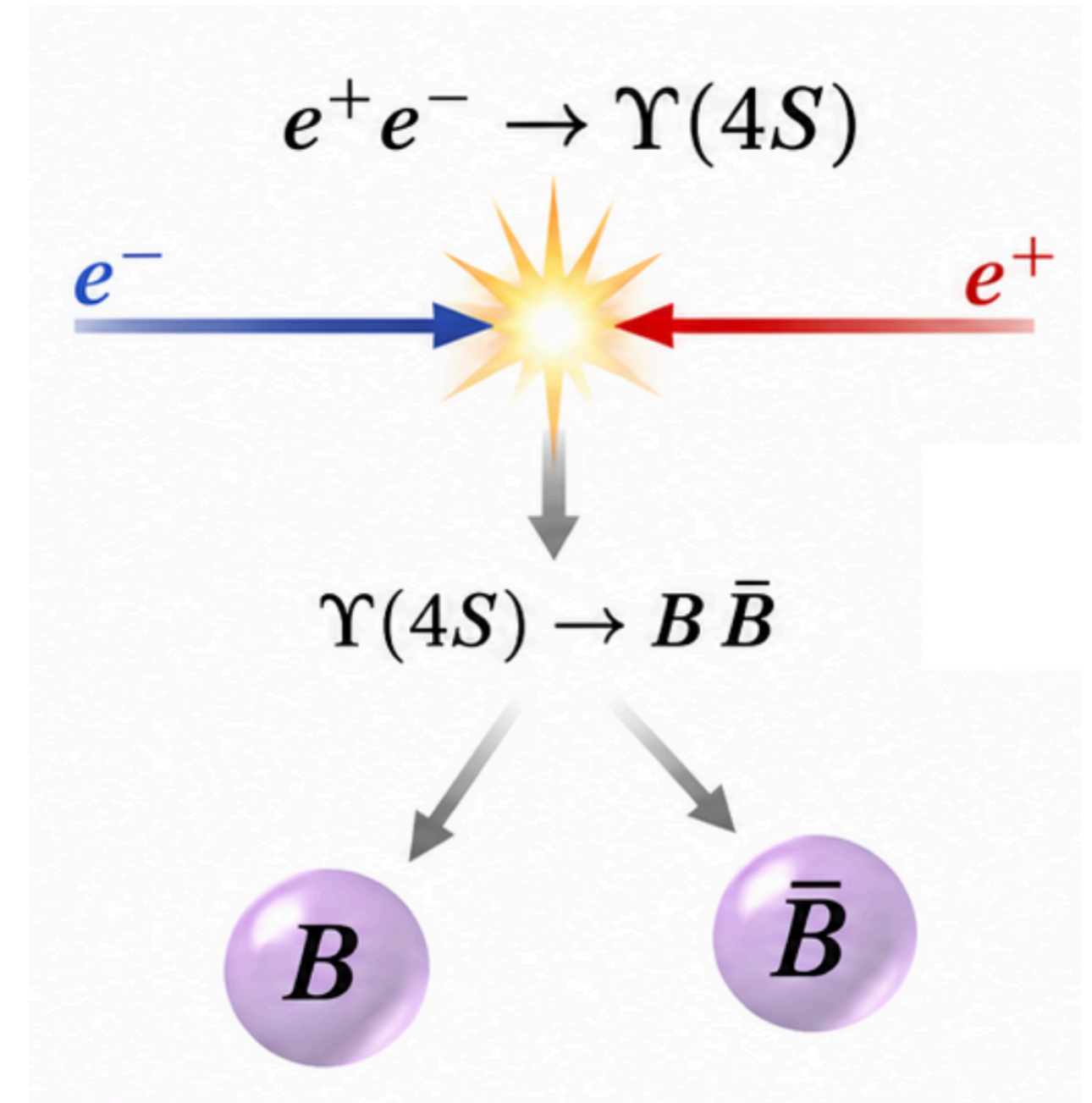
$$\mathcal{B}_{s\gamma}^{\text{exp}} = (3.49 \pm 0.19) \times 10^{-4}$$

[CLEO; 01][Babar; 12][Belle; 15][Belle II; 22]

- Theory prediction was just updated [Misiak, et al. ; 26]

$$\mathcal{B}_{s\gamma}^{\text{SM}} = (3.54 \pm 0.14) \times 10^{-4}$$

- ▶ Some NNLO charm-mass effects [Misiak, et al. ; 26]
- ▶ Four- and five-body contributions [KB, Huber, Moos ; 25]



Theory prediction

- $\bar{B} \rightarrow X_s \gamma$ photon-energy spectrum in the endpoint region: $p_+ \equiv m_b - 2E_\gamma = \mathcal{O}(\Lambda_{QCD})$

$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb} V_{ts}^*|^2}{2\pi^4} \bar{m}_b^2(\mu) E_\gamma^3 \left[|H_\gamma(\mu)|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega J^{(n)}(m_b(\omega + p_+), \mu) S(\omega, \mu) \right. \\ \left. + \frac{1}{m_b} \sum_{i \leq j} \text{Re} [C_i^*(\mu) C_j(\mu)] F_{ij}(E_\gamma) + \mathcal{O}\left(\frac{1}{m_b^2}\right) \right]$$

[Benzke, Lee, Neubert, Paz; 10]

Theory prediction

- $\bar{B} \rightarrow X_s \gamma$ photon-energy spectrum in the endpoint region: $p_+ \equiv m_b - 2E_\gamma = \mathcal{O}(\Lambda_{QCD})$

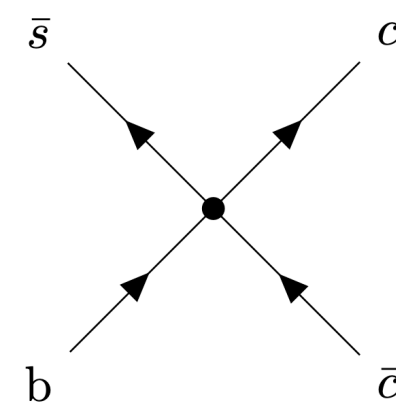
$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb} V_{ts}^*|^2}{2\pi^4} \bar{m}_b^2(\mu) E_\gamma^3 \left[|H_\gamma(\mu)|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega J^{(n)}(m_b(\omega + p_+), \mu) S(\omega, \mu) \right. \\ \left. + \frac{1}{m_b} \sum_{i \leq j} \text{Re} [C_i^*(\mu) C_j(\mu)] F_{ij}(E_\gamma) + \mathcal{O}\left(\frac{1}{m_b^2}\right) \right]$$

Resolved contribution

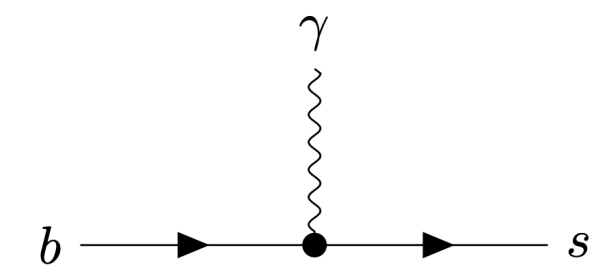
[Benzke, Lee, Neubert, Paz; 10]

- Largest theoretical uncertainty from $Q_1 - Q_7$

$$Q_1 = (\bar{d}_i^\alpha \gamma_\mu P_L q^\beta) (\bar{q}^\beta \gamma^\mu P_L d_i^\alpha)$$



$$Q_7 = em_{d_i} (\bar{d}_j \sigma^{\mu\nu} P_R d_i) F_{\mu\nu}$$

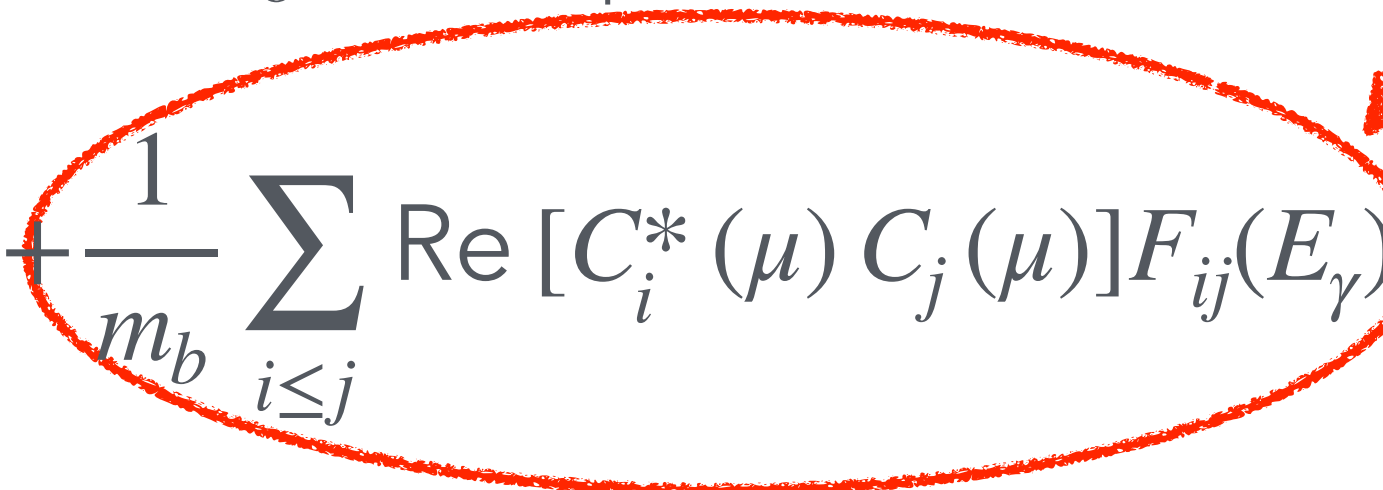


Theory prediction

- $\bar{B} \rightarrow X_s \gamma$ photon-energy spectrum in the endpoint region: $p_+ \equiv m_b - 2E_\gamma = \mathcal{O}(\Lambda_{QCD})$

$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb} V_{ts}^*|^2}{2\pi^4} \bar{m}_b^2(\mu) E_\gamma^3 \left[|H_\gamma(\mu)|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega J^{(n)}(m_b(\omega + p_+), \mu) S(\omega, \mu) \right. \\ \left. + \frac{1}{m_b} \sum_{i \leq j} \text{Re} [C_i^*(\mu) C_j(\mu)] F_{ij}(E_\gamma) + \mathcal{O}\left(\frac{1}{m_b^2}\right) \right]$$

Resolved contribution



[Benzke, Lee, Neubert, Paz; 10]

- Largest theoretical uncertainty from $Q_1 - Q_7$
 - Large scale ambiguity [Gunawardana, Paz; 20] [Benzke, Hurth; 20] [Benzke, Garzelli, Hurth; 25]

⇒ **NLO analysis and Renormalisation Group evolution will reduce this ambiguity**

Factorisation at LO

- LO diagram of $\mathcal{O}_1 - \mathcal{O}_7$ interference [Benzke, Lee, Neubert, Paz; 10]

$$d\Gamma(\bar{B} \rightarrow X_s \gamma) \propto \text{Disc}_{\text{rest.}} \left[i \int d^4x \langle \bar{B} | \mathcal{H}_{\text{eff}}^\dagger(x) \mathcal{H}_{\text{eff}}(0) | \bar{B} \rangle \right]$$

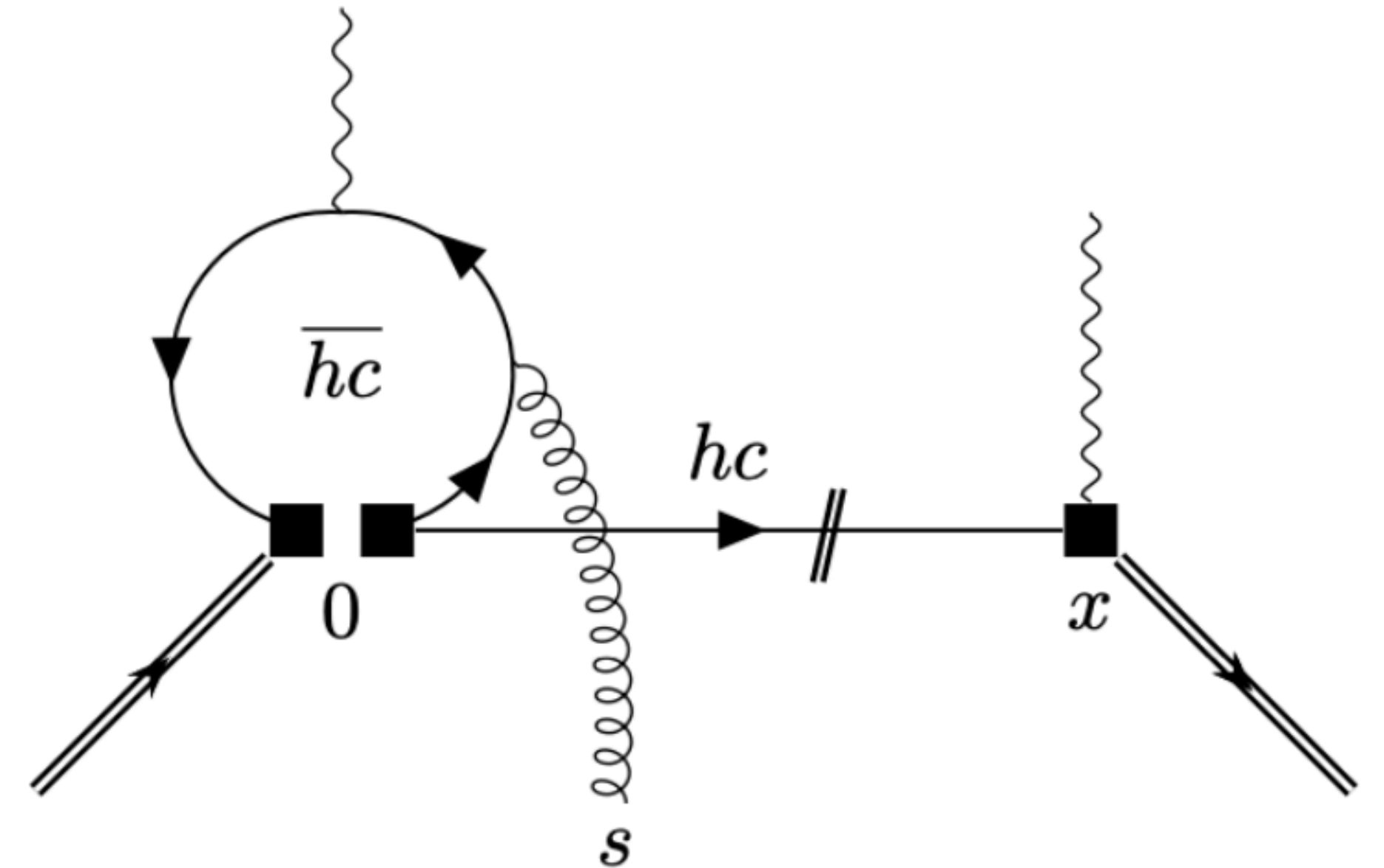
- Momentum scaling

$$b \sim (1, 1, 1) m_b \quad \bar{hc} \sim (1, \lambda^2, \lambda) m_b$$

$$hc \sim (\lambda^2, 1, \lambda) m_b \quad s \sim (\lambda^2, \lambda^2, \lambda^2) m_b$$

- Factorisation theorem

$$d\Gamma(\bar{B} \rightarrow X_s \gamma) \sim H \cdot J^{(\bar{n})} \otimes J^{(n)} \otimes g_{17}$$

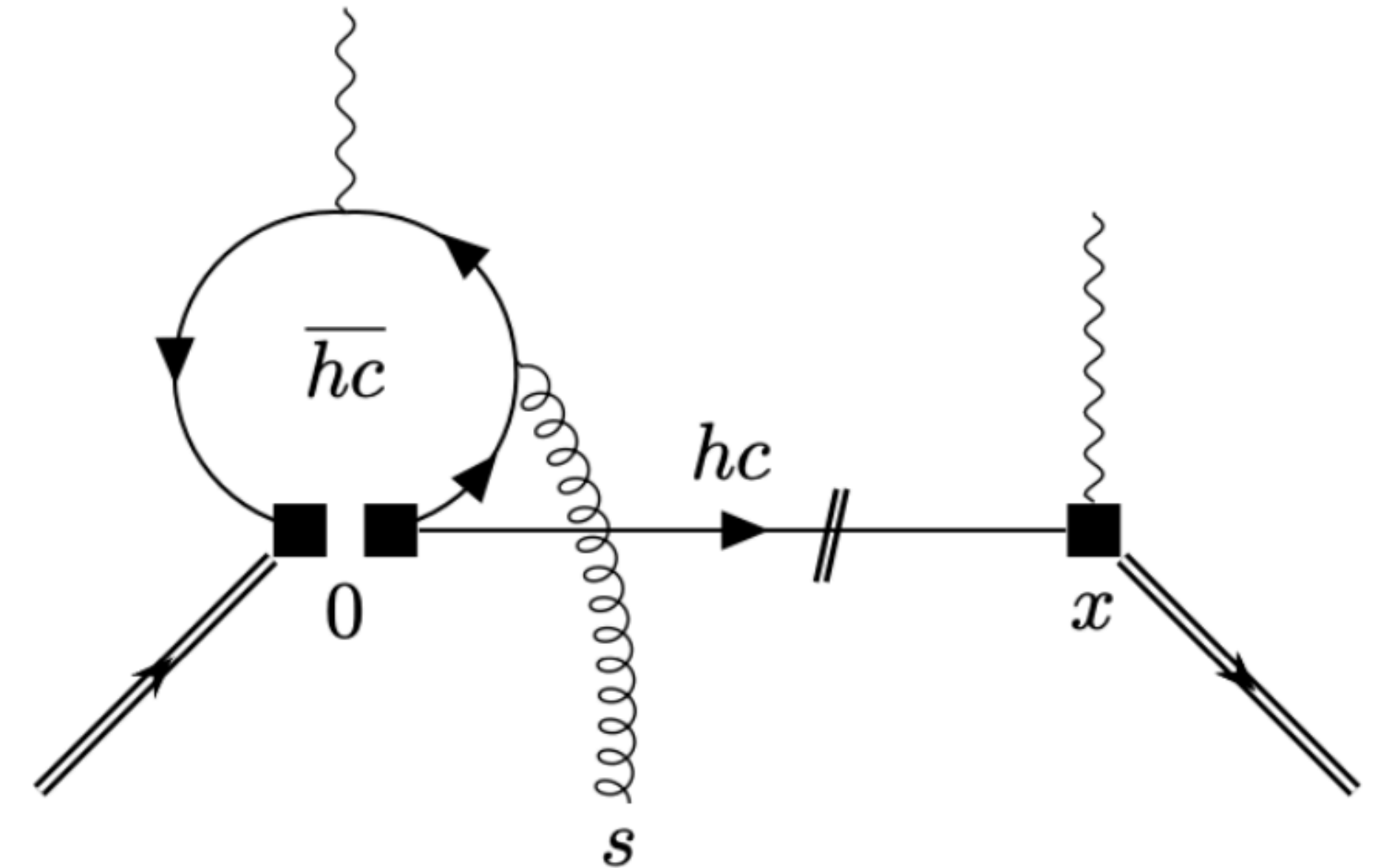


Factorisation at LO

[Benzke, Lee, Neubert, Paz; 10]

$$d\Gamma(\bar{B} \rightarrow X_s \gamma) \sim \textcolor{red}{H} \cdot \textcolor{blue}{J}^{(\bar{n})} \otimes \textcolor{green}{J}^{(n)} \otimes \textcolor{violet}{g}_{17}$$

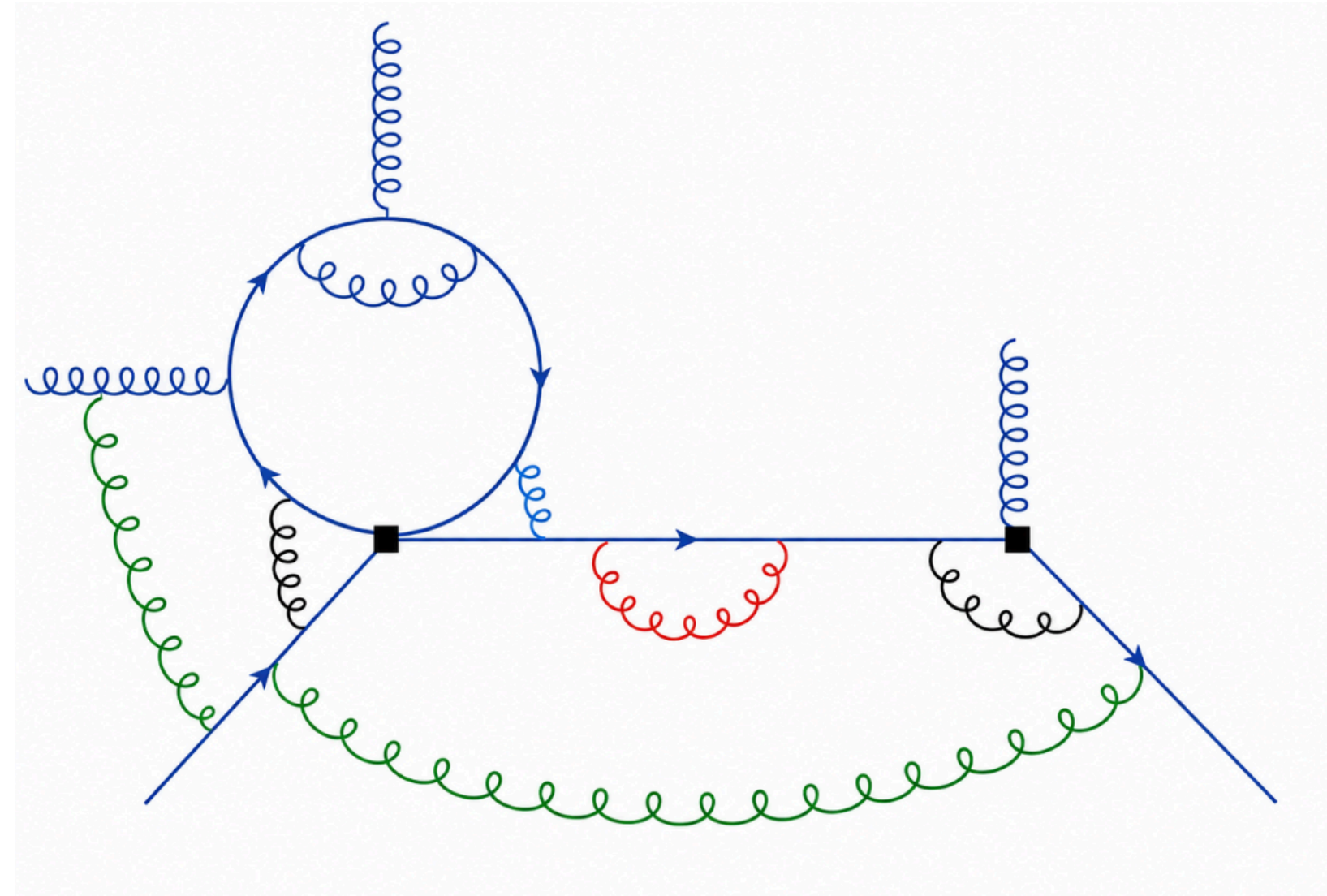
- I. Hard function H (perturbative)
- II. Anti-hard collinear jet function $J^{(\bar{n})}$ (perturbative)
- III. Collinear jet function $J^{(n)}$ (perturbative)
- IV. Subleading shape function g_{17} (non-perturbative)



$$d\Gamma \sim \textcolor{red}{1} \cdot \text{Re} \int_{-\infty}^{\bar{\Lambda}} d\omega \int_{-\infty}^{\infty} d\omega_1 \frac{1}{\omega_1 + i\eta} \left[1 - F \left(\frac{m_c^2 - i\eta}{2E_\gamma \omega_1} \right) \right] \textcolor{green}{\delta}(\omega + p_+) \textcolor{violet}{g}_{17}(\omega, \omega_1)$$

Factorisation at NLO

- In SCET we can compute the ingredients piecewise

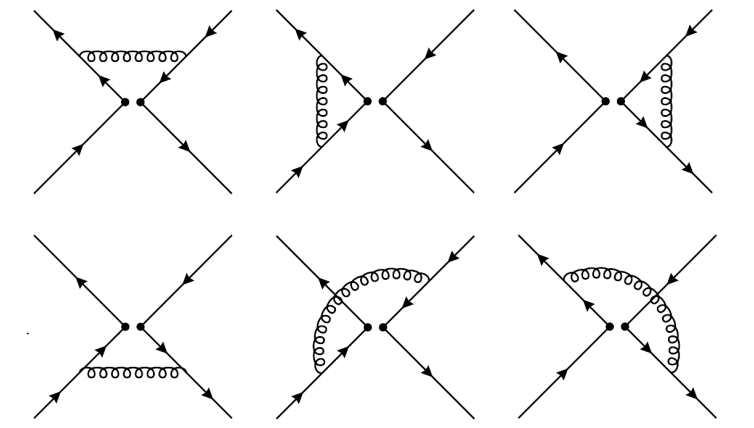


- Factorisation theorem at NLO [Bartocci, Böer, Hurth; wip]

$$d\Gamma(\bar{B} \rightarrow X_s \gamma) \sim \int_0^1 du H(u; \mu) \int_{-\infty}^{\infty} d\omega_1 J^{(\bar{n})}(u, \omega_1; \mu) \int_{-\infty}^{\bar{\Lambda}} d\omega J^{(n)}(\omega; \mu) g_{17}(\omega, \omega_1; \mu)$$

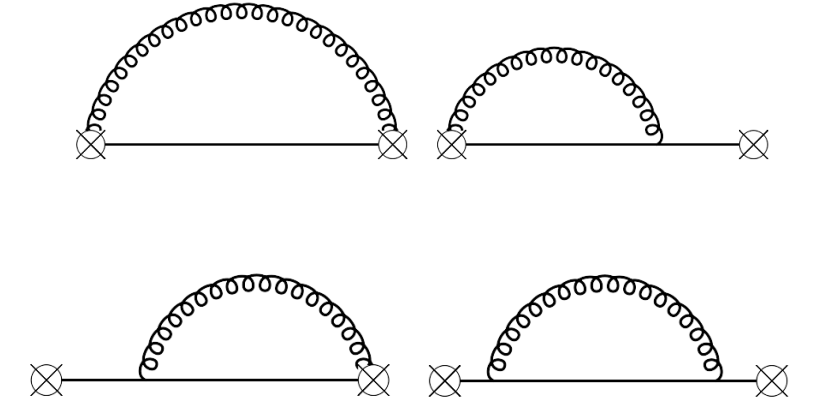
Hard function

[Beneke et al.; 09]



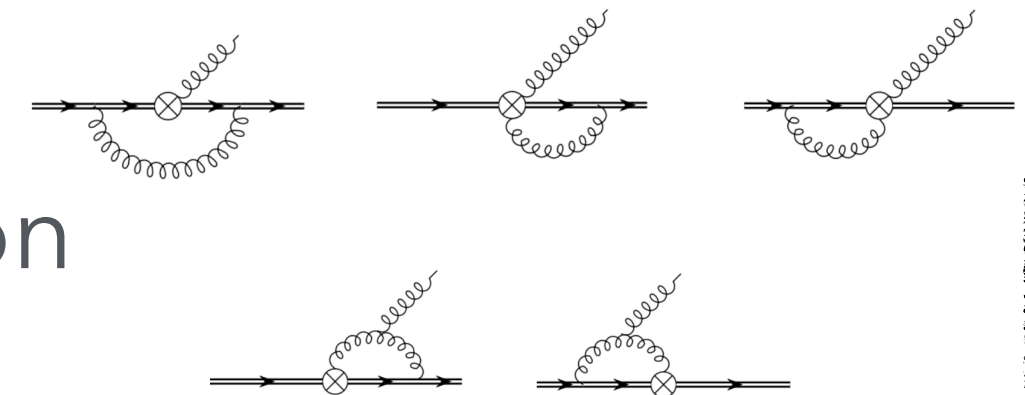
Jet function

[Becher, Neubert; 06]



Shape function

[Bartocci, Böer, Hurth; 24]



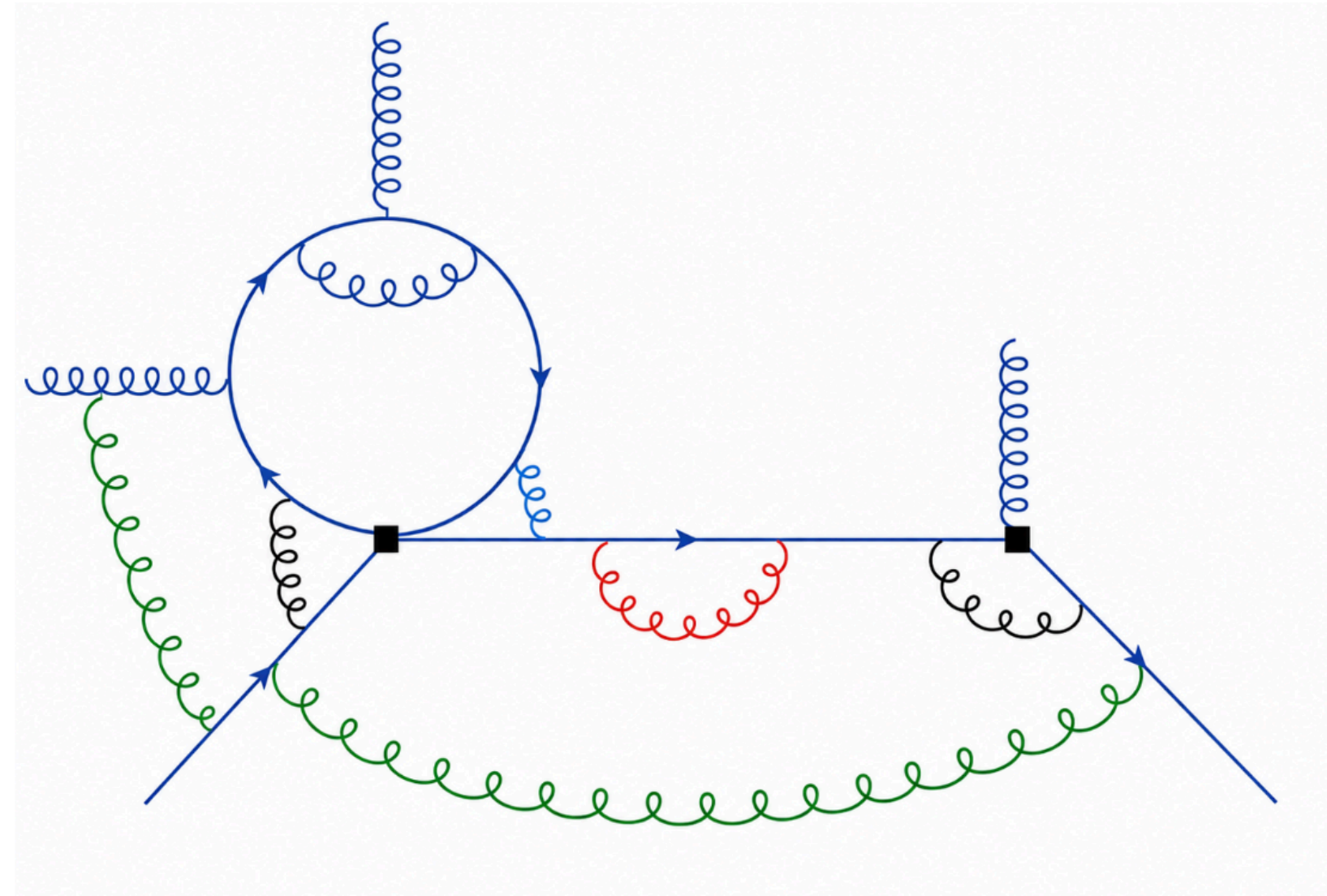
„Penguin“-jet function

[Bartocci, Böer, KB, Hurth; wip]

This talk

Factorisation at NLO

- In SCET we can compute the ingredients piecewise

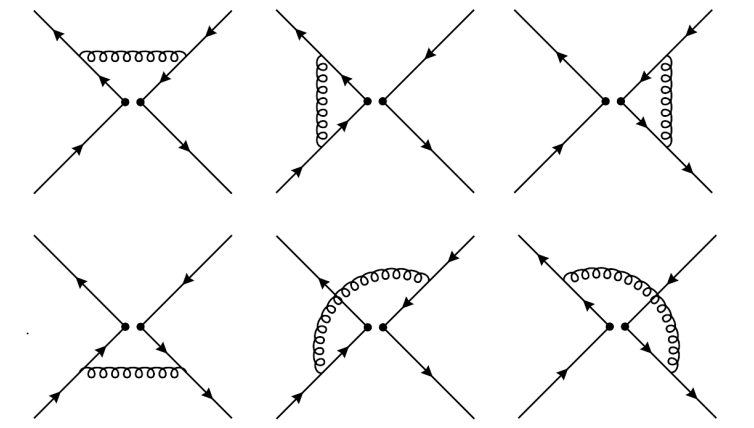


- Factorisation theorem at NLO [Bartocci, Böer, Hurth; wip]

$$d\Gamma(\bar{B} \rightarrow X_s \gamma) \sim \int_0^1 du H(u; \mu) \int_{-\infty}^{\infty} d\omega_1 J^{(\bar{n})}(u, \omega_1; \mu) \int_{-\infty}^{\bar{\Lambda}} d\omega J^{(n)}(\omega; \mu) g_{17}(\omega, \omega_1; \mu)$$

Hard function

[Beneke et. al.; 09]



Side comment

Jet function

[Becher, Neubert; 06]

$$Q_{1,2}^q \rightarrow \mathcal{O}_{1,2}^q$$

$$\mathcal{O}_i^q = [\bar{\chi}_{hc}(0) \dots \chi_{hc}(0)] [\bar{\chi}_{hc}(sn) \dots h_v(0)]$$

- Two anti-hard collinear fields separated on the light cone

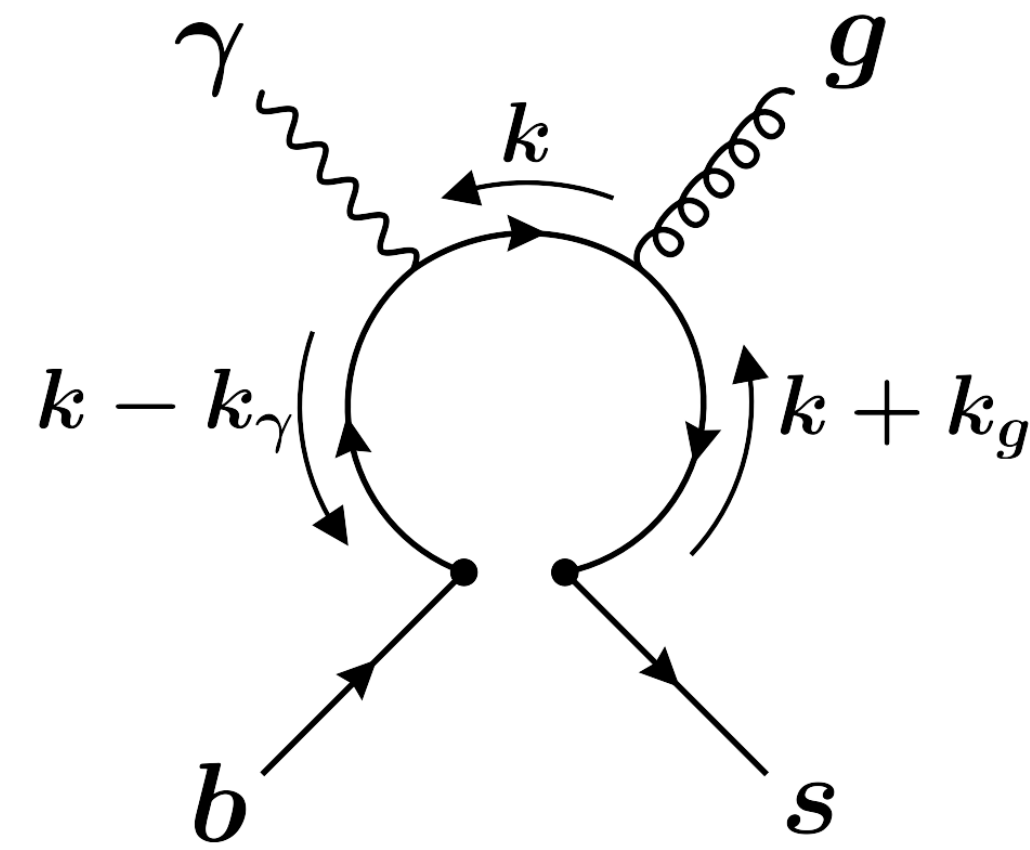
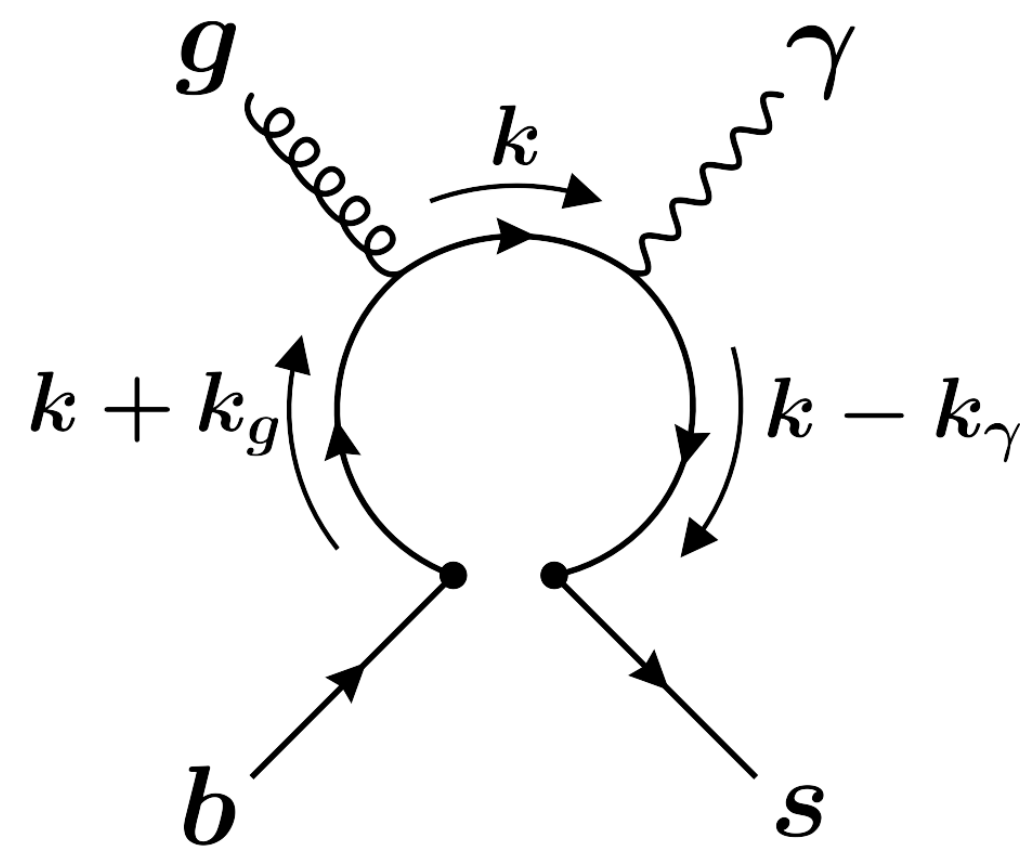
- E_γ distributed between quark and antiquark $\rightarrow$ Energy fraction u

„Penguin“-jet function

[Bartocci, KB, Hurth; wip]

The „penguin“-jet function

- The „penguin“-jet function is extracted from the anti-hard collinear region of $b \rightarrow s\gamma g$
- At LO :



$$k_g^\mu = \omega_1 n^\mu$$

$$k_\gamma^\mu = E_\gamma \bar{n}^\mu$$

$$s = (k_g + k_\gamma)^2$$

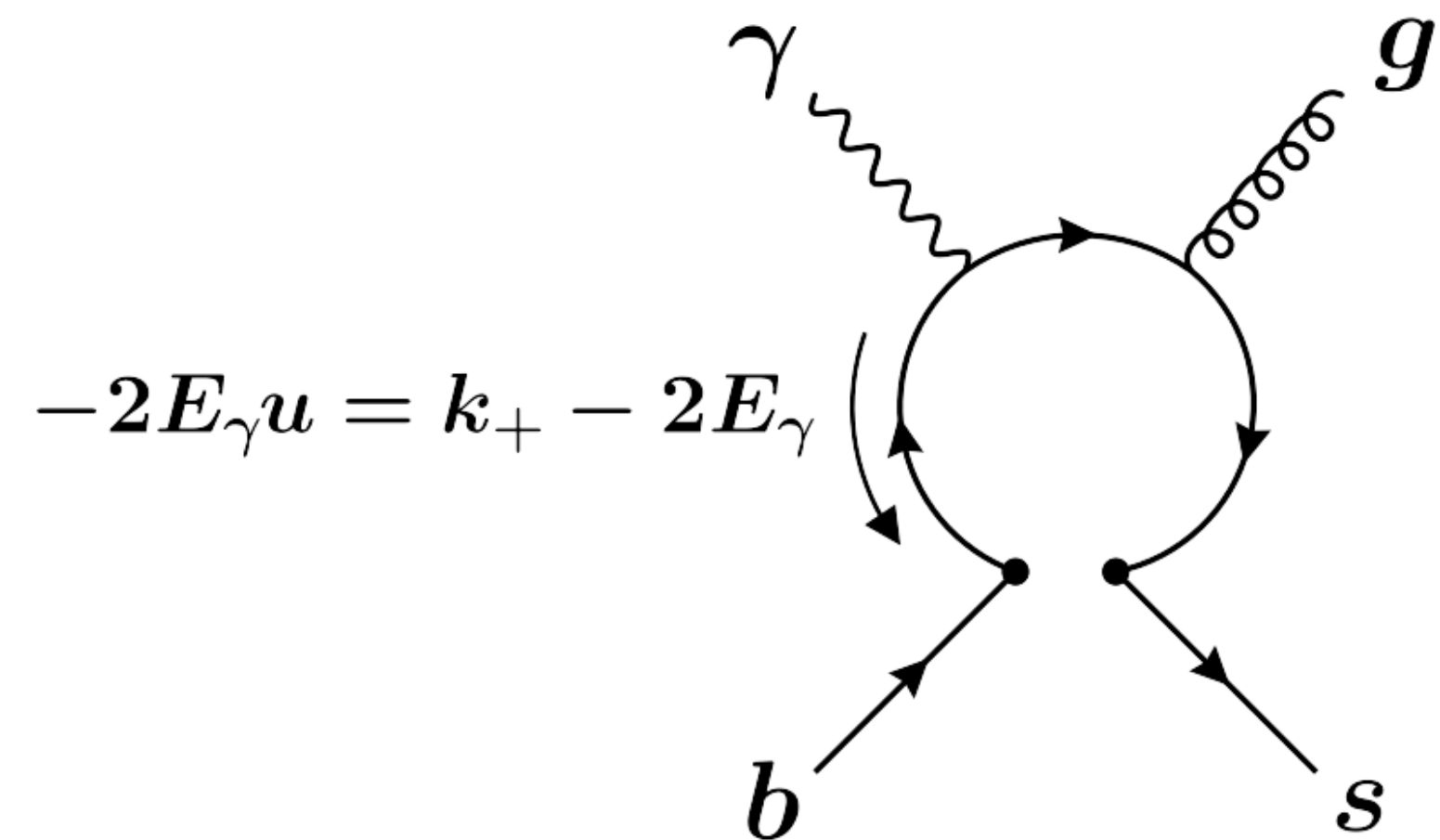
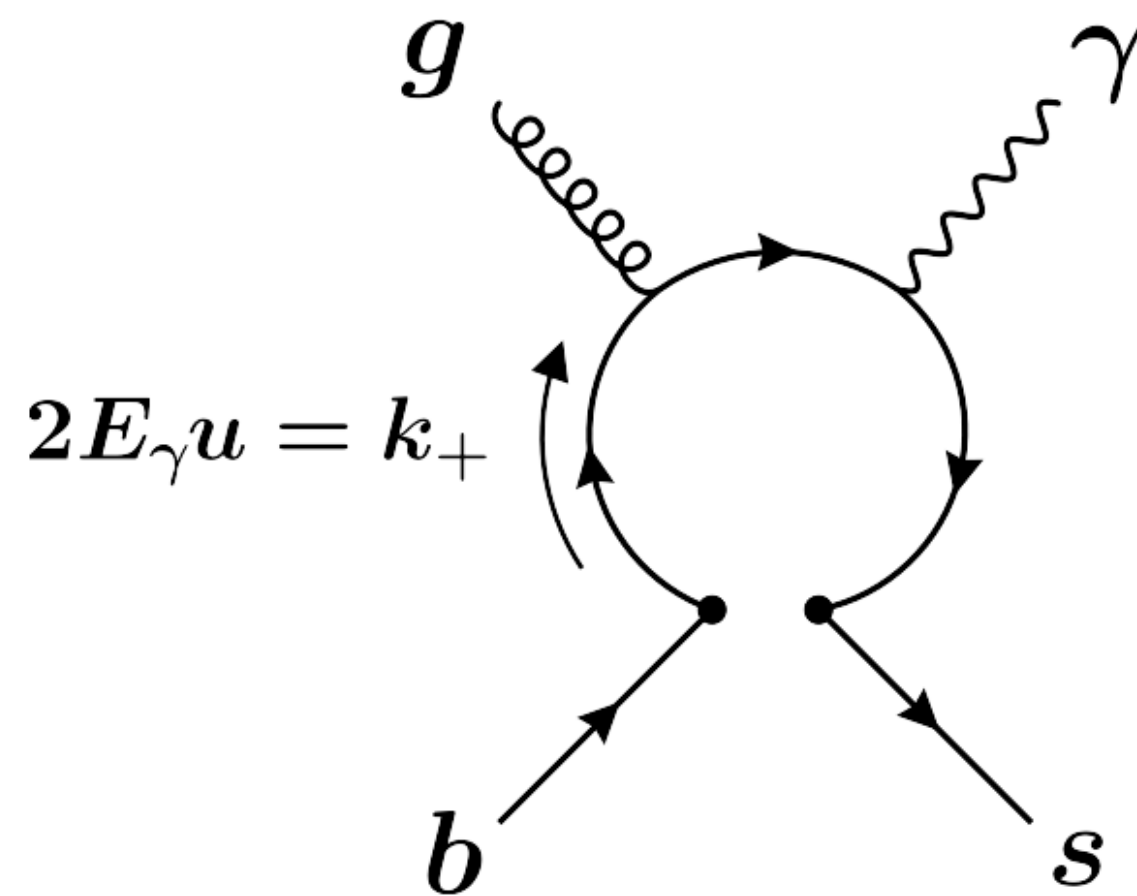
- Massive charm loop $\rightarrow$ Introduce dimensionless quantity $x = \sqrt{1 - \frac{4m_c^2}{s}}$

The „penguin“-jet function

- Additional complication due to factorisation theorem

$$d\Gamma(\bar{B} \rightarrow X_s \gamma) \sim \int_0^1 du H(\boldsymbol{u}; \mu) \int_{-\infty}^{\infty} d\omega_1 J^{(\bar{n})}(\boldsymbol{u}, \omega_1; \mu) \int_{-\infty}^{\bar{\Lambda}} d\omega J^{(n)}(\omega; \mu) g_{17}(\omega, \omega_1; \mu)$$

- Energy constraint on k_+



The „penguin“-jet function

- At LO still possible with direct integration [Bartocci (Thesis); 26]

$$\mathcal{A}_1(u, x) \sim \frac{\Gamma(\varepsilon)}{1 - \varepsilon} (m_c^2)^{-\varepsilon} E_\gamma \left[4\varepsilon(1 - x^2)^\varepsilon (3 + x^2)^{-\varepsilon} + (1 - x^2)u\bar{u} \left(1 - \left(1 - \frac{4\bar{u}u}{1 - x^2} \right)^{-\varepsilon} \right) \right] \\ \times \bar{\chi}_{hc,L} \left[(1 - u(1 - \varepsilon))\not{\epsilon}_{\perp,g}^* \not{\epsilon}_{\perp,\gamma}^* - (1 - \bar{u}(1 - \varepsilon))\not{\epsilon}_{\perp,\gamma}^* \not{\epsilon}_{\perp,g}^* \right] \not{b}_L$$

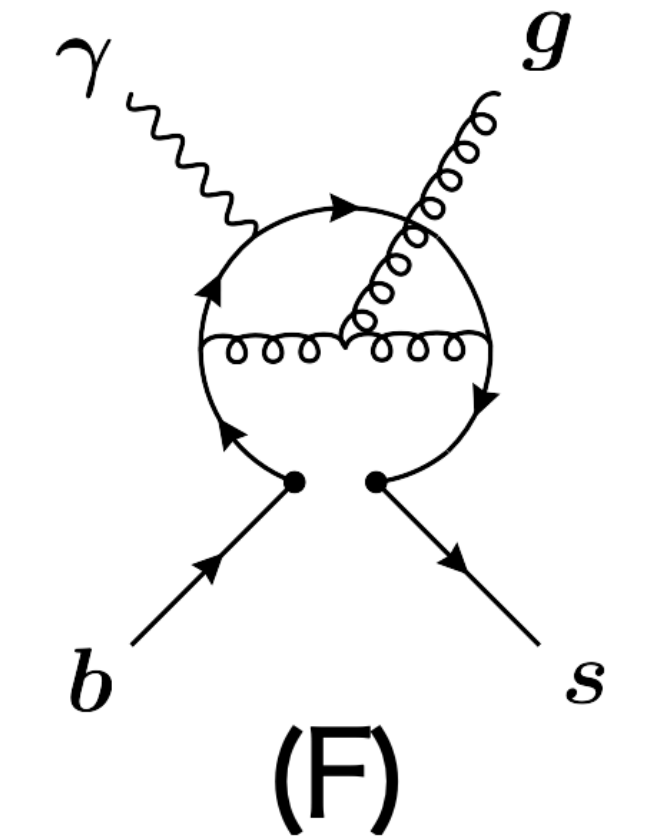
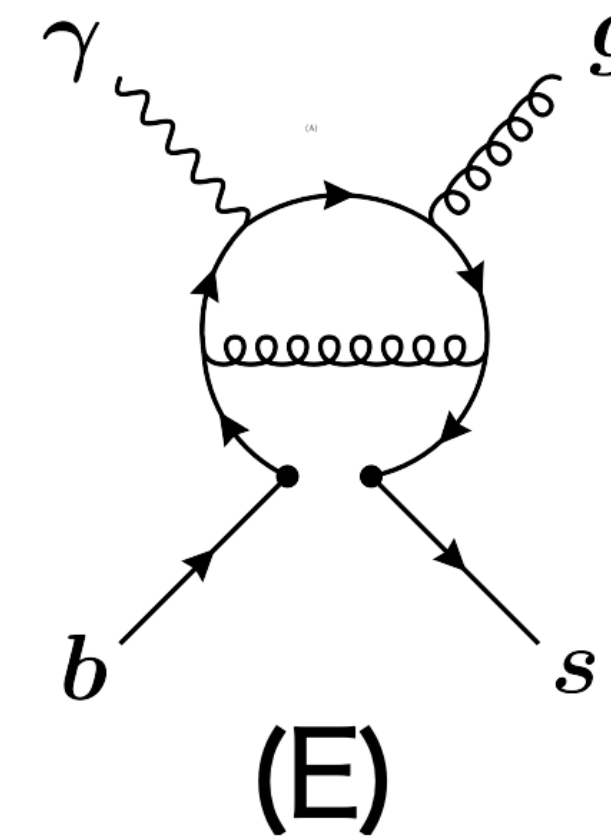
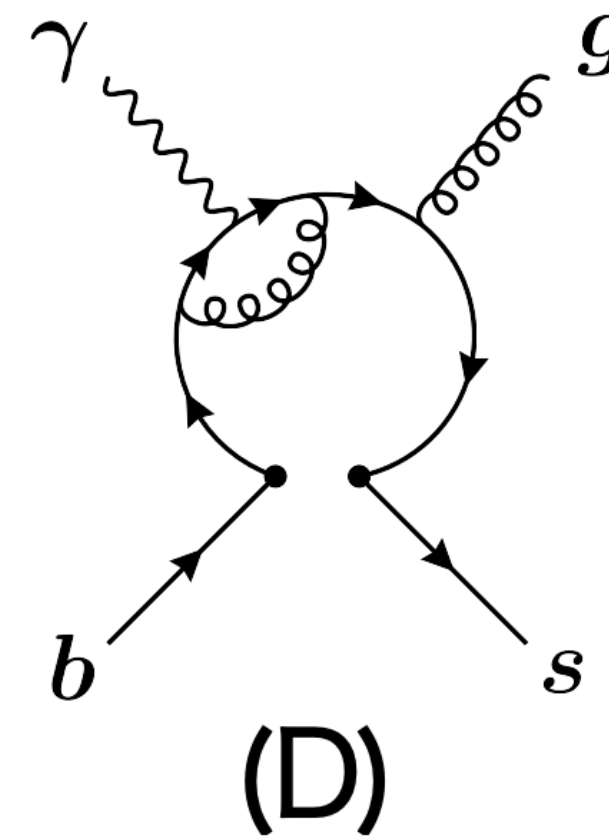
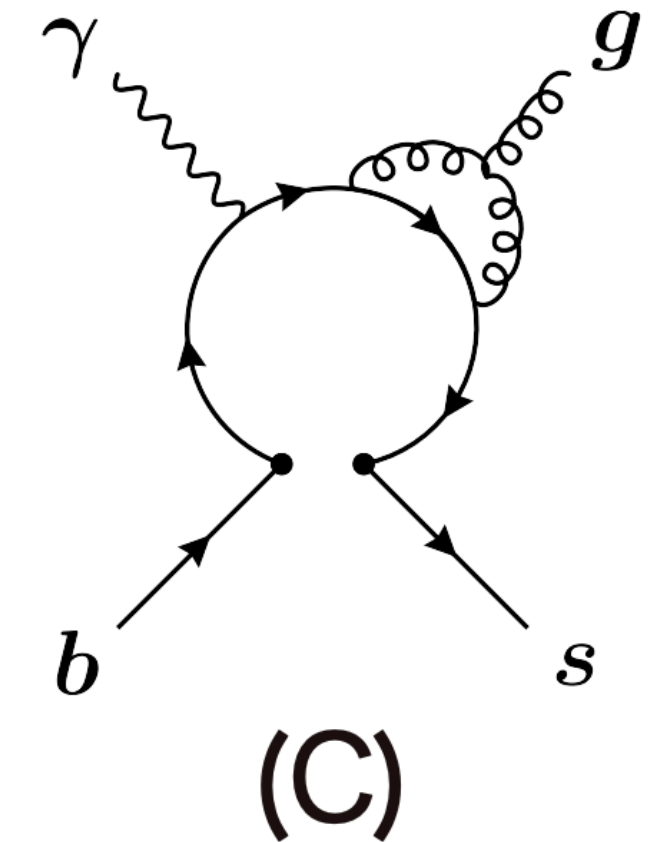
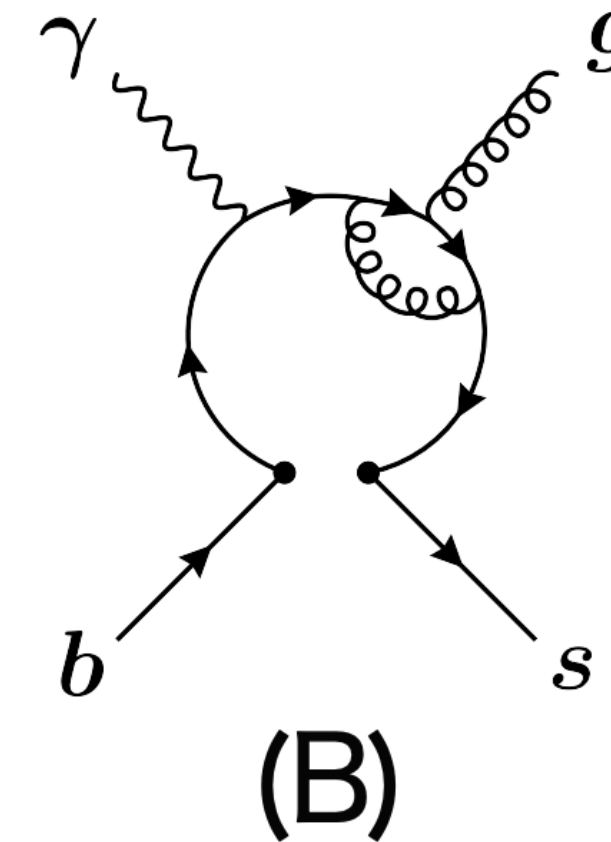
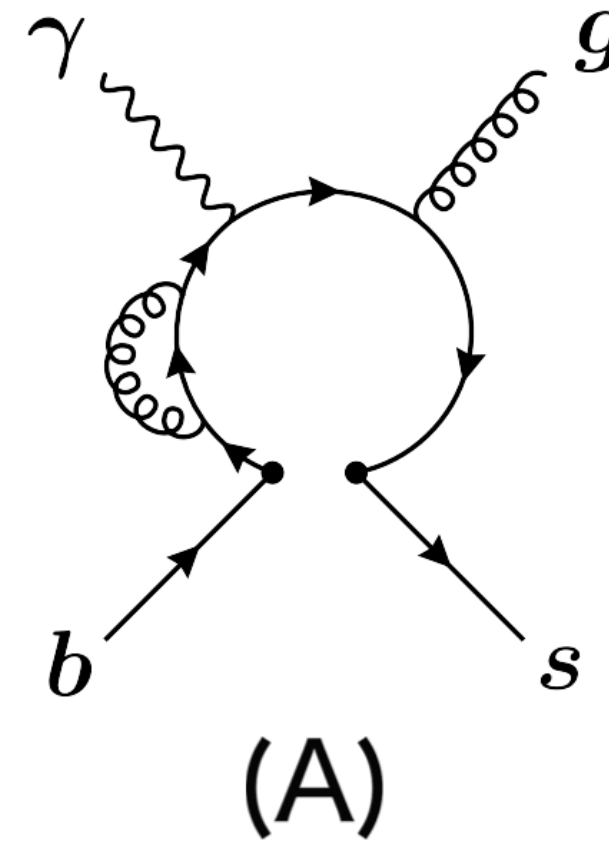
- Smooth in $m_c \rightarrow 0$ limit
- In general more Dirac structures possible in massive amplitude
 - ▶ Remember momentum scaling: $E_\gamma \sim 1$, $\omega_1 \sim \lambda^2$, $m_c \sim \lambda$
 - ▶ All other structures are power suppressed

The „penguin“-jet function

- At NLO: Two distinct categories
- Category I:
- Additional gluon is not connected to external quarks
- Only anti-hard collinear region present

$$k_g^\mu = \omega_1 n^\mu \quad k_\gamma^\mu = E_\gamma \bar{n}^\mu \quad s = (k_g + k_\gamma)^2$$

$$x = \sqrt{1 - \frac{4m_c^2}{s}}$$

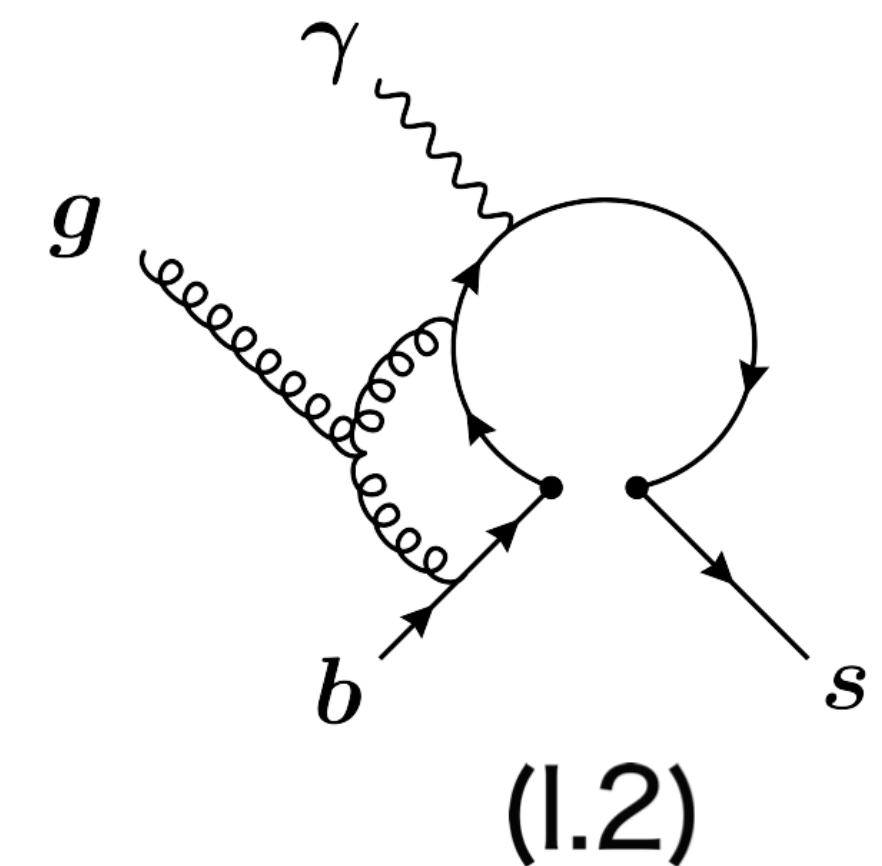
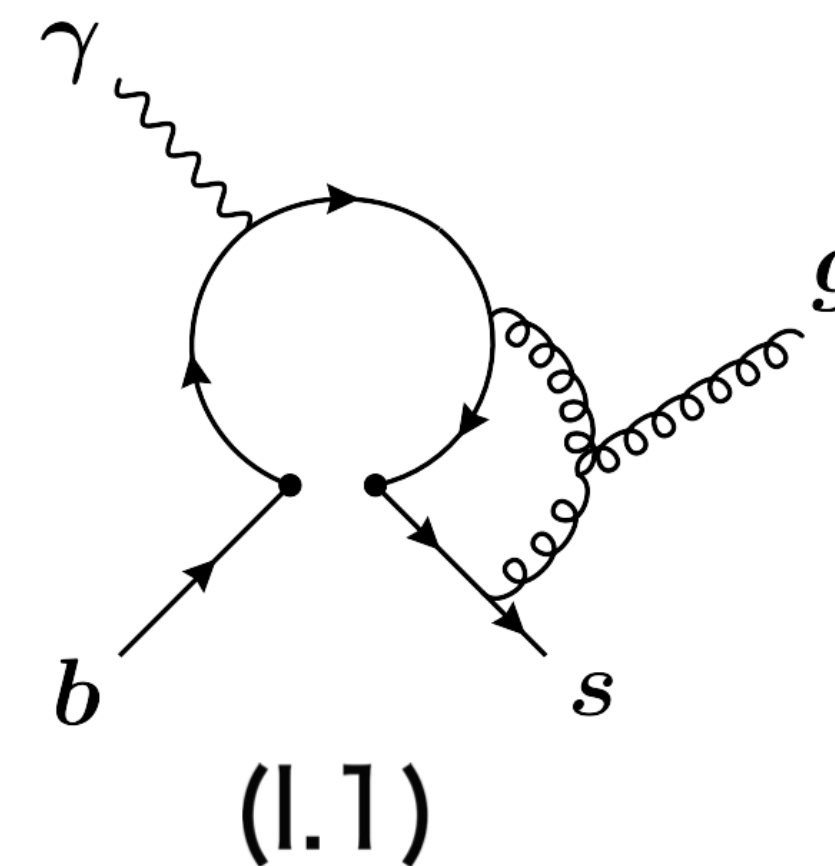
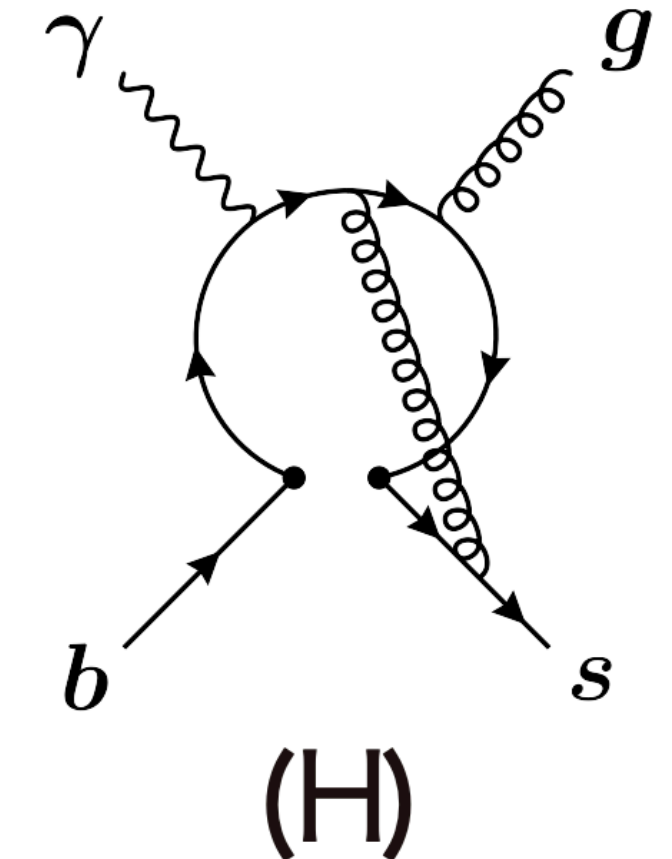
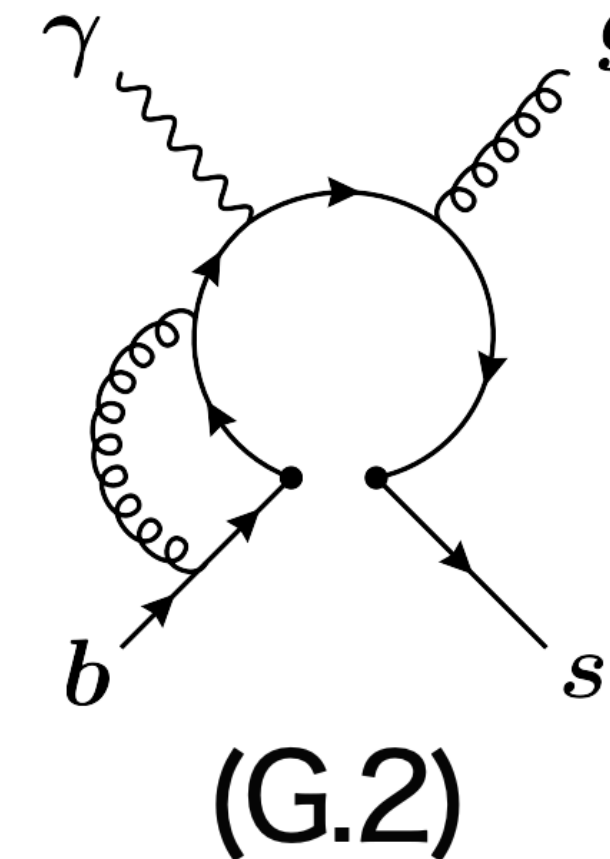
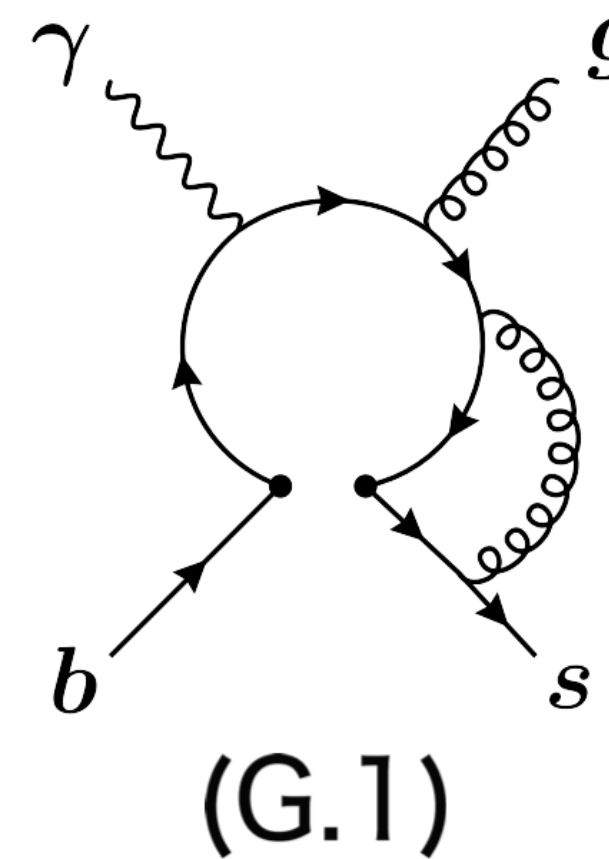


The „penguin“-jet function

- At NLO: Two distinct categories
- Category II:
- Additional gluon is connected to external quarks
- Multiple regions contribute

$$k_g^\mu = \omega_1 n^\mu \quad k_\gamma^\mu = E_\gamma \bar{n}^\mu \quad s = (k_g + k_\gamma)^2$$

$$x = \sqrt{1 - \frac{4m_c^2}{s}}$$



The „penguin“-jet function

- For category II we have to expand in anti-hard collinear region
- Example:

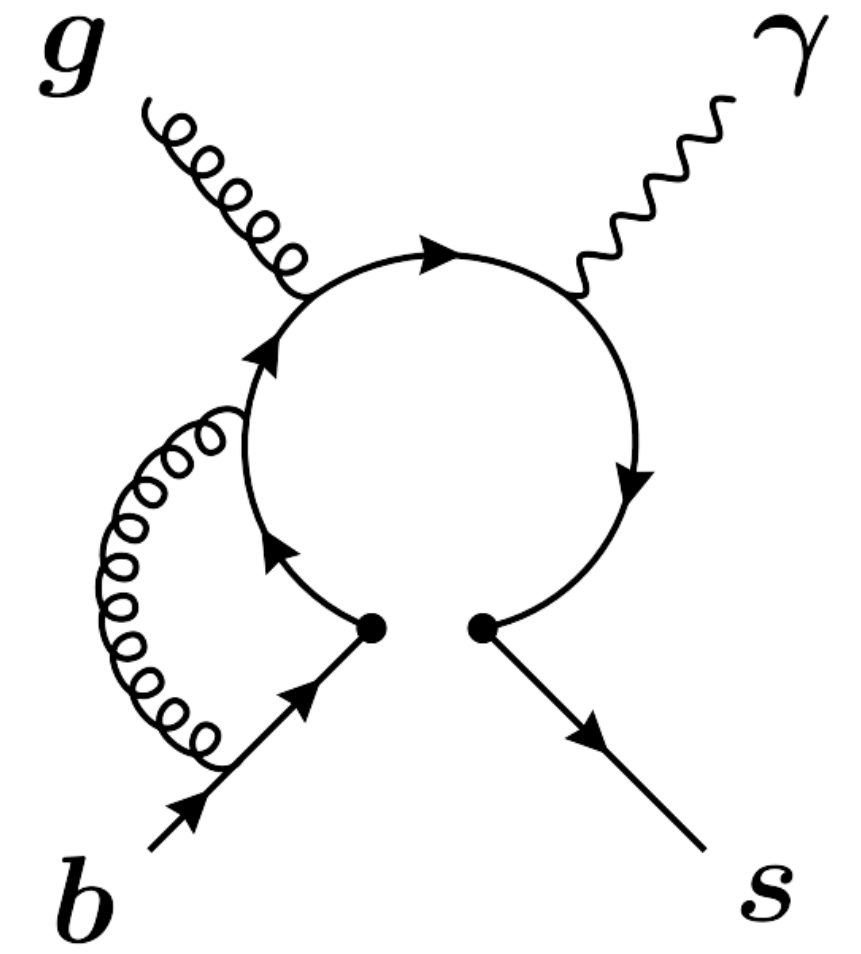
$$\mathcal{A}_2 = \int \frac{d^d k}{(2\pi)^d} \frac{d^d l}{(2\pi)^d} \bar{u}_{hc,L} \cdots \frac{\not{k}_b + \not{l} - m_b}{(k_b + l)^2 - m_b^2 + i\eta} \cdots b_L$$

- ▶ Remember momentum scaling:

$$k_b \sim (1,1,1) m_b \quad l \sim (1,\lambda^2,\lambda) m_b \Rightarrow \frac{\not{k}_b + \not{l} - m_b}{(k_b + l)^2 - m_b^2 + i\eta} b_L = \frac{\not{s}}{2m_b} b_L + \mathcal{O}(\lambda)$$

$$\mathcal{A}_2^{(0)} = \int \frac{d^d k}{(2\pi)^d} \frac{d^d l}{(2\pi)^d} \bar{u}_{hc,L} \cdots \frac{l_+ + k_+}{l_+ + i\eta} \cdots b_L + \mathcal{O}(\lambda)$$

- Do this for all diagrams in category II
 - ▶ Sum of diagrams $\sim C_A$



The „penguin“-jet function

Initial amplitude

- In total 18 diagrams

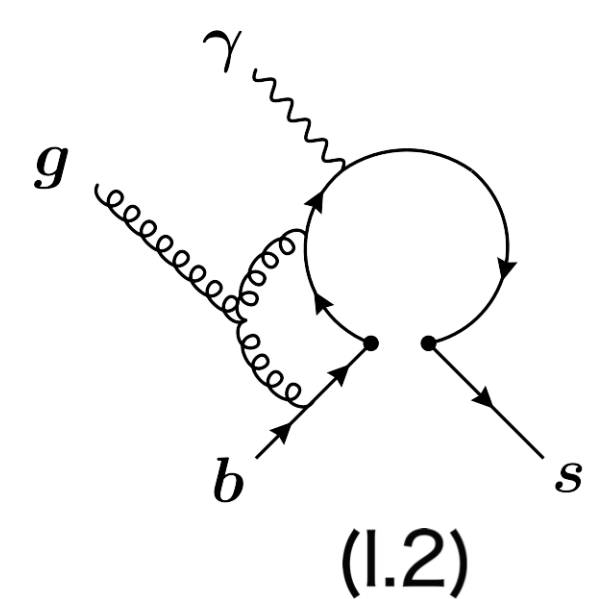
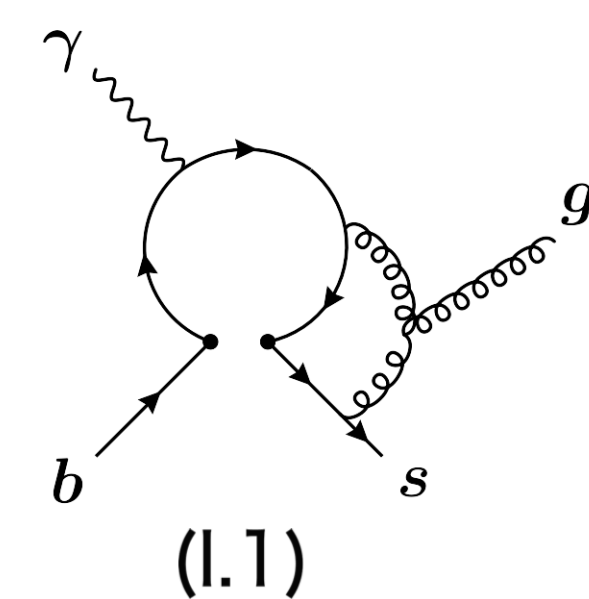
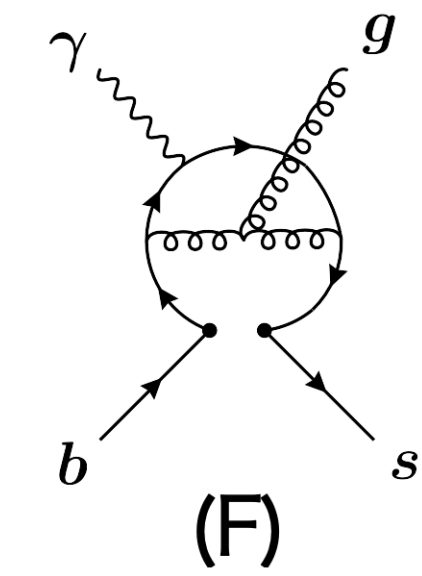
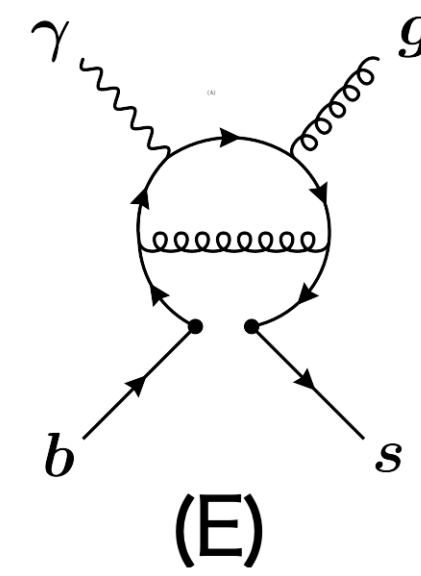
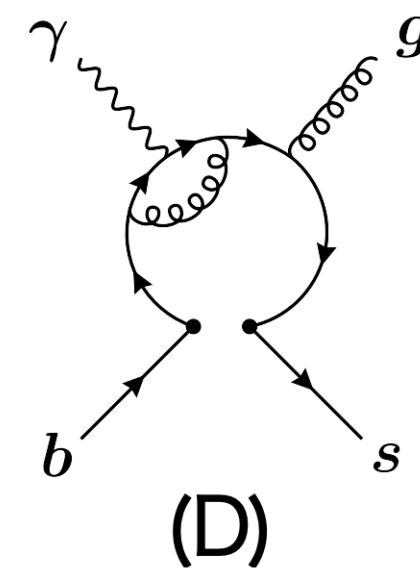
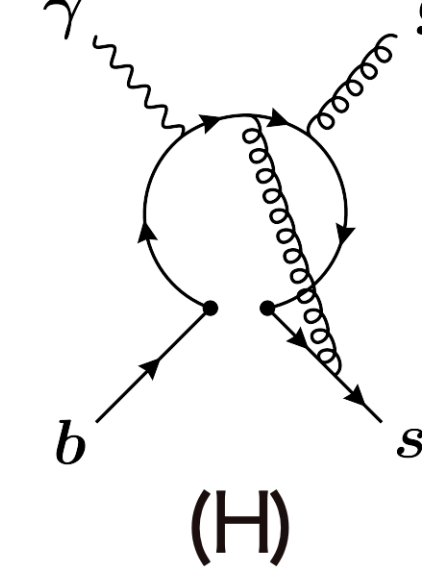
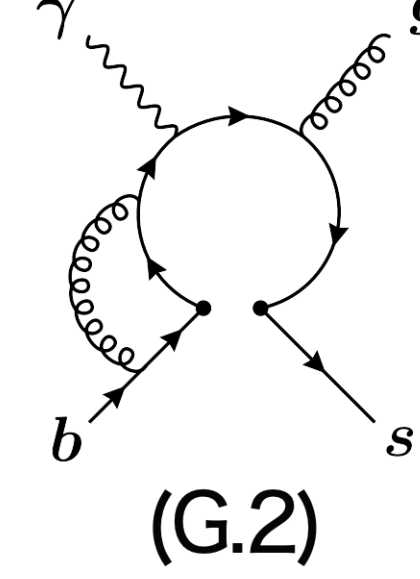
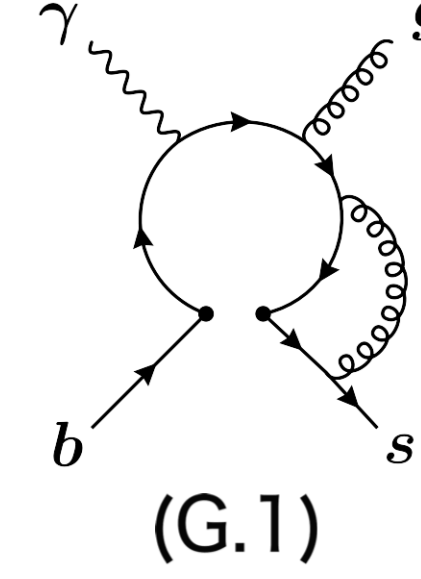
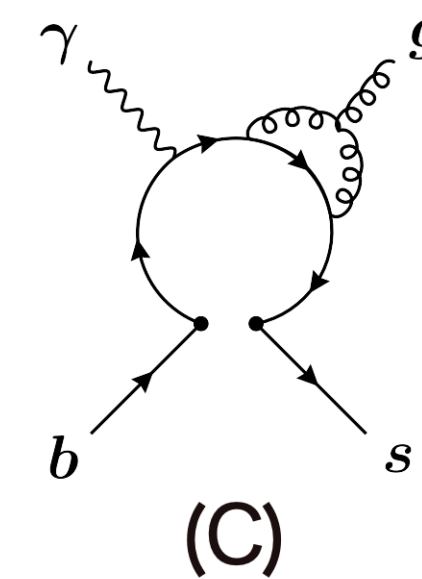
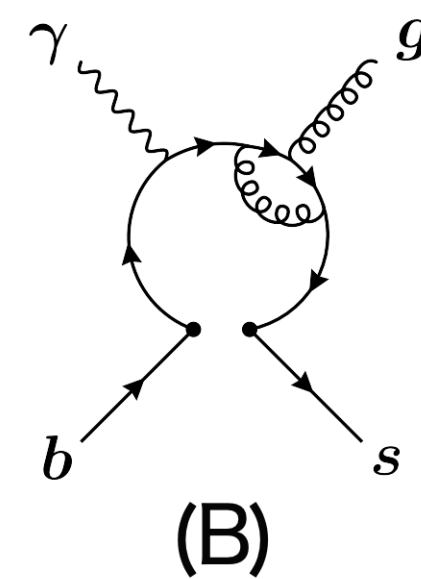
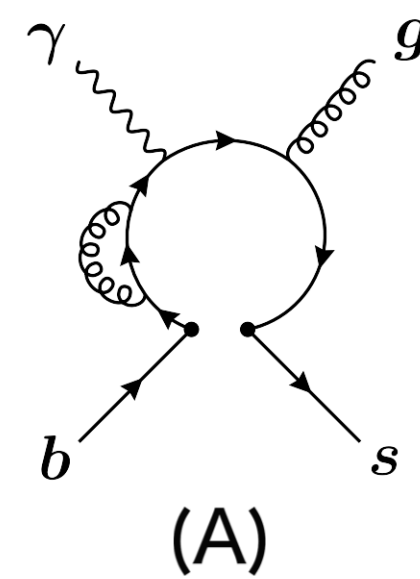
u-dependent delta function

IBP reduction

DE in canonical form

Boundary condition

Result



The „penguin“-jet function

Initial amplitude

u-dependent delta function

IBP reduction

DE in canonical form

Boundary condition

Result

- All integrals still contain delta function from energy constraint

$$\delta\left(u - \frac{k_+}{2E_\gamma}\right) \quad \text{or} \quad \delta\left(\bar{u} - \frac{k_+}{2E_\gamma}\right)$$

- Reverse unitarity [Anastasiou,Melnikov; 02]

$$\delta\left(u - \frac{k_+}{2E_\gamma}\right) = \frac{iE_\gamma}{\pi} \left[\frac{1}{k_+ - 2E_\gamma u + i\eta} - \frac{1}{k_+ - 2E_\gamma u - i\eta} \right]$$

- Reduce to scalar amplitude via FeynCalc [Shtabovenko,et al.; 23]
 - In total 9 unique topologies with $\mathcal{O}(3500)$ scalar integrals

The „penguin“-jet function

Initial amplitude

u-dependent delta function

IBP reduction

DE in canonical form

Boundary condition

Result

- Perform IBP with KIRA [Lange, Usovitsch, Wu; 25]

- Reintroduction of delta function leads to

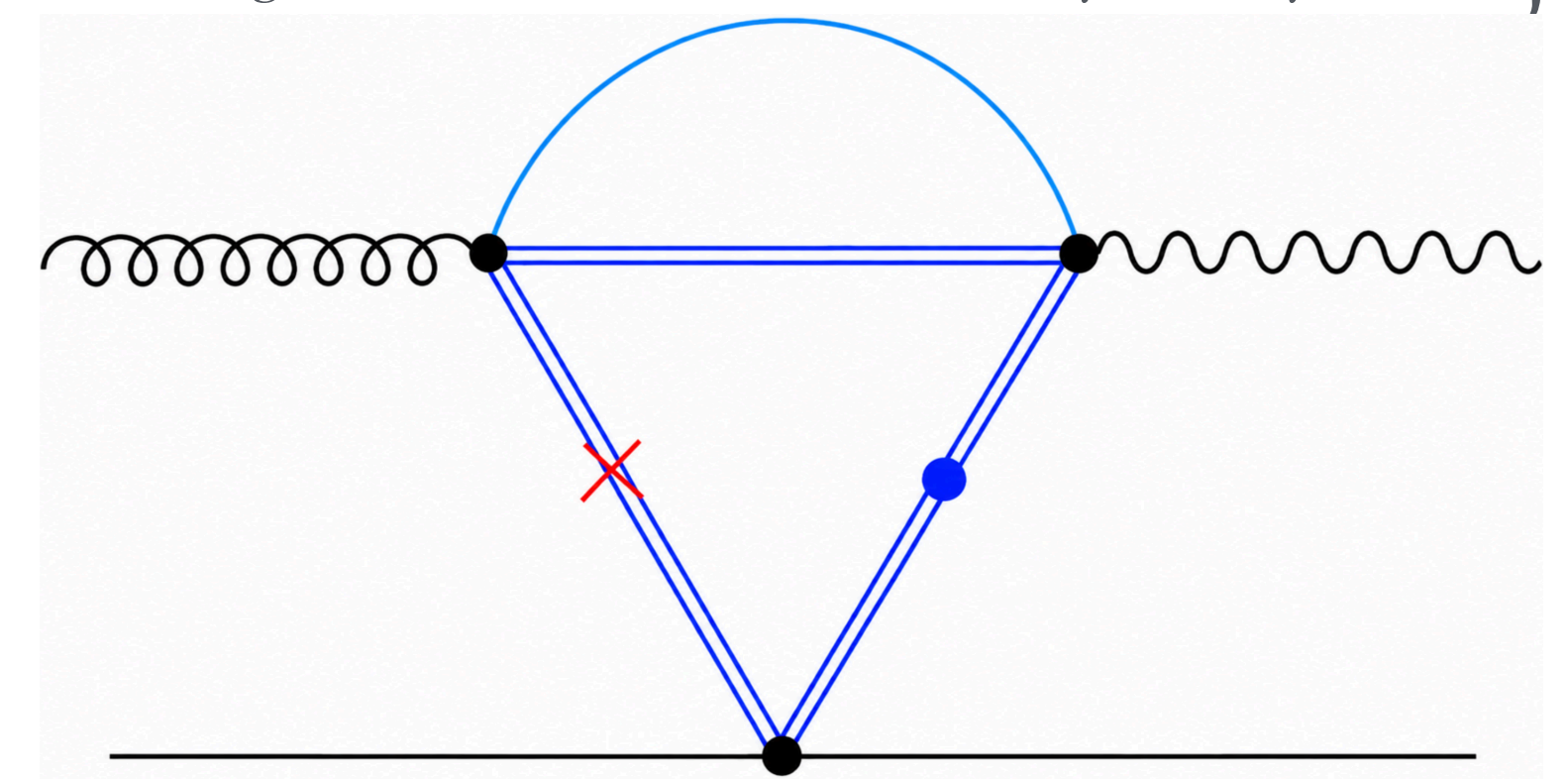
$$(k_+ - 2uE_\gamma)^n \delta(k_+ - 2uE_\gamma) = 0, \quad n = 1, 2, \dots$$

- Obtain 7 integral topologies with 105 MI

$$\{\mathcal{T}_A, \mathcal{T}_B, \mathcal{T}_C, \mathcal{T}_D, \mathcal{T}_E, \mathcal{T}_F, \mathcal{T}_G\} \rightarrow \{45, 8, 31, 7, 2, 11, 1\}$$

$$\mathcal{T}_G = \left\{ l^2, k^2 - m_c^2, (k + k_g)^2 - m_c^2, (k - k_\gamma)^2 - m_c^2, \right. \\ \left. (k + l + k_g)^2 - m_c^2, l_+, k_+ - 2uE_\gamma, l \cdot k_\gamma, l \cdot k \right\}$$

$$F_G[1, 1, 0, 2, 1, 0, 1, 0, 0] \rightarrow$$



The „penguin“-jet function

Initial amplitude

u-dependent delta function

IBP reduction

DE in canonical form

Boundary condition

Result

- Solve the MIs via methods of differential equations

► Bring system in canonical form

$$dF_i(u, x) = A(u, x, \varepsilon) F_i(u, x) \rightarrow dG_i(u, x) = \varepsilon \tilde{A}(u, x) G_i(u, x)$$

► Found with CANONICA or Libra

[Meyer; 17]

[Lee; 20]

- We observe „one“ difficulty with the alphabet

$$\{u, u - 1, x + 1, x, x - 1, 2u - x - 1, 2u + x - 1, 4u - x^2 + 1, 4u - x^2 - 3\}$$

- Can we obtain the x dependency in a different fashion ?

The „penguin“-jet function

Initial amplitude

$$F_i(u, x) = \tilde{F}_i(x) \delta(k_+ - 2E_\gamma u) \rightarrow \int_0^1 du F_i(u, x) = \tilde{F}_i(x)$$

[Gituliar; 17] [Moch, Magerya; 21][KB, Huber, Moos; 25]

u-dependent delta function

IBP reduction

DE in canonical form

Boundary condition

Result

- Now we can solve both sides and compare to fix $C_j(x)$

$$\int_0^1 du u^a T_{ij}(u, x, \varepsilon) \left[\int_0^u du' g_{u,j}(u', x) + C_j(x) \right] = (2E_\gamma)^{-1-a} k_+^a \tilde{F}_i(x)$$

- Reduce rhs via IBPs and set up new DE system

- In total $\mathcal{O}(30)$ new MI to compute with alphabet: $\{x+1, x, x-1\}$

The „penguin“-jet function

Initial amplitude

u-dependent delta function

IBP reduction

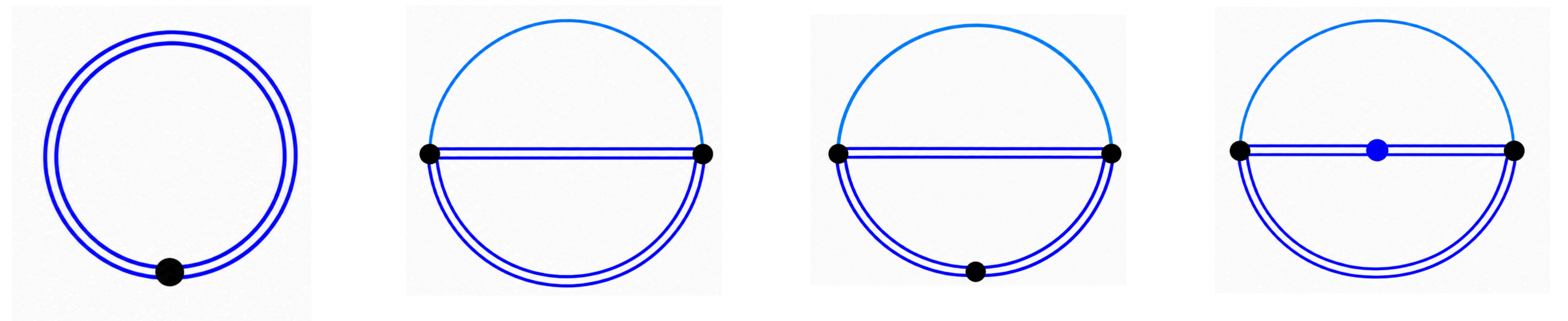
DE in canonical form

Boundary condition

Result

- Solve the boundary conditions in the asymptotic limit $x \rightarrow 1$
 - We use method of regions for computation

- In case of family $\mathcal{T}_G$



- We have solved all necessary boundary conditions

The „penguin“-jet function

Initial amplitude

u-dependent delta function

IBP reduction

DE in canonical form

Boundary condition

Result

- What is the status of the computation ?
- Finished computing all $\tilde{F}_i(x)$ $i = \{B, D, E, F, G\}$
- Solution falls into the classes of HPLs
 - We expect the same for $i = \{A, C\}$

The „penguin“-jet function

Initial amplitude

u-dependent delta function

IBP reduction

DE in canonical form

Boundary condition

Result

- What is the status of the computation ?
- Finished computing all $\tilde{F}_i(x)$ $i = \{B, D, E, F, G\}$
- Solution falls into the classes of HPLs
 - We expect the same for $i = \{A, C\}$
- What is left to do ?
- Finish computation of $\tilde{F}_i(x)$
- Fix boundary function $C_i(x)$
- Check that poles match massless case

Summary

NLO analysis

Large scale ambiguity from $Q_1 - Q_7$

Summary

NLO analysis

Large scale ambiguity from $Q_1 - Q_7$

Factorisation theorem

Formulation in SCET at LO and NLO

Summary

NLO analysis

Large scale ambiguity from $Q_1 - Q_7$

Factorisation theorem

Formulation in SCET at LO and NLO

„Penguin“-jet function

Outlined the computation for
analytic result

Summary

NLO analysis

Large scale ambiguity from $Q_1 - Q_7$

Factorisation theorem

Formulation in SCET at LO and NLO

„Penguin“-jet function

Outlined the computation for analytic result

What is next ?

1. Finish the two-loop computation
2. Full Pheno analysis