

Three-Loop QCD Corrections to Scattering Amplitudes of a Higgs Boson and Three Partons

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QCD@LHC · 1 October 2026

University of Oxford

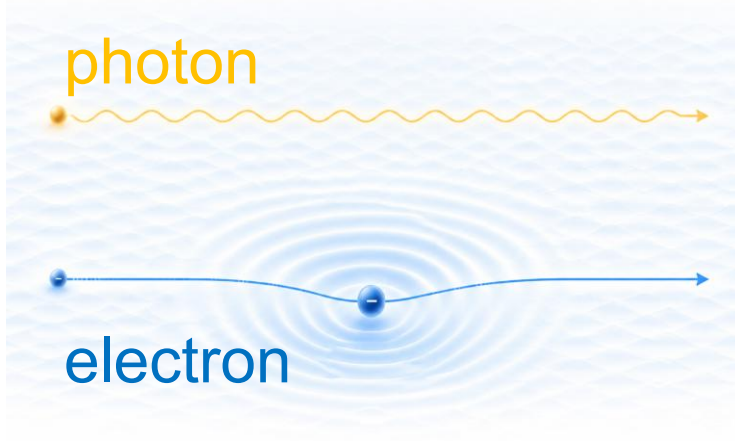
Based on

2607.12033: XG, Bernhard Mistlberger, Michael Ruf.

2504.06490: Xiang Chen, XG, Bernhard Mistlberger.

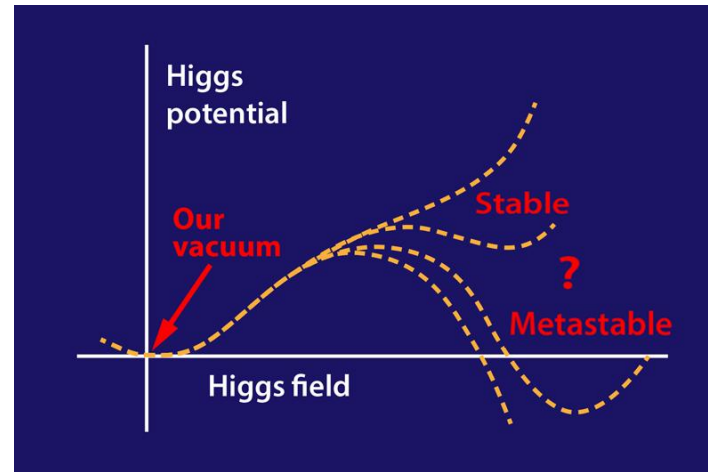


Why Higgs



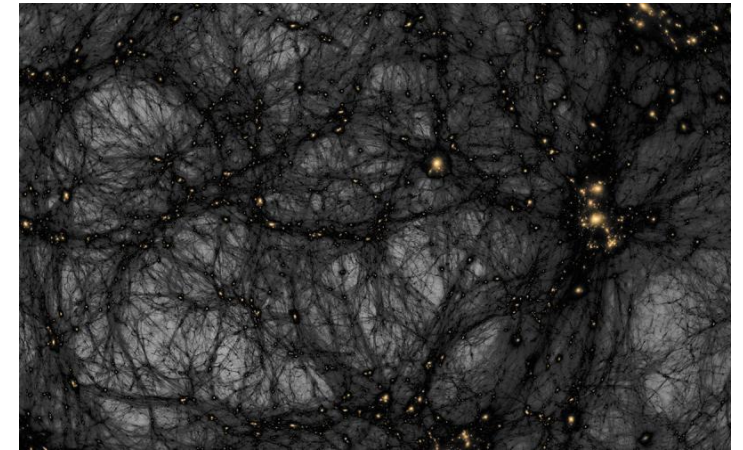
The origin of mass

Electroweak symmetry breaking



The stability of our universe

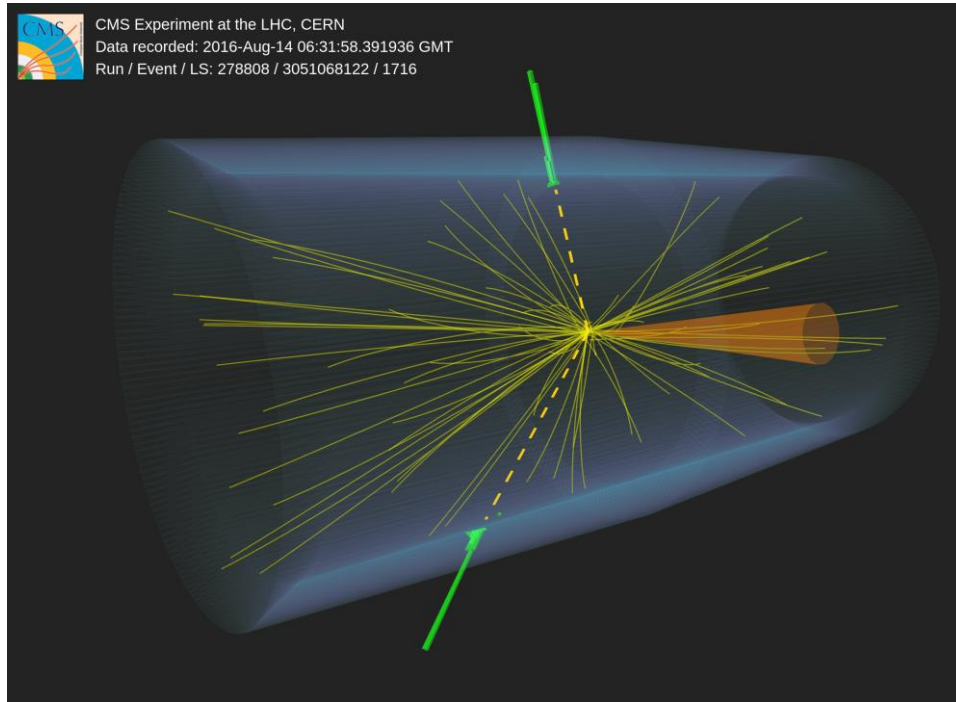
Higgs potential



Unique portal to new physics

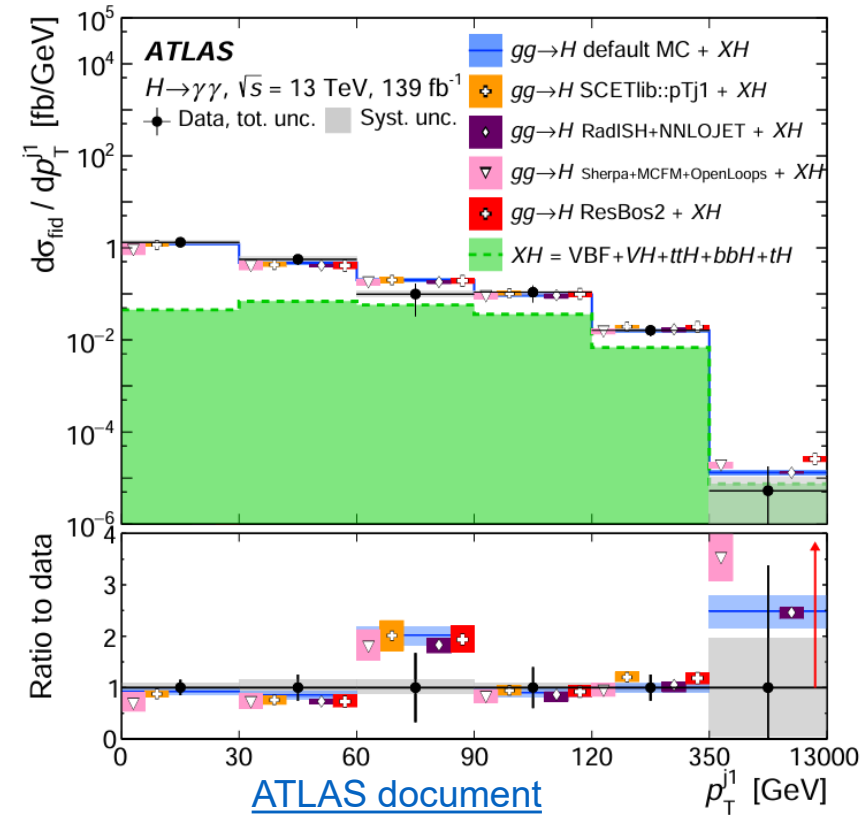
Composite Higgs?
Couples to new particles?

Higgs bosons are often produced with energetic jets



[CMS Experiment](#)

$H(\gamma\gamma) + j$



ATLAS/CMS measures the Higgs-plus-jet cross section over a wide range of jet transverse momenta.

Precise QCD predictions are needed to interpret this spectrum.

H+J status

Chen et al., 2110.06953

➤ Experiment

- Current typical differential uncertainty: 10-15%

ATLAS, 2004.03969; 2202.00487

- Expected (HL-LHC) uncertainty: $\leq 9.5\%$

Huss et al., 2504.06689

➤ Theory

- NNLO_{HEFT}

Boughezal et al., 1302.6216

X. Chen et al., 1607.08817

- NLO full-top-mass

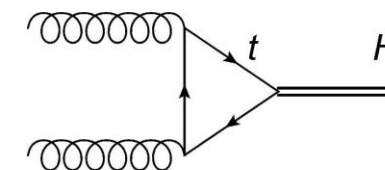
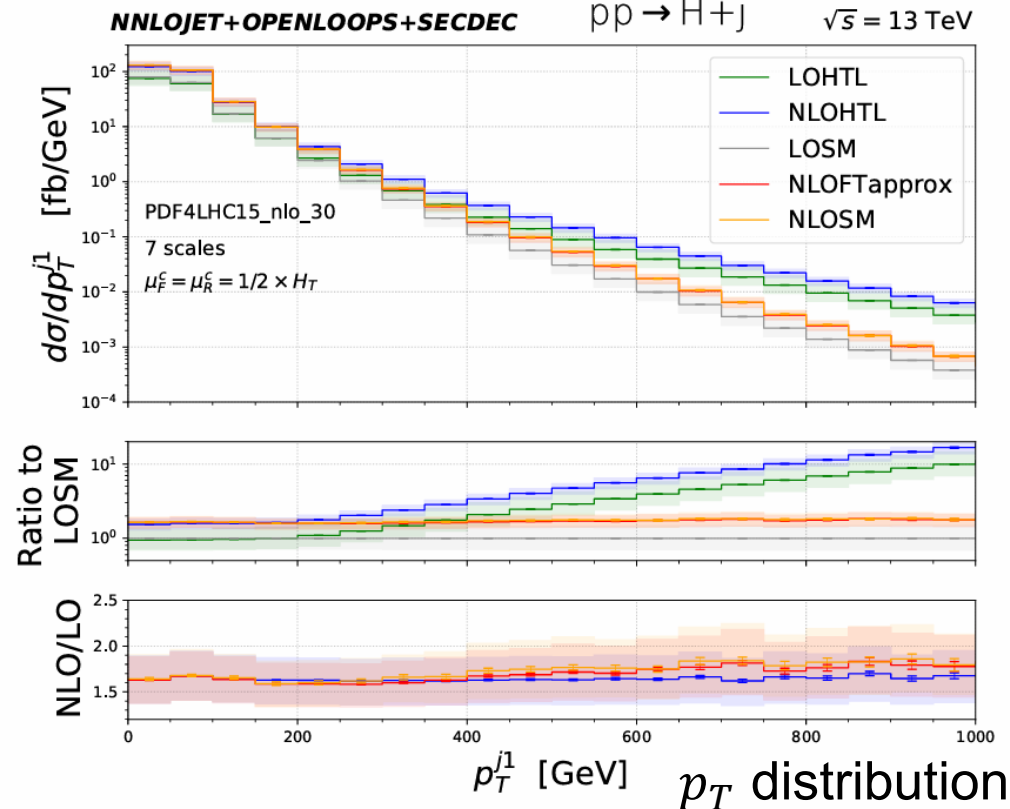
Jones, Kerner, Luisoni, 1802.00349

- Large p_T region is sensitive to new physics

- High energy expansion; Full-top mass + HEFT K factor at large p_T

Neumann 1802.02981

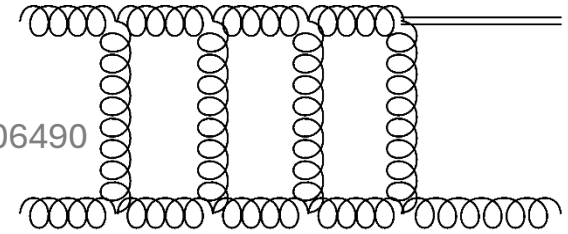
N³LO_{HEFT} is important to quantify the systematic uncertainty of the hybrid theory setup widely used by LHC experiments.



Towards a N^3LO calculation

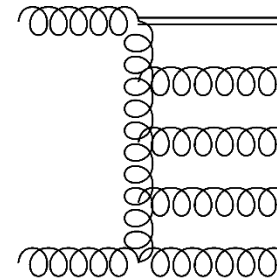
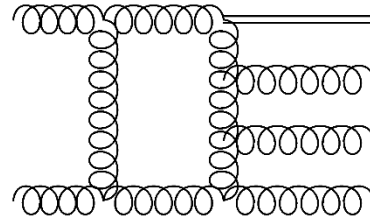
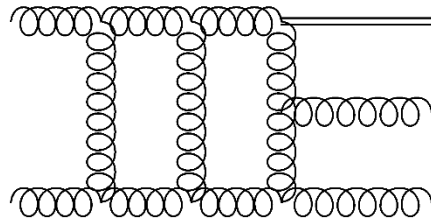
➤ Virtual

- Three loop (This talk) **Leading color** ✓ Chen, XG, Mistlberger, 2504.06490
- Lower loops through high-order ϵ ✓



➤ Real emissions

Badger et al., 0607139; 0909.4475; 0910.4481;
Dixon et al., 0906.0008;
Gehrmann et al., 1112.3554; 2301.10849



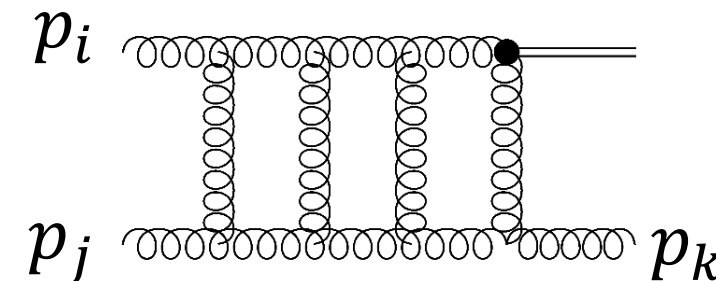
two-loop $Hb\bar{b}gg$ Badger et al., 2412.06519

two-loop $Hgggg$ (leading color) [Laurentis, et al., 2605.04009](#)

➤ Subtraction and Mento Carlo...

Three-loop H+J amplitudes

$$A^{(3)} = \sum_{i=1}^{\mathcal{O}(10^5)} f_i \times I_i$$



50 GB file size

43183 Feynman diagrams,
6 million integrals

After symmetry



46 topologies,
200k integrals

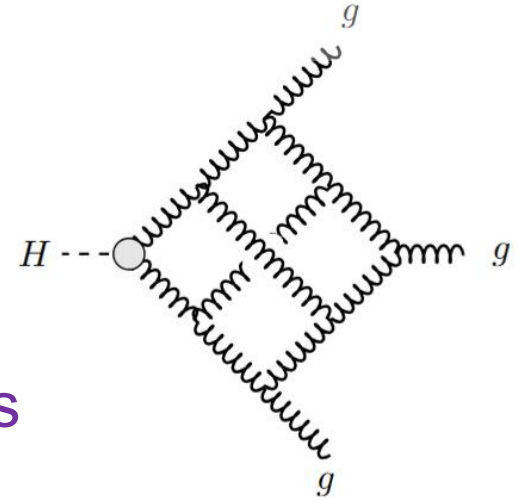
$g g \rightarrow H g$
 $q g \rightarrow H q$
 $q \bar{q} \rightarrow H g$

Qgraf: Nogueira et al., Comput. Phys. 1993

CalcLoop: Ma et al., <https://gitlab.com/multiloop-pku/calclloop>

Bottleneck: Integral evaluation

Feynman integral is the main bottleneck:



15 indices: 10 propagators and 5 irreducible scalar products

$$\int \frac{d^d l_1 d^d l_2 d^d l_3 [(l_2 + p_1)^2]^{-a_{11}} [(l_3 + p_1)^2]^{-a_{12}} [(l_1 + p_2)^2]^{-a_{13}} [(l_3 + p_2)^2]^{-a_{14}} [(l_1 - p_3)^2]^{-a_{15}}}{[l_1^2]^{a_1} [(l_1 + p_1)^2]^{a_2} [l_2^2]^{a_3} [(l_2 + p_2)^2]^{a_4} [l_3^2]^{a_5} [(l_3 + p_3)^2]^{a_6} [-l_1 + l_3 + p_3]^2^{a_7} [(l_2 + l_3)^2]^{a_8} [-l_1 + l_2 + l_3 - p_1]^2^{a_9} [-l_1 + l_2 + l_3 + p_2 + p_3]^2^{a_{10}}}$$

Encounter up to 6 numerator powers

The fundamental issue is not the millions of integrals, it's the complexity.

Integration-by-part (IBP) reduction

$$0 = \int \prod_i^L \frac{d^d l_i}{i \pi^{d/2}} \frac{\partial}{\partial l^\mu} \frac{v^\mu}{D_1^{a_1} \dots D_{15}^{a_{15}}}$$

$$I[a_1, a_2, \dots, a_{15}] \equiv \int \prod_i^L \frac{d^d l_i}{i \pi^{d/2}} \frac{1}{D_1^{a_1} \dots D_{15}^{a_{15}}}$$

→ $c_1 I_1 + c_2 I_2 + \dots + c_n I_n = 0$

Chetyrkin et al., NPB(1981)
K.G. Chetyrkin and A.L. Kataev, 1981

c_i are rational function coefficients I_i are Feynman integrals

Gaussian elimination, Laporta algorithm

Laporta, 0102033

200k integrals
numerator power 6

After using relations

→

2236 master integrals
numerator power 2

Fewer & simpler integrals to compute!

Challenges in IBP reduction

Complicated intermediate expression

Simple input => **intermediate expression swell** => (relatively) simple output

Resource-consuming due to large scale of linear equations

e.g. 5×10^8 equations in four-loop
QED form factors

Laporta, 1910.01248

O(TB) RAM

e.g. V+J (similar to H+J, but simpler)

Mella, Pos LL2024

Months of runtime

e.g. Five-loop QCD beta function uses
 $O(10^6)$ core*h.

Baikov, Chetyrkin, Kuhn, 1606.08659



Selected developments in IBP reduction

Syzygy

Eliminate unnecessary integrals in the equation,
delete duplicate computation

Gluza, Kajda, Kosower, 1009.0472

NeatIBP, Wu, Boehm, Ma et al., 2305.08783

Finite Field

Solve the linear system in finite field (**numerical sampling**)
and reconstruct the analytic results.

Manteuffel, Schabinger, 1406.4513

FiniteFlow, Peraro, 1905.08019

Firefly, Klappert, Lange, 1904.00009

Spanning-cut/sector

Divide the linear system into subsystems,
divide and conquer.

Larsen, Zhang, et. al., 1511.01071,
1805.01873, 2104.06866

Blade, XG, Liu, Ma, Wu. 2405.14621

Block-triangular form

Recognize and exploit block
structures in the relations.

Liu, Ma, 1801.10523

XG, Liu, Ma, 1912.09294

Blade, XG, Liu, Ma, Wu. 2405.14621

Improved seeding

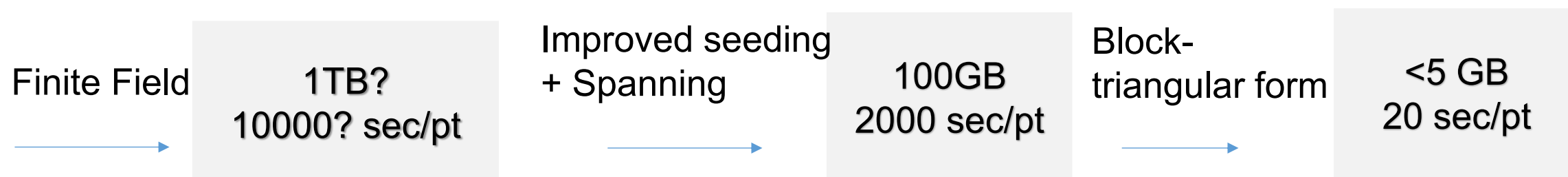
Avoid generating unnecessary
seeds (relations) from the beginning.

Blade, XG, Liu, Ma et al., 2405.14621

Driesse, Jakobsen, Mogull et al., 2403.07781

Bern, Herrmann, Roiban et al., 2406.01554

IBP reduction summary



We made the reduction feasible on realistic computing resources with Blade.

- **Reduce memory usage significantly!**
- **Orders-of-magnitude speedup.**

Several hundreds of thousands core*hours



Link: <https://gitee.com/multiloop-pku/blade>

项目信息

Block-triangular form improved Feynman integral decomposition .

Blade: XG, Liu, Ma, Wu. 2405.14621

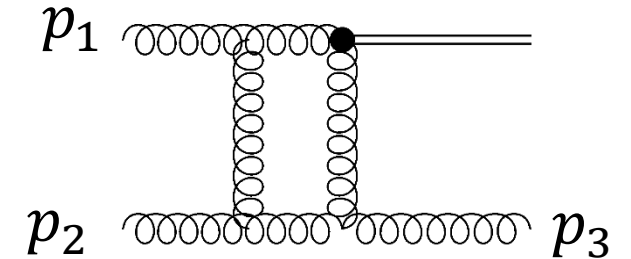
A new version is coming!

Compute master integrals using differential equations

Use IBP to construct differential equations of master integrals

$$\frac{\partial I[1,1,1,1]}{\partial t} = -\frac{(s+t)}{2t} I[1,2,1,1] + \dots$$

$$\text{IBP} = \frac{s+t}{s t} I[1,1,1,1] + \dots$$



$$\frac{\partial}{\partial t} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = B_t(t, \epsilon) \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}$$

$$B_t = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \# & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \# & \# & \# & \# \end{pmatrix}$$

$$t = (p_1 - p_3)^2$$

$$u = (p_2 - p_3)^2$$

$$d = 4 - 2\epsilon$$

Same for $B_u(t, u, \epsilon)$

Solve DEs analytically or using series expansion

Canonical differential equation (CDE)

Henn, 1304.1806

Canonical differential equation makes the analytic solution transparent

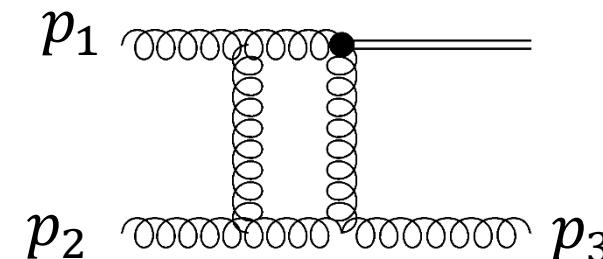
Find a basis transformation $\vec{I} = T \vec{f}$

such that ϵ is factorized

$$d \vec{f} = \epsilon \sum_k A_k d\text{Log}(\alpha_k) \vec{f}$$

Solve CDE in terms of Lauren expansion ϵ

$$\begin{aligned} f_4 = & 1 - \epsilon [\log(1 - t - u) + \log(u)] \\ & + \epsilon^2 \left[\frac{1}{2} \log^2(1 - t - u) + \frac{1}{2} \log^2(u) + \log(1 - t - u) \log(u) + \text{Li}_2(t + u) + \text{Li}_2(1 - u) - \frac{\pi^2}{6} \right] \\ & + O(\epsilon^3) \end{aligned}$$



$$d = 4 - 2\epsilon$$

Introducing ϵ is a calculation trick. We only need first a few orders.

Hybrid strategy

Lee algorithm + *CANONICA*

Lee, 1411.0911;
Meyer, 1611.01087, 1705.06252

Univariate slice + Lee

$$\gamma(\lambda) = (t_0 + a\lambda, u_0 + b\lambda)$$

$$d\vec{f}(\gamma(\lambda)) = C\vec{f} = \epsilon \sum_k A_k d\text{Log}(\alpha_k(\gamma(\lambda))) \vec{f}$$

Reconstruct A_k and α_k

Make ansatz of T

$$TC = BT - dT$$

Solve linear equations;
Degree bound is known.

Efforts in the literature:

Gehrmann et al., 2410.19088
Di Vita et al., 1408.3107
Canko et al., 2112.14275
Henn et al., 2302.12776
Syrrakos et al., 2307.08432
XG, Mistlberger, Ruf, [2511.13845](#)
XG, Mistlberger, Ruf, 2607.12033

$$\vec{I} = T \vec{f}$$

H+J masters

The canonical form is extremely simple

4 GB DE $\rightarrow$ 5 MB Canonical DE

$$d\vec{f} = \epsilon \sum_k A_k d\text{Log}(\alpha_k) \vec{f}$$

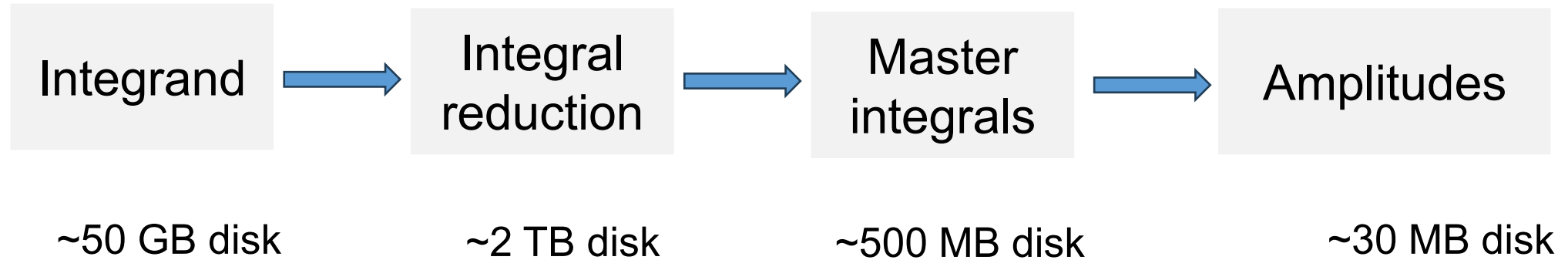
$\alpha_k \in \{t, 1-t, t-(1-u)^2, 1-t-tu, \frac{r+t}{r-t}\} + \text{permutations}$

- 20 symbol letters (two algebraic)
- One square root ($r = \sqrt{-u v w}$)
- The same letters as that in the leading color

Results

Compact analytic expressions in multiple polylogarithms, through ϵ^6

Assemble the amplitude



Analytic form in terms of multiple polylogarithms

Validation

Numerical Check • Collinear Limit • UV&IR Structure • N=4 SYM Structure

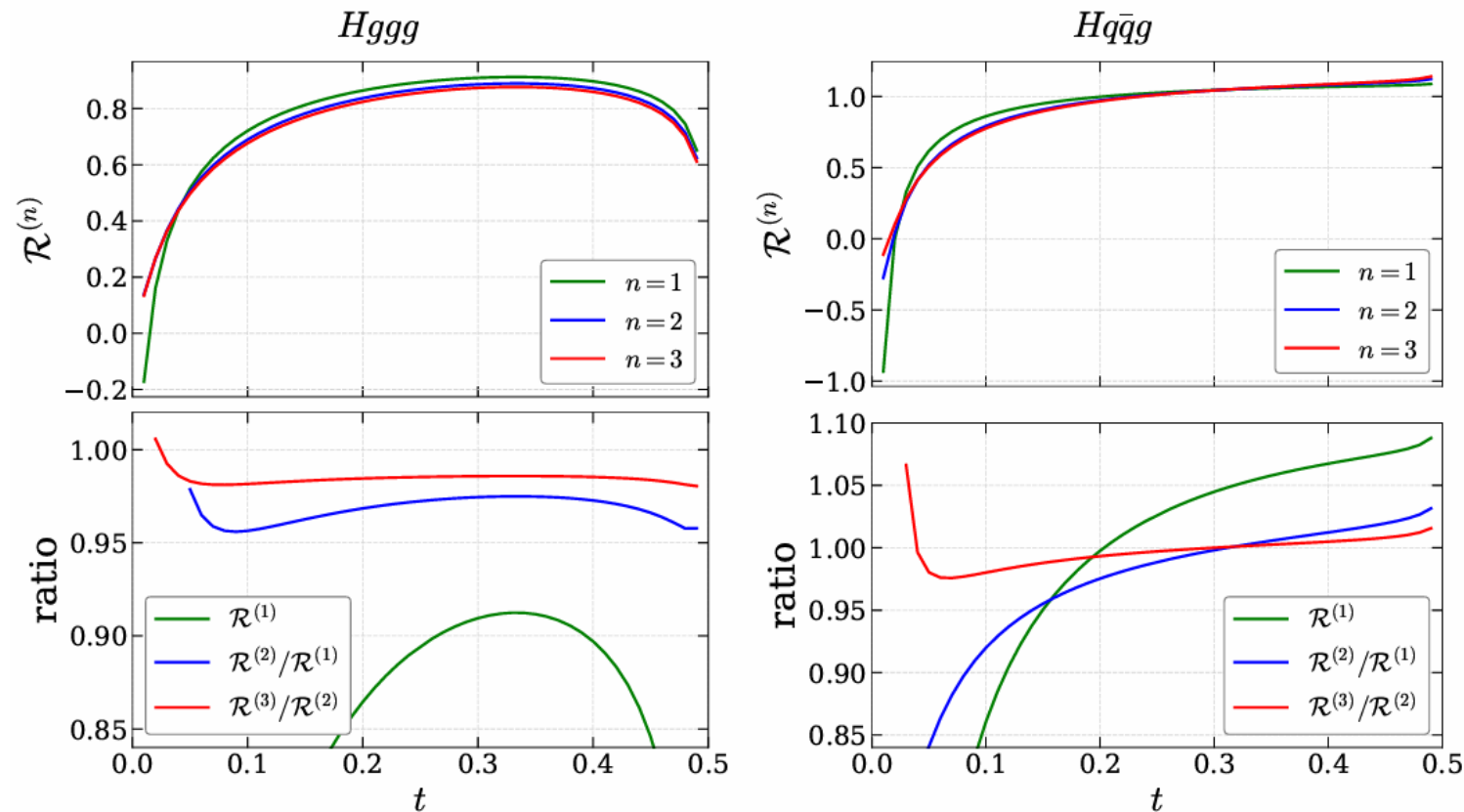
AMFlow: Liu, Ma. [2201.11669](#)

XG, Herzog, Ma, Mistlberger,
Suresh, [2408.03019](#)

Almelid, Duhr, Gardi, [1507.00047](#)
Becher, Neubert, [1908.11379](#)

Dixon et al., [2012.12286](#); [2204.11901](#)
Gehrmann et al., [2410.19088](#)
XG, Mistlberger, Ruf, [2511.13845](#)

Numerical results of the amplitudes



Good convergence behavior

~2% corrections at three-loop
(not a physical prediction yet)

Easy to evaluate

ready for phenomenology
applications

Numerical results of interfered amplitudes at different perturbative order, along the $t = u$ line.

One H + jet amplitude unlocks three Higgs predictions

LHC

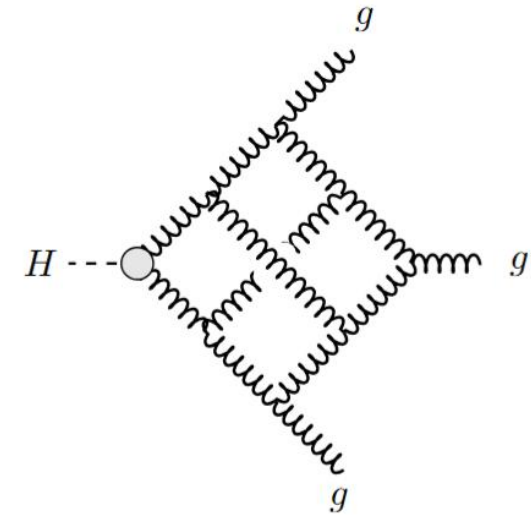
$N^3\text{LO}$ Higgs + jet observables

Combined with the remaining real-radiation ingredients.

Future e^+e^- colliders

$N^3\text{LO}$ $H \rightarrow 3$ jets

The same amplitude is reused through crossing symmetry.



Inclusive Higgs production

A building block for $N^4\text{LO}$

Integrating over final states feeds the total rate.

=> Adi's talk

Bernhard Mistlberger, Adi Suresh, [LoopFest2026 Talk](#)

One three-loop object connects differential, decay, and inclusive precision.

The calculation unlocks a reusable three-loop toolkit

3 loops • $2 \rightarrow 2$ scattering • 1 massive leg

Phenomenology

$b \bar{b} \rightarrow Hg$ • $W/Z + \text{jet}$

Reuse the integral basis in other one-mass processes.

T. Gehrmann, P. Jakubčík, C. C. Mella et al., 2307.15405

XG, Mistlberger, Ruf, To appear

Amplitude theory

$N = 4$ SYM form factors

Inputs for bootstrap programs beyond leading color.

Gehrmann et al., 2410.19088

XG, Mistlberger, Ruf, 2511.13845

Next frontier

Four-loop / $N^4\text{LO}$ Higgs production

The same techniques make the next order approachable.

Lee et al. [2202.04660](#)

Mistlberger, Suresh, [2504.10574](#)

The deliverable is not only one amplitude—it is reusable mathematical infrastructure.

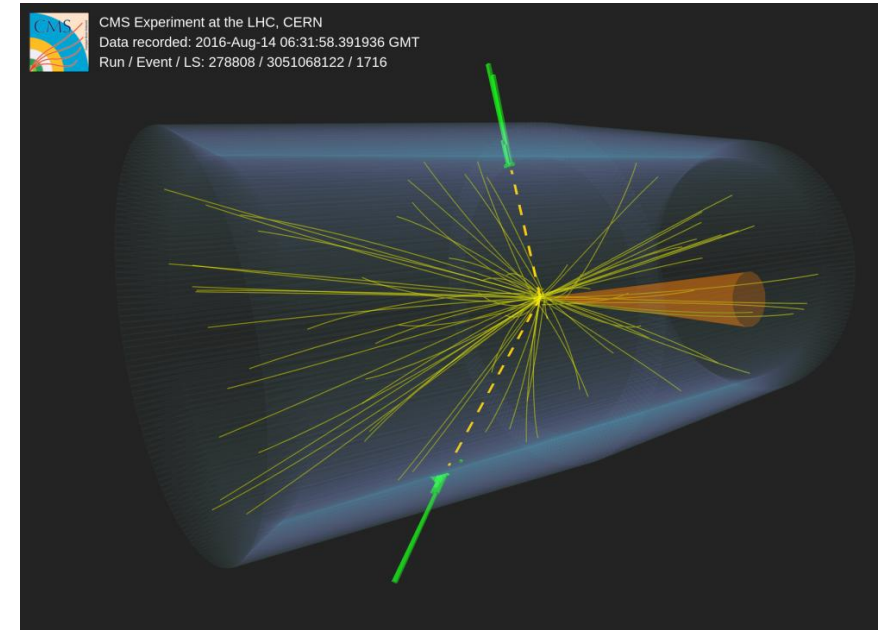
The public codes, e.g. Blade, will benefit the broader community.

Conclusions

- We complete the three-loop QCD corrections to the scattering amplitude of a Higgs boson and three partons.
- These results provide essential ingredients for testing the Standard Model and searching for new physics.
- We incorporate cutting-edge techniques to tackle the computational complexity of Feynman integrals.

Looking ahead:

N3LO H+J predictions



$$p p \rightarrow H(\gamma\gamma) + j$$

Thank you!

Block-triangular form

Liu, Ma, 1801.10523
XG, Liu, Ma, 1912.09294

Difficulty of IBP reduction: **highly coupled • auxiliary integrals**

Key idea:

1. Strict block upper-triangular form.

Solve by blocks

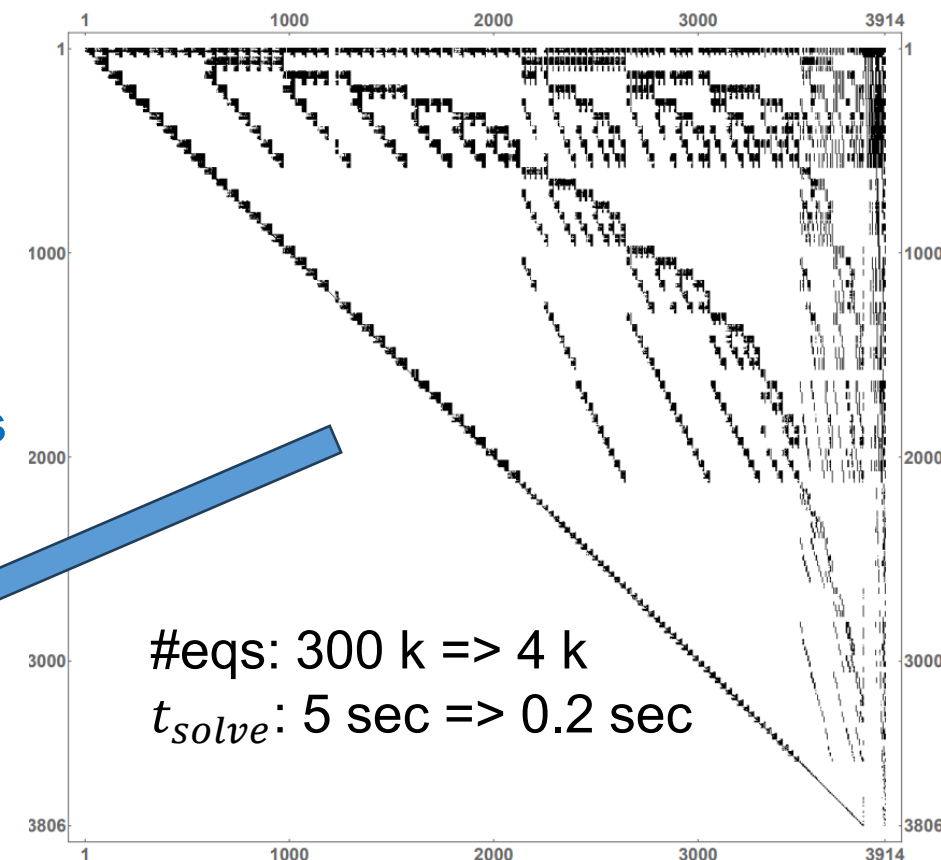
2. Search/fit relations among needed integrals

Orders of magnitude fewer equations

$$\begin{aligned} t^2 I[1,1,-2] + u^2 t I[1,1,-1] + \epsilon t^4 I[1,1,0] + u^2 I[0,1,-1] &= 0 \\ u^2 I[1,1,-2] + t^2 u I[1,1,-1] + \epsilon^2 (u^2 t^2 + t^4) I[1,1,0] &= 0 \end{aligned}$$

Each row is an equation

Each column represents an integral $I[\vec{a}]$



An example of block-triangular form

How to search relations

➤ Make polynomial ansatzes

$$a_0 I_1 + (a_1 u + a_2 t) I_2 + (a_3 s^2 + a_4 u t + a_5 t^2) I_3 + \dots = 0$$

- a_i are unknown constants

➤ Numeric IBP as Input

$$I_i(\epsilon, u_0, t_0) = \sum_{j=1}^n c_{ij}(\epsilon_0, u_0, t_0) M_j$$

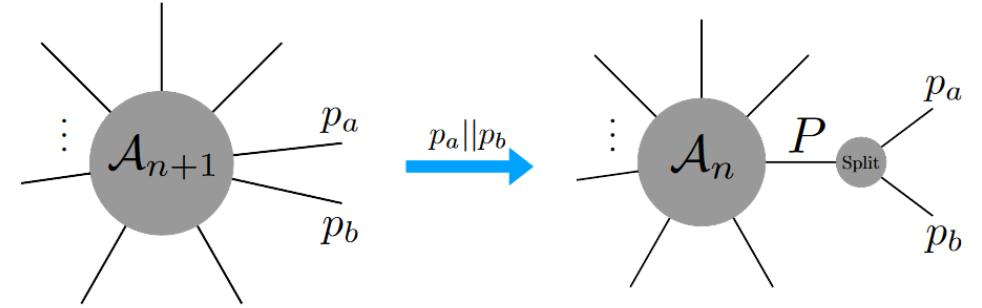
$$a_0 \begin{pmatrix} c_{11} M_1 \\ c_{12} M_2 \\ \dots \end{pmatrix} + (a_1 u_0 + a_2 t_0) \begin{pmatrix} c_{21} M_1 \\ c_{22} M_2 \\ \dots \end{pmatrix} + (a_3 u_0^2 + a_4 u_0 t_0 + a_5 t_0^2) \begin{pmatrix} c_{31} M_1 \\ c_{32} M_2 \\ \dots \end{pmatrix} + \dots = 0$$

- $c_{ij}, \epsilon_0, u_0, t_0$: numeric values in finite field
- M_i are independent $\rightarrow$ Coefficients must vanish
- Each master integral generate an equation of unknowns a_i
- Solve for $a_i \rightarrow$ find relations among integrals

Validation: Collinear limit

1. A universal limit provides a powerful check.

$$A_{Hggg} \rightarrow \text{Split} \times A_{Hgg}$$



XG Herzog, Ma, Mistlberger, Suresh,
2408.03019

The amplitude factorizes.

1. Two particles moving in the same direction (collinear).
2. Other particles cannot resolve the inner structure of the two collinear particles

The splitting amplitudes are universal.

Validation: structural and numerical checks

2. UV and IR divergence (ϵ poles) match universal predictions.

Almelid, Duhr, Gardi, [1507.00047](#)

Becher, Neubert, [1908.11379](#)

3. Independent numerical checks at one phase space point

$$\left(t = \frac{1}{3}, u = \frac{1}{3}\right).$$

AMFlow: Liu, Ma. [2201.11669](#)

4. Leading transcendental structure matches known analytic structures
($\mathcal{N}=4$ sYM)

Dixon et al., [2012.12286](#); [2204.11901](#)

Gehrmann et al., [2410.19088](#)

XG, Mistlberger, Ruf, [2511.13845](#)