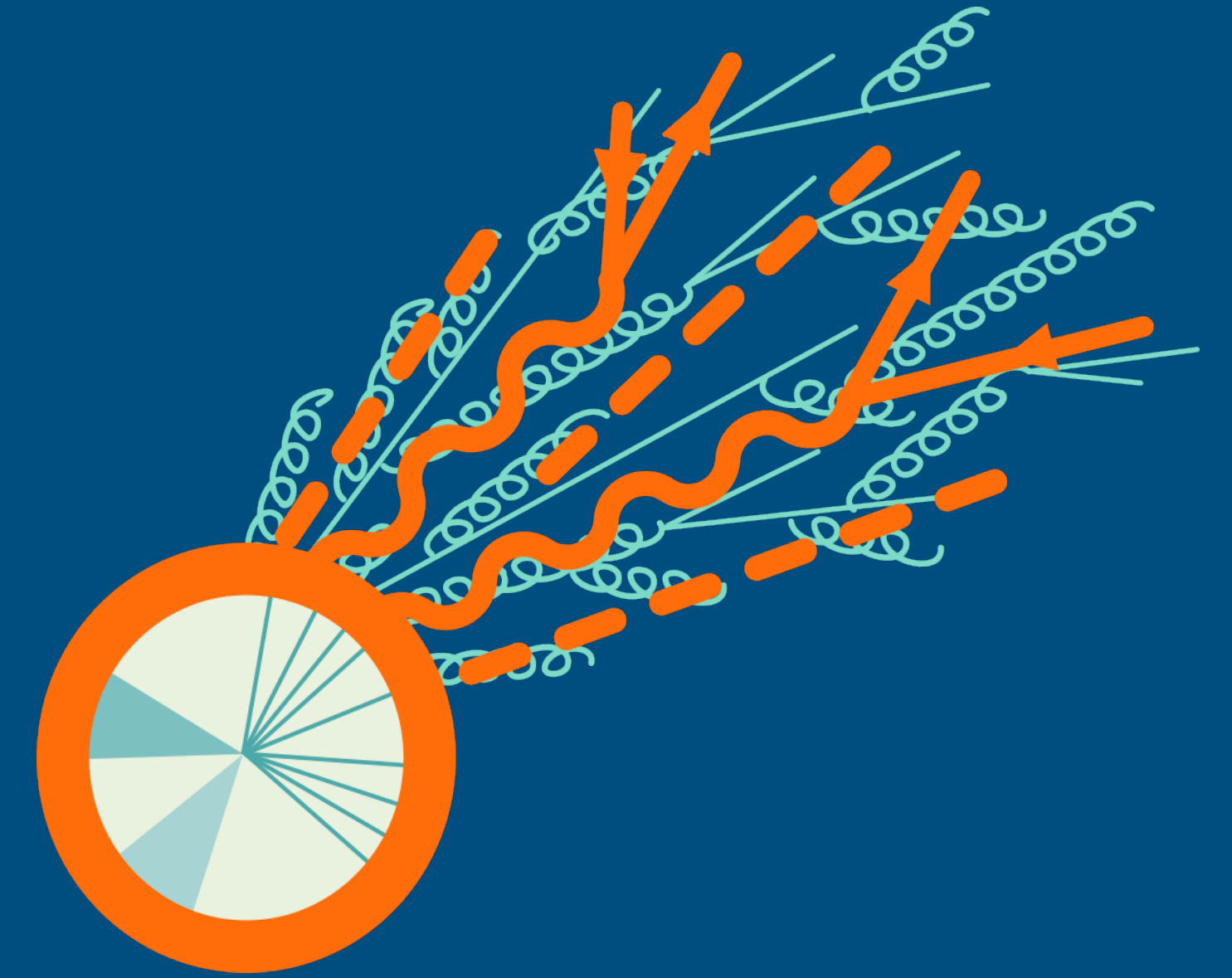


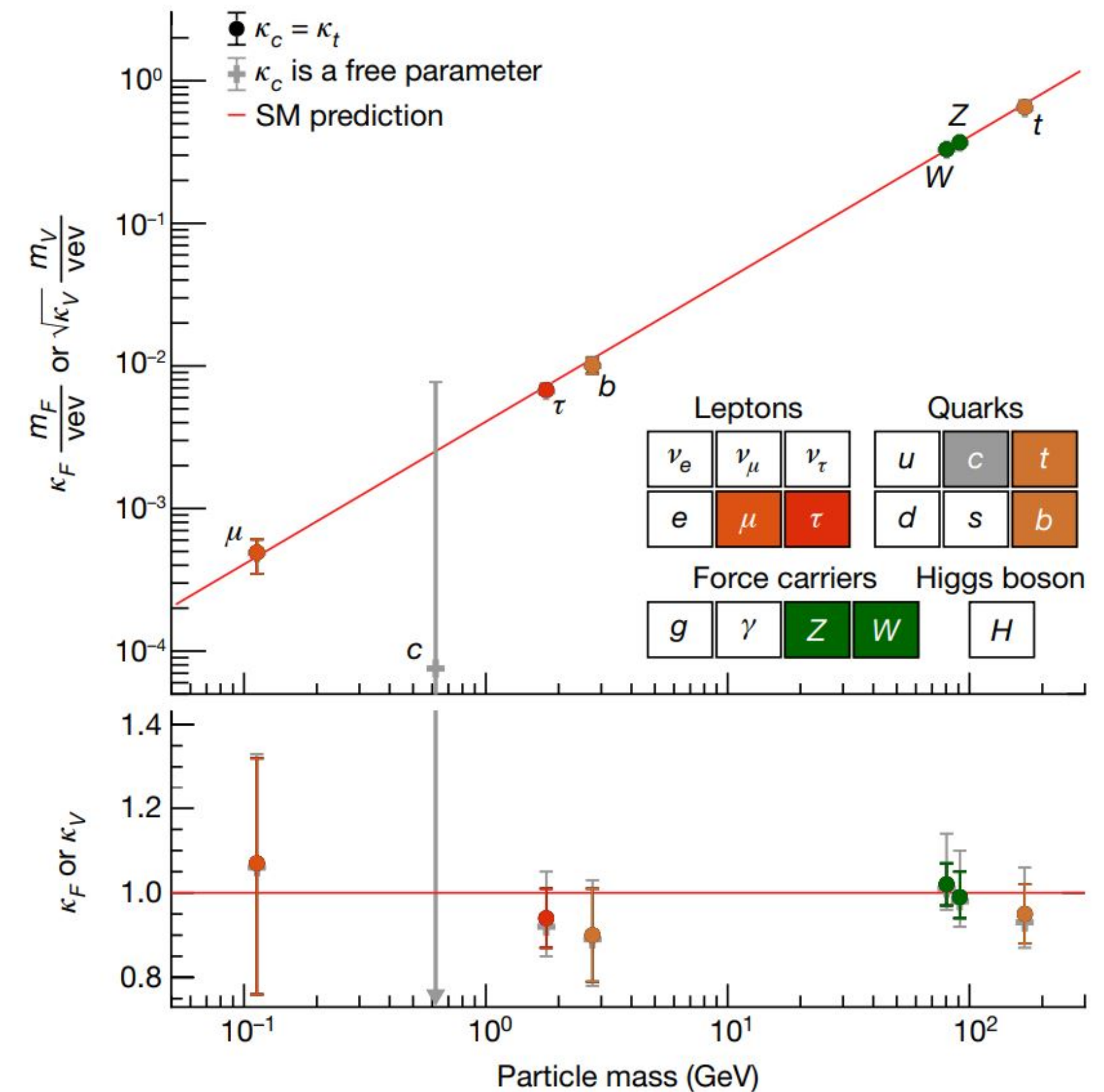
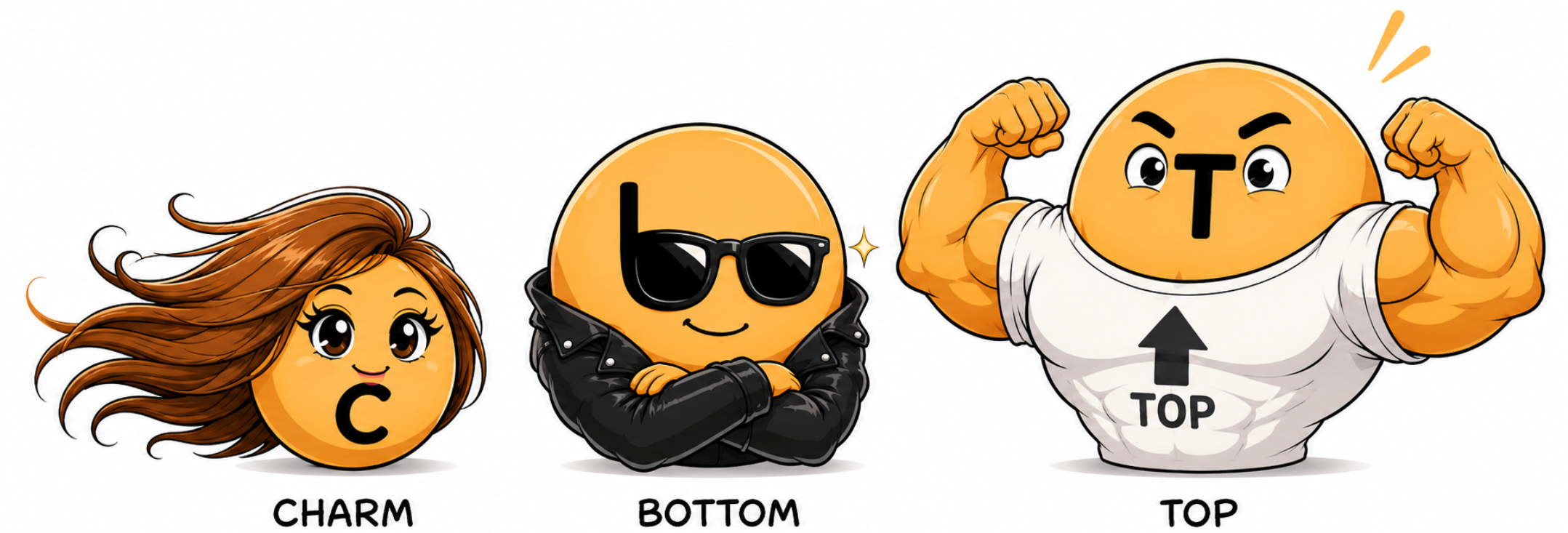


# Substructure of Heavy Quark Jets

Based on results of [arxiv:2606.20798](https://arxiv.org/abs/2606.20798) in collaboration with L. Mai, S. Marzani, D. Reichelt, S. Schumann, L. Stoecker, and on work in progress with L. Bason, S. Devoto, S. Marzani, P.F. Monni and G. Ridolfi.

# Introduction

- All the fundamental particles acquire mass through the Higgs mechanism: SM prediction  $Y_f \propto m_f$
- Heavy flavor processes offer a more robust test of pQCD
- $b - c$  quarks constitute portal to non perturbative physics
- Recent measurements on jss observable for H-F jets:
  - Lund b jet plane by LHCb collaboration,
  - $\theta_g, z_g$  observable for charm jets by ALICE,
  - Upcoming measurements at ATLAS and CMS for b-jet substructure to expose dead-cone effect

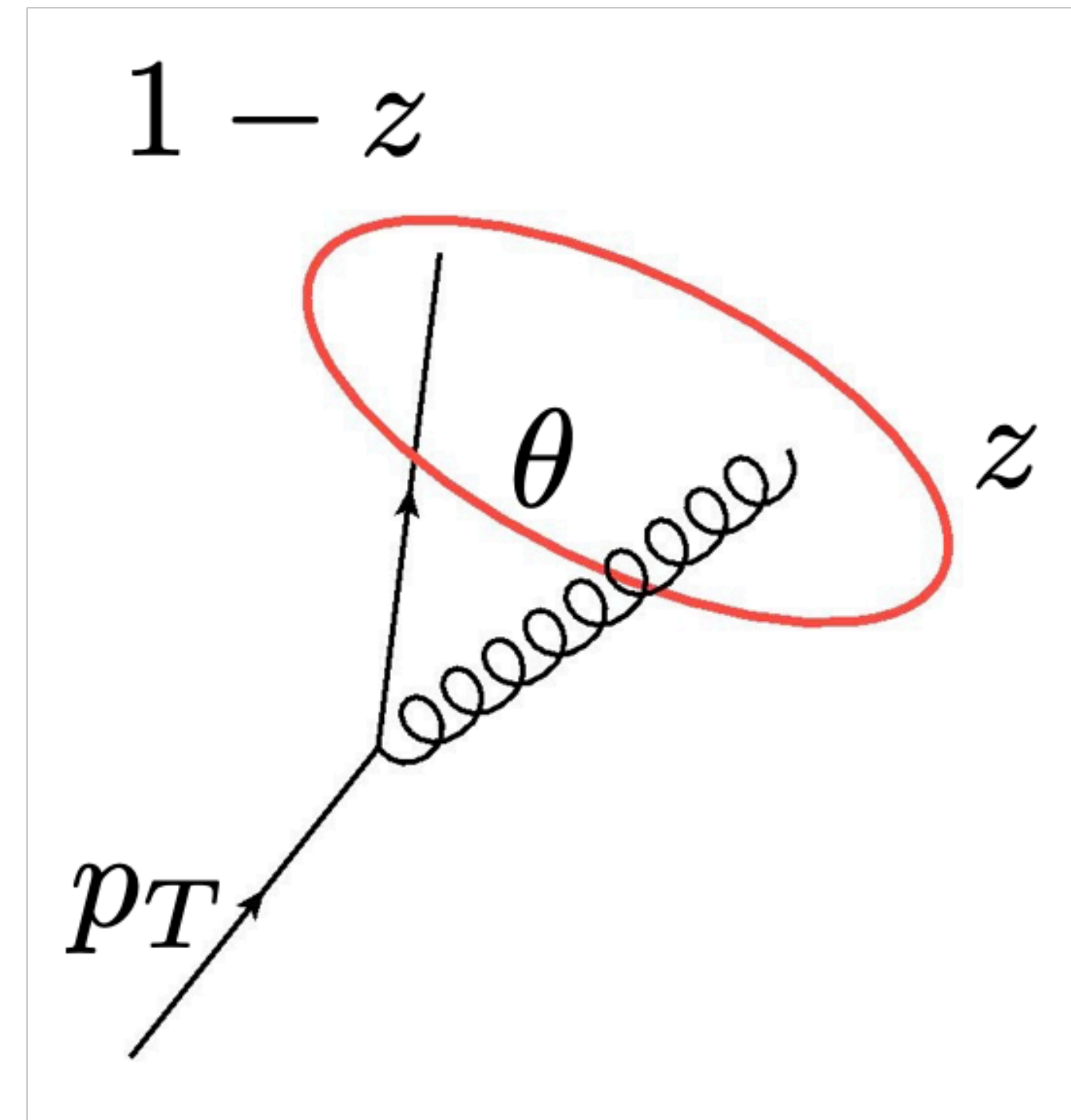


# Jet angularities at hadron colliders

- We want to study jet angularities (sensitive to radiation pattern, i.e. to HF jet substructure).
- In case of massless theory with just one emission
$$\lambda_\alpha \simeq z\theta^\alpha$$
- Many more possible definitions when massive particles are involved:

$$\lambda_{\alpha} = \sum_i \frac{p_{ti}}{p_T} \left( \frac{\Delta R_i}{R_0} \right)^{\alpha}, \quad \dot{\lambda}_{\alpha} = \sum_i \frac{p_{ti}}{p_T} \left( \frac{2p_i \cdot n}{p_{ti} R_0} \right)^{\frac{\alpha}{2}}, \quad \dot{\lambda}_{\alpha,0} = \sum_i \frac{p_{ti}}{p_T} \left( \frac{2p_i \cdot n_0}{p_{ti} R_0} \right)^{\frac{\alpha}{2}}$$

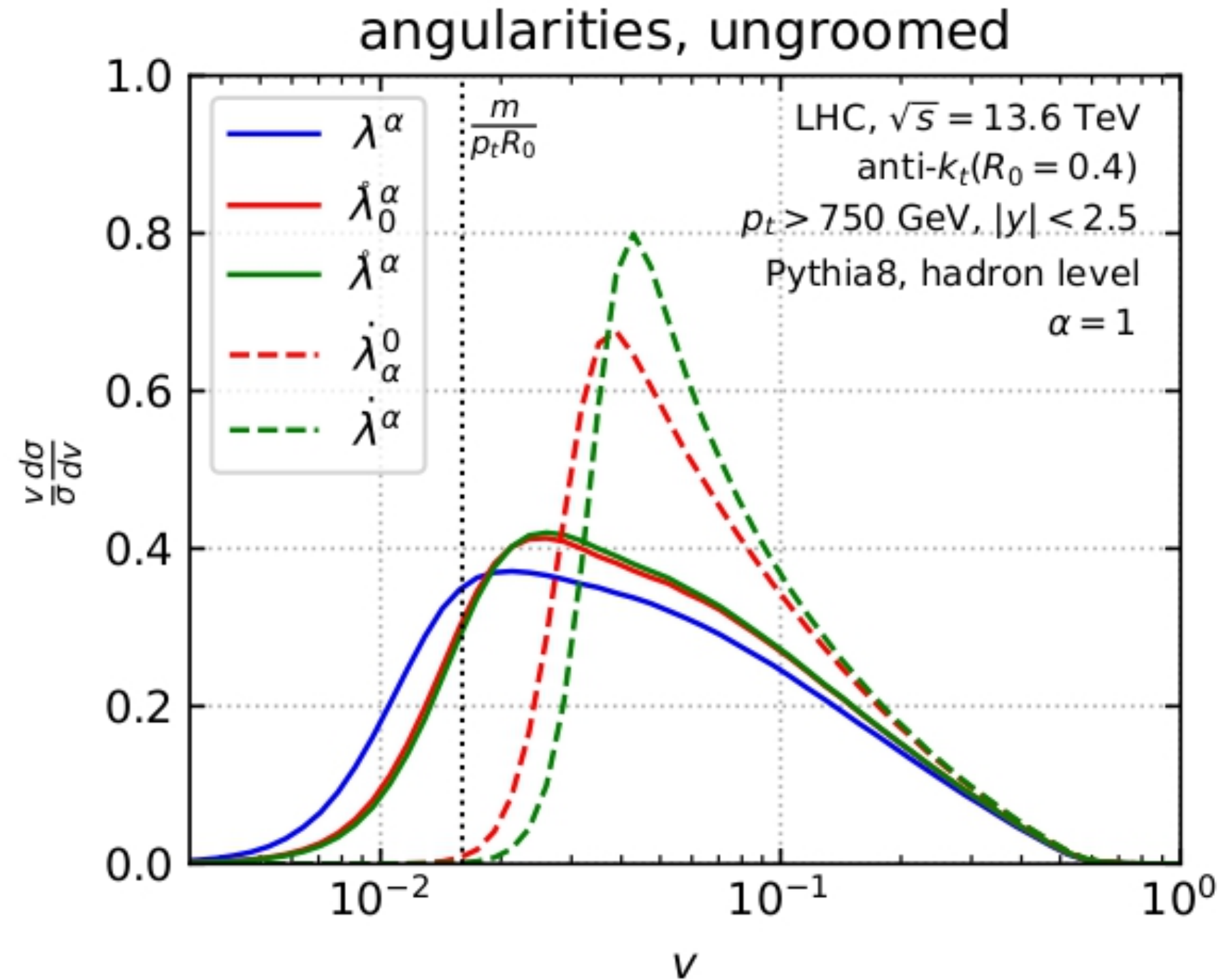
- All these variables coincide in the massless case.
- All these variables do not coincide when one take the quasi-collinear limit ( $\theta_{k,\text{jet}} \sim m/p_T$ )



$n(n_0)$  is a massive (massless) 4 vector aligned with the WTA axis

See also (C. Lee, P. Shrivastava, V. Vaidya), for SCET analyses of ECFs.

# Jet angularities definitions: Monte Carlo study



$$\lambda_\alpha = \sum_i \frac{p_{ti}}{p_T} \left( \frac{\Delta R_i}{R_0} \right)^\alpha$$

P.K. Dhani et al.

$$\lambda_\alpha^\circ \simeq \sum_i \frac{p_{ti}}{p_T} \left( \frac{m_i^2}{p_{ti}^2 R_0^2} + \frac{m_n^2}{p_T^2 R_0^2} + \frac{\Delta R_i^2}{R_0^2} \right)^{\frac{\alpha}{2}}$$

$$\lambda_{\alpha,0}^\circ \simeq \sum_i \frac{p_{ti}}{p_T} \left( \frac{m_i^2}{p_{ti}^2 R_0^2} + \frac{\Delta R_i^2}{R_0^2} \right)^{\frac{\alpha}{2}}$$

- The circled observables exhibits a larger peak than  $\lambda_\alpha$  : more sensitive to mass effects as mass enter both at amplitude and at substructure observable level
- The mass contribution in on  $\lambda_\alpha$  distribution is only “dynamical” as mass effects enter only at amplitude level. More sensitive to suppression of collinear radiation

# Resummation in the CAESAR formalism

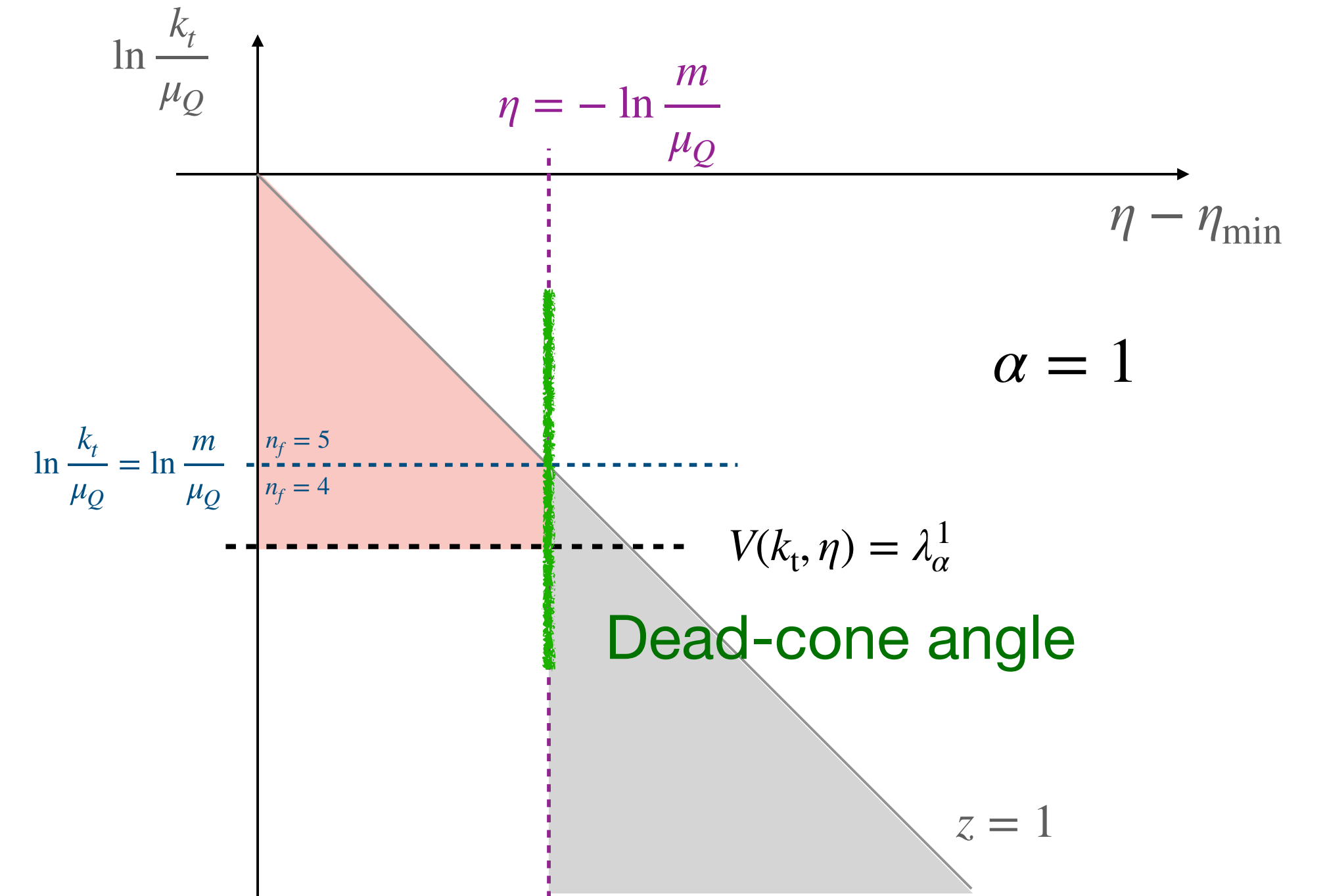
Resummation performed using the CAESAR formalism (A. Banfi, G. Salam, G. Zanderighi)

$$\Sigma_{\text{res}}(v) = \sum_{\delta} \int d\mathcal{B}_{\delta} \frac{d\sigma_{\delta}}{d\mathcal{B}_{\delta}} \exp \left[ - \sum_{l \in \delta} R_l^{\mathcal{B}_{\delta}}(v) \right] \mathcal{S}^{\mathcal{B}_{\delta}}(v) \mathcal{P}^{\mathcal{B}_{\delta}}(v) \mathcal{F}^{\mathcal{B}_{\delta}}(v) \mathcal{H}^{\delta}(\mathcal{B}_{\delta}),$$

D. Reichelt

Here,  $R$  denotes the massive radiator:

$$R = \int_0^1 dz \int_{z^2 m^2}^{\mu_Q^2} \frac{dk_t^2}{k_t^2} \frac{\alpha_S^{\text{CMW}}(k_t^2)}{2\pi} P_{gb}(z, k_t^2 - z^2 m^2) \times \Theta_{\alpha} \times \Theta(\eta - \eta_{\min})$$



$\Theta_{\alpha}$  = observable constraint

$\eta_{\min}$  = lower bound in rapidity

$P_{gb}$  = massive splitting function

# Resummation in the CAESAR formalism for groomed jets

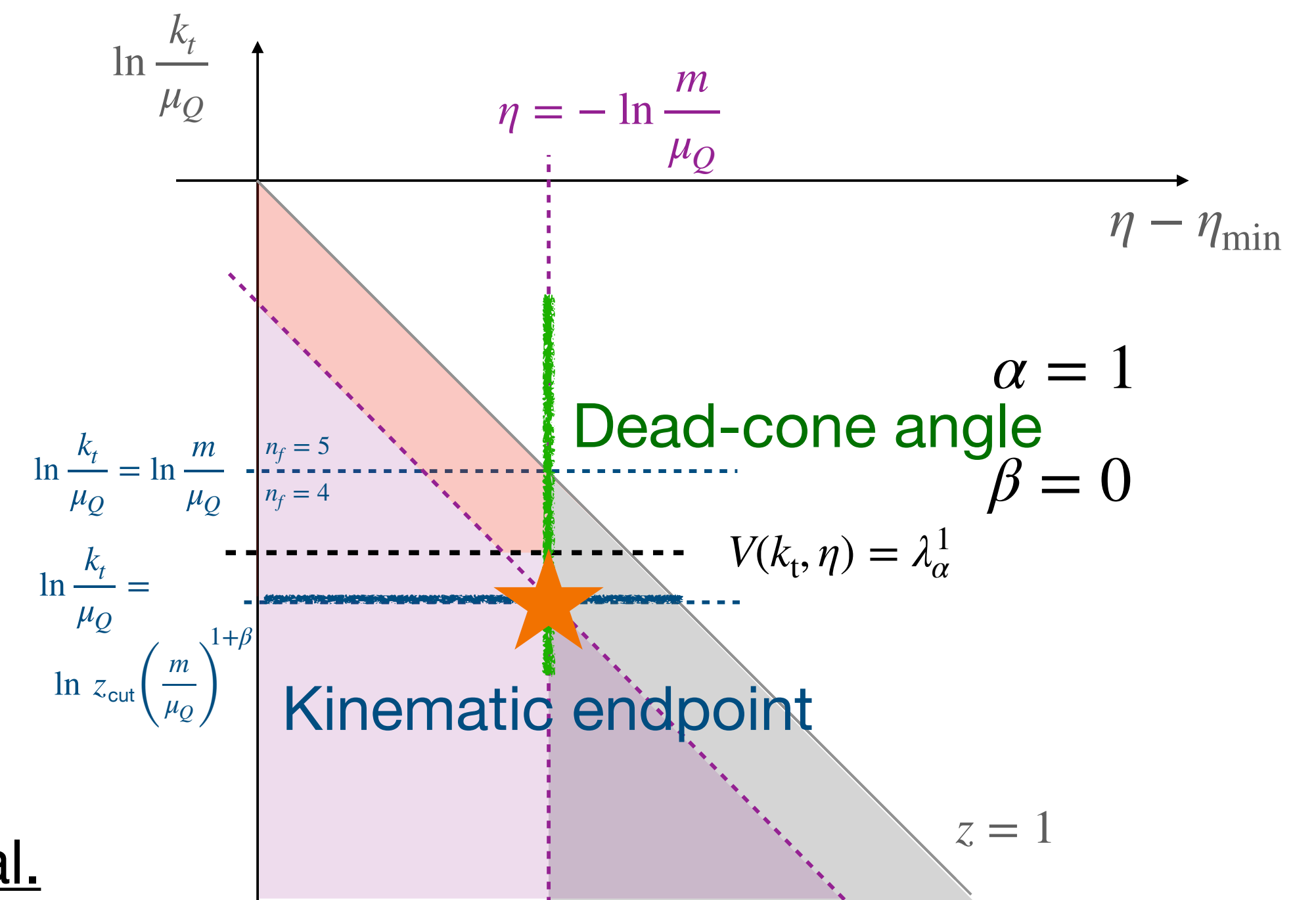
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$$\Sigma_{\text{res}}(v) = \sum_{\delta} \int d\mathcal{B}_{\delta} \frac{d\sigma_{\delta}}{d\mathcal{B}_{\delta}} \exp \left[ - \sum_{l \in \delta} R_l^{\mathcal{B}_{\delta}(v)} \right] \mathcal{S}^{\mathcal{B}_{\delta}(v)} \mathcal{P}^{\mathcal{B}_{\delta}(v)} \mathcal{F}^{\mathcal{B}_{\delta}(v)} \mathcal{H}^{\delta}(\mathcal{B}_{\delta}),$$

D. Reichelt, J. Baron et al.

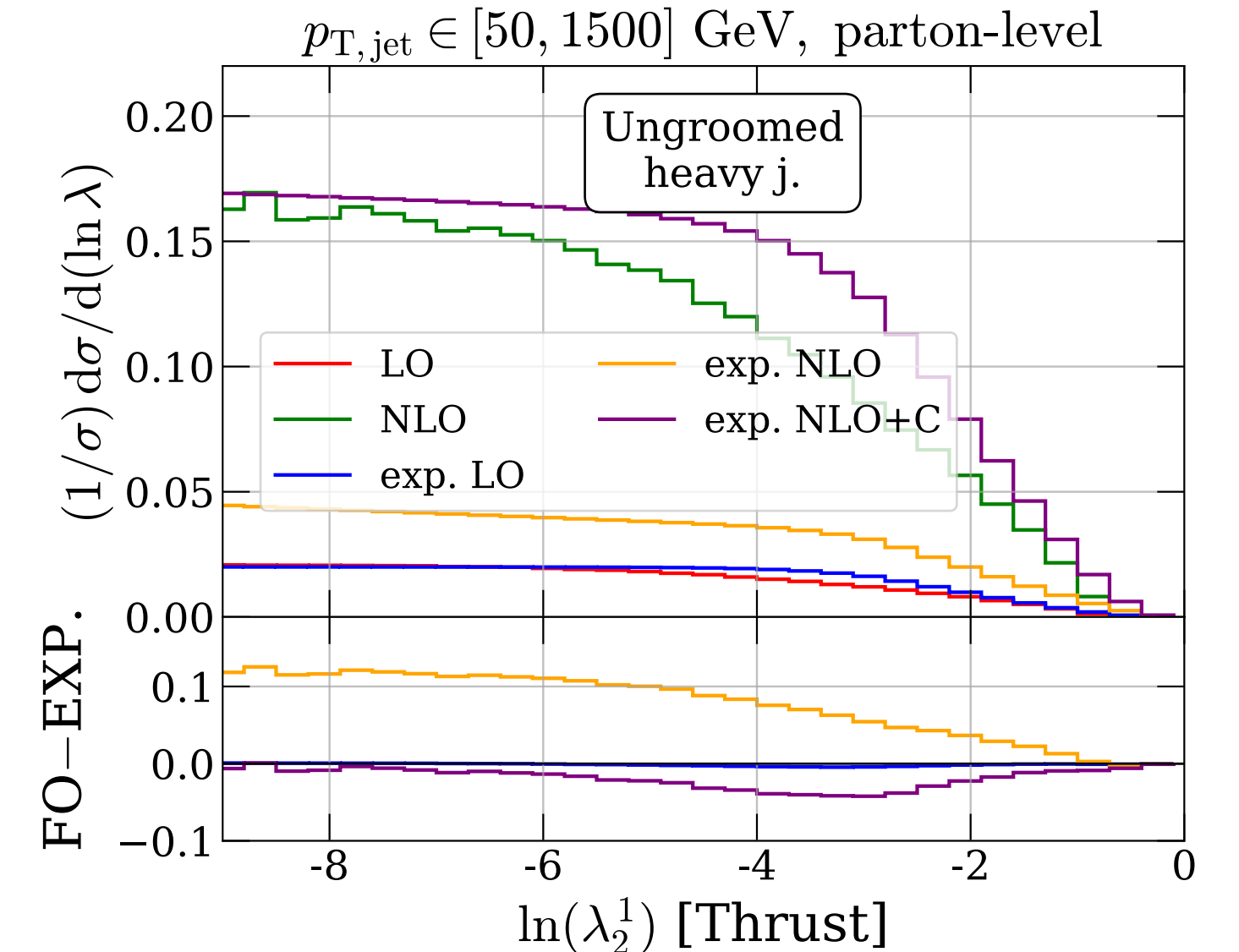
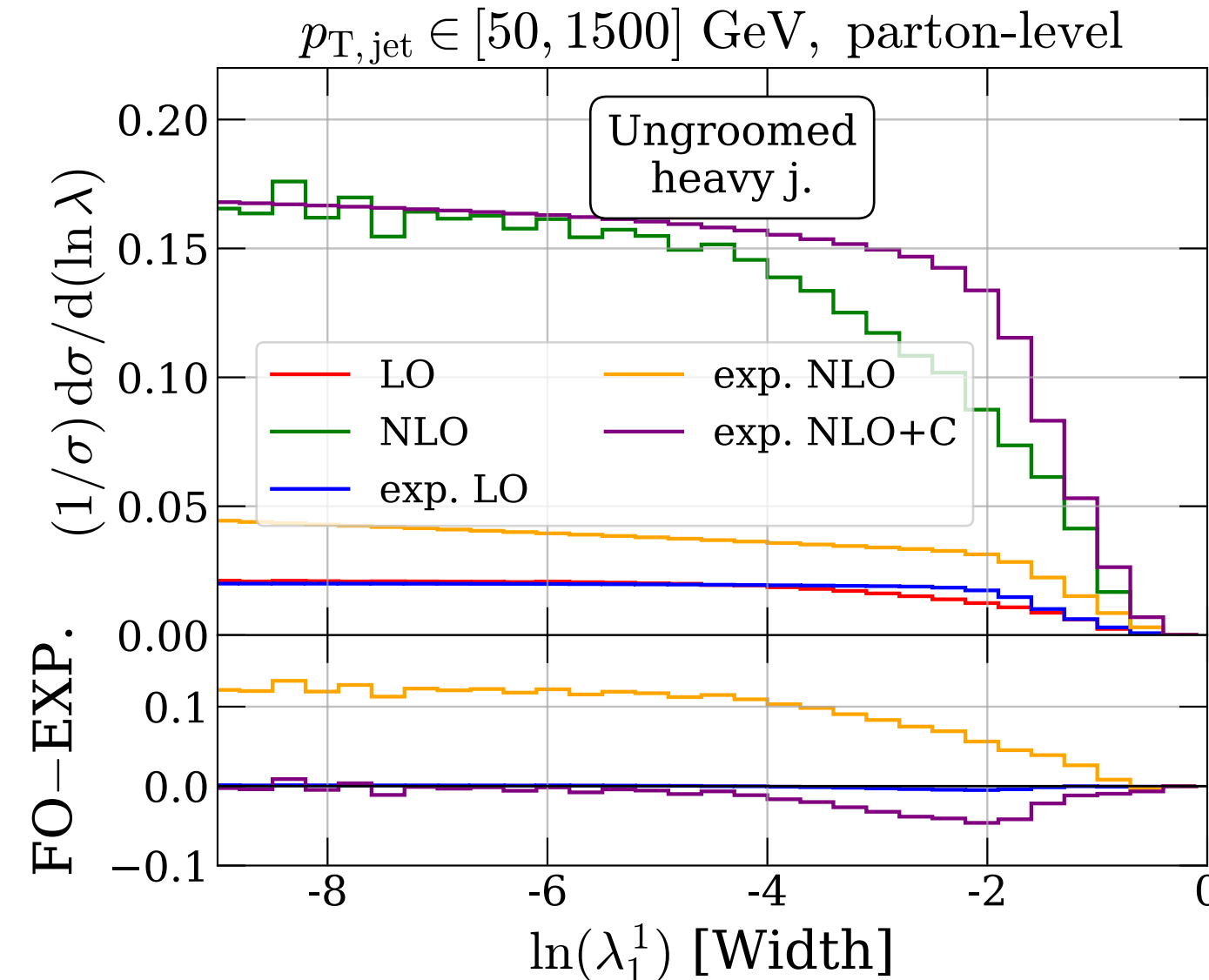
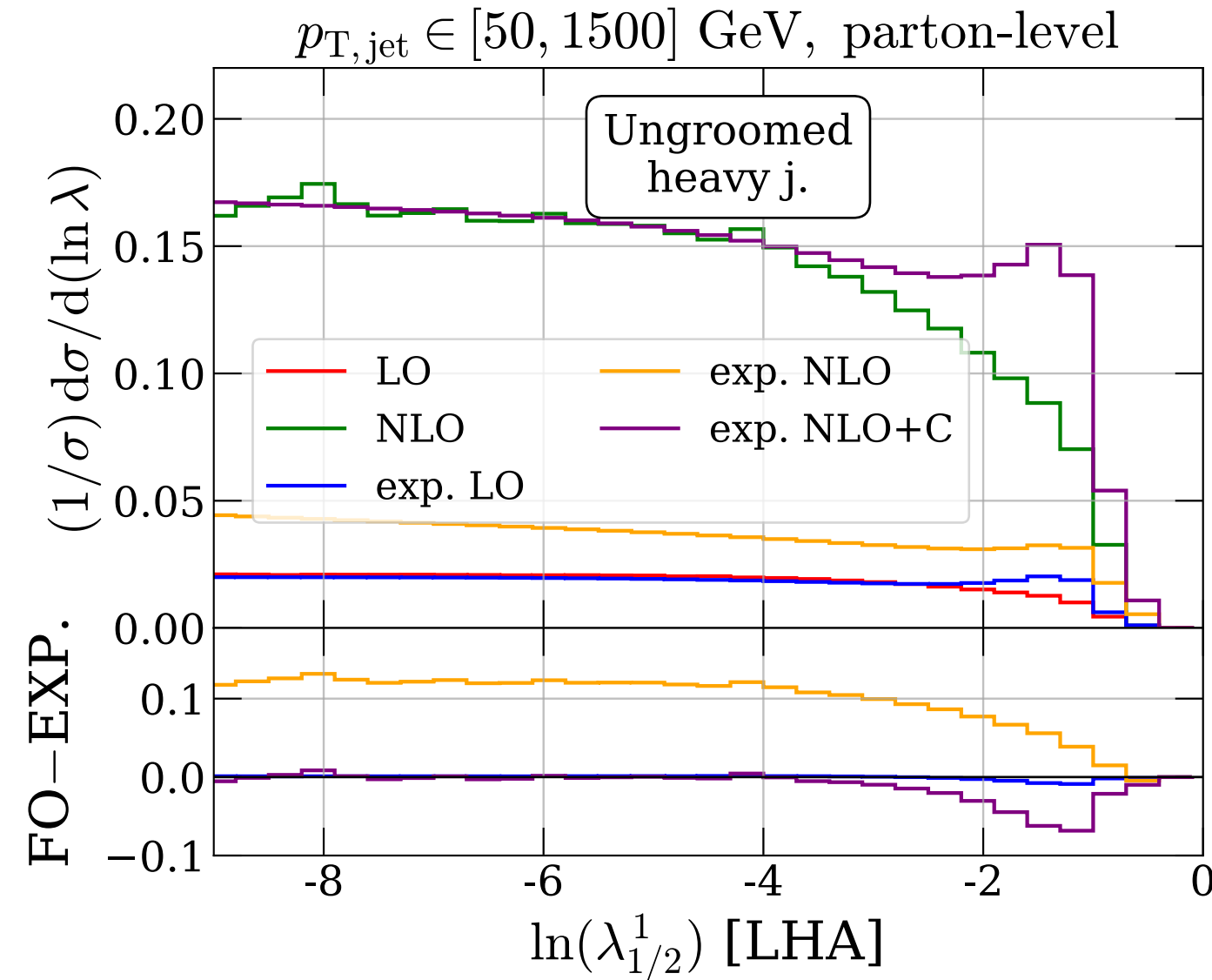
Here,  $R$  denotes the massive radiator:

$$R = \int_0^1 dz \int_{z^2 m^2}^{\mu_Q^2} \frac{dk_t^2}{k_t^2} \frac{\alpha_S^{\text{CMW}}(k_t^2)}{2\pi} P_{gb}(z, k_t^2 - z^2 m^2) \times \Theta_\alpha \times \Theta(\eta - \eta_{\min}) \times \Theta_{\text{SD}}$$

 $\Theta_\alpha = \text{observable constraint}$  $\Theta_{\text{SD}}$  = soft drop constraint $\eta_{\min}$  = lower bound in rapidity

$P_{gb}$  = massive splitting kernel

# Validation against fixed order



$$pp \rightarrow Z + b$$

Comparison between expansion of the resummation at NLO against fixed order

- Distribution overlapped at leading (the extremely small offset is driven by power corrections in the mass and in the jet radius)
- At NLO the difference is given by a constant off set  $C$  obtain through matching.

$$\Sigma(\lambda_\alpha^1) \sim e^{-\alpha_s L}$$

The distribution is single logarithmic for  $\lambda_\alpha^1 \ll (m/\mu_Q)^\alpha$

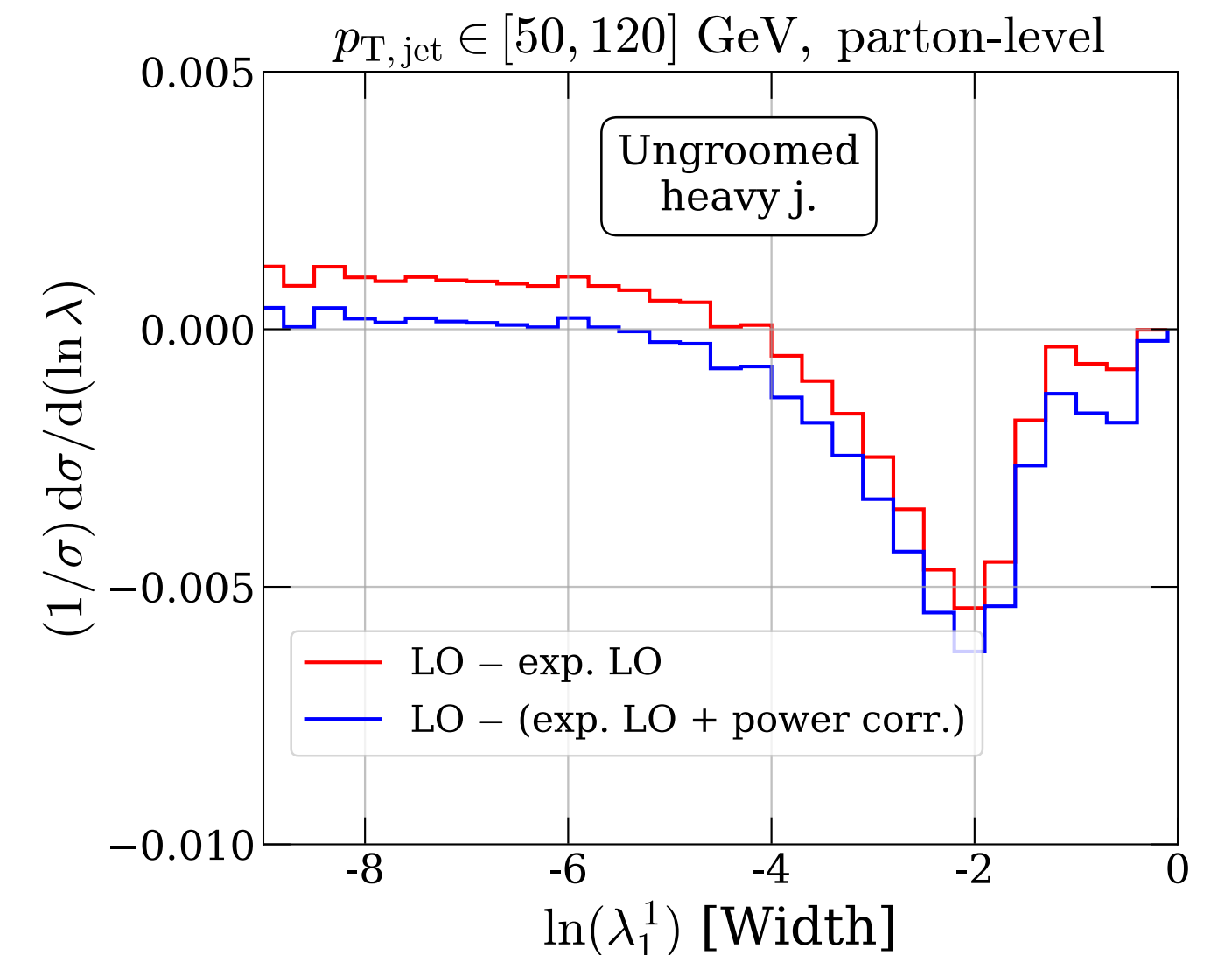
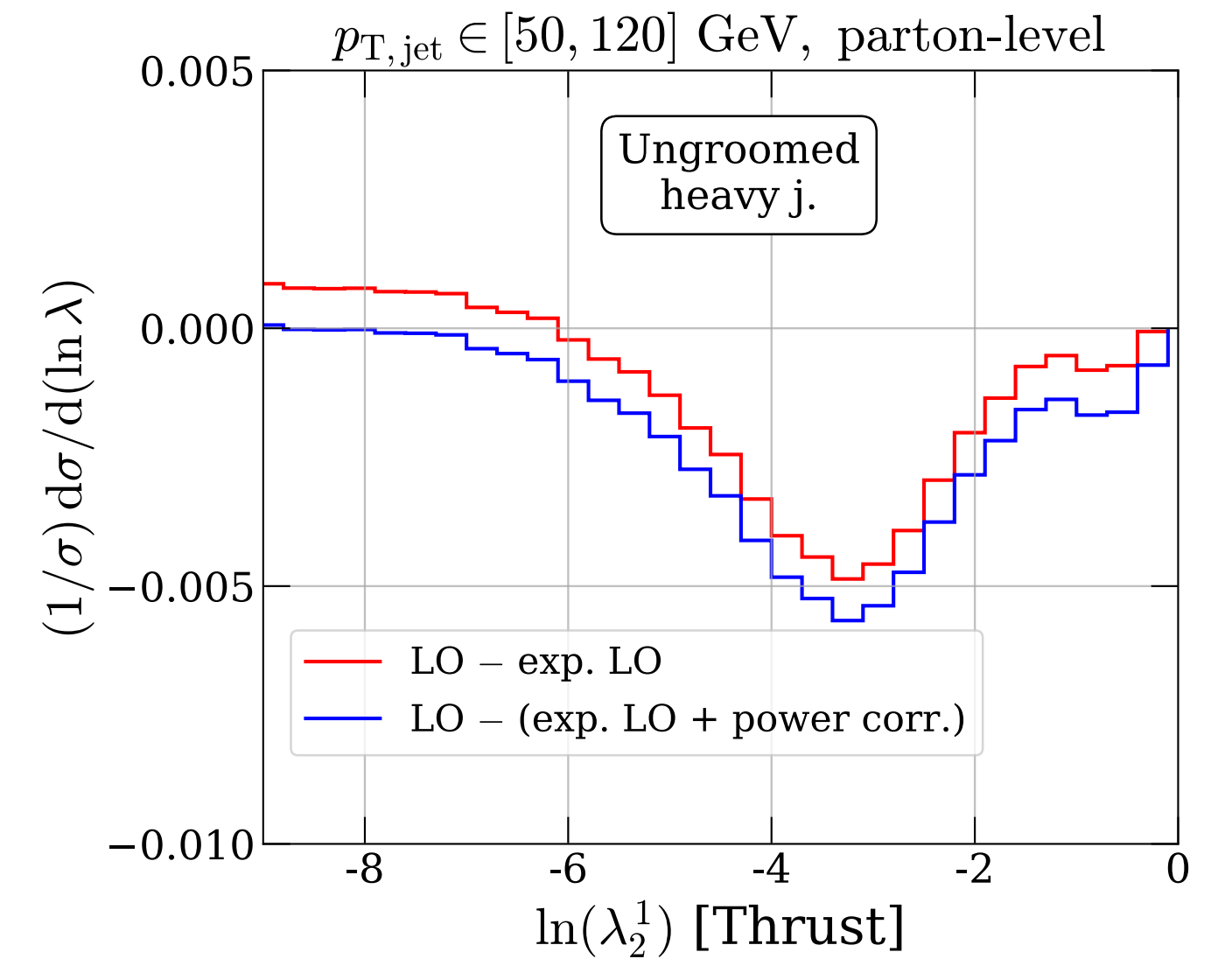
# Effects of leading power corrections

- In the small observable limit at order  $\alpha_s$ , the leading power corrections arise only from the calculation of soft function
- Jet radius power corrections computed in M. Dasgupta et al.

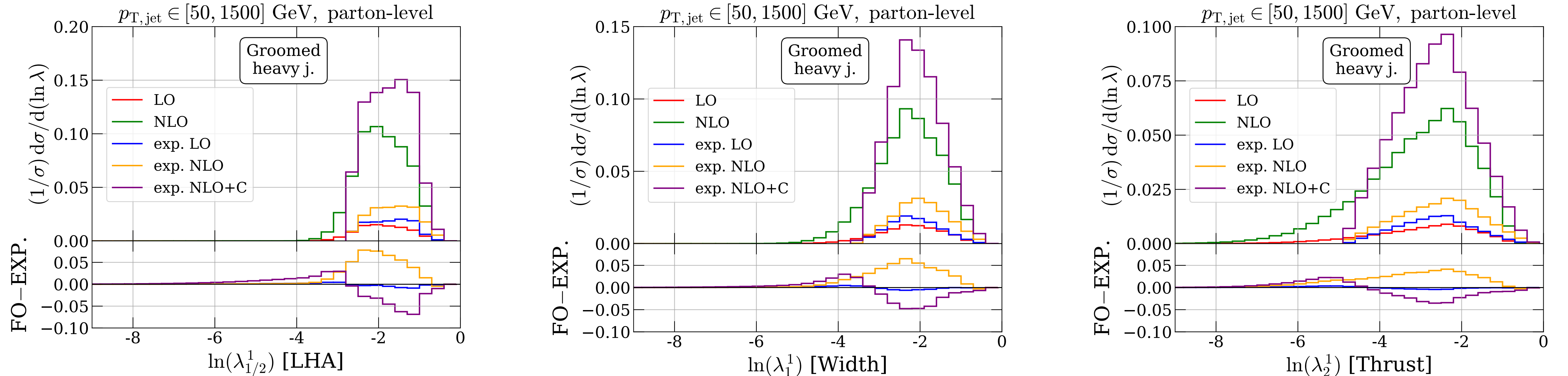
$$\frac{d\sigma}{d\ln\lambda_\alpha^1} = \lambda_\alpha^1 \sum_{ij} \mathbf{T}_i \cdot \mathbf{T}_j \int d^3k w_{ij} \delta(\mathcal{V}(k_t, \eta) - \lambda_\alpha^1) \Theta(\eta - \eta_{\min})$$

$$w_{ij} = \frac{(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} - \frac{p_i^2}{2(p_i \cdot k)^2} - \frac{p_j^2}{2(p_j \cdot k)^2} \quad \text{Massive Eikonal}$$

Power corrections computed in the limit where  $m/p_T \sim \Delta R_k$



# Validation against fixed order: groomed case



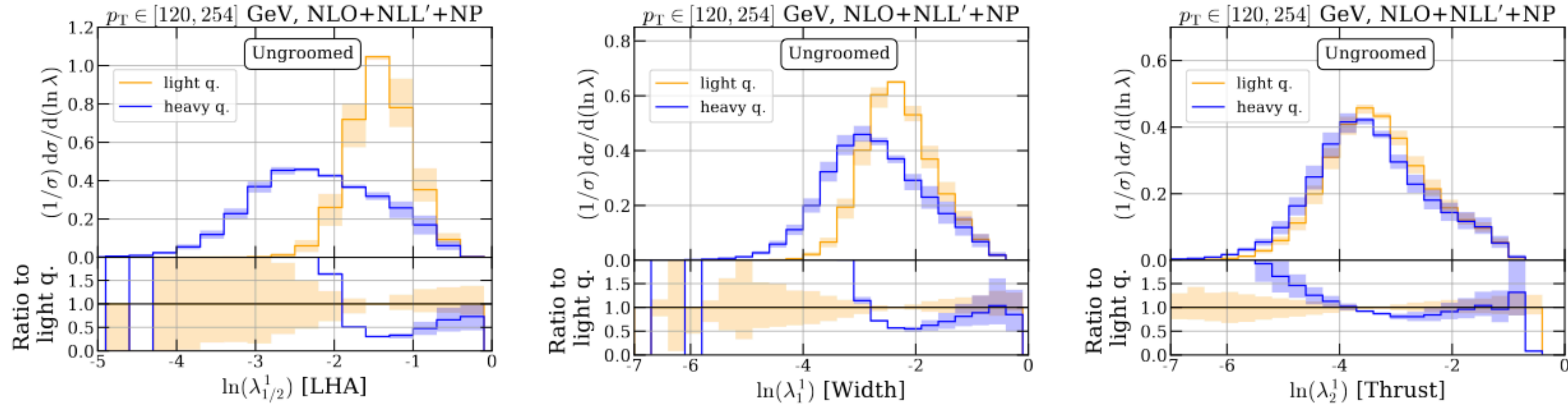
$$pp \rightarrow Z + b$$

Comparison between expansion of the resummation at NLO against fixed order for  $\beta = 0$

- In principle  $\mathcal{O}(z_{\text{cut}})$  effects, but very small, not observed numerically.
- The distribution tend to 0 in the asymptotic limit: the cumulative becomes constant.
- Negative derivative of the radiator over a part of the spectrum.

$$\Sigma(\lambda_\alpha^1) \sim e^{-\alpha_s L_\mu}, L_\mu = \log \frac{\mu_Q}{m}$$

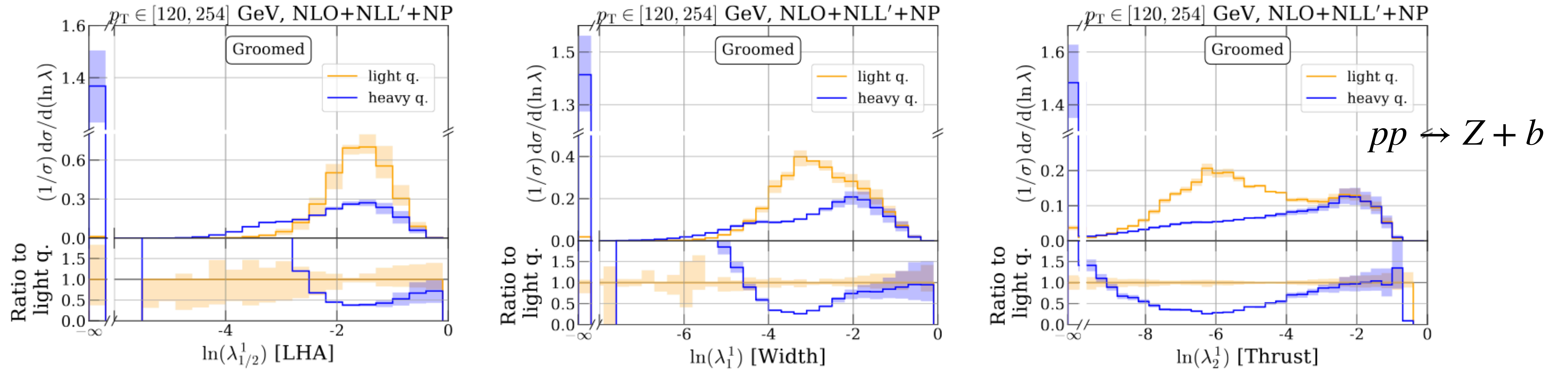
# $b$ -jet vs light quark jet: ungroomed case



$$pp \rightarrow Z + b$$

- Mass effects scale as  $\sim m^\alpha$ : H-F distribution exhibits a larger deviation from the light quark distribution.
- Longer tail: reduced Sudakov suppression compared to the massless case  $\Sigma_b \sim e^{-\alpha_s L}$ ,  $\Sigma_l \sim e^{-\alpha_s L^2}$ .
- Comparison performed with stable  $B$  hadrons (P.K. Dhani et al.) See talk by S. Schumann!
- Non perturbative corrections implemented with the transfer matrix approach (Reichelt et al.)

# $b$ -jet vs light quark jet: groomed case



- Huge peak at the end of the distribution, already present at parton level:  $\delta(\lambda_\alpha^1)$  contribution to the resummation (jets composed of a single parton)
- Non perturbative effects smears out this effect, but configurations remain in which the jet has only the  $B$  hadron
- This effect can be removed by using Soft Drop in tagging mode, as suggested in [S. Caletti, A.G., S. Marzani](#). In this case it is natural to consider ratio  $b/l$  distributions

# Towards higher accuracy: scale of the coupling and CMW scheme

In order to improve the accuracy of our calculation one the missing ingredients is the running coupling:

$$\alpha_s^2(\mu^2)[dk] | M_{1\text{-loop,N.A.}}(k) + \alpha_s^2(\mu^2) \int [dk_1][dk_2] | M_{\text{N.A.}}(k_1, k_2) \delta^3(k - k_1 - k_2)$$

$$= \alpha_s^2(\mu^2)[dk] | M(k, \mu) |^2 \left( \beta_0 \ln \frac{k_t^2}{\mu^2} + \frac{K^{\text{CMW}}}{2\pi} \right)$$

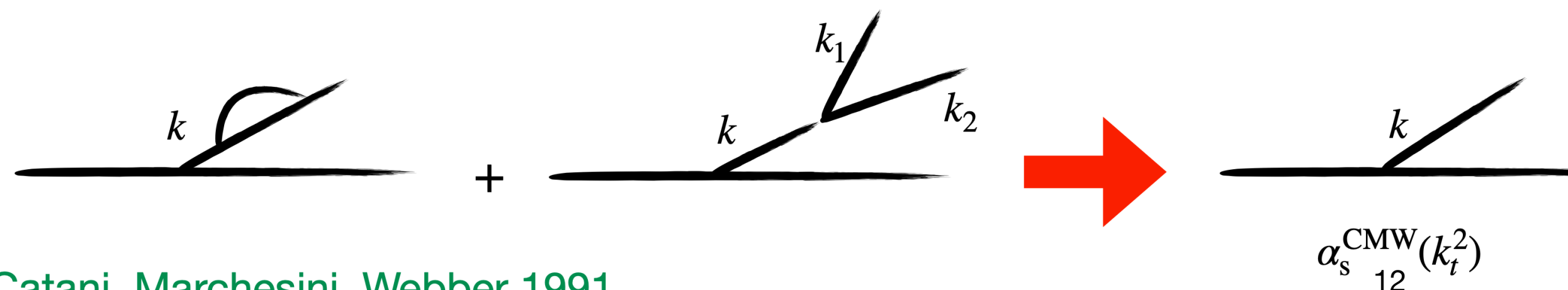
- This prescription is perfectly fine even if the emitter is massive at NLL accuracy as shown in for fragmentation (A.G., S. Marzani and G. Ridolfi)

- Used for recent developments for NLL shower with massive particles (M. Van Beekved et al)

- It does not work at NNLL

$$\alpha_s(k_t^2) = \alpha_s(\mu^2) \left( 1 + \alpha_s \beta_0 \ln \frac{k_t^2}{\mu^2} \right)$$

$$\alpha_s^{\text{CMW}} = \alpha_s^{\overline{\text{MS}}} \left( 1 + K^{\text{CMW}} \frac{\alpha_s^{\overline{\text{MS}}}}{2\pi} \right)$$



Slide taken from G. Zanderighi

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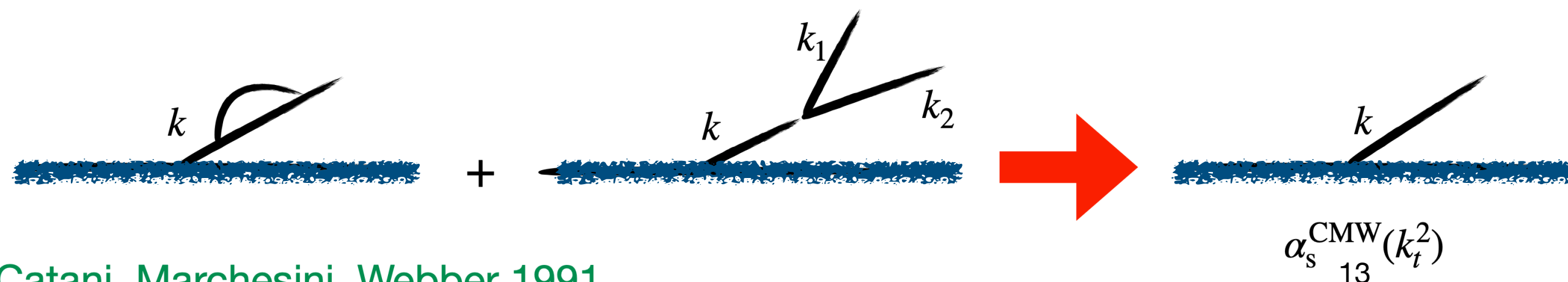
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**Repeat the calculation for massive emitter**

# Sketch of the calculation for the effective coupling

The effective coupling can be determined from the calculation of the web function ([A. Banfi, B. El-Menoufi, P.F. Monni](#))

$$w_{\mathcal{Q},\text{NLO}} = \mathcal{N}_\epsilon(k_T) \left\{ \frac{1}{(2\pi)^{d-1}} \delta_+(k^2) |J_{\mathcal{Q}}(k)|_{\text{NLO}}^2 + \int \frac{d^d k_1}{(2\pi)^{d-1}} \frac{d^d k_2}{(2\pi)^{d-1}} \delta_+(k_1^2) \delta_+(k_2^2) \delta^d(k - k_1 - k_2) \left( \frac{1}{2} W_{\mathcal{Q}}^{gg}(k_1, k_2)_{\text{LO}} + n_f W_{\mathcal{Q}}^{q\bar{q}}(k_1, k_2)_{\text{LO}} \right) \right\}.$$

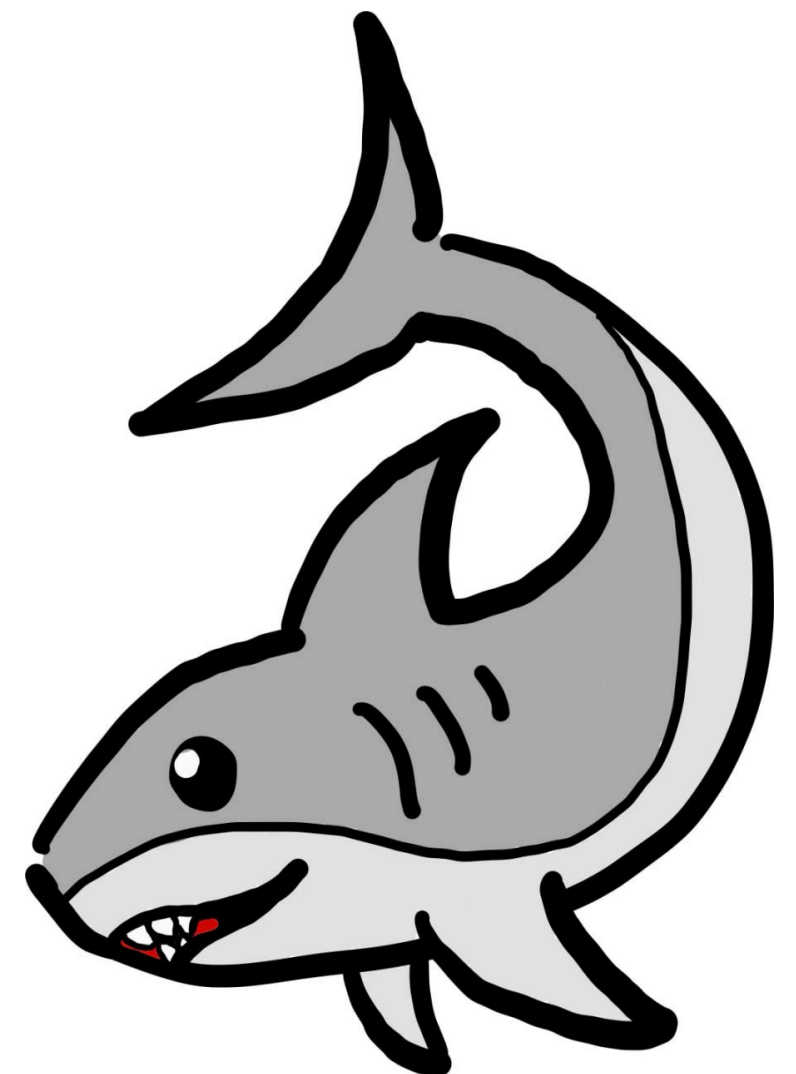
Virtual correction of the tree level eikonal current: [Catani and Grazzini, I. Bierenbaum, M. Czakon and A. Mitov, M. Czakon and D. Heymes](#)

Tree level current for the emission of two soft partons

The effective coupling is then computed in terms of the web function as ([Catani et al.](#)):

$$\mathcal{A}_{\mathcal{Q}}(\alpha_s(\mu^2), k_t^2) = \frac{1}{2} \int dk^2 w_{\mathcal{Q}}(k)$$

- Alternative approach: inverse Fourier transform of the fully differential soft function computed in position space ([Ze Long Liu and P.F. Monni](#))
- Performed cross-check against the  $q_T$  soft function obtained in [Catani et al.](#) and [S. Devoto and J. Mazzitelli](#)



# Running coupling scale(s!)

- In the massless case, the previous definition of effective coupling is not ambiguous:  $\frac{p_1 \cdot p_2}{2(p_1 \cdot k)(p_2 \cdot k)} = \frac{1}{k_t^2 + k^2}$  The eikonal factor depends only on the  $k_t$  and on the invariant mass of the emission
- In the massive case, the eikonal is not simply the  $k_t$  of the emission. The definition of the effective coupling depends on which kinematical property of the emission we fix:

Scale that appear massless case

$$\mathcal{A}_Q(\alpha_S(\mu^2), k_t^2, y) = \frac{1}{2} \int dk^2 w_Q(k) \delta \left( y - \log \frac{k_+}{k_-} \right) \longrightarrow \sim \alpha_S(\mu^2) \left( 1 + \beta_0 \alpha_S(\mu^2) \log \frac{k_t^2}{\mu^2} \right)$$

$$\mathcal{A}_Q(\alpha_S(\mu^2), k_t^2, z) = \frac{1}{2} \int dk^2 w_Q(k) \delta \left( z - \frac{k_+}{p_{1+}} \right) \longrightarrow \sim \alpha_S(\mu^2) \left( 1 + \beta_0 \alpha_S(\mu^2) \log \frac{k_t^2 + z^2 m_1^2}{\mu^2} \right)$$

Scale that appear in quasi collinear factorization (Catani et al.)

- The difference between the two is @NNLL accuracy

# Physical interpretation

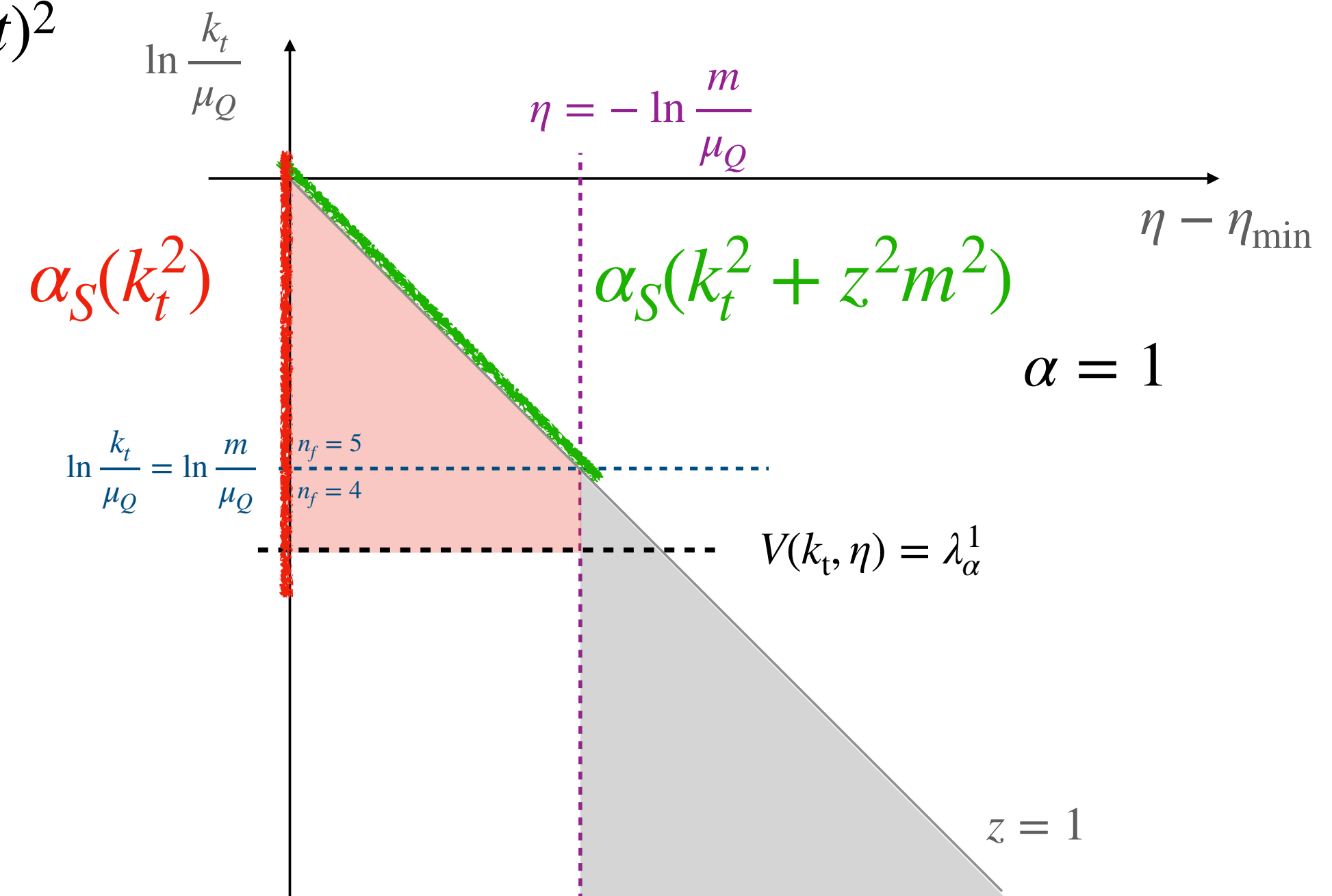
This happens because of the different dependence on the invariant mass in the two cases:

$$\frac{1}{(k^2)^{1+\epsilon}} \left( \frac{2(p_1 \cdot p_2)}{(p_1 \cdot k)(p_2 \cdot k)} - \frac{m_1^2}{(p_1 \cdot k)^2} - \frac{m_2^2}{(p_2 \cdot k)^2} \right) \sim \frac{t^{2+\epsilon}}{(1-t)^{1+\epsilon}} \quad \text{@ fixed rapidity} \quad t = k_t^2/k^2$$

$$\frac{1}{(k^2)^{1+\epsilon}} \left( \frac{2(p_1 \cdot p_2)}{(p_1 \cdot k)(p_2 \cdot k)} - \frac{m_1^2}{(p_1 \cdot k)^2} - \frac{m_2^2}{(p_2 \cdot k)^2} \right) \sim \frac{t^{2+\epsilon}}{(1-t)^{1+\epsilon}} \frac{1}{(k_t^2 + z^2 m_1^2 t)^2} \quad \text{@ fixed momentum fraction}$$

Heuristic interpretation:

- at large angle  $k_+ \sim k_- \implies \mu^2 = k_t^2$
- In the collinear region  $k_- \ll k_+ \sim p_{1+} \implies \mu^2 = k_t^2 + z^2 m_1^2$



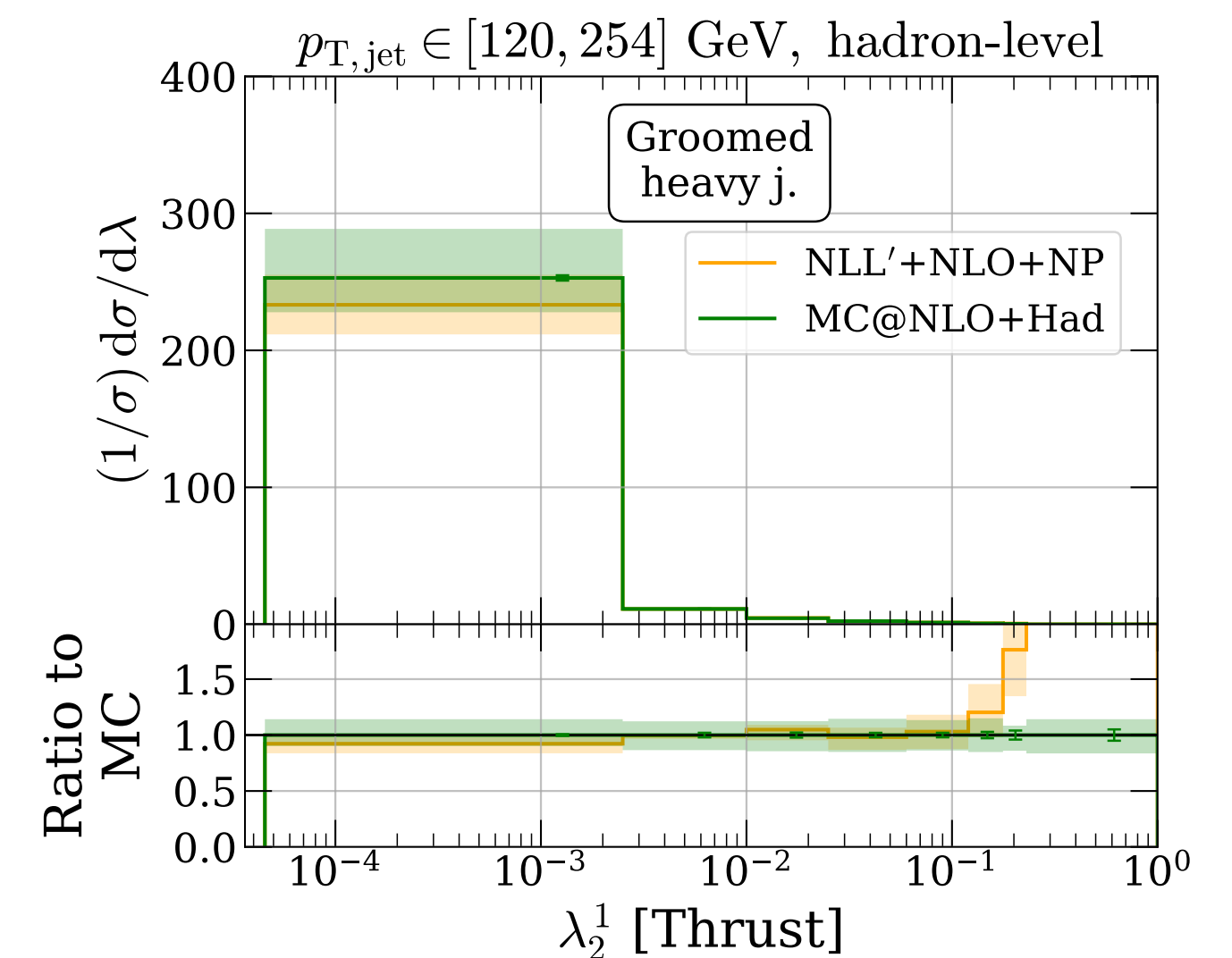
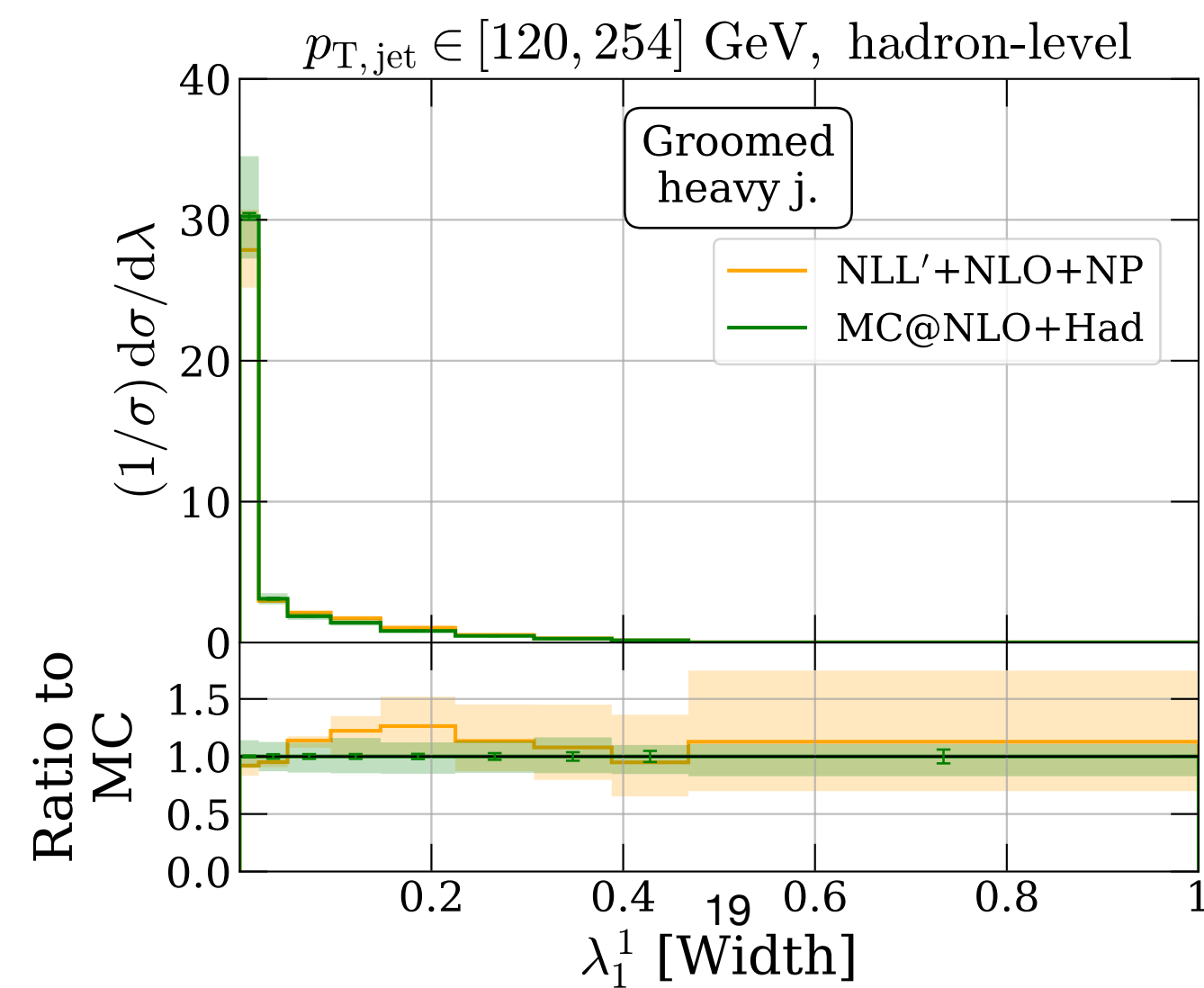
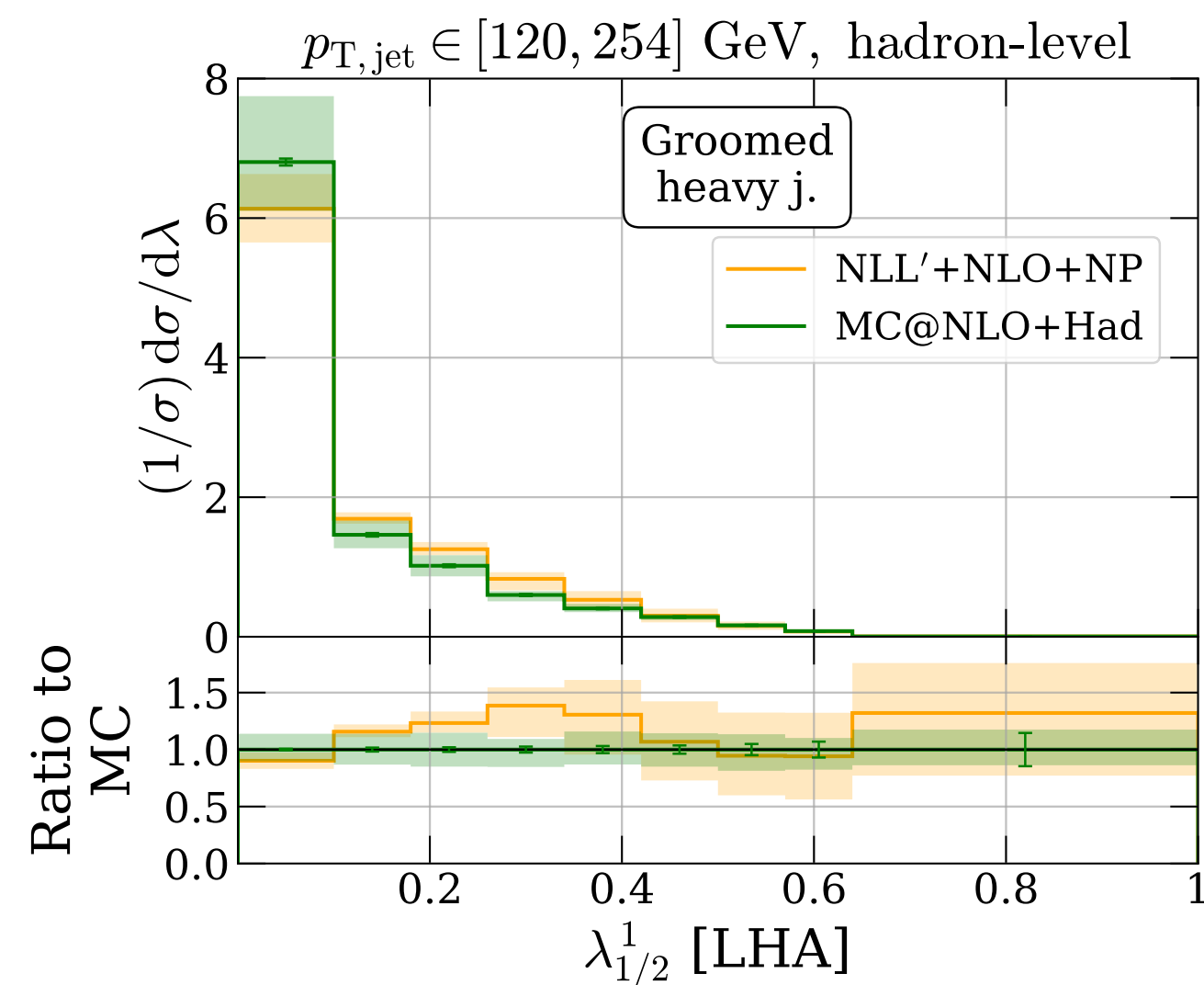
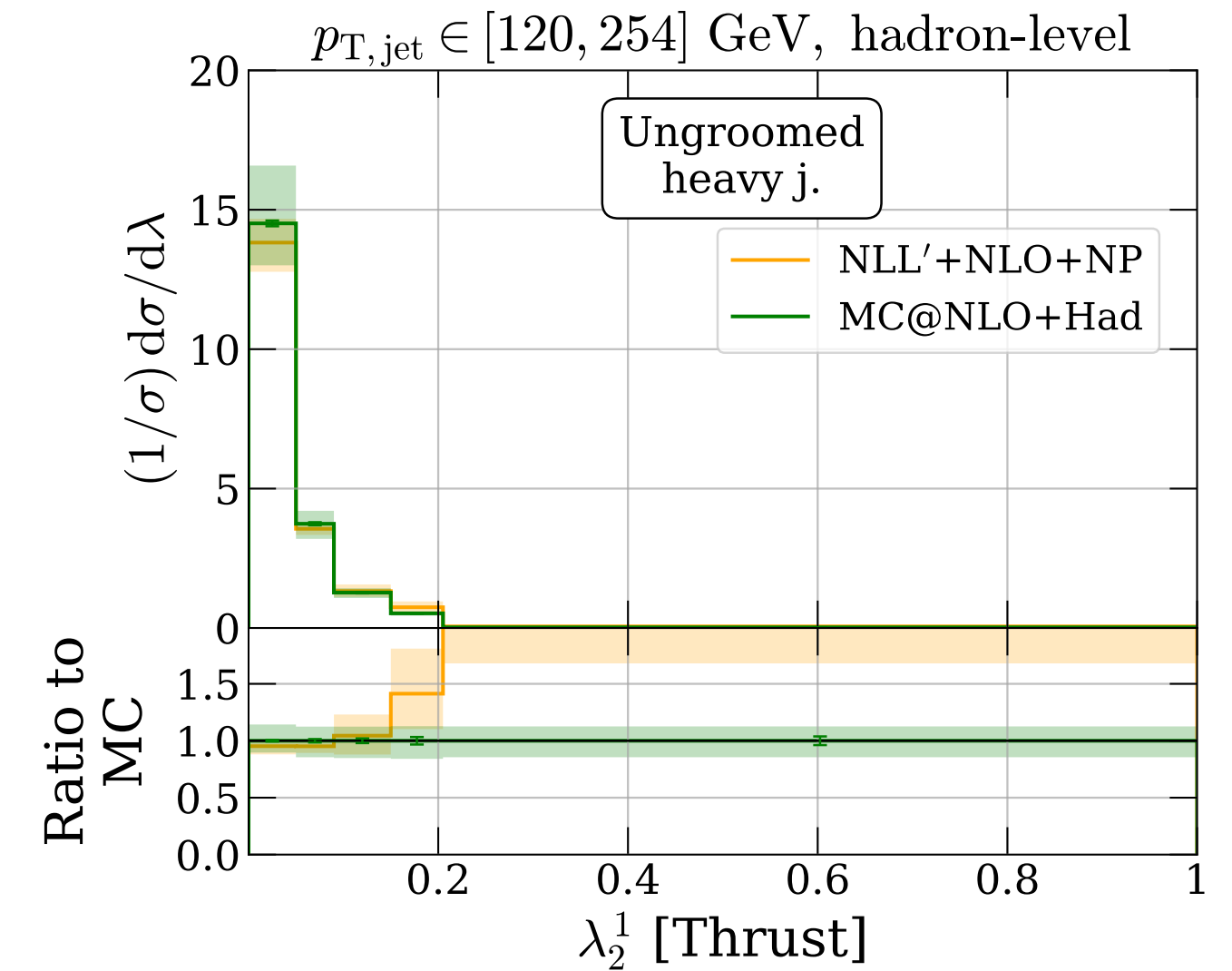
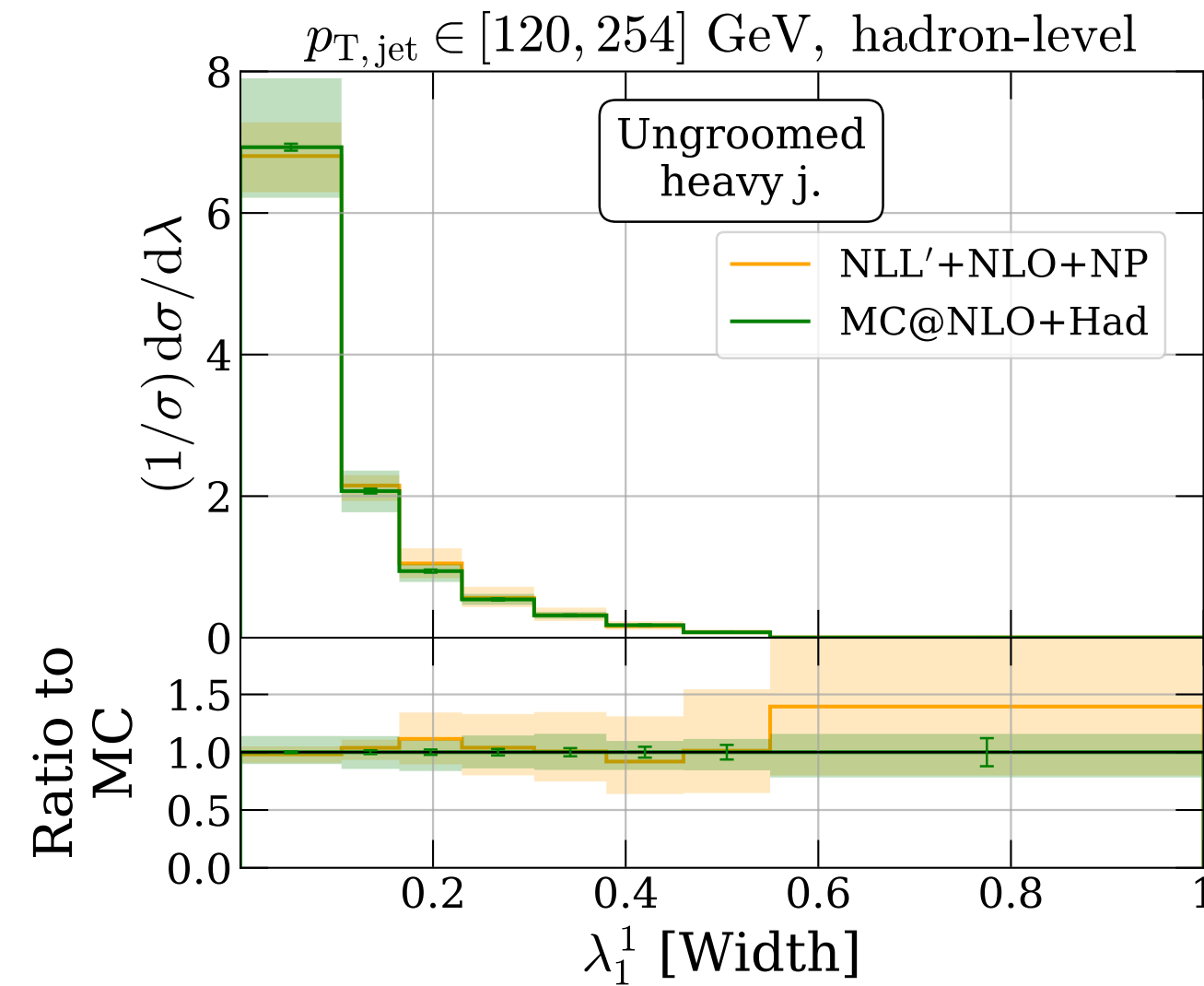
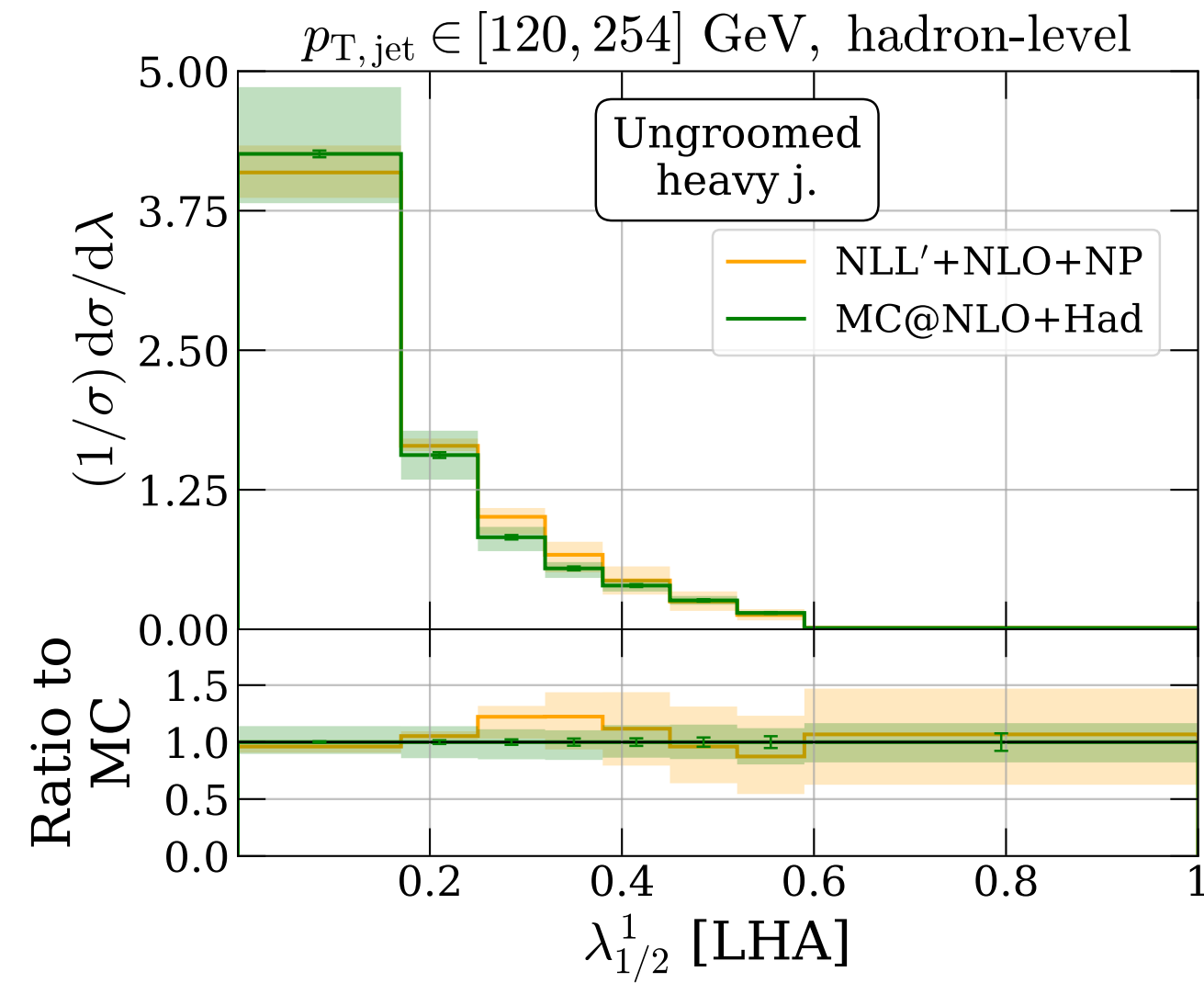
# Conclusion and outlook

- Examined recent results on jet angularities for H.F. jets
- Examined choice of the resummation scale in the case of massive emitters
- Comparison against two loop soft anomalous dimension (A. Manteuffel, R. Schabinger and Hua Xing Zu ,N. Kidonakis )
- Test our prescription for substructure observables

*Thanks for your attention!!!*

# Backup slides

# Comparison against SHERPA MC@NLO

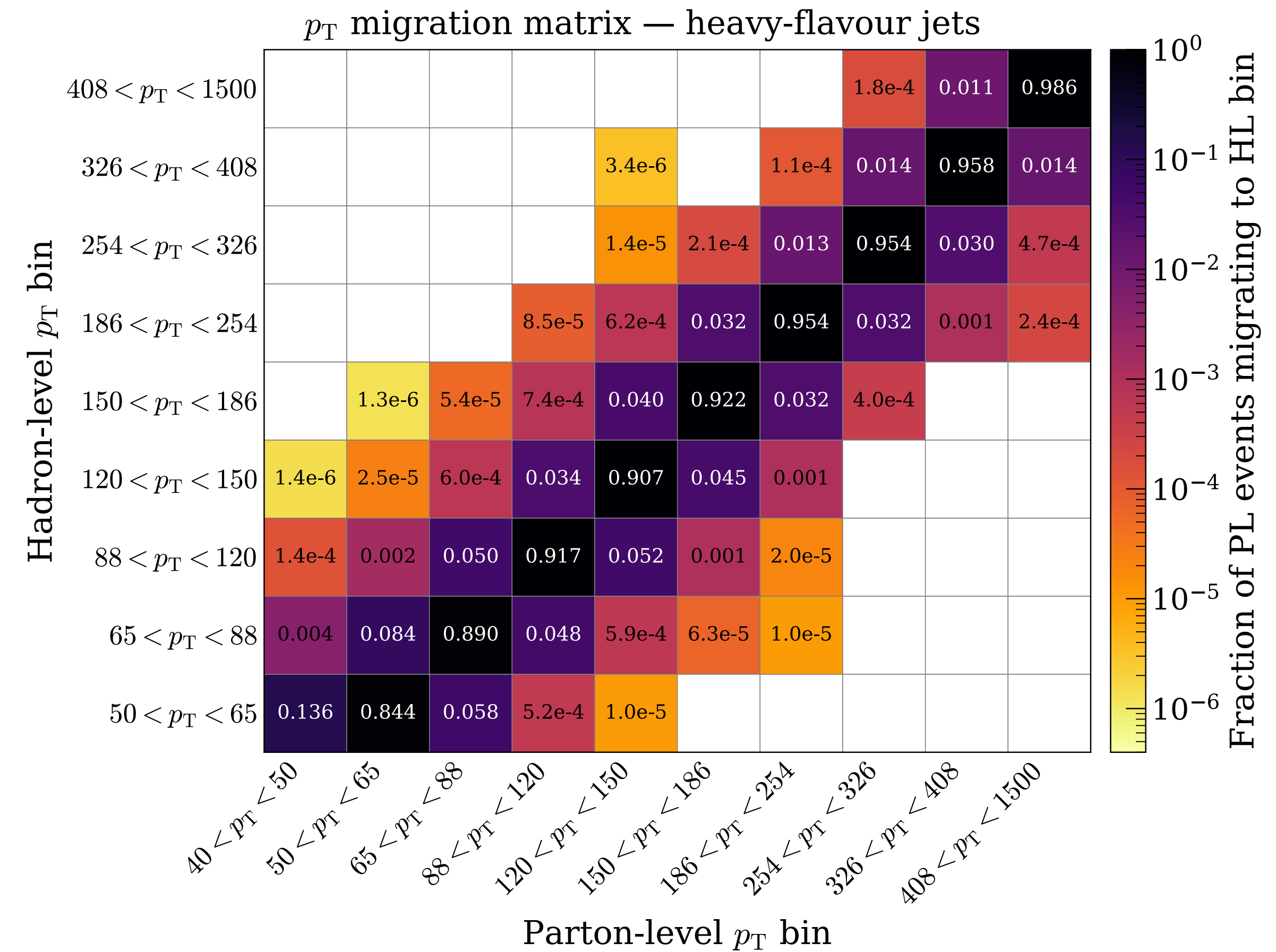


# Transfer matrix approach

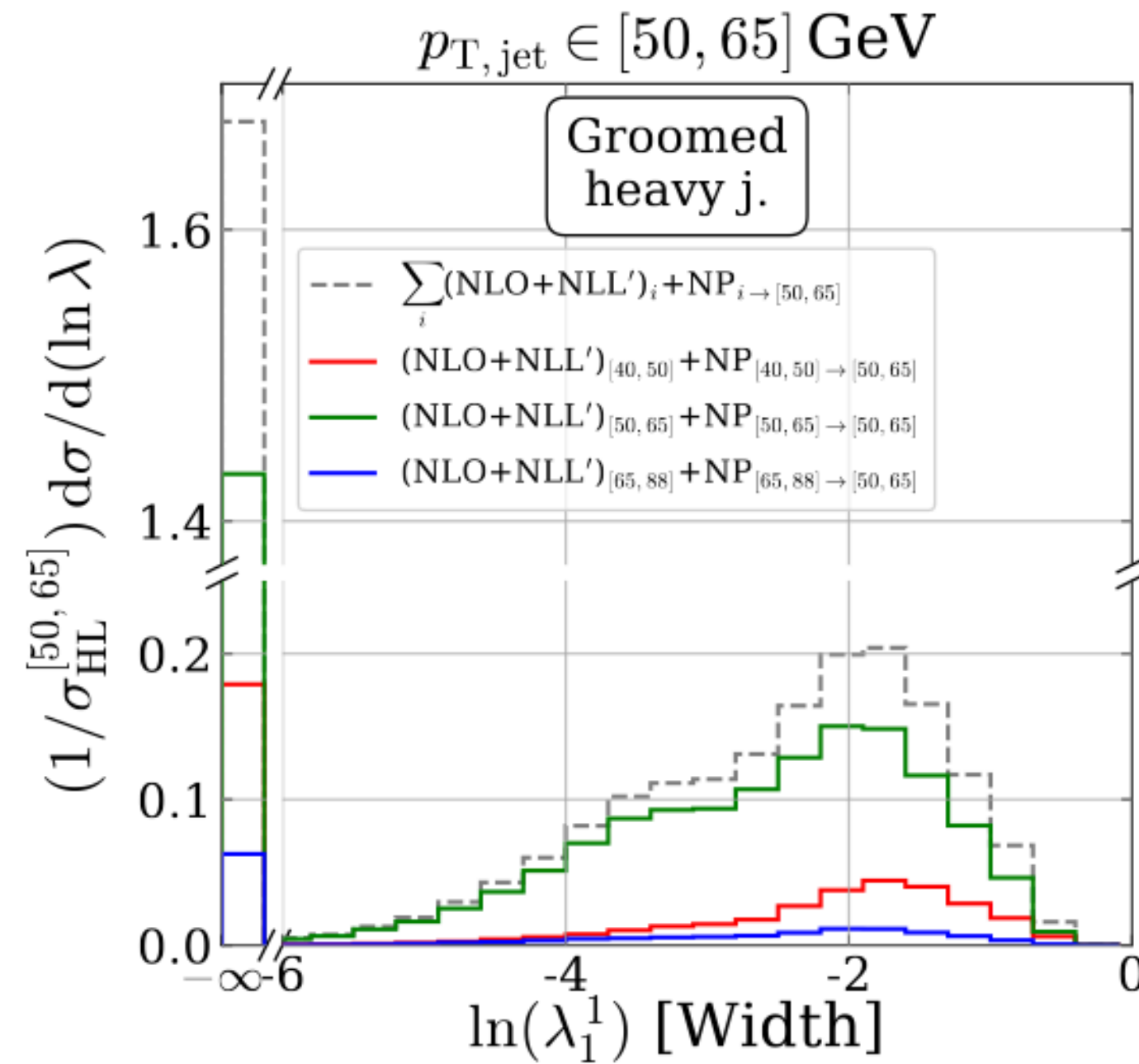
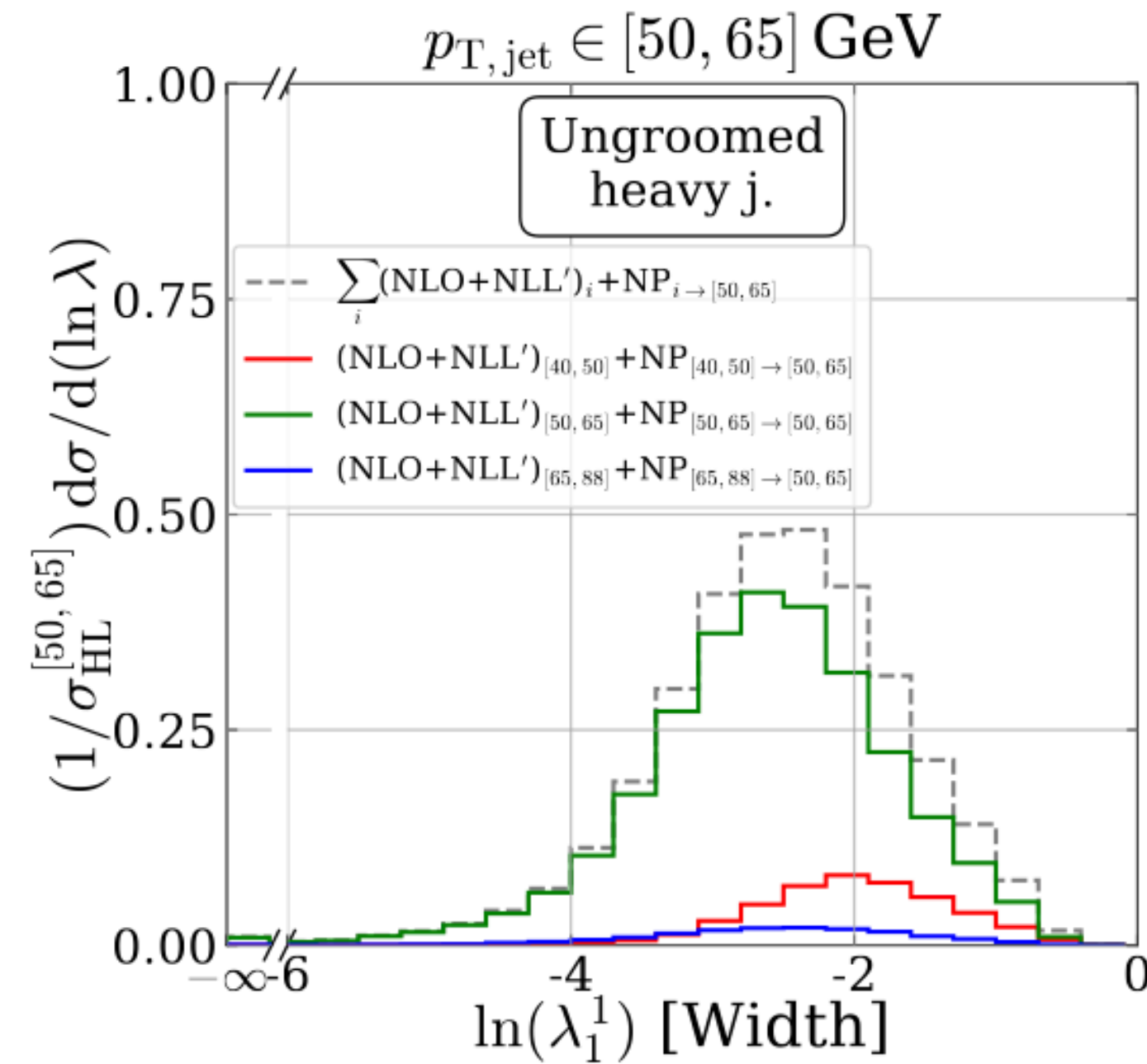
Hadronisation correction included with the transfer matrix approach (Reichelt et al.)

$$\mathcal{T}(\vec{v}^h | \vec{v}^p) = \frac{\int d\mathcal{P} \frac{d\sigma^p}{d\mathcal{P}} \delta^{(m)}(\vec{v}^p - \vec{V}(\mathcal{P})) \delta^{(m)}(\vec{v}^h - \vec{V}(\mathcal{H}(\mathcal{P})))}{\int d\mathcal{P} \frac{d\sigma^p}{d\mathcal{P}} \delta^{(m)}(\vec{v}^p - \vec{V}(\mathcal{P}))}.$$

- We consider distribution of kinematic variables and observables to determine the transfer operator
- Migration of the cross section in different  $p_T$  bins from parton to hadron level due to UE and hadronization corrections
- Transfer matrix almost diagonal in high  $p_T$  bins, due to the fact that we keep the  $B$  hadron stable (less fragmentation product).



# Hadronisation corrections via transfer matrix

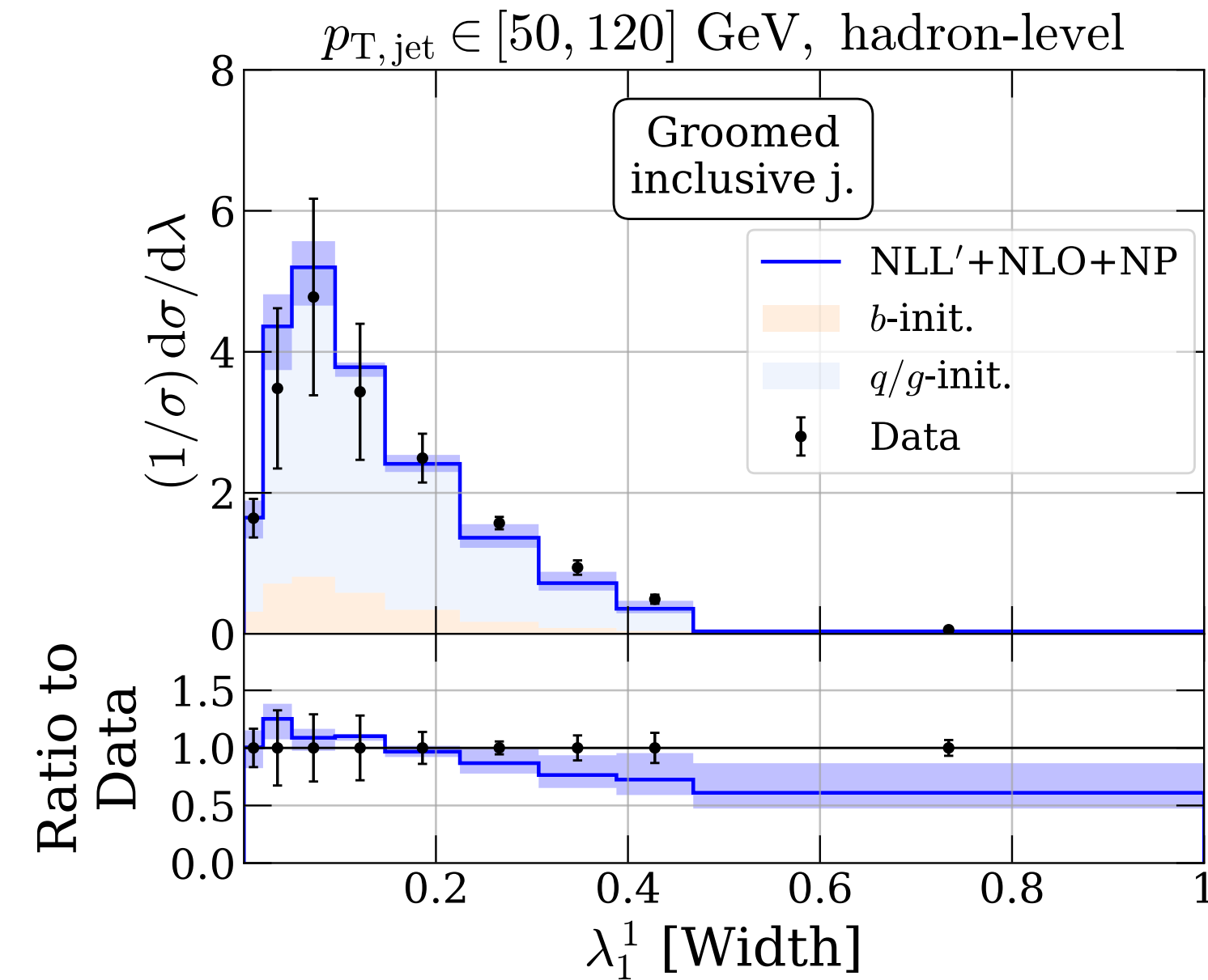
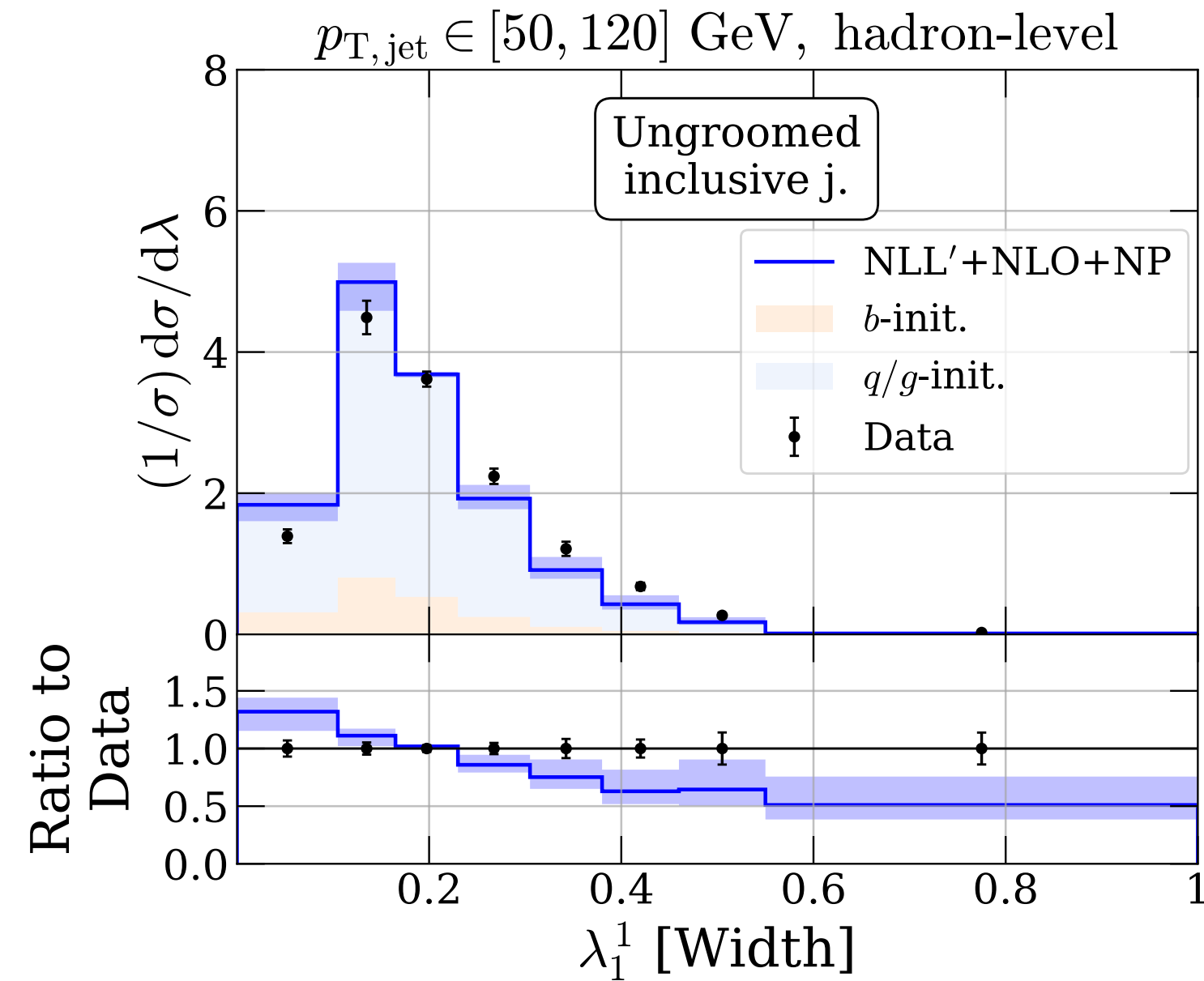


$$\mathcal{P}([65, 88] \rightarrow [50, 65]) = 12.8 \%$$

$$\mathcal{P}([40, 50] \rightarrow [50, 65]) = 7.6 \%$$

- For both cases, at small  $p_T$  the transfer matrices have rather large off-diagonal elements, populating significantly larger observable values at hadron than at parton level.
- In the groomed case at hadron level we have a more pronounced peak at  $\lambda_\alpha^1 = 0$

# Comparison to CMS data for untagged jets



Comparison against CMS data for the inclusive case for NLL+NL+NP angularity distributions

- We refine the theoretical framework of S. Caletti et al. by incorporating contributions from  $b$ -initiated jets and by including running coupling effects in the light flavour case
- Blue band: light quarks + gluon, Orange band:  $b$ -jets. Good agreement with data, confirming the results of S. Caletti et al.