



Complete NLO corrections to off-shell $t\bar{t}$ production in the $\ell + j$ channel

Leon Mans

In collaboration with: Daniel Stremmer, Malgorzata Worek

Based on: JHEP 07 (2026) 010

Collaborative Research Center TRR 257



Particle Physics Phenomenology after the Higgs Discovery

*QCD@LHC,
Oxford,
29.09.26*

With funding from the:

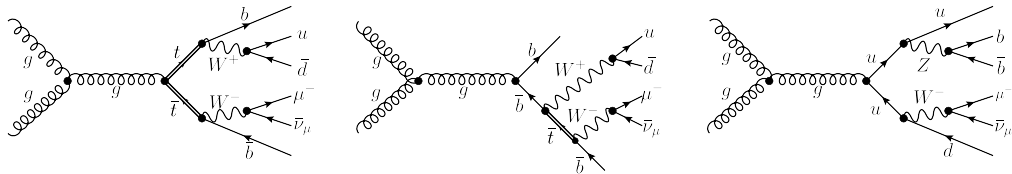


Federal Ministry
of Research, Technology
and Space

Motivation for $t\bar{t} \rightarrow \ell + j$ with NLO QCD+EW Corrections

- ▶ $\ell + j$ is one of the *highest-statistics* decay modes (BR $\sim 30\%$, of all $t\bar{t}$ events, excluding τ) $\Rightarrow$ key channel for precision differential measurements.
- ▶ With only one neutrino, the top quarks are easier to reconstruct. Off-shell effects can be important near kinematic boundaries and in distribution tails, and can affect top-mass extractions (Heinrich et al., JHEP 07 (2018) 129).
- ▶ Electroweak Sudakov logarithms can give $\mathcal{O}(-10\%)$ corrections in the boosted regime, crucial for new-physics searches (Czakon et al., JHEP 10 (2017) 186).

Full off-shell = resonant diagrams + non-resonant diagrams + interference + Breit-Wigner effects



(Feynman diagrams done with FEYNGAME, Bündgen et al., Comput.Phys.Commun. 314 (2025) 109662)

Theory Status: Production and Decays

Stable tops: NNLO QCD, combined with NLO EW

Czakon, Heymes, Mitov, Phys. Rev. Lett. 116 (2016) 082003; Catani et al., JHEP 07 (2019) 100.

QCD+EW: Czakon et al., JHEP 10 (2017) 186.

Dilepton decays: NNLO QCD in production and decay (NWA)

Behring et al., Phys. Rev. Lett. 123 (2019) 082001; Czakon, Mitov, Poncelet, JHEP 05 (2021) 212.

Both decay channels: NNLO QCD + PS in production, LO spin-correlated decays

Mazzitelli et al., Phys. Rev. Lett. 127 (2021) 062001; JHEP 04 (2022) 079.

Off-shell dilepton production: NNLO QCD with a two-loop DPA

Buonocore et al., JHEP 10 (2025) 195.

Full Off-Shell Predictions at NLO

Dilepton channel

NLO QCD

Bevilacqua et al., JHEP 02 (2011) 083;

Denner et al., Phys. Rev. Lett. 106 (2011) 052001;

Denner et al., JHEP 10 (2012) 110;

Heinrich et al., JHEP 06 (2014) 158.

NLO EW

Denner, Pellen, JHEP 08 (2016) 155.

Lepton+jets channel

NLO QCD

Denner, Pellen, JHEP 02 (2018) 013.

This work

Complete NLO QCD+EW

All Born orders and channels

$\ell + j$ Production Process

$$pp \rightarrow \ell^- \bar{\nu}_\ell j_b j_b jj + X$$

$$\ell^- = e^-, \mu^-$$

- ▶ j_b : b -tagged jets
- ▶ j : Light-flavor jets
- ▶ X = Additional radiation from higher-order corrections:
 - ▶ QCD: extra gluons or jets
 - ▶ EW: photons from initial or final state radiation
- ▶ Additional radiation is included inclusively. We do not require an additional resolved jet or photon.

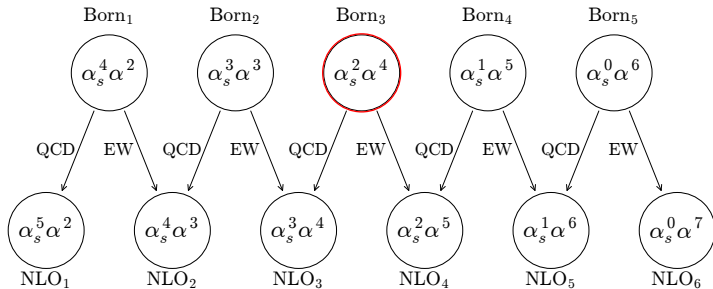
Born-Level Contributions to $pp \rightarrow \ell^- \bar{\nu}_\ell j_b j_b j j$

Born₁: $\mathcal{O}(\alpha_s^4 \alpha^2)$

QCD-dominated. No doubly-resonant $t\bar{t}$.

Born₂: $\mathcal{O}(\alpha_s^3 \alpha^3)$

Photon-initiated, suppressed by photon PDFs. Also includes interference.



Born₃: $\mathcal{O}(\alpha_s^2 \alpha^4)$

Dominant contribution, including doubly-resonant $t\bar{t}$.

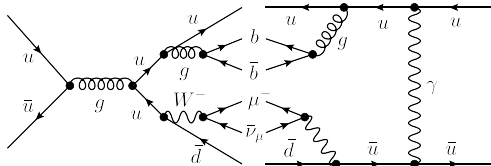
Born₄: $\mathcal{O}(\alpha_s \alpha^5)$

Photon-initiated or interference contributions, including doubly-resonant $t\bar{t}$.

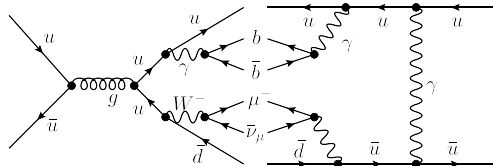
Born₅: $\mathcal{O}(\alpha^6)$

Same $t\bar{t}$ resonances as Born₃. Suppressed by EW couplings and the absence of gluon PDFs.

Born-Level Contributions to $pp \rightarrow \ell^- \bar{\nu}_\ell j_b j_b j j$

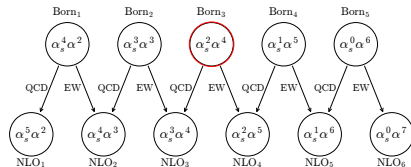


Left diagram: Born₁ Right diagram: Born₃
Interference $\Rightarrow$ Born₂

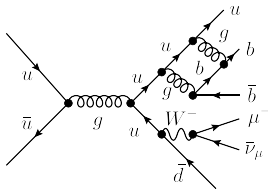


Left diagram: Born₃ Right diagram: Born₅
Interference $\Rightarrow$ Born₄

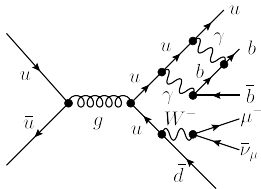
- Include **all resonant and non-resonant diagrams**
- **Photons** and **bottom quarks** in the initial state
- All possible light-jet flavors, not just W decays like $u\bar{d}$ or $c\bar{s}$



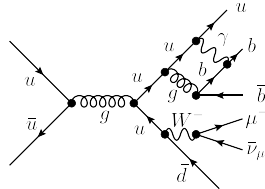
Loop Contributions and Real Radiation



QCD loops

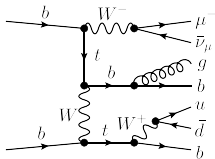


EW loops

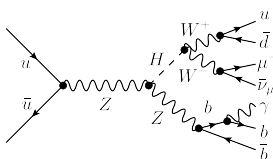


Mixed loops

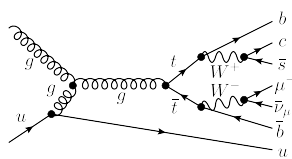
⇒ IR cancellation requires diagrams of different coupling orders.



NLO gluon radiation



NLO photon radiation



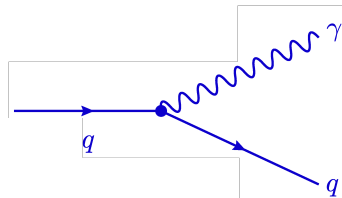
init./final state splitting

Democratic Clustering and Photon Fragmentation

- ▶ Cluster photons and QCD partons together with anti- k_T .
- ▶ Separate hadronic jets from photon-dominated clusters:

$$z_\gamma = \frac{p_{T,\gamma}}{p_{T,\gamma} + p_{T,i}}, \quad z_{\gamma,\text{cut}} = 0.7.$$

- ▶ $z_\gamma < 0.7$: hadronic jet; $z_\gamma > 0.7$: photon.
A hard photon plus a soft gluon must not count as a hadronic jet.
- ▶ The separation restricts collinear $q \rightarrow q\gamma$ radiation.
- ▶ Absorb the remaining collinear singularity into the **parton-to-photon fragmentation function** $D_{q \rightarrow \gamma}$.



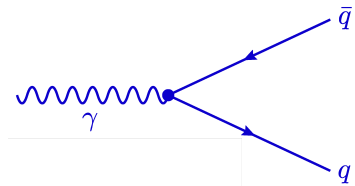
Non-perturbative input

Finite fragmentation contribution fitted to ALEPH photon-in-jet data.

Glover et al., Z. Phys. C 62 (1994) 311; Denner et al., JHEP 01 (2015) 094; ALEPH, Z. Phys. C 69 (1996) 365.

Photon-to-Jet Conversion

- ▶ A collinear $\gamma \rightarrow q\bar{q}$ pair can be clustered into a light jet.
- ▶ The corresponding final state with a photon instead of this jet does not satisfy the Born-level jet requirement.
- ▶ Its virtual corrections therefore cannot cancel the collinear singularity within the selected process.
- ▶ The **photon-to-jet conversion function** $D_{\gamma \rightarrow \text{jet}}$ supplies the required contribution.



Non-perturbative input

$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ from
 $e^+e^- \rightarrow \text{hadrons}$ data.

$$d\sigma_{X+\text{jet}} = d\sigma_{X+q\bar{q}}^{\text{pert}} + d\sigma_{X+\gamma}^{\text{Born}} \int_0^1 dz D_{\gamma \rightarrow \text{jet}}^B(z).$$

Denner et al., Phys. Lett. B 798 (2019) 134951; Keshavarzi et al., Phys. Rev. D 97 (2018) 114025.

Computational Framework

Full off-shell matrix elements in the complex-mass scheme

RECOLA

Born and real-emission matrix elements
One-loop amplitudes and tensor coefficients
COLLIER: tensor integrals
CutTools + OneLOop: OPP alternative

HELAC-Dipoles

QCD and QED infrared subtraction
Nagy–Soper subtraction with QED extension
Catani–Seymour subtraction for cross-checks
Local and integrated subtraction terms



Phase-space integration: KALEU + PARNI

Adaptive multi-channel integration and importance sampling



Integrated cross sections and differential distributions

RECOLA: Actis et al., CPC 214 (2017) 140; COLLIER: Denner et al., CPC 212 (2017) 220.

HELAC-Dipoles: Czakon et al., JHEP 08 (2009) 085; Nagy–Soper: Bevilacqua et al., JHEP 10 (2013) 204.

QED extension: Stremmer, Worek, JHEP 07 (2024) 091.

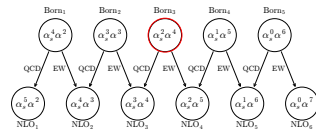
Integrated Fiducial Cross-Sections

Mans, Stremmer, Worek, JHEP 07 (2026) 010

$$\mu_0 = H_T/4, \quad |M_{jj} - m_W| < 20 \text{ GeV}$$

$$\sqrt{s} = 13.6 \text{ TeV}, \quad \ell^- = e^-, \mu^-$$

	Order	σ_i [pb]	Ratio to Born ₃
Born ₁	$\mathcal{O}(\alpha_s^4 \alpha^2)$	0.138(2)	0.44%
Born ₂	$\mathcal{O}(\alpha_s^3 \alpha^3)$	0.0002(1)	0.00%
Born ₃	$\mathcal{O}(\alpha_s^2 \alpha^4)$	31.264(4)	100.00%
Born ₄	$\mathcal{O}(\alpha_s^1 \alpha^5)$	0.1336(1)	0.43%
Born ₅	$\mathcal{O}(\alpha_s^0 \alpha^6)$	0.0968(1)	0.31%
NLO ₁	$\mathcal{O}(\alpha_s^5 \alpha^2)$	+ 0.144(4)	+ 0.46%
NLO ₂	$\mathcal{O}(\alpha_s^4 \alpha^3)$	- 0.0022(4)	- 0.01%
NLO ₃	$\mathcal{O}(\alpha_s^3 \alpha^4)$	- 1.90(2)	- 6.10%
NLO ₄	$\mathcal{O}(\alpha_s^2 \alpha^5)$	+ 0.112(2)	+ 0.36%
NLO ₅	$\mathcal{O}(\alpha_s^1 \alpha^6)$	+ 0.1864(6)	+ 0.60%
NLO ₆	$\mathcal{O}(\alpha_s^0 \alpha^7)$	+ 0.00228(2)	+ 0.01%
LO		27.241(2) ^{+30.7%} _{-22.0%}	87%
Born		31.634(4) ^{+30.7%} _{-22.0%}	101%
NLO _{QCD}		29.72(2) ^{+1.4%} _{-6.0%}	95%
NLO		30.16(2) ^{+1.5%} _{-5.6%}	96%

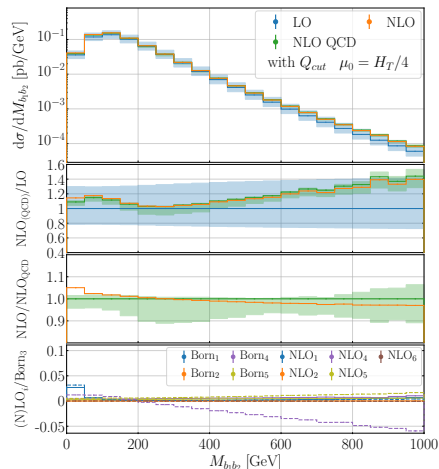
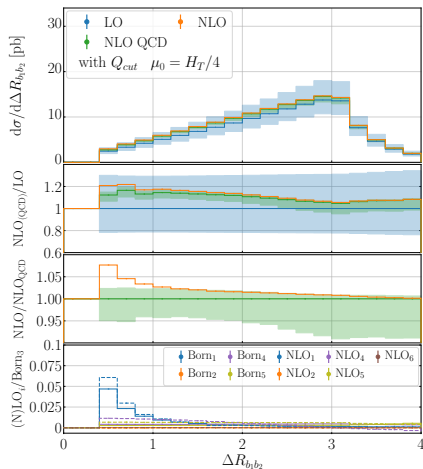


- ▶ The W -mass window suppresses QCD backgrounds.
- ▶ Born₃ and NLO₃ dominate. Each subleading term is below 1% of Born₃.
- ▶ Complete NLO is 1.5% above NLO QCD; scale uncertainty is about 6%.

LO uses Γ_t^{LO} ; Born and NLO use Γ_t^{NLO} .

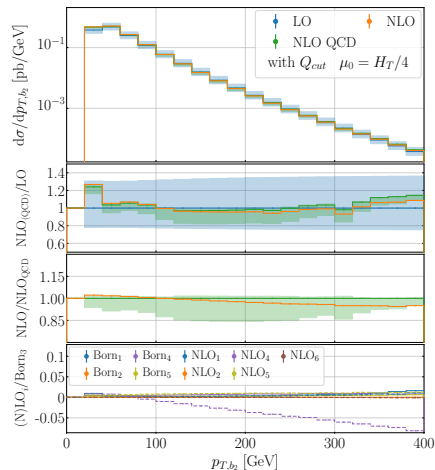
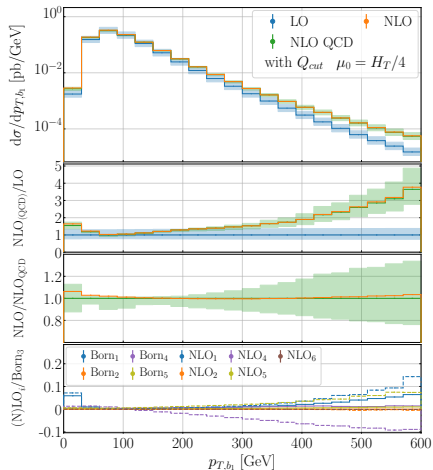
$$\text{Born} = \sum_{i=1}^5 \text{Born}_i, \quad \text{NLO}_{\text{QCD}} = \text{Born} + \text{NLO}_3, \quad \text{NLO} = \text{Born} + \sum_{i=1}^6 \text{NLO}_i.$$

Kinematics of the b -Jet Pair



- ▶ Small $\Delta R_{b_1 b_2}$: QCD backgrounds with $g \rightarrow b\bar{b}$ are enhanced; $NLO_1 \simeq +6\%$, $\Delta_{QCD} \simeq +8\%$.
- ▶ $M_{b_1 b_2}$: $\Delta_{QCD} \simeq +5\%$ at low mass; in the tail, $NLO_4 \simeq -6\%$ from EW Sudakov effects; $\Delta_{QCD} \simeq -3\%$.

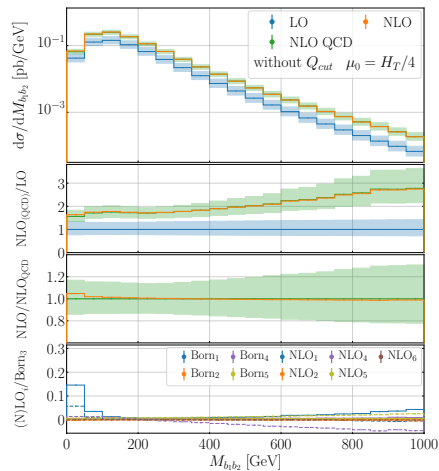
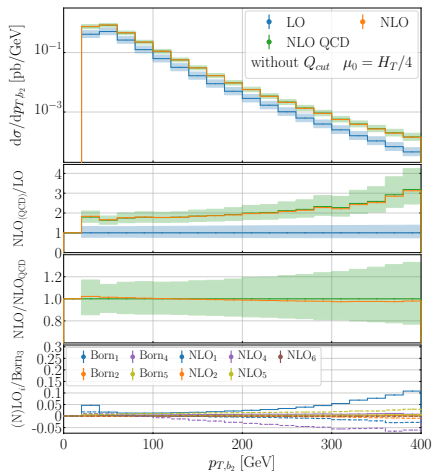
Transverse Momenta of the b Jets



- $p_{T,b1}$: $NLO_1 \simeq +14\%$, $NLO_4 \simeq -9\%$, $NLO_5 \simeq +7\%$; $\Delta_{QCD} \simeq +3\%$ in the tail.
- $p_{T,b2}$: $NLO_4 \simeq -8\%$; $\Delta_{QCD} \simeq -5\%$, comparable to the scale uncertainty.

Removing the Mass Window Enhances Backgrounds

Mans, Stremmer, Worek, JHEP 07 (2026) 010



- ▶ Without the cut, tail K -factors rise from 1.1 to 3.1 (p_{T,b_2}) and 1.4 to 2.7 ($M_{b_1b_2}$).
- ▶ Largest NLO QCD scale variations grow from 4.5%/11% to 33%/31%, with larger Born₁ backgrounds.

Summary and Outlook

- ▶ Complete NLO QCD+EW predictions for off-shell $t\bar{t}$ production in the **lepton + jets channel**, including all Born orders and partonic channels.
- ▶ Fragmentation and photon-to-jet conversion functions give an **infrared-safe description** of photons and light jets.
- ▶ **Integrated cross sections:** Born₃ and NLO₃ dominate. With the W -mass window, individual subleading terms are below 1% of Born₃.
- ▶ **Differential distributions:** QCD backgrounds and EW Sudakov effects alter shapes.
Complete NLO vs. NLO QCD: about +8% at small $\Delta R_{b_1 b_2}$ and −5% in the p_{T,b_2} tail.
- ▶ The same framework can be applied to complete off-shell NLO predictions for $t\bar{t}H$, $t\bar{t}t\bar{t}$ and $t\bar{t}W^\pm$ with hadronic W decays.

BACKUP

Setup and PDF Choice

- ▶ **Collider:** pp at $\sqrt{s} = 13.6$ TeV (LHC Run III).
- ▶ **Scheme:** 5FS, massless b , diagonal CKM.
- ▶ **Coupling:** G_μ scheme with $G_\mu = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}$.
- ▶ **PDF:** NNPDF3.1luxQED_NLO with $\alpha_s(m_Z) = 0.118$ (NNPDF 17).
- ▶ Photon content treated consistently with NLO EW corrections.
- ▶ Lepton flavours: $\ell^- = e^-, \mu^-$.

SM Masses and Widths

Input	Mass [GeV]	Width [GeV]
W (on shell)	80.377	2.085
Z (on shell)	91.1876	2.4955
H	125.0	4.07×10^{-3}
t	172.5	1.4580658 (LO), 1.3518591 (NLO QCD+EW)

- Convert the on-shell W/Z inputs to pole values:

$$m_V = \frac{m_V^{\text{OS}}}{\sqrt{1 + (\Gamma_V^{\text{OS}}/m_V^{\text{OS}})^2}}, \quad \Gamma_V = \frac{\Gamma_V^{\text{OS}}}{\sqrt{1 + (\Gamma_V^{\text{OS}}/m_V^{\text{OS}})^2}}.$$

- Finite widths are incorporated through the complex-mass scheme.

Dynamic Scales, Jet Algorithm and Cuts

Scale choices

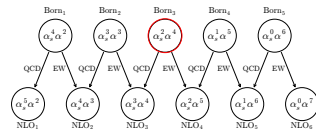
- ▶ Central: $\mu_R = \mu_F = \mu_0$.
- ▶ *Alternative:* $\mu_0 = E_T/4$, with
$$E_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_{\bar{t}}^2 + p_{T,\bar{t}}^2}.$$
- ▶ *Default:* $\mu_0 = H_T/4$, $H_T = p_{T,\ell} + p_{T,b_1} + p_{T,b_2} + p_{T,j_1} + p_{T,j_2} + p_T^{\text{miss}}$.
- ▶ Uncertainty: seven-point variation of μ_R and μ_F by factors 1/2 and 2, omitting the antipodal choices.

Jet algorithm & cuts

- ▶ anti- k_T algorithm ($R = 0.4$)
- ▶ Democratic clustering with $z_{\gamma,\text{cut}} = 0.7$.
- ▶ Acceptance: $p_T > 25 \text{ GeV}$, $|y| < 2.5$ for ℓ^- , b -jets, light jets.
- ▶ Separation: $\Delta R_{bb,jb,\ell b,\ell j,jj} > 0.4$.
- ▶ $\ell^- = e^-, \mu^-$. Event topology: $\ell^- \bar{\nu}_\ell b\bar{b}jj$ with ≥ 2 b -jets, ≥ 2 light jets.
- ▶ Optional W -mass window Q_{cut} :
$$|M_{jj} - M_W| < 20 \text{ GeV}.$$

$$\mu_0 = H_T/4, \quad \text{no } W\text{-mass window}$$

	Order	σ_i [pb]	Ratio to Born ₃
Born ₁	$\mathcal{O}(\alpha_s^4 \alpha^2)$	0.778(4)	2.40%
Born ₂	$\mathcal{O}(\alpha_s^3 \alpha^3)$	0.0021(1)	0.01%
Born ₃	$\mathcal{O}(\alpha_s^2 \alpha^4)$	32.388(4)	100.00%
Born ₄	$\mathcal{O}(\alpha_s^1 \alpha^5)$	0.1365(2)	0.42%
Born ₅	$\mathcal{O}(\alpha_s^0 \alpha^6)$	0.09980(6)	0.31%
NLO ₁	$\mathcal{O}(\alpha_s^5 \alpha^2)$	+ 0.220(6)	+ 0.68%
NLO ₂	$\mathcal{O}(\alpha_s^4 \alpha^3)$	− 0.0172(8)	− 0.05%
NLO ₃	$\mathcal{O}(\alpha_s^3 \alpha^4)$	+ 16.56(2)	+ 51.15%
NLO ₄	$\mathcal{O}(\alpha_s^2 \alpha^5)$	+ 0.192(2)	+ 0.59%
NLO ₅	$\mathcal{O}(\alpha_s^1 \alpha^6)$	+ 0.276(1)	+ 0.85%
NLO ₆	$\mathcal{O}(\alpha_s^0 \alpha^7)$	+ 0.00235(2)	+ 0.01%
LO		28.896(2) ^{+31.2%} _{−22.2%}	89%
Born		33.404(4) ^{+31.2%} _{−22.2%}	103%
NLO _{QCD}		49.98(2) ^{+15.1%} _{−13.5%}	154%
NLO		50.64(2) ^{+14.2%} _{−13.2%}	156%



- Born₁ contributes 2.4% of Born₃.
- $K = \text{NLO}/\text{LO} \simeq 1.75$.
- Complete NLO is 1.3% above NLO QCD; scale uncertainty is about 14%.

LO uses Γ_t^{LO} ; Born and NLO use Γ_t^{NLO} .

$$\text{Born} = \sum_{i=1}^5 \text{Born}_i, \quad \text{NLO}_{\text{QCD}} = \text{Born} + \text{NLO}_3, \quad \text{NLO} = \text{Born} + \sum_{i=1}^6 \text{NLO}_i.$$

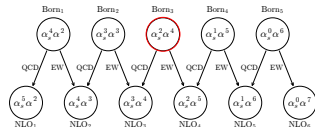
Integrated Cross-Sections: $\mu_0 = E_T/4$

Mans, Stremmer, Worek, JHEP 07 (2026) 010

$$\mu_0 = E_T/4, \quad |M_{jj} - m_W| < 20 \text{ GeV}$$

	Order	σ_i [pb]	Ratio to Born ₃
Born ₁	$\mathcal{O}(\alpha_s^4 \alpha^2)$	0.1270(2)	0.43%
Born ₂	$\mathcal{O}(\alpha_s^3 \alpha^3)$	0.00032(2)	0.00%
Born ₃	$\mathcal{O}(\alpha_s^2 \alpha^4)$	29.824(2)	100.00%
Born ₄	$\mathcal{O}(\alpha_s \alpha^5)$	0.1302(1)	0.44%
Born ₅	$\mathcal{O}(\alpha_s^0 \alpha^6)$	0.10086(4)	0.34%
NLO ₁	$\mathcal{O}(\alpha_s^5 \alpha^2)$	+ 0.152(2)	+ 0.51%
NLO ₂	$\mathcal{O}(\alpha_s^4 \alpha^3)$	- 0.0016(2)	- 0.01%
NLO ₃	$\mathcal{O}(\alpha_s^3 \alpha^4)$	+ 0.48(1)	+ 1.61%
NLO ₄	$\mathcal{O}(\alpha_s^2 \alpha^5)$	+ 0.1074(2)	+ 0.36%
NLO ₅	$\mathcal{O}(\alpha_s \alpha^6)$	+ 0.1782(4)	+ 0.60%
NLO ₆	$\mathcal{O}(\alpha_s^0 \alpha^7)$	+ 0.00242(1)	+ 0.01%
LO		25.991(1) ^{+30.3%} _{-21.8%}	87%
Born		30.182(2) ^{+30.3%} _{-21.8%}	101%
NLO _{QCD}		30.66(1) ^{+1.6%} _{-4.8%}	103%
NLO		31.10(1) ^{+1.7%} _{-5.0%}	104%

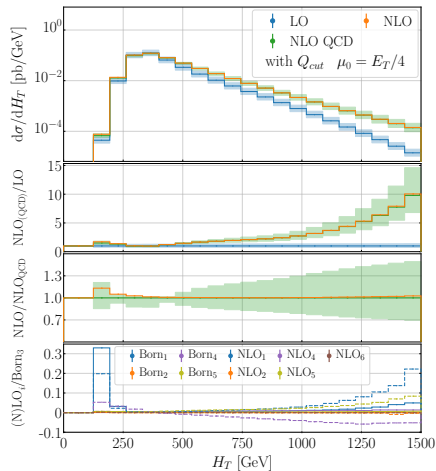
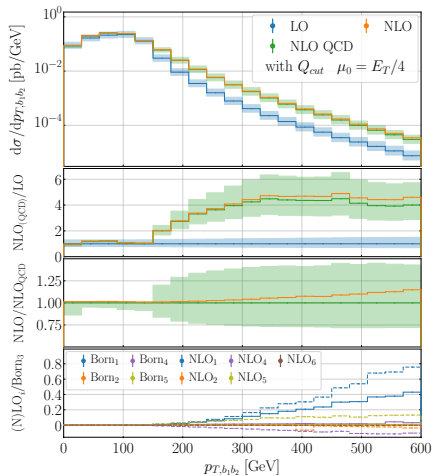
$$\text{Born} = \sum_{i=1}^5 \text{Born}_i, \quad \text{NLO}_{\text{QCD}} = \text{Born} + \text{NLO}_3, \quad \text{NLO} = \text{Born} + \sum_{i=1}^6 \text{NLO}_i.$$



- ▶ $\sigma_{\text{NLO}} = 31.10(1) \text{ pb}$: +3.1% relative to $H_T/4$, within scale uncertainties.
- ▶ $K = \text{NLO}/\text{LO} \simeq 1.20$.
- ▶ Subleading contributions remain similar in relative size.

LO uses Γ_t^{LO} ; Born and NLO use Γ_t^{NLO} .

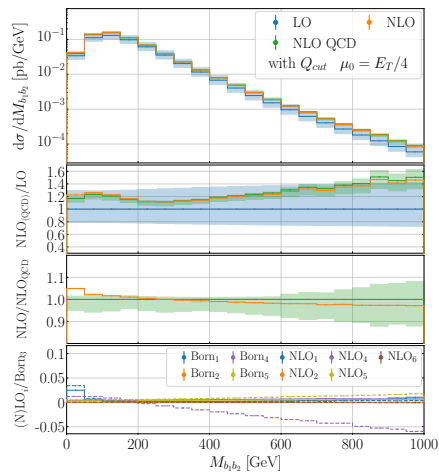
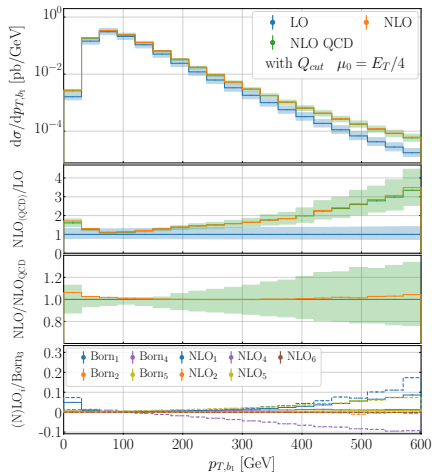
Differential Distributions: $\mu_0 = E_T/4$



- ▶ p_{T,b_1b_2} : tail K -factor decreases from about 5.3 ($H_T/4$) to 4.6 ($E_T/4$).
- ▶ H_T : tail K -factor increases from about 8.5 to 10; the pattern of subleading terms persists.

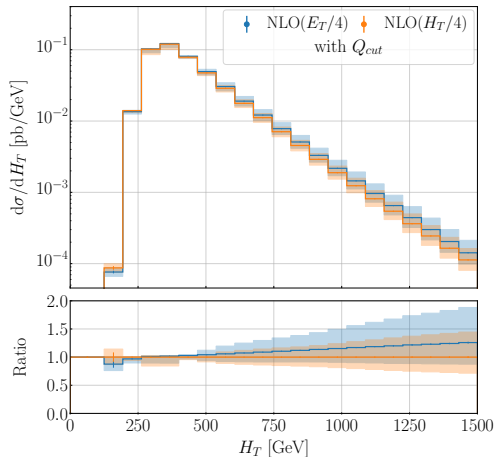
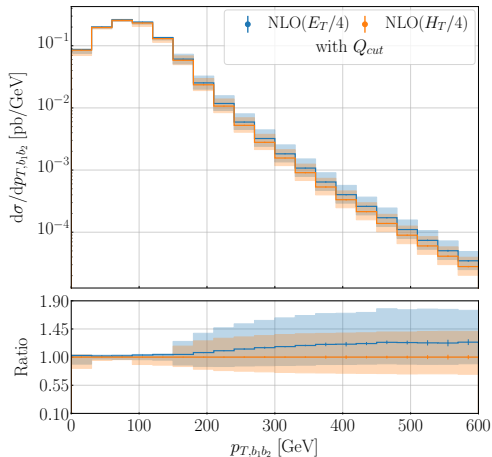
Differential Distributions: $\mu_0 = E_T/4$

Mans, Stremmer, Worek, JHEP 07 (2026) 010



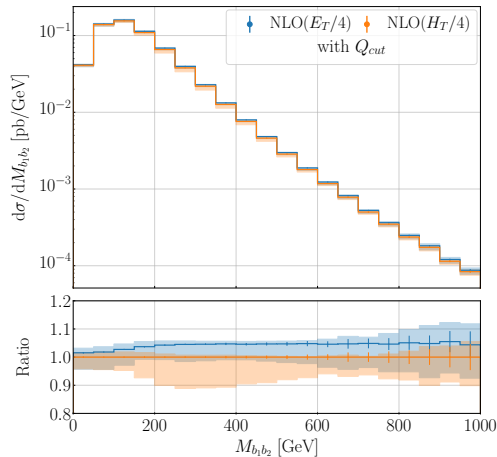
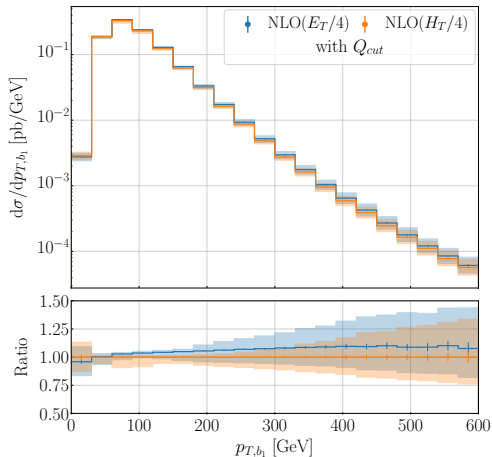
- p_{T,b_1} and $M_{b_1 b_2}$: subleading contributions change only modestly with the scale choice.
- The qualitative interplay of QCD backgrounds and electroweak corrections is unchanged.

Scale Comparison: $H_T/4$ and $E_T/4$

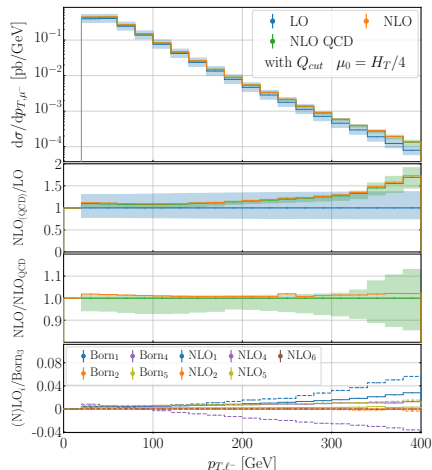
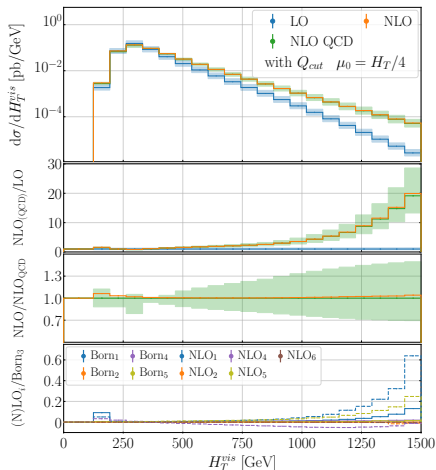


- ▶ $E_T/4$ gives harder complete-NLO spectra: up to 24% for p_{T,b_1b_2} and 26% for H_T .
- ▶ Both predictions agree within their scale-variation bands.

Scale Comparison: $H_T/4$ and $E_T/4$

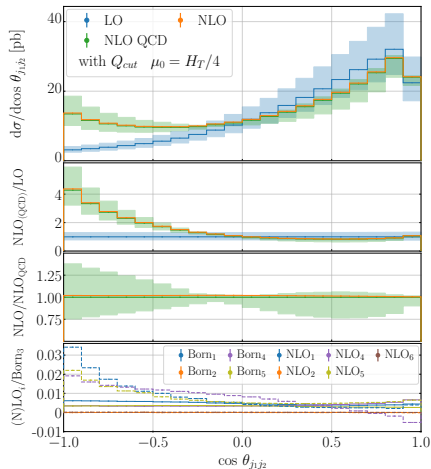
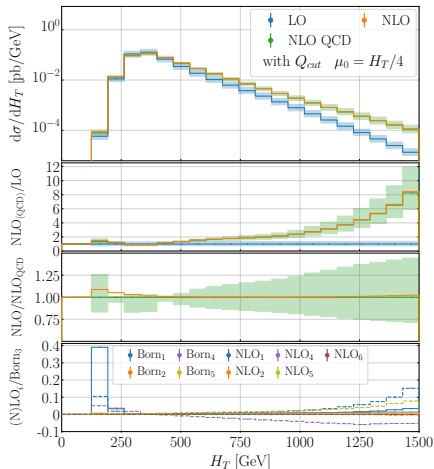


- Tail differences are about 10% for p_{T,b_1} and 5% for $M_{b_1 b_2}$.
- Both predictions agree within their scale-variation bands.



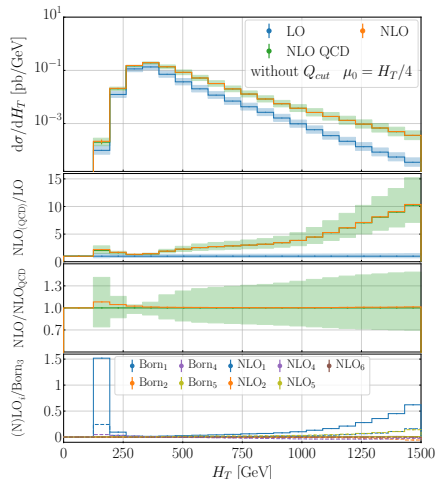
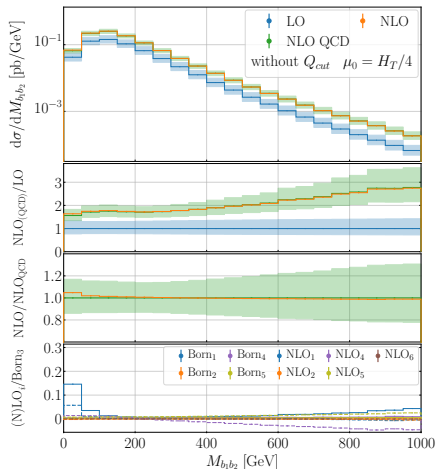
- $H_T^{\text{vis}} = p_{T,\ell} + p_{T,b_1} + p_{T,b_2} + p_{T,j_1} + p_{T,j_2}$: tail K -factor reaches about 20.
- $p_{T,\ell}$: tail K -factor about 1.7, with a 2% shift relative to NLO QCD.

Additional Distributions with Q_{cut}



- ▶ $H_T = H_T^{\text{vis}} + p_T^{\text{miss}}$: tail K -factor reaches about 8.5.
- ▶ For back-to-back light jets, $\cos \theta_{j_1 j_2} \simeq -1$, the K -factor reaches about 4.

Additional Distributions without Q_{cut}



- ▶ Removing the W -mass window enhances QCD corrections and backgrounds.
- ▶ Tail corrections relative to LO reach about +175% for $M_{b_1 b_2}$ and +940% for H_T .

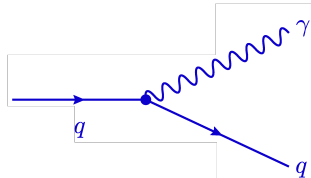
Fragmentation Function and Collinear Cancellation

$$d\hat{\sigma}_{\text{jet}+\gamma} = d\hat{\sigma}_{\text{jet}+\gamma}^{\text{demo}} - \sum_j d\hat{\sigma}_j^{\text{Born}} \otimes D_{j \rightarrow \gamma}^B.$$

$$D_{q \rightarrow \gamma}^B(z) = D_{q \rightarrow \gamma}(z, \mu_{\text{Fr}}^2) - \frac{\alpha}{2\pi} \Gamma_{q \rightarrow \gamma}^{(0)}(z, \mu_{\text{Fr}}^2) + \mathcal{O}(\alpha^2).$$

$$\Gamma_{q \rightarrow \gamma}^{(0)}(z, \mu_{\text{Fr}}^2) = -\frac{Q_q^2}{\epsilon} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{\mu_R^2}{\mu_{\text{Fr}}^2} \right)^\epsilon P_{q \rightarrow \gamma}(z).$$

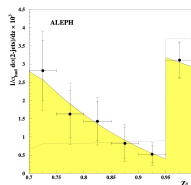
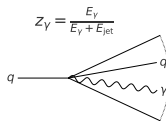
- ▶ The factorization kernel cancels the collinear singularity from real photon emission.
- ▶ The RGE evolution of $D_{q \rightarrow \gamma}$ cancels the μ_{Fr} dependence of the counterterm at this order.



Denner et al., JHEP 01 (2015) 094.

Non-Perturbative Fragmentation Input

- ▶ Finite contribution fitted to ALEPH $e^+e^- \rightarrow \text{jets} + \gamma$ data.
- ▶ Select at least one jet with a large photon energy fraction, $z_\gamma > 0.7$.
- ▶ The fit fixes the non-perturbative input at μ_0 .



$$D_{q \rightarrow \gamma}^{\text{ALEPH}, \overline{\text{MS}}}(1-z, \mu_{\text{Fr}}^2) = \frac{\alpha Q_q^2}{2\pi} \left[\frac{1+z^2}{1-z} \ln\left(\frac{\mu_{\text{Fr}}^2}{z^2 \mu_0^2}\right) + C \right],$$

$$\mu_0 = 0.14 \text{ GeV}, \quad C = -13.26.$$

Data and figures: ALEPH, Z. Phys. C 69 (1996) 365–378.

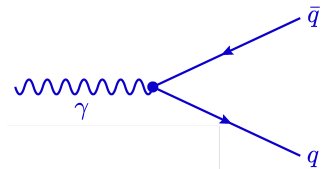
Photon-to-Jet Conversion and Collinear Cancellation

$$d\sigma_{X+\text{jet}} = d\sigma_{X+q\bar{q}}^{\text{pert}} + d\sigma_{X+\gamma}^{\text{Born}} \int_0^1 dz D_{\gamma \rightarrow \text{jet}}^B(z).$$

$$D_{\gamma \rightarrow \text{jet}}^B(z) = D_{\gamma \rightarrow \text{jet}}(z, \mu_{\text{Fr}}^2) - \frac{\alpha}{2\pi} \Gamma_{\gamma \rightarrow \text{jet}}^{(0)}(z, \mu_{\text{Fr}}^2) + \mathcal{O}(\alpha^2).$$

$$\Gamma_{\gamma \rightarrow \text{jet}}^{(0)}(z, \mu_{\text{Fr}}^2) = -\frac{N_c \sum_q Q_q^2}{\epsilon} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{\mu_R^2}{\mu_{\text{Fr}}^2} \right)^\epsilon P_{\gamma \rightarrow \text{jet}}(z).$$

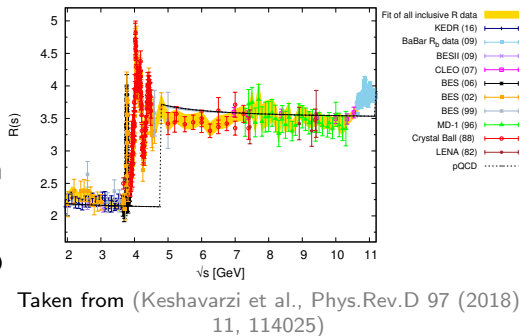
- ▶ The factorization kernel cancels the collinear $\gamma \rightarrow q\bar{q}$ singularity.
- ▶ The finite conversion contribution is constrained by hadronic vacuum-polarization data.



Denner et al., Phys. Lett. B 798 (2019) 134951.

Non-Perturbative Input to Photon-to-Jet Conversion

- ▶ Photon vacuum polarization from $q\bar{q}$ loops drives the running of $\alpha(M_Z^2)$; non-perturbative at low s .
- ▶ Use experimental $R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$ in a dispersion relation to compute $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$.
- ▶ Integrate $R(s)$: data at low s + pQCD at high $s \Rightarrow$ precise $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$.



$$\int_0^1 dz D_{\gamma \rightarrow \text{jet}}(z, \mu_{\text{Fr}}^2) = \Delta\alpha_{\text{had}}^{(5)}(M_Z^2) + \sum_q N_{c,q} \frac{Q_q^2 \alpha}{3\pi} \left[\ln \left(\frac{\mu_{\text{Fr}}^2}{M_Z^2} \right) + \frac{5}{3} \right]$$

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = (276.11 \pm 1.11) \times 10^{-4}$$