

State-of-the-art predictions for four top production with invariant-mass threshold resummation

Tommaso Saracco

based on work done with

Melissa van Beekveld, Anna Kulesza & Michele Lupattelli

[[2505.10381](#)][[26XX.XXXX](#)]

QCD@LHC 2026
Oxford

- 01 Motivation**
- 02 Resummation**
- 03 Theoretical uncertainties**
- 04 Results**

01 Motivation

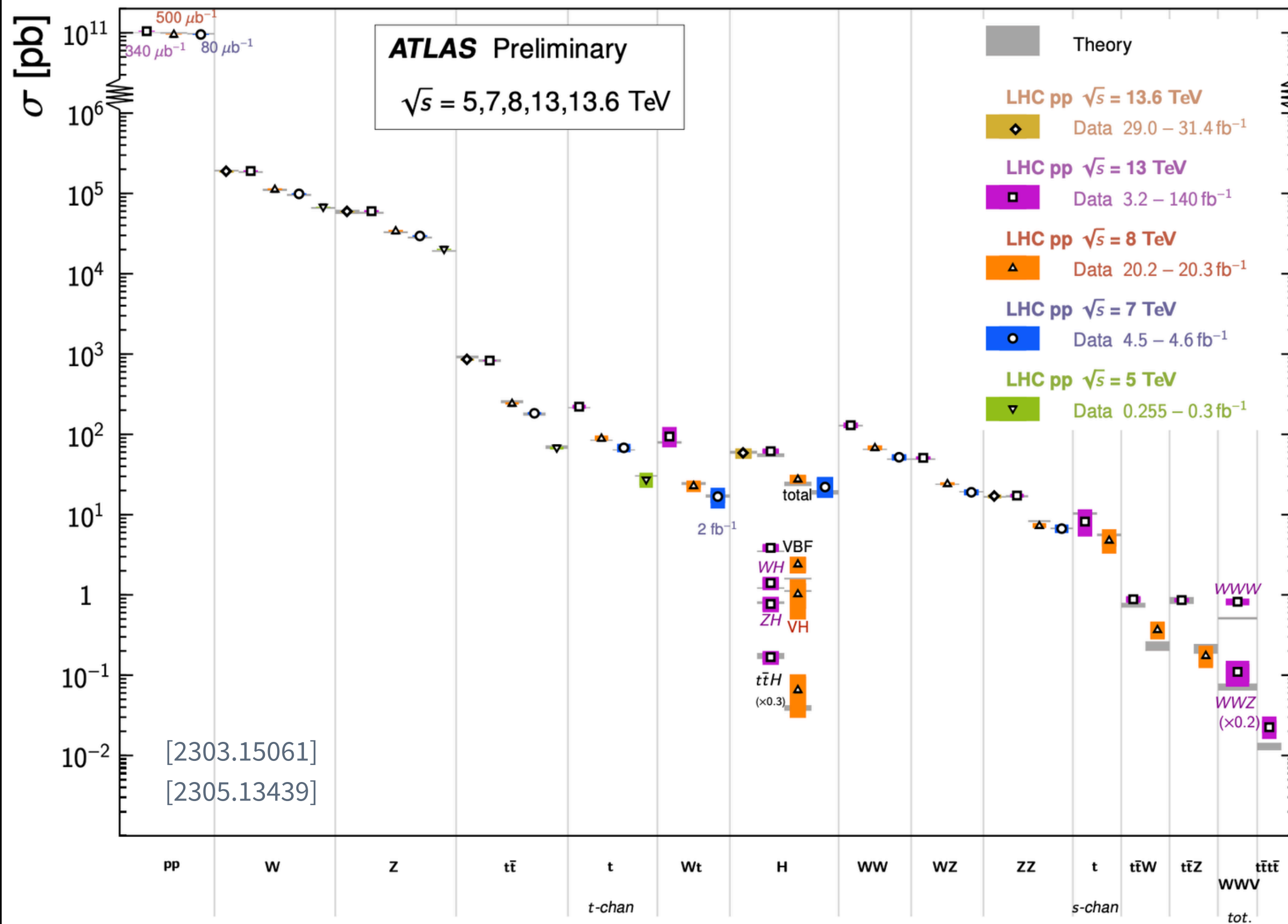
02 Resummation

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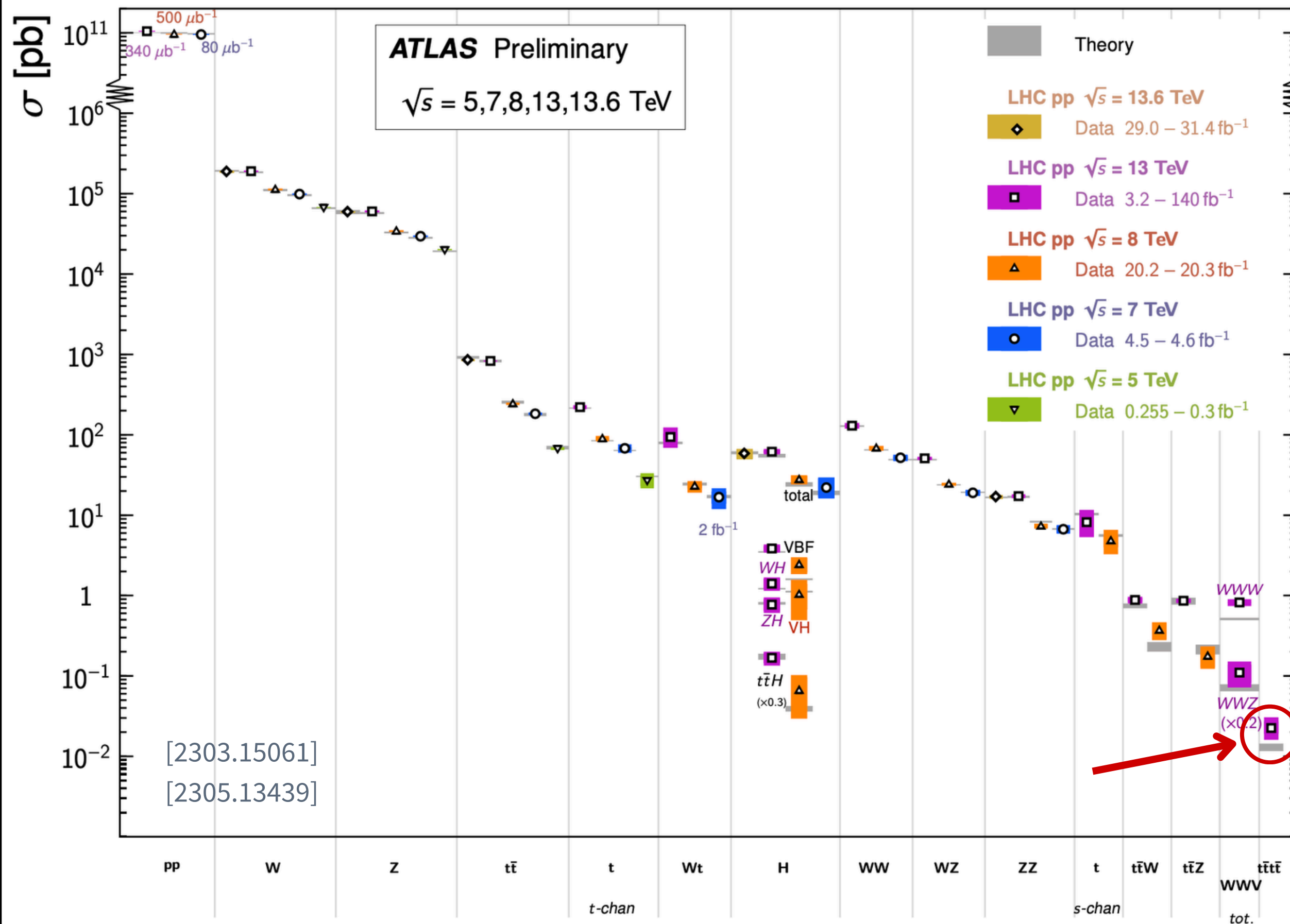
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Status: June 2024



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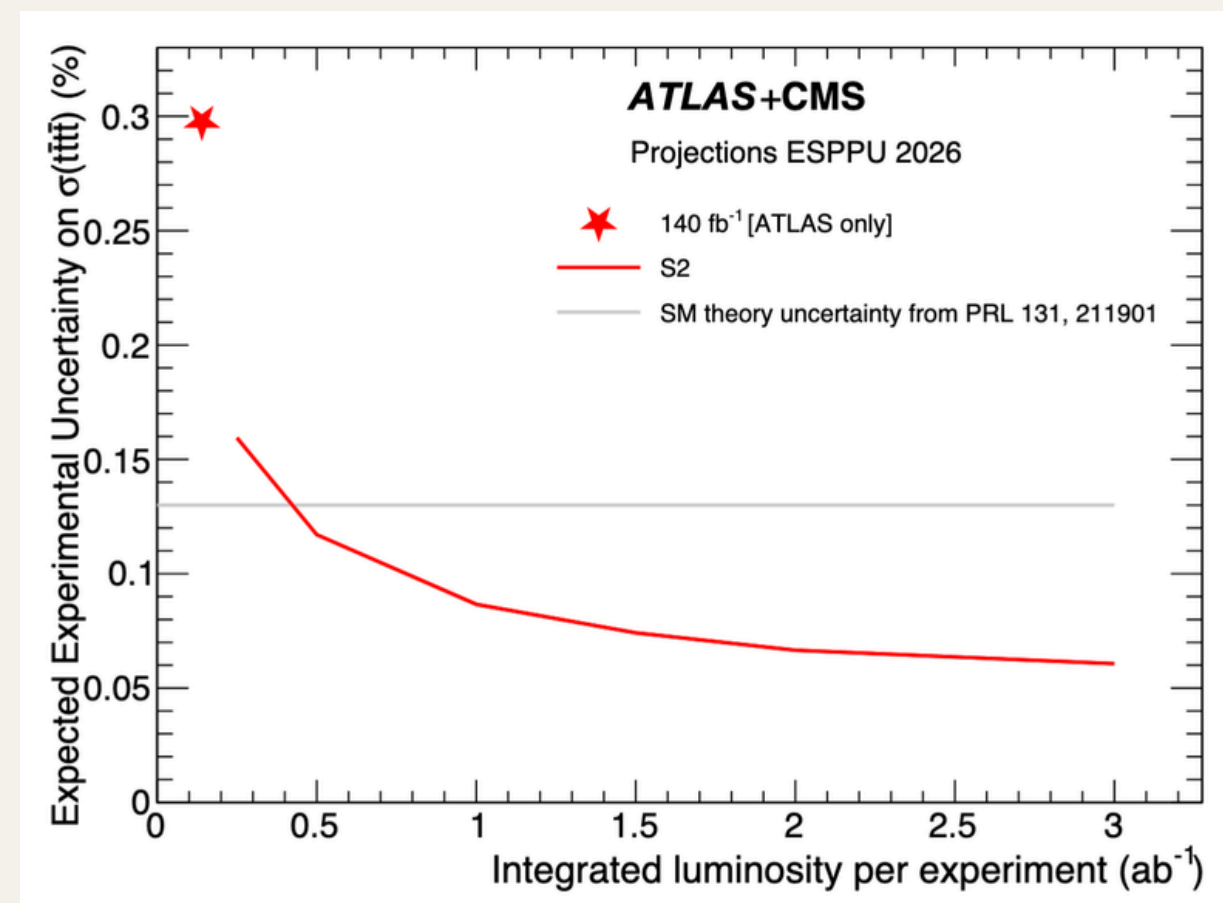


Motivation

HL-LHC

new data will decrease experimental uncertainties
and will be sensible to distributions

[2504.00672]



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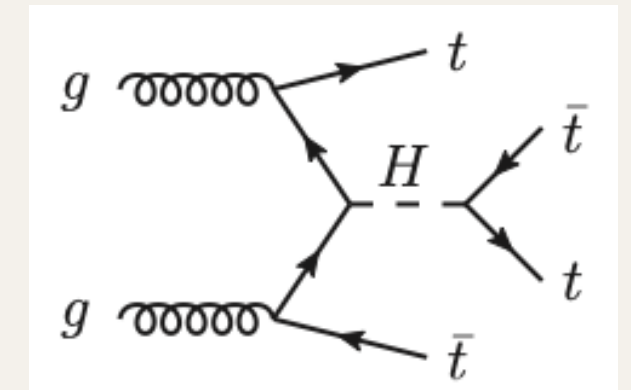
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strong sensitivity to the Yukawa coupling
+ constrain Higgs width

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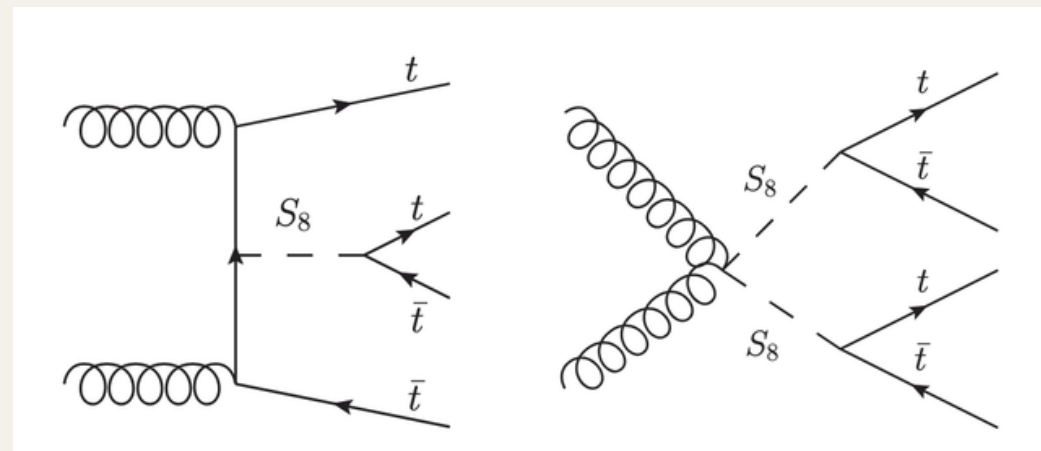
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EFT CONSTRAINTS

constrain four-heavy fermion operators

e.g. [1708.05928] [1901.05965] [2105.00006] [2208.04962], ...

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Resummation

Perturbative expansion in powers of α_s

$$\frac{d\sigma}{dQ} = c^{(0)} + \alpha_s c^{(1)} + \alpha_s^2 c^{(2)} + \dots$$

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→ Full NLO QCD+EW result: [1711.02116]

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NNLO currently out of reach

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some ratio of scales

if $\log(\star) \sim \frac{1}{\alpha_s} \implies$ the expansion in α_s becomes meaningless

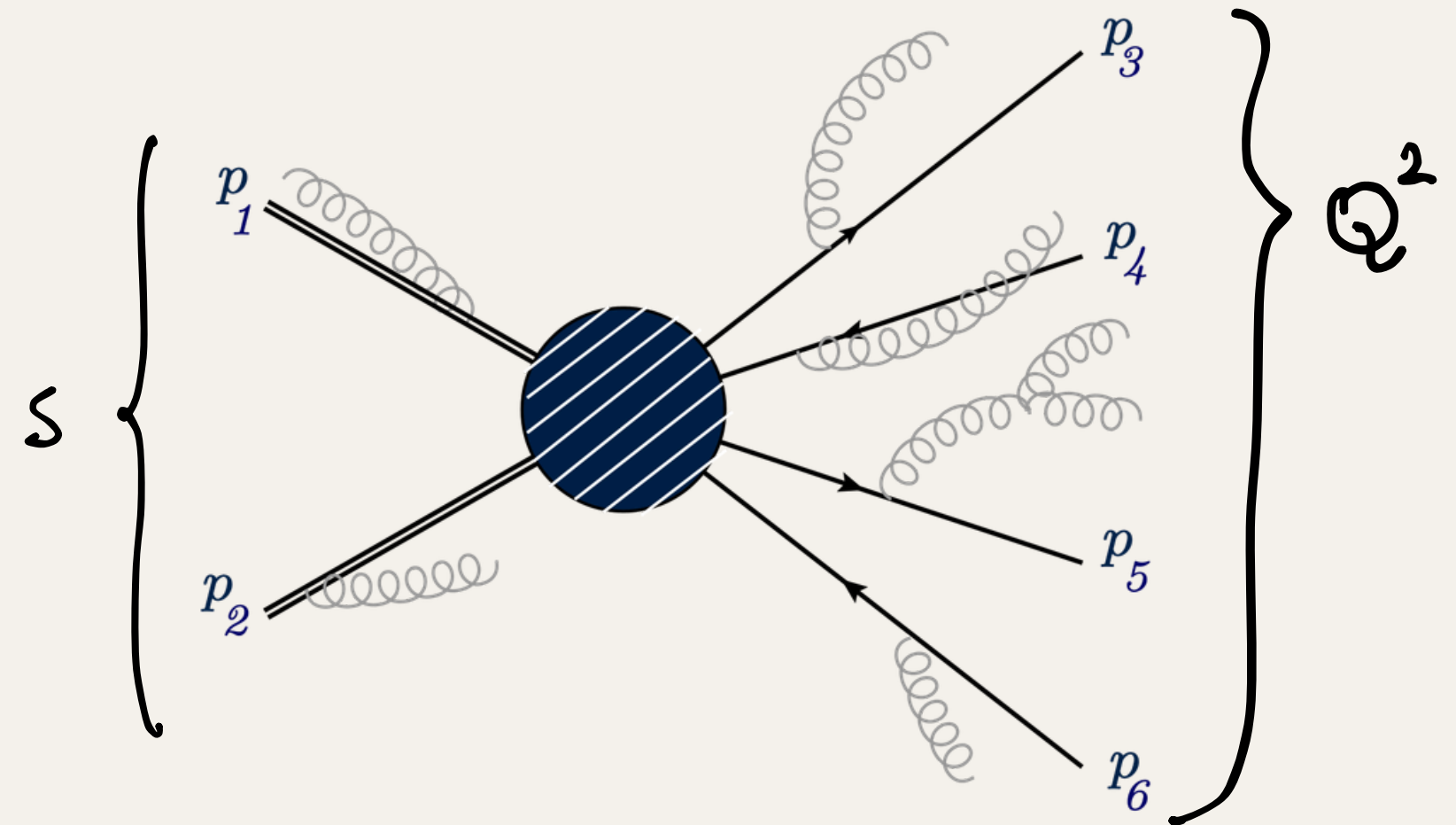
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Large Logarithms

The logarithms become large when all the emitted gluons are soft: $\hat{\rho} \equiv Q^2/s \rightarrow 1$

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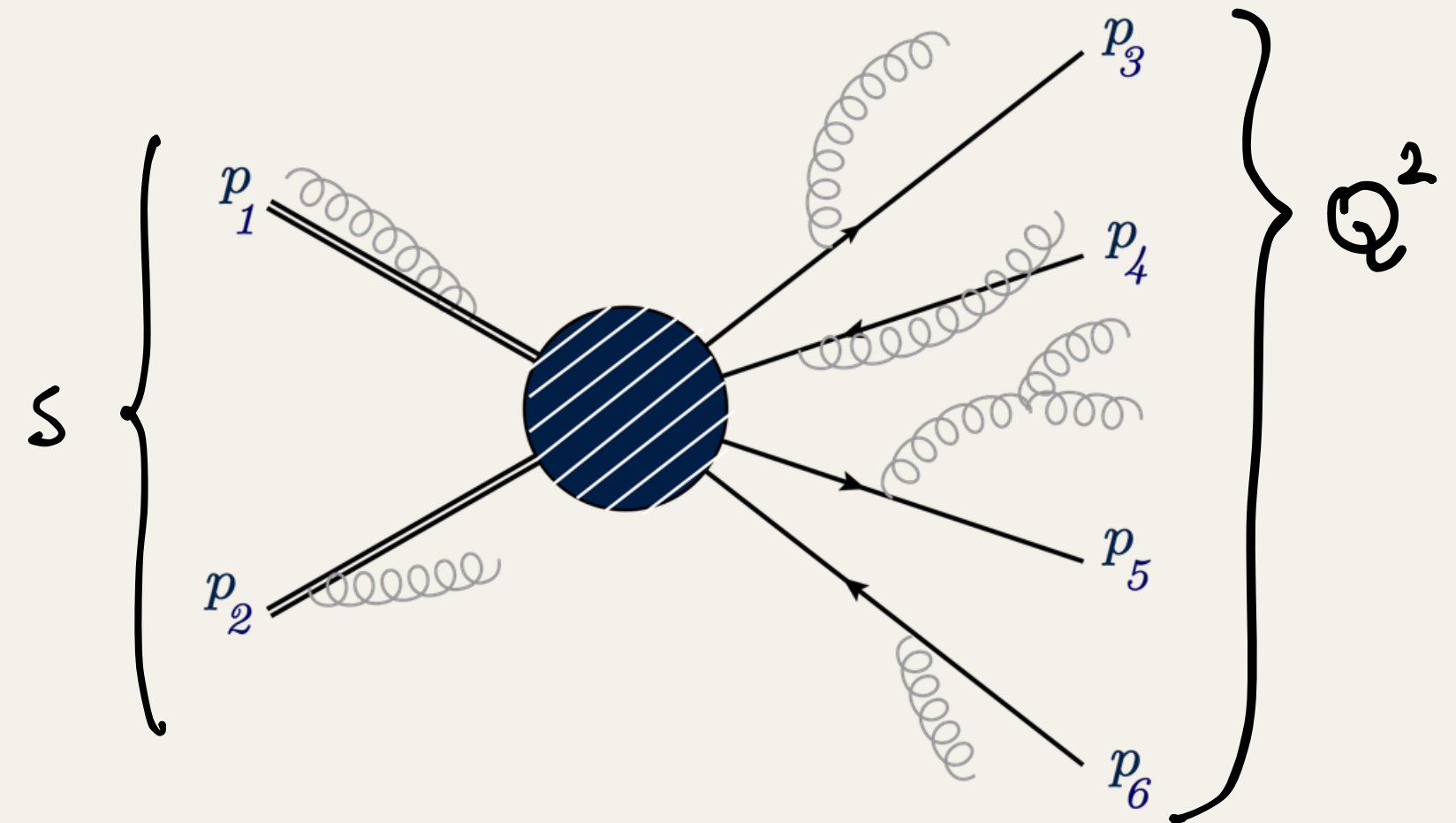
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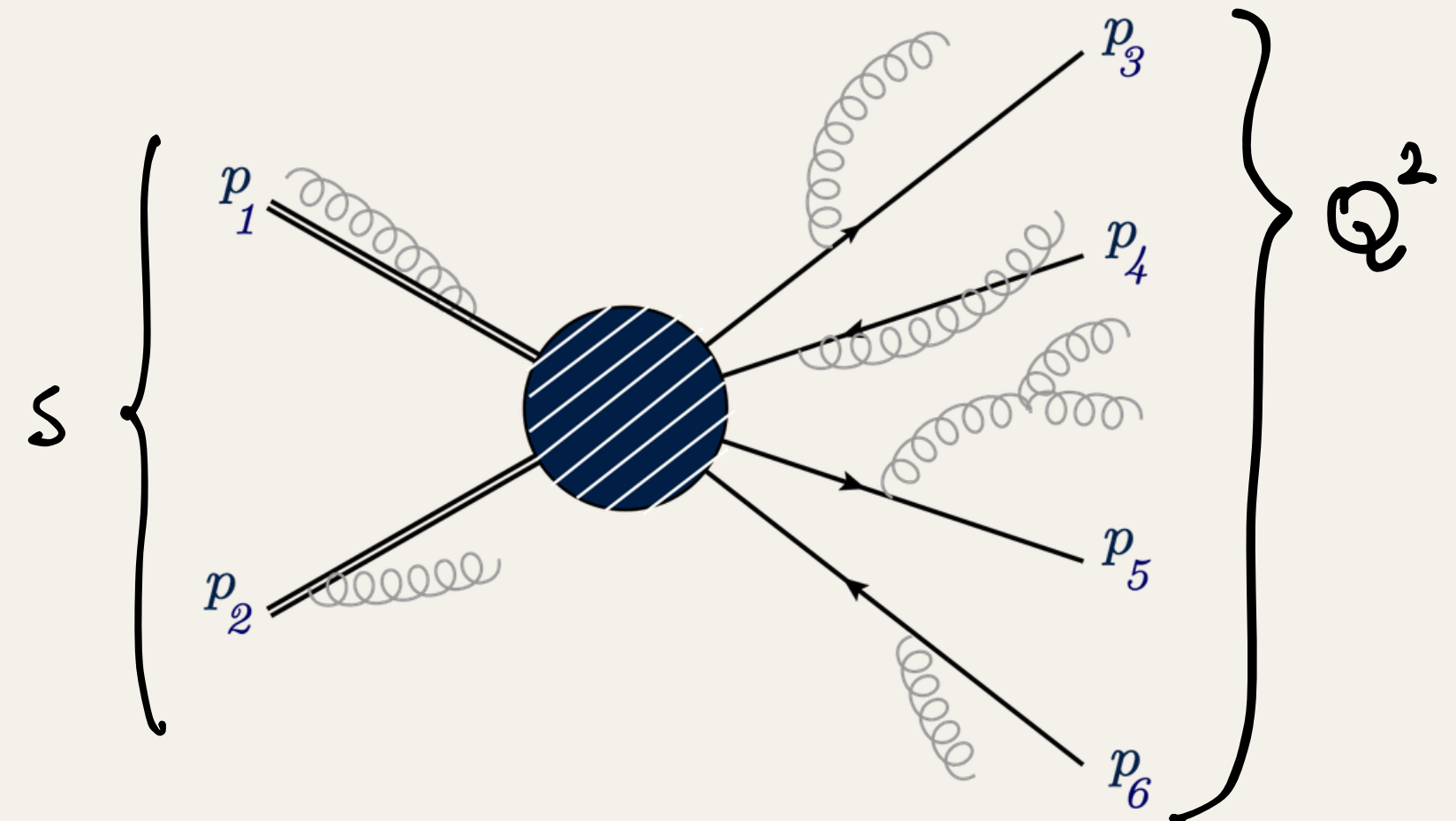
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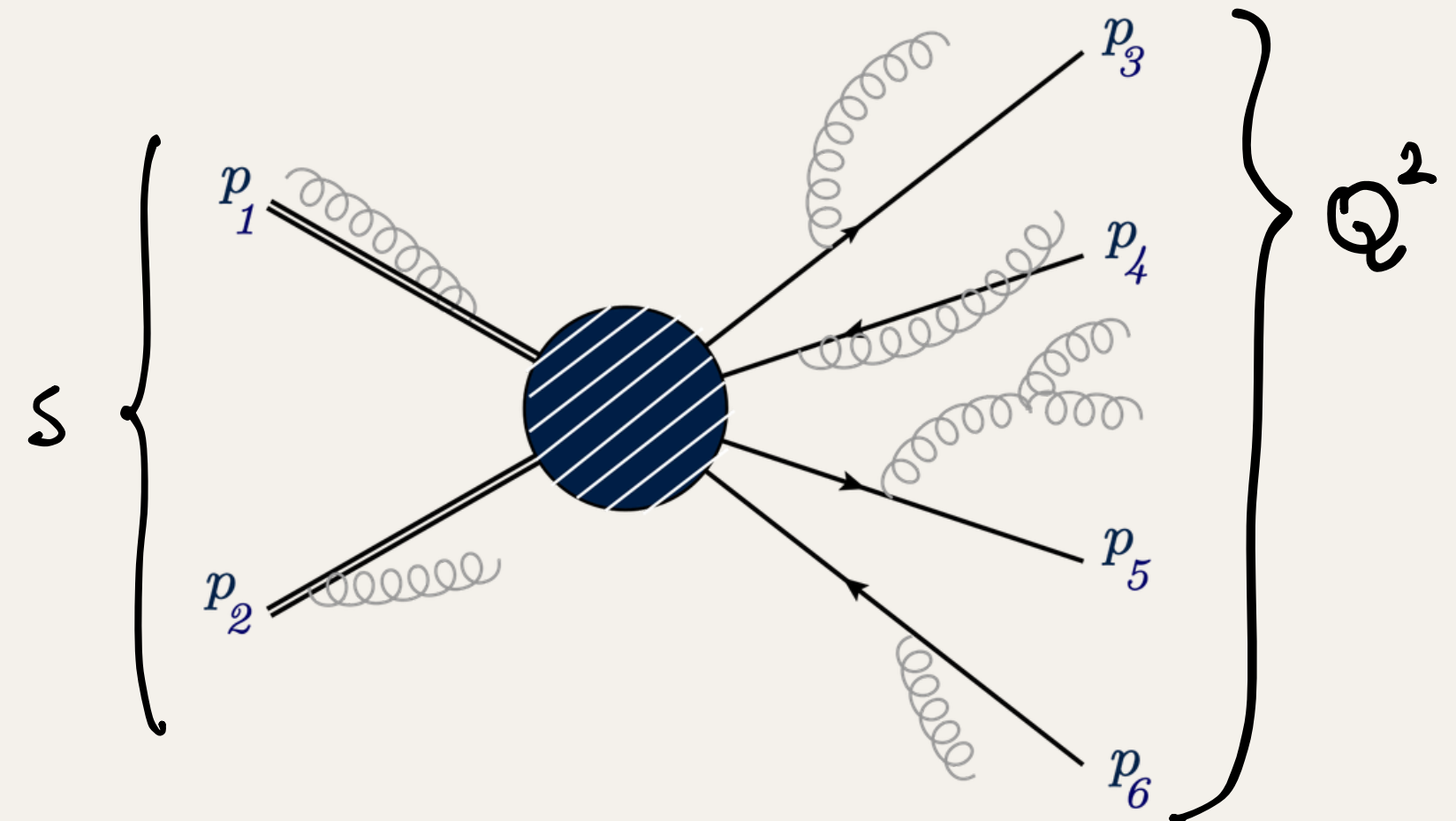
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In this limit, the soft and collinear contributions can be **factorised** in the cross section

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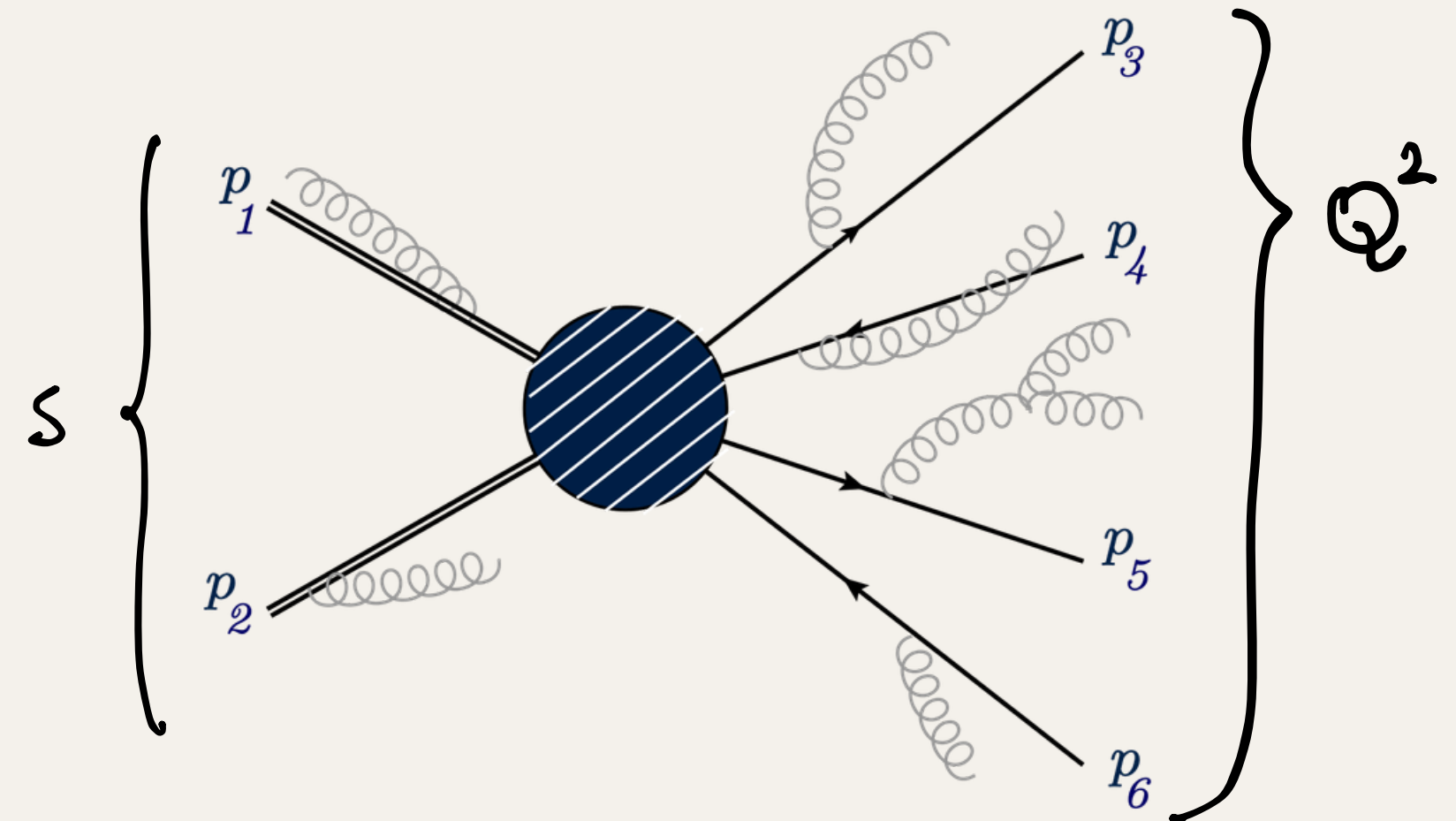
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In this limit, the soft and collinear contributions can be **factorised** in the cross section

these logarithms can be **resummed to all orders**

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Resummation

Mellin transform

$$\hat{\rho} \equiv Q^2/s$$

$$\rho \equiv Q^2/S$$

To achieve the factorisation of the phase space, we work in Mellin space:

$$\frac{d\tilde{\sigma}_{pp \rightarrow t\bar{t}t\bar{t}}(N)}{dQ} = \int_0^1 d\rho \rho^{N-1} \frac{d\sigma_{pp \rightarrow t\bar{t}t\bar{t}}(\rho)}{dQ}$$

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The Mellin transform maps the threshold logarithms into logarithms of the Mellin moment N

$$\left[\frac{\log^m(1 - \hat{\rho})}{1 - \hat{\rho}} \right]_+ \longmapsto \sum_{k=0}^{m+1} a_{km} (\log N + \gamma_E)^k + \mathcal{O}\left(\frac{1}{N}\right)$$

and the threshold limit becomes $\hat{\rho} \rightarrow 1 \quad \longmapsto \quad N \rightarrow \infty$

Resummation

Resummed cross section

In Mellin space we get the factorised cross section:

$$\frac{d\tilde{\sigma}_{pp \rightarrow t\bar{t}t\bar{t}}^{\text{NNLL}}(N)}{dQ} = \sum_{i,j} \tilde{f}_i(N+1) \tilde{f}_j(N+1) \frac{d\tilde{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{NNLL}}(N)}{dQ}$$

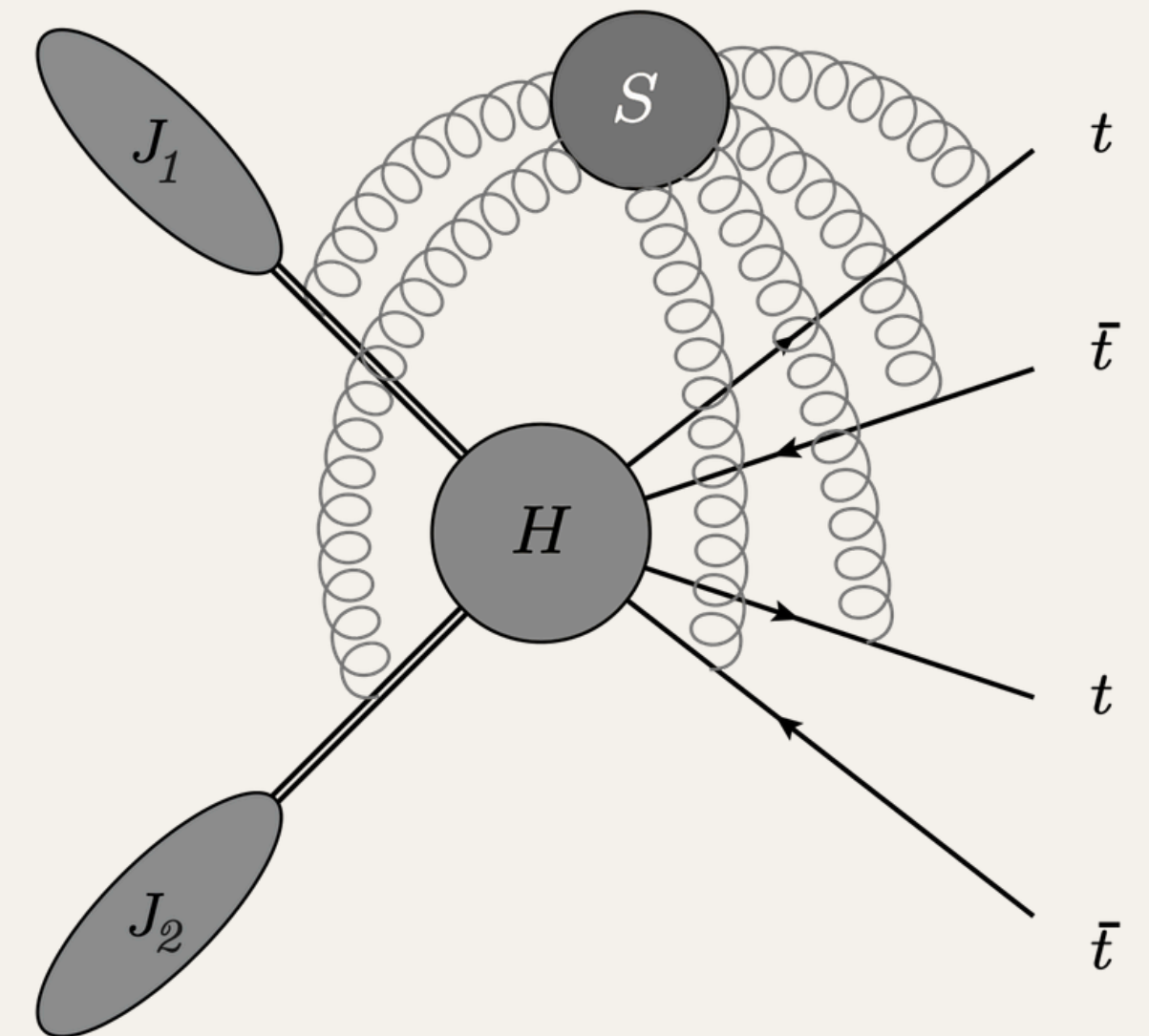
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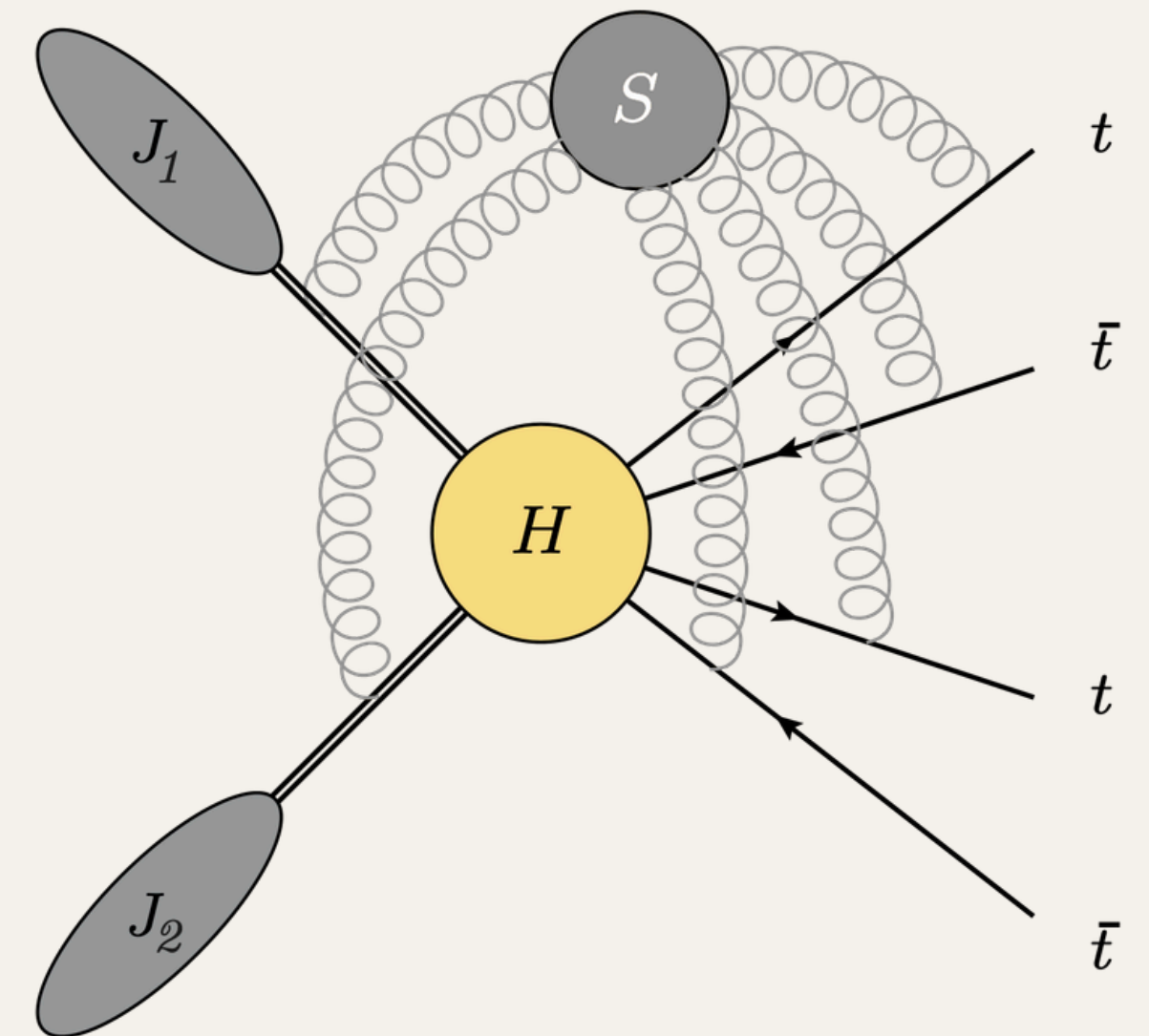
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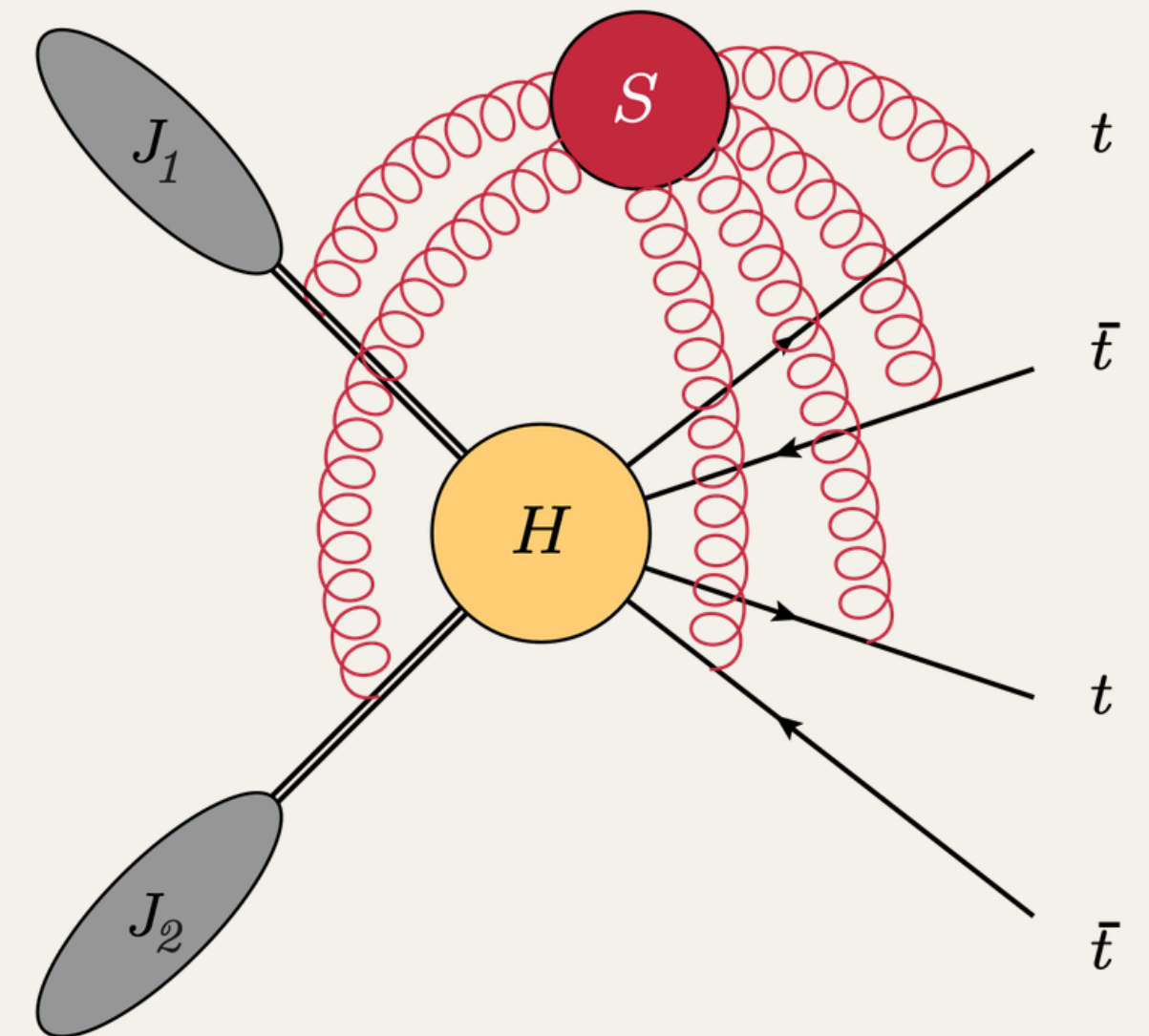
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Soft function:

wide-angle soft emissions

[9705234] [9604313] [9803241]



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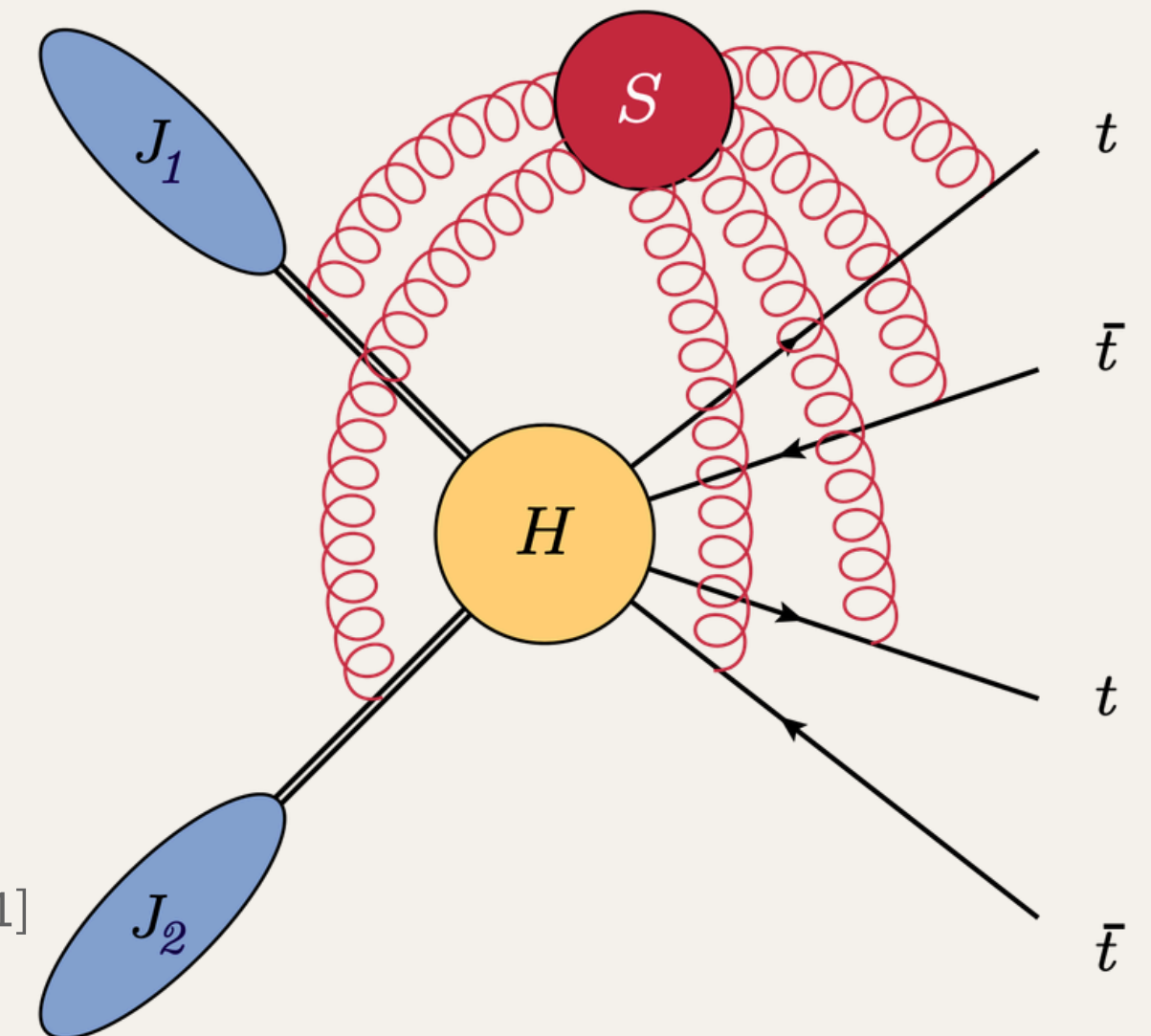
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[9705234] [9604313] [9803241]

Jet functions:

collinear and soft-collinear
enhancements see e.g. [0306211]



Resummation

Matching to fixed-order

At the end, we need to go back to momentum space **inverting the Mellin transform**
(numerically, using Minimal Prescription) [9604351]

$$\frac{d\sigma_{pp \rightarrow t\bar{t}t\bar{t}}^{\text{NNLL}}(\rho)}{dQ} = \sum_{i,j} \int_{\mathcal{C}_{\text{MP}}} \frac{dN}{2\pi i} \rho^{-N} \tilde{f}_i(N+1) \tilde{f}_j(N+1) \frac{d\tilde{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{NNLL}}(N)}{dQ}$$

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We then match to NLO using **additive matching**:

$$d\sigma^{\text{NLO+NNLL}} = d\sigma^{\text{NLO}} + \left[d\sigma^{\text{NNLL}} - d\sigma^{\text{NNLL}}|_{\text{NLO}} \right].$$

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Theoretical uncertainties

1. Scale uncertainty:

we use seven-point scale variation, we vary the renormalisation and factorisation scale as

$$\left(\frac{\mu_R}{\mu_0}, \frac{\mu_F}{\mu_0} \right) \in \{ (0.5, 0.5), (0.5, 1), (1, 0.5), (1, 1), (1, 2), (2, 1), (2, 2) \}.$$

Theoretical uncertainties

2. PDF uncertainty:

we use PDF4LHC21_40_pdfas, and the uncertainty is computed with the symmhessian method

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4. top-quark mass variation:

vary from the central value $m_t = 172.5 \text{ GeV}$ by $\pm 1 \text{ GeV}$

Theoretical uncertainties

5. Terms of order $\mathcal{O}(\alpha_s^2)$ formally beyond NNLL:

When we compute

$$\tilde{\sigma}^{\text{res}} = \text{Tr} [\mathbf{H} \mathbf{S}] \Delta_i \Delta_j$$

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→ we consider the addition of these terms as part of our theoretical uncertainty

Theoretical uncertainties

6. Choice on resumming N or $\bar{N}$

When we take the Mellin transform of a plus distribution we get logarithms of N

$$\left[\frac{\log^m(1 - \hat{\rho})}{1 - \hat{\rho}} \right]_+ \longrightarrow \sum_{k=0}^{m+1} a_{km} (\log N + \gamma_E)^k + \mathcal{O}\left(\frac{1}{N}\right)$$

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From this, a possibility is to *rearrange* the resummation in terms of $\bar{N} \equiv N e^{\gamma_E}$ instead of N

This helps capturing subleading logarithmic contributions

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→ this is a *choice* in our resummation procedure, so we account for it as part of our uncertainty

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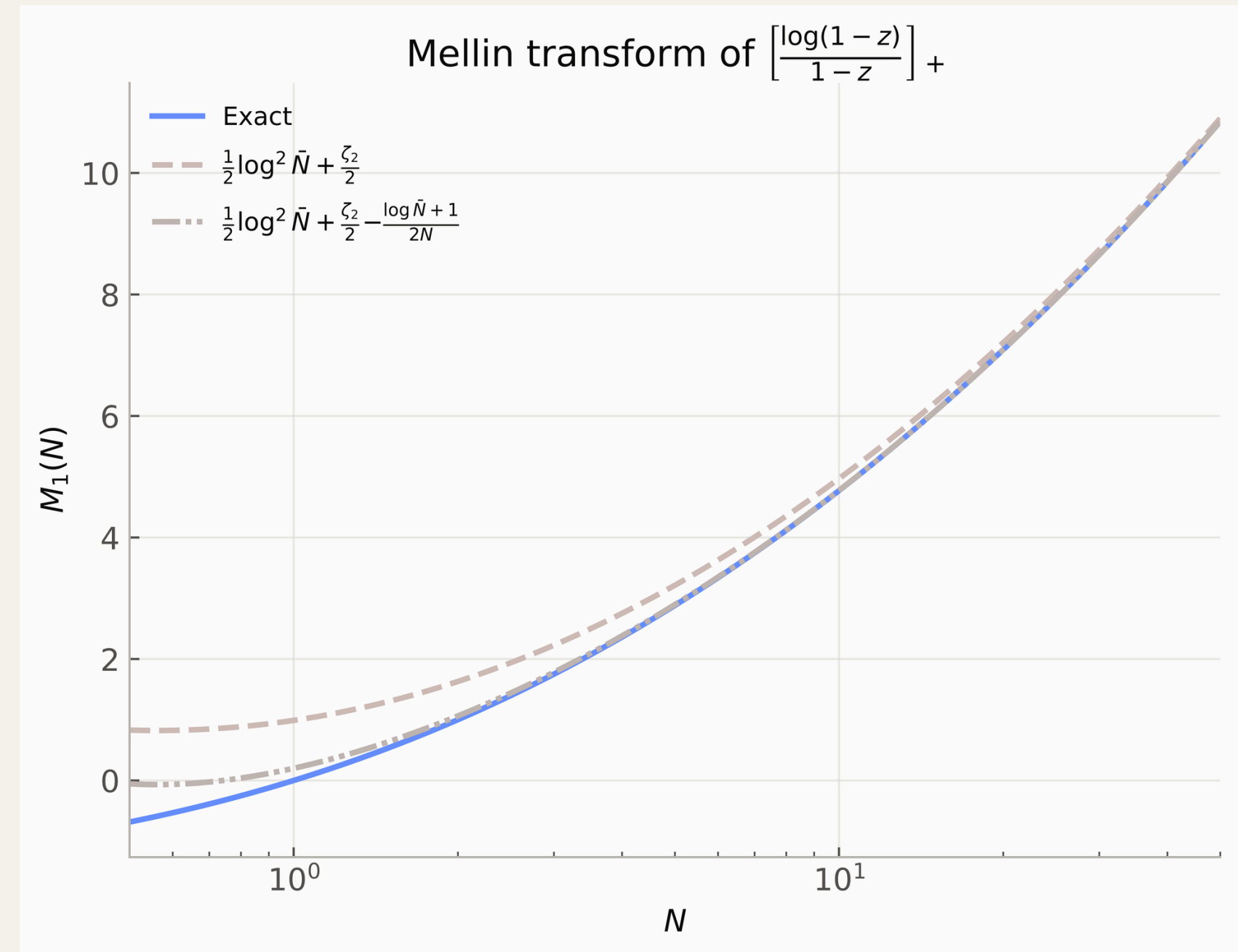
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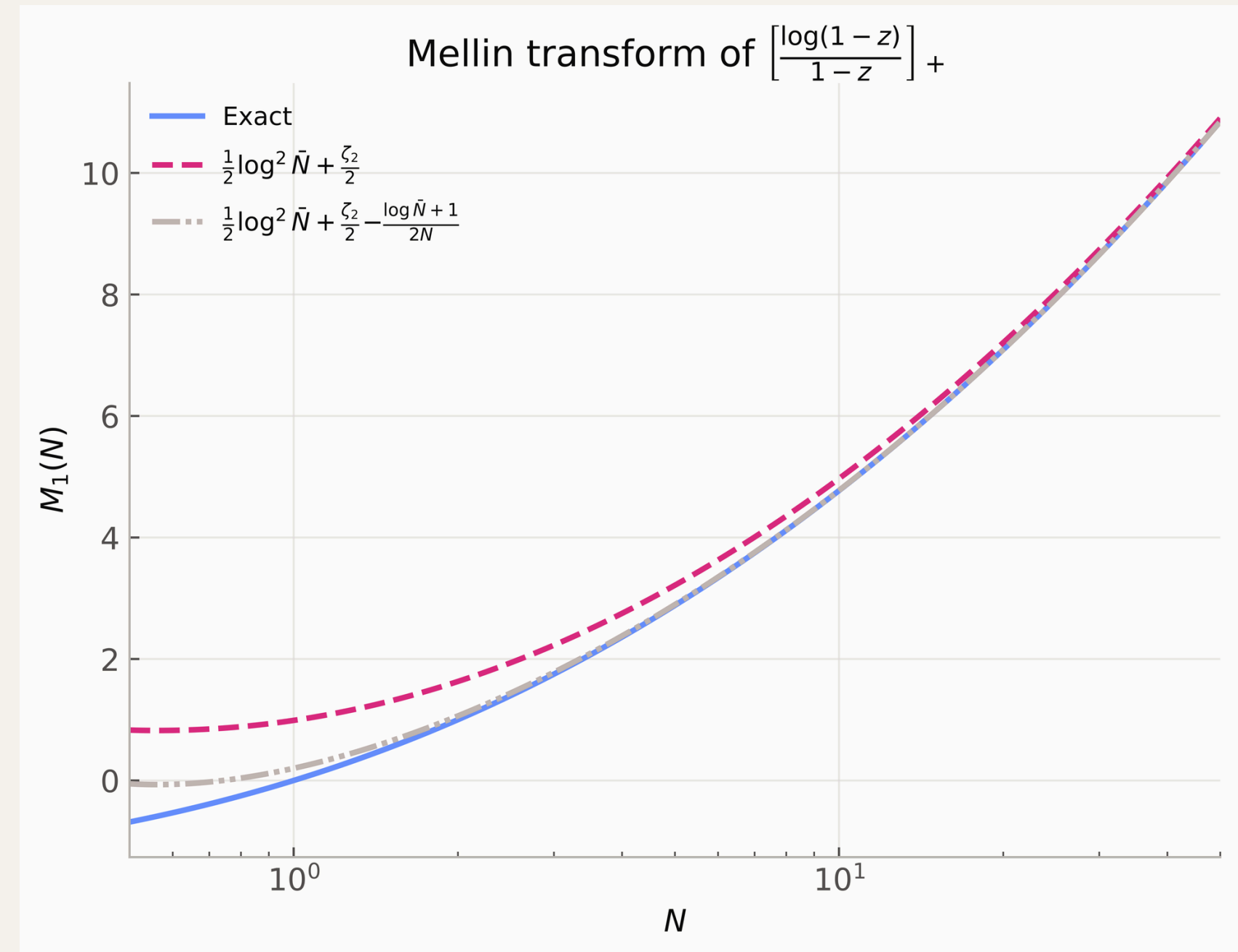
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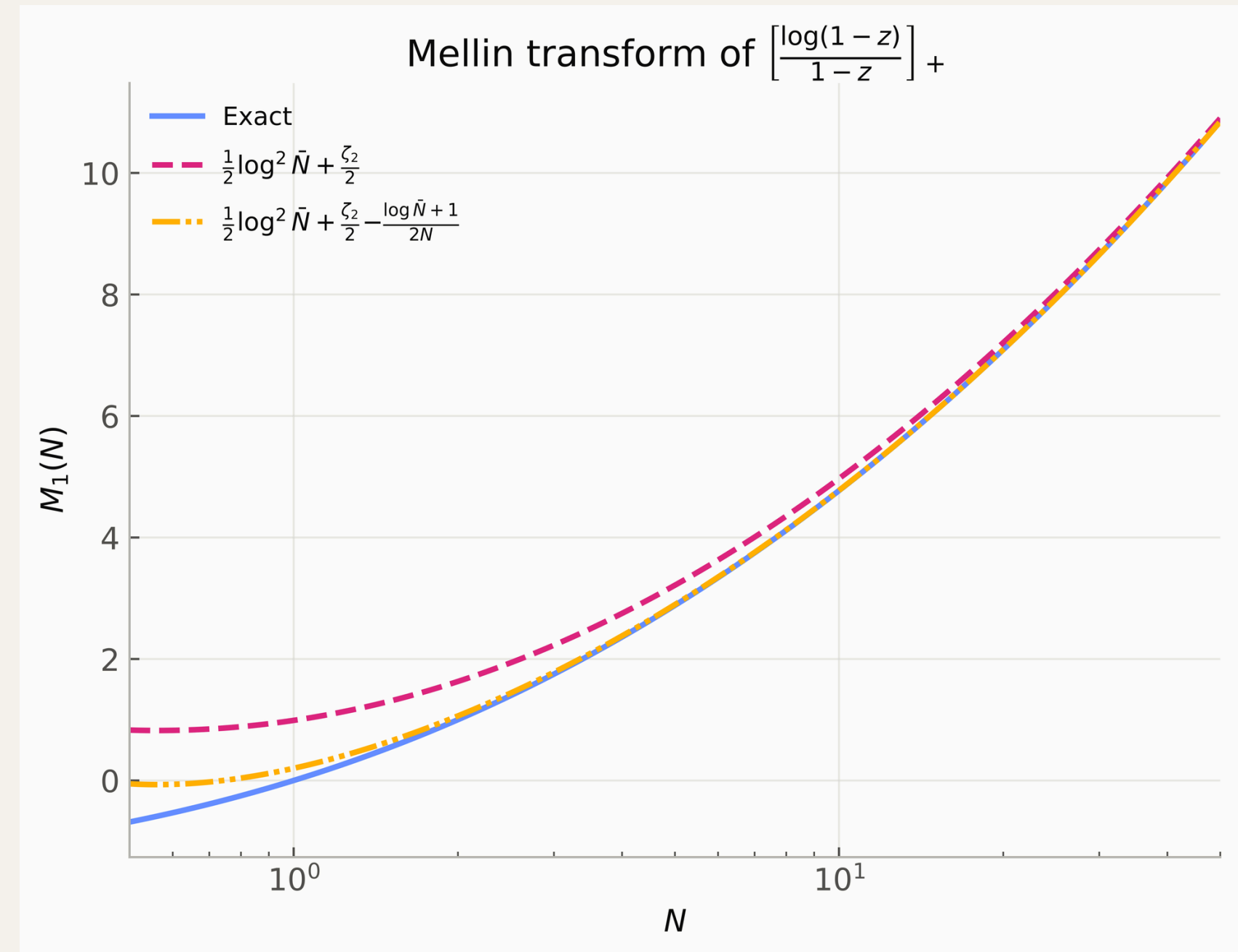
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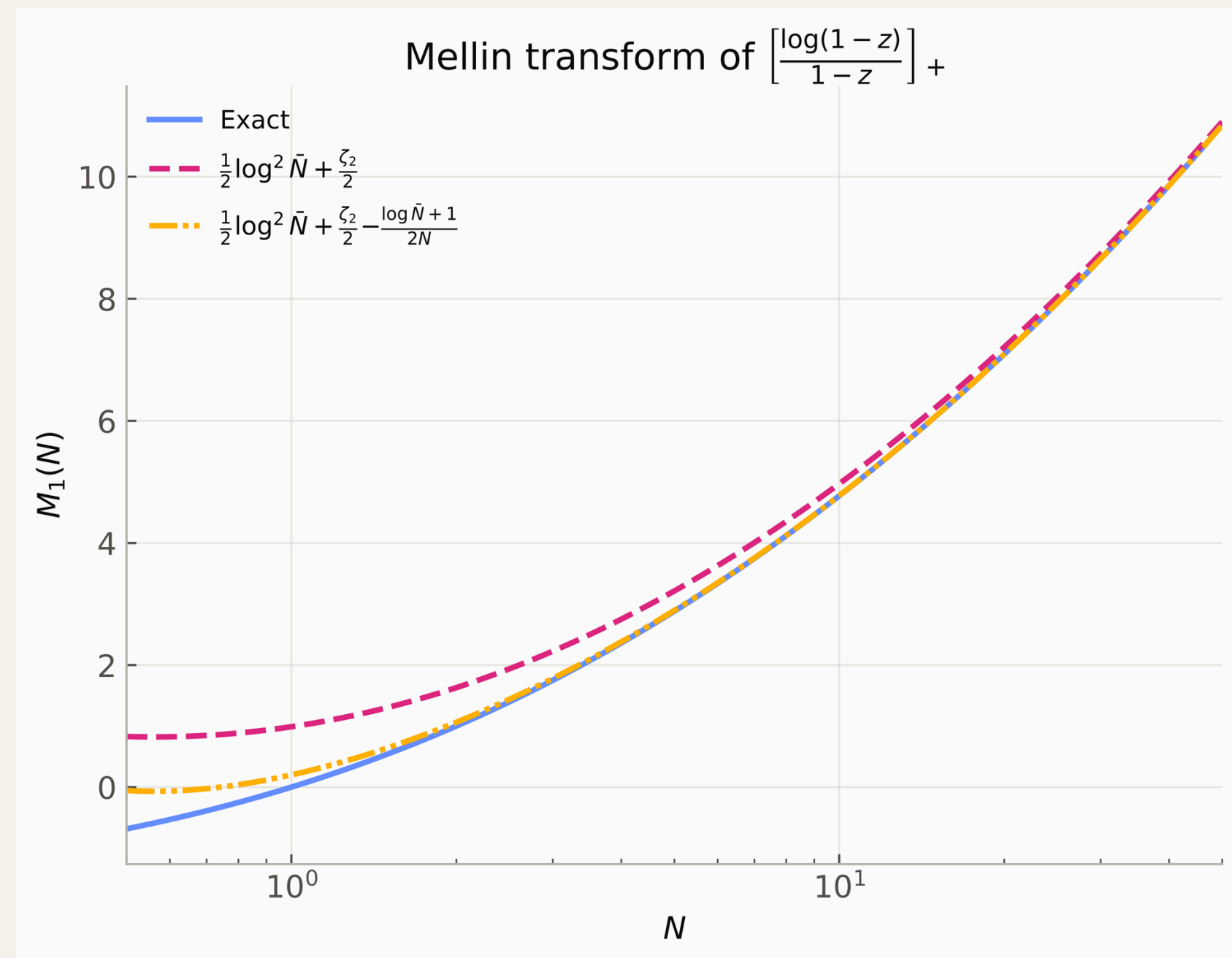
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LL



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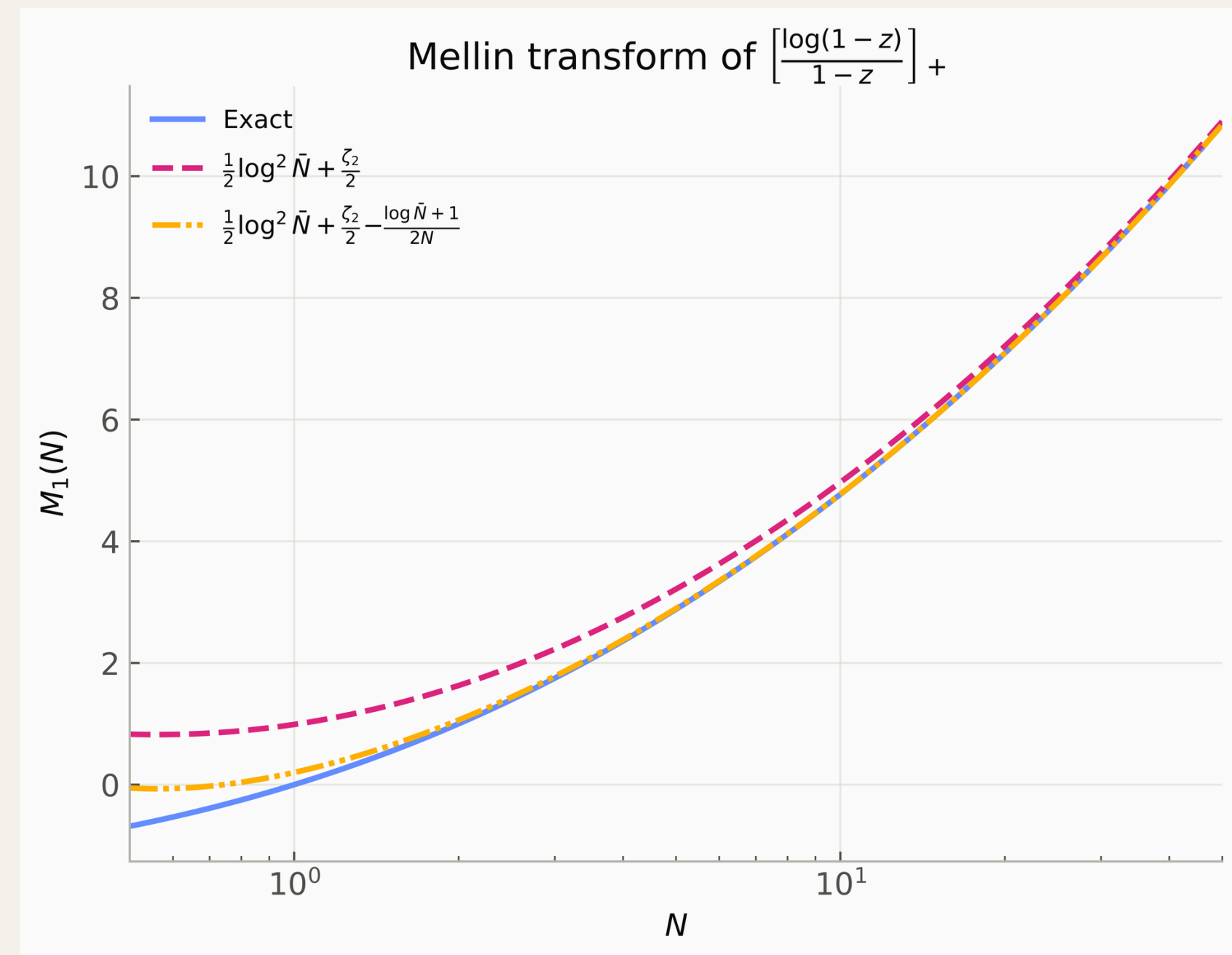
7a. When we take the Mellin transform we typically neglect $\mathcal{O}(1/N)$ terms, as they are subleading in the threshold limit

$$\left[\frac{\log^m(1 - \hat{\rho})}{1 - \hat{\rho}} \right]_+ \longrightarrow \sum_{k=0}^{m+1} a_{km} (\log N + \gamma_E)^k + \mathcal{O}\left(\frac{1}{N}\right)$$

E.g the Mellin transform for the distribution with $m=1$:

$$\left[\frac{\log(1 - \hat{\rho})}{1 - \hat{\rho}} \right]_+ \longrightarrow \boxed{\frac{1}{2} \log^2 \bar{N} + \frac{1}{2} \zeta_2 - \frac{1}{2N} \log \bar{N} - \frac{1}{2N}}$$

LL
LL_{1/N}



Theoretical uncertainties

7. Impact of power-suppressed $\mathcal{O}(1/N)$ terms

7b. Also, we can include approximate-NNLO corrections by analytically inverting the Mellin transform of the NNLO expansion of the resummed cross section,

$$d\sigma^{\text{aNNLO}} = \int_c \frac{dN}{2\pi i} \rho^{-N} d\tilde{\sigma}^{\text{NNLL}} \big|_{\mathcal{O}(\alpha_s^2)\text{-term only}}$$

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$$P_k(z) \equiv \left[\frac{\log^k(1-z)}{1-z} \right]_+ \quad M_k(N) \equiv \int_0^1 dz z^{N-1} P_k(z) .$$

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$$\begin{aligned} 1 &\longleftrightarrow \delta(1-z) , \\ \log \bar{N} &\longleftrightarrow -P_0(z) , \\ \log^2 \bar{N} &\longleftrightarrow 2P_1(z) - \zeta_2 \delta(1-z) , \\ \log^3 \bar{N} &\longleftrightarrow -3P_2(z) + 3\zeta_2 P_0(z) - 2\zeta_3 \delta(1-z) , \\ \log^4 \bar{N} &\longleftrightarrow 4P_3(z) - 12\zeta_2 P_1(z) + 8\zeta_3 P_0(z) + 3(\zeta_2^2 - 2\zeta_4) \delta(1-z) . \end{aligned}$$

Theoretical uncertainties

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matching for the central prediction was:

$$d\sigma^{\text{NLO+NNLL}} = d\sigma^{\text{NLO}} + \left[d\sigma^{\text{NNLL}} - d\sigma^{\text{NNLL}}|_{\text{NLO}} \right].$$

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We now match to include power-suppressed terms

$$d\sigma^{\text{NLO+NNLL+power-suppr.}} = d\sigma^{\text{NLO}} + d\sigma^{\text{aNNLO}} + \left[d\sigma^{\text{NNLL+LL}_{1/N}} - d\sigma^{\text{NNLL+LL}_{1/N}} \big|_{\text{NNLO}} \right].$$

01 Motivation

02 Resummation

03 Theoretical uncertainties

04 Results

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We get results for the invariant-mass distribution and the total cross section,
for a centre of mass energy of 13.6 TeV

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Results

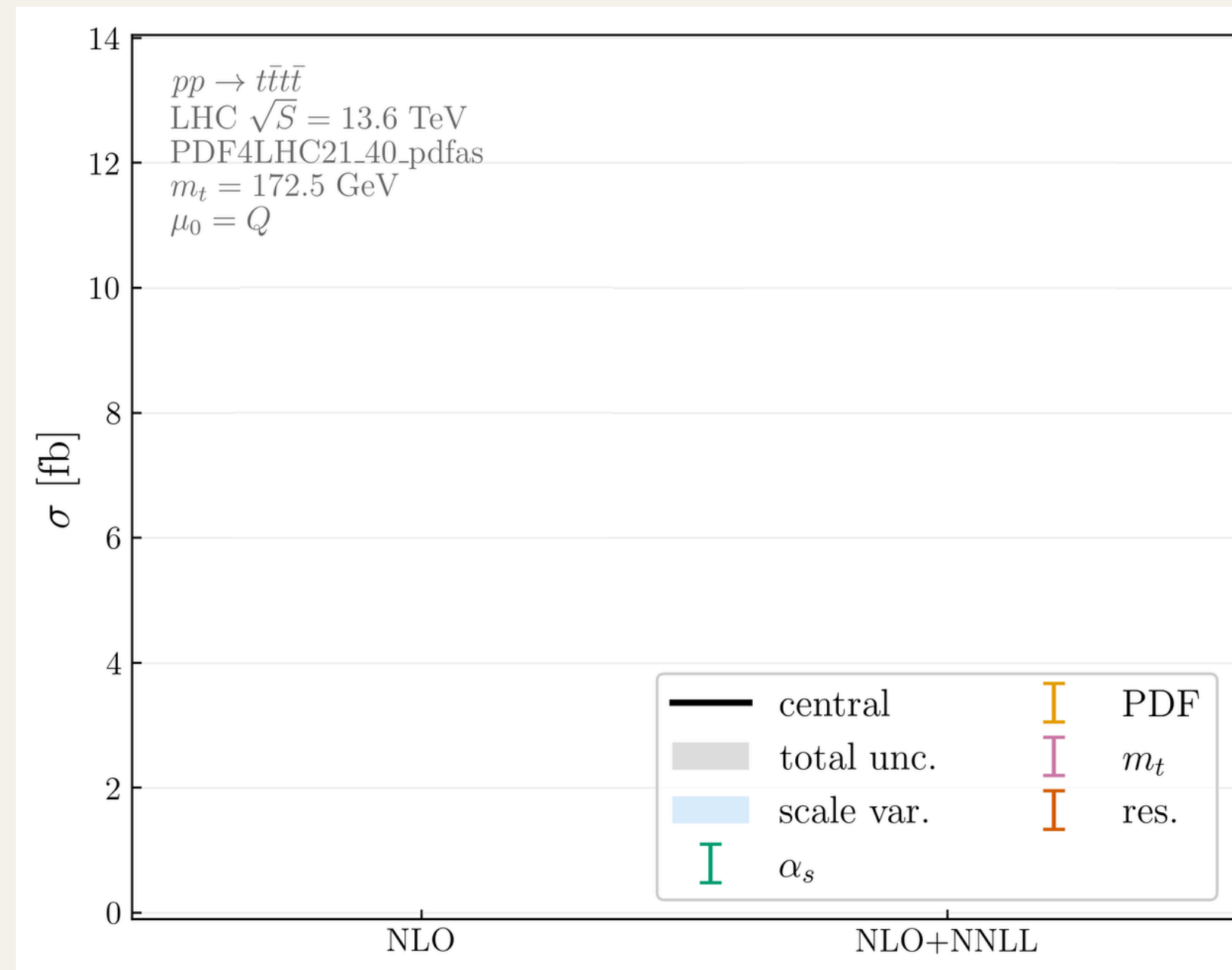
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We choose the central scale as $\mu_0 = Q$

Results

Total cross section

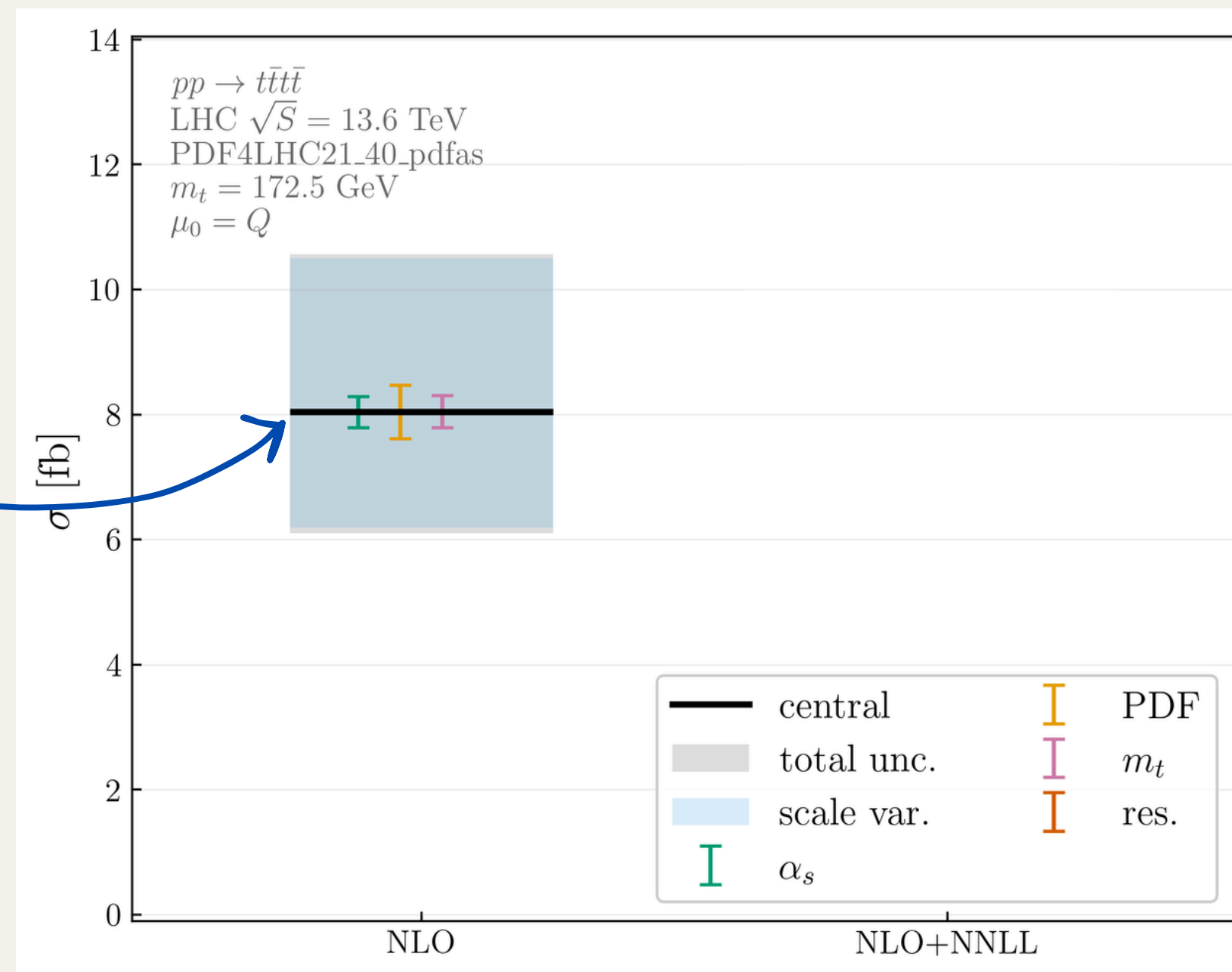


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result @ NLO
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[1405.0301] [1804.10017]



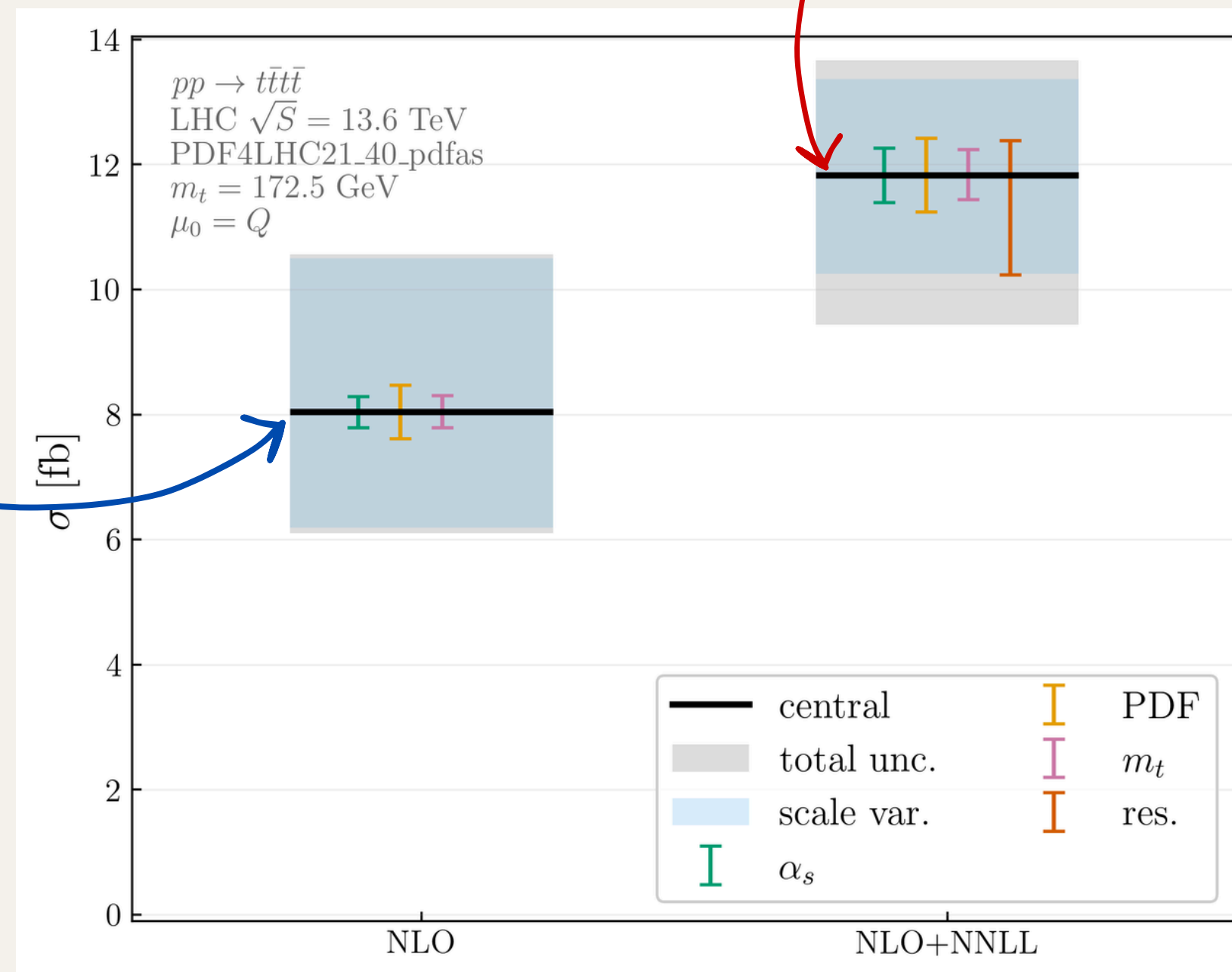
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Resummed result @
NLO+NNLL accuracy

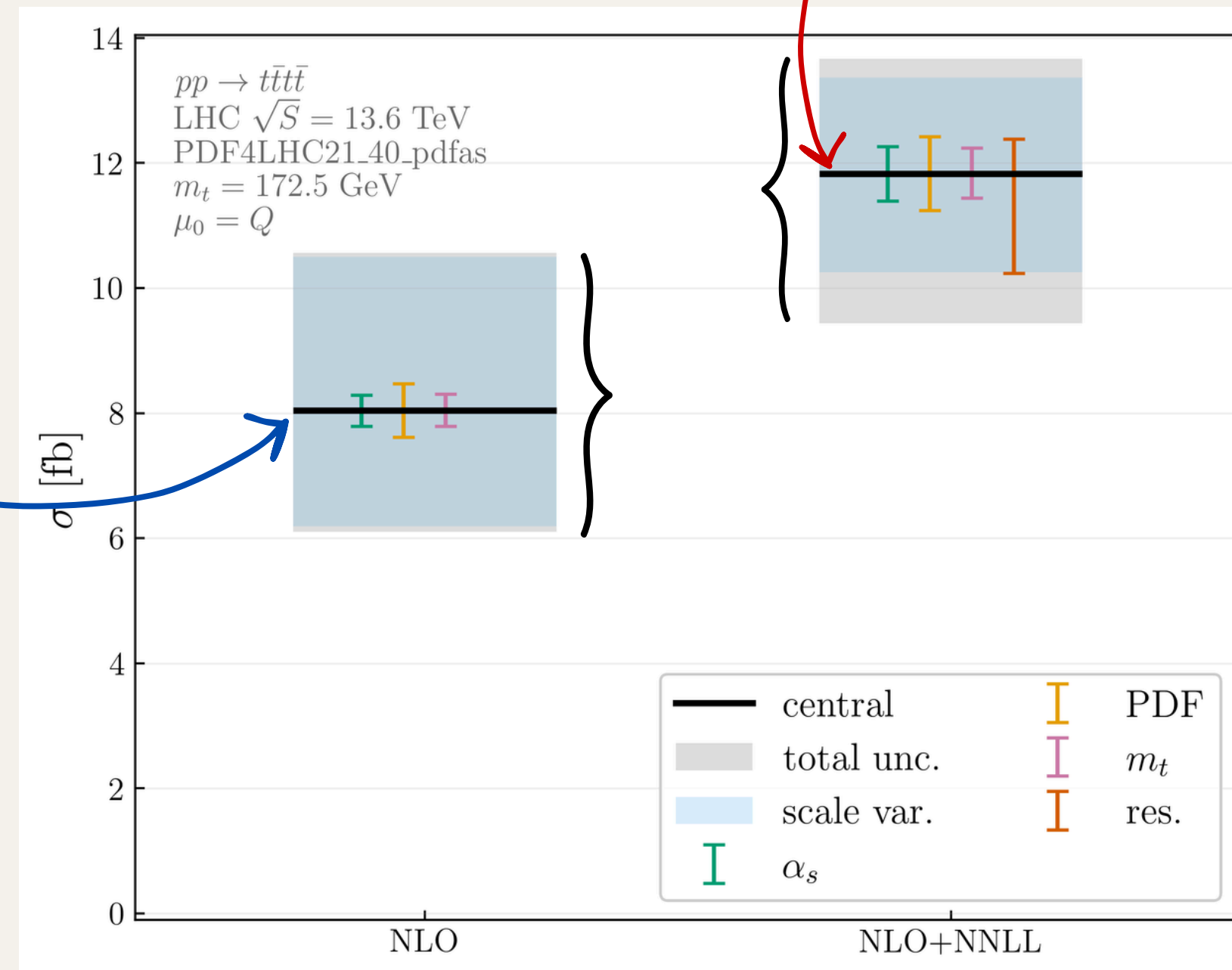


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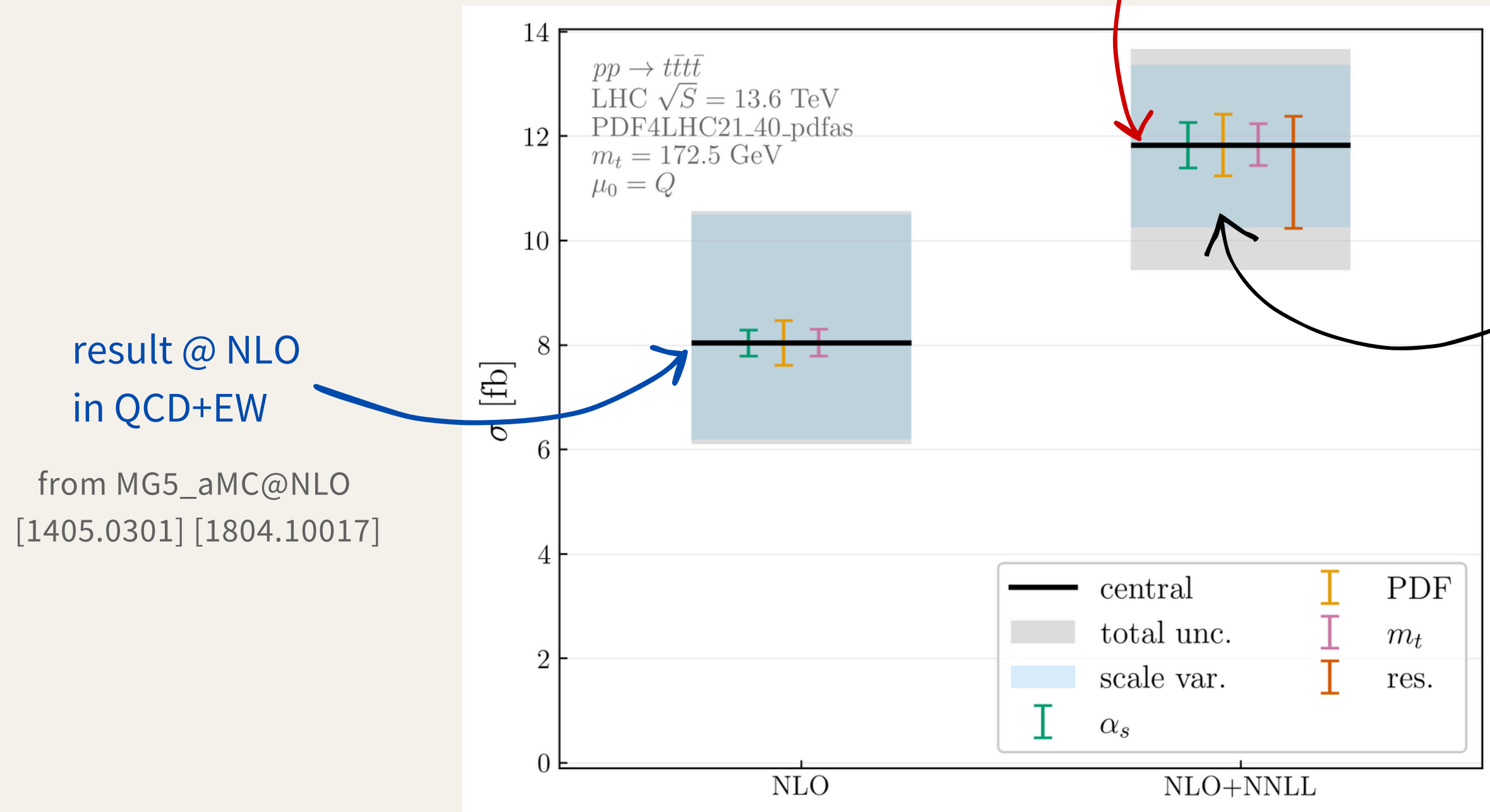


Resummed result @
NLO+NNLL accuracy

1. Reduction of overall relative
uncertainty

Results

Total cross section

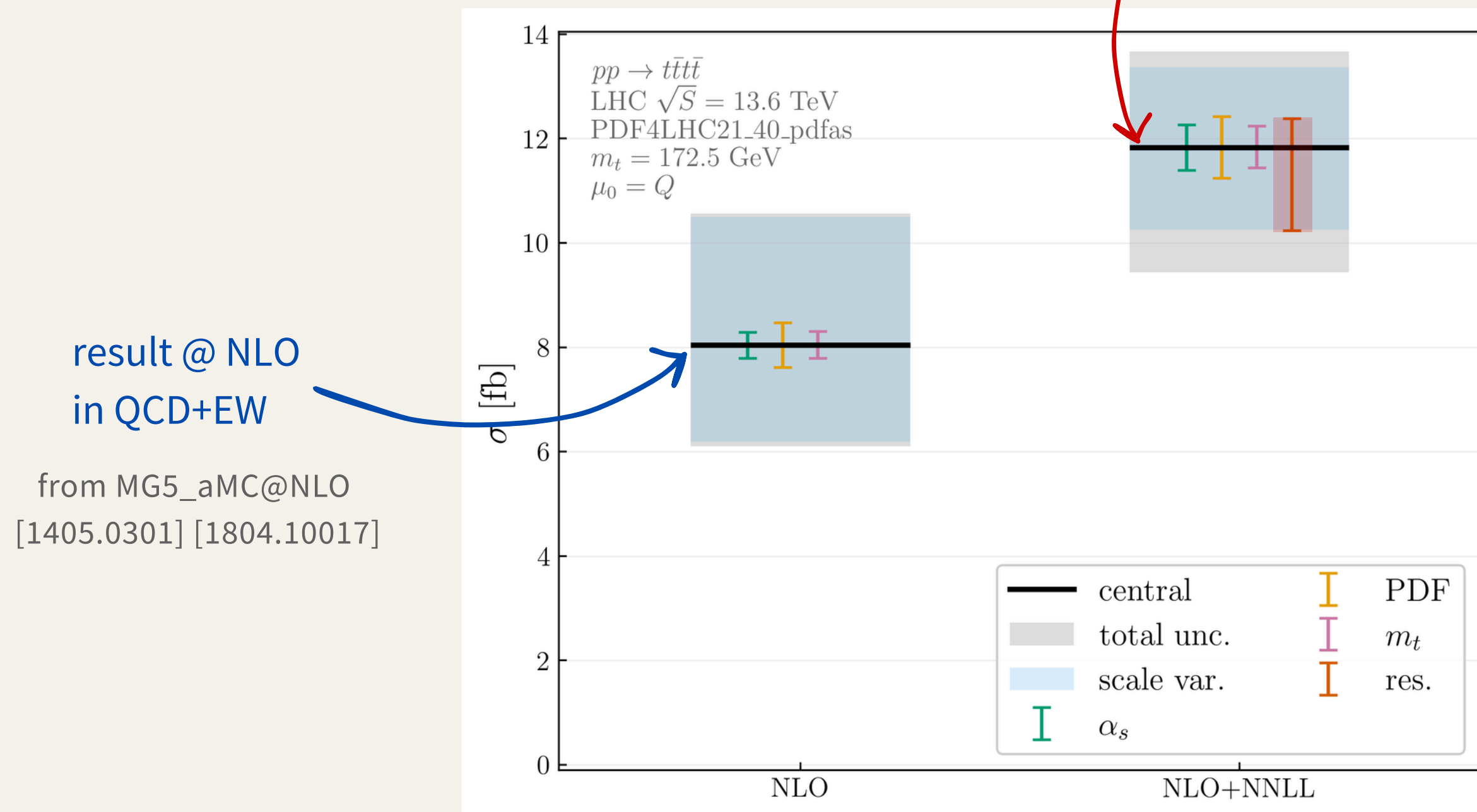


1. Reduction of overall relative uncertainty

2. Scale variation is still the dominant source of uncertainty

Results

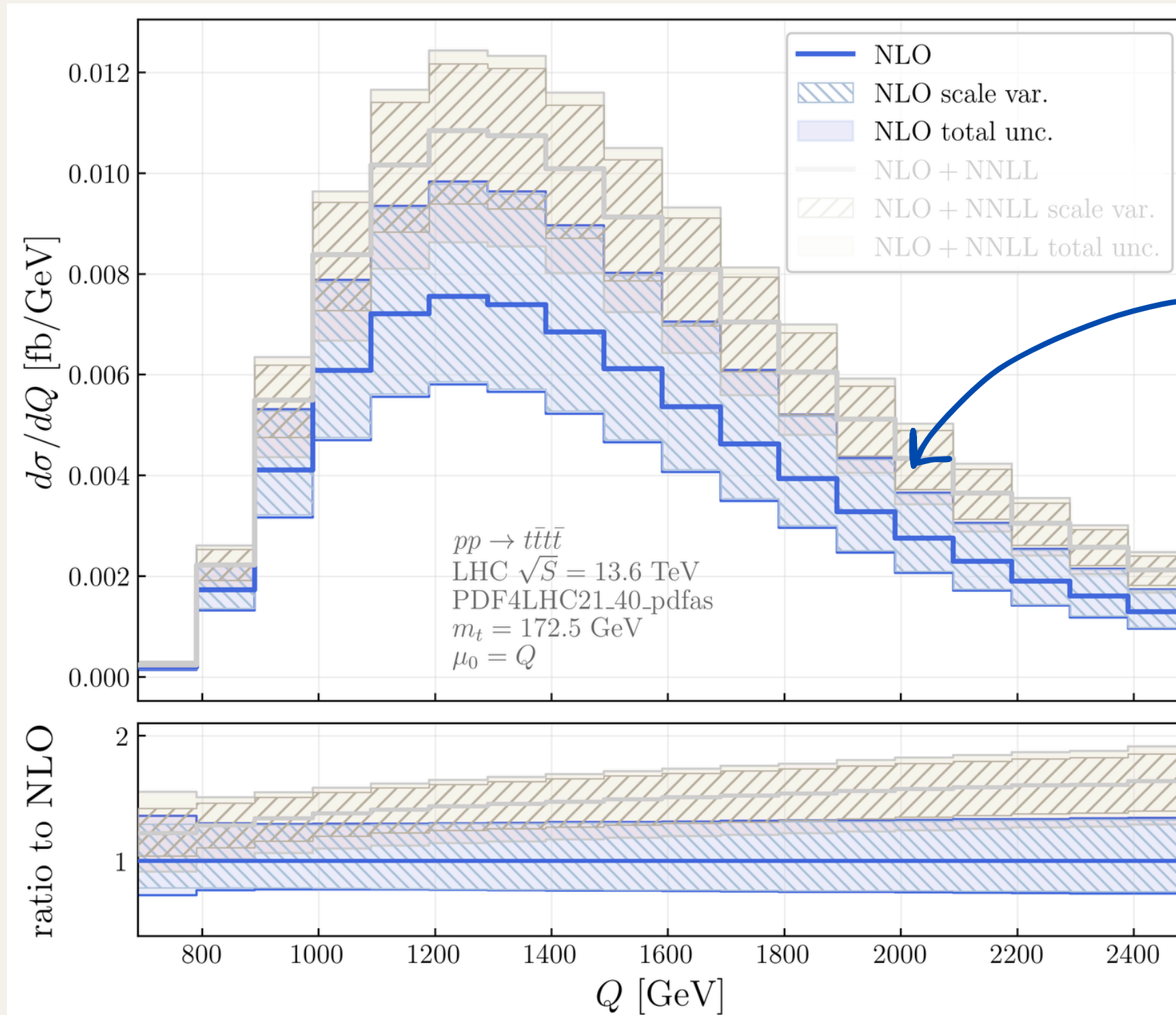
Total cross section



1. Reduction of overall relative uncertainty
2. Scale variation is still the dominant source of uncertainty
3. The biggest uncertainty from resummation is coming from the power-suppressed contributions

Results

Invariant-mass distribution

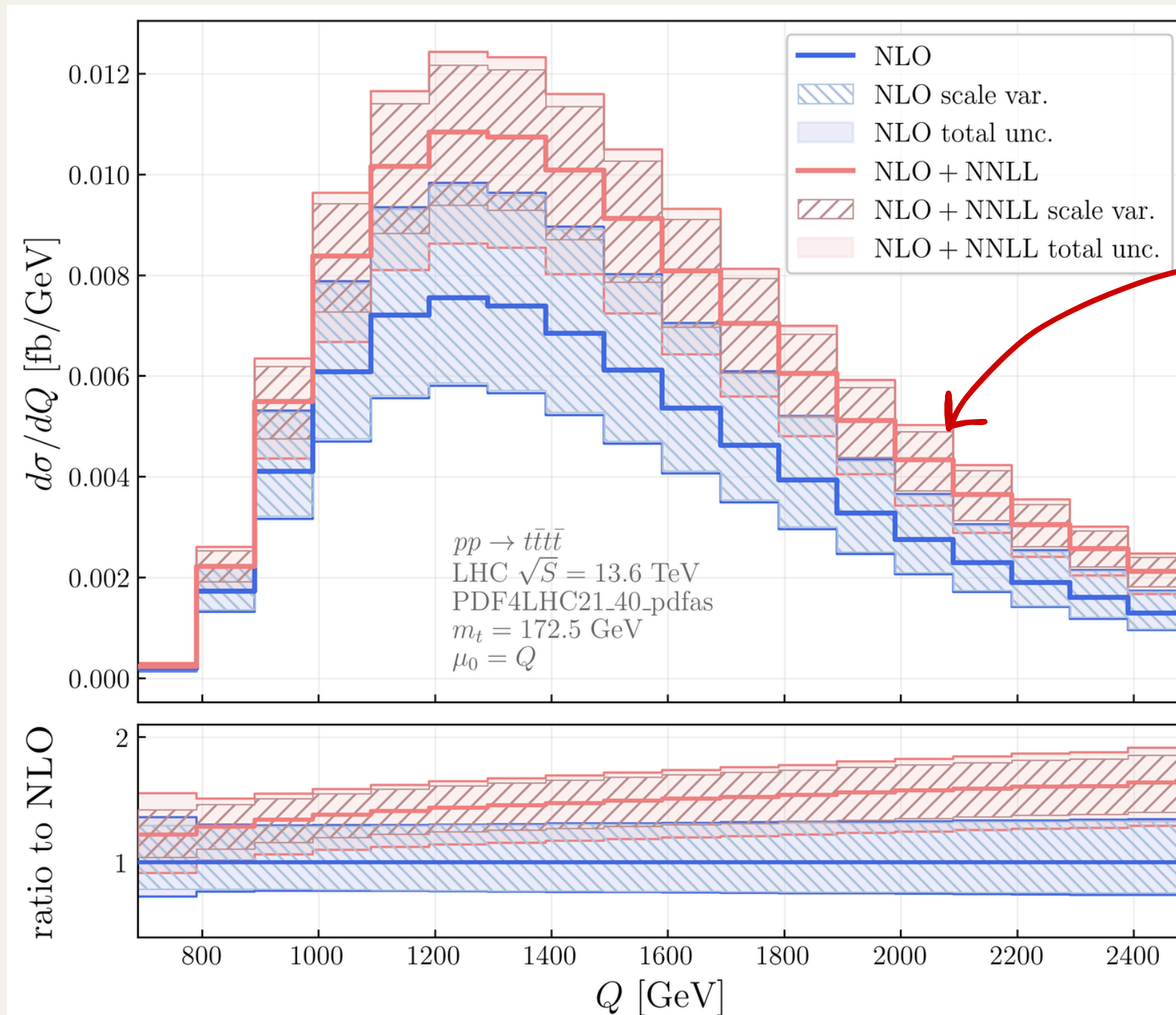


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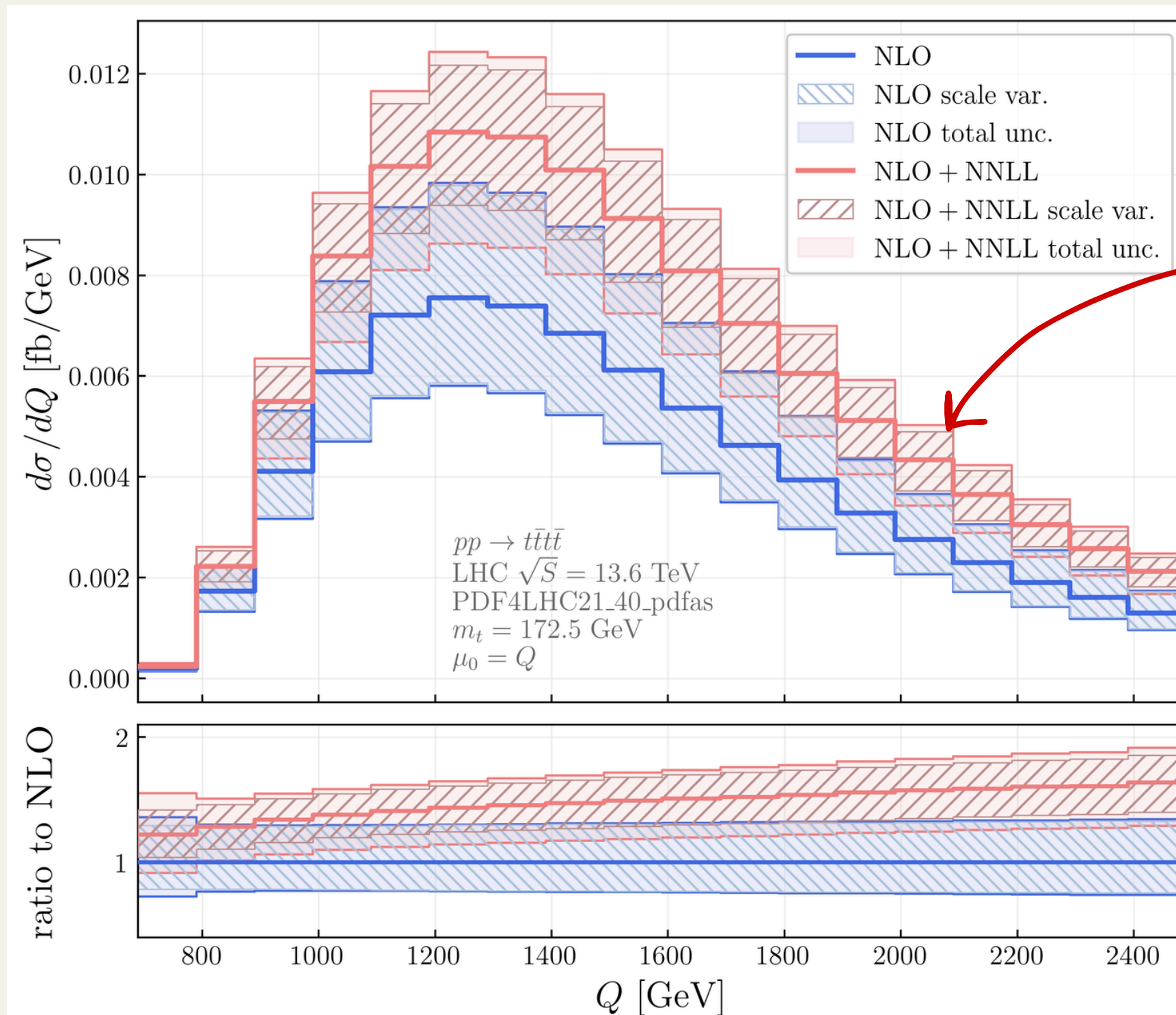
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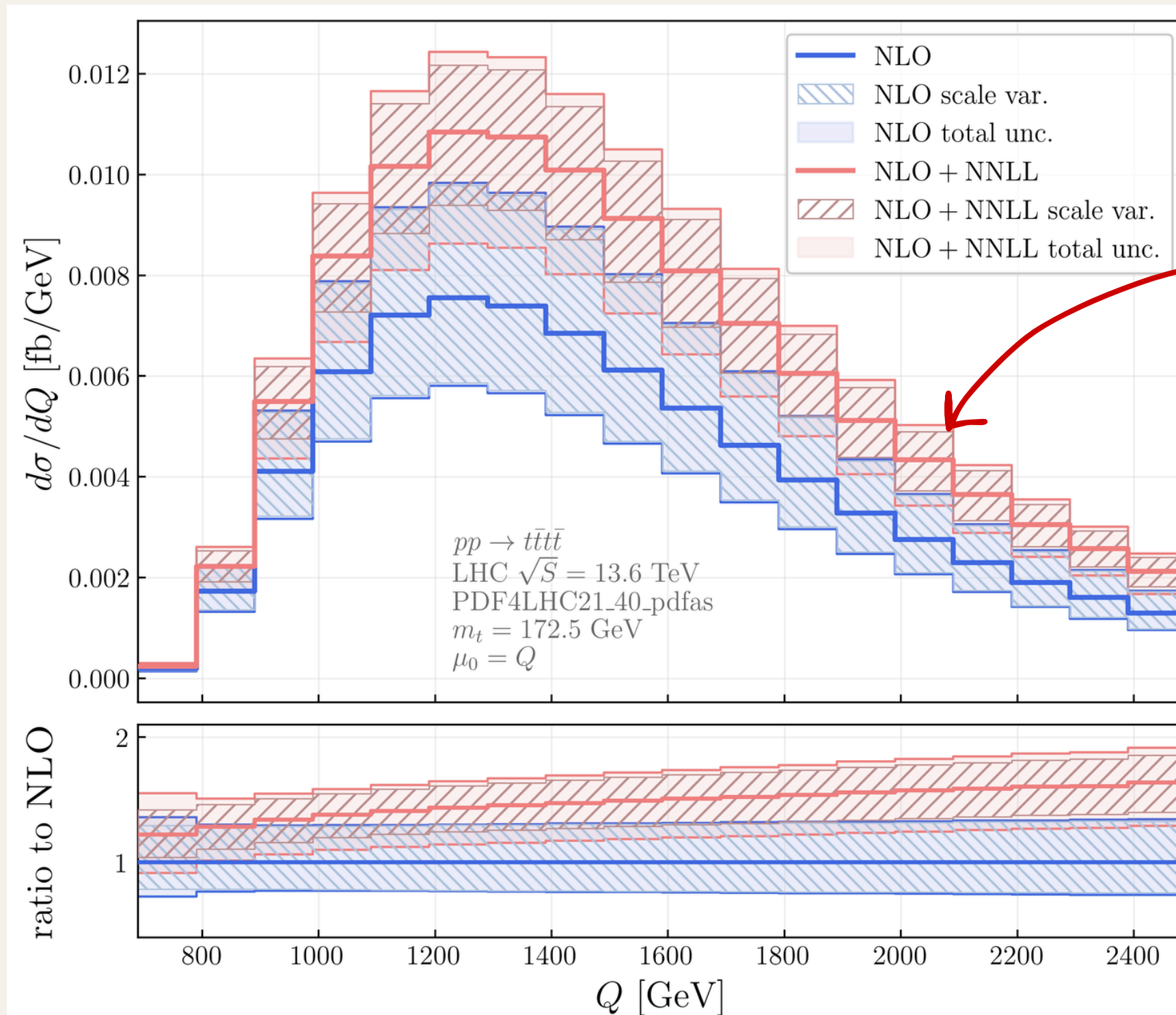


Resummed result @
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The resummed result (NLO+NNLL)
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Results

Invariant-mass distribution



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The resummed result (NLO+NNLL)
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We can also clearly see a **shape modification**

Summary

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- We carried out a detailed estimation of the uncertainties
- Soft gluon corrections modify the shape of the invariant mass distribution
- To do: study on top-mass renormalisation-scheme dependence

Back-up

Resummation

Scale choice

Logarithmic counting
in the resummation
exponents

$$\begin{aligned} \text{LL} : & \alpha_s^n L^{n+1} \\ \text{NLL} : & \alpha_s^n L^n \\ \text{NNLL} : & \alpha_s^n L^{n-1} \\ & \dots \end{aligned}$$

Scale logarithms that are present in NLL exponents are produced by using the one-loop running coupling in the LL contributions

$$\underbrace{\alpha_s(Q^2)}_{\text{LL}} \log^2 N \simeq \underbrace{\alpha_s(\mu_R^2)}_{\text{LL}} \log^2 N - \underbrace{2b_0 \alpha_s^2(\mu_R^2)}_{\text{NLL}} \log^2 N \log \left(\frac{Q^2}{\mu_R^2} \right) + \mathcal{O}(\text{NNLL})$$

The threshold limit is $N \rightarrow \infty$ but numerically it is typically not big for 4top

Numerically, on average, $\log N \sim \log \left(\frac{Q}{\mu_R} \right)$ $\mu_R \neq Q \implies$ The NLL term has typically the same numerical size of a LL term!

The effect is **even bigger at NNLL** as higher powers are involved, and it cannot be neglected

The central scale choice is chosen to be $\mu_0 = Q$ so this effect does not impact our results

Resummation

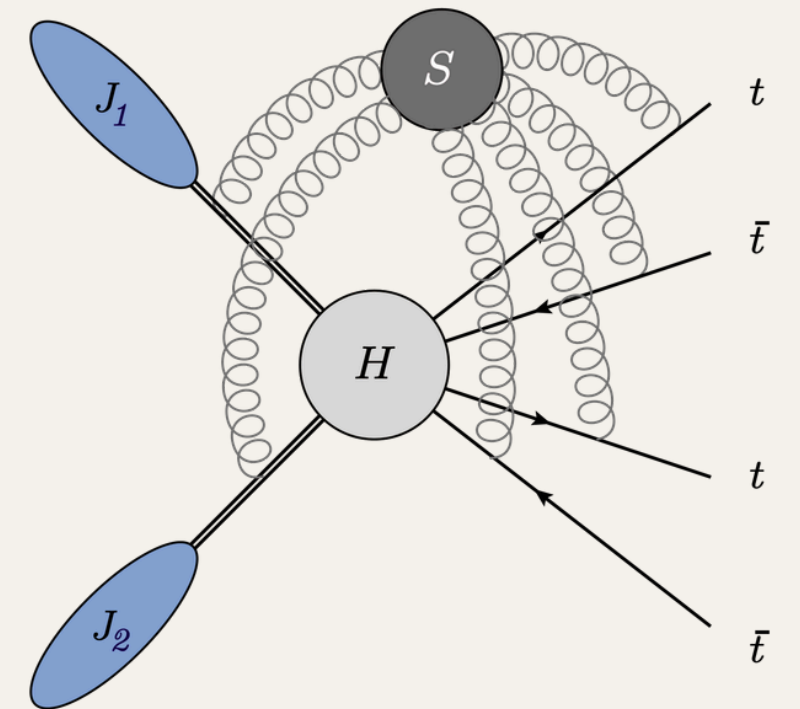
Resummed cross section

$$\frac{d\tilde{\sigma}^{\text{NNLL}}_{ij \rightarrow t\bar{t}t\bar{t}}}{dQ} = \text{Tr} [\mathbf{H} \mathbf{S}] \Delta_i \Delta_j$$

Jet functions:
collinear and soft-collinear
enhancements

Known analytically see e.g. [0306211]

$$\Delta_i = \exp \left\{ \underbrace{\frac{1}{\alpha_s} g_1(\alpha_s \log N)}_{\text{needed at LL}} + \underbrace{g_2(\alpha_s \log N)}_{\text{needed at NLL}} + \underbrace{\alpha_s g_3(\alpha_s \log N) \dots}_{\text{needed at NNLL}} \right\}$$



Resummation

Resummed cross section

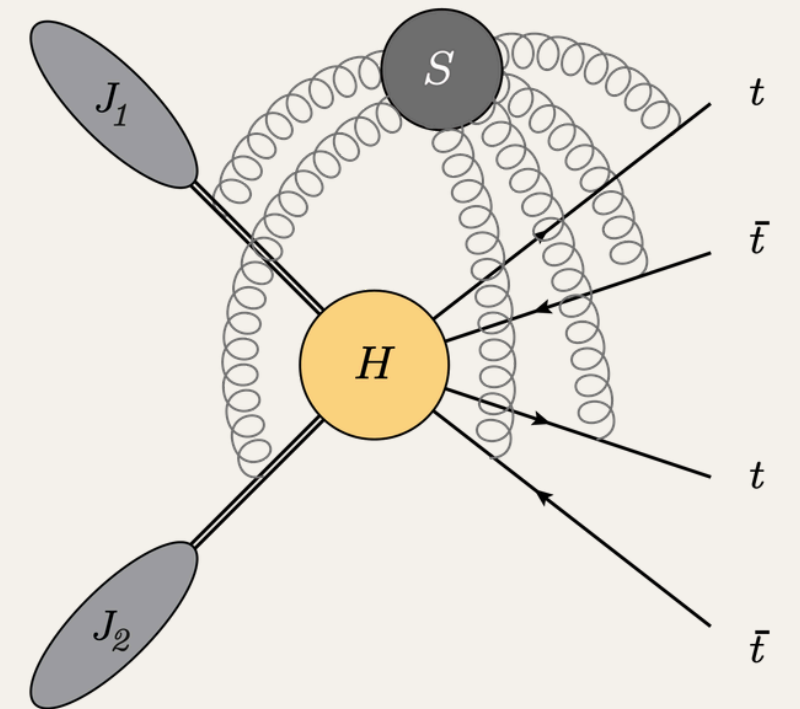
$$\frac{d\tilde{\sigma}^{\text{NNLL}}_{ij \rightarrow t\bar{t}t\bar{t}}}{dQ} = \text{Tr} [\mathbf{H} \mathbf{S}] \Delta_i \Delta_j$$

Hard function: contains the hard contributions

It is a matrix in colour space

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s}{\pi} \mathbf{H}^{(1)} + \dots$$

Extracted from OpenLoops [1907.13071]



Resummation

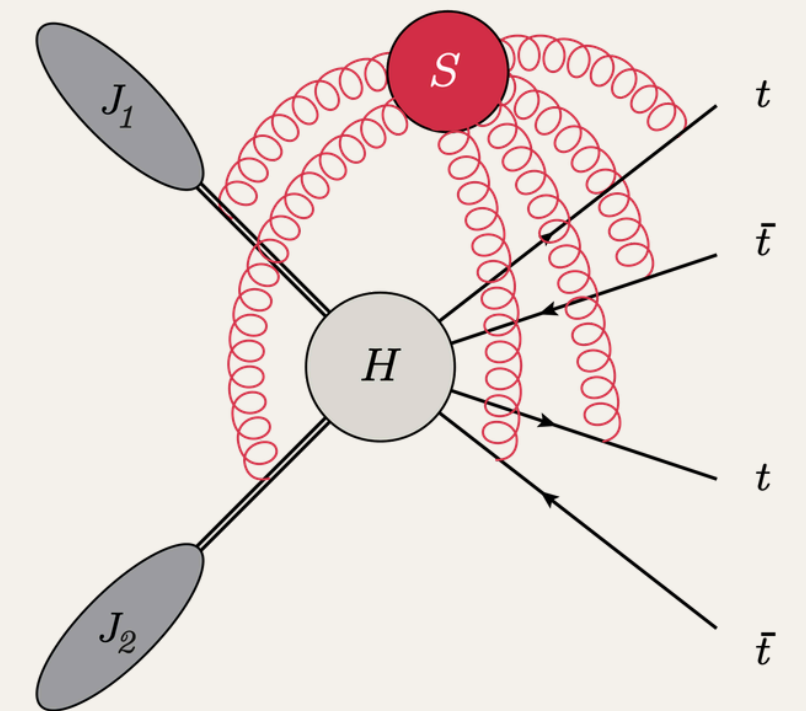
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Soft function:
wide-angle soft enhancements

The soft function is made of **soft-gluon correlators**

It is a **matrix in colour space**, modifying the colour of the hard process



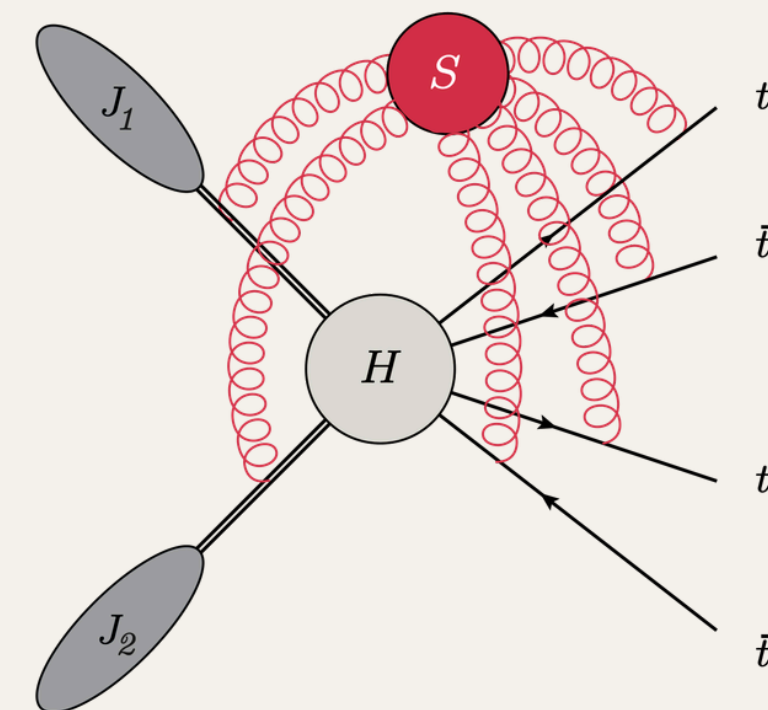
[9705234] [9604313] [9803241]

Resummation

Resummed cross section

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Soft function:
wide-angle soft enhancements



[9705234] [9604313] [9803241]

The soft function $\mathbf{S} = \bar{\mathbf{U}} \tilde{\mathbf{S}} \mathbf{U}$ is the solution of a matrix RGE

evolution matrix $\mathbf{U} = \mathcal{P} \exp \left[\frac{1}{2} \int_{\mu_R^2}^{Q^2/\bar{N}^2} \frac{d\mu^2}{\mu^2} \mathbf{\Gamma}(\mu^2, \alpha_s(\mu^2)) \right]$ where $\mathbf{\Gamma}$ is the soft anomalous dimension matrix

$$\mathbf{\Gamma} = \underbrace{\frac{\alpha_s}{\pi} \mathbf{\Gamma}^{(1)}}_{\text{needed at NLL}} + \underbrace{\left(\frac{\alpha_s}{\pi} \right)^2 \mathbf{\Gamma}^{(2)}}_{\text{needed at NNLL}} + \dots$$

boundary condition $\tilde{\mathbf{S}} = \underbrace{\tilde{\mathbf{S}}^{(0)}}_{\substack{\text{needed at LL} \\ \text{and NLL}}} + \underbrace{\frac{\alpha_s}{\pi} \tilde{\mathbf{S}}^{(1)}}_{\substack{\text{needed at NLL'} \\ \text{and NNLL}}} + \dots$

Soft anomalous dimension for 2->n

see e.g. [0904.1021]

$$\begin{aligned}\Gamma_{2 \rightarrow n}^{(1)} = & \mathbf{T}_1 \cdot \mathbf{T}_2 \gamma_{\text{cusp}}^{(0)} \log \left(\frac{\mu^2}{-s_{12}} \right) + \sum_{i=1}^2 \gamma_i^{(0)} \mathbf{1} + \sum_{I=3}^n \gamma_I^{(0)} \mathbf{1} \\ & - \frac{1}{2} \sum_{I,J=3}^n \mathbf{T}_I \cdot \mathbf{T}_J \gamma_{\text{cusp}}^{(0)}(\beta_{IJ}) + \sum_{i=1}^2 \sum_{J=3}^n \mathbf{T}_i \cdot \mathbf{T}_J \gamma_{\text{cusp}}^{(0)} \log \left(\frac{m_J \mu}{-s_{iJ}} \right)\end{aligned}$$

where

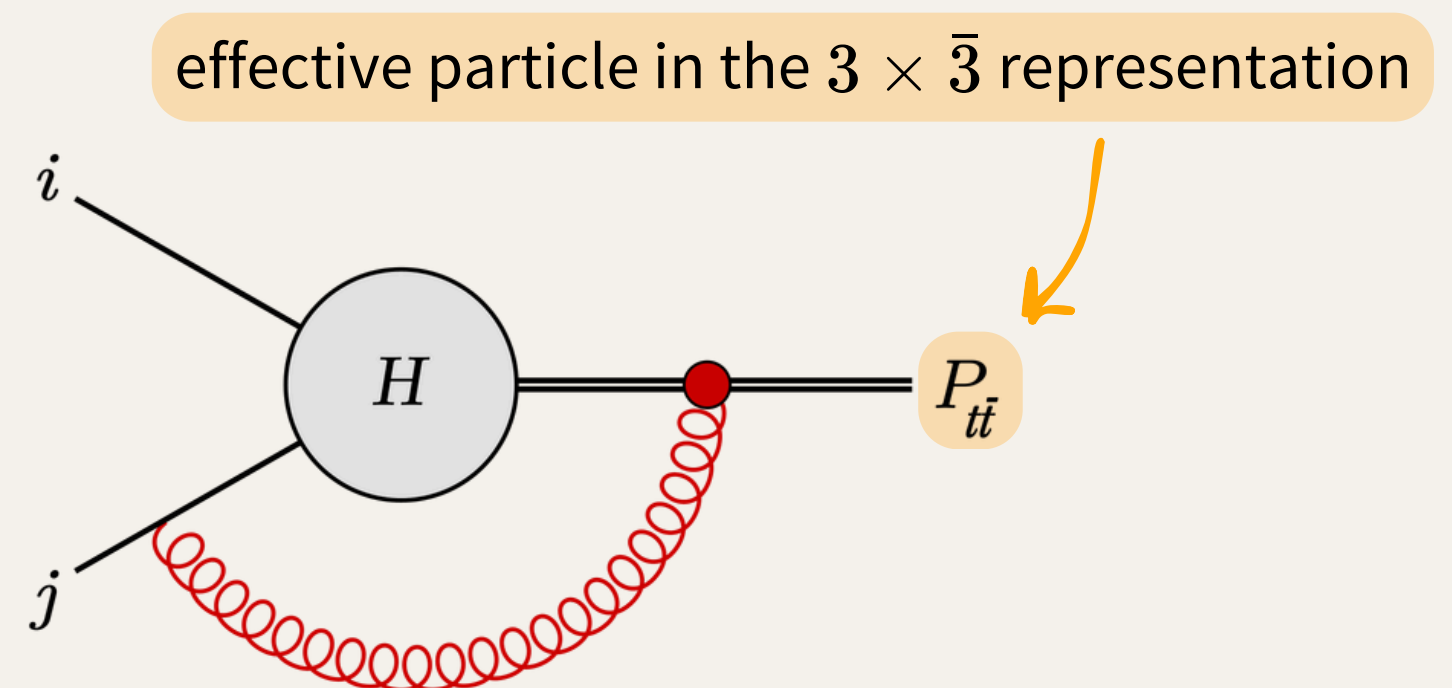
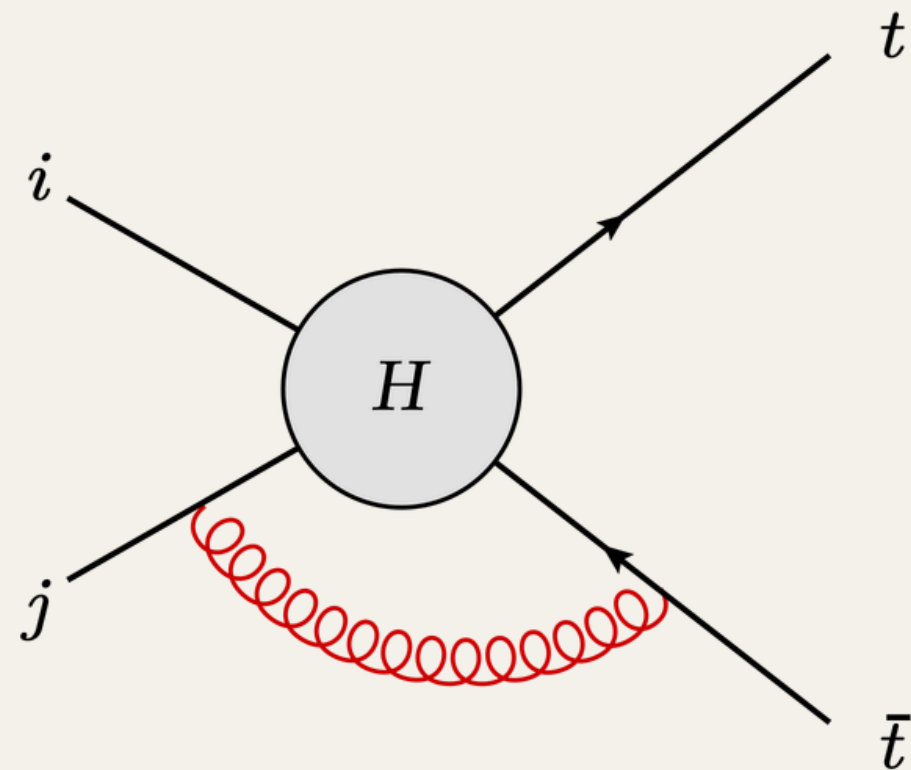
$$\gamma_{\text{cusp}}^{(0)}(\beta_{IJ}) = \gamma_{\text{cusp}}^{(0)} \beta_{IJ} \coth \beta_{IJ} ,$$

$$\cosh \beta_{IJ} \equiv -\frac{p_I \cdot p_J}{m_I m_J} - i0 , \quad \beta_{IJ} = b_{IJ} - i\pi .$$

Case of $t\bar{t}$

If the two final particles are emitted at rest $p_3 \simeq p_4 \simeq (m_t, 0, 0, 0)$ the soft function becomes diagonal.
Physically: in this limit soft gluons are sensitive only to the total colour charge of the final-state system

[0907.1443], [0907.1790]

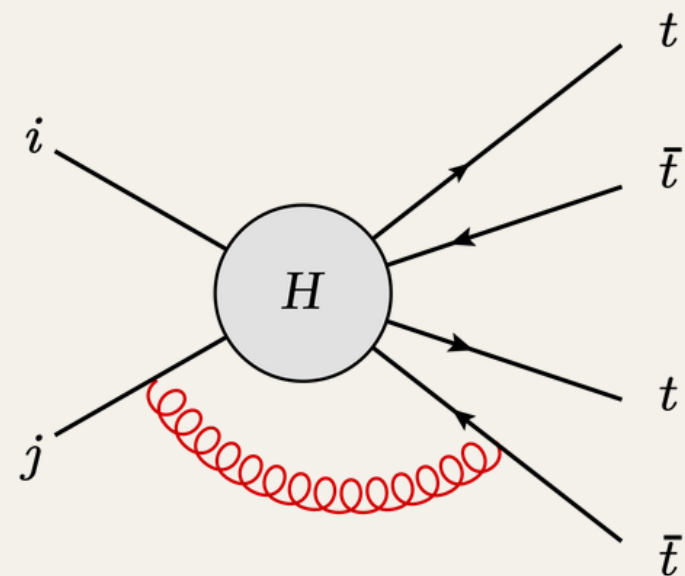


The soft function in this phase-space region
can be built directly, and it is way simpler

Novelties

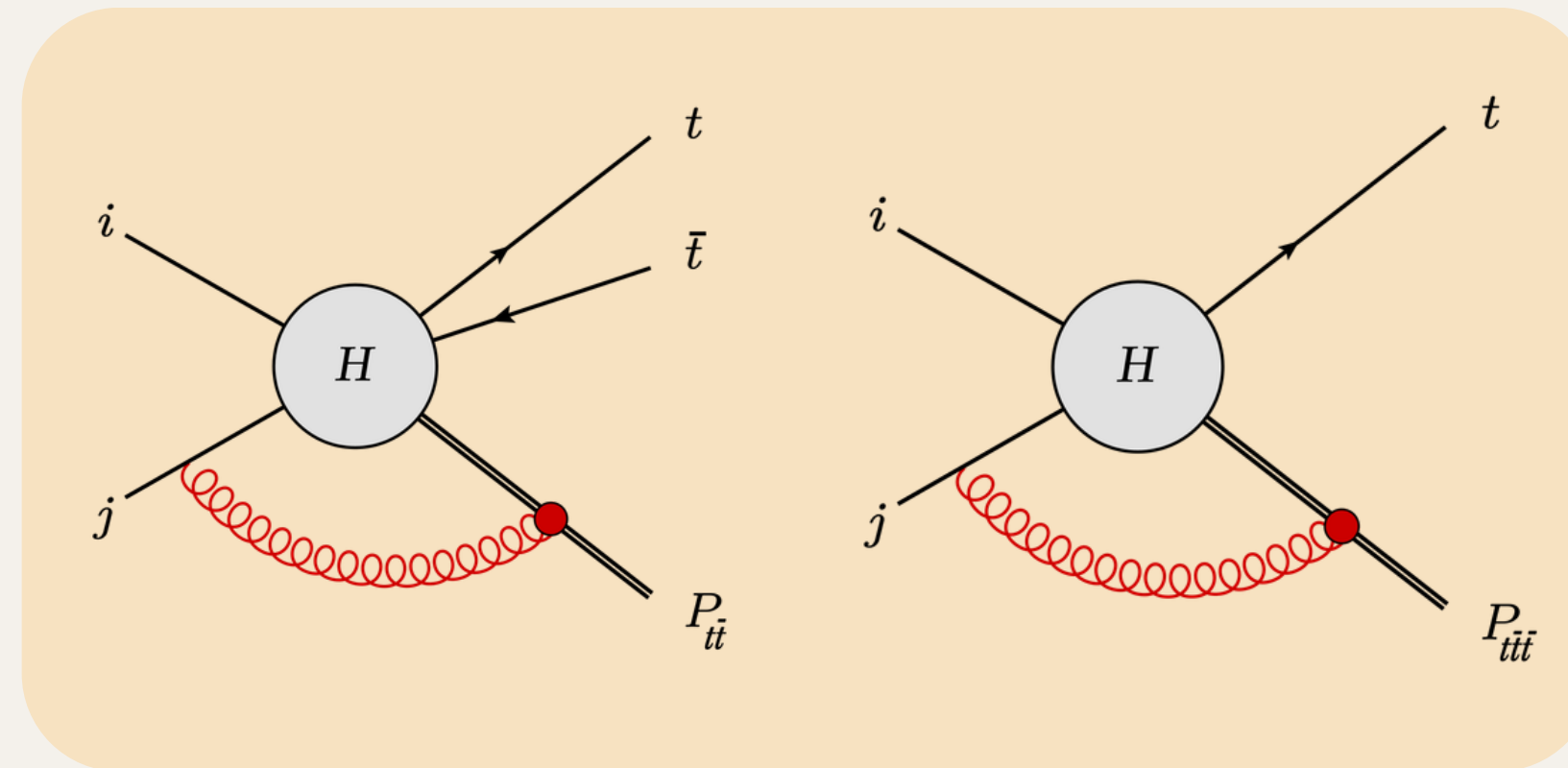
4top is the first process with more than two final-state coloured particles to which threshold resummation is applied

4top has a rich kinematical structure: we can have different subsets of particles emitted at rest



General kinematics
**invariant-mass threshold
resummation**

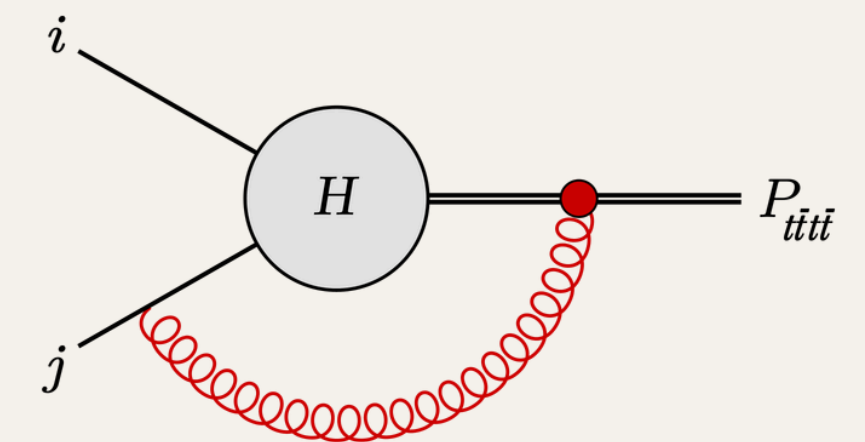
Full soft anomalous dimension



New possibilities



These configurations contain divergences!



Limit of **absolute-mass
threshold resummation**

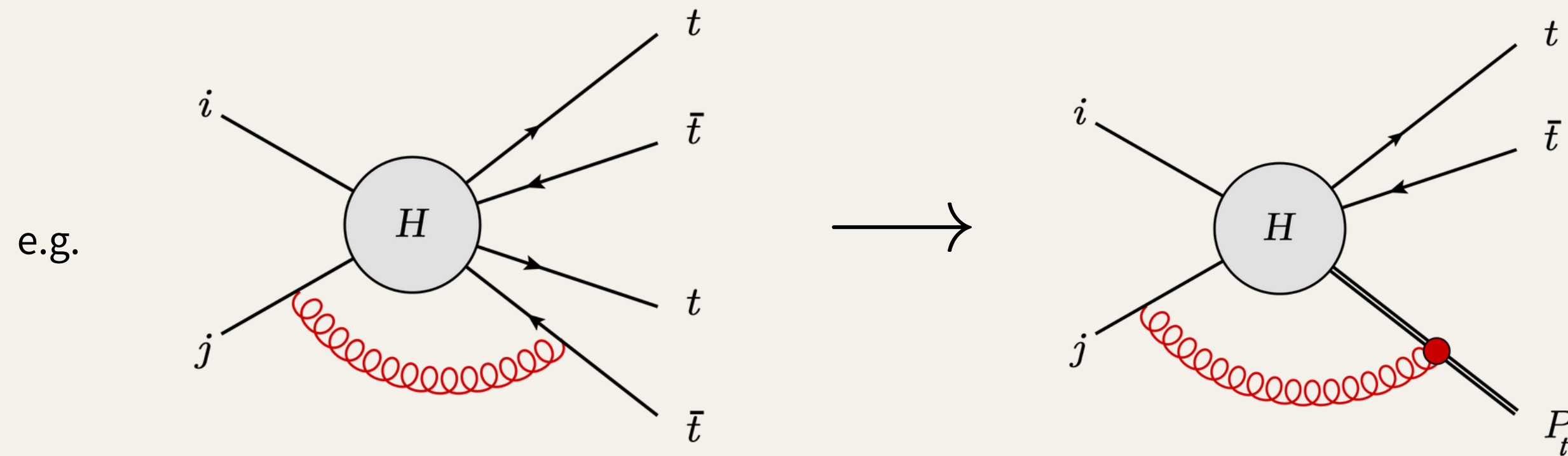
[2212.03259]

*Diagonal soft anomalous dimension
in the colour-flow basis*

[9801375]

Novelties

Coulomb singularities



→ there are **Coulomb singularities** in the soft anomalous dimension associated with the two top quarks being produced at rest
 [0904.1021] [0907.1790] [0907.1443] [2211.08838]

If *all* the final particles emitted at rest,
Coulomb poles cancel out
 between amplitude and complex conjugate at NLL

Not a problem
 for 4top if *all* the final particles
 are emitted at rest (absolute-
 mass threshold limit)

[2212.03259]

Not a problem
 for ttbar resummation

Novelties

Coulomb singularities

The limit $d=4$ (to extract IR divergences) and the limit in which the final particles are produced at rest **do not commute**

If **the limit in which the final particles are produced at rest is taken *before* extracting IR divergences,**
Coulomb poles are not present anymore in the soft function
and we account for them at fixed order, as part of **H**.

[0907.1790]

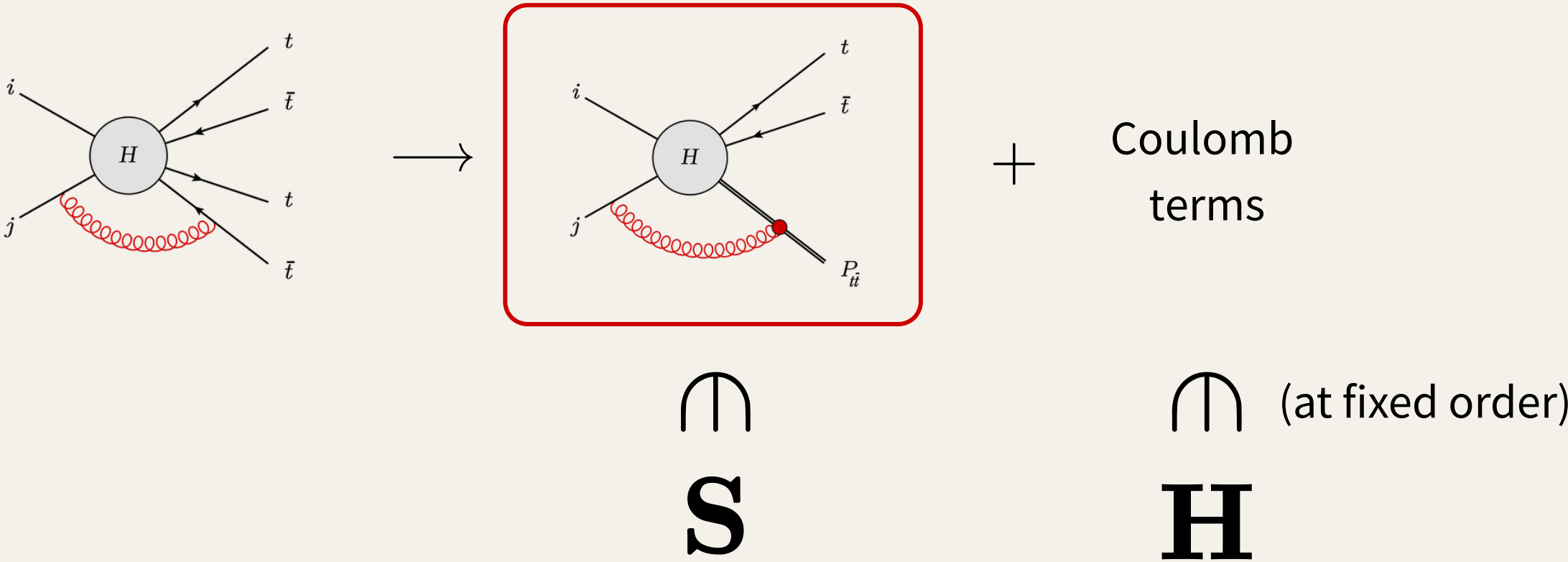
[0907.1443]

[0904.1021]

[1007.5414]

[2211.08838]

compute this directly



Resummation

Mellin space

The convolution with the PDFs is treated by going to **Mellin space**.

[9604351]

$$\tilde{\sigma}(N) = \int_0^1 d\rho \, \rho^{N-1} \sigma(\rho)$$

**Different choices of
the Mellin variable**
are possible

**Absolute-mass
threshold resummation**
[2212.03259]

$$\rho_{m_t} \equiv \frac{(4m_t)^2}{s}$$

**Invariant-mass
threshold resummation**
[this work]

$$\rho_Q \equiv \frac{Q^2}{s}$$

Resummation

Which threshold resummation?

**Absolute-mass
threshold resummation**
[2212.03259]

Mellin variable

$$\rho \equiv \frac{(4m_t)^2}{s}$$

What we get

corrections only to the
total cross section

What we resum

$$\log^k (1 - \rho)$$

**Invariant-mass
threshold resummation**
[this work]

$$\rho \equiv \frac{Q^2}{s}$$

corrections to the **invariant-mass distribution**
and to the **total cross section**
for all the values of Q^2

$$\left[\frac{\log^k (1 - \rho)}{1 - \rho} \right]_+$$